High precision Gravitational Waves

Storming the GW Frontier

KITP, April 19th, 2022

Radiative contributions to conservative binary dynamics

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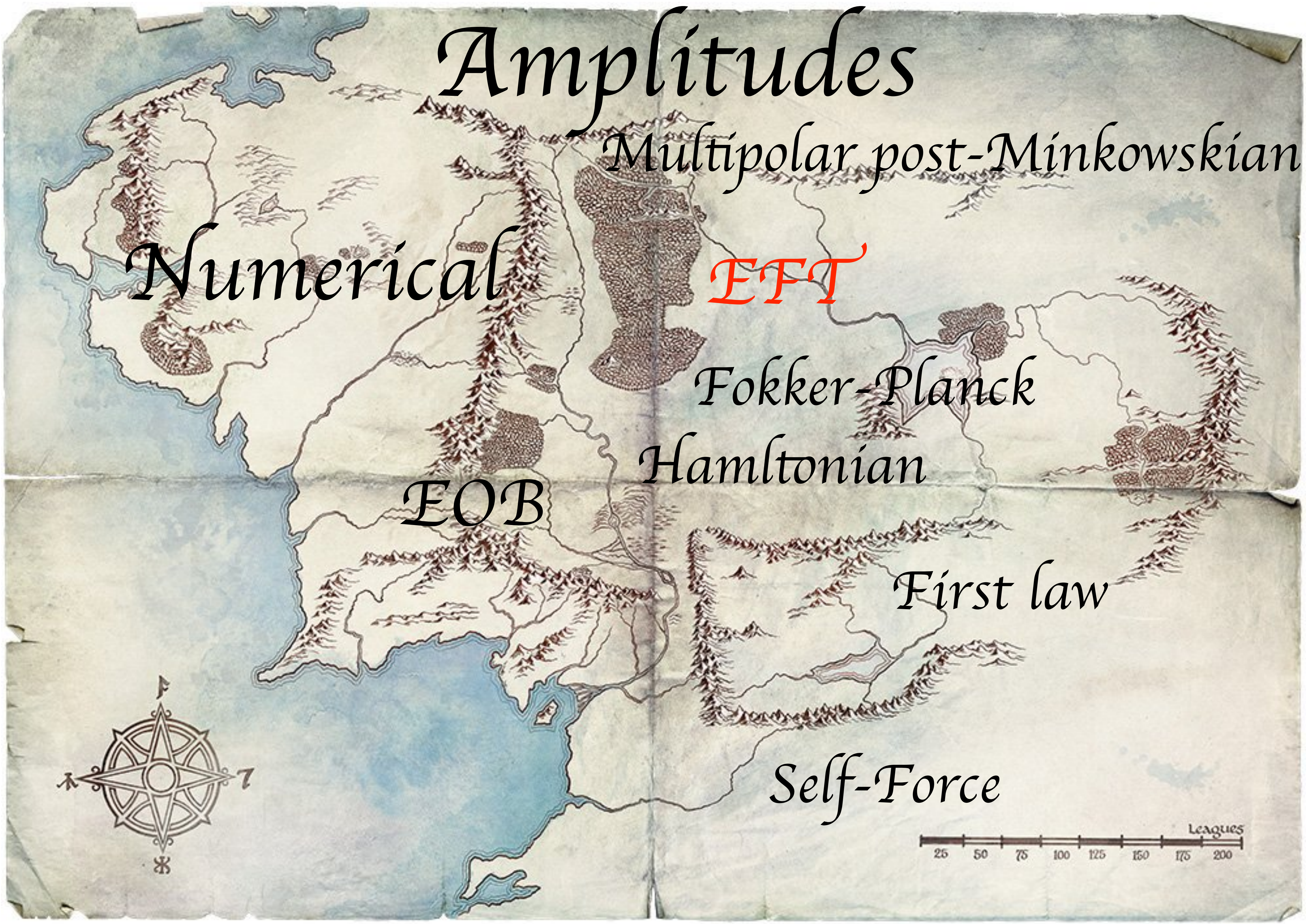
Multipolar post-Minkowskian
Post-Newtonian
Leagues
25 50 75 100 125 150 175 200



Amplitudes
Multipolar post-Minkowskian
Numerical
EFT
Fokker-Planck
Hamltonian
EOB
First law
Self-Force
Leagues
25 50 75 100 125 150 175 200

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5PN conservative:
two *almost* complete derivations

TUTTIFRUTTI

Bini Damour Geralico

$\mathcal{H}_{5PN}$ modulo 2 coefficients

$\mathcal{H}_{6PN}$ modulo 4+2 coefficients

EFT (NRGR)

Blumlein Maier Marquard Schaefer

potential modes

+

SF Sturani

SF Mastrolia Sturani Sturm Torres

radiative contributions*

=

$\mathcal{L}_{5PN}$

*contains fishy memory contributions

Mismatch in scattering angle predictions

Bini Damour Geralico 2107.08896

$$\frac{1}{2}\chi^{\text{cons,tot}}(j,\gamma,\nu) = \sum_{n\geq 1} \frac{\chi_n^{\text{cons,tot}}(\gamma,\nu)}{j^n}$$

at 5PN

$$\chi_{4,5,6} = \chi_{4,5,6}^{Schw} + \nu\chi_{4,5,6}^{\nu} + \nu^2\chi_{4,5,6}^{\nu^2}$$

OK

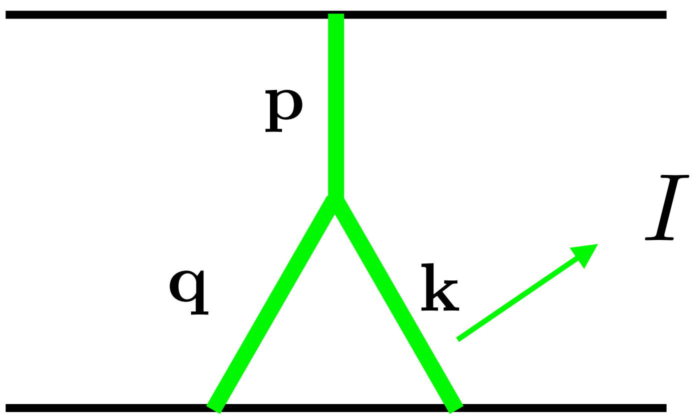
$\chi_{5,6}^{\nu^2}$ depend on the two 5PN undetermined coefficients

mismatch* in $\chi_4^{\nu^2}$

*memory is $\mathcal{O}(\nu^2)$

EFT, method of regions

exact propagators = near + far



$$I = \frac{e^{i\mathbf{k}.\mathbf{x}}}{\mathbf{k}^2 - (k_0 + i\mathrm{a})^2}$$

$$N = \frac{e^{i\mathbf{k}.\mathbf{x}}}{\mathbf{k}^2} \sum \left(\frac{k_0^2}{\mathbf{k}^2} \right)^n$$

$$F = \frac{1}{\mathbf{k}^2 - (k_0 + i\mathrm{a})^2} \sum \frac{(i\mathbf{k}.\mathbf{x})^n}{n!}$$

$I = N$ for $|\mathbf{k}| \gg k_0$

$I = F$ for $|\mathbf{k}| \ll 1/r$

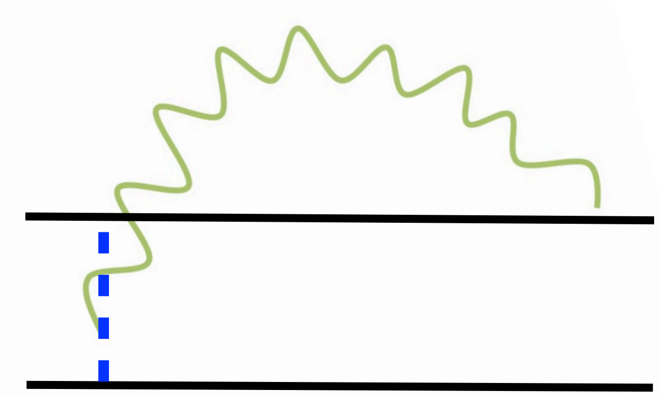
$$\int_{\mathbf{k}} I = \int_{\mathbf{k}} (N + F)$$ + scaleless

scale separation + momentum conservation:

SF Sturani 2103.03190

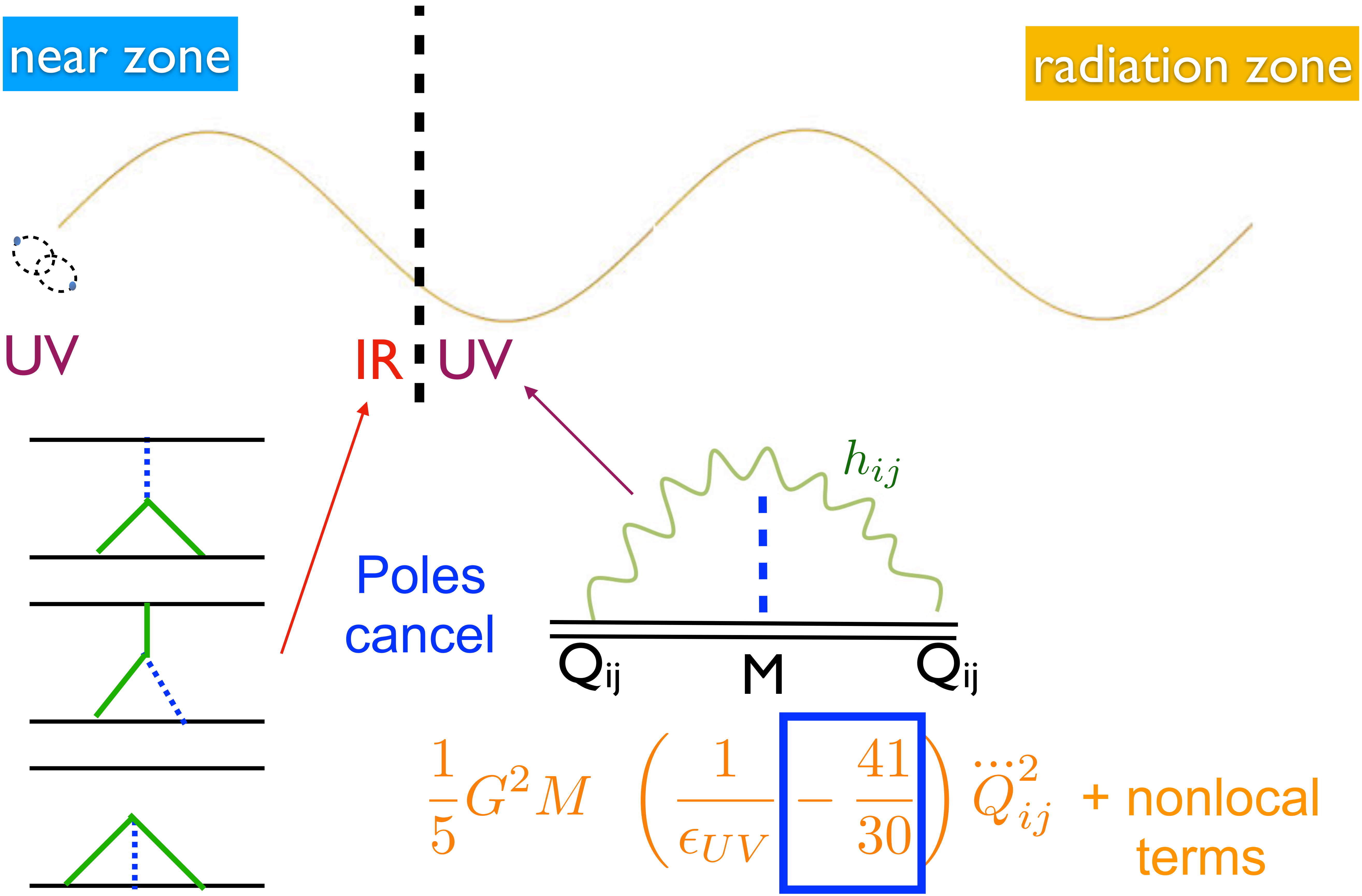
$$\int_{\mathbf{pkq}} I_p I_k I_q = \int_{\mathbf{pkq}} (N_p N_k N_q + F_p F_k F_q)$$

