Stable MHD–Equilibria in Young Neutron Stars
Magnetars or Radio–Pulsars

M. Rheinhardt & U. Geppert

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Rôle of ***B*** "around" a NS's birth

Tightly Connected Phases

- SN–Progenitor
- Core–Collapse
- Proto–Neutron Star (X)
- Young Neutron Star (X)

$\boldsymbol{B}(t)$

- all phases "short"
- $\boldsymbol{B}(t)$: strongly dependent
  on initial conditions
- state at the end of each phase
  =
  initial conditions of the next

Proto–Neutron Star Dynamo

Necessity & Conditions

To understand the observed $\boldsymbol{B}$ of neutron stars:

- $\boldsymbol{B}$ to be amplified during transition from SN–Progenitor to Neutron Star
- vigorous convection + high conductivity
- **DYNAMO !**
- $\boldsymbol{v}_{\rm convection}(\boldsymbol{r}, t)$ available (Janka, Fryer)

Proto–Neutron Star Dynamo

A semi-realistic PNS-dynamo

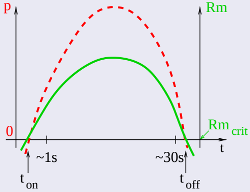
- Flow pattern from Keil 97, $Rm = VL/\eta$ very large
- $\dot{\boldsymbol{B}} = \Delta \boldsymbol{B} + Rm\,\mathrm{curl}(\boldsymbol{v} \times \boldsymbol{B})$
- solution numerically resolvable for $Rm \lesssim 10^4$
  but: real $Rm \sim 10^{19}$
- judging the **flow geometry** for dynamo capability
- determination of critical Rm (for which $\dot{\boldsymbol{B}} = \boldsymbol{0}$)
- $\sim$ of growth rates for $Rm \gtrsim Rm_{\mathrm{crit}}$

Viability & Impediments

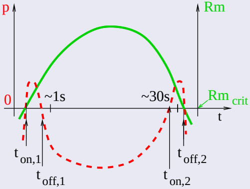
- $Rm_{\mathrm{crit}} \approx 5000 \ldots 10000$
- flow not dynamo–friendly
- $Rm_{\mathrm{crit}} <<< Rm$
- no monotonous relation growth rate p of $\boldsymbol{B}$ vs. Rm
- typically: p first grows with Rm,
  $\rightarrow$ maximum $\rightarrow p \leq 0$
  $\curvearrowright$ dynamo ability lost for high Rm
- reason: flux expulsion from convective cells

PNS-dynamo: Basic scenarios

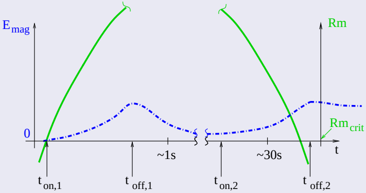
- fast dynamo: $p \longrightarrow > 0$ if $Rm \to \infty$



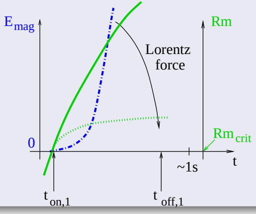
- slow dynamo: $p \longrightarrow \leq 0$ if $Rm \to \infty$



- slow $\boldsymbol{B}$ growth: $p_{\rm conv} > p_{\boldsymbol{B}}$



- rapid $\boldsymbol{B}$ growth: $p_{\boldsymbol{B}} > p_{\rm conv}$



Stable MHD–Equilibria in Young Neutron Stars

Birth of Neutron Stars in Supernovae

- NSs born with $B \sim 10^{15}$G
- few of them seen as magnetars (AXP, SGR)
- $E_{\mathrm{mag}} \sim E_{\mathrm{rot}}, E_{\mathrm{therm}}$, released in GRBs
- How does the establishment of $\boldsymbol{B}_{\mathrm{Magnetar}}$ proceed?

