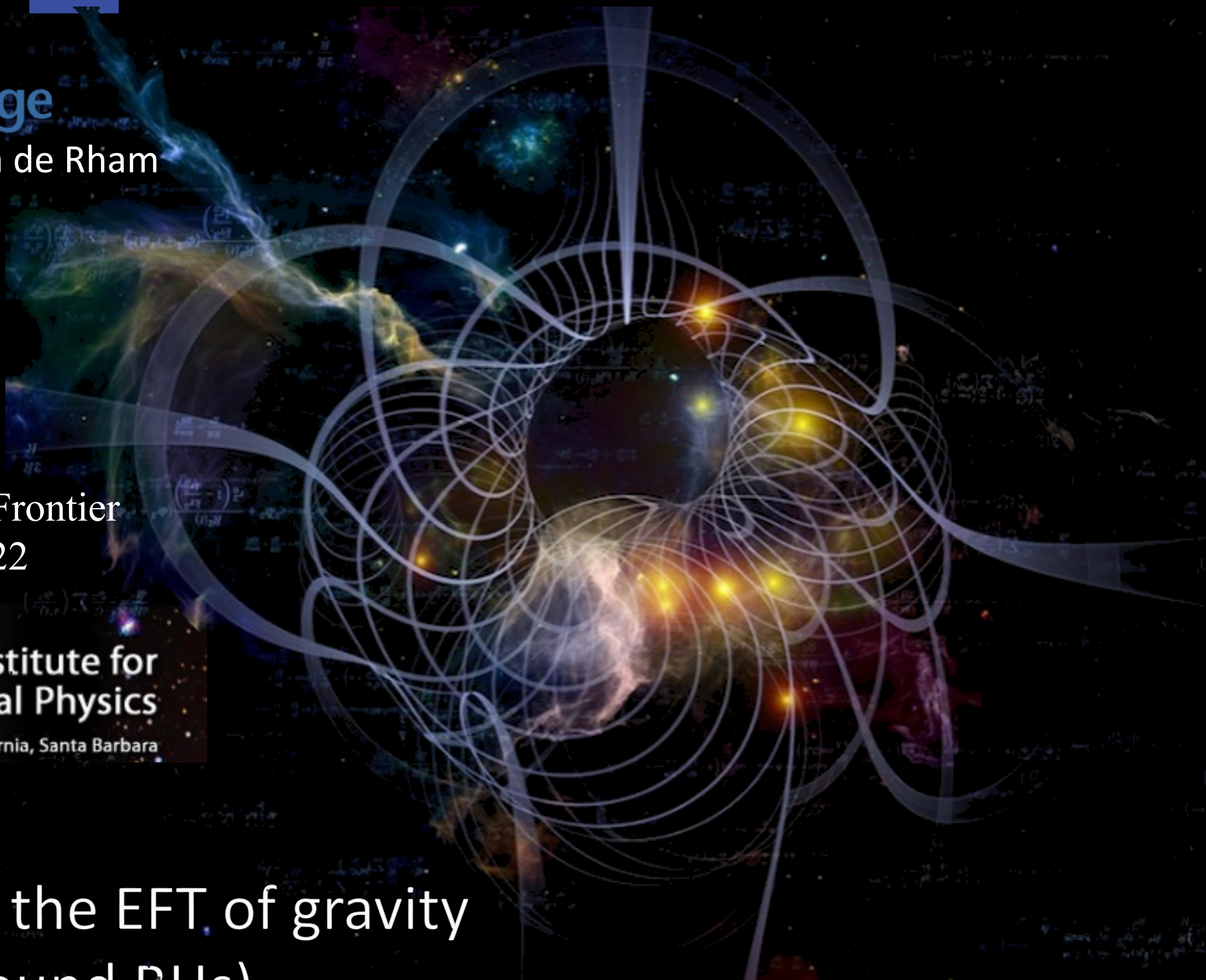
Imperial College London
Claudia de Rham

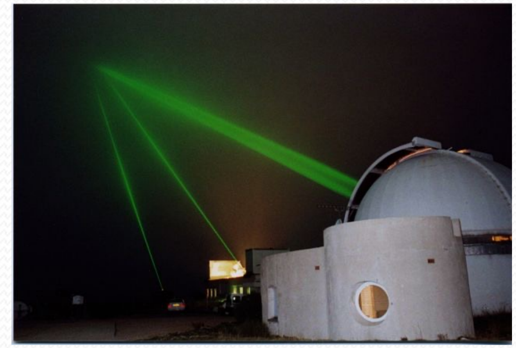
KITP
Storming the GW Frontier
25th April 2022

The Kavli Institute for Theoretical Physics
University of California, Santa Barbara



Causality in the EFT of gravity (around BHs)

GR as an EFT

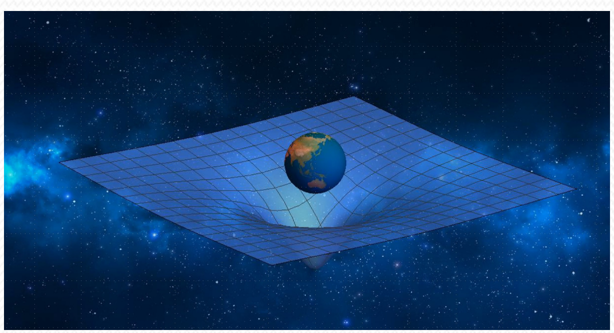


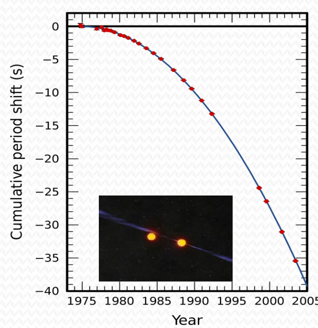
GR extremely well tested in a variety of systems

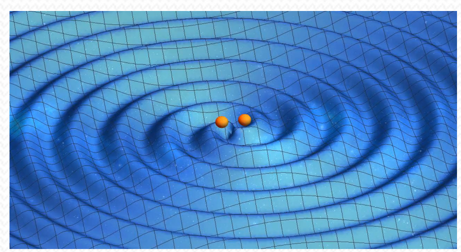
But we know GR is leading term in an EFT

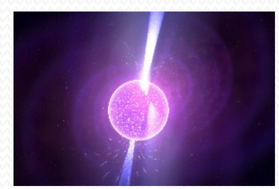
What can we expect to learn about Gravity (with GWs) ?

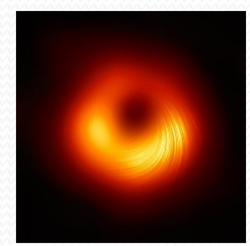
(very optimistically & with theoretical priors)