+ scaleless + dissipative term



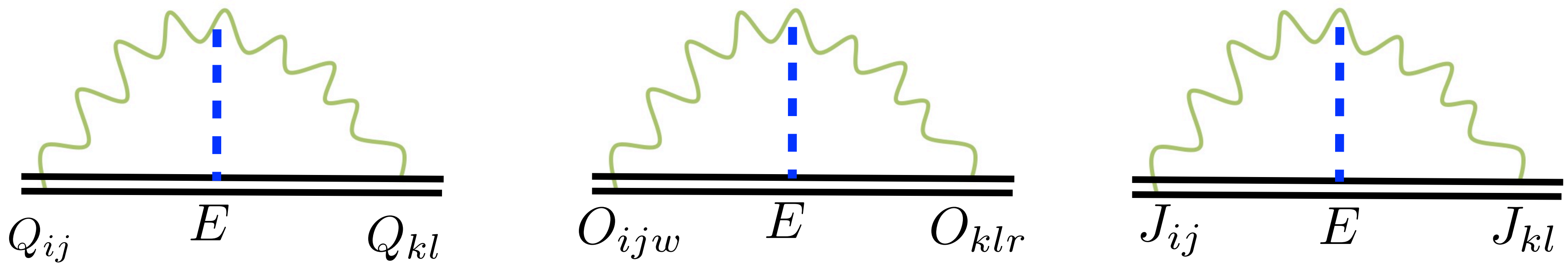
NNF

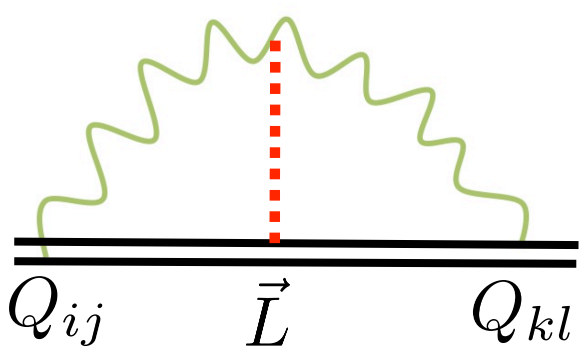
Tails at 4PN....


near zone
radiation zone
UV
IR
UV
Poles cancel
h_{ij}
Q_{ij}
M
Q_{ij}
$\frac{1}{5}G^2M\left(\frac{1}{\epsilon_{UV}}-\frac{41}{30}\right)\dddot{Q}_{ij}^2$ + nonlocal terms

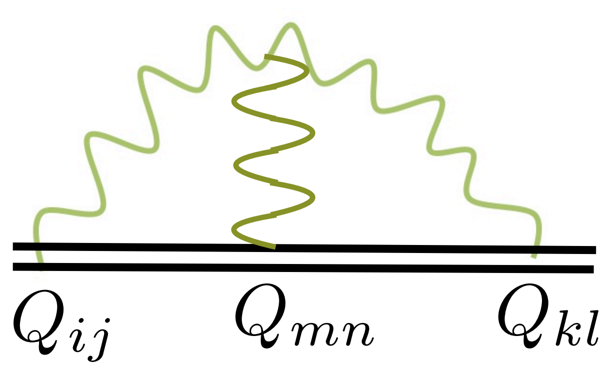
Hereditary terms at 5PN

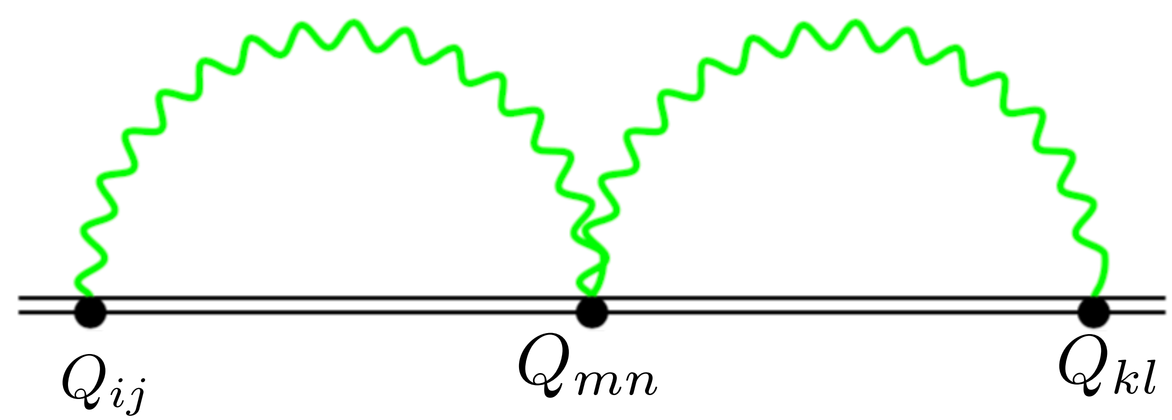
$\mathcal{O}(\nu)$



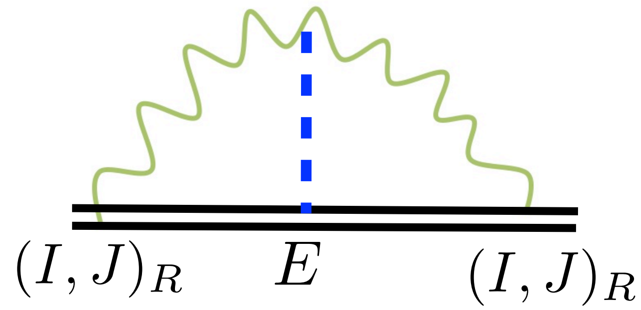


$\mathcal{O}(\nu^2)$





Tails at 4PN….and beyond



Almeida SF Sturani 2110.14146

$$\mathcal{S}^{(e,r)}_{tail} \simeq -G_N^2 E \frac{2^{r+2}(r+3)(r+4)}{(r+1)(r+2)(2r+5)!} \int \frac{\mathrm{d}p_0}{2\pi} (p_0^2)^{r+3} I^{ijR}(p_0) I^{ijR}(-p_0) \left[\frac{1}{\tilde{\varepsilon}} - \gamma_r^{(e)}\right]$$

$$\gamma_r^{(e)} \equiv \frac{1}{2}\left(H_{r+\frac{5}{2}} - H_{\frac{1}{2}} + 2H_r + 1\right) + \frac{2}{(r+2)(r+3)} = \left\{\frac{41}{30}, \frac{82}{35}, \frac{1819}{630}, \ldots\right\}$$

$$\frac{1}{\tilde{\varepsilon}} \equiv \frac{1}{\varepsilon} + \log\left(\frac{p_0^2 e^{\gamma}}{\pi \mu^2}\right)$$

$$\mathcal{S}^{(m,r)}_{tail} \simeq -G_N^2 E \frac{2^{r+4}(r+2)(r+4)}{(r+1)(r+3)^2(2r+5)!} \times \int \frac{\mathrm{d}p_0}{2\pi} (p_0^2)^{r+3} J_{l|jRk}(p_0) J_{n|aRk}(-p_0) \left[\delta_{ja}\delta_{ln} + (r+1)\delta_{jn}\delta_{la}\right] \left[\frac{1}{\tilde{\varepsilon}} - \gamma_r^{(m)}\right]$$

$$\gamma_r^{(m)} \equiv \frac{1}{2}\left(H_{r+\frac{5}{2}} - H_{\frac{1}{2}} + 2H_{r+3} + 2\right) - \frac{2r^2 + 13r + 22}{(r+2)(r+3)(r+4)} = \left\{\frac{49}{20}, \frac{22}{7}, \frac{4541}{1260}, \ldots\right\}$$

3-dim vs. d-dim magnetic quadrupole

Henry, Faye, Blanchet 2105.10876

$$J_{l|jk} \underset{d\to 3}{\longrightarrow} \epsilon^{ilk} J_{ij}$$

SF Sturani 1907.02869

$$-\frac{16G_N^2E}{135}\int \mathrm{d}t \left(\dddot{J}_{ij}\right)\left(d\delta_{ai}\delta_{bj}-\delta_{ab}\delta_{ij}\right)\left(\dddot{J}_{ab}\right)\times\left(\frac{1}{\tilde{\varepsilon}}-\frac{49}{20}\right) = -\frac{16G_N^2E}{45}\int \mathrm{d}t\left(\frac{1}{\tilde{\varepsilon}}-\frac{127}{60}\right)\left(\dddot{J}_{ij}\right)^2$$

$\mathcal{S}_{tail}+\mathcal{S}_{near}$ is finite so in principle one can use either definitions *provided* being consistent on both regions

But in the computation of $\mathcal{S}_{near}$ it is more natural **not to** introduce ϵ_{ijk}

In-in formalism

(irrelevant for potential modes *and tails*)

Galley Tiglio 0903.1122

$$e^{iS_{eff}[x]} = \int \mathcal{D}[h] e^{iS_{tot}[x,h]}$$

$$S_{tot}[x,h] \to S_{tot}[x_1,h_1] - S_{tot}[x_2,h_2]$$

$$x_- = x_1 - x_2$$
$$x_+ = \frac{1}{2}(x_1 + x_2)$$
$$\langle h_- h_+ \rangle = G_{ret}$$

$$\left.\frac{\delta S_{eff}[x_\pm]}{\delta x_-}\right|_{x_-=0} = 0$$

if $S_{eff}[x_\pm] = S_{eff}[x_1] - S_{eff}[x_2]$

the dynamics is conservative

and one can avoid using Keldish variables

Tails at 4PN revisited

Galley Leibovich Porto Ross 1511.07379

$$W_{\rm tail}[\boldsymbol{x}_a^{\pm}] = \frac{2G_N^2 M}{5}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega^6\, I_-^{ij}(-\omega) I_+^{ij}(\omega)\left[-\frac{1}{(d-4)_{\rm UV}} - \gamma_E + \log\pi - \log\frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi\,{\rm sign}(\omega)\right]. \qquad (3.3)$$

$$\omega^6\, I_-^{ij}(-\omega) I_+^{ij}(\omega) \to \dddot{I}_1^{ij}(t)\dddot{I}_1^{ij}(t) - \dddot{I}_2(t)^{ij}\dddot{I}_2^{ij}(t)$$

conservative part same as **SF Sturani 1111.5488**

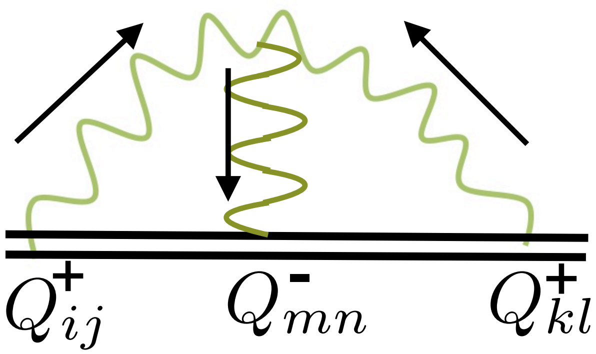
$$\frac{1}{5}G^2 M^2\left(\frac{1}{\epsilon_{UV}} - \frac{41}{30}\right)\dddot{Q}_{ij}^2$$ + nonlocal terms

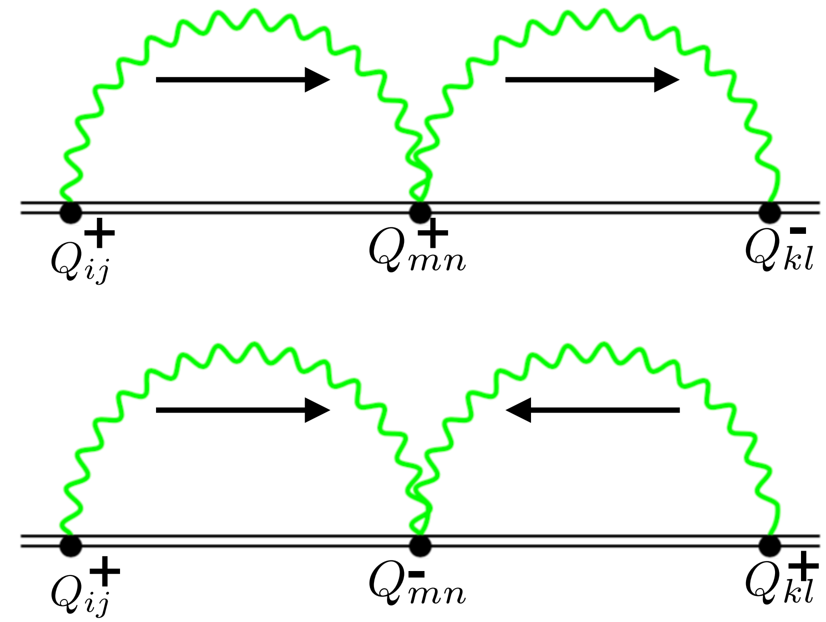
$$\left(\boldsymbol{a}_a^j\right)_{\rm diss}(t) = -\frac{4G_N^2 M}{5}\,\boldsymbol{x}_a^i(t)\ {\rm PV}\int_{-\infty}^{\infty} dt'\, I^{ij(6)}(t')\left[\frac{1}{t-t'}\right]. \qquad (3.9)$$

dissipative non-local radiation reaction

Memory and double emission

finite and local-in-time





$$\frac{1}{35}\int \left[8\left(\dddot{Q}^+\right)^2\ddot{Q}^- + 7\left(\ddddot{Q}^+\right)^2 Q^- - 12\dddot{Q}^+\ddot{Q}^+\dddot{Q}^- - 14\ddddot{Q}^+ Q^+\dddot{Q}^-\right]$$

as in **Blumlein Maier Marquard Schaefer 2110.13822**

$$\int_t \left[Q_-^{(4)}Q_+^{(4)}Q_+ - \frac{1}{2}Q_+^{(4)}Q_+^{(4)}Q_-\right]$$

but this is **not** in the form

$$\alpha\int_t [(\ddddot{Q}_{ik}\ddddot{Q}_{jl}Q_{kl})_1 - (\ddddot{Q}_{ik}\ddddot{Q}_{jl}Q_{kl})_2] + \beta\int_t \left[\left(\dddot{Q}_{ik}\dddot{Q}_{jl}\ddot{Q}_{kl}\right)_1 - \left(\dddot{Q}_{ik}\dddot{Q}_{jl}\ddot{Q}_{kl}\right)_2\right]$$

presence of dissipative contributions

ambiguity in definition of conservative part attempt to disentangle conservative and dissipative dynamics

$$\left.\frac{\delta S_{eff}[x_\pm]}{\delta x_-}\right|_{x_-=0} = 0$$

$$\mathbf{a}_a^k = G_N^2 \mathbf{x}_a^i \left[\bar{\alpha}_8^{c,d} Q_{ij}{}^{(8)} Q_{jk} + \bar{\alpha}_7^{c,d} Q_{ij}{}^{(7)} \dot{Q}_{jk} + \bar{\alpha}_6^{c,d} Q_{ij}{}^{(6)} \ddot{Q}_{jk} + \bar{\alpha}_5^{c,d} Q_{ij}{}^{(5)} \dddot{Q}_{jk} + \bar{\alpha}_4^{c,d} \ddddot{Q}_{ij} \dddot{Q}_{jk} + (ij) \leftrightarrow (jk)\right]$$
$$-\frac{2}{3} G_N \mathbf{x}_a^k \left(\bar{\alpha}_8^{c,d} Q_{ij}{}^{(8)} Q_{ij} + \bar{\alpha}_7^{c,d} Q_{ij}{}^{(7)} \dot{Q}_{ij} + \bar{\alpha}_6^{c,d} Q_{ij}{}^{(6)} \ddot{Q}_{ij} + \bar{\alpha}_5^{c,d} Q_{ij}{}^{(5)} \dddot{Q}_{ij} + \bar{\alpha}_4^{c,d} \ddddot{Q}_{ij} \ddddot{Q}_{ij}\right)$$

energy balance equation → Flux + Schott term

$$\dot{M} = -\sum_a m_a \mathbf{v}_a \cdot \mathbf{a}_a = \mathcal{F} + \dot{E}_{Schott}$$
$$\equiv G_N^2 \dot{Q}_{ik} \left(-\frac{6}{5} Q_{ij}{}^{(8)} Q_{jk} - \frac{24}{5} Q_{ij}{}^{(7)} \dot{Q}_{jk} - 8\, Q_{ij}{}^{(6)} \ddot{Q}_{jk} - 8\, Q_{ij}{}^{(5)} \dddot{Q}_{jk} - \frac{36}{5} \ddddot{Q}_{ij} \ddddot{Q}_{jk}\right)$$

incompatible with

Arun Blanchet Iyer Qusailah 0711.0302

$$\mathcal{F} = \frac{2}{5} Q_{ik}^{(3)} \left(\frac{1}{7} Q_{ij}^{(6)} Q_{jk} - \frac{4}{7} Q_{ij}^{(5)} Q_{jk}^{(1)} - Q_{ij}^{(4)} Q_{jk}^{(2)} - \frac{2}{7} Q_{ij}^{(3)} Q_{jk}^{(3)}\right)$$

…to be continued…

Conclusions, so far

Radiative terms
give unavoidable contributions
to conservative binary dynamics starting from 4PN

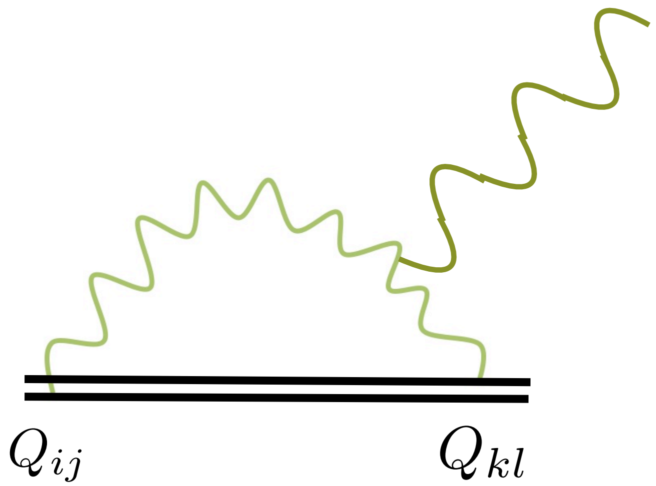
Tail terms are well understood at all PN's

We cannot claim the same
about memory and double emission terms,
which are relevant at 5PN

If we understand this
(and deal with some new integrals)
the road is paved to go 6PN or even 7PN

…and also incompatible with

$h^{TT} \propto$



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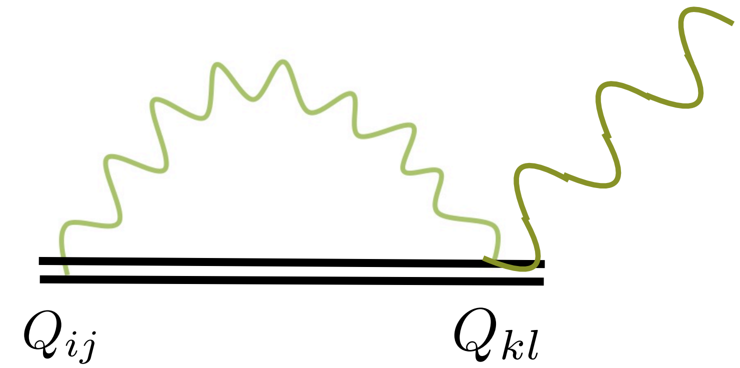

and

$$\mathcal{F} = r^2 \int \mathrm{d}\Omega \dot{h}_{ij}^{TT} \dot{h}_{ij}^{TT}$$