Storming the Gravitational Wave Frontier

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Lessons from the Ultra-Relativistic Frontier

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Introduction

- Even neglecting spin, the two-body scattering problem in GR has a large parameter space.
- We may take its "coordinates" as: (m_1, m_2, J, E_{cm})
- In the classical limit (h->0) GR has no intrinsic length or mass scale and physical observables can only depend non-trivially on 3 independent (for scattering) dimensionless quantities. In G=c=1 units we may take

them to be $(m_1/b, m_2/b, v)$ or

$(m_1 m_2/b^2 , \ \nu = m_1 m_2/(m_1+m_2)^2 , \ \sigma = (1- v^2)^{-1/2})$, or …

- Observables such as deflection angle, time delay, waveforms, memory, radiated energy… do depend in a complicated way on ALL these variables.

- The full problem is very hard, of course. One looks at limits in which it simplifies.
- Two of them have been investigated for some time:
  1. The post-Newtonian (PN) expansion in powers of v/c;
  2. The probe-limit expansion in $\nu \sim m_2/m_1$
- A more recent popular expansion is the one in powers of G: the so-called post-Minkowskian (PM) expansion supposedly exact in v and ν at each order in G.
- It is close to the particle-physicist's heart since it corresponds to the loop expansion in QFT.

- Actually, the HE community has been interested in the gravitational 2-body problem since the late '80s ('t Hooft, Amati-Ciafaloni-GV, Muzinich & Soldate,….) although with completely different motivations:

1. See the emergence of classical and quantum gravity effects through thought-experiments in flat spacetime.
2. Construct a unitary S-matrix describing the formation and decay of a BH in (say) a string-string collision => solution to Hawking's information puzzle.

- In that context transplanckian energy is needed for the collision to be able to form a black hole larger than l_P.
- It also allows to justify a semiclassical approximation. URL (not a uniform resource locator) unavoidable!

- What was completely missed at the time is that, in some limit, massive, astrophysical black holes can be thought of as elementary particles (no hair, just mass and spin).
- Of course, for BH's the non-relativistic or mildly relativistic regimes are the most relevant ones. Should we then forget about the URL? (My) answer is NO!
- In 1710.10599, Damour argued that useful input for the two-body problem can be obtained from the URL of gravitational scattering and gave an example (see below).

- Giving other examples of

"Lessons from the UR frontier"

will be the main aim of this talk.

- All this rests on an essential property of gravity: the absence of collinear singularities making the massless limit well defined (Weinberg 1965).
- The massless (point particle) limit has a one-dimensional parameter-space given by E_{cm}/b (or θ_s).
- As we will see, one surprise (?) is that the UR frontier is much richer and (at least) 3-dimensional.

Outline

- **URL and deflection angle**
  - triviality of URL @ 2PM
  - the 3PM puzzle and its resolution
  - new problems @ 4PM
- **URL and radiation**
  - ACV07 energy crisis & its partial resolution
  - The Kovacs-Thorne (D'Eath) bound
- **Improved eikonal operator in soft limit**
  - wave-forms, memory (see CH's talk)
  - A rich UR frontier & non-analyticity in G
- **Beyond the soft-radiation limit**

URL & deflection angle @ 2&3PM

(DHRV* 2008.12743)

Reminder: the elastic eikonal "phase" defined by

$$S(E, b) = \exp(2i\delta) \; ; \; \delta = \delta_0 + \delta_1 + \delta_2 + \dots \; ; \; \delta_n = \mathcal{O}(G^{n+1})$$

gives the scattering angle and Shapiro time delay as derivatives of Re 2δ w.r.t. impact parameter and energy, respectively.

On the other hand, Im 2δ > 0 is related to the opening of inelastic channels and to the consequent suppression of the elastic one.

* Di Vecchia, Heissenberg, Russo, GV

ACV90 results up to 3PM (D=4, GR, massless)

1PM $$2\delta_0 = -\frac{Gs}{\hbar}\log b^2$$ classical

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2PM $$2Re\delta_1 = \frac{12G^2 s}{\pi b^2}\log s\ ;\ \ Im\delta_1 = 0$$ quantum and non-universal

Damour's use of URL: URL -> 0 & 2PM in classical limit

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3PM

deflection $$2Re\delta_2 = \frac{4G^3 s^2}{\hbar b^2}$$ classical, finite

radiation $$Im\delta_2 \sim \frac{G^3 s^2}{\hbar b^2}\log s \log\frac{b^2}{\lambda^2}$$ classical, divergent

A puzzle @ 3PM

- In 1901.04424/1908.01493 an impressive calculation by BCRSSZ led to the first 3PM (i.e. 2-loop) result (in GR for two massive scalars).
- Checked to be consistent up to "6PN" (integer) order but presented a puzzle.
- The high-energy (or just the massless) limit of the BCRSSZ result exhibited a logarithmic divergence in contrast with the finite result by ACV90.

