# *Exploring the Root of Gravity*


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**KITP Program 2022:**

**High-Precision Gravitational Waves**

**Based on recent work with:**

Bern, Carrasco, Chiodaroli, HJ, Roiban [1909.01358, 2203.13013];

Andi Brandhuber, Gang Chen, HJ, Gab Travaglini, Congkao Wen [2111.15649];

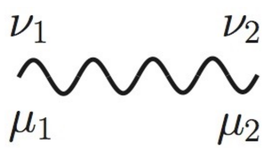
Maor Ben-Shahar, HJ [2112.11452];

Chiodaroli, HJ, Pichini [2107.14779]

# Textbook perturbative gravity is complicated!

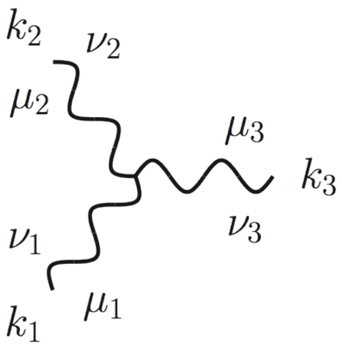
$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

DeWitt ('67)




$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2+i\epsilon}$$

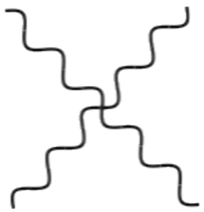
de Donder gauge



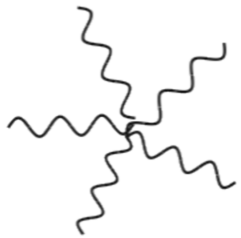

$$\begin{aligned} = \mathrm{sym}[ & -\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1_{\mu_1}}k_{1_{\nu_2}}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3}) \\ & + P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1_{\mu_2}}k_{1_{\nu_3}}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1_{\nu_2}}k_{2_{\mu_1}}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3}) \\ & + P_3(k_{1_{\mu_3}}k_{2_{\nu_3}}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1_{\mu_3}}k_{1_{\nu_3}}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1_{\mu_2}}k_{2_{\nu_3}}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3}) \\ & + 2P_3(k_{1_{\mu_2}}k_{2_{\mu_1}}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})] \end{aligned}$$

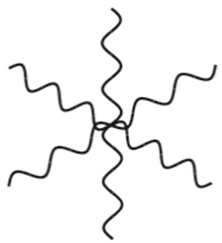
After symmetrization $\sim$ 100 terms !

higher order vertices...



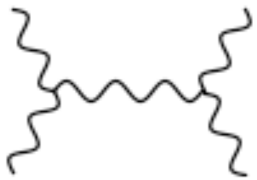
$\sim 10^3$ terms



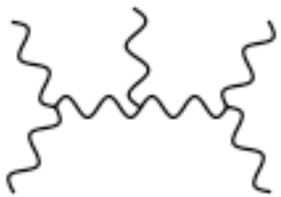


. . .

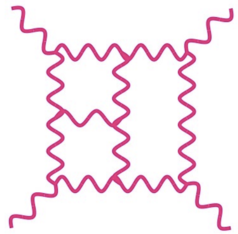
complicated diagrams:



$\sim 10^4$ terms



$\sim 10^7$ terms



$\sim 10^{21}$ terms

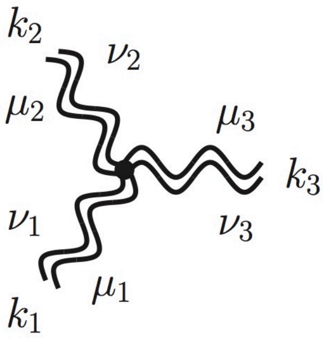
# On-shell simplifications

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

Yang-Mills polarization

$$|\text{spin } 2\rangle \sim |\text{spin } 1\rangle \otimes |\text{spin } 1\rangle$$

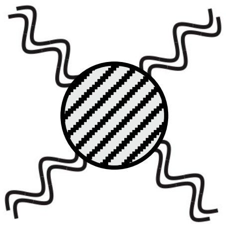
On-shell 3-graviton vertex:




$$= \Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3} + \text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3} + \text{cyclic}\Big)$$

Yang-Mills vertex

Gravity scattering amplitude:



Yang-Mills amplitude

$$M^{\text{GR}}_{\text{tree}}(1,2,3,4) = \frac{st}{u}\Big[A^{\text{YM}}_{\text{tree}}(1,2,3,4)\Big]^2$$

Gravity processes = “squares” of gauge theory ones: **KLT, BCJ, CHY**

# Kawai-Lewellen-Tye Relations ('86)

String theory tree-level identity: closed string ~ (left open string) × (right open string)


$$A_n \sim \int \frac{dx_1 \cdots dx_n}{\mathcal{V}_{abc}} \prod_{1\le i<j\le n} |x_i - x_j|^{k_i\cdot k_j} \exp\left[\sum_{i<j}\left(\frac{\epsilon_i\cdot\epsilon_j}{(x_i-x_j)^2} + \frac{k_i\cdot\epsilon_j - k_j\cdot\epsilon_i}{(x_i-x_j)}\right)\right]\Bigg|_{\rm multi-linear}$$

KLT relations emerge after nontrivial world-sheet integral identities

Field theory limit $\Rightarrow$ gravity theory ~ (YM theory) × (YM theory)

$$M_4^{\rm tree}(1,2,3,4) = -is_{12}A_4^{\rm tree}(1,2,3,4)\,\widetilde{A}_4^{\rm tree}(1,2,4,3)$$

$$M_5^{\rm tree}(1,2,3,4,5) = is_{12}s_{34}A_5^{\rm tree}(1,2,3,4,5)\,\widetilde{A}_5^{\rm tree}(2,1,4,3,5) + is_{13}s_{24}A_5^{\rm tree}(1,3,2,4,5)\,\widetilde{A}_5^{\rm tree}(3,1,4,2,5)$$

gravity states are products of YM states:

$$|2\rangle = |1\rangle \otimes |1\rangle$$

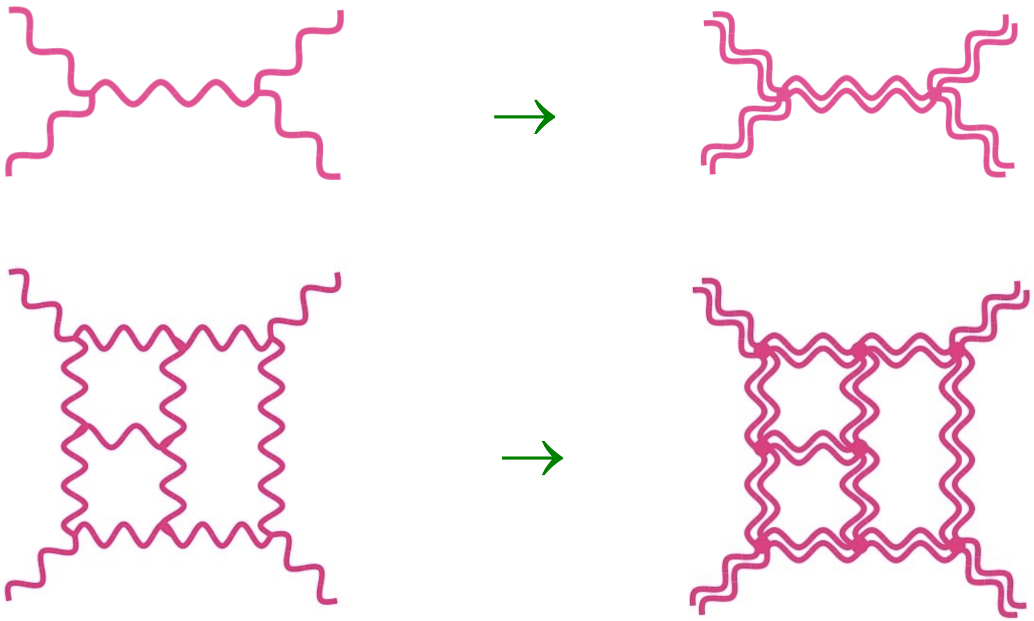
$$|3/2\rangle = |1\rangle \otimes |1/2\rangle$$

etc…

# Squaring of YM theory – the double copy

Gravity processes = squares of gauge theory ones - entire S-matrix

Yang-Mills                Gravity



squared numerators

(BCJ double copy)

E.g. pure Yang-Mills → Einstein gravity + dilaton + axion

4D YM + massless quarks → Pure 4D Einstein gravity

# Example: axion-dilaton gravity

Consider double copy of *D*-dimensional pure YM:

States:

$$(\varepsilon^h)^{ij}_{\mu\nu} = \varepsilon^{((i}_{\mu}\varepsilon^{j))}_{\nu} \qquad \text{(graviton)}$$

$$(\varepsilon^B)^{ij}_{\mu\nu} = \varepsilon^{[i}_{\mu}\varepsilon^{j]}_{\nu} \qquad (B\text{-field})$$

$$(\varepsilon^\phi)_{\mu\nu} = \frac{\varepsilon^i_\mu \varepsilon^j_\nu \delta_{ij}}{D-2} \qquad \text{(dilaton)}$$

Amplitudes consistent with the theory:

$$S = \int d^D x \sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2(D-2)}\partial^\mu\phi\partial_\mu\phi + \frac{1}{6}e^{-4\phi/(D-2)}H^{\lambda\mu\nu}H_{\lambda\mu\nu}\right]$$

In 4D this is axion-dilaton gravity:

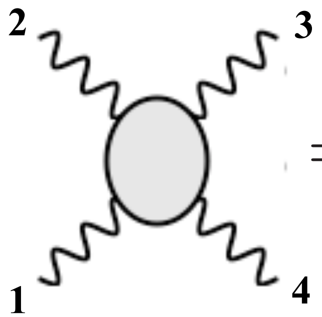
$$S = \int d^4 x \sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{4}\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}e^{-2\phi}\partial_\mu\chi\partial^\mu\chi\right]$$

Symmetry $\chi \to -\chi$, $\phi \to -\phi$ allows for consistent truncation of scalars

# The (Square-)Root of Gravity

# Color-kinematics duality

Consider Yang-Mills 4p tree amplitude:




$$= \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

color factors: $c_s = f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic numerators:

$$n_s = \Big[(\varepsilon_1 \cdot \varepsilon_2) p_1^\mu + 2(\varepsilon_1 \cdot p_2)\varepsilon_2^\mu - (1 \leftrightarrow 2)\Big]\Big[(\varepsilon_3 \cdot \varepsilon_4) p_{3\mu} + 2(\varepsilon_3 \cdot p_4)\varepsilon_{4\mu} - (3 \leftrightarrow 4)\Big]$$
$$+ s\big[(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) - (\varepsilon_1 \cdot \varepsilon_4)(\varepsilon_2 \cdot \varepsilon_3)\big] \,,$$

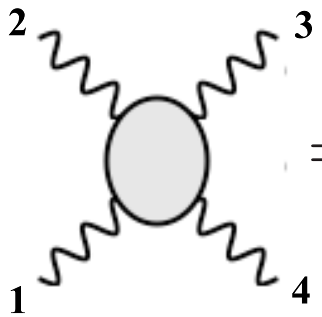
consider gauge transformation $\delta A_\mu = \partial_\mu \phi$

$$n_s\Big|_{\varepsilon_4 \to p_4} = s\Big[(\varepsilon_1 \cdot \varepsilon_2)\big((\varepsilon_3 \cdot p_2) - (\varepsilon_3 \cdot p_1)\big) + \text{cyclic}(1,2,3)\Big] \equiv s\,\alpha(\varepsilon, p)$$

(individual diagrams not gauge inv.)

# Color-kinematics duality

Consider Yang-Mills 4p tree amplitude:




$$= \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

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kinematic numerators:

$$n_s = \Big[(\varepsilon_1\cdot\varepsilon_2)p_1^\mu + 2(\varepsilon_1\cdot p_2)\varepsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\varepsilon_3\cdot\varepsilon_4)p_{3\mu} + 2(\varepsilon_3\cdot p_4)\varepsilon_{4\mu} - (3\leftrightarrow 4)\Big]$$
$$+s\Big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\Big],$$

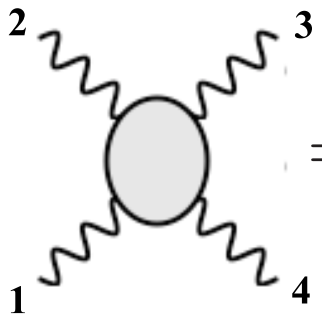
consider linearized gauge transformation $\delta A_\mu = \partial_\mu \phi$

$$\left.\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right|_{\varepsilon_4\to p_4} = \underbrace{(c_s + c_t + c_u)}_{=0}\,\alpha(\varepsilon, p)$$

= 0 Jacobi identity

# Color-kinematics duality

Consider Yang-Mills 4p tree amplitude:




$$= \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

color factors: $c_s = f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic numerators:

$$n_s = \Big[(\varepsilon_1 \cdot \varepsilon_2) p_1^\mu + 2(\varepsilon_1 \cdot p_2)\varepsilon_2^\mu - (1 \leftrightarrow 2)\Big]\Big[(\varepsilon_3 \cdot \varepsilon_4) p_{3\mu} + 2(\varepsilon_3 \cdot p_4)\varepsilon_{4\mu} - (3 \leftrightarrow 4)\Big]$$
$$+ s\big[(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) - (\varepsilon_1 \cdot \varepsilon_4)(\varepsilon_2 \cdot \varepsilon_3)\big] ,$$

$c_s + c_t + c_u = 0$ Jacobi Id. (gauge invariance)

$\Leftrightarrow$

$n_s + n_t + n_u = 0$ kinematic Jacobi Id. (diffeomorphism inv.)

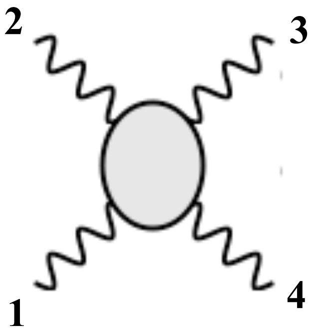
BCJ ('08)

# Double copy

Color and kinematics are dual...

$$c_s + c_t + c_u = 0 \qquad \Leftrightarrow \qquad n_s + n_t + n_u = 0$$

...replace color by kinematics $\quad c_i \to n_i \quad$ BCJ double copy




$$= \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

← gravity ampl.

Properties of ampl:
- spin-2 scattering $\qquad \varepsilon_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$
- 2-derivative interactions $\qquad \partial_\mu \to \partial_\mu \partial_\nu$
- diffeomorphism inv. $\qquad \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

$$\left.\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right|_{\varepsilon_4^{\mu\nu} \to p_4^\mu \varepsilon_4^\nu + p_4^\nu \varepsilon_4^\mu} = 2(n_s + n_t + n_u)\, \alpha(\varepsilon, p) = 0$$

# What is the Kinematic Algebra ?

– YM numerators obey Jacobi Id. → a kinematic algebra should exist!

– Algebra *may* dramatically simplify GR calculations!

**What is known?**

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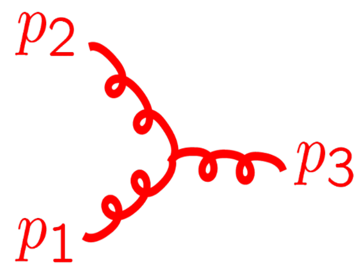
**Self dual YM in light-cone gauge:** Monteiro, O'Connell ('11)




Generators of area-preserving diffeomorphisms:

$$L_k = e^{-ik\cdot x}(-k_w\partial_u + k_u\partial_w)$$

Lie Algebra: $[L_{p_1}, L_{p_2}] = iX(p_1,p_2)L_{p_1+p_2} = iF_{p_1p_2}{}^{k}L_k$

YM vertex

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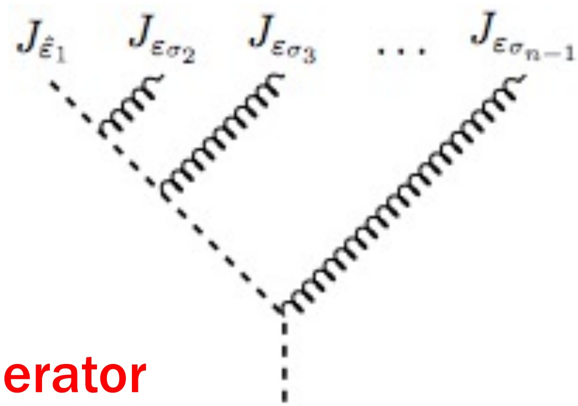
Beyond the simplest helicity sectors (NMHV)

Chen, HJ, Teng, Wang [1906.10683, 2104.12726]




$$J_{\hat{\varepsilon}_1}(p) \star J_{\varepsilon_i}(p_i) = \varepsilon_i\cdot p\, J_{\hat{\varepsilon}_1}(p+p_i) - \frac{1}{2}J_{\hat{\varepsilon}_1\otimes\varepsilon_i\otimes(p+p_i)}(p+p_i)$$

vector generator          tensor generator

# Cheung-Shen Lagrangian

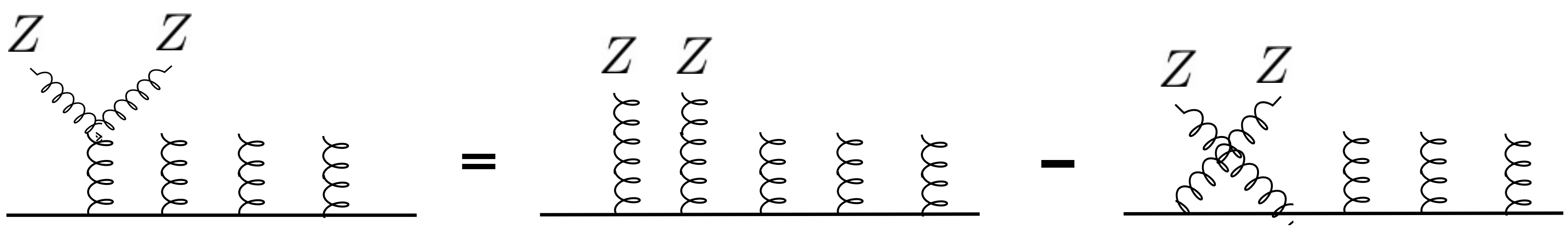
Cubic Lagrangian that manifests color-kinematics duality, gives:
→ NLSM pions at tree level
→ YM trees for MHV sector

Cheung, Shen ('16)

$$\mathcal{L}_{\rm CS} = Z^{a\mu}\Box X^a_\mu + \frac{1}{2}Y^a\Box Y^a - g f^{abc} Z^{a\mu}\left(Z^{b\nu}X^c_{\mu\nu} + Y^b\partial_\mu Y^c\right)$$

Jacobi Id. manifest:

$$X^a_{\mu\nu} = \partial_\mu X^a_\nu - \partial_\nu X^a_\mu \,;$$




NLSM pions: external states $Y^a$ or $\partial_\mu Z^{a\mu}$

Gives all YM numerator terms of type: $n^{\rm YM} \sim \; (\varepsilon_1\cdot\varepsilon_n)\prod_{i,j}(\varepsilon_i\cdot p_j)$

sufficient for MHV amplitude: Chen, HJ, Teng, Wang

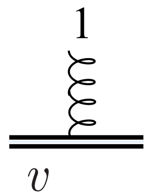
# Hopf algebra structure and heavy mass EFT

## Gauge invariant BCJ numerators from heavy-quark limit

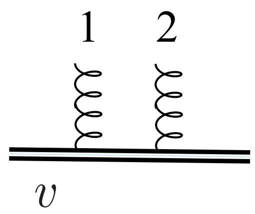
Brandhuber, Chen, HJ, Travaglini, Wen '21

$$F_i^{\mu\nu} := p_i^\mu \varepsilon_i^\nu - \varepsilon_i^\mu p_i^\nu$$
$$V_\tau^{\mu\nu} := v^\mu \sum_{j\in\tau} p_j^\nu$$

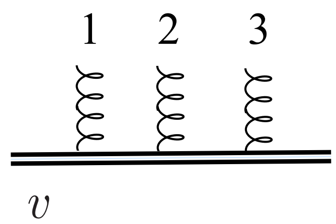
$$\mathcal{N}(1,v) = v\cdot\varepsilon_1\,,$$




$$\mathcal{N}(12,v) = -\frac{v\cdot F_1\cdot F_2\cdot v}{2v\cdot p_1}\,,$$




$$\mathcal{N}(123,v) = \frac{v\cdot F_1\cdot F_2\cdot F_3\cdot v}{3v\cdot p_1} - \frac{v\cdot F_1\cdot F_2\cdot V_{12}\cdot F_3\cdot v}{3v\cdot p_1 v\cdot p_{12}} - \frac{v\cdot F_1\cdot F_3\cdot V_1\cdot F_2\cdot v}{3v\cdot p_1 v\cdot p_{13}}$$

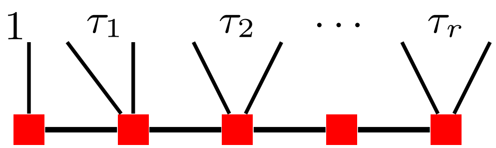



## YM numerators at any multiplicity given by an associative Hopf algebra

$$\mathcal{N}(12\ldots n{-}2, v) := \langle T_{(1)} \star T_{(2)} \star \cdots \star T_{(n-2)} \rangle$$

**Quasi-shuffle product:** $T_{(12)} \star T_{(3)} = -T_{(123)} + T_{(12),(3)} + T_{(13),(2)}$

$$\langle T_{(1\tau_1),(\tau_2),\ldots,(\tau_r)} \rangle :=$$



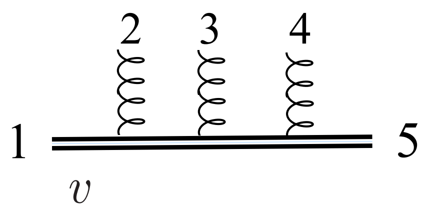

$$= \frac{v\cdot F_{1\tau_1}\cdot V_{\Theta(\tau_2)}\cdot F_{\tau_2}\cdots V_{\Theta(\tau_r)}\cdot F_{\tau_r}\cdot v}{(n{-}2)v\cdot p_1 v\cdot p_{1\tau_1}\cdots v\cdot p_{1\tau_1\tau_2\cdots\tau_{r-1}}}$$

# Heavy mass numerators for PM calculations

Efficient calculations for Post Minkovskian corrections:

Brandhuber, Chen, Travaglini, Wen [2108.04216]

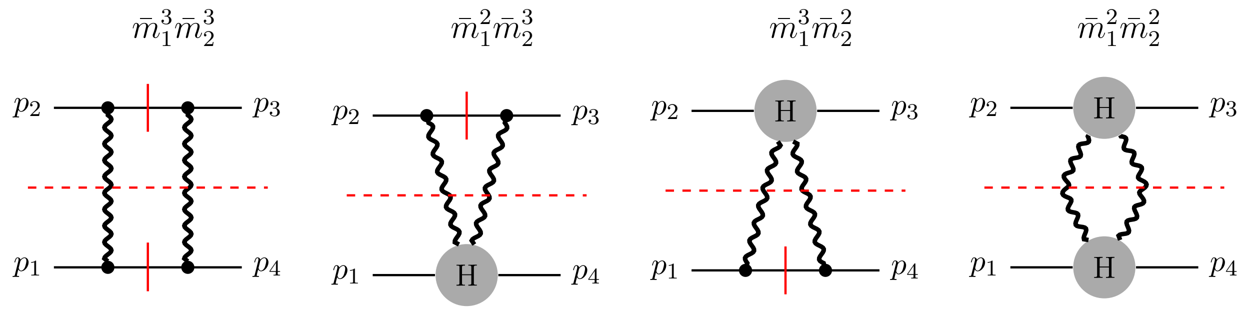
$$A_5^{\mathrm{YM-M}}(234, v) = \frac{\mathcal{N}_5([[2,3],4],v)}{s_{234}s_{23}} + \frac{\mathcal{N}_5([2,[3,4]],v)}{s_{234}s_{34}}$$



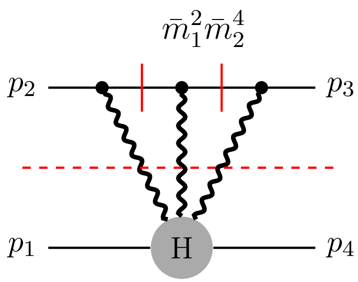

Double copy for massive scalar (Schwarzschild BH):

$$A_5^{\mathrm{GR-M}}(234, v) = \frac{\left[\mathcal{N}_5([[2,3],4],v)\right]^2}{s_{234}s_{23}} + \frac{\left[\mathcal{N}_5([[2,4],3],v)\right]^2}{s_{234}s_{24}} + \frac{\left[\mathcal{N}_5([[3,4],2],v)\right]^2}{s_{234}s_{34}}$$

2PM:




3PM:




+ more

Reproduces 3PM calculations of Bern, Cheung, Roiban, Shen, Solon, Zeng ('20) and Kälin, Liu, Porto ('20)

# Progress on the Kinematic Algebra ?

Recent surprise:

Ben-Shahar, HJ

A complete QFT with straightforward kinematic algebra at tree and loop level.

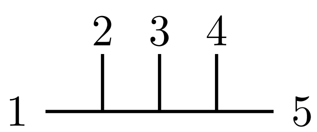
Generators $$L^\mu(p) = e^{ip\cdot x}\Delta^{\mu\nu}\partial_\nu$$

3D transversality "projector" $$\Delta^{\mu\nu}(p) = i\epsilon^{\rho\mu\nu}p_\rho$$

Infinite-dimensional kinematic Lie algebra $$[L^\mu(p_1), L^\nu(p_2)] = F^{\mu\nu}{}_\rho\, L^\rho(p_1+p_2)$$

Kinematic structure constants $$F^{\mu_1\mu_2}{}_\nu(p_1,p_2) = \Delta^{\rho\mu_1}(p_1)\epsilon_{\rho\nu\sigma}\Delta^{\sigma\mu_2}(p_2)$$

BCJ numerators




$$= \mathrm{tr}\Big([[[L^{\mu_1}(p_1), L^{\mu_2}(p_2)], L^{\mu_3}(p_3)], L^{\mu_4}(p_4)], L^{\mu_5}_{\mathrm{amp}}(p_5)\Big)$$

$$= F^{\mu_1\mu_2}{}_\nu F^{\nu\mu_3}{}_\rho F^{\rho\mu_4\mu_5}\delta^3(p_1+p_2+p_3+p_4+p_5)\ ,$$

Lie algebra of 3D volume-preserving diffeomorphisms!

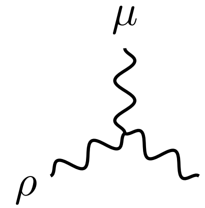
# Chern-Simons theory – off-shell C/K duality

Ben-Shahar, HJ

Pure Chern-Simons theory (tree-level action)

$$S = \frac{k}{4\pi}\int \mathrm{Tr}\Big(A\wedge dA + \frac{2i}{3}A\wedge A\wedge A\Big)$$

cubic Feynman rules: $A_\mu \sim\sim A_\rho = -\frac{\epsilon_{\mu\nu\rho}p^\nu}{p^2}$




$= -\frac{\epsilon^{\mu\nu\rho}}{\sqrt{2}}$

in Lorenz gauge these obeys color-kinematics duality off shell !

Chern-Simons is toplogical → amplitudes vanish,
but off-shell correlation fn's are non-zero

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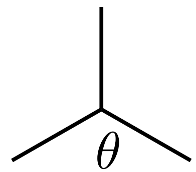
Quantum CS action includes Faddeev-Popov ghosts.
Can be packaged into superfield: $\Psi = c + \theta_\mu A^\mu + \theta_\mu\theta_\nu C^{\mu\nu} + \theta_1\theta_2\theta_3 a$

Feynman rules: $\theta \xrightarrow[p]{} \tilde{\theta} = \frac{p\cdot\vartheta}{p^2}\delta^3(\theta-\tilde{\theta})$




$= i\int d^3\theta$

again obeys color-kinematics duality!

# Double Copy Theories

# Example: pure GR

Pure 4D Einstein gravity: $\mathcal{S} = \frac{1}{2}\int d^4x\sqrt{g}R$

HJ, Ochirov

Does not match YM² spectrum: $\mathrm{YM} \otimes \mathrm{YM} = \mathrm{GR} + \phi + a$

Deform YM theories with massless fundamental quarks

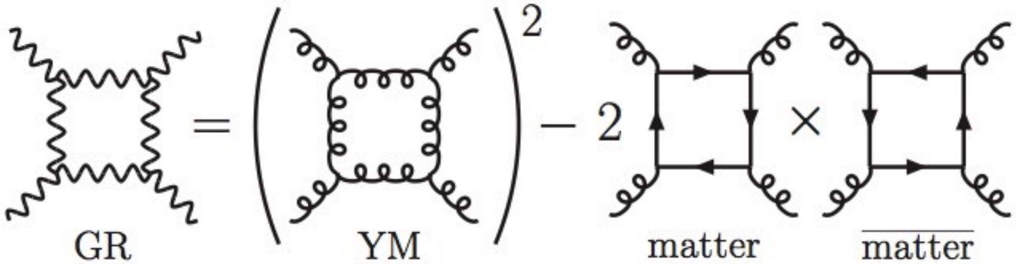
$$(\mathrm{YM} + \mathrm{quark}) \otimes (\mathrm{YM} + n_f\ \mathrm{quarks})$$
$$= \mathrm{GR} + 2(n_f + 1)\ \mathrm{scalars}$$

Anti-align the spins of the quarks → gives scalars in GR

e.g. $\phi = q \otimes \bar{q} + \bar{q} \otimes \bar{q}$

$a = q \otimes \bar{q} - \bar{q} \otimes \bar{q}$

become ghosts if $n_f = -1$



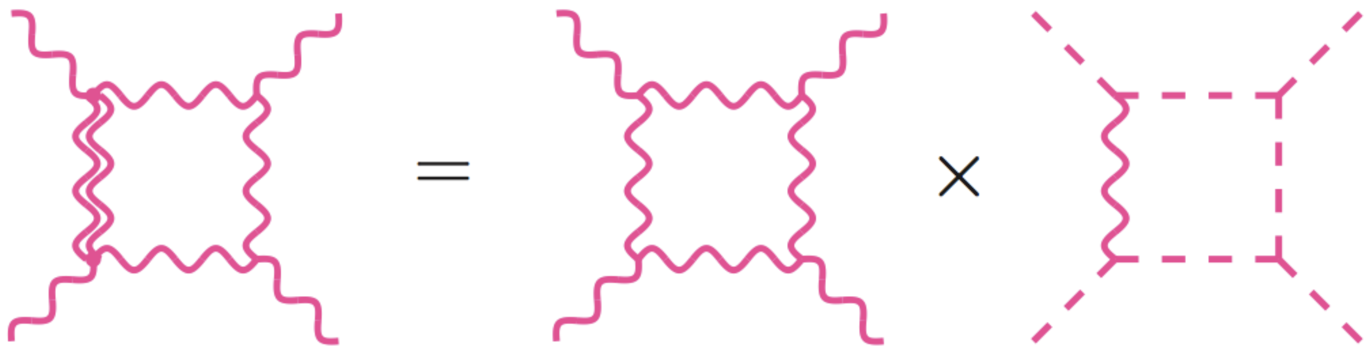
# Example: YM-Einstein theory

## GR+YM amplitudes are "heterotic" double copies

$$\mathrm{GR} + \mathrm{YM} = \mathrm{YM} \otimes (\mathrm{YM} + \phi^3)$$

Chiodaroli, Gunaydin, HJ, Roiban

$$h^{\mu\nu} \sim A^\mu \otimes A^\nu$$
$$A^{\mu a} \sim A^\mu \otimes \phi^a$$



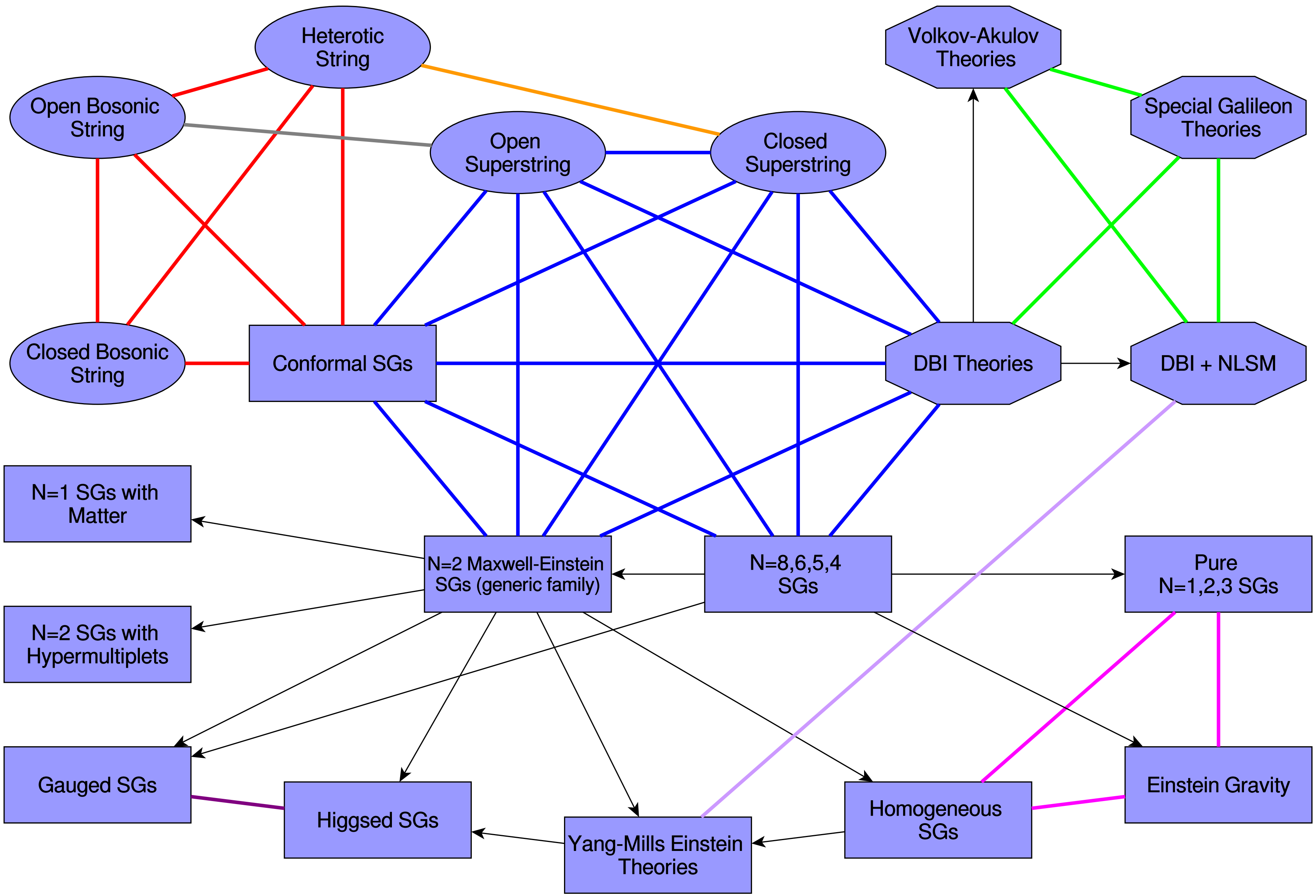
$\mathcal{N}$= 0,1,2,4 YM-E supergravity

$\mathcal{N}$= 0,1,2,4 SYM

YM + $\phi^3$

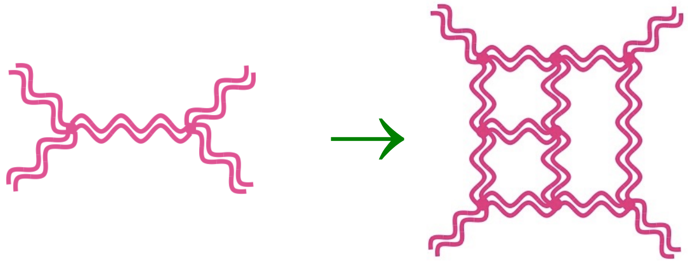
- $\mathcal{N}$=0,1,2 YM-E all have axion-dilaton states $\rightarrow$ $g, \theta$ parameters
- Construction extends to SSB (Coulomb branch) Chiodaroli, Gunaydin, HJ, Roiban ('15)

# Web of double-copy constructible theories




See reviews [1909.01358], [2203.13013] – Bern, Carrasco, Chiodaroli, HJ, Roiban

# Generalizations of C/K & double copy

Trees → loops:  Bern, Carrasco, HJ ('10)

→ Theories that are not truncations of *N*=8 SG HJ, Ochirov; Chiodaroli, Gunaydin, Roiban,...
→ Theories with fundamental matter HJ, Ochirov; Chiodaroli, Gunaydin, Roiban,...
→ Spontaneously broken theories (gauge/susy) Chiodaroli, Gunaydin, HJ, Roiban
→ Form factors Boels, Kniehl, Tarasov, Yang & CFT correlators Farrow, Lipstein, McFadden
→ Gravity off-shell symmetries from YM Anastasiou, Borsten, Duff, Hughes, Nagy,...
→ Classical (black hole) solutions Luna, Monteiro, O'Connell, White; Ridgway, Wise; Goldberger,...
→ Gravitational radiation/potential Luna, Monteiro, Nicholson, O'Connell, White; Goldberger,.. Bern, Cheung, Roiban, Solon; Bjerrum-Bohr et al. ..
→ Amplitudes in curved background Adamo, Casali, Mason, Nekovar
→ CHY scattering eqs, twistor strings Cachazo, He, Yuan, Skinner, Mason, Geyer, Adamo, Monteiro,..
→ Scalar EFTs: NLSM, DBI, Galileon Cachazo, He, Yuan; Du, Chen; Cheung, Shen; Elvang et al.
→ New double copies for string theory Mafra, Schlotterer, Stieberger, Taylor, Broedel, Carrasco... ... Azevedo, Marco Chiodaroli, HJ, Schlotterer
→ Conformal gravity HJ, Nohle; Mogull, Teng
→ Celestial amplitudes Casali, Puhm

# Exception that proves the rule…

Not all gauge theories obey color-kinematics duality

Imagine the double copy:

$$\mathrm{YM} \otimes (\mathrm{YM} + \mathcal{N} \text{ adjoint fermions}) \stackrel{?}{=} \mathrm{GR} + \mathcal{N}\ \Psi_{3/2}$$

According to conventional wisdom $\Psi_{3/2}$ must be a gravitino and $\mathcal{N} \leq 8$ is the number of supersymmetries

What goes wrong? The theory

$$\mathrm{YM} + \mathcal{N} \text{ adjoint fermions} + \ldots$$

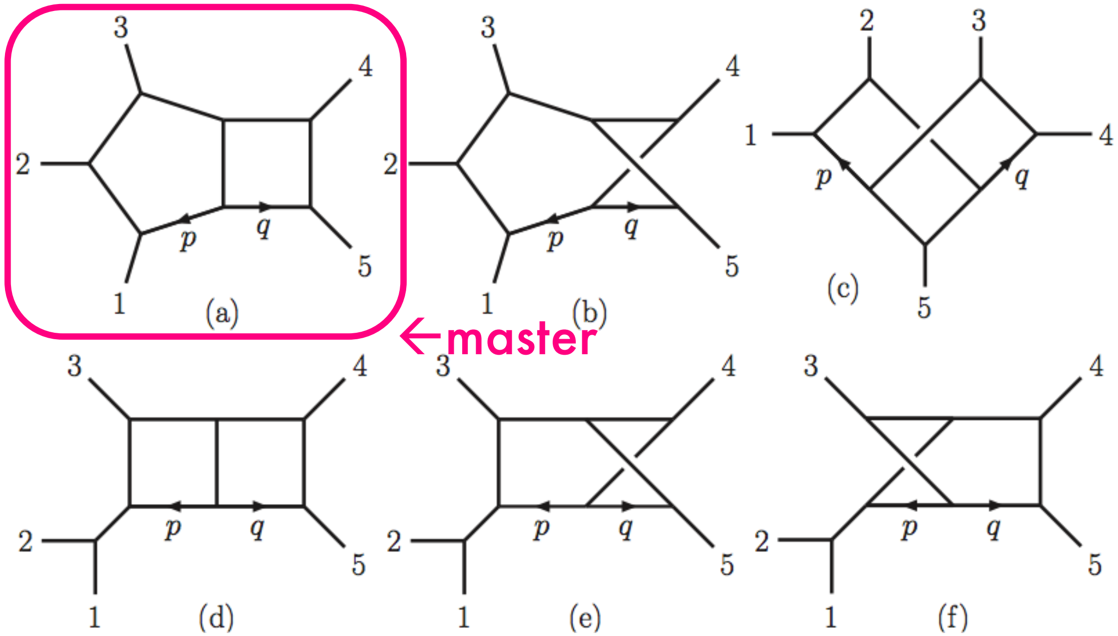
only obeys color-kinematics duality if supersymmetric → $\mathcal{N} \leq 4$

Kinematic Jacobi Id. → Fierz Id. that enforces SUSY

Chiodaroli, Jin, Roiban

# Multiloop calculations w/ duality and double copy

# Example: 2-loop 5-pts $\mathcal{N}=4$ SYM & $\mathcal{N}=8$ SG




Carrasco, HJ
1106.4711

color-kinematics duality
+ unitarity method
→ simple numerators

| $\mathcal{I}^{(x)}$ | $\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator |
|---|---|
| (a),(b) | $\frac{1}{4}\Big(\gamma_{12}(2s_{45}-s_{12}+\tau_{2p}-\tau_{1p})+\gamma_{23}(s_{45}+2s_{12}-\tau_{2p}+\tau_{3p})$ $+2\gamma_{45}(\tau_{5p}-\tau_{4p})+\gamma_{13}(s_{12}+s_{45}-\tau_{1p}+\tau_{3p})\Big)$ |
| (c) | $\frac{1}{4}\Big(\gamma_{15}(\tau_{5p}-\tau_{1p})+\gamma_{25}(s_{12}-\tau_{2p}+\tau_{5p})+\gamma_{12}(s_{34}+\tau_{2p}-\tau_{1p}+2s_{15}+2\tau_{1q}-2\tau_{2q})$ $+\gamma_{45}(\tau_{4q}-\tau_{5q})-\gamma_{35}(s_{34}-\tau_{3q}+\tau_{5q})+\gamma_{34}(s_{12}+\tau_{3q}-\tau_{4q}+2s_{45}+2\tau_{4p}-2\tau_{3p})\Big)$ |
| (d)-(f) | $\gamma_{12}s_{45}-\frac{1}{4}\Big(2\gamma_{12}+\gamma_{13}-\gamma_{23}\Big)s_{12}$ |

$$\tau_{ip}=2k_i\cdot p$$

direct calculation in GR would naively give $\sim 10^{18}$ terms

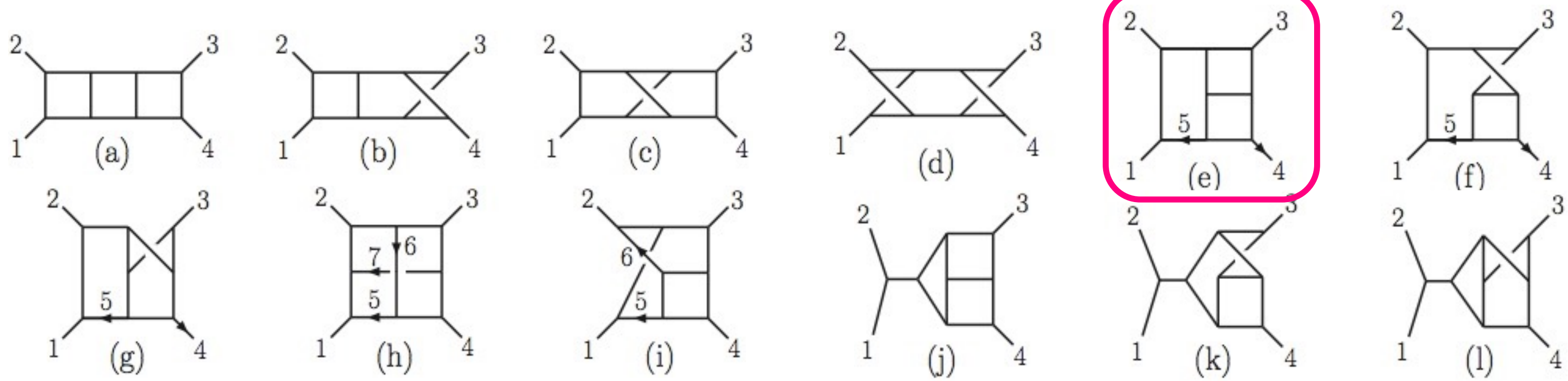
# 3-loop $\mathcal{N}$=8 SG & $\mathcal{N}$=4 SYM

**Color-kinematics dual form:**

Bern, Carrasco, HJ

$$N^{(\mathrm{e})} = s(\tau_{45} + \tau_{15}) + \frac{1}{3}(t - s)(s + \tau_{15} - \tau_{25})$$

$\tau_{ij} = 2k_i \cdot l_j$



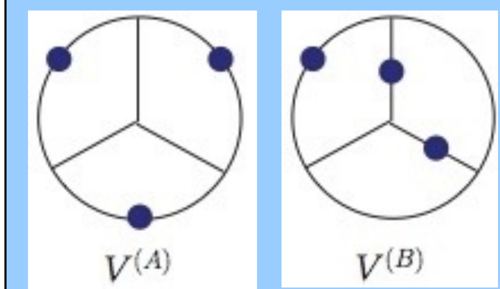

**Used to compute UV divergences (in 6*D*):**

Bern, Carrasco, Dixon, HJ, Roiban

$$\mathcal{A}^{(3)}\Big|_{\text{pