Solving Dissipative Dynamics from Poincare Invariance and Soft theorems

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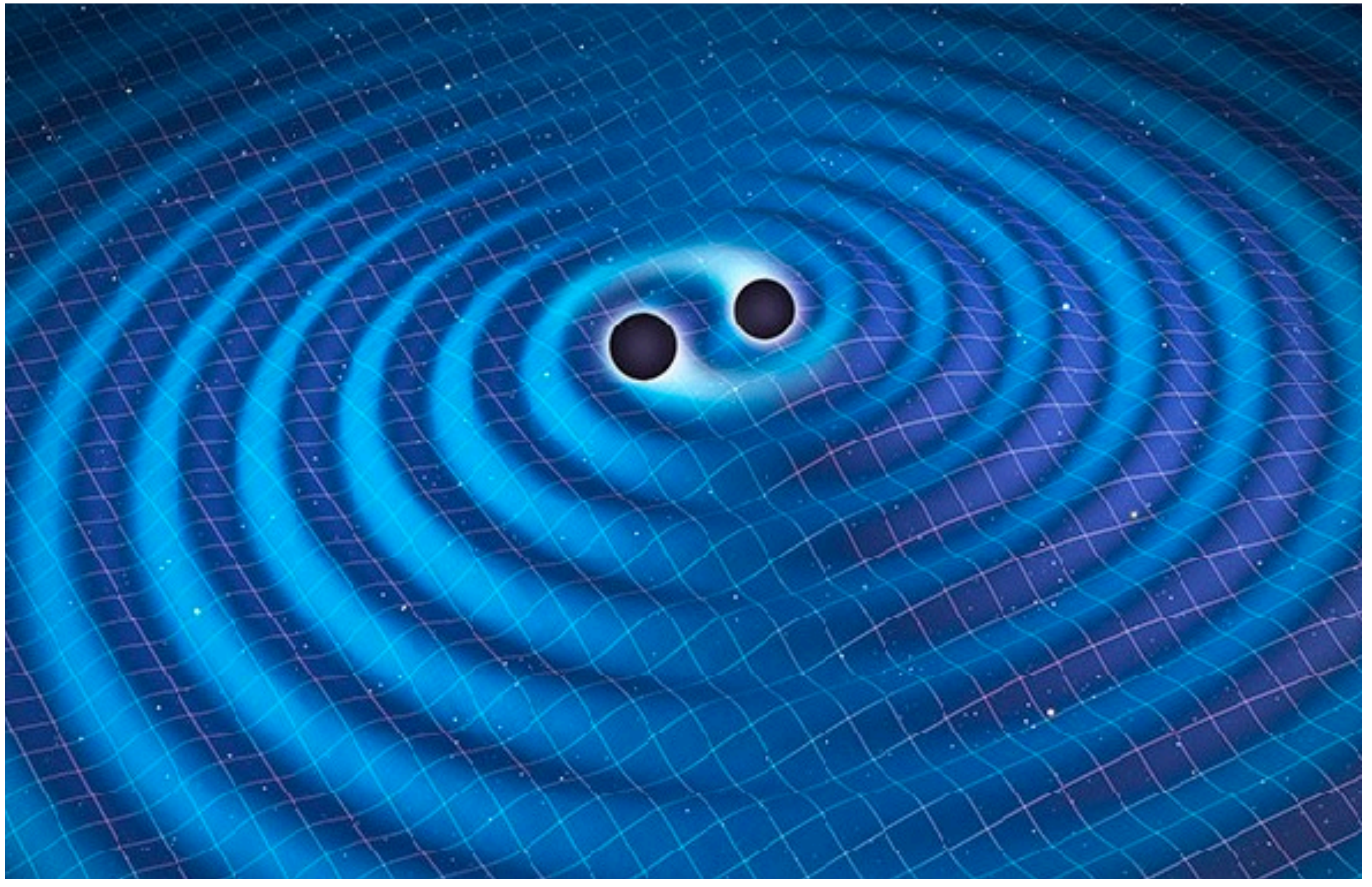
based on 2203.04283 with Manohar and Ridgway

“Can we ***analytically*** *solve the* ***full*** *binary dynamics in* ***perturbation*** *theory, or even* ***nonperturbatively****?”*



Symmetry $\longrightarrow$ Dynamics

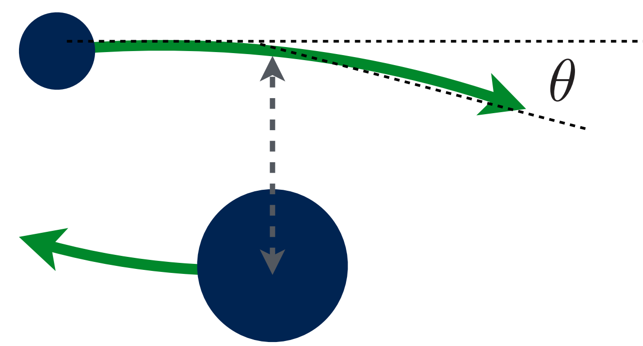
$$V(r, \vec{p}^2, \vec{p} \cdot \hat{r})$$

Effective Field Theory

[Goldberger, Rothstein '04]

Symmetry $\longrightarrow$ Conservative Dynamics

Lorentz invariance



$$\theta(J, E) \qquad V(r, \vec{p}^{\,2}, \cancel{\vec{p}\cdot\hat{r}})$$

Effective Field Theory

[Goldberger, Rothstein ’04]

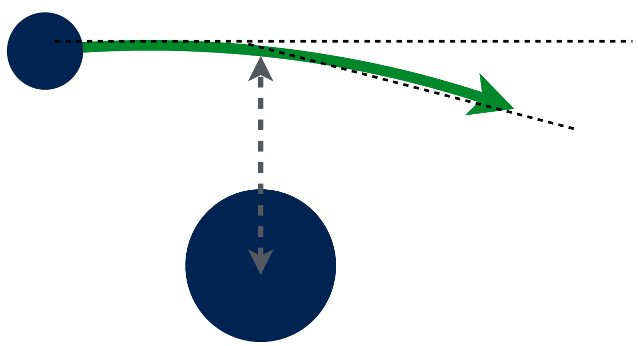
[Buonanno, Damour] [Damour]

[Neill, Rothstein] [Cheung, Rothstein, Solon]

Symmetry $\longrightarrow$ **Conservative Dynamics**

Lorentz invariance

$$\gamma = \sigma \equiv \frac{p_1 \cdot p_2}{m_1 m_2}$$



$$\theta(J, E)$$

$$V(r, \vec{p}^{\,2})$$

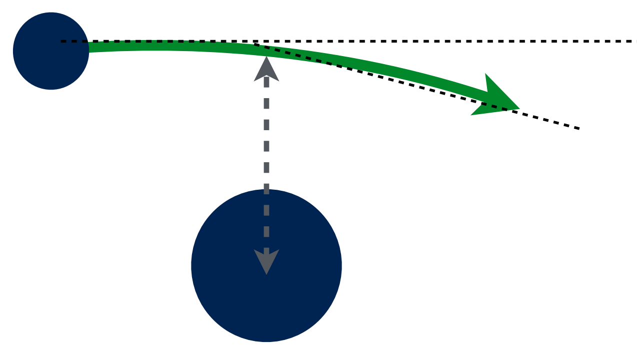
[Buonanno, Damour] [Damour]

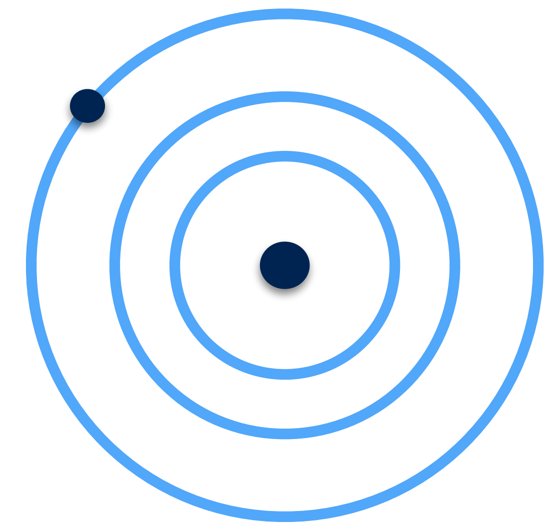
[Neill, Rothstein] [Cheung, Rothstein, Solon]

Symmetry $\longrightarrow$ Conservative Dynamics

$$\gamma = \sigma \equiv \frac{p_1 \cdot p_2}{m_1 m_2}$$

Lorentz invariance





$\theta(J, E)$ $V(r, \vec{p}^{\,2})$

Scattering Amplitudes

Generalized unitarity [Bern, Dixon, Dunbar, Kosower]…

GR=YM² [Kawai, Lewellen, Tye][Bern, Carrasco, Johansson]…

Effective Field Theory

[Goldberger, Rothstein ’04]

See Snowmass white paper [Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng, 2204.05194]
and many other talks in this program

Conservative Dynamics

- Impressive progress from both traditional and new methods
- Higher order potential, spin, tidal effects

[Bini, Damour, Geralco]
[Blumlein, Maier, Marquard, Schafer]
[Foffa, Sturani, +Mastrolia, Strum, Torres]

0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
$(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\, v^{10}$	$+\, v^{12}$	$+\, v^{14}$	$+\, \ldots)$	G^1
	$(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\, v^{10}$	$+\, v^{12}$	$+\, \ldots)$	G^2
		$(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\, v^{10}$	$+\, \ldots)$	G^3
			$(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\, \ldots)$	G^4
				$(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, \ldots)$	G^5
					$(1$	$+\, v^2$	$+\, v^4$	$+\, \ldots)$	G^6