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BCRSSZ = Bern, Cheung, Roiban, Shen, Solon, Zeng

The ACV90 argument (m=0)

- Combining:
  - Real analyticity: $A^*(s^*,t) = A(s,t)$
  - Asymptotics => fixed-t dispersion relations.
  - Xing symmetry: $A(s,t) = A(u,t)$
  - Perturbative Unitarity

an explicit calculation of $Im\delta_2$ from the inelastic (3-particle) cut of the two-loop amplitude gives the quoted result for $Re\delta_2$ from

$$2Re\delta_2 = \frac{\pi}{2\log s}(2Im\delta_2) - \frac{\delta_0}{s}(2\nabla\delta_0)^2$$

$$2Re\delta_2 = \frac{\pi}{2\log s}(2Im\delta_2) - \frac{\delta_0}{s}(2\nabla\delta_0)^2$$

The logarithmically growing term in $Im\delta_2$ has an IR divergence which, however, cancels against the δ_0 term. This yields the finite ACV result for $Re\delta_2$

(By contrast, in BCRSSZ $Im\delta_2$ grows like $\log^2 s$ and this implies their (in)famous log s in $Re\delta_2$)

- In 2008.12743 DHRV extended the ACV90 argument to massive UR case & confirmed the ACV result.

- Then confirmed it by computing the full amplitude in N=8 SUGRA including contributions from the full soft (rather than just the potential) integration region.

3PM eikonal in N=8 SUGRA

$$\mathrm{Re}(\delta_2) = \frac{2G^3(2m_1m_2\sigma)^2}{\hbar b^2}\left[\frac{\sigma^4}{(\sigma^2-1)^2} - \cosh^{-1}(\sigma)\left(\frac{\sigma^2}{\sigma^2-1} - \frac{\sigma^3(\sigma^2-2)}{(\sigma^2-1)^{5/2}}\right)\right]$$

P-MRZ/BCRSSZ

ACV-limit

New

cancel @ large σ

$$2m_1m_2\sigma = s - m_1^2 - m_2^2$$

$$\cosh^{-1}(\sigma) \sim \log\sigma \text{ as } \sigma \to \infty$$

NB: new & old terms behave quite differently in their PN-expansion (σ->1) but cancel in URL

- When we presented this result at a workshop in Aug. 2020, Damour immediately grasped the physical meaning of what we had found:
- Our half-integer PN terms meant that we had added to the conservative dynamics the effect of radiation on the eikonal phase, the so-called radiation reaction.
- A couple of months later, using a smart shortcut, Damour extended the result to GR (see below).
- A bit later, using a different shortcut, DHRV gave another simple derivation of both the N=0 and the N=8 result for the radiation reaction.
- More confirmations given last year by extracting the RR from full-fledged two-loop calculations.

Damour's result for GR

1PN

2PN(BCRSSZ)

$$2Re\delta_2 = \frac{2G^3 m_1 m_2 s}{\hbar b^2 (\sigma^2 - 1)^{3/2}} \left(12\sigma^4 - 10\sigma^2 + 1\right)$$
$$- \frac{4G^3 m_1^2 m_2^2}{\hbar b^2 (\sigma^2 - 1)^{1/2}} \left(\frac{\sigma(14\sigma^2 + 25)}{3} + (4\sigma^4 - 12\sigma^2 - 3) \frac{cosh^{-1}(\sigma)}{(\sigma^2 - 1)^{1/2}} \right)$$
$$+ \frac{2G^3 m_1^2 m_2^2 (2\sigma^2 - 1)^2}{\hbar b^2 (\sigma^2 - 1)^2} \left(\frac{8 - 5\sigma^2}{3} + \sigma(2\sigma^2 - 3) \frac{cosh^{-1}(\sigma)}{(\sigma^2 - 1)^{1/2}} \right)$$

2.5PN

UR-limit: log s terms become subleading &

$(48 - 56/3 - 40/3)\sigma^2 = 16\sigma^2$ => ACV90!

UNIVERSALITY OF THE MASSLESS LIMIT!

Two shortcuts to Radiation Reaction

I. RR from linear response

(T. Damour 2010.01641, see CH's talk)

II. RR from soft theorems

(DHRV 2101.05772, see CH's talk)

New challenges @ 4PM

- New challenges appear at 4PM (= 3 loop) order
- A partial result ("conservative part") has been obtained by Bern et al in 2101.07254.
- Unfortunately, it exhibits the same shortcomings as the 3PM conservative result, only worsened.
- Not only the UR (or zero mass) limit is even more singular than at 3 PM. Even at finite σ the result is IR divergent (coeff. related to E^{rad}, see below)
- Therefore at 4 PM adding RR is absolutely essential for recovering a finite result at any σ!
- The IR divergence has now been cancelled but a full 4PM answer is still unavailable as of this talk.

Gravitational Radiation and UR "energy crises"

I. A first radiation puzzle and its (partial) resolution

An "energy crisis"

(ACV 0712.1209, J.Wosiek & GV 0805.2973)

Graviton spectrum @ $\frac{Gs}{\hbar}\frac{R^2}{b^2} \sim \langle n_{gr} \rangle \gg 1$

$$R \equiv 2G\sqrt{s} \;,\;\; \theta_s \sim \frac{2R}{b}$$

$$\frac{dE_{gr}}{d^2k \; d\omega} = Gs \;\; R^2 \;\; exp\left(-|k||b| - \omega \frac{R^3}{b^2}\right) \;\; ; \;\;\; \Rightarrow \frac{E_{gr}}{\sqrt{s}} \sim 1$$

even @ small θ_s => E-crisis.

Two approaches

1. A classical GR calculation
(A. Gruzinov & GV, 1409.4555)

2. An amplitude-based (quantum) calculation
(CC&Coradeschi & GV, 1512.00281, Ciafaloni, Colferai & GV, 1812.08137)

NB: 2. goes over to 1. in the classical limit in spite of the two completely different methods!
Both limited to small θ_s and θ.

The classical limit (NB: a resummation in G!)

Frequency + angular spectrum ($s = 4E^2$, $R = 4GE$)

$$\frac{dE^{GW}}{d\omega\ d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2\ ;\ \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s\ ;\ \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega \mathbf{x}\cdot\tilde{\boldsymbol{\theta}}} \left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy \qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

Re ζ^2 and Im ζ^2 correspond to the usual (+, x) GW polarizations, ζ^2, ζ^{*2} to the two circular ones.

Analytic results: a Hawking knee
(& an unexpected bump, not today)

For $b^{-1} < \omega < R^{-1}$ the GW-spectrum is almost flat in ω

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi} \theta_s^2 E^2 \log(\omega R)^{-2}$$

Below $\omega = b^{-1}$ it "freezes", giving the expected zero-frequency limit (ZFL) (Smarr 1977)

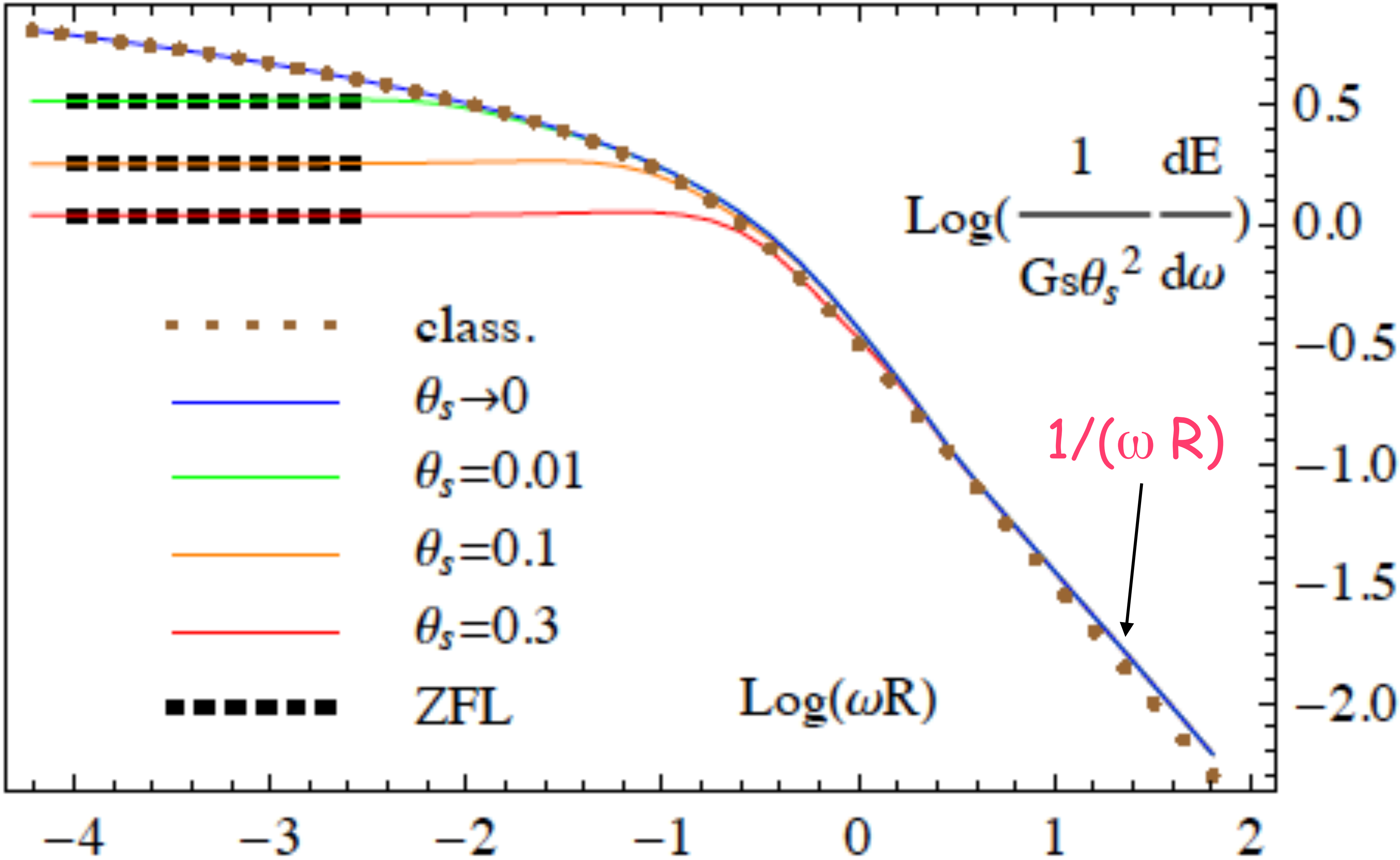
$$\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi} \theta_s^2 E^2 \log(\theta_s^{-2})$$

