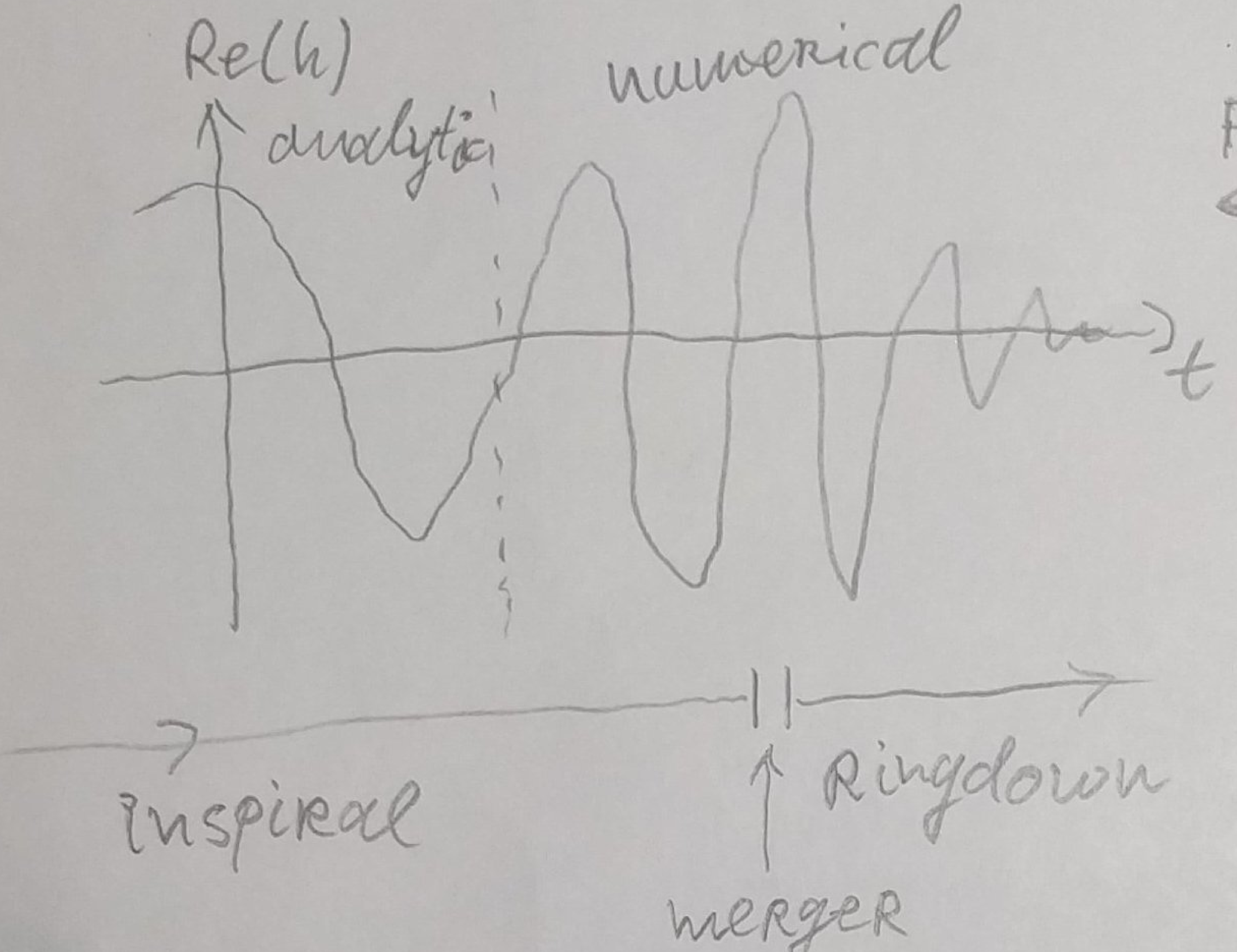
Waveforms (of CBC)

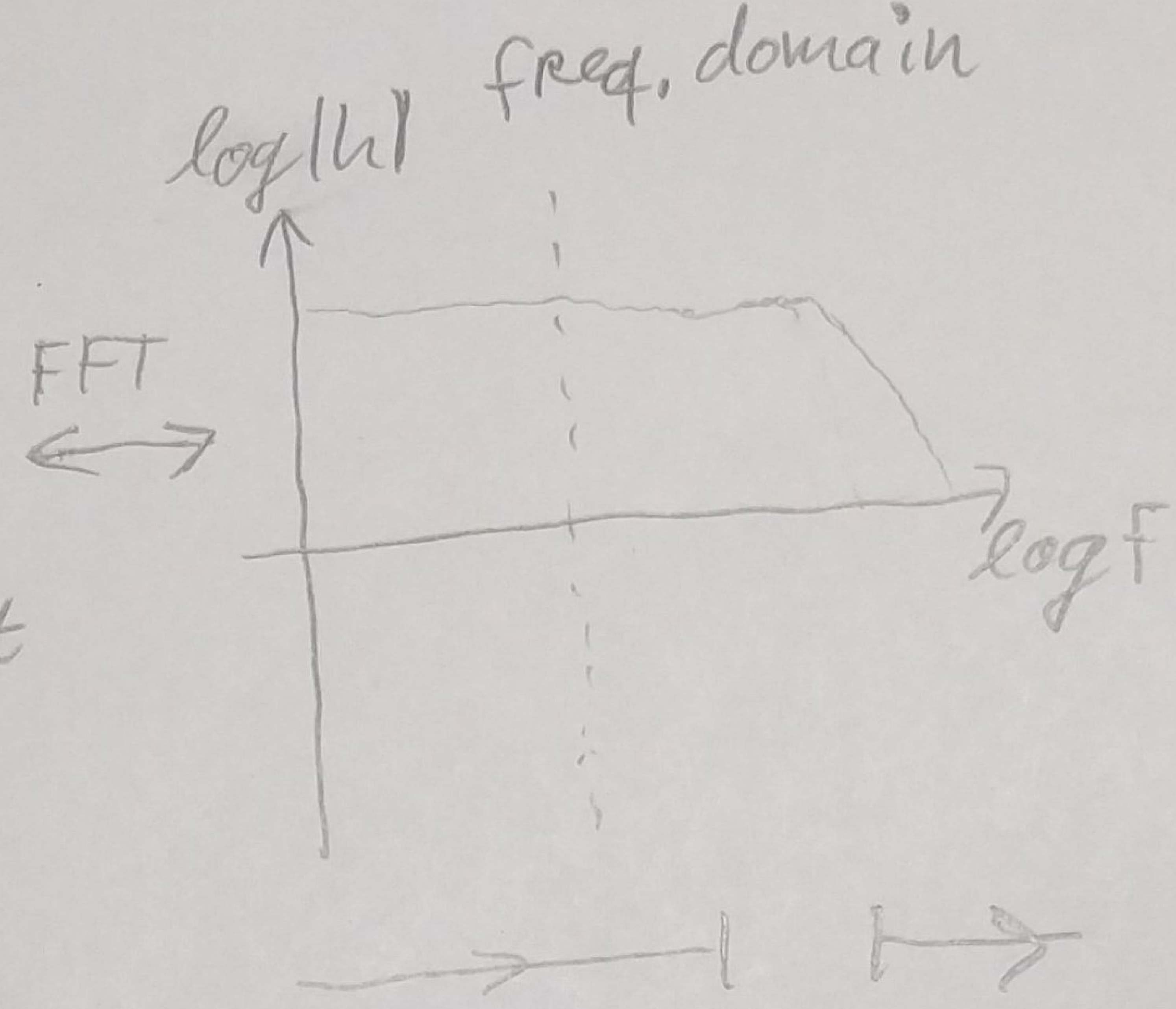
why? → data analysis

$$p(h \mid d, H) \stackrel{\uparrow \text{Bayes}}{=} \frac{p(h \mid H)\, p(d \mid h, H)}{p(d \mid H)}$$

(freq. domain) waveform

time dom., $h = h_+ + i h_\times$





TEOBResumS
SEOBNR
IMRPhenomT

IMRPhenom
(TaylorF2)

↳ Effective One Body (EOB) waveforms

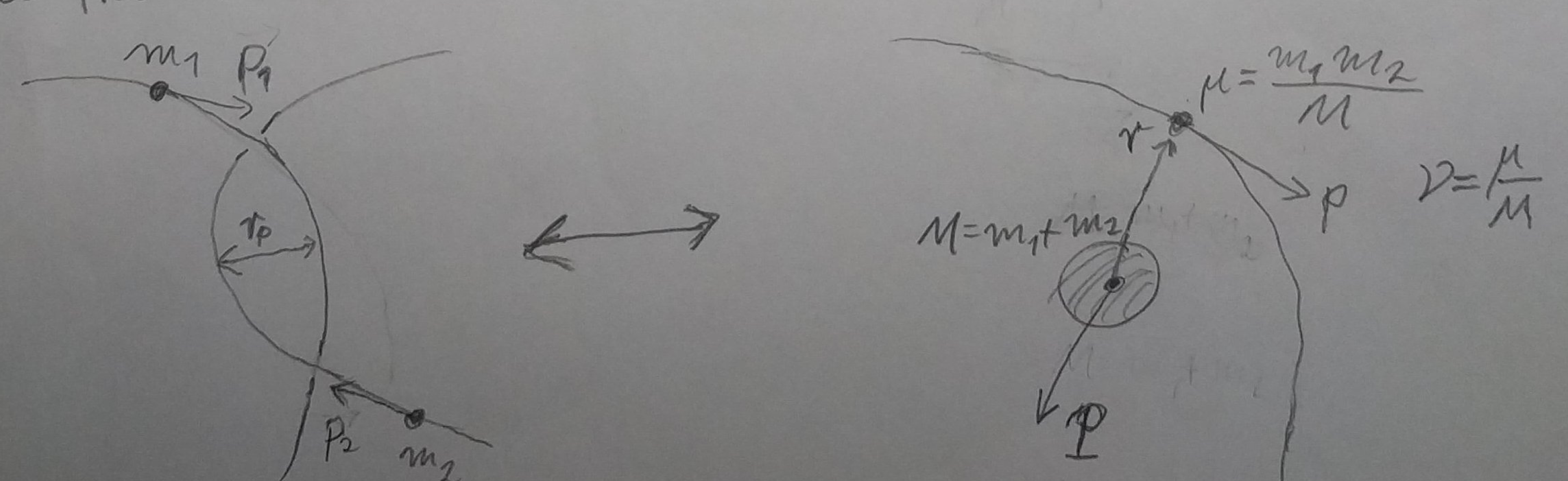
↳ building blocks:

- conservative dynamics → Hamiltonian
- Radiation Reaction
- waveform modes / harmonics
- Ringdown attachment

trajectories

Hamiltonian H

idea from Newtonian binaries



Relativistic:

$$H=\sqrt{M^2\left[1+2\nu\left(\frac{H_e}{\mu}-1\right)\right]+\vec{P}^2}$$

$H_e(\vec{r},\vec{p})$: "effective"/relative Ham.

0th order ⇔ straight lines

$$H_e=\sqrt{\mu^2+\vec{p}^2}$$

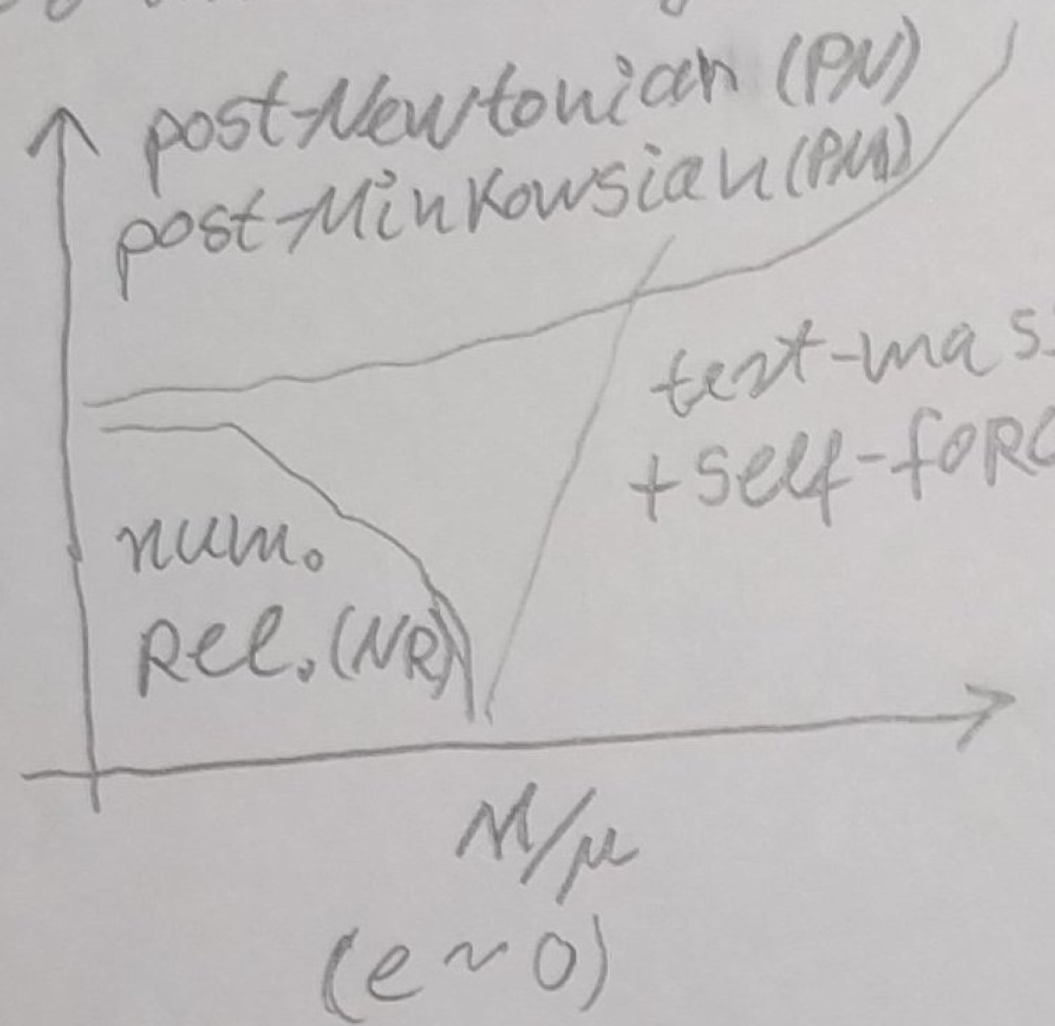
cf. $E_{COM}=\sqrt{M^2[1+2\nu(\gamma-1)]}$

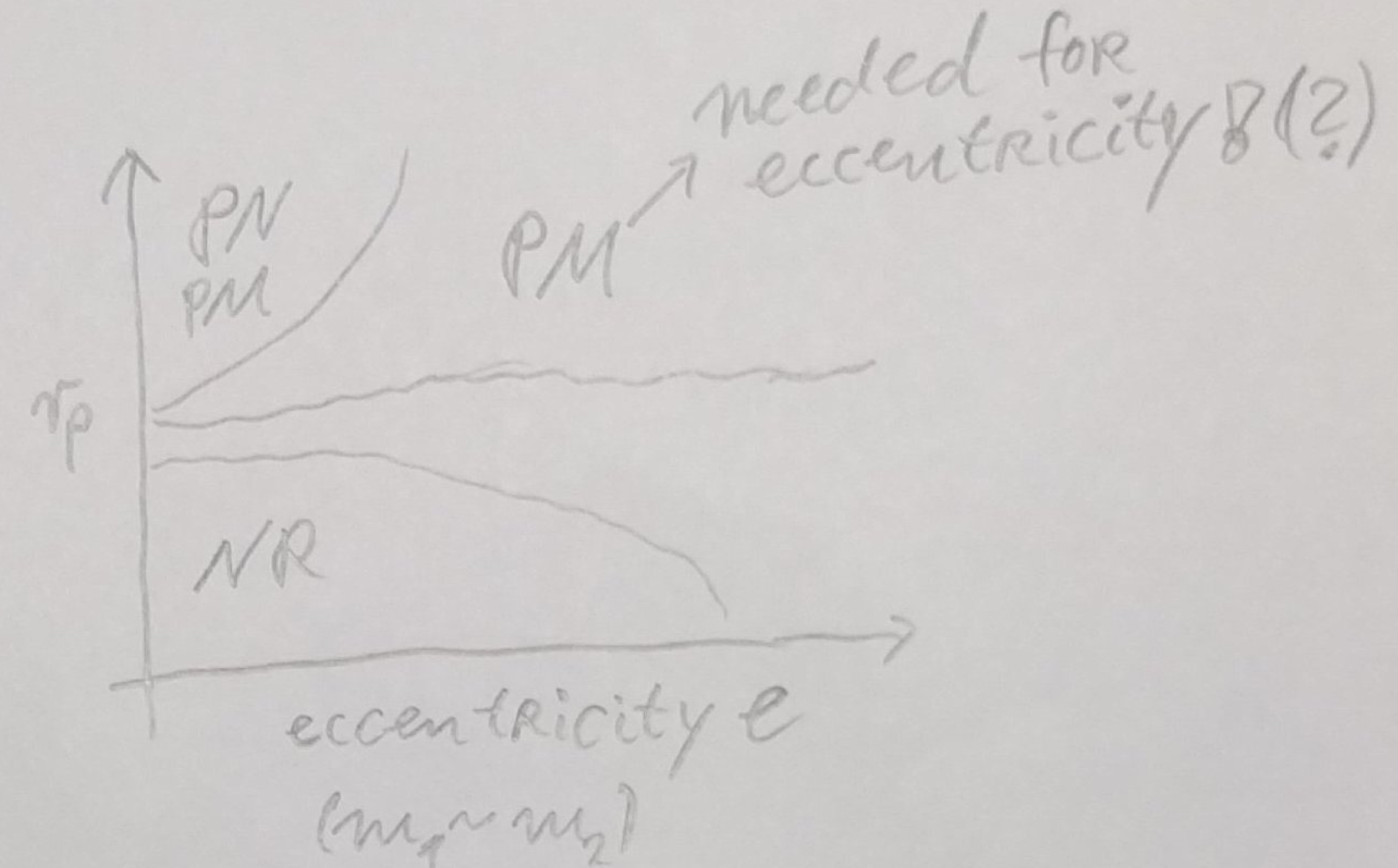
↳ basic kinematics ($H_e=\mu\gamma$)