[Bern, Cheung, Roiban, **CHS**, Solon, Zeng '19]

[Bern, Parra-Martinez, Roiban, Ruf, **CHS**, Solon, Zeng '21]
[Dlapa, Kalin, Liu, Porto, '21]

All orders in velocity at G^3 and G^4

Symmetry $\longrightarrow$ Dissipative Dynamics

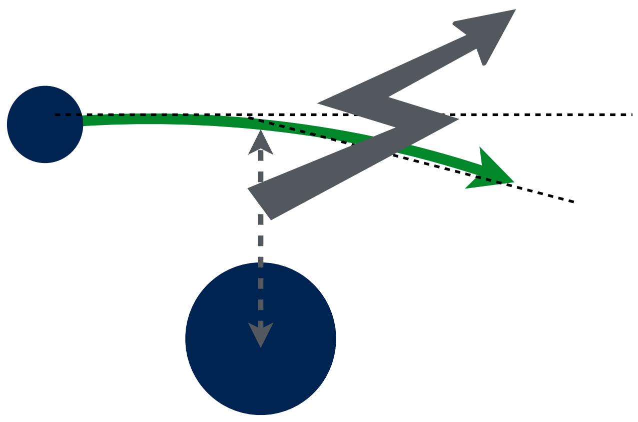
Starts at 2.5PN!!

State of the art: (partial) 4.5PN

$$F_{\rm RR}(r, \vec{p}^{\,2}, \vec{p}\cdot\hat{r})$$

[Burke, Throne '69]

Symmetry → Dissipative Dynamics



[Kovacs, Throne ’77]

[Goldberger, Ridgway ’16]…

See Snowmass white paper [Buonanno, Khalil, O'Connell, Roiban, Solon, 2204.05194]
and talks by Gralla, Heissenberg, Moynihan, Parra-Martinez, Porto, Mogull

Dissipative Dynamics in Scattering

- Double copy structure
  [Goldberger, Ridgway] [**CHS**] [Vazquez-Holm, Carrasco]…
- Radiative contribution to binary deflections
  [Amati, Ciafaloni, Veneziano][Bini, Damour] [Damour] [Bini, Damour, Geralico]
  [Di Vecchia, Heissenberg, Russo, Veneziano][Herrmann, Parra-Martinez, Ruf, Zeng]…
- Radiated energy via classical or quantum (KMOC) methods
  [Herrmann, Parra-Martinez, Ruf, Zeng]
  [Jakobsen, Mogull, Plefka, Steinhoff] [Mougiakakos, Riva, Vernizzi]
- Radiated angular momentum
  [Damour][Bini, Damour, Geralico] [Di Vecchia, Heissenberg, Russo, Veneziano]
  [Jakobsen, Mogull, Plefka, Steinhoff][Mougiakakos, Riva, Vernizzi][Gralla, Lobo]
- Waveforms
  [Jakobsen, Mogull, Plefka, Steinhoff][Mougiakakos, Riva, Vernizzi]
  [Cristofoli, Gonzo, Kosower, O'Connell] [Britto, Gonzo, Jehu] [Cristofoli, Gonzo, Moynihan, O'Connell, Ross]
- Boundary to bound map [Cho, Kalin, Porto]

Dissipative Dynamics in Scattering

- Double copy structure
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- Radiative contribution to binary deflections
  [Amati, Ciafaloni, Veneziano][Bini, Damour] [Damour] [Bini, Damour, Geralico]
  [di Vecchia, Heissenberg, Russo, Veneziano][Herrmann, Parra-Martinez, Ruf, Zeng]…

What can we learn about bounded binaries?

- Radiated energy
  [Herrmann, Parra-Martinez, Ruf, Zeng]
  [Jakobsen, Mogull, Plefka, Steinhoff] [Mougiakakos, Riva, Vernizzi]
- Radiated angular momentum
  [Damour][Bini, Damour, Geralico]
  [Jakobsen, Mogull, Plefka, Steinhoff][Mougiakakos, Riva, Vernizzi][Gralla, Lobo]

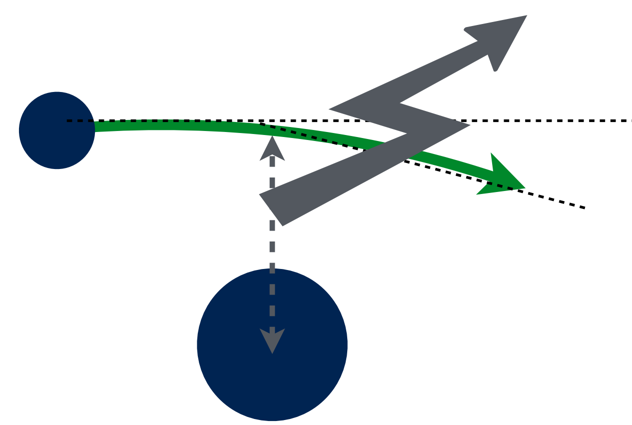
Can we leverage symmetries again?

- Waveform
  [Jakobsen, Mogull, Plefka, Steinhoff][Mougiakakos, Riva, Vernizzi]
  [Cristofoli, Gonzo, Kosower, O'Connell] [Britto, Gonzo, Jehu] [Cristofoli, Gonzo, Moynihan, O'Connell, Ross]

Pioneering idea: [Iyer, Will]

Symmetry $\longrightarrow$ Dissipative Dynamics

Poincare invariance



E, J

$F_{\rm RR}(r, \vec{p}^{\,2})$

[Manohar, Ridgway, **CHS**, 2203.04283]

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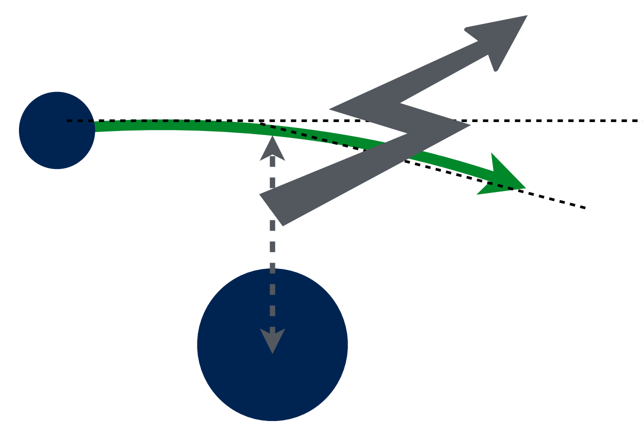
Symmetry $\longrightarrow$ Dissipative Dynamics

E, J



Poincare invariance

$F_{\mathrm{RR}}(r, \vec{p}^{\,2})$

2.5PN **3.5PN** **4.5PN**

$$(v^3 + v^5 + v^7 + v^9 + \ldots)\ G^2$$
$$(v + v^3 + v^5 + v^7 + \ldots)\ G^3$$
$$(v + v^3 + v^5 + \ldots)\ G^4$$

How to calculate radiated angular momentum?

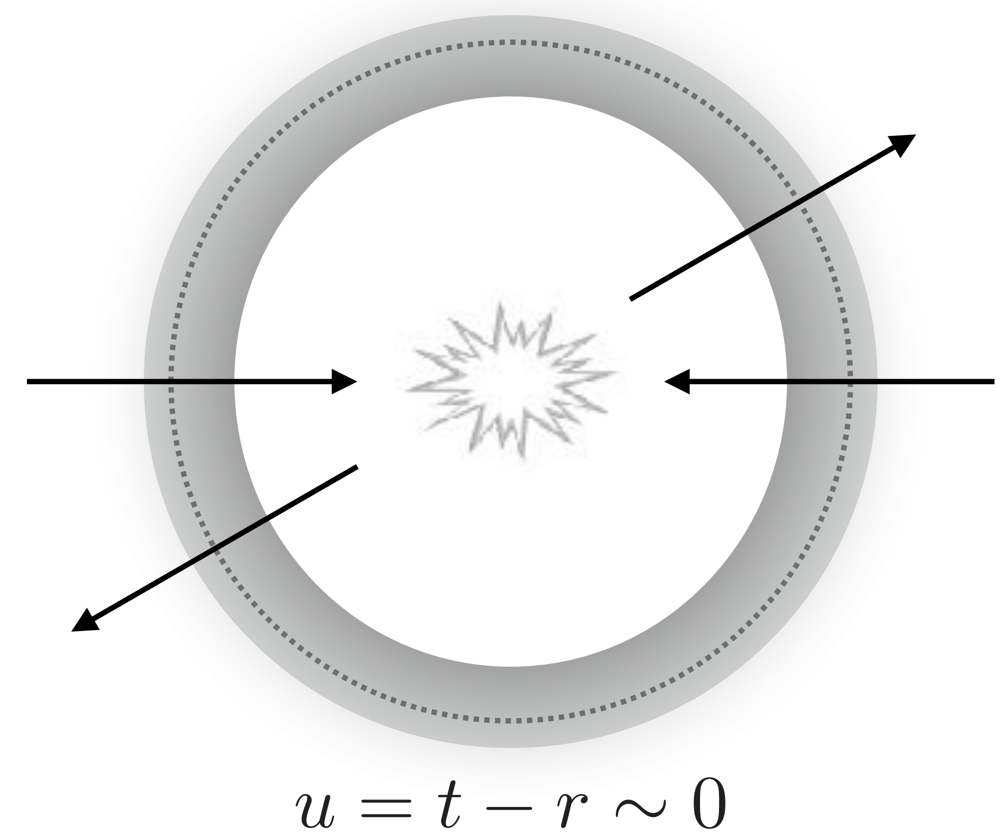
*We do not intend to resolve the BMS subtlety. But please ask during the discussion.

