The basics of the Advective-Acoustic Cycle

Janka et al. (2004)

Chanaud & Powell (1965)

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Outline

1. Instability of the stalled accretion shock during core collapse

2. The advective-acoustic cycle: a new instability?

3. Understanding simple toy models
   - why is there an advective-acoustic coupling?
   - why a low frequency, low l instability?
   - why transverse rather than radial?

4. Conclusion: back to the core-collapse problem
Instabilities during the phase of stalled accretion shock

- Convection in the gain region, low l
  (Herant, Benz & Colgate 1992)

- l=1,2 SASI in an adiabatic flow:
  vortical-acoustic cycle (Blondin, Mezzacappa & DeMarino 2003)
  or purely acoustic mechanism (Blondin & Mezzacappa 2006) ?

- Neutron star kick resulting from the l=1 instability of convection and/or vortical-acoustic cycle

- New explosion mechanism driven by acoustic waves, initiated by the advective-acoustic cycle (Burrows et al. 2005)

  convection ?
  SASI 2003 ?
  SASI 2006 ?
  advective-acoustic cycle ?
Contribution of the convective instability to a mode l=1?
Asymmetry without convection: adiabatic simulation of a stalled accretion shock on a neutron star

Evidence for a vortical-acoustic cycle (Blondin, Mezzacappa & DeMarino 2003)

A purely acoustic cycle? (Blondin & Mezzacappa 2005)

Linear stability analysis

l=0: Houck & Chevalier (1992)
l=0,1: Galletti (PhD Thesis 2005)
Blondin & Mezzacappa (2005)
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Aero-acoustic instabilities

- advected perturbations
- acoustic feedback

• vortical-acoustic cycle

• entropic-acoustic cycle

vibrations in Ariane 5:
segmented solid propergol
J. of Sound and Vibration 230, 761

impinging shear layers
Rockwell, D. 1983, AIAA J., 21, 645

whistling kettle
Chanaud & Powell (1965)
J. Acoust.Soc. Am. 37, 902

combustion

rumble instability of ramjet combustors
Abouseif, Keklak & Toong (1984)
Combustion Science and Technology, 36, 83
Analytical study of the advective-acoustic coupling in a radial, accelerated flow


Bondi-Hoyle-Lyttleton accretion; unstable
Beyond the eigenspectrum

- identification of the 2 cycles \((Q, \tau_Q)\) and \((R, \tau_R)\)

 Isothermal flow (Foglizzo 2002)

Adveetive-acoustic cycle

- Efficiency
- Duration

\[
\frac{Q}{\tau_Q} \equiv \frac{Q_{sh}}{\tau_{sh}} \frac{Q_{\nabla}}{\tau_{\nabla}}
\]

Purely acoustic cycle

- Efficiency
- Duration

\[
\frac{R}{\tau_R} \equiv \frac{R_{sh}}{\tau_{sh}} \frac{R_{\nabla}}{\tau_{\nabla}}
\]

\[Q e^{i\omega \tau_Q} + R e^{i\omega \tau_R} = 1\]
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Advective-acoustic coupling: hand waving

\[ \Delta E = \delta m(h_2 - h_1) \]

advection of entropy

advection of vorticity

(Foglizzo & Tagger 2000)
Adveotive-acoustic coupling: illustrative simulations

2 isothermal simulations  (thanks to F. Masset)

-> accelerated flow \((M_{in}=0.1, M_{out}=0.7)\)

\[
\omega < k_x c (1 - M_{out}^2)^{\frac{1}{2}} < k_x c (1 - M_{in}^2)^{\frac{1}{2}}
\]

in: evanescent
out: evanescent

-> decelerated flow \((M_{in}=0.7, M_{out}=0.1)\)

\[
k_x c (1 - M_{in}^2)^{\frac{1}{2}} < \omega < k_x c (1 - M_{out}^2)^{\frac{1}{2}}
\]

in: propagate
out: evanescent

propagating wave: \(\omega > k_x c (1 - M^2)^{\frac{1}{2}}\)

evanescent wave: \(\omega < k_x c (1 - M^2)^{\frac{1}{2}}\)

\[
\omega^2 = (k_x^2 + k_z^2) c^2
\]

\[
k_z^2 = \frac{\omega^2}{c^2} - k_x^2
\]
Compact approximation

\[ \Delta z_{\nabla} \to 0 \]

scale free

\[ \tau_{\nabla} \equiv \int_{z_{\nabla} - \frac{\Delta z_{\nabla}}{2}}^{z_{\nabla} + \frac{\Delta z_{\nabla}}{2}} \frac{d z}{|v|} \]

validity of the compact approximation

\[ \tau_{\nabla} \ll \frac{2\pi}{\omega} \]

\[ k_x \Delta z_{\nabla} \ll 1 \]