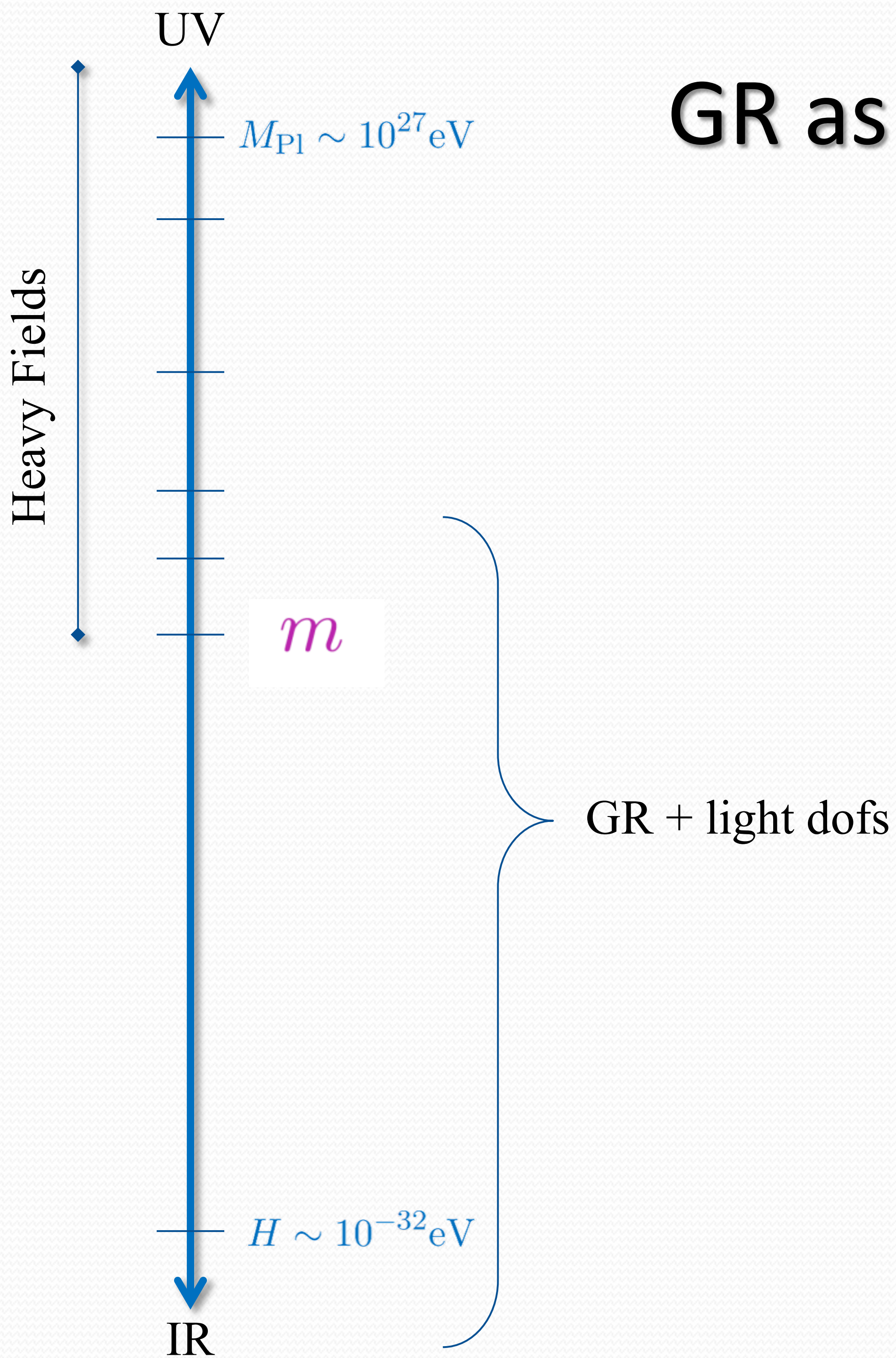


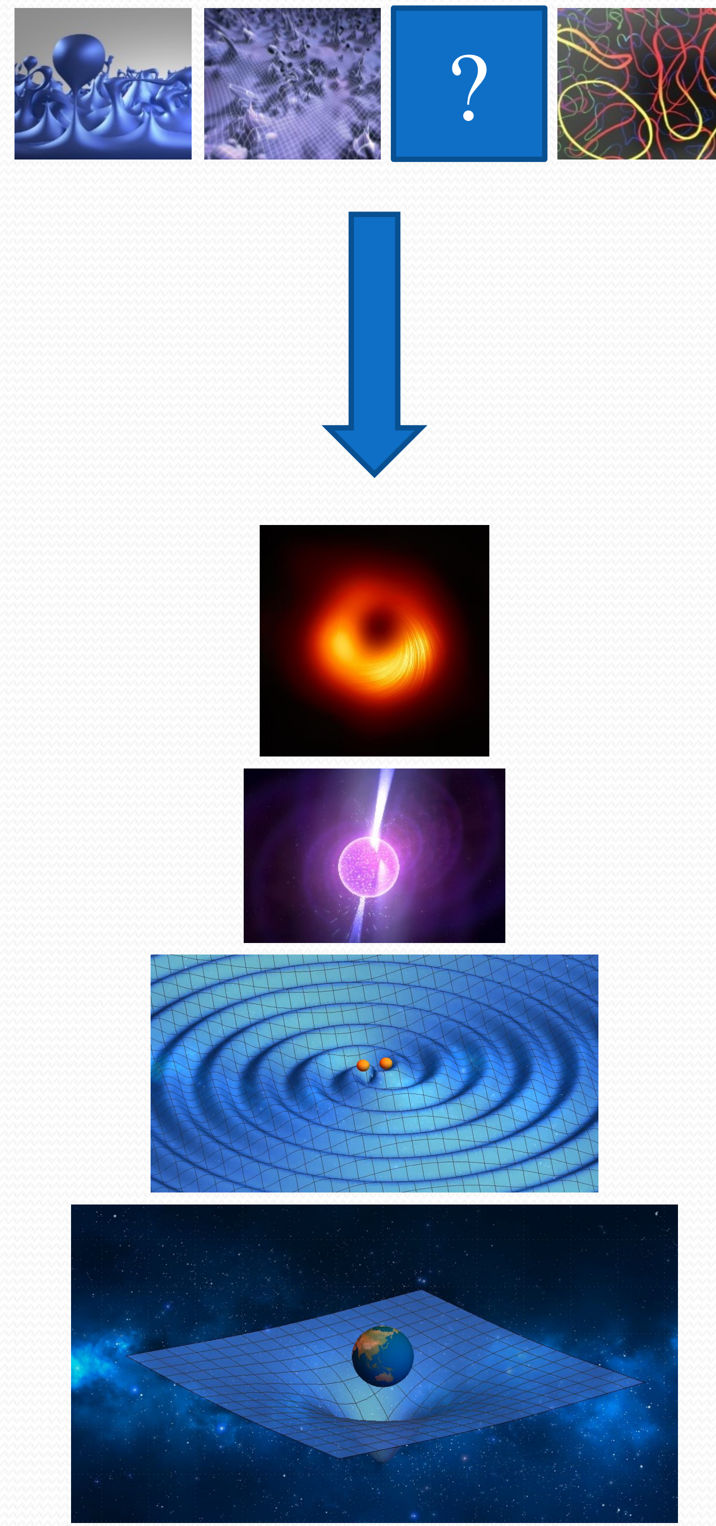


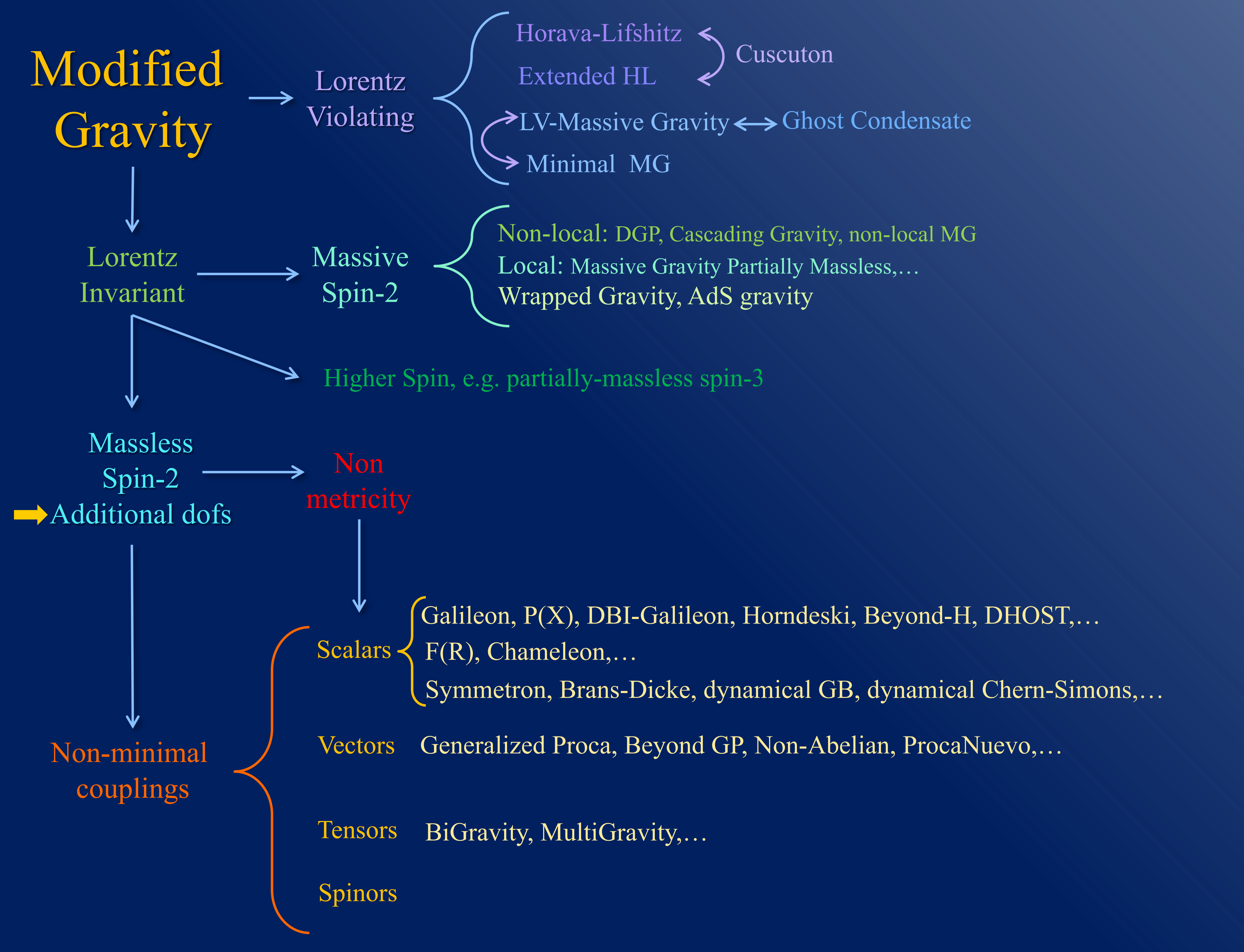




GR as an EFT


UV
$M_{\rm Pl} \sim 10^{27}{\rm eV}$
Heavy Fields
m
GR + light dofs
$H \sim 10^{-32}{\rm eV}$
IR


?



Modified Gravity
Lorentz Violating
Horava-Lifshitz
Extended HL
Cuscuton
LV-Massive Gravity
Ghost Condensate
Minimal MG
Lorentz Invariant
Massive Spin-2
Non-local: DGP, Cascading Gravity, non-local MG
Local: Massive Gravity Partially Massless,…
Wrapped Gravity, AdS gravity
Higher Spin, e.g. partially-massless spin-3
Massless Spin-2
Additional dofs
Non metricity
Scalars
Galileon, P(X), DBI-Galileon, Horndeski, Beyond-H, DHOST,…
F(R), Chameleon,…
Symmetron, Brans-Dicke, dynamical GB, dynamical Chern-Simons,…
Non-minimal couplings
Vectors
Generalized Proca, Beyond GP, Non-Abelian, ProcaNuevo,…
Tensors
BiGravity, MultiGravity,…
Spinors

Modified Gravity

Lorentz Violating
- Horava-Lifshitz
- Extended HL
- Cuscuton
- LV-Massive Gravity ⟷ Ghost Condensate
- Minimal MG

Lorentz Invariant

Massless Spin-2

➡ **Additional dofs**

Non metricity

New dof usually couple weakly to matter
(no direct screening but unnatural fine-tuned coupling)

$$\mathcal{L} = -\frac{1}{2}Z(\phi_b,\rho_b)(\partial\delta\phi)^2 - \frac{1}{2}m^2(\phi_b,\rho_b)\delta\phi^2 + \frac{\beta(\phi_b,\rho_b)}{M_{\rm Pl}}\delta\phi\delta\rho$$