Radiated Poincare Charges

- Consider the final state of scattering.
  The radiated linear and angular momentum are

$$P^\mu = \int \mathrm{d}^3 x \, T^{\mu 0}$$

$$J^{\mu\nu} = \int \mathrm{d}^3 x \, x^{[\mu} T^{\nu]0}$$



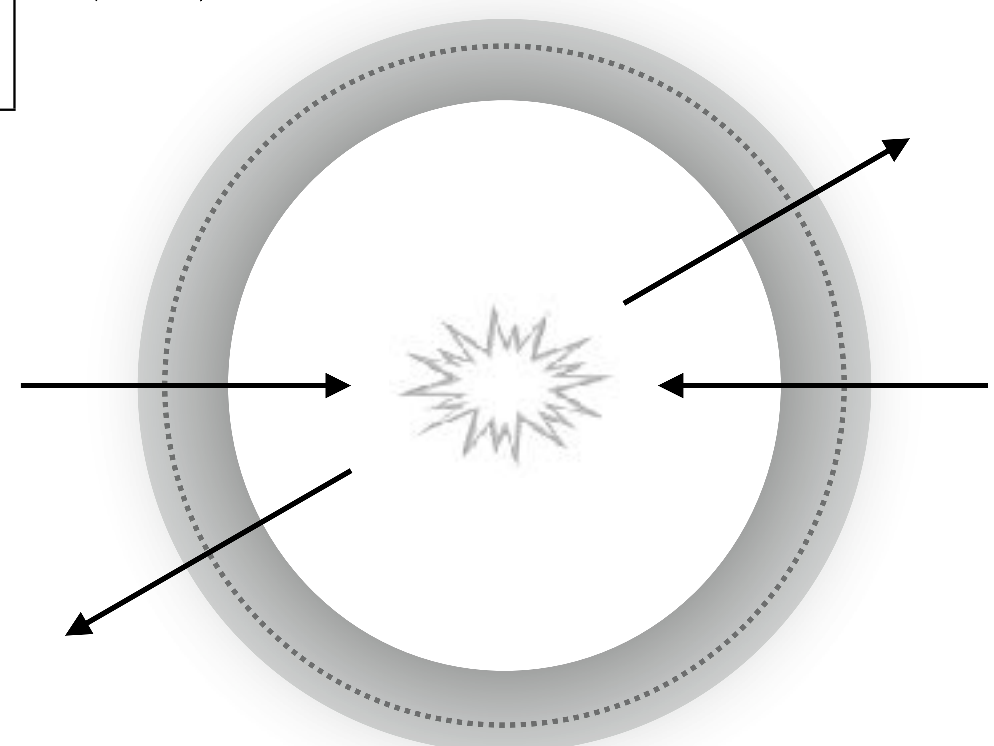
Radiated Poincare Charges

- Textbook formula for angular momentum

$$J^i = \frac{c^2}{32\pi G} \int d^3x \left[-\epsilon^{ikl} \dot{h}^{\mathrm{TT}}_{ab} x^k \partial^l h^{\mathrm{TT}}_{ab} + 2\epsilon^{ikl} h^{\mathrm{TT}}_{ak} \dot{h}^{\mathrm{TT}}_{al} \right] . \quad (2.51)$$

How to see gauge invariance?

Why not covariant?



$u = t - r \sim 0$

Radiated Poincare Charges

- Radiation in momentum space

$$A_\mu(x) = \int \widetilde{dk}\left(P_{\mu\nu}\,\mathcal{J}^\nu(k)\,e^{-ik\cdot x} + \text{c.c.}\right),$$

$$h_{\mu\nu}(x) = \sqrt{8\pi G}\int \widetilde{\mathrm{d}k}\left(P_{\mu\nu\rho\sigma}\,\mathcal{T}^{\rho\sigma}(k)\,e^{-ik\cdot x} + \text{c.c.}\right)$$

Lorentz-invariant phase space

sources (current or stress-energy pseudotensor)

gauge-dependent tensor

$$P_{\mu\nu} = \eta_{\mu\nu}$$

$$P_{\mu\nu\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

- Do not impose standard transverse projection for the Coulomb mode (free-particle waveform)

- Do not use creation/annihilation operators for the same reason

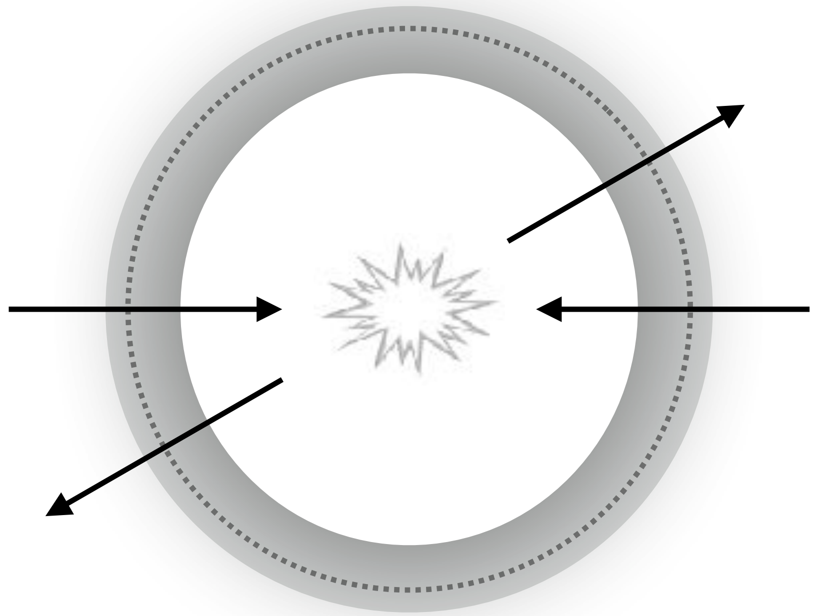
Radiated Poincare Charges

- Linear momentum

$$h_{\mu\nu}(x) = \sqrt{8\pi G}\int \widetilde{\mathrm{d}k}\,\left(P_{\mu\nu\rho\sigma}\,\mathcal{T}^{\rho\sigma}(k)\,e^{-ik\cdot x} + \text{c.c.}\right) \longrightarrow P^\mu = \int \mathrm{d}^3x\,T^{\mu 0}$$

$$P^\mu = 8\pi G \int \widetilde{\mathrm{d}k}\,k^\mu \left(\mathcal{T}^{*\rho\sigma}(k)\mathcal{T}_{\rho\sigma}(k) - \frac{\mathcal{T}^{*\rho}_{\rho}(k)\mathcal{T}^{\sigma}_{\sigma}(k)}{D-2}\right)$$

Phase space integral momentum polarization sum



$u = t - r \sim 0$

Radiated Poincare Charges

- New formula in GR for radiated angular momentum

$$P^\mu = 8\pi G \int \widetilde{dk}\, k^\mu \left(\mathcal{T}^{*\rho\sigma}(k)\mathcal{T}_{\rho\sigma}(k) - \frac{\mathcal{T}_\rho^{*\rho}(k)\mathcal{T}_\sigma^{\sigma}(k)}{D-2} \right), \qquad \mathcal{L}^{\mu\nu} = ik^\mu \frac{\partial}{\partial k_\nu} - ik^\nu \frac{\partial}{\partial k_\mu}$$

$$J^{\mu\nu} = 8\pi G \int \widetilde{dk} \left(\mathcal{T}^{*\rho\sigma}(k)\mathcal{L}^{\mu\nu}\mathcal{T}_{\rho\sigma}(k) - \frac{\mathcal{T}_\rho^{*\rho}(k)\mathcal{L}^{\mu\nu}\mathcal{T}_\sigma^{\sigma}(k)}{D-2} + 2i\, \mathcal{T}^{*\rho[\mu}(k)\mathcal{T}^{\nu]}{}_\rho(k) \right)$$

[Manohar, Ridgway, **CHS**]

- Fully covariant
- Time dependence and gauge choice cancel

[Manohar, Ridgway, **CHS**]

Radiated Poincare Charges

- New formula in GR for radiated angular momentum

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- Poincare algebra:

$$x^{\mu} \to x^{\mu} + a^{\mu} \qquad\qquad P^{\mu} \to P^{\mu}$$

$$\mathcal{T}^{\mu\nu}(k) \to \mathcal{T}^{\mu\nu}(k)e^{ik\cdot a} \quad \longrightarrow \quad J^{\mu\nu} \to J^{\mu\nu} + a^{[\mu}P^{\nu]}$$

[Manohar, Ridgway, **CHS**]

Radiated Poincare Charges

- New formula in GR for radiated angular momentum

$$P^\mu = 8\pi G \int \widetilde{dk}\, k^\mu \left(\mathcal{T}^{*\rho\sigma}(k)\mathcal{T}_{\rho\sigma}(k) - \frac{\mathcal{T}^{*\rho}_{\rho}(k)\mathcal{T}^{\sigma}_{\sigma}(k)}{D-2} \right), \qquad \mathcal{L}^{\mu\nu} = ik^\mu \frac{\partial}{\partial k_\nu} - ik^\nu \frac{\partial}{\partial k_\mu}$$

$$J^{\mu\nu} = 8\pi G \int \widetilde{dk} \left(\underline{\mathcal{T}^{*\rho\sigma}(k)\mathcal{L}^{\mu\nu}\mathcal{T}_{\rho\sigma}(k)} - \underline{\frac{\mathcal{T}^{*\rho}_{\rho}(k)\mathcal{L}^{\mu\nu}\mathcal{T}^{\sigma}_{\sigma}(k)}{D-2}} + \underline{2i\, \mathcal{T}^{*\rho[\mu}(k)\mathcal{T}^{\nu]}{}_{\rho}(k)} \right)$$