Above $\omega = R^{-1}$ drops, becomes "scale-invariant"

Hawking knee!

$$\frac{dE^{GW}}{d\omega} \sim \theta_s^2 \frac{E}{\omega}$$

(CCCV 1512.00281)



The "scale-invariant" spectrum gives a log ω^* sensitivity in the total radiated energy for a cutoff at $\omega = \omega^*$

Using, with some motivations, $\omega^* \sim R^{-1} \theta_s^{-2}$ one gets (to leading-log accuracy & neglecting largish θ contr.s):

$$\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi}\ \theta_s^2\ \log(\theta_s^{-2})$$

The URL E-crisis is thus almost solved: we need to go beyond some approximations made in G&V or CCCV, find the actual value of ω^*, and also extend the method to arbitrary θ.

The Kovacs-Thorne (D'Eath) bound

- Before embarking in those non-trivial calculations of the URL we (G&V) checked the literature and asked some experts, including num. rel. guys.
- Each time, after some initial optimism, the feedback was disappointing...
- Instead, we found Kovacs & Thorne's warning on the limit of validity of their 1977 result.

THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

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ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$(\text{angle of gravitational deflection of stars' orbits}) \ll (1 - v^2/c^2)^{1/2} .$$

$\theta_s\, \sigma^{1/2} \ll 1$ in our notations

I will refer to $\theta_s\, \sigma^{1/2} = 1$ as the KT bound

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High-speed black-hole encounters and gravitational radiation

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(Received 15 March 1977)

Encounters between black holes are considered in the limit that the approach velocity tends to the speed of light. At high speeds, the incoming gravitational fields are concentrated in two plane-fronted shock regions, which become distorted and deflected as they pass through each other. The structure of the resulting curved shocks is analyzed in some detail, using perturbation methods. This leads to calculations of the gravitational radiation emitted near the forward and backward directions. These methods can be applied when the impact parameter is comparable to $Gc^{-2}M\gamma^2$, where M is a typical black-hole mass and γ is a typical Lorentz factor (measured in a center-of-mass frame) of an incoming black hole. Then the radiation carries power/solid angle of the characteristic strong-field magnitude c^5G^{-1} within two beams occupying a solid angle of order γ^{-2}. But the methods are still valid when the black holes undergo a collision or close encounter, where the impact parameter is comparable to $Gc^{-2}M\gamma$. In this case the radiation is apparently not beamed, and the calculations describe detailed structure in the radiation pattern close to the forward and backward directions. The analytic expressions for strong-field gravitational radiation indicate that a significant fraction of the collision energy can be radiated as gravitational waves.

KT bound $M\gamma \sim \sqrt{s}$; $\gamma \sim \sqrt{\sigma}$ Beyond KT bound!

2. New incarnations of the "energy crisis"

- A recent $O(G^3)$ calculation of the total E^{rad} (HP-MRZ*, 2101.07255) has confirmed KT's result leading to an "energy crisis" similar to the one found in ACV07 (and "solved" as discussed above).

HP-MRZ= Hermann, Parra-Martinez, Ruf, Zeng

HP-MRZ, 2101.07255 (confirmed in DHRV, 2104.03256)

$$E^{rad} = \frac{\pi G^3 m_1^2 m_2^2 (m_1 + m_2)}{b^3 \sqrt{s}} \left[f_1(\sigma) + f_2(\sigma) \log \frac{\sigma + 1}{2} + f_3(\sigma) \frac{\sigma \cosh^{-1} \sigma}{2\sqrt{\sigma^2 - 1}} \right]$$

where in $\mathcal{N} = 8$

$$f_1 = \frac{8\sigma^6}{(\sigma^2-1)^{\frac{3}{2}}}, \qquad f_2 = -\frac{8\sigma^4}{\sqrt{\sigma^2-1}}, \qquad f_3 = \frac{16\sigma^4(\sigma^2-2)}{(\sigma^2-1)^{\frac{3}{2}}},$$

while in GR

$$f_1 = \frac{210\sigma^6 - 552\sigma^5 + 339\sigma^4 - 912\sigma^3 + 3148\sigma^2 - 3336\sigma + 1151}{48(\sigma^2-1)^{\frac{3}{2}}},$$

$$f_2 = -\frac{35\sigma^4 + 60\sigma^3 - 150\sigma^2 + 76\sigma - 5}{8\sqrt{\sigma^2-1}},$$

$$f_3 = \frac{(2\sigma^2-3)(35\sigma^4 - 30\sigma^2 + 11)}{8(\sigma^2-1)^{\frac{3}{2}}},$$

$$\frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \sqrt{\frac{\sigma}{\nu}} \ ; \ \text{for } \sigma \to \infty$$

Another "energy crisis" @ fixed θ_s

NB: log σ cancels

- A warning sign can already be found in the 3PM result in the ZFL for the URL:

$$\frac{dE^{rad}}{d\omega} \to \frac{Gs}{\pi}\theta_s^2 \log(\sigma) \Rightarrow \frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \log(\sigma)$$

- In this case, however, Weinberg et al. tell us how to fix the problem.
- One can directly study the ZFL for massless scattering and, as we have already seen, the result is quite different (and finite!):

$$\frac{dE^{rad}}{d\omega} \to \frac{Gs}{\pi}\theta_s^2 \log(\theta_s^{-2})$$

- The price to pay is that it is non-polynomial in G!