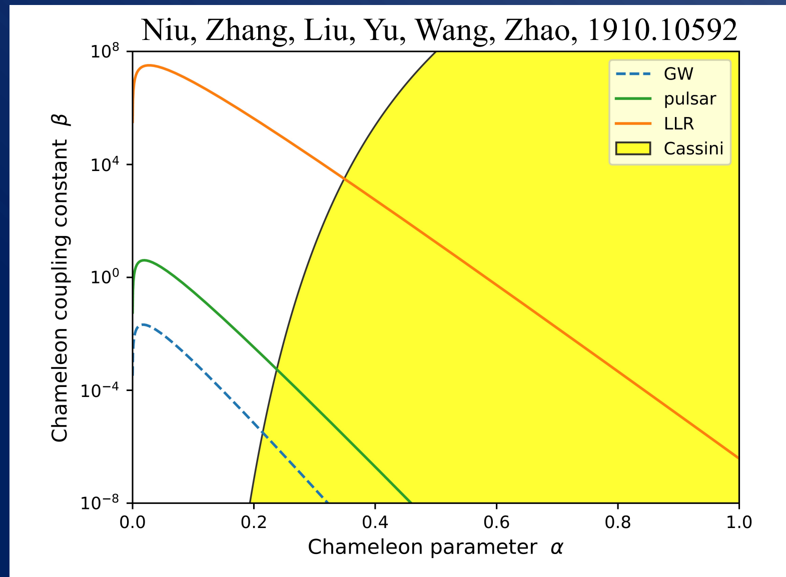
Non-minimal couplings

- Scalars
  - Galileon, P(X), DBI-Galileon, Horndeski, Beyond-H, DHOST,…
  - F(R), Chameleon,…
  - Symmetron, Brans-Dicke, dynamical GB, dynamical Chern-Simons,…
- Vectors: Generalized Proca, Beyond GP, Non-Abelian, ProcaNuevo,…
- Tensors: BiGravity, MultiGravity,…
- Spinors

Modified Gravity

New dof screened via the Chameleon mechanism

See Berti, Yagi, Yunes, 1801.03208, 1801.03587



Lorentz Invariant

Massless Spin-2

Additional dofs

Non metricity

Non-minimal couplings

Scalars

- Galileon, P(X), DBI-Galileon, Horndeski, Beyond-H, DHOST,…
- F(R), Chameleon
- Symmetron, Brans-Dicke, dynamical GB, dynamical Chern-Simons,…

$$\mathcal{L} = -\frac{1}{2} Z(\phi_b, \rho_b)(\partial\delta\phi)^2 - \frac{1}{2} m^2(\phi_b, \rho_b)\delta\phi^2 - \frac{\beta(\phi_b, \rho_b)}{M_{\rm Pl}}\delta\phi\delta\rho$$

Modified Gravity

New dof screened via the Vainshtein mechanism

Lorentz Invariant → Massive Spin-2

$$\mathcal{L} = -\frac{1}{2}Z(\phi_b,\rho_b)(\partial\delta\phi)^2 - \frac{1}{2}m^2(\phi_b,\rho_b)\delta\phi^2 + \frac{\beta(\phi_b,\rho_b)}{M_{\rm Pl}}\delta\phi\delta\rho$$

E.g. application of amplitude methods to weakly coupled region in these EFTs

$$V = -\frac{m_1^2 m_2^2}{M_{\rm Pl}^2 E_1 E_2 r}\left(1 + \frac{r^3_{*,a,b}}{r^3} + \frac{r^6_{*,a,b}}{r^6}\left(1+\frac{|p|^2}{E^2_{a,b}}\right)+\cdots\right)$$

Mariana Carrillo-González, CdR, Andrew Tolley, 2107.11384

Massless Spin-2

➡ Additional dofs

Non-minimal couplings

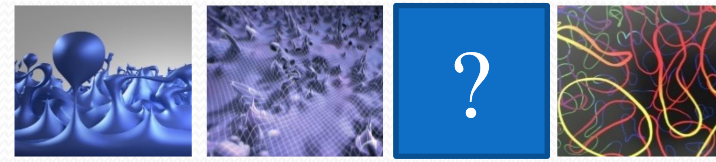
- Scalars
  - Galileon, P(X), DBI-Galileon, Horndeski, Beyond-H, DHOST
  - F(R), Chameleon
  - Symmetron, Brans-Dicke, dynamical GB, dynamical Chern-Simons,…
- Vectors
  - Generalized Proca, Beyond GP, Non-Abelian, ProcaNuevo,…
- Tensors
  - BiGravity, MultiGravity,…
- Spinors

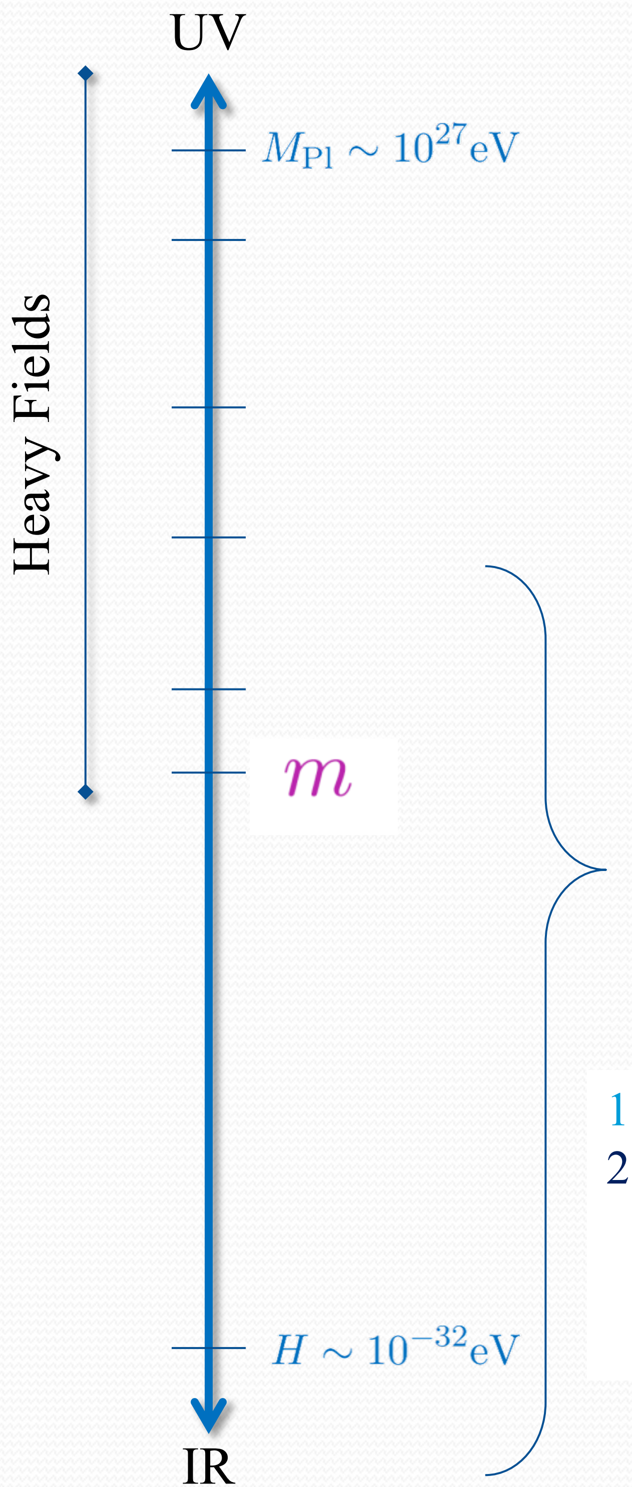
New dof screened via the Vainshtein mechanism

Monopole & dipole radiation suppressed
Emission dominated by quadrupole with corrections starting at 6PN

Dar, CdR, Deskins, Giblin & Tolley, 1808.02165

GR as an EFT

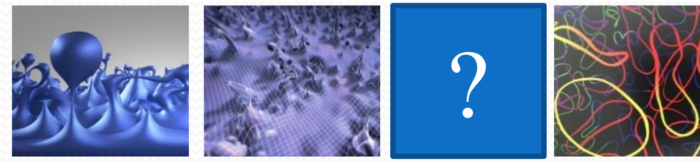




Potential IR modifications of gravity

1. Violation of Lorentz invariance (dangerous propagation to SM)
2. New light degrees of freedom in cohesive form (bosons)
   - either violation of BH no hair theorem or equivalence principle (need of screening)
   - or technically unnatural fine-tuned couplings

GR as an EFT



UV

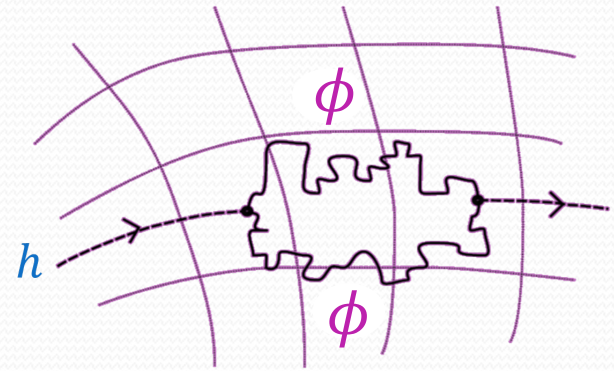
$M_{\rm Pl} \sim 10^{27}\mathrm{eV}$

Heavy Fields

High-energy theory with gravity and light & heavy modes ϕ potentially including an infinite tower of higher spins

m

Integrate out heavy modes ϕ



Adapted from Hollowood & Shore

Low-energy EFT of gravity valid below m + light modes (e.g. photon)

Have to content ourselves with parameterizing our lack of knowledge

$$\mathcal{L}_{\mathrm{EFT}<m} = \frac{M_{\rm Pl}^2}{2}\left[R + \frac{R^2 + R_{\mu\nu}^2 + \mathrm{GB}}{m^2} + \frac{\mathrm{Riem}^3}{m^4} + \frac{\mathrm{Riem}^4}{m^6} + \cdots\right] + \mathcal{L}[\psi_{\rm light}, g_{\mu\nu}]$$

IR

EFT of gravity

$$\mathcal{L}_{\mathrm{EFT}<m} = \frac{M_{\mathrm{Pl}}^2}{2}\left[R + \frac{R^2 + R_{\mu\nu}^2 + \mathrm{GB}}{m^2} + \frac{\mathrm{Riem}^3}{m^4} + \frac{\mathrm{Riem}^4}{m^6} + \cdots\right] + \mathcal{L}[\psi_{\mathrm{light}}, g_{\mu\nu}]$$

Consider solutions in the vacuum (BH solutions)

$$\mathcal{L}_{\mathrm{EFT}<m}^{(\mathrm{vacuum})} = \frac{M_{\mathrm{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\mathrm{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\mathrm{Riem}^4 + \cdots\right]$$

Anything else ≡ *low-energy* modification of gravity

EFT of gravity

Consider solutions in the vacuum (BH solutions)

$$\mathcal{L}^{(\mathrm{vacuum})}_{\mathrm{EFT}<m} = \frac{M_{\mathrm{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\mathrm{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\mathrm{Riem}^4 + \cdots\right]$$

dim-6 dim-8

- Inspiralling GWs

For dim-8, Endlich, Gorbenko, Huang & Senatore, 1704.01590 JHEP (2017)
For dim-6 & 8, Accettulli Huber, Brandhuber, De Angelis &Travaglini, 2012.06548 PRD (2021)
…

- QNMs

For dim-8, Cardoso, Kimura, Maselli & Senatore, 1808.08962 PRL (2018)
For dim-6, CdR, Francfort, Zhang, 2005.13923, PRD (2020)
For dim-6 & -8 in rotating BHs, Cano, Fransen, Hertog & Maenaut, 2110.11378, PRD (2022)
…

- LVK waveform Constraints

For dim-8, Sennett, Brito, Buonanno, Gorbenko & Senatore, 1912.09917, PRD (2020)
For dim-6, to come…

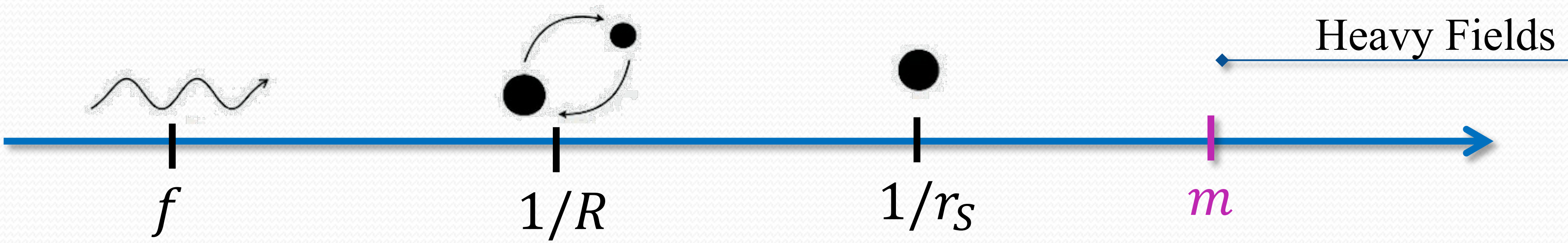
- Bending

Andreas Brandhuber, Gabriele Travaglini, 1905.05657, JHEP (2020)

$$\mathcal{L}^{(\text{vacuum})}_{\text{EFT}<m} = \frac{M_{\text{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\text{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\text{Riem}^4 + \cdots\right]$$

dim-6 dim-8

Observability



$$m r_S \gg 1$$

$$r_s \sim \frac{M_{\text{BH}}}{M_{\text{Pl}}^2} \sim G M_{\text{BH}}$$

With BH ringdown, fractional correction to QNM: $\delta = \frac{\omega_{\text{EFT}} - \omega_{\text{GR}}}{\omega_{\text{GR}}}$

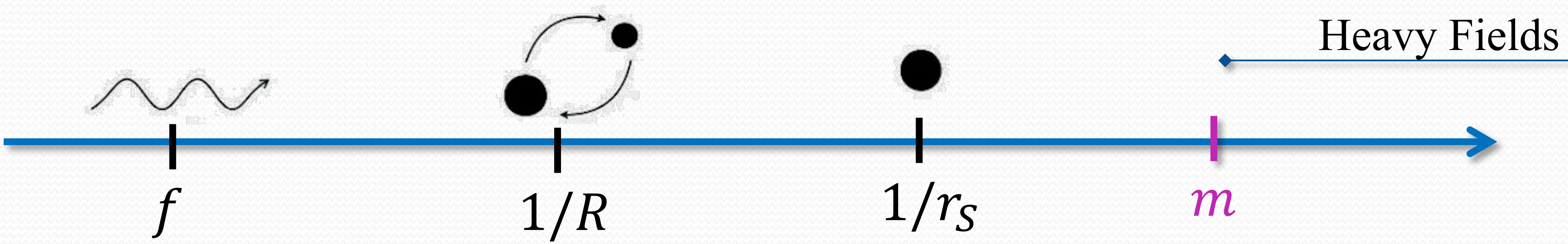
$$\delta_{\omega_{D8}} \sim (m r_S)^{-6} \ll \delta_{\omega_{D6}} \sim (m r_S)^{-4} \ll 1$$

Potentially observable in the future ???

Would need to significantly reduce fractional error on BH QNfrequencies

$$\mathcal{L}^{(\text{vacuum})}_{\text{EFT}<m} = \frac{M_{\text{Pl}}^2}{2}\left[R + \underbrace{\frac{\mathcal{C}_3}{m^4}\text{Riem}^3}_{\text{dim-6}} + \underbrace{\frac{\mathcal{C}_4}{m^6}\text{Riem}^4}_{\text{dim-8}} + \cdots\right]$$

Observability



$$m r_S \gg 1$$

With BH ringdown, fractional correction to QNM:

$$\delta_{\omega_{D8}} \sim (m r_S)^{-6} \ll \delta_{\omega_{D6}} \sim (m r_S)^{-4} \ll 1$$

With finite size effects in inspiralling GWs: (phase correction)

$$\delta\Psi^{\text{SPA}}_{D6} \sim \frac{v^{10}}{(r_s m)^4}$$

Accettulli Huber, Brandhuber, De Angelis &Travaglini, 2012.06548

(same order as tidal effects for NS, Cheung&Solon, 2006.06665)

$$\delta\Psi^{\text{SPA}}_{D8} \sim \frac{v^{16}}{(r_s m)^6}$$

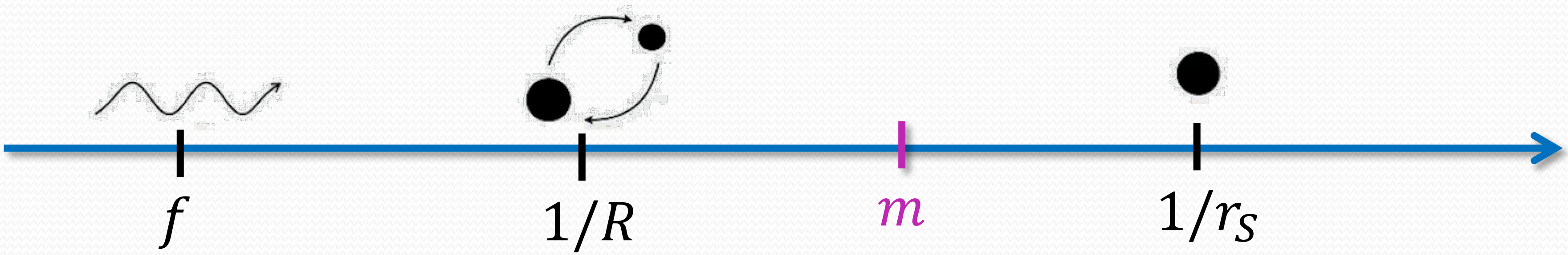
Endlich, Gorbenko, Huang & Senatore, 1704.01590

using SPA, Buonanno, Iyer, Ochsner, Pan & Sathyaprakash, 0907.0700

$$\mathcal{L}^{(\text{vacuum})}_{\text{EFT}<m} = \frac{M_{\text{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\text{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\text{Riem}^4 + \cdots\right]$$

dim-6 dim-8

Observability



Sennett, Brito, Buonanno, Gorbenko & Senatore, 1912.09917, PRD (2020)

$$m r_s \gg (r_s f)^{2/3}$$

With **early** finite size effects in inspiralling GWs:

$$\delta\Psi^{\text{SPA}}_{D6} \sim \frac{v^{10}}{(r_s m)^4}$$

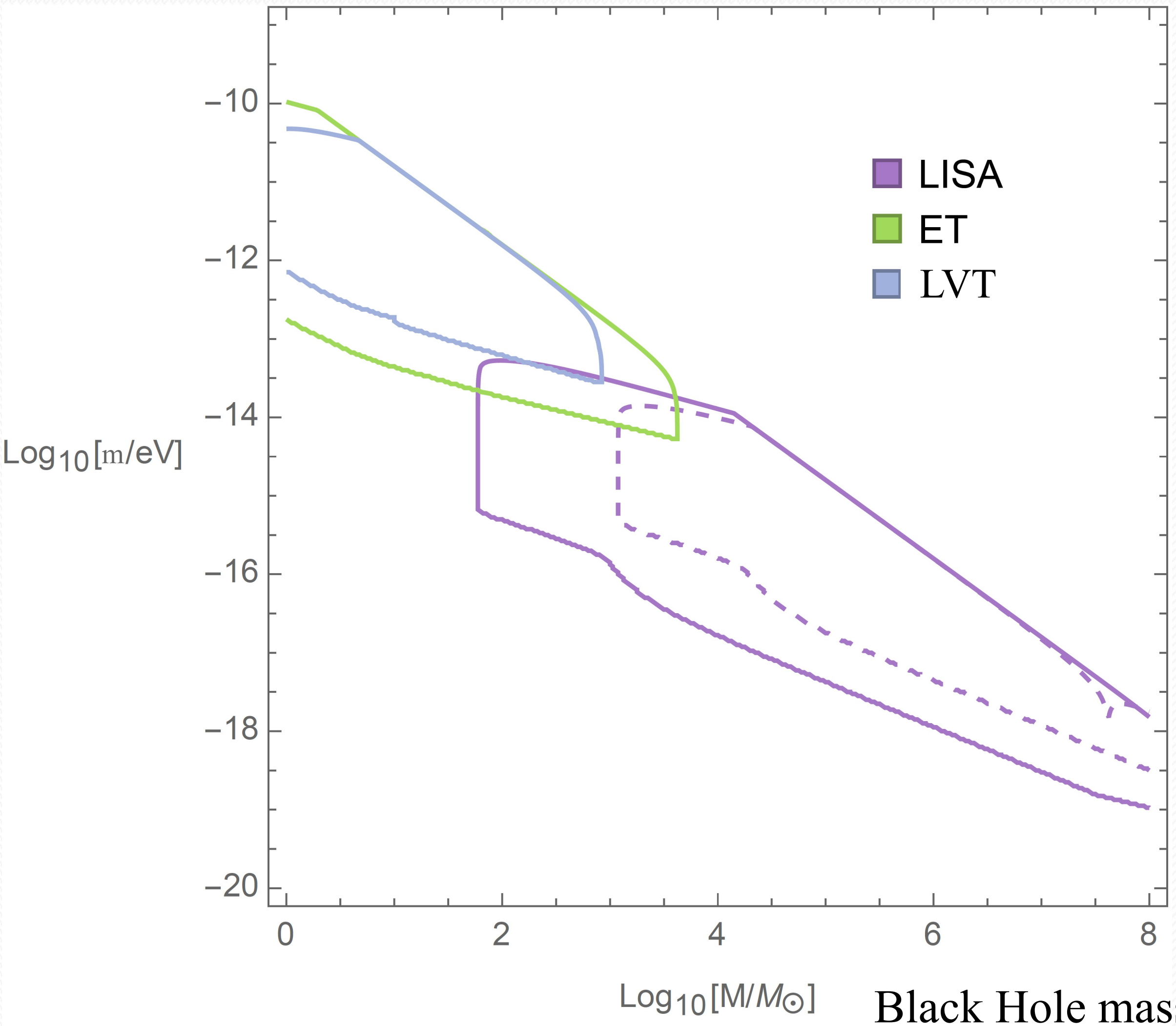
$$\delta\Psi^{\text{SPA}}_{D8} \sim \frac{v^{16}}{(r_s m)^6}$$

Endlich, Gorbenko, Huang & Senatore, 1704.01590
Accettulli Huber, Brandhuber, De Angelis &Travaglini, 2012.06548

$$\Delta\Psi^{\text{SPA}}_{D6} \sim \frac{1}{(r_s m)^4}\left[(r_s f_f)^{5/3} - (r_s f_i)^{5/3}\right]$$

Main question is how smoothly & rapidly waveform returns to GR upon exiting regime of validity

Observability of Dim-6 EFT



Black Hole mass

ET & LVT, BHs at 300Mpc
LISA, BHs at 3Gpc and 26Gpc

$$\Delta\Psi^{\rm SPA}_{D6} \sim \frac{1}{(r_s m)^4}\left[(r_s f_f)^{5/3} - (r_s f_i)^{5/3}\right]$$

$$r_s f_f \sim (m r_s)^{3/2}$$

Increasing m,
means reducing EFT contributions

Decreasing m,
means EFT runs out of control faster
(f_f smaller) hence less data available

Tolley, Zhang, CdR 2112.05054, PRL (2022)

Theoretical Constraints

$$\mathcal{L}^{(\text{vacuum})}_{\text{EFT}<m} = \frac{M_{\text{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\text{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\text{Riem}^4 + \cdots\right]$$

dim-6 dim-8

For dim-8:

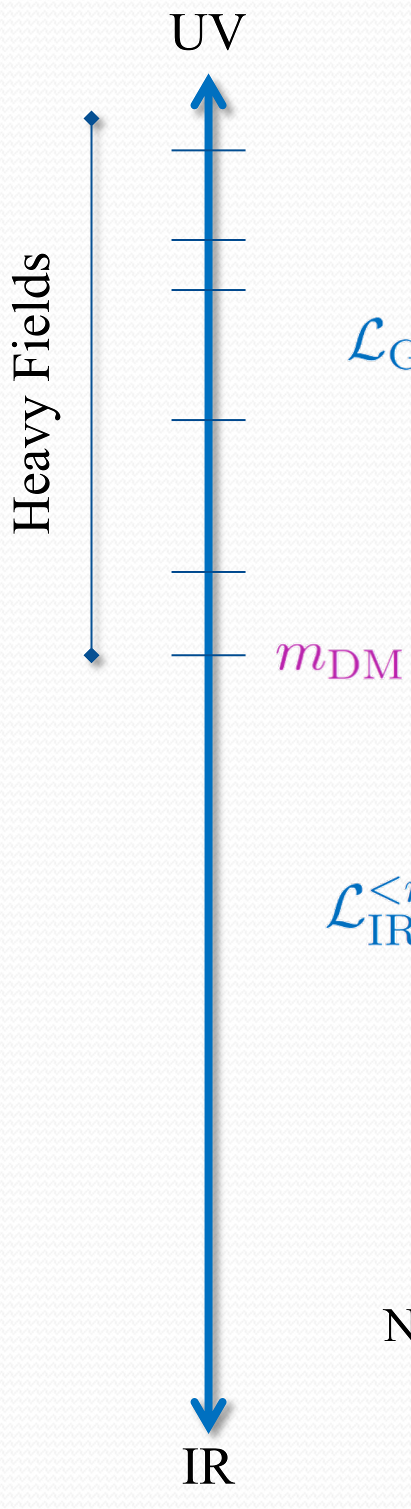
Gruzinov & Kleban, hep-th/0612015, CQG (2007), (superluminalities)
Bellazzini, Cheung & Remmen, 1509.00851, PRD (2016) (S-matrix positivity)

For dim-6 & 8:

Bern, Kosmopoulos & Zhiboedov, 2103.12728, J. Phys. A (2021) (S-matrix positivity)
CdR, Tolley, Zhang, 2112.05054, PRL (2022) (low-energy causality)
Caron-Huot, Li, Parra-Martinez Simmons-Duffin, 2201.06602 (S-matrix positivity)

- Constraints on the scale
- Constraints on the coefficients from S-matrix positivity bounds
- Constraints on the coefficients from causality about BHs

Constraints on the scale



$$\mathcal{L}_{\rm GR+SM+light\ DM} = \frac{M_{\rm Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu}^2 - \bar{\psi}(i\not{\nabla} + m_{\rm DM})\psi + \cdots$$

Integrate out fields of mass$\geq m_{\rm DM}$

$$\mathcal{L}^{<m}_{\rm IR\ EFT} = \frac{M_{\rm Pl}^2}{2} \left[R + \frac{\mathcal{C}_6}{m^4} {\rm Riem}^3 + \cdots \right] - \frac{1}{4} F_{\mu\nu}^2$$

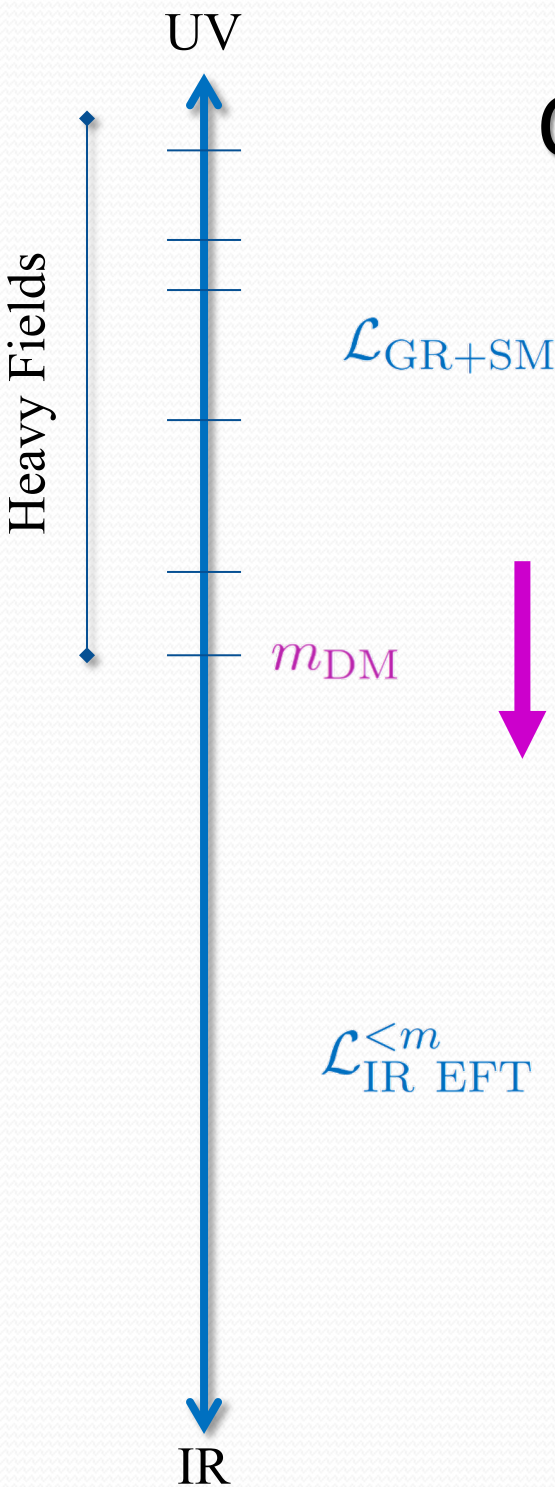
Routinely consider models for which scale m would be low

$$m \sim (m_{\rm DM} M_{\rm Pl})^{1/2} \sim 10^3 {\rm eV}$$

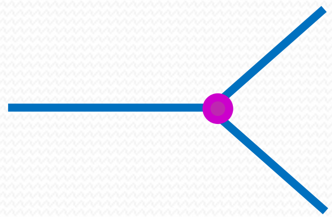
No necessarily issues with LHC physics or other known high energy processes simply need to go back to partial UV description