- Gauge invariance: $\mathcal{T}^{\mu\nu}(k) \to \mathcal{T}^{\mu\nu}(k) + k^\mu \epsilon^\nu(k) + k^\nu \epsilon^\mu(k)$

- No physical separation of “orbital” and “spin” angular momentum

[Jaffe, Manohar]

Leading Order: Zero-Frequency Limit

(Weinberg Soft theorem, memory, …)

[Weinberg]
see Carlo's talk

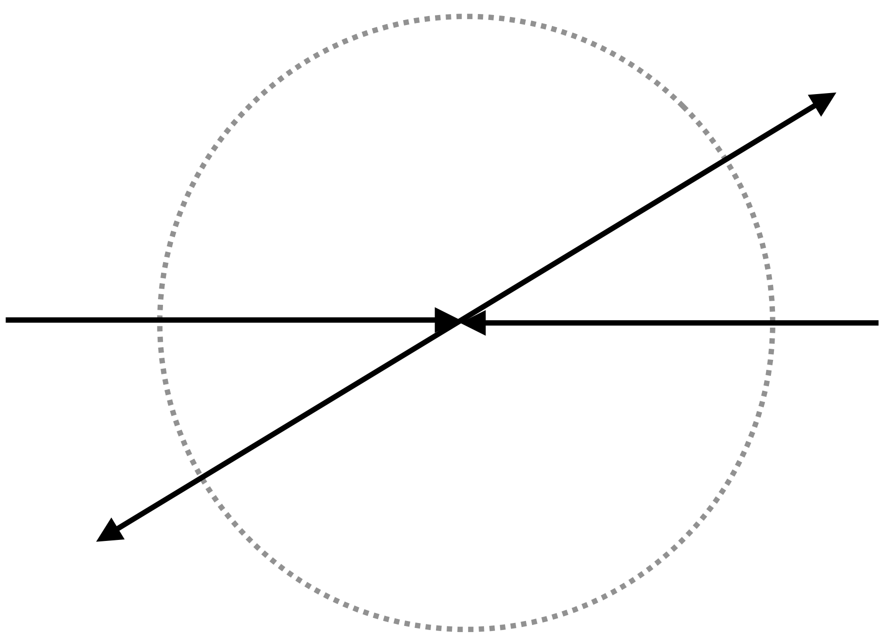
Stress-energy Pseudotensor

- Zoom out the time scale so collision occurs at t=0 (zero-frequency limit)

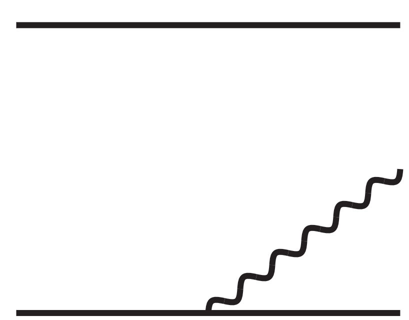
$$\mathcal{T}^{\mu\nu}(k)\big|_{\omega\to 0^+} = \underbrace{-i\pi\delta(\omega)\sum_a \frac{p_a^\mu p_a^\nu}{E_a - \hat{\mathbf{k}}\cdot\mathbf{p}_a}}_{\text{Free particles (Coulomb mode)}} + \underbrace{\frac{1}{\omega + i0}\sum_a \left(\frac{p_a^\mu p_a^\nu}{E_a - \hat{\mathbf{k}}\cdot\mathbf{p}_a}\right)\Bigg|_i^f}_{\text{deflection turned on at t=0}}$$

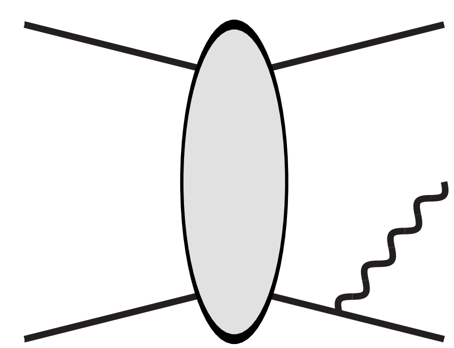
Free particles
(Coulomb mode)

deflection turned on at t=0



$\delta(u = t - r)$





Stress-energy Pseudotensor

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Free particles
(Coulomb mode)

deflection turned on at t=0

- Coulomb modes are **physical but *NOT transverse!***

$$A_\mu \sim J_\mu / r$$
$$\vec{E}, \vec{B} \sim 1/r^2$$

- Valid for ***arbitrary*** deflection

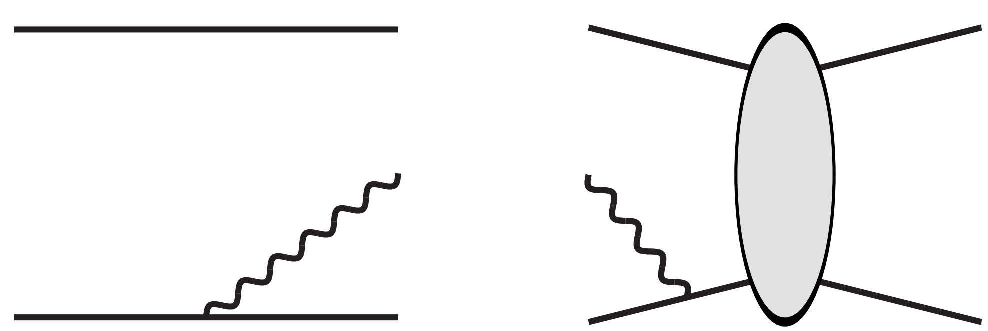
Radiated angular momentum

- Zero-frequency limit

$$P^\mu = 0,$$

$$J^{\mu\nu} = 8\pi G \int \widetilde{dk} \left(\mathcal{T}^{*\rho\sigma}(k) \mathcal{L}^{\mu\nu} \mathcal{T}_{\rho\sigma}(k) - \frac{\mathcal{T}^{*\rho}_{\rho}(k) \mathcal{L}^{\mu\nu} \mathcal{T}^{\sigma}_{\sigma}(k)}{D-2} + 2i\, \mathcal{T}^{*\rho[\mu}(k) \mathcal{T}^{\nu]}{}_{\rho}(k) \right)$$

$$d\omega\, \omega \quad \delta(\omega) \qquad \frac{1}{\omega}$$



Radiated angular momentum

- Zero-frequency limit

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- There is nothing wrong with zero energy and non-zero angular momentum

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} \sim \int d\Omega r^2 \, \mathbf{r} \times (\mathbf{E} \times \mathbf{B})$$

$$\frac{1}{r^2} \qquad \frac{1}{r}$$

See also talks by Gralla and Heissenberg

Radiated angular momentum

- Leading order in deflection angle θ

$$P^{\mu} = 0$$

$$\frac{J^{12}_{\mathrm{CM},2}}{\mathsf{J}_{\mathrm{CM}}} = 2 \times \frac{J^{12}_{\mathrm{rest},2}}{\mathsf{J}_{\mathrm{rest}}} = 2 m_1 m_2 \mathcal{I}(\sigma)\, \theta$$

$$\frac{J^{02}_{\mathrm{CM},2}}{(E_1 - E_2) b} = \frac{J^{02}_{\mathrm{rest},2}}{(m_1 - m_2 \sigma) b} = m_1 m_2 \mathcal{I}(\sigma)\, \theta$$

- Model independent (GR, with spin, dilaton gravity, supergravity, etc)

- Radiated Angular momentum ***is positive*** when scattering is ***attractive***

See also talks by Gralla and Heissenberg

Comparison

- J^{12} at G^2 agrees with [Damour]
- Fully agrees arbitrary deflection [Di Vecchia, Heissenberg, Russo]
- J^{0i} at G^2 agrees w/ [Gralla, Lobo] (modulo a potential extra term)
- Disagree with the textbook formula in the rest frame by x2
  [Jakobsen, Mogull, Plefka, Steinhoff][Mougiakakos, Riva, Vernizzi]

Textbook v.s. Our formula

Textbook formula
TT part of metric

$$\frac{J^{12}_{\mathrm{rest},2}}{\mathrm{J}_{\mathrm{rest}}} = \frac{J^{12}_{\mathrm{CM},2}}{\mathrm{J}_{\mathrm{CM}}}$$