URL, radiation & eikonal
(see also CH's talk)

When radiation is included the eikonal phase needs to be upgraded to an hermitian operator in order to account for inelastic channels while preserving unitarity

The above puzzles pushed us to propose an eikonal operator implicitly containing some resummation of perturbation theory, reproducing the correct ZFL, and possibly valid (far?) beyond it.

An improved eikonal operator in the soft-graviton limit

(DHRV 2204.02378)

- We start from Weinberg's soft theorem in momentum space (a multiplication!)

$$S^{(M)}_{s.r.,N} = \prod_{r=1}^{N} f_{j_r}(k_r)\, S^{(M)}(\sigma, Q)$$

$$f_j(k) = \varepsilon_j^{*\mu\nu}(k) F_{\mu\nu}(k)\,, \qquad F^{\mu\nu}(k) = \sum_n \frac{\kappa\, p_n^\mu p_n^\nu}{p_n \cdot k}$$

- We then go over to b-space by FT (=> a convolution) and arrive at following operator eikonal:

$$S_{s.r.}(\sigma, b; a, a^\dagger) = \exp\left(\frac{1}{\hbar}\int_{\vec{k}}\sum_j \left[\tilde{f}_j(k)a_j^\dagger(k) - \tilde{f}_j^*(k)a_j(k)\right]\right) e^{i\,\mathrm{Re}\,2\delta(\sigma,b)}$$

where in the f~ we have to use the replacement:

$$q \to -i\hbar\frac{\partial}{\partial b}$$

- Since Reδ is O(h[-1]) the classical limit is obtained by replacing the quantum q with the classical Q:

$$q \to Q = \hbar\,\frac{\partial(Re2\delta)}{\partial b} = Q^{\text{class}}(\sigma, b)$$

- At this point we can compute various radiative observables, like the waveforms, the (linear) memory, and the radiated-energy spectrum in the soft limit (but at arbitrary c.o.m. velocity).
- Let me concentrate on the latter (more in CH's talk).

Features of the ZFL in the URL

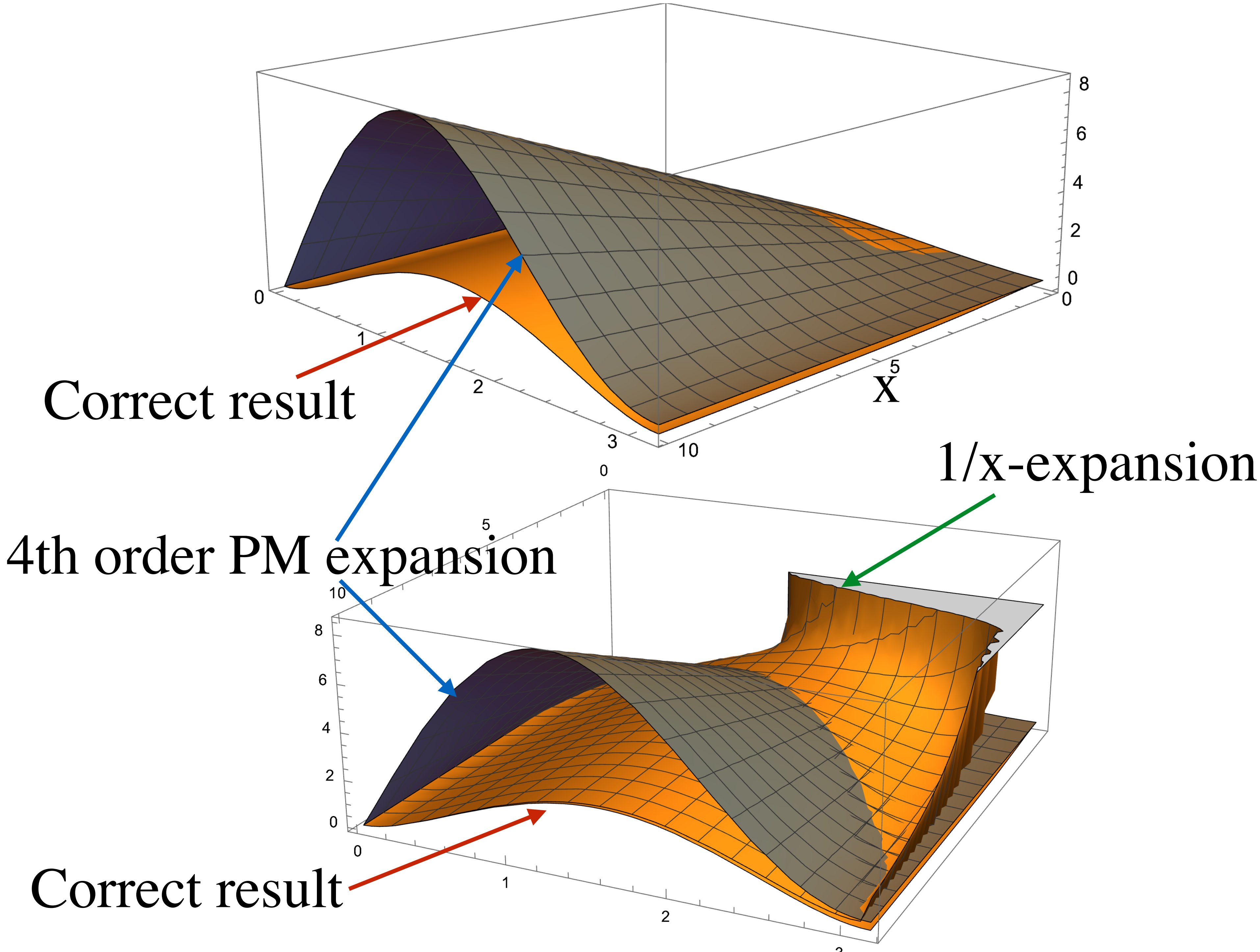
- Rich structure of UR limit emerging
- In URL the ZFL depends non trivially on two "scaling variables": $x_i = Q/2m_i$. One combination is of course related to ν, the other is new, e.g. taken to be $\theta_s^2 \sigma$.
- Dependence on G is non-analytic & a PM expansion in powers of G (or of the x_i) has a finite radius of convergence, given by ($x_1=1$, $x_2=1$).
- Reason: a singularity at the unphysical points