\[ Q_{\nabla} = \frac{M_{\text{out}} + \mu_{\text{out}}}{1 + \mu_{\text{out}} M_{\text{out}}} \frac{1}{\mu_{\text{out}} c_{\text{out}}^2 + \mu_{\text{in}} M_{\text{in}} M_{\text{out}}} \left[ 1 - \frac{c_{\text{in}}^2}{c_{\text{out}}^2} + \frac{k_x^2 c_{\text{in}}^2}{\omega^2} (M_{\text{in}}^2 - M_{\text{out}}^2) \right] \]

longitudinal

transverse

\[ \mu^2 = 1 - \frac{k_x^2 c_{\text{in}}^2}{\omega^2} \]

cut-off frequency

\[ \omega_{\nabla} \equiv \frac{1}{\tau_{\nabla}} \]

-> analytical upper bound of the coupling efficiency
The simplest example of an advective-acoustic instability

-parallel adiabatic flow, localized coupling ($\gamma, M_1, \Delta \Phi, \Delta z_\gamma$)

-2-D perturbations

$$Qe^{i\omega \tau Q} + Re^{i\omega \tau R} = 1$$

\[Q_e \equiv \frac{4\mu_{in} (1 - M_{in}^2) \left(1 - \frac{1}{\gamma} \right) M_{out} + \mu_{out}}{\mu_{out} \mu_{in} M_{out} M_{in}} \left[ 1 - \frac{\gamma^2}{\mu_{out}^2} + \frac{\mu_{in}^2}{\mu_{out}^2} (M_{in}^2 - M_{out}^2) \right] \left( \gamma + 1 \right) \left( 1 - \mu_{in} M_{in} \right) \left( \mu_{in}^2 + 2\mu_{in} M_{in} + M_{in}^2 \right)$$

\[\mu^2 = 1 - \frac{k_z^2}{\beta_1^2 (1 - M^2)}\]
Effect of the size $\Delta z$ of the coupling region

compact approximation confirmed

a low frequency instability

$\frac{k_x c (1 - M^2)^{1/2}}{\omega} \leq \omega \ll \omega_{\nabla}$

a low $n_x$ instability

why not $n_x=0$ ?

why so much irregularity ?

a possible benchmark test

$M_1=5$, $\gamma=4/3$, $T_{in}/T_{out}=0.75$
Comparison of $n_x=0$ and $n_x>0$ modes

- no vorticity in 1-D: the instability of the mode $n_x=0$ relies only on temperature gradients
- transverse modes benefits from the vortical-acoustic coupling

\[ Q_c = \frac{\mathcal{M}_{in}}{\mathcal{M}_{in}} \left( 1 - \mathcal{M}^2_{in} \right) \left( 1 - \frac{1}{\mathcal{M}^2_{in}} \right) \mathcal{M}_{out} + \mu_{out} \left[ \frac{1 - \frac{c^2_{out}}{c^2_{in}}}{(\gamma + 1)(1 - \mu_{in} \mathcal{M}_{in})(\mu_{in}^2 + 2 \mu_{in} \mathcal{M}_{in} + \mathcal{M}_{1}^{-2})} \right] \]

At the shock, the coupling efficiency is also maximum for transverse perturbations:

\[ \mu_{sh}^2 \propto \pm \frac{1}{\mathcal{M}^2_{1}} \]
Contribution of the acoustic cycle

(Foglizzo 2002)

\[ Q e^{i \omega \tau_Q} + R e^{i \omega \tau_R} = 1 \]

global dispersion relation

The acoustic cycle alone is stable: \[ |R| \leq 1 \]

Its contribution can be either constructive or destructive

\[ |Q| + |R| < 1 \] stable
\[ |Q| - |R| > 1 \] unstable

\[ \alpha \equiv \frac{|R|}{|Q|} \frac{\tau_Q}{\tau_R} \]

\[ \frac{1}{\tau_Q} \log \{(1 - \alpha)|Q|\} \leq \omega_i \leq \frac{1}{\tau_Q} \log \left\{ \frac{|Q|}{1 - \alpha} \right\} \]

-> the stability threshold is very sensitive to geometrical factors
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Conclusion

Simple toy model of the advective-acoustic instability in a decelerated flow

- gradient cut-off -> low frequency \[ k_x c (1 - M^2)^{1/2} \leq \omega \ll \omega_{\text{v}} \]
- acoustic evanescence -> low \( n_x \)
- vorticity -> transverse rather than longitudinal
- acoustic cycle -> sensitive to geometrical parameters

Benchmark test for numerical simulations

- advection of vorticity, numerical viscosity ?

Relevance to the core-collapse problem ?

- gravity, geometry, photodissociation, heating, cooling

- detailed comparison with numerical simulations: ongoing effort (Blondin, Scheck et al. 2006)