Our formula
stress-energy pseudotensor

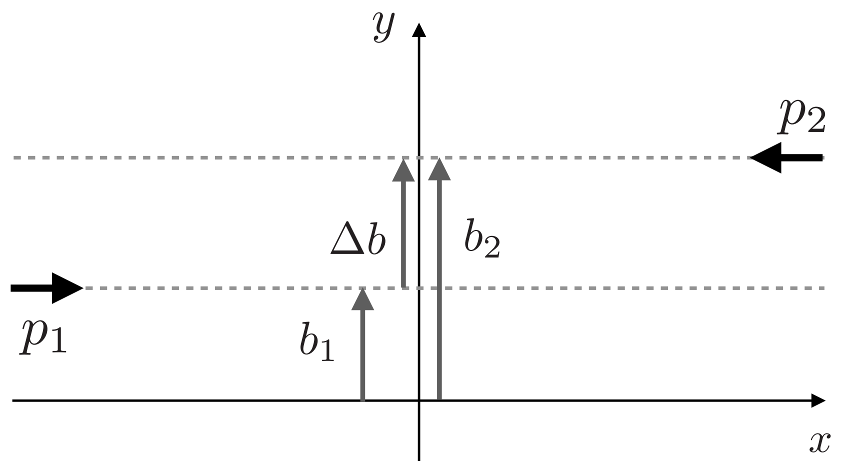
$$\frac{J^{12}_{\mathrm{rest},2}}{\mathrm{J}_{\mathrm{rest}}} = \frac{1}{2} \times \frac{J^{12}_{\mathrm{CM},2}}{\mathrm{J}_{\mathrm{CM}}}$$

- Both agree in the CM frame
- Independent checks: general covariance and 3.5PN RR force

[Jaranowski, Schafer; Nissanke, Blanchet]

General Structure

- Form factors parametrization:



$$P^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3 \Delta b^\mu,$$

$$J^{\mu\nu} = \bar{b}^{[\mu} \left(F_1 p_1^{\nu]} + F_2 p_2^{\nu]} + F_3 \Delta b^{\nu]} \right) + \Delta b^{[\mu} \left(G_1 p_1^{\nu]} - G_2 p_2^{\nu]} \right) + H_{12}\, p_2^{[\mu} p_1^{\nu]}$$

$$F_1 \overset{m_1 \leftrightarrow m_2}{=} F_2, \qquad G_1 \overset{m_1 \leftrightarrow m_2}{=} G_2,$$
$$F_3 \overset{m_1 \leftrightarrow m_2}{=} -F_3, \qquad H_{12} \overset{m_1 \leftrightarrow m_2}{=} -H_{12},$$

- Only assume Lorentz covariance, Poincare algebra, and $1 \leftrightarrow 2$ symmetry
- Form factors are functions of $m_{1,2}, |\Delta b|, \sigma$

[Manohar, Ridgway, **CHS**]

Nonperturbative result

CM Frame

$$\left.\frac{J^{12}_{\rm CM}}{\mathsf{J}_{\rm CM}}\right|_{\omega=0} = G_1 + G_2$$

Rest Frame

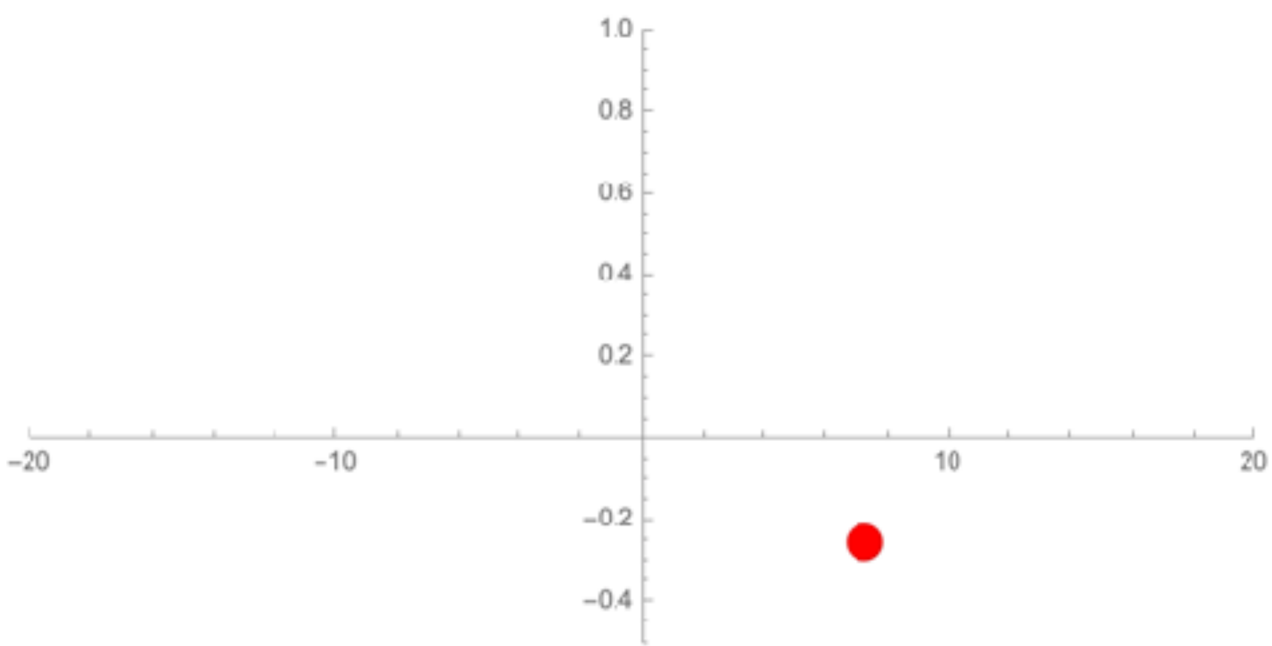
$$\left.\frac{J^{12}_{\rm rest}}{\mathsf{J}_{\rm rest}}\right|_{\omega=0} = G_2$$

- This is an exact relation
- Since $G_1 = G_2$ at this order, our answer agrees with this general prediction

Crosscheck with Burke-Throne

- Burke-Throne force at G^2:

$$\mathbf{a_1} = -\mathbf{a_2} = \frac{4G^2 m_1 m_2}{5r^3}\left(3v^2 v_r \hat{\mathbf{r}} - v^2 \mathbf{v}\right)$$

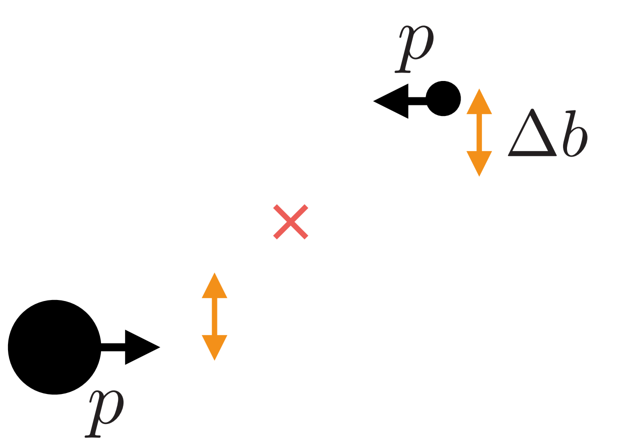


- Final energy is the same as initial
- Impact parameter shrinks equally
- Non-decoupling of heavy particle

Radiation Reaction Force

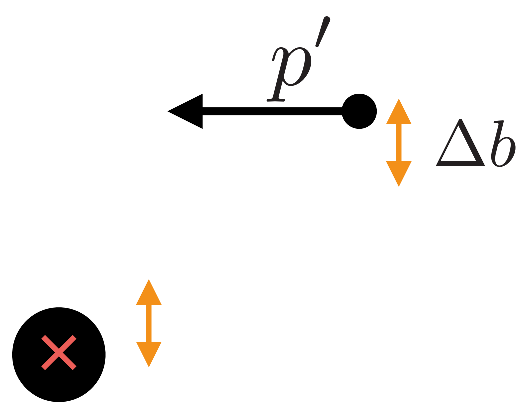
- Burke-Throne force at G²:

CM Frame:



$$\frac{J^{12}_{\mathrm{CM},2}}{\mathrm{J}_{\mathrm{CM}}} = \frac{2p\Delta b}{pb} = 2 \times \frac{\Delta b}{b}$$

Rest Frame:



$$\frac{J^{12}_{\mathrm{rest},2}}{\mathrm{J}_{\mathrm{rest}}} = \frac{p'\Delta b}{p'b} = \frac{\Delta b}{b}$$

Back reaction is important!

$$J^{\mu\nu} = 8\pi G \int \widetilde{dk} \left(\mathcal{T}^{*\rho\sigma}(k) \mathcal{L}^{\mu\nu} \mathcal{T}_{\rho\sigma}(k) - \frac{\mathcal{T}_{\rho}^{*\rho}(k) \mathcal{L}^{\mu\nu} \mathcal{T}_{\sigma}^{\sigma}(k)}{D-2} + 2i\, \mathcal{T}^{*\rho[\mu}(k) \mathcal{T}^{\nu]}{}_{\rho}(k) \right)$$

Our formula agrees with covariance and Burke-Throne force

Precision Frontier:

Radiated angular momentum at G^3

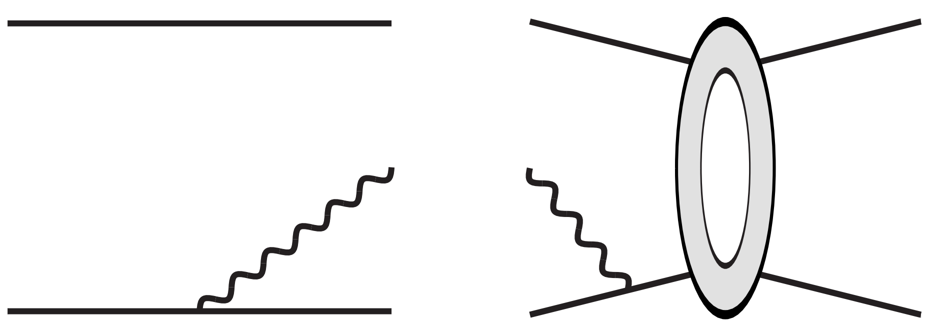
Radiated Poincare Charges

- State of the art precision at G^3

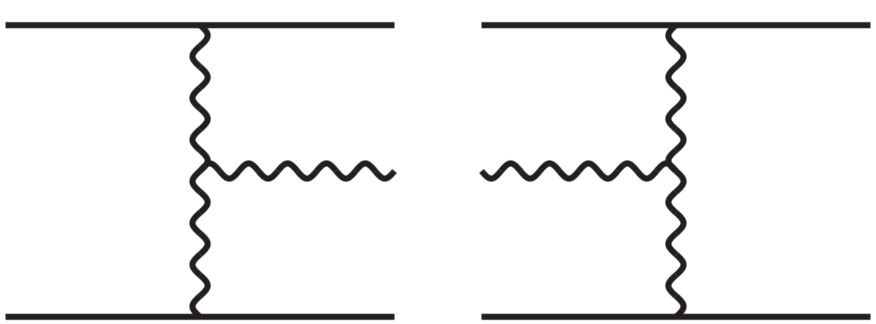
$P^\mu \rightarrow$ Known [Herrmann, Parra-Martinez, Ruf, Zeng]

$J^{\mu\nu} \rightarrow$ Both zero and finite frequency contributions

Soft Theorem



Double Copy & Generalized Unitarity



- Same as before, just use G^2 impulses
  [Westpfahl 80's]

- Waveform from 2-to-3 amplitude via KMOC
- Resum velocity expansion from $O(v^{60})$ series
  [See Parra-Martinez's talk]

New Results in General Relativity

- New results for G^3 radiated angular momentum

$$J^{12}_{\mathrm{rest},3} = b m_1 m_2^2 \left(\underline{m_1 \mathcal{C}(\sigma)} + \underline{(m_1 + m_2)\mathcal{D}(\sigma)} \right),$$

$$\mathcal{I}(\sigma) = -\frac{16}{3} + \frac{2\sigma^2}{\sigma^2-1} + \frac{(2\sigma^2-3)}{\sigma^2-1}\frac{\sigma \operatorname{arcsinh}\left(\sqrt{\frac{\sigma-1}{2}}\right)}{\sqrt{\sigma^2-1}}$$

$$\frac{\mathcal{E}(\sigma)}{\pi} = f_1 + f_2 \log\left(\frac{\sigma+1}{2}\right) + f_3 \frac{\sigma \operatorname{arcsinh}\left(\sqrt{\frac{\sigma-1}{2}}\right)}{\sqrt{\sigma^2-1}}$$

$$\frac{\mathcal{C}(\sigma)}{\pi} = g_1 + g_2 \log\left(\frac{\sigma+1}{2}\right) + g_3 \frac{\sigma \operatorname{arcsinh}\left(\sqrt{\frac{\sigma-1}{2}}\right)}{\sqrt{\sigma^2-1}}$$

$$\mathcal{D}(\sigma) = \frac{3\pi(5\sigma^2-1)}{8}\mathcal{I}(\sigma)$$

$$f_1 = \frac{210\sigma^6 - 552\sigma^5 + 339\sigma^4 - 912\sigma^3 + 3148\sigma^2 - 3336\sigma + 1151}{48(\sigma^2-1)^{3/2}}$$

$$f_2 = -\frac{35\sigma^4 + 60\sigma^3 - 150\sigma^2 + 76\sigma - 5}{8\sqrt{\sigma^2-1}}$$

$$f_3 = \frac{(2\sigma^2-3)(35\sigma^4 - 30\sigma^2 + 11)}{8(\sigma^2-1)^{3/2}}$$

$$g_1 = \frac{105\sigma^7 - 411\sigma^6 + 240\sigma^5 + 537\sigma^4 - 683\sigma^3 + 111\sigma^2 + 386\sigma - 237}{24(\sigma^2-1)^2}$$

$$g_2 = \frac{35\sigma^5 - 90\sigma^4 - 70\sigma^3 + 16\sigma^2 + 155\sigma - 62}{4(\sigma^2-1)}$$

$$g_3 = \frac{-(2\sigma^2-3)(35\sigma^5 - 60\sigma^4 - 70\sigma^3 + 72\sigma^2 + 19\sigma - 12)}{4(\sigma^2-1)^2}$$

- As the form factors show, radiated energy enters when translating from rest to CM frame

$$\frac{J^{12}_{\mathrm{CM},3}}{\mathrm{J_{CM}}} = \frac{m_1 m_2 (m_1+m_2)}{\sqrt{\sigma^2-1}} \left[\underline{\mathcal{C}(\sigma)} + \underline{2\mathcal{D}(\sigma)} - \frac{m_1 m_2 \sqrt{\sigma^2-1}}{E^2} \underline{\mathcal{E}(\sigma)} \right]$$

- Elucidate the relation originally found by Bini, Damour, Geralico when considering G^4 scattering

New Results in General Relativity

- New results for G^3 radiated angular momentum

$$\frac{J_3}{\pi} = \frac{28}{5}p_\infty^2 + \left(\frac{739}{84} - \frac{163}{15}\nu\right)p_\infty^4 + \left(-\frac{5777}{2520} - \frac{5339}{420}\nu + \frac{50}{3}\nu^2\right)p_\infty^6$$

$$+ \left(\frac{115769}{126720} + \frac{1469}{504}\nu + \frac{9235}{672}\nu^2 - \frac{553}{24}\nu^3\right)p_\infty^8 + \ldots$$

[Bini, Damour, Geralico '21]

[Manohar, Ridgway, **CHS**]

2.5PN		**3.5PN**		**4.5PN**					
$(v^3$	$+$	v^5	$+$	v^7	$+$	v^9	$+$	$\ldots)$	G^2
$(v$	$+$	v^3	$+$	v^5	$+$	v^7	$+$	$\ldots)$	G^3
		$(v$	$+$	v^3	$+$	v^5	$+$	$\ldots)$	G^4

New Results in General Relativity

- Predict for G^4 odd-in-v impulses via Bini-Damour formula

$$\Delta p_{\perp,4} = \nu M^5 \left(\frac{G}{b}\right)^4 \left(c_{b,4}^{\text{cons}} + c_{b,4}^{\text{rr,even}} + c_{b,4}^{\text{rr,odd}}\right)$$

Conservative; even in v

Dissipative; even in v
Unknown!!

Dissipative; odd in v

[Manohar, Ridgway, **CHS** ‘22]

[Bern, Parra-Martinez, Roiban, Ruf, **CHS**, Solon, Zeng, ’21]
[Dlapa, Kalin, Liu, Porto ’21]

$$c_{b,4}^{\text{rr,odd}} = \nu\left[\frac{\sigma(6\sigma^2-5)}{\sigma^2-1} - \frac{m_1}{M}\frac{2\sigma^2-1}{(\sigma+1)}\right]\frac{\mathcal{E}(\sigma)}{p_\infty} - \frac{\nu(2\sigma^2-1)}{\sigma^2-1}\left[\frac{3\pi(5\sigma^2-1)}{2}\mathcal{I}(\sigma) + \mathcal{C}(\sigma) + 2\mathcal{D}(\sigma)\right] \quad (19)$$

- Using ideas from factorization in EFT,
simply a G^4 problem into mostly leading order inputs

Precision Binary Dynamics

- State-of-the-art dynamics without using Einstein Eq.

$$\dot{\boldsymbol{x}} = \frac{\partial H}{\partial \boldsymbol{p}}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{F}_{\mathrm{RR}}$$

Conservative

All orders in v to $\mathbf{G}^4$

bootstrap from the scattering angle

[Bern, Cheung, Roiban, **CHS**, Solon, Zeng, '19]

[Bern, Parra-Martinez, Roiban, Ruf, **CHS**, Solon, Zeng, '21]

[Dlapa, Kalin, Liu, Porto '21]