$$x_i^2=-1 : Q^2 = - 4 m_i^2$$

corresponding to t-channel thresholds

- This defines quantitatively the KT bound!
- Only the truly massless limit ($m_i \ll Q$) is universal!

Divergence of URL expansions ($dE/d\theta$, $\nu = 1/4$)



$$\sigma = \frac{s - m_1^2 - m_2^2}{2m_1 m_2} \; ; \; \sigma_Q = -\frac{u - m_1^2 - m_2^2}{2m_1 m_2} = \sigma - \frac{Q^2}{2m_1 m_2}$$

GR

$$\lim_{\omega\to 0} \frac{dE^{\mathrm{gr}}}{d\omega} = \frac{4G}{\pi}\Bigg[2m_1 m_2\left(\sigma^2 - \tfrac{1}{2}\right)\frac{\operatorname{arccosh}\sigma}{\sqrt{\sigma^2-1}} - 2m_1 m_2\left(\sigma_Q^2 - \tfrac{1}{2}\right)\frac{\operatorname{arccosh}\sigma_Q}{\sqrt{\sigma_Q^2-1}}$$

$$+\frac{m_1^2}{2} - m_1^2\left(\left(1+\frac{Q^2}{2m_1^2}\right)^2 - \frac{1}{2}\right)\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_1^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_1^2}\right)^2-1}}$$

$$+\frac{m_2^2}{2} - m_2^2\left(\left(1+\frac{Q^2}{2m_2^2}\right)^2 - \frac{1}{2}\right)\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_2^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_2^2}\right)^2-1}}\Bigg]_{Q=2p\sin\frac{\Theta_s}{2}}$$

in URL

$$\frac{4G}{\pi}\Bigg[1 + \frac{1}{8x_1^2} + \frac{1}{8x_2^2} + \log(\theta_s^{-2}) + \log(16x_1x_2)$$

$$-\frac{\left(1+x_1^2+\frac{1}{8x_1^2}\right)\cosh^{-1}(1+2x_1^2)}{\sqrt{(1+2x_1^2)^2-1}} - \frac{\left(1+x_2^2+\frac{1}{8x_2^2}\right)\cosh^{-1}(1+2x_2^2)}{\sqrt{(1+2x_2^2)^2-1}}\Bigg]$$

and in N=8-SUGRA

$$\lim_{\omega\to 0}\frac{dE^{\mathcal{N}=8}}{d\omega}=\frac{4G}{\pi}\Bigg[2m_1m_2\sigma^2\frac{\operatorname{arccosh}\sigma}{\sqrt{\sigma^2-1}}-2m_1m_2\sigma_Q^2\frac{\operatorname{arccosh}\sigma_Q}{\sqrt{\sigma_Q^2-1}}$$

$$-\frac{(Q^2)^2}{4m_1^2}\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_1^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_1^2}\right)^2-1}}-\frac{(Q^2)^2}{4m_2^2}\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_2^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_2^2}\right)^2-1}}\Bigg]_{Q=2p\sin\frac{\Theta_s}{2}}$$

becoming in URL

$$\frac{4G}{\pi}\left[1+\log(\theta_s^{-2})+\log(16x_1x_2)-x_1^2\frac{\cosh^{-1}(1+2x_1^2)}{\sqrt{(1+2x_1^2)^2-1}}-x_2^2\frac{\cosh^{-1}(1+2x_2^2)}{\sqrt{(1+2x_2^2)^2-1}}\right]$$

Universality broken at finite x_i, recovered only for x_i going to infinity

The URL beyond the ZFL
(DHRV, in preparation)

- We have only considered the leading order in $\theta_s \ll 1$.
- Fast fall-off above $\omega^* \sim b^{-1}\,\theta_s^{-3}$ ($\sim b^{-1}\,\sigma^{-3/2}$) appears to be confirmed.
- A table summarizing the preliminary situation is given below.

