Measuring Eccentricity w/o Measuring Eccentricity

W/ Zhong-Zhi Xianyu

Eccentricity Papers

- 1.**Eccentricity Without Measuring Eccentricity: Discriminating Among Stellar Mass Black Hole Binary Formation Channels**
  Lisa Randall, Zhong-Zhi Xianyu. Jul 4, 2019. 7 pp.
  e-Print: **arXiv:1907.02283 [astro-ph.HE] |**
- 2.**Observing Eccentricity Oscillations of Binary Black Holes in LISA**
  Lisa Randall, Zhong-Zhi Xianyu (Harvard U.). Feb 22, 2019. 6 pp.
  e-Print: **arXiv:1902.08604 [astro-ph.HE] |**
- 3.**A Direct Probe of Mass Density Near Inspiraling Binary Black Holes**
  Lisa Randall, Zhong-Zhi Xianyu (Harvard U.). May 14, 2018. 7 pp.
  Published in **Astrophys.J. 878 (2019) no.2, 75**
  DOI: 10.3847/1538-4357/ab20c6
  e-Print: **arXiv:1805.05335 [gr-qc] |**
- 4.**An Analytical Portrait of Binary Mergers in Hierarchical Triple Systems**
  Lisa Randall, Zhong-Zhi Xianyu (Harvard U.). Feb 15, 2018. 19 pp.
  Published in **Astrophys.J. 864 (2018) no.2, 134**
  DOI: 10.3847/1538-4357/aad7fe
  e-Print: **arXiv:1802.05718 [gr-qc] |**
- 5.**Induced Ellipticity for Inspiraling Binary Systems**
  Lisa Randall (Harvard U., Phys. Dept.), Zhong-Zhi Xianyu (Harvard U. & Harvard U., Dept. Math.). Aug 28, 2017. 13 pp.
  Published in **Astrophys.J. 853 (2018) no.1, 93**
  DOI: 10.3847/1538-4357/aaa1a2
  e-Print: **arXiv:1708.08569 [gr-qc] |**

Just a Mention: real time KL

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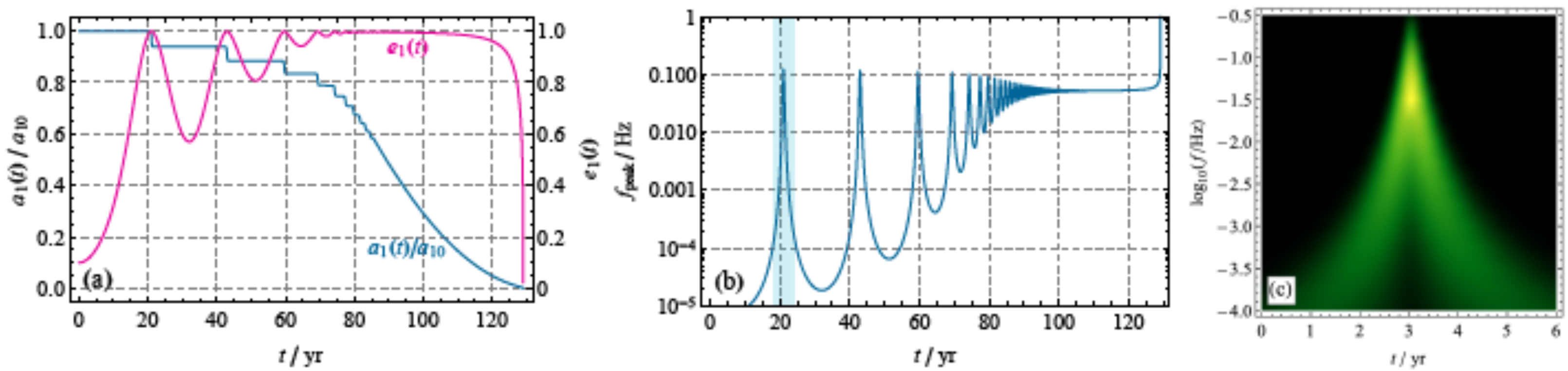

FIG. 1: An example of KL BBH in a galactic center. (a) The semi-major axis $a_1(t)$ and the eccentricity $e_1(t)$ as functions of the time. (b) The peak frequency f_p of the GWs emitted by this BBH as a function of time. (c) An illustration of GW spectrum in frequency-time domain, in a 6-year period marked by the shaded strip in (b). In this example, we take $m_0 = m_1 = 30M_\odot$, $m_2 = 4 \times 10^6 M_\odot$, $a_{10} = 0.2$AU, a_2=150AU, $e_{10} = e_2 = 0.1$ and $I_0 = 89.9°$. The maximal eccentricity $e_{1\max}$ reached in this example is $\epsilon_{1\min} = 1 - e_{1\max}^2 = 0.0022$ and the merger time is $\tau = 129$yr.