Dissipative

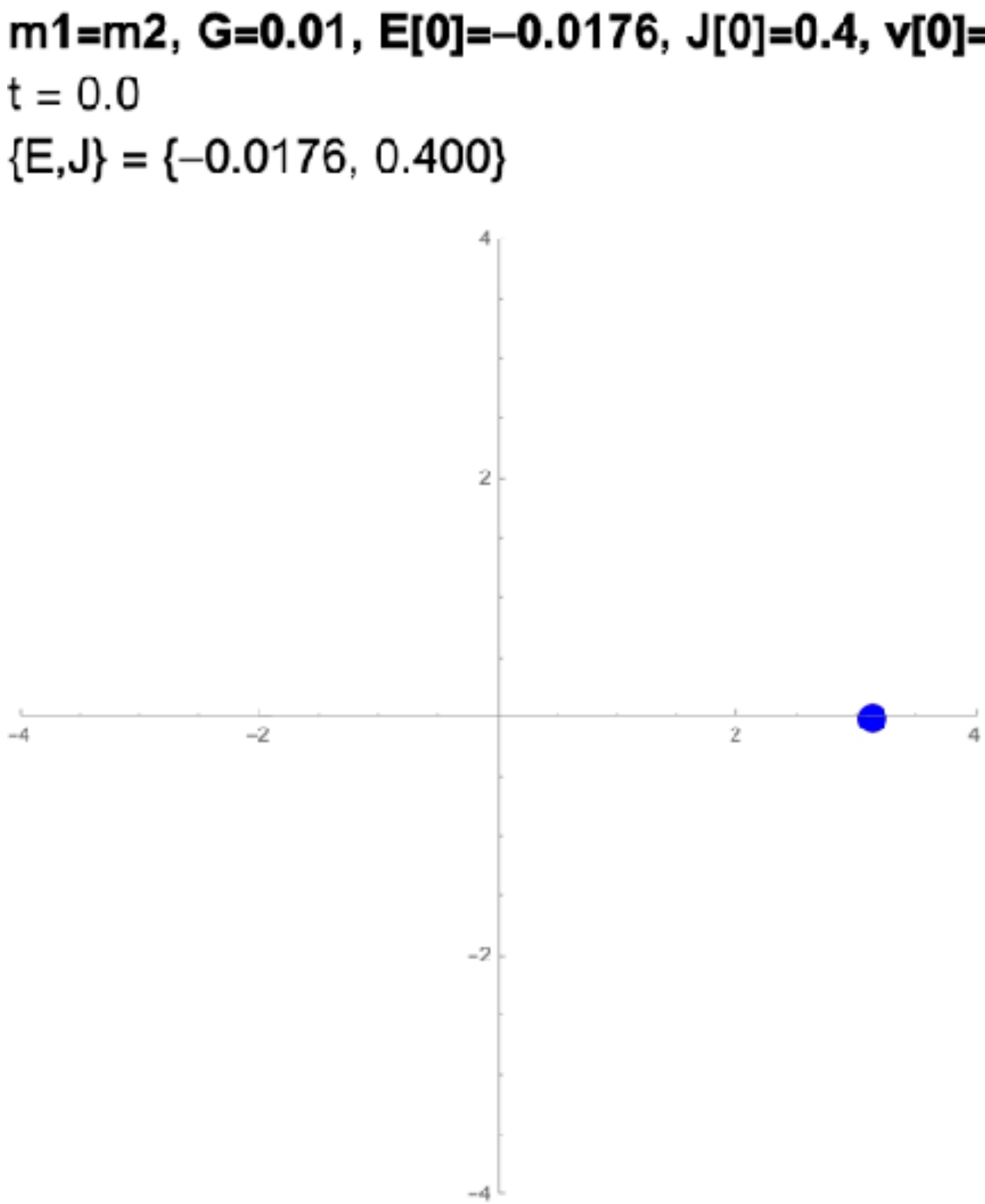
All orders in v to $\mathbf{G}^3$

bootstrap from radiated E and J

[Manohar, Ridgway, **CHS** '22]

[Bern, Cheung, Roiban, **CHS**, Solon, Zeng, ’19]
[Bern, Parra-Martinez, Roiban, Ruf, **CHS**, Solon, Zeng, ’21] [Dlapa, Kalin, Liu, Porto ’21]
[Manohar, Ridgway, **CHS** ‘22]

Precision Binary Dynamics



$$\dot{\boldsymbol{x}} = \frac{\partial H}{\partial \boldsymbol{p}}$$

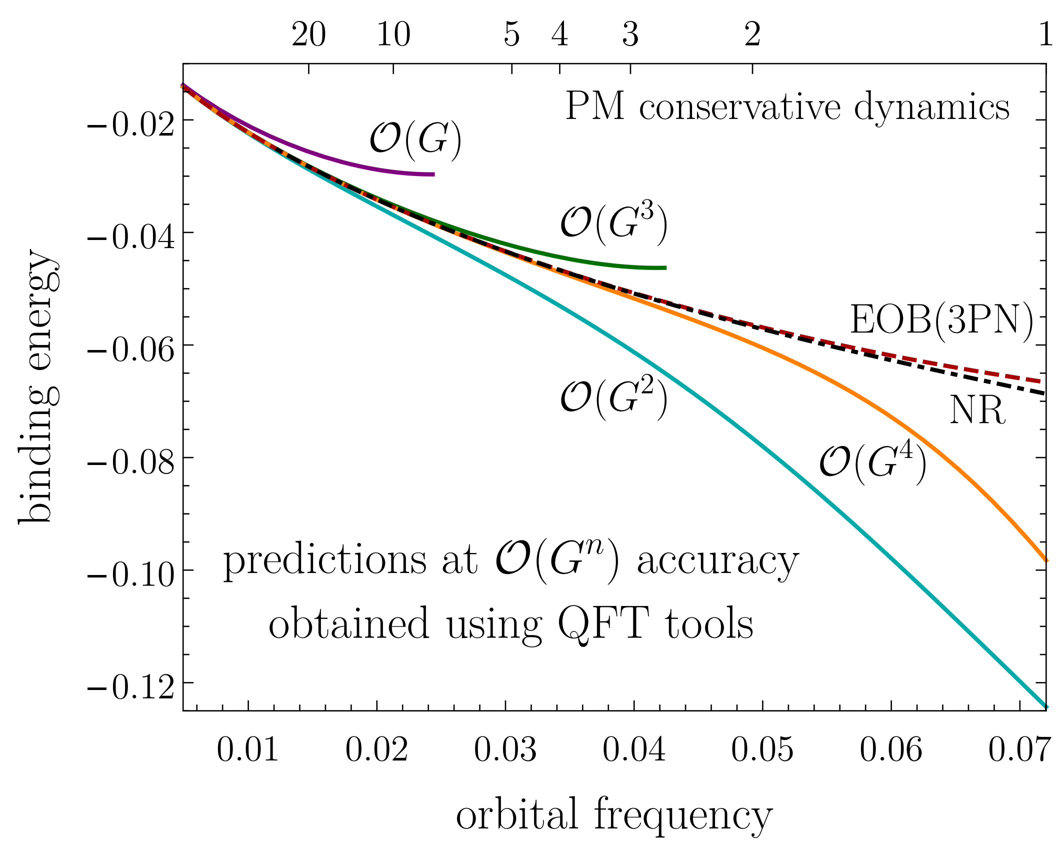
$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{F}_{\rm RR}$$

*Caveat: illustration only
Don’t trust a plot made by theorists

Precision Binary Dynamics

Only Conservative PM effect included so far

Can dissipation bring closer to NR?



Snowmass white paper [Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng, 2204.05194]
[Khalil, Buonanno, Steinhoff, Vines, 2204.05047]

Summary

- We solve dissipative dynamics using Poincare invariance and soft theorem
- We derive a new formula for radiated angular momentum
  - Agreement found w/ covariance and known force
- We give a nonperturbative form-factor parametrization
- We calculate all Poincare charges to G^3
- We bootstrap the dissipative force from Poincare invariance to G^3
- New prediction at G^4

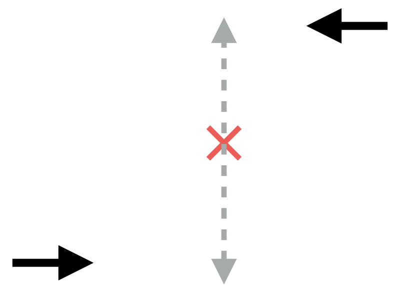
Thank you

Backups

CM Frame v.s. Rest frame

CM Frame:

$$p_2^\mu = (E_2, -|\mathbf{p}|, 0, 0)$$

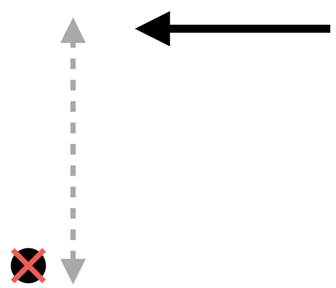


$$p_1^\mu = (E_1, |\mathbf{p}|, 0, 0)$$

$$\mathsf{J}_{\mathrm{CM}} = |\mathbf{p}| b$$

Rest Frame:

$$p_2^\mu = (\sigma m_2, -\sqrt{\sigma^2 - 1} m_2, 0, 0)$$



$$p_1^\mu = (m_1, 0, 0, 0)$$

$$\mathsf{J}_{\mathrm{rest}} = \sqrt{\sigma^2 - 1} m_2 b$$

- They are related by boost and translation

Precision Binary Dynamics

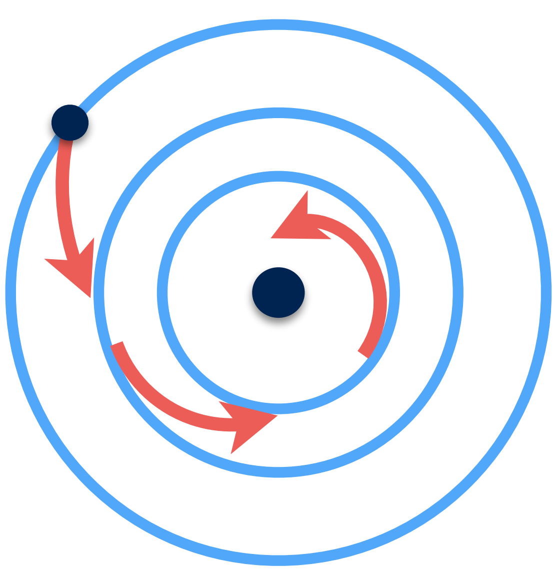
- State-of-the-art EOM all orders in v to G^3

$$H(r, p^2)$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right),$$

$$c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1 - 2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\left[\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{3\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} + \frac{2\nu^3(3 - 4\xi)\sigma\left(1 - 2\sigma^2\right)^2}{\gamma^4\xi^3} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2} + \frac{\nu^4(1 - 2\xi)\left(1 - 2\sigma^2\right)^3}{2\gamma^6\xi^4}\right].$$



$$\mathbf{F}_{\mathrm{RR}} = c_r p_r \hat{\mathbf{r}} + c_p \mathbf{p}$$

$$c_r = \frac{G^2}{r^3} c_{r,2}\left(\mathbf{p^2}\right) + \frac{G^3}{r^4} c_{r,3}\left(\mathbf{p^2}\right) + \ldots,$$