UR frontier @ different ω

	soft ($\omega b < 1$)	interm. ($1 < \omega b < \sigma^{1/2}$) ($1 < \omega b < 1/\theta_s$)	hard ($\sigma^{1/2} < \omega b < \sigma^{3/2}$) ($\theta_s^{-1} < \omega b < \theta_s^{-3}$)
below KT	$\theta_s^3 \;\log\sigma$ (same)	$\theta_s^3 \;\log\left(\frac{\sigma}{\omega^2 b^2}\right)$ $(\Delta E/\sqrt{s} = \theta_s^3\sqrt{\sigma})$	preliminary $\theta_s^3\sqrt{\sigma}\;(\omega b)^{-1-\Delta}$ $\Delta E/\sqrt{s} = \theta_s^3\sqrt{\sigma}$
above KT	$\theta_s^3 \;\log\theta_s^{-2}$ (same)	$\theta_s^3 \;\log\left(\frac{\theta_s^{-2}}{\omega^2 b^2}\right)$ $(\Delta E/\sqrt{s} = \theta_s^2)$	$\theta_s^2\;(\omega b)^{-1}$ $\Delta E/\sqrt{s} = \theta_s^2 \log\theta_s^{-2}$ G&V, CCCV

$$\frac{1}{\sqrt{s}}\frac{dE^{rad}}{d\omega b} \; ; \; \frac{\Delta E^{\rm rad}}{\sqrt{s}}$$

under scrutiny

Conclusions

- I have sketched why I believe that the URL of gravitational scattering is both useful and fun. But that limit is also interesting on its own:

1. UR collisions of light particles in the very early Universe may have generated an interesting stochastic background of GW's (Cf. Weinberg's 1965 calculation of GW's from NR thermal collisions in the sun).
2. Having developed further our computational tools, we may try to come back to the (35 years-old) goal of understanding how information is encoded in the S-matrix for the collapse regime of trans-planckian-energy collisions.

Additional slides

A classical GR approach

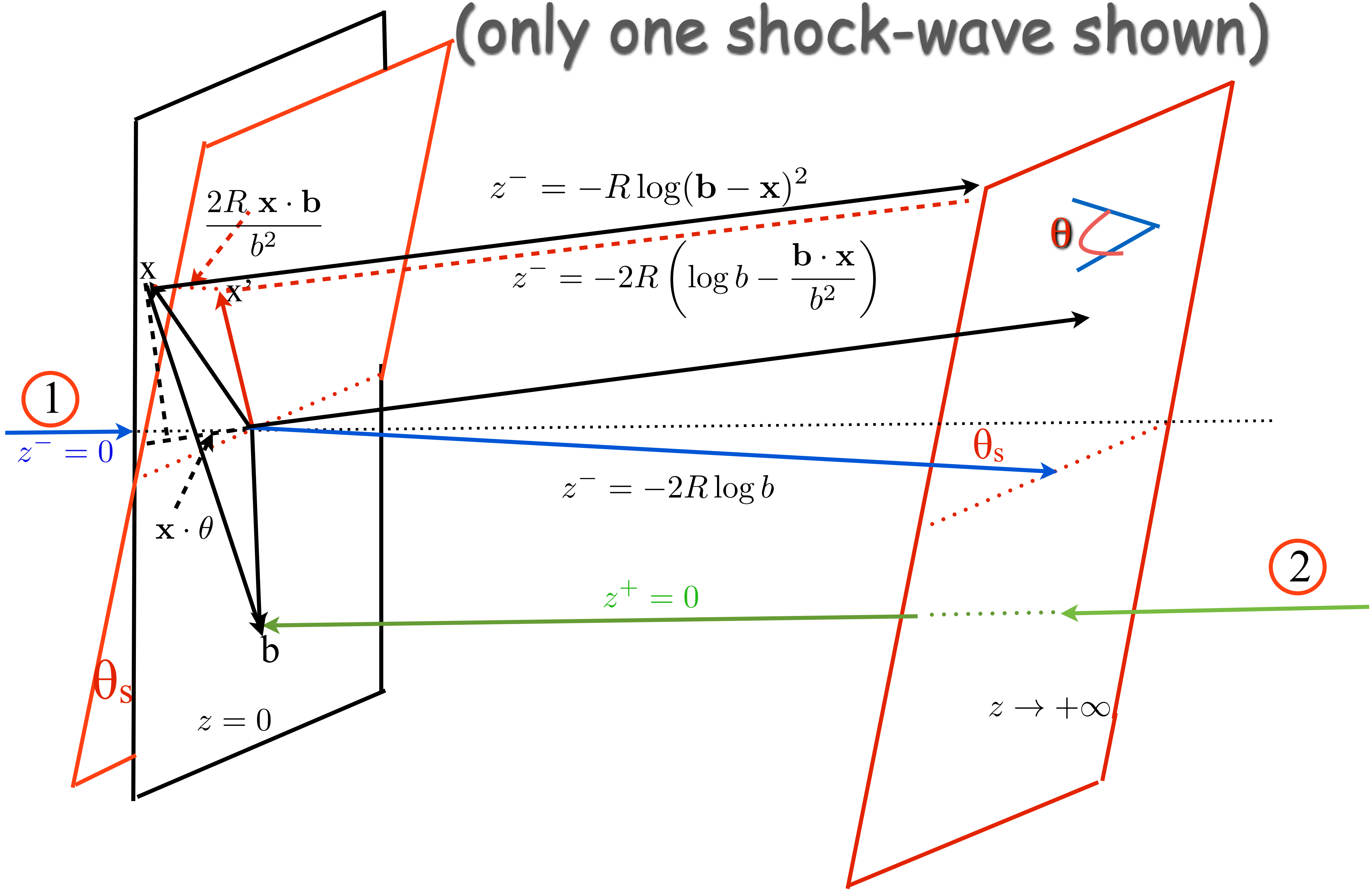
(A. Gruzinov & GV, 1409.4555)