EOB: deformed test-body Ham. for H_e, $\vec{P}=0$

↳ drastic simplification of PN Ham.s ✓

interlude: methods to get h





Radiation reaction (RR)

add RR-force to Ham. EOM: $(\theta=\frac{\pi}{2},\ p_\theta=0)$ (missing: $\dot{\vec{P}}=\dots$)

$$\dot{r}=\frac{\partial H}{\partial p_r} \qquad \dot{p}_r=-\frac{\partial H}{\partial r}+\underline{f_r}$$

$$\omega=\dot{\phi}=\frac{\partial H}{\partial \underset{\shortparallel\, p_\phi}{L}} \qquad \dot{L}=-\underbrace{\frac{\partial H}{\partial \phi}}_{0}+\underline{f_\phi}$$ ⇝ angular mom. loss

⇝ energy loss: $\dot{E}=\frac{dH}{dt}=\frac{\partial H}{\partial r}\dot{r}+\frac{\partial H}{\partial p_r}\dot{p}_r+\frac{\partial H}{\partial L}\dot{L}$

$$=f_r\dot{r}+f_\phi\omega$$

fluxes $\mathcal{F}_E, \mathcal{F}_L$ from h in PN ⇝ $\dot{E}, \dot{L}$ ⇝ f_r, f_ϕ { choices to make, should be resummed!

equal on average

(circular: $\underset{\text{choice}}{f_r=0}$, $f_\phi=-\mathcal{F}_L=-\frac{\mathcal{F}_E}{\omega}$)

waveform modes ($e=0$)

$$h = \sum_{l,m} h_{lm} \; {}_{-2}Y^{lm}$$

resum h_{lm} ⇝ factorization

$$h_{lm} = h_{lm}^{\text{Newt.}} \underbrace{S_{\text{eff}}^{lm}}_{\text{leading logs}} \underbrace{T_{lm}}_{} \underbrace{(\rho_{lm})^{l}}_{\text{since } h_{lm} \sim (r_{\text{harmonic}})^{l}} \overbrace{e^{i\delta_{lm}}}^{\text{"the rest"}} f_{lm}^{NQC}$$

NQC → non-quasi-circular corrections

S_{eff}^{lm} → $\left.\begin{cases} H_{\text{e}}/\ldots & l+m \text{ even} \\ L/\ldots & l+m \text{ odd} \end{cases}\right\}$ "effective binary source"