A Direct Probe of Mass Density Near Inspiraling Binary Black Holes

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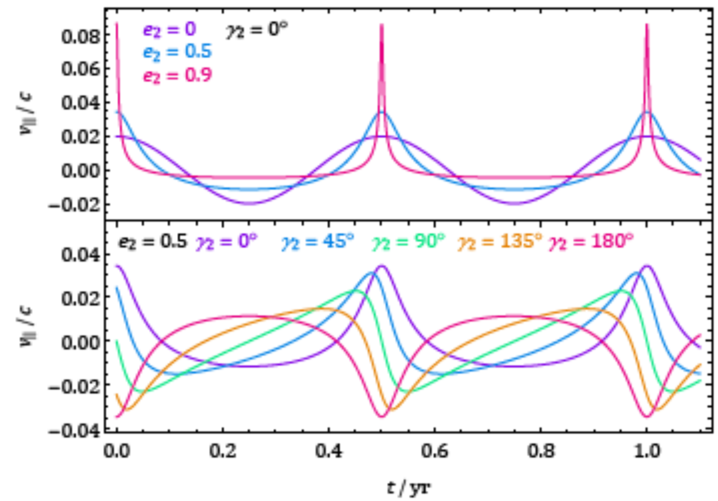


Figure 1. The longitudinal velocity $v_{2\parallel}(t)$ of the BBH barycenter. In both panels $m_0 = m_1 = 10M_\odot$, $m_2 = 4 \times 10^6 M_\odot$ (the mass of Sgr A*), $a_2 = 100$AU, $I_2 = 90°$. Note that the inclination $I_2 = 90°$ optimizes the effect so better velocity sensitivity will be important in general cases.