Based on Huygens superposition principle in Fraunhofer's approximation (needs $\theta \ll 1$)

For gravity this includes in an essential way gravitational time delay in the (AS) shock-wave metric.

In pictures

(only one shock-wave shown)



A quantum-amplitudes approach (CCCV, 1512.00281, CCV,1812.08137)

Emission from external and internal legs throughout the whole ladder (with its suitable phase) has to be taken into account for not-so-soft gravitons.

One should also take into account the (finite) difference between the (infinite) Coulomb phase of a final 3-particle state and that of an elastic 2-particle state.

When this is done (so far again for $\theta \ll 1$), the GR result of G&V is recovered for $\hbar\omega/E \to 0$!

In terms of 3PM sc. angle

$$\chi_{3\mathrm{PM}} = \frac{2G^3(2m_1m_2\sigma)^3}{J^3}\left(S + \frac{(2m_1m_2\sigma)}{s}(B + A + C)\right)$$

$$S = -\frac{1}{3}\frac{\sigma^3}{(\sigma^2-1)^{3/2}}$$ ← "Schwarzshild."

$$B = -\cosh^{-1}(\sigma)$$ ← P-MRZ (2PN)

$$A = \frac{\sigma^2}{\sigma^2-1}$$ ← ACV-limit

$$C = \cosh^{-1}(\sigma)\frac{\sigma(\sigma^2-2)}{(\sigma^2-1)^{3/2}}$$

cancel @ large σ (B and C)

1.5PN ! (A and C)

half-integer PN!

I. RR from linear response

- In 2010.01641 Damour derived the RR part of the defl. angle in GR via a smart shortcut.
- Used a previous linear response formula with Bini (1210.2834) relating RR to radiated energy and angular momentum.
- He argued that, at 3PM, only latter at $O(G^2)$ enters
- He then computed J^{rad} at $O(G^2)$ and got the RR correction to the BCRSSZ deflection angle recovering smoothness and the ACV90 UR limit.
- Damour's result has been confirmed by other more direct techniques.
- Yet, it raises another puzzle (at least for some!)

Which is the true J^{rad}?
Which is the relevant J^{rad}?

- How can one radiate angular momentum w/out also radiating, at the same order in G, energy?
- Looks puzzling at the quantum level if one associates E^{rad} and J^{rad} with the E & J of emitted gravitons.
- G. Vilkovisky and myself have been looking into this question recently (2201.11607).

Which is the "true" J^{rad}?

- The definition of angular momentum, and of its loss, is affected by ambiguities related to BMS transformations (see e.g. Bonga Poisson)
- The shear in the Bondi-Sachs metric is affected by BMS supertranslations and, under mild conditions, can be gauged away. This fixes a "canonical Bondi frame (CBF)". We found the ST removing Damour's shear.
- According to Ashtekar et al. () the angular momentum of the system at u = - \infty coincides with J^{ADM} only in the CBF.
- In that gauge J^{rad} is $O(G^3)$, just like E^{rad}.

Which is the "relevant"*) J^{rad}?

- GV^2 also found, however, another Bondi frame (that we dubbed "intrinsic") with the property that its light cones coincide, asymptotically, with those originating from the worldline of the c. o. m.
- We believe this to be the reason why the Bini-Damour formula should be used with J^{rad} computed in the intrinsic Bondi gauge (IBG).
- Indeed J^{rad} computed in the IBG is nothing but Damour's J^{rad}.

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*) for the linear response argument

Diagram with branch point at $t = 4\ m_1^2$