Stability of $\boldsymbol{B}(\boldsymbol{r}, t)$ in a Conducting Fluid

Early Work

- Prendergast '56: $\boldsymbol{B}^{tor} \sim \boldsymbol{B}^{pol}$
- Tayler et al. '73: $\boldsymbol{B}^{tor}$ always unstable with respect to $m = 1$ perturbations, $p \sim 1/\tau_\mathrm{A}$
- Pitts & Tayler '85: $\boldsymbol{B}^{tor} + \text{uniform } \boldsymbol{B}^{pol}$: rotation may stabilize AND destabilize $\Longrightarrow$ "rotation is unlikely to stabilize general $\boldsymbol{B}$ "

Recent Work

- Braithwaite & Nordlund, $\sim$ & Spruit '05: $\boldsymbol{B}$ in Ap–stars generally unstable
- Braithwaite & Spruit '05: $\boldsymbol{B}^{tor} \sim \boldsymbol{B}^{pol}$ torus–like configuration may be stable if concentrated in a very small inner core region of the NS

The effect of rotation on the stability of poloidal/toroidal field configurations has not yet been considered.

Stable MHD–Equilibria in Young Neutron Stars

What Does "Young" Mean?

- Immediately after PNS–phase
- Whole NS liquid, $T \gtrsim 10^{10}$
- $\tau_{\mathrm{cryst}} \sim \tau_{\mathrm{SF}} \sim 1000$ sec,

... let's take Thompson & Duncan '93 seriously:

- Rapid rotation: $P = 0.6 \ldots 60$ ms
- Highly magnetized: $B \gtrsim 10^{15}$G

Model Assumptions

- Rigid rotation: Ott et al. '05: viscous ($\boldsymbol{B}$) dissipation in PNS: $\Omega(r) \Longrightarrow \Omega_{\mathrm{rigid}}$
- Incompressibility: $v_{\mathrm{flow}} \ll c_{\mathrm{s}} = 8 \cdot 10^8 \cdots \lesssim c$, $\tau_{\mathrm{s}} \ll \tau_{\mathrm{flow}}$
- Constant density (?): flow concentrated within the core $\Longrightarrow$ anelastic approximation

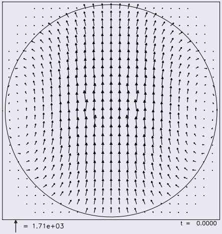
Stable MHD–Equilibria in Young Neutron Stars

Timescales

- Rotation: $P \sim$ millisecond
- Ohmic decay:
  $\sigma \approx 10^{24}\mathrm{s}^{-1}, \quad R \approx 10^6\mathrm{cm}$
  $\Longrightarrow \tau_{\mathrm{Ohm}} \sim 1.4 \cdot 10^{17}\mathrm{s} \approx 4.4 \cdot 10^8\mathrm{yrs}$ ⇔ problem of extremely large Rm
- Viscous dissipation:
  $\tau_{\mathrm{visc}} = R^2/\nu \lesssim \tau_{\mathrm{Ohm}}$
- MHD waves:
  $\tau_{\mathrm{A}} = \sqrt{4\pi\rho}R/B \approx 0.05\mathrm{s}$
  $(\rho = 2 \cdot 10^{14}\mathrm{g\ cm}^{-3})$
  $\Rightarrow \tau_{\mathrm{A}} \sim 10^{-25}\tau_{\mathrm{Ohm,visc}}$: will help to get reliable results for stability!

$\boldsymbol{B}(\boldsymbol{r}, t = 0)$

a) Poloidal magnetostatic equilibrium



b) internal uniform, external potential (Flowers & Rudermann '77)

- different angles $\alpha(\boldsymbol{B}, \boldsymbol{\Omega})$, $6\ \mathrm{ms} \le P \le 0.6\mathrm{s}$

Mathematical model

Equations (dimensionless)

induction equation + "BC"

$$\frac{\partial \boldsymbol{B}}{\partial t} = \Delta\, \boldsymbol{B} + \mathrm{curl}(\boldsymbol{u} \times \boldsymbol{B}) \quad \text{in } V'$$

$$\mathrm{rot}\, \boldsymbol{B} = \boldsymbol{0} \qquad \text{in } V \setminus V'$$

$$\mathrm{div}\, \boldsymbol{B} = 0 \qquad \text{in } V$$

$$[\boldsymbol{B}] = \boldsymbol{0} \qquad \text{on } \partial V'$$

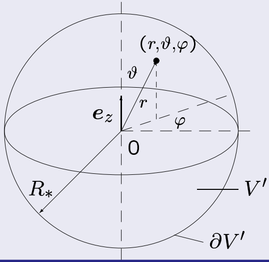
momentum equation + BC

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla p - (\boldsymbol{u}\nabla)\boldsymbol{u} + Pm\,\Delta\, \boldsymbol{u} - 2\Omega \boldsymbol{e}_z \times \boldsymbol{u} + \mathrm{rot}\, \boldsymbol{B} \times \boldsymbol{B}$$

$$\mathrm{div}\, \boldsymbol{u} = 0 \qquad \text{in } V'$$

$$\boldsymbol{u} \cdot \boldsymbol{r} = \left(\underset{\sim}{D}(\boldsymbol{u}) \cdot \boldsymbol{r}\right)_{\vartheta,\varphi} = 0 \quad \text{on } \partial V$$