This is just a loop example, could consider tree-level from higher-spin instead

Constraints on the scale

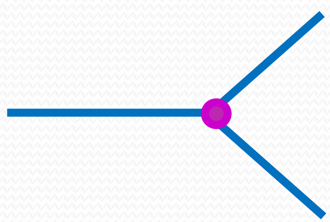


$$\mathcal{L}_{\rm GR+SM+light\ DM} = \frac{M_{\rm Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu}^2 - \bar{\psi}(i\nabla\!\!\!/ + m_{\rm DM})\psi + \cdots$$



$$\mathcal{A}_{h\to hh}^{E \gtrsim m_{\rm DM}} \sim \frac{E^2}{M_{\rm Pl}} \left[1 + \ln\left(1 + \frac{m_{\rm DM}^2}{E^2}\right) + \cdots \right]$$

$$\mathcal{L}_{\rm IR\ EFT}^{<m} = \frac{M_{\rm Pl}^2}{2} \left[R + \frac{\mathcal{C}_6}{m^4} {\rm Riem}^3 + \cdots \right] - \frac{1}{4} F_{\mu\nu}^2$$



$$\mathcal{A}_{h\to hh}^{E \ll m_{\rm DM}} \sim \frac{E^2}{M_{\rm Pl}} \left[1 + \left(\frac{E}{m}\right)^4 + \left(\frac{E}{m}\right)^6 + \cdots \right]$$

Constraints on the scale

$$\mathcal{L}^{(\mathrm{vacuum})}_{\mathrm{EFT}<m} = \frac{M_{\mathrm{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\mathrm{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\mathrm{Riem}^4 + \cdots\right]$$

dim-6 dim-8

- Requires scale $m \lll$ SM physics
  Theoretically ok so long as *no direct coupling to SM fields*,
  physics should be in a decoupled dark sector (not necessarily DM/DE though)

- For dim-6 operators, scale m can be tight to scale of SUSY breaking in that dark sector ($C_3/m = 0$ for susy sector)

- Very small window of opportunity to probe these operators with GW physics. But so far window still theoretically open…

S-matrix positivity bounds

$$\mathcal{L}^{(\text{vacuum})}_{\text{EFT}<m} = \frac{M_{\text{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\text{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\text{Riem}^4 + \cdots\right]$$

Demanding a unitary, local, Lorentz invariant and causal/analytic high energy completion imposes a positivity constraints on the 2-2 elastic scattering amplitude

$$\partial_s^2 \mathcal{A}_{hh\to hh} > 0$$

Pham and Truong 1985
Ananthanarayan, Toublan and Wanders, 1994
Adams et. al. 2006

In fact, statement of unitarity translates into an infinite number of constraints

$$\partial_s^{2n}\partial_t^k \mathcal{A}_{hh\to hh} > \# \qquad \forall\ n \geq 1\,, k \geq 0$$

CdR, Melville, Tolley & Zhou, 1702.06134

s: center of mass energy^2
t: momentum transfer

S-matrix positivity bounds

$$\mathcal{L}^{(\mathrm{vacuum})}_{\mathrm{EFT}<m} = \frac{M_{\mathrm{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\mathrm{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\mathrm{Riem}^4 + \cdots\right]$$

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CdR, Melville, Tolley & Zhou, 1702.06134

$$M_{\mathrm{Pl}}^2\mathcal{A} \sim s^2\left[-\frac{1}{t} + \mathcal{C}_3\frac{t}{m^4} + \cdots\right] + \frac{s^4}{m^6}\left[\mathcal{C}_4 + \mathcal{C}_3^2\frac{t}{m^2} + \cdots\right]$$

S-matrix positivity bounds

$$\mathcal{L}^{(\mathrm{vacuum})}_{\mathrm{EFT}<m} = \frac{M_{\mathrm{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\mathrm{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\mathrm{Riem}^4 + \cdots\right]$$

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t-channel pole (graviton exchange) spoils applicability of leading order positivity bounds alternative is to impose bounds at finite impact parameter Caron-Huot, Mazac, Rastelli & Simmons-Duffin, 2102.08951, consistent with infrared causality bounds found in 1909.00881, 2007.01847, 2112.05031 (with Chen, Margalit & Tolley)

S-matrix positivity bounds

$$\mathcal{L}^{(\mathrm{vacuum})}_{\mathrm{EFT}<m} = \frac{M_{\mathrm{Pl}}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}\mathrm{Riem}^3 + \frac{\mathcal{C}_4}{m^6}\mathrm{Riem}^4 + \cdots\right]$$

In fact, statement of unitarity translates into an infinite number of constraints

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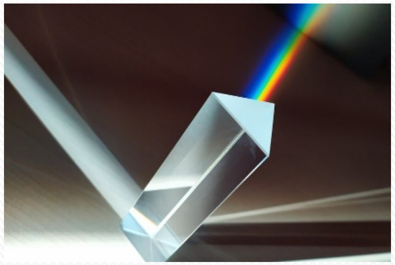
CdR, Melville, Tolley & Zhou, 1702.06134

$$M_{\mathrm{Pl}}^2\mathcal{A} \sim s^2\left[-\frac{1}{t} + \mathcal{C}_3\frac{t}{m^4} + \cdots\right] + \frac{s^4}{m^6}\left[\mathcal{C}_4 + \mathcal{C}_3^2\frac{t}{m^2} + \cdots\right]$$

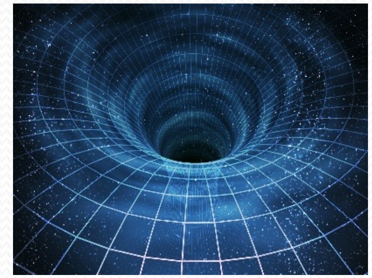
So far, all positivity bounds involve dim-8 operators or beyond (bounds on $\mathcal{C}_4, \mathcal{C}_3^2, \cdots$)

Bellazzini, Cheung & Remmen, 1509.00851, PRD (2016)
Bern, Kosmopoulos & Zhiboedov, 2103.12728, J. Phys. A (2021)
Caron-Huot, Li, Parra-Martinez Simmons-Duffin, 2201.06602

Constraints from Causality about BHs



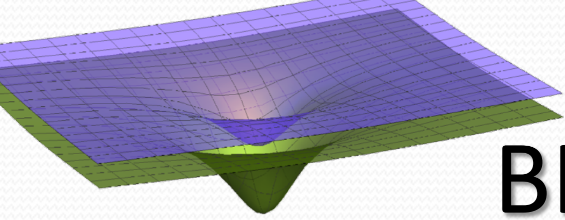
Equivalent of prism: BH geometry
Equivalent of refractive index: EFT corrections
Equivalent of light: GWs



Requirement: No resolvable time advance wrt null geodesics (setup by local metric)

Note: departure from luminality is below realm of detectability (for now…)

These are *theoretical* bounds, demanding consistency with causality.
When they are, their impact should be observed through other effects.



BHs in the EFT of Gravity

$$\mathcal{L}_{\rm EFT} = \frac{M_{\rm Pl}^2}{2}\left[R + \frac{\mathcal{C}_3}{m^4}{\rm Riem}^3 + \cdots\right]$$

$$\epsilon = \frac{1}{(mr_s)^4} = \left(\frac{M_{\rm Pl}^2}{mM_{\rm BH}}\right)^4 \ll 1$$

Schwarzschild metric (GR, no EFT corrections)

$$\bar{\gamma}_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu = -f{\rm d}t^2 + \frac{1}{f}{\rm d}r^2 + r^2{\rm d}\Omega^2$$

New BH metric (including effects from EFT operators)

$$\gamma_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu = -\left(f + \epsilon\delta f_t\right){\rm d}t^2 + \frac{1}{f+\epsilon\delta f_r}{\rm d}r^2 + r^2{\rm d}\Omega^2$$

$$\delta f \sim \left(\frac{r_s}{r}\right)^6$$

At low-frequency, GWs see yet a different metric…

Regge–Wheeler, Zerilli master mode

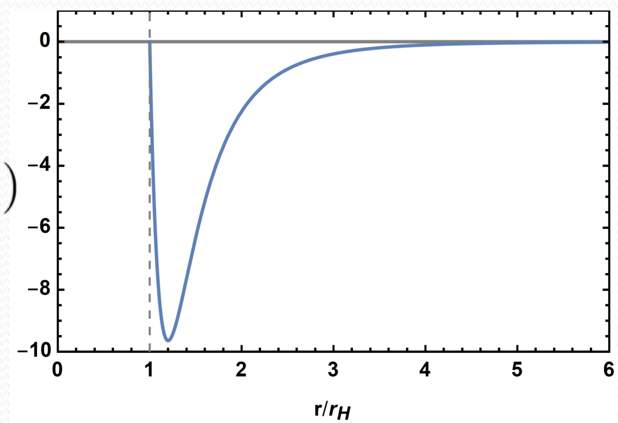
$$g_{\mu\nu} = \left(\gamma_{\mu\nu} + h^{o/e}_{\mu\nu}\right) \mathrm{d}x^\mu \mathrm{d}x^\nu$$

$$h^{o/e} \to \Psi^{\pm}_{\omega\ell}(r) e^{-i\omega t} Y_\ell(\theta)$$ master mode for GWs

In Tortoise coordinates, $\dfrac{\mathrm{d}^2\Psi^\pm}{\mathrm{d}r_*^2} = -\left[\omega^2 - V^\pm_{\mathrm{GR}}(r,\ell) - \epsilon\, \mathcal{C}_3 V^\pm(r,\ell,\omega)\right] \Psi^\pm$

$$\frac{\mathrm{d}^2\Psi^\pm}{\mathrm{d}r_*^2} + \frac{\omega^2}{c_s^2} - \left[V^\pm_{\mathrm{GR}}(r,\ell) + \epsilon\, \mathcal{C}_3 V^\pm(r,\ell)\right] \Psi^\pm = 0$$

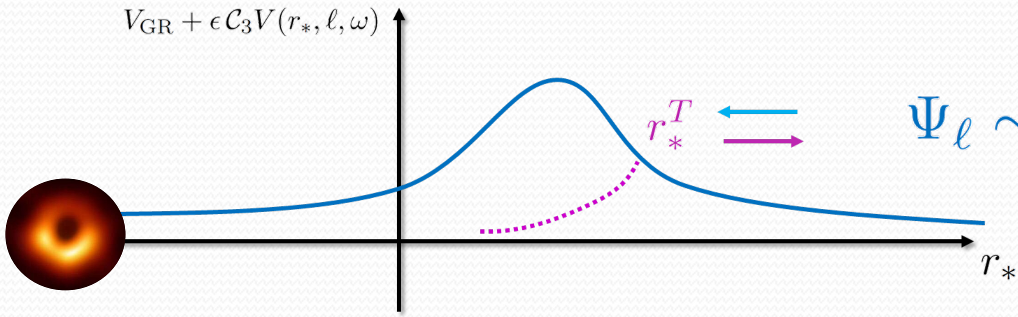
$(c_s - 1)/(\epsilon\, \mathcal{C}_3)$



$$\epsilon = \frac{1}{(m r_s)^4} = \left(\frac{M_{\mathrm{Pl}}^2}{m M_{\mathrm{BH}}}\right)^4 \ll 1$$

Scattering of GWs about a BH

EFT RWZ eq.: $$\frac{d^2\Psi^\pm}{dr_*^2} = -\left[\omega^2 - V_{\rm GR}^\pm(r,\ell) - \epsilon\, \mathcal{C}_3 V^\pm(r,\ell,\omega)\right]\Psi^\pm$$



$$\Psi_\ell \sim e^{2i\delta_\ell} e^{i\omega r_*} - (-1)^\ell e^{-i\omega r_*}$$

Phase Shift: $$\delta_\ell = \int_{r_*^T}^{\infty} dr_* \left(\sqrt{\omega^2 - V_{\rm GR} - \epsilon\, \mathcal{C}_3 V} - \omega\right) - \omega r_*^T + \frac{\pi}{2}\left(\ell + \frac{1}{2}\right)$$

Time Delay: $$T_\ell = 2\frac{\partial \delta_\ell}{\partial \omega} = T_\ell^{\rm GR} + \underbrace{\epsilon\, \mathcal{C}_3 \delta t_\ell^{\rm EFT}}_{\Delta T_\ell^{\rm EFT}} + \mathcal{O}(\epsilon^2)$$

$$\epsilon = \frac{1}{(m r_s)^4} = \left(\frac{M_{\rm Pl}^2}{m M_{\rm BH}}\right)^4 \ll 1$$

Resolvable time-delay/advance

Time Delay: $$T_\ell = 2\frac{\partial \delta_\ell}{\partial \omega} = T_\ell^{\rm GR} + \underbrace{\epsilon\, \mathcal{C}_3 \delta t_\ell^{\rm EFT}}_{\Delta T_\ell^{\rm EFT}} + \mathcal{O}(\epsilon^2)$$

A negative $\Delta T^{\rm EFT} < 0$ would suggest a propagation outside the light-cone as set by the local geometry and as seen by other species (including photons)

Time advance is only meaningful if it can be resolvable i.e. $\Delta T^{\rm EFT} < -\omega^{-1}$

Naively to make time advance resolvable, we could either increase wave frequency or probe higher curvature region, or increase ϵ

Intuitively, there should be some limit to how much we can push the system.
EFT *should break down* at "high-energy" but how do we diagnose this?
Energy is not scalar…

$$k_\mu k^\mu = 0$$

Statement $\omega \ll m$ isn't meaningful here

Validity of the EFT

In GR, $\Box_g h = 0$

In EFT, $\Box_g h = \frac{\mathcal{C}}{m^4}\text{Riem}^2\nabla^2 h + \sum_n \frac{1}{m^{4n}}\text{Riem}^n\nabla^{2(n+1)}h$

EFT valid if $\left|\text{Riem}^n\right| \ll m^{2n}$ and $\left|\left(R^a{}_{bcd}k^b k^d\right)^n\right| \ll m^{4n}$ → $\omega \ll r_s m^2$

We cannot probe the EFT with a particle of arbitrarily high "momentum"

what is meant by that is weighted by curvature

No equivalent constraint on flat ST (and also no SL)

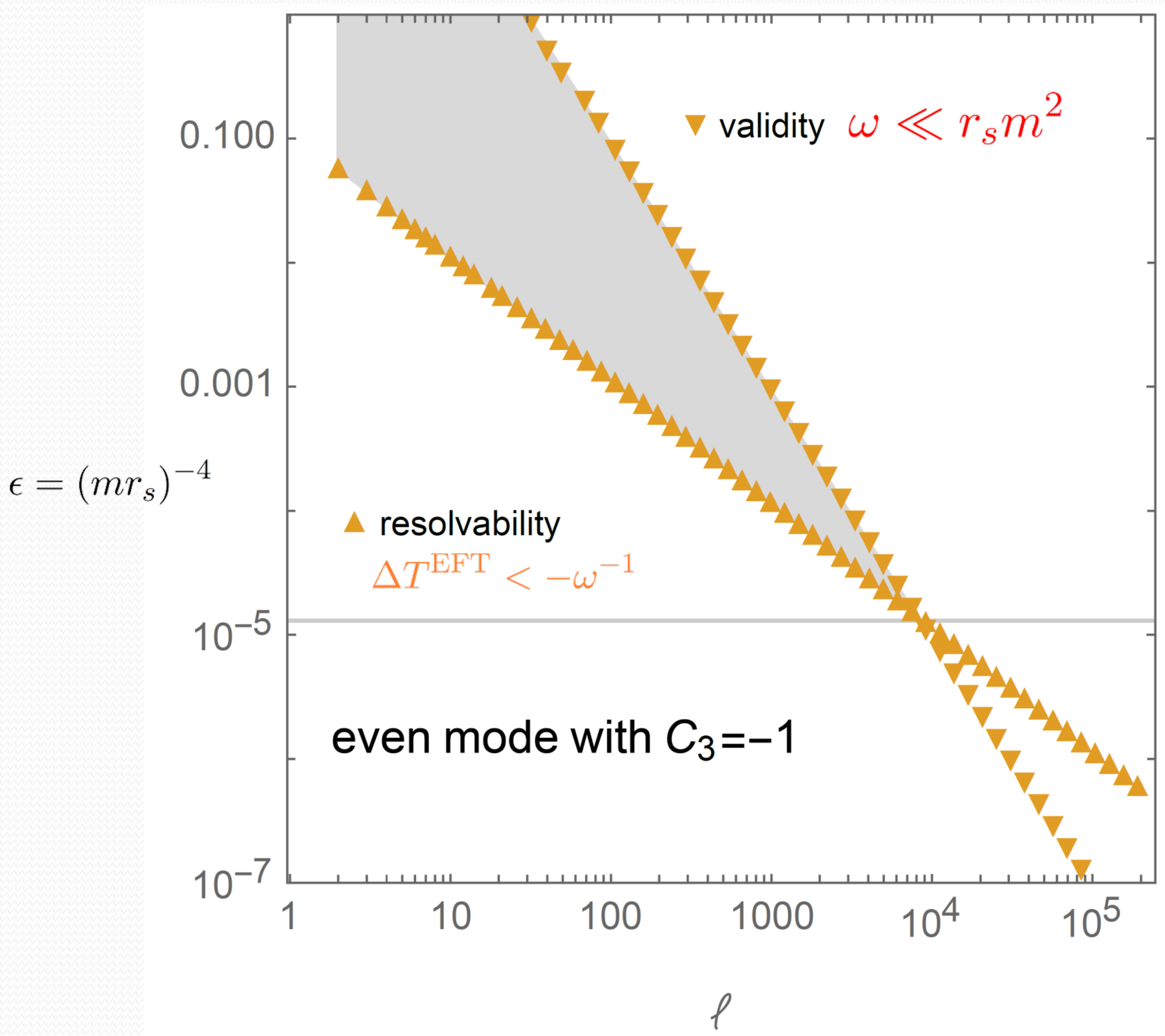
There is a resolvable violation of causality if: $\Delta T^{\text{EFT}} < -\omega^{-1}$

(+ everything else under control) while $\omega \ll r_s m^2$

$\left|\text{Riem}^n\right| \ll m^{2n}$

Resolvable time advance

If $C_3 < 0$, there is an upper limit on ϵ, (lower limit on m) to avoid causality violating situations



Constraint on dim-6 coefficient

If $\mathcal{C}_3 \neq 0,\ \mathcal{C}_3 < 0 \quad \Rightarrow \quad \epsilon = (m r_s)^{-4} < 10^{-5} \quad \Rightarrow \quad m \gtrsim 0.1\, r_s^{-1} \quad \forall$ BH

A priori r_s could be as small as fundamental scale of QG, implying that we can never have $C_3 < 0$ (aside from loop-suppressed effects)

In maximally supersymmetric & heterotic string theories, $C_3 = 0$,

for bosonic string theory, $\mathcal{C}_3 = \frac{1}{6}\left(\frac{\alpha'}{4}\right)^2 > 0$

$C_3 \geq 0$ is consistent with known string theory realizations but more general (does not assume any specific realization, nor string theory)