$$c_p = \frac{G^2}{r^3} c_{p,2}\left(\mathbf{p^2}\right) + \frac{G^3}{r^4} c_{p,3}\left(\mathbf{p^2}\right) + \ldots,$$

$$c_{r,2}\left(\mathbf{p^2}\right) = -3 c_{p,2}\left(\mathbf{p^2}\right),$$

$$c_{p,2}\left(\mathbf{p^2}\right) = -\frac{\nu^2 M^4}{E_1 E_2}\left(2\sigma^2 - 1\right)\mathcal{I}(\sigma)$$

$$\mathcal{I}(\sigma) = -\frac{16}{3} + \frac{2\sigma^2}{\sigma^2-1} + \frac{(2\sigma^2-3)}{\sigma^2-1}\frac{\sigma\operatorname{arcsinh}\left(\sqrt{\frac{\sigma-1}{2}}\right)}{\sqrt{\sigma^2-1}}$$

$$\frac{\mathcal{E}(\sigma)}{\pi} = f_1 + f_2 \log\left(\frac{\sigma+1}{2}\right) + f_3 \frac{\sigma\operatorname{arcsinh}\left(\sqrt{\frac{\sigma-1}{2}}\right)}{\sqrt{\sigma^2-1}}$$

$$\frac{\mathcal{C}(\sigma)}{\pi} = g_1 + g_2 \log\left(\frac{\sigma+1}{2}\right) + g_3 \frac{\sigma\operatorname{arcsinh}\left(\sqrt{\frac{\sigma-1}{2}}\right)}{\sqrt{\sigma^2-1}}$$

$$\mathcal{D}(\sigma) = \frac{3\pi(5\sigma^2-1)}{8}\mathcal{I}(\sigma)$$

$$f_1 = \frac{210\sigma^6 - 552\sigma^5 + 339\sigma^4 - 912\sigma^3 + 3148\sigma^2 - 3336\sigma + 1151}{48(\sigma^2-1)^{3/2}}$$

$$f_2 = -\frac{35\sigma^4 + 60\sigma^3 - 150\sigma^2 + 76\sigma - 5}{8\sqrt{\sigma^2-1}}$$

$$f_3 = \frac{(2\sigma^2-3)(35\sigma^4 - 30\sigma^2 + 11)}{8(\sigma^2-1)^{3/2}}$$

$$g_1 = \frac{105\sigma^7 - 411\sigma^6 + 240\sigma^5 + 537\sigma^4 - 683\sigma^3 + 111\sigma^2 + 386\sigma - 237}{24(\sigma^2-1)^2}$$

$$g_2 = \frac{35\sigma^5 - 90\sigma^4 - 70\sigma^3 + 16\sigma^2 + 155\sigma - 62}{4(\sigma^2-1)}$$

$$g_3 = \frac{-(2\sigma^2-3)(35\sigma^5 - 60\sigma^4 - 70\sigma^3 + 72\sigma^2 + 19\sigma - 12)}{4(\sigma^2-1)^2}$$

$$c_{p,3}(\mathbf{p^2}) = -\frac{2p_\infty J^{12}_{\mathrm{CM},3}}{\pi\xi E J_0} + \left(2\xi E c'_{p,2}(\mathbf{p^2}) - \left(2 - \frac{p_\infty^2(1-3\xi)}{\xi^2E^2}\right)\frac{J^{12}_{\mathrm{CM},2}}{2p_\infty J_0}\right)c_{H,1}(\mathbf{p^2}) - p_\infty c'_{H,1}(\mathbf{p^2})\frac{J^{12}_{\mathrm{CM},2}}{J_0}$$

$$c_{r,3}(\mathbf{p^2}) = \frac{8}{\pi p_\infty}\left(\frac{p_\infty^2}{J_0E\xi}J^{12}_{\mathrm{CM},3} - E_{\mathrm{CM},3}\right) + \left(-6\xi E c'_{p,2}(\mathbf{p^2}) + 2\left(1 + \frac{p_\infty^2(1-3\xi)}{\xi^2E^2}\right)\frac{J^{12}_{\mathrm{CM},2}}{p_\infty J_0}\right)c_{H,1}(\mathbf{p^2}) + 4p_\infty c'_{H,1}$$

Common concerns

- Can zero-energy radiation carries angular momentum?
- Is radiated angular momentum infrared finite
(due to 1/r potential in 4D)?
- Are distribution functions (e.g. delta functions) well-defined?
- Is there BMS ambiguity on angular momentum?

….[Veneziano, Vilkovisky]

Need to analyze each question by calculation
scalar, EM, gravity

Common concerns

- Scalar theory: fixed spacetime & no gauge $S_{\text{int}} = g \int d\tau \phi(x(\tau))$
- Momentum space formula: $J^{\mu\nu} = \int \widetilde{dk} \, (4\pi J(k))^* \left(ik^{[\mu}\partial^{\nu]}\right) (4\pi J(k))$
- Position space calculation $J^{\mu\nu} = \int \mathrm{d}^3x \, x^{[\mu}T^{\nu]0}$
- Radiation reaction force $m\ddot{\mathbf{x}} = \frac{g}{12\pi}\dddot{\mathbf{x}}$
  - Not derived from 1/r expansion and applies to **arbitrary trajectory**
  - Bini-Damour relation on N=8
    [Di Vecchia, Heissenberg, Russo]

We find agreement

Common concerns

- Electromagnetism: fixed spacetime but with gauge freedom
- Momentum space formula: $J^{\mu\nu} = \int \widetilde{dk} \left(-\mathcal{J}^{*\rho}(k)\, \mathcal{L}^{\mu\nu} \mathcal{J}_\rho(k) - i\mathcal{J}^{*[\mu}(k)\mathcal{J}^{\nu]}(k) \right)$
- Position space calculation $J^{\mu\nu} = \int \mathrm{d}^3x\, x^{[\mu} T^{\nu]0}$
- Abraham-Lorentz-Dirac force: $m\frac{d^2x^\mu}{d\tau^2} = \frac{2}{3}\alpha q^2 \left(\dddot{x}^\mu - \dot{x}^\mu (\dddot{x}^\nu \ddot{x}_\nu) \right)$
  - Not derived from 1/r expansion and applies to **arbitrary trajectory**
  - Bini-Damour relation on electromagnetism and N=8
    [Sakeh, Vines, Steinhoff, Buonanno][Bern, Gatica, Herrmann, Luna, Zeng]
    [Di Vecchia, Heissenberg, Russo]

We find agreement

Common concerns

- Gravity: asymptotic flat spacetime with gauge freedom
- Momentum space formula: $J^{\mu\nu} = 8\pi G \int \widetilde{dk} \left(\mathcal{T}^{*\rho\sigma}(k)\mathcal{L}^{\mu\nu}\mathcal{T}_{\rho\sigma}(k) - \frac{\mathcal{T}_{\rho}^{*\rho}(k)\mathcal{L}^{\mu\nu}\mathcal{T}_{\sigma}^{\sigma}(k)}{D-2} + 2i\,\mathcal{T}^{*\rho[\mu}(k)\mathcal{T}^{\nu]}{}_{\rho}(k) \right)$
- Position space calculation $J^{\mu\nu} = \int \mathrm{d}^3x\, x^{[\mu}T^{\nu]0}$
- 3.5PN radiation reaction force
  - Bini-Damour relation at G^3

We find agreement

Common concerns

- Can zero-energy radiation carries angular momentum? **Yes**
- Is radiated angular momentum infrared finite
  (due to 1/r potential in 4D)? **Yes**
- Are distribution functions (e.g. delta functions) well-defined? **Yes**

All of above can be answered in scalar theory

- Is there BMS ambiguity on angular momentum? **Maybe, I don’t know**
  - But need to explain the match to ALD force in electromagnetism
  - Need to explain the match to Burke-Throne force in GR

See Snowmass white paper [Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng, 2204.05194]
and many other talks in this program

Conservative Dynamics

- Impressive progress from both traditional and new methods

- Higher order potential
  [Bern, Cheung, Parra-Martinez, Roiban, Ruf, **CHS**, Solon, Zeng]
  [Bini, Damour, Geralico] [Blumlein, Maier, Marquard, Schafer] [Dlapa, Kalin, Liu, Porto]
  [Bjerrum-Bohr, Cristofoli, Damgaard, Festuccia, Plante, Vanhove] [di Vecchia, Heissenberg, Russo, Veneziano]
  [Kosower, Maybee, O'Connell] [Damgaard, Haddard, Helset] [Jakobsen, Mogull, Plefka, Steinhoff]
  [Brandhuber, Chen, Travaglini, Wen] [Kol, O'Connell, Telem]….
- Spin
  [Vaidya] [Vines] [Guevara, Ochirov, Vines] [Chung, Huang, Kim, Lee] [Aoude, Haddard, Helset]
  [Bern, Luna, Roiban, **CHS**, Zeng][Bern, Kosmopoulos, Luna, Roiban, Teng]
  [Steinhoff, Levi] [Levi, Von Hippel, McLeod] [Liu, Porto, Yang]
  [Maybee, O'Connell, Vines] [Jakobsen, Mogull, Plefka, Steinhoff] [Chiodaroli, Johansson, Pichini]…
- Tidal effects
  [Bini, Damour][Cheung, Solon][Kalin, Liu, Porto][Aoude, Haddard, Helset]
  [Bern, Parra-Martinez, Roiban, **CHS**, Sawyer] [Cheung, Shah, Solon]…