Geometry



Parameters

$Pm = \nu/\eta$ – magnetic Prandtl number

Ω – normalized angular velocity

$q_P = P/\tau_A$ – relative rotation period

Solution characteristics

Symmetries

- with resp. to equatorial plane:

  "S": $\boldsymbol{B}$, $\boldsymbol{u}$, p symmetric

  $$\text{“}A\text{”}: \begin{cases} \boldsymbol{B} & \text{antisymmetric} \\ \boldsymbol{u},\ p & \text{symmetric} \end{cases}$$

- about an axis:
  expansion in Fourier series

  $$\sum_{m=0}^{\infty} \ldots e^{im\varphi}$$

  axisymmetric
  if only $m = 0$ term

characterizing quantities

"parity" $$\Pi = \frac{E^{\text{sym}} - E^{\text{anti}}}{E^{\text{sym}} + E^{\text{anti}}}$$

"non–axisymmetry" $$M = 1 - \frac{E^{\text{ax}}}{E}$$

$\longrightarrow$ for poloidal equilibrium:
$\Pi = -1$, $M = 0$

"ohmic reference energy" $E^{\text{Ohm}}_{\text{mag}}$

denormalization with

$$R = 10^6\ \text{cm}\,,\ \ \rho = 2 \cdot 10^{14}\ \text{g}\,\text{cm}^{-3}\,,$$

$$B(0) = 10^{15}\ \text{G}$$

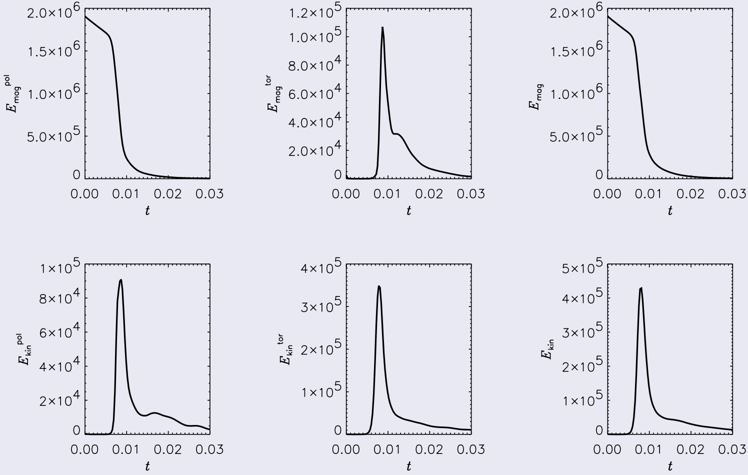
Results for dipolar magnetostatic equilibrium:

for $t = \tau_{\text{decay}}$:

α	q_P	$E_{\text{mag}}/E_{\text{mag}}^{\text{Ohm}}$	$E_{\text{kin}}/E_{\text{mag}}$	Π_{mag}	M_{mag}	symmetry
0	∞	0.00018	4.67	-0.06	0.4	mixed
	12	0.00035	6.6	-0.5	0.2	mixed
	1.2	0.076	0.00065	-0.99	0.06	mixed
	0.12	0.98	0.00003	-1	0	A0
45	1.2	0.075	0.001	-0.46	0.27	mixed
	0.12	0.82	0.00005	-0.24	0.38	mixed
90	12.	0.0033	0.64	0.5	0.77	mixed
	1.2	0.043	0.003	0.88	0.81	mixed
	0.12	0.14	0.00014	1	1	S1