Predicts that *every "standard" tree-level UV completion must have* $C_3 \geq 0$

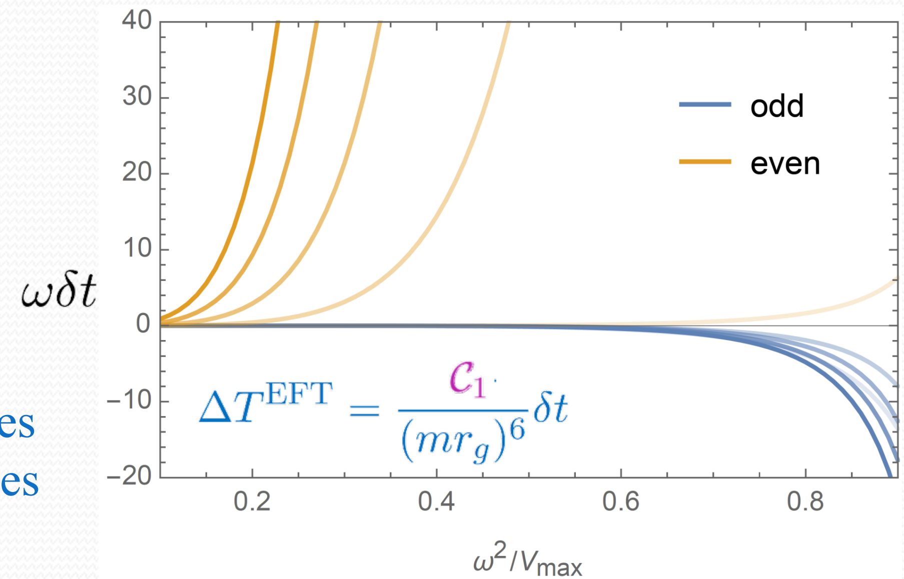
Tolley, Zhang, CdR 2112.05054

Causality of dim-8 Operators

$$\mathcal{L}_{\rm EFT}^{\rm (tree)} = \frac{M_{\rm Pl}^2}{2}\left[R + \mathcal{C}_1 \frac{\left(\mathrm{Riem}^2\right)^2}{m^6} + \mathcal{C}_2 \frac{\left(\mathcal{E}\mathrm{Riem}^2\right)^2}{m^6}\right]$$

If $C_2 = 0$

If $C_1 > 0$, resolvable causality violation for odd modes
If $C_1 < 0$, resolvable causality violation for even modes



Causality implies that if $C_2 = 0$ then $C_1 = 0$ (ie cannot have $C_2 = 0$ and $C_1 \neq 0$)

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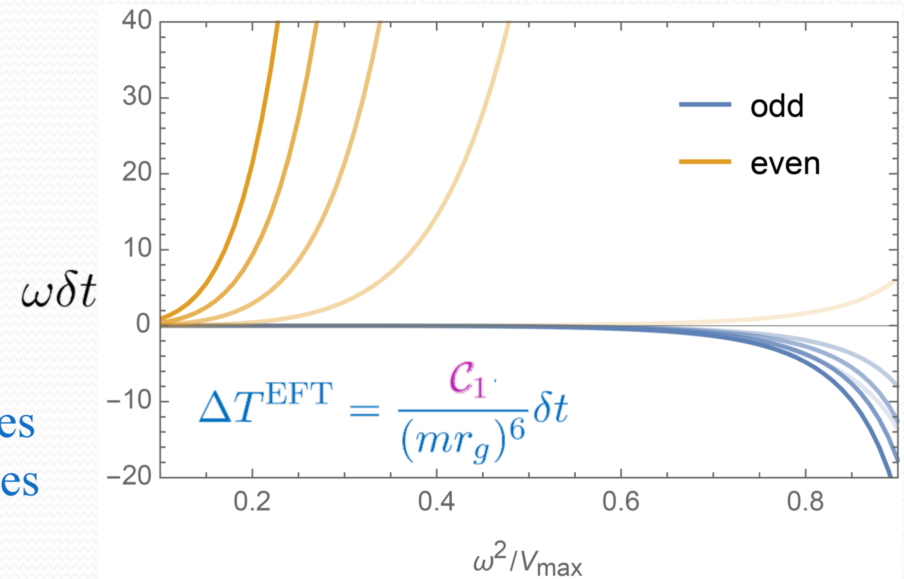


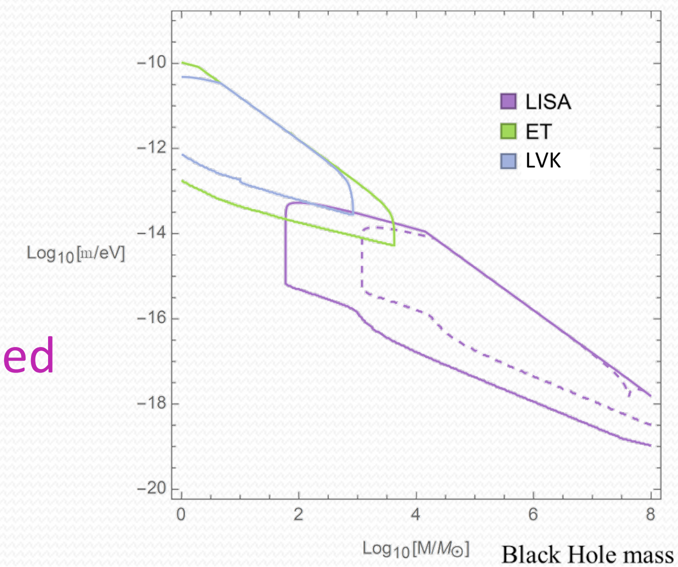
Causality implies that if $C_2 = 0$ then $C_1 = 0$ (ie cannot have $C_2 = 0$ and $C_1 \neq 0$)
$C_1 < 0$ always pathological
If $C_1 > 0$ then $C_2 > 0$

Tolley, Zhang, CdR 2112.05054

consistent with positivity bounds further found in
Caron-Huot, Li, Parra-Martinez & Simmons-Duffin 2201.06602

Summary: EFT of Gravity

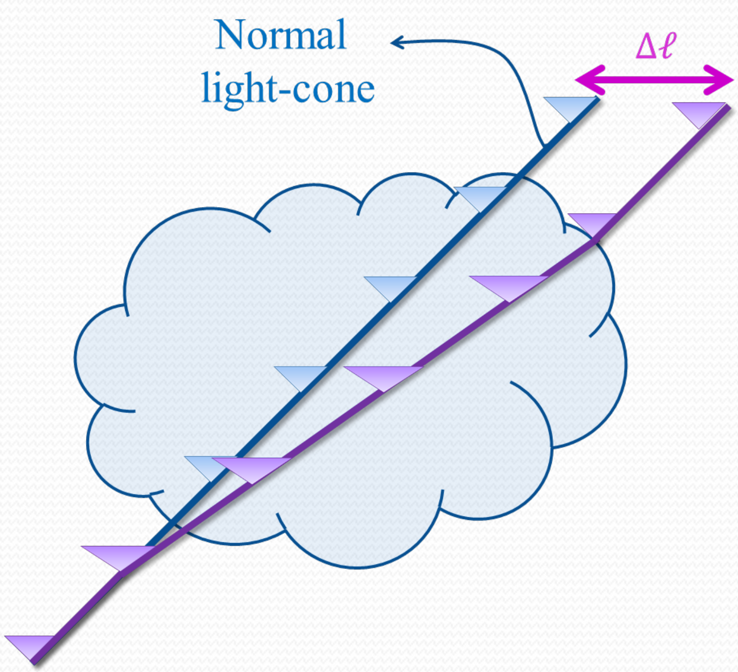
- EFT of gravity provides a systematic way to parameterize our lack of knowledge (in an agnostic way)
- Theoretical constraints complement existing and upcoming observational constraints
- EFT corrections are expected to be extremely suppressed but could potentially provide a window of opportunity to connect with Dark Sector



- For dim-6 operators, can be tight to scale of SUSY breaking in dark sector
- Causality conditions impose constraints on EFT coefficients which are consistent with all known string theory realizations, but are more generic (remain agnostic on precise completion).
- Even when applied to situations at energy scales well-separated from quantum gravity, violation of these constraints would have strong implications for our understanding of high-energy physics

No Gravity

As soon as a "*substance*" allows the tiniest SL, nothing prevents us from stacking it so as to end up with a macroscopic time advance



With Gravity

Anything living on the spacetime *inexorably* **curves** the geometry.
There is a limit to "stacking" leading to a maximal unresolvable time advance

$$\left|\left(R^a{}_{bcd}k^b k^d\right)^n\right| \ll m^{4n}$$

For any low-energy EFT of gravity arising from integrating loops of heavy fields, the amount of superluminality is so small that it can never build up to lead to a violation of causality

$$\left|\Delta T_\ell^{\rm EFT}\right| \ll \sqrt{\frac{r_s}{b}}\omega^{-1} \lesssim \omega^{-1}$$

Unresolvable…

No meaningful propagation outside lightcone

Living with Superluminality & Negativity

- Lesson 1: Imposing subluminality priors only makes sense in a frame where gravity can be decoupled

- Lesson 2: In the frame where matter and gravity can decouple, superluminality is consistent with causality so long as

$$\lim_{M_{\rm Pl}\to\infty} |c_s^2 - 1| \sim M_{\rm Pl}^{-\alpha} \quad \text{with} \quad \alpha \geq 2$$

- Lesson 3: An amount of "allowed" SL is directly connected to a level of "positivity"-violation in gravitational EFTs

$$\mathcal{A}(s,t) \sim -\frac{s^2}{M_{\rm Pl}^2 t} + \frac{\mathcal{C}}{M^4} s^2 + \cdots \quad \Longrightarrow \quad \mathcal{C} > -\frac{M^2}{M_{\rm Pl}^2} \times \mathcal{O}(1)$$

e.g. Causality Constraints on EFTs

Causality Constraints on Corrections to the Graviton Three-Point Coupling

Camanho, Edelstein, Maldacena and Zhiboedov 1407.5597

An example of a theory that is constrained by these considerations is given by the action

$$S = l_p^{2-D} \int d^D x \sqrt{g} \left[R + \alpha \left(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2 \right) \right] + \beta R^3 + \gamma R^4 + \cdots$$

where the second term is the Lanczos-Gauss-Bonnet term.[2] The constant α has dimensions of length squared. For $\alpha \gg l_p^2$, we will show that the theory is not causal. Furthermore, there is no way to make it causal by adding local higher curvature terms. In fact, our

(…) unless we are in weakly coupled string theory in which higher spins enter

In almost all subsequent literature, the lesson learnt seems to have been $\alpha = 0$ and focus on other operators with $\gamma \gg l_p^6$

Is this the right approach???

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If a weakly coupled string theory completion resolves these issues,
HOW does this manifest itself within the low-energy EFT?

The only way it can do so within the low energy EFT
is through the local EFT operators