https://arxiv.org/pdf/1802.05718.pdf Analytical computation of eccentricity

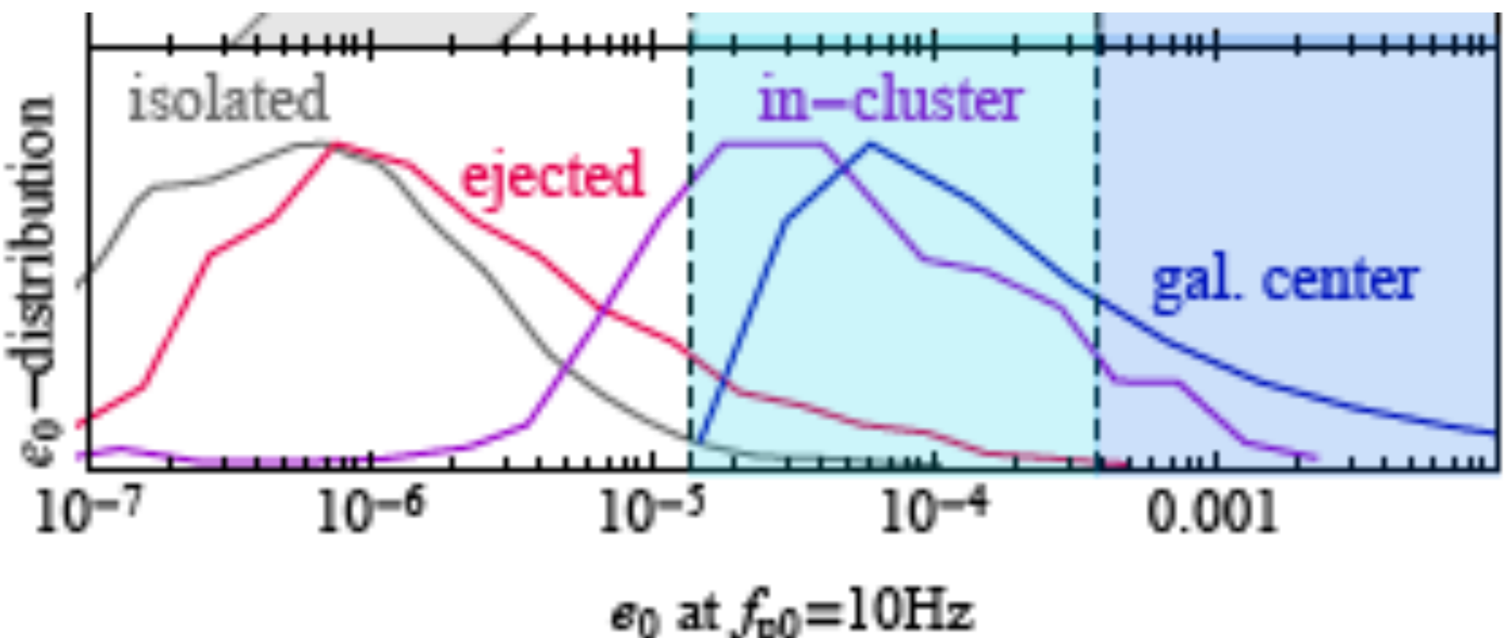
from the cusp model (68) with m? = M, α = 7/4, ρ0 = 106M/pc3 and a20 = 0.1pc. We perform the integration (73) for several sets of initial distributions and show the resultant probability P(e) in Fig. 10. For the cusp model, we take the same background profile as the one we take for Fig. 9, while the core profile corresponds to replacing α = 7/4 by α = 1/2. It is clear from the figure that the cusp profile tends to produce a more elliptic distribution than a core model when other parameters are fixed. More interestingly, lighter binaries tend to gain more eccentricity in NC than heavier binaries, which means there is an anti-correlation between the binary mass and eccentricity in this formation channel. This is different from the binaries in GCs where the mass has little impact on eccentricity distribution [30], and is in contrast to what is claimed in [22] who considered an alternative eccentric BBH formation channel with direct two-body encounter and found that the binary mass and the eccentricity is positively correlated. Though we have yet to analyze such situations, such parameter-dependence might ultimately be used to distinguish different formation scenarios. In general, it is clear that most binaries in NCs will have small eccentricities. A careful

LISA/LIGO and eccentricity

- Number in LISA affected by
  - Peak gravitational frequency
  - Enhanced number density required to produce LIGO observed rate
  - Reduced S/N
- Observe number of binaries as a function of frequency
- Determine formation channel based on predicted expected eccentricity
- BUT DON'T NEED TO HAVE ECCENTRIC TEMPLATES
- Though more information with modest eccentricities detected

LIGO eccentricity predictions

- Eccentricity predicted for LIGO very correlated with black hole binary origin
- Dynamical channels clearly predicted to have larger eccentricity in LIGO window
- Break at about 10-5
- Most values too small to be observed at LIGO
- But project back to higher values in LISA



Peak Frequency

- Depends on

$$f_p \simeq \frac{\sqrt{Gm}(1+e)^\gamma}{\pi[a(1-e^2)]^{3/2}}, \qquad \gamma = 1.1954.$$

Periapsis

Means highly eccentric binaries radiate at higher frequencies

Important to use peak frequency (not orbital frequency) as relevant LISA variable

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{64}{5}\frac{G^3\mu m^2}{c^5 a^3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}},$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{304}{15}\frac{G^3\mu m^2}{c^5 a^4}\frac{e(1+\frac{121}{304}e^2)}{(1-e^2)^{5/2}},$$

where G is Newton's constant and c is the speed of li
Eliminating t from Peters' equation, we get a relation
tween $a(t)$ and $e(t)$ for a binary with initial value $(a_0,$

$$\frac{a}{a_0} = \frac{\mathcal{G}(e)}{\mathcal{G}(e_0)}, \quad \mathcal{G}(e) \equiv \frac{e^{12/19}}{1-e^2}\left(1+\frac{121}{304}e^2\right)^{870/2299}.$$

Eliminate a:

$$\frac{f_p}{f_{p*}} = \frac{\mathcal{H}(e)}{\mathcal{H}(e_*)}, \qquad \mathcal{H}(e) \equiv \frac{(1+e)^\gamma}{[(1-e^2)\mathcal{G}(e)]^{3/2}}$$