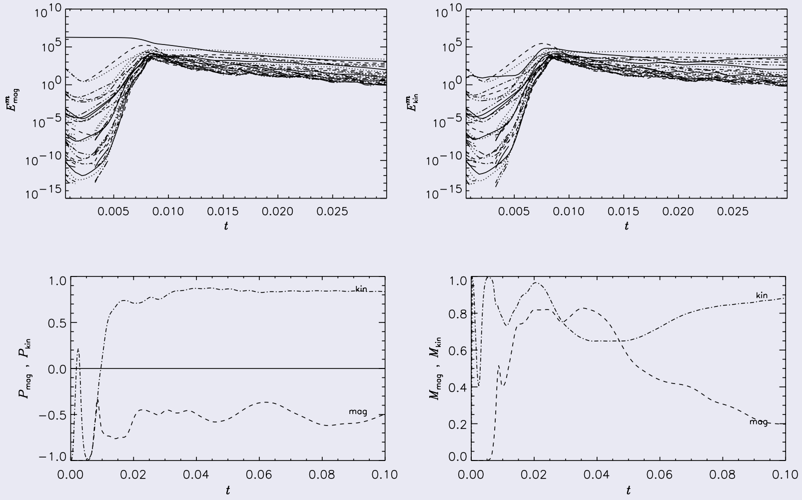
Unstable decay: $P = 0.6$ s, $\alpha = 0$

Energies:



Unstable decay: $P = 0.6$ s, $\alpha = 0$

Spectra, parities, non-axisymmetries:



Rotation-stabilized decay: $P = 6$ ms, $\alpha = 0$

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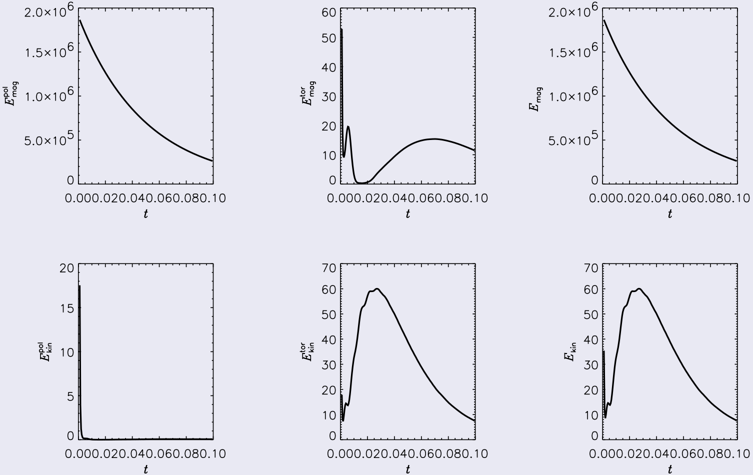

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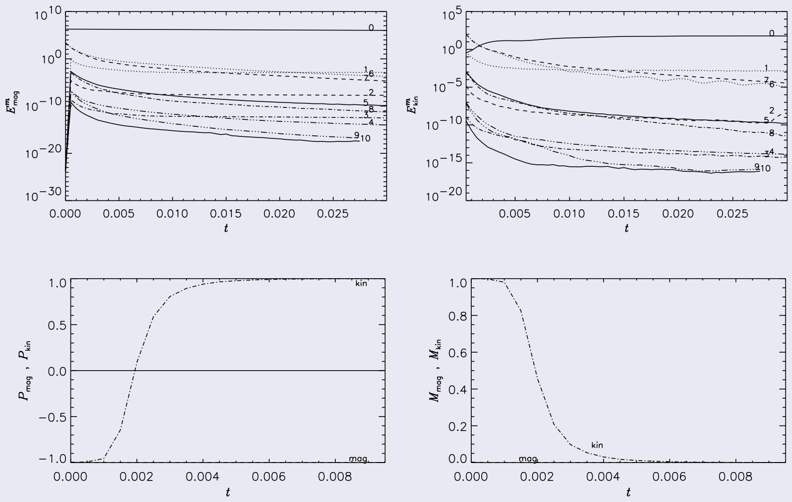

Conclusions

- neutron stars born with $B \gtrsim 10^{15}$G:
  - those with $P \lesssim 5$ ms AND $\alpha \lesssim 45^o \Rrightarrow$ magnetars
  - those with $P \gtrsim 10$ ms AND/OR $\alpha \gtrsim 45^o \Rrightarrow$ standard pulsars
  - No other stabilizing effect (torus–like ***B*** configuration, density stratification), only **RAPID ROTATION** can maintain magnetar field!
- neutron stars born with $B \sim 10^{12} \ldots 10^{13}$G, $P \leq 60$ ms: always stabilized!