Notice eccentricity in LIGO determines min frequency where it radiates

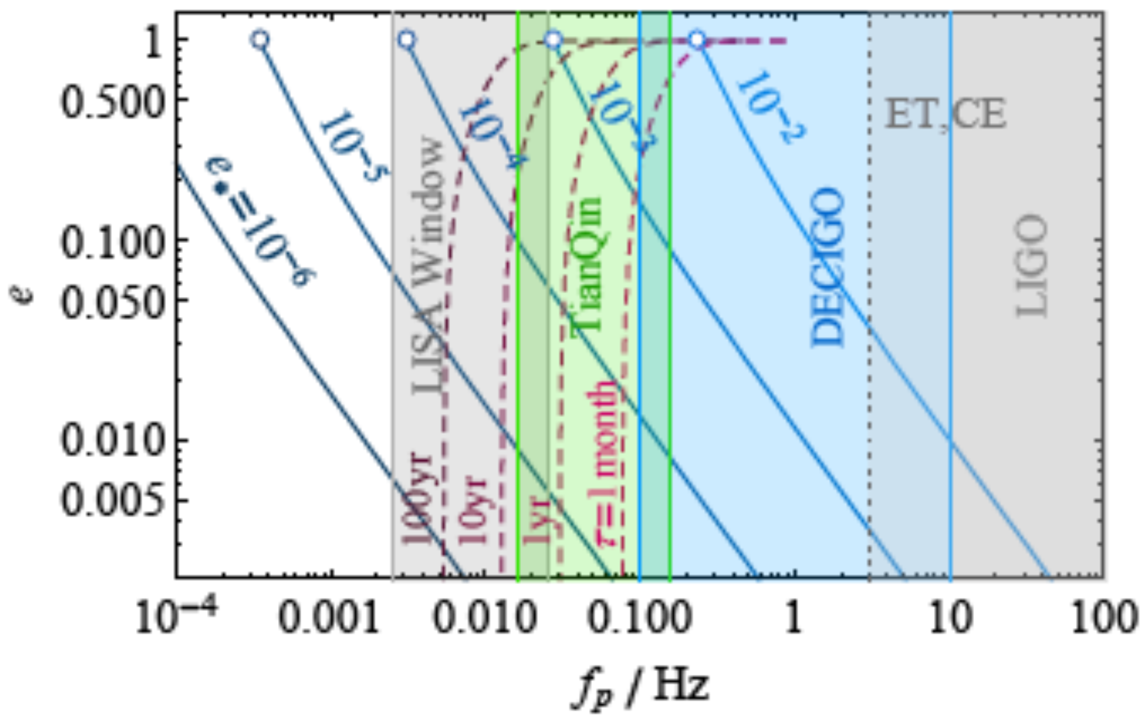


FIG. 1: The binary eccentricity e as a function of the peak GW frequency f_p. The five blue solid curves correspond to five reference values $e_* = 10^{-n}$ $(n = 2, 3, 4, 5, 6)$ at $f_{p*} = 10\text{Hz}$, respectively. The four dashed magenta curves show the time τ to coalescence of binaries with $m_1 = m_2 = 30M_\odot$. The shaded strips show the frequency ranges covered by several GW telescopes.

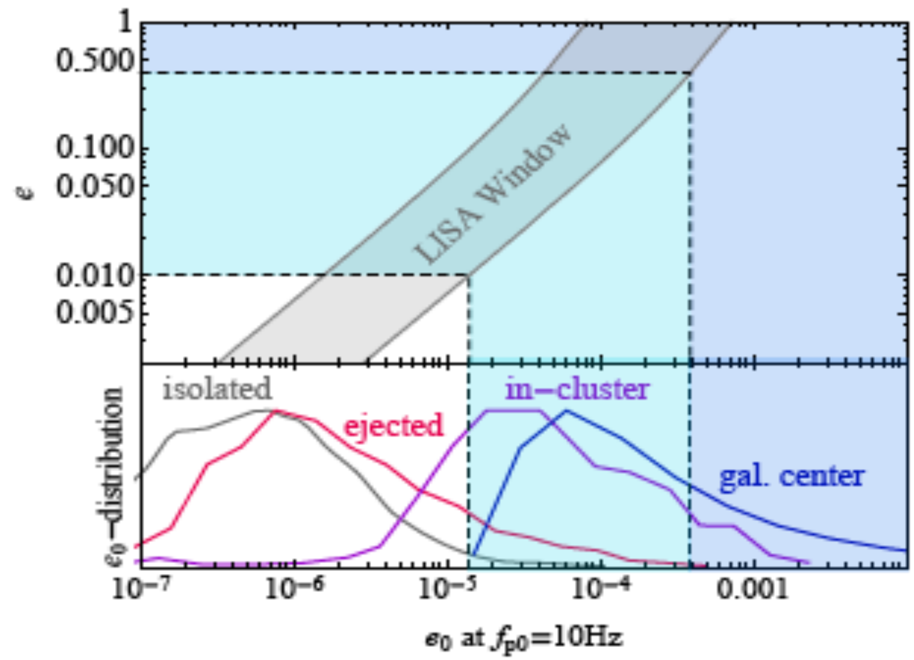


FIG. 2: UPPER: The eccentricity e in the LISA window (grey strip, same as in Fig. 1) versus the eccentricity e_* at f_{p*} =10Hz. BBHs to the lower-left of the black dashed lines could be seen in LISA if LISA is able to measure e up to 0.01 or 0.4, respectively. LOWER: Eccentricity distributions from several channels at 10Hz. The four curves corresponds to the isolated channel [3], the ejected binaries from globular clusters and the in-cluster mergers [7], and binaries from galactic centers [8]. All curves are normalized at their peak values and the overall heights do not represent relative fractions of channels.

To get numbers need

- Rate
- S/N
- Both as function of eccentricity

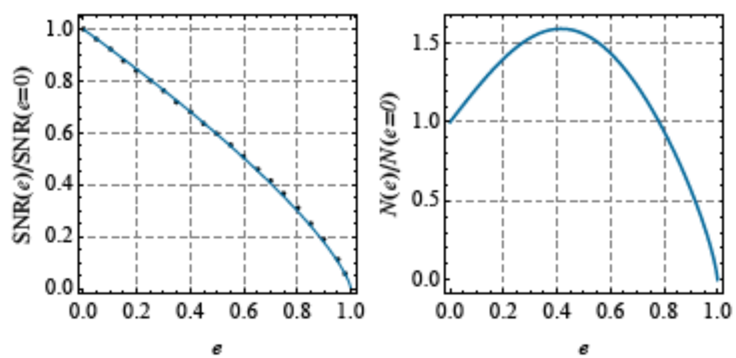


FIG. 3: LEFT: The SNR of a non-chirping eccentric binary as a function of eccentricity e, with peak frequency f_p and all other parameters fixed. The black dots are calculated from summation over harmonics and the blue curve shows the simplified formula (11). RIGHT: The relative enhancement/suppression of expected number of BBHs in LISA due to finite eccentricity.

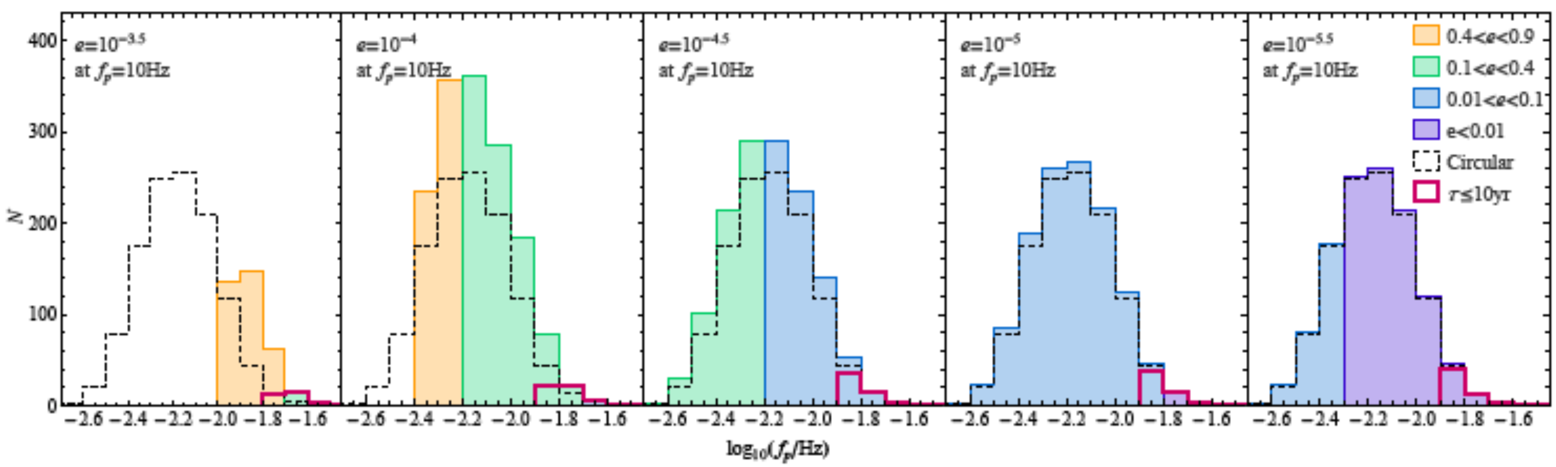


FIG. 4: The number of resolvable ($\varrho > 8$) BBHs in LISA with N2A5 configuration [11] and 10yr observation. In all panels, we use dashed black lines to show a circular distribution with $e_* = 0$, which serves as a basis to which we compare number distribution with finite e_*. In each panel, we choose a different e_* at 10Hz ranging from $10^{-3.5}$Hz to $10^{-5.5}$Hz. The purple, blue, green, and orange shadings correspond to $e_{\rm cut} = 0.01, 0.1, 0.4, 0.9$, respectively. The binaries enclosed by magenta lines merge in 10 years so are possible for joint detection with ground GW telescopes.

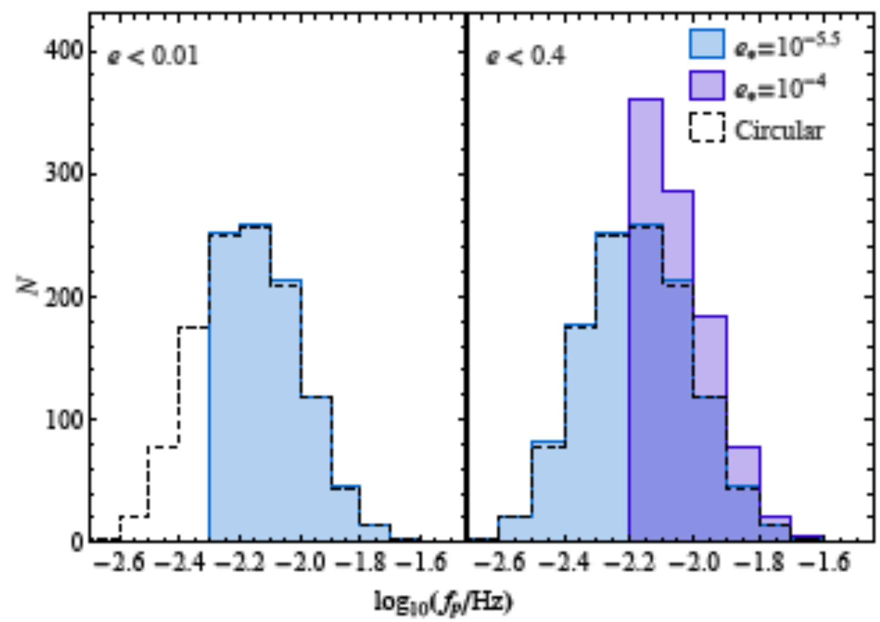


FIG. 5: Rearrangement of histograms in Fig. 4 to highlight the effects of eccentricity e_* on the number of resolvable binaries. The left (right) panel shows the resolvable number with eccentricity in LISA smaller than 0.01 (0.4). The blue and purple shadings correspond to $e_* = 10^{-5.5}$ and 10^{-4}, respectively.

Conclusion

- Number of events
  - As function of frequency
- Can give big insights into eccentricity distribution
  - Hence formation channel
- Some ranges of eccentricities will be seen with templates
  - Some lost entirely (sufficiently large e*)
- Amazing that LISA has just the right frequency range to distinguish dynamical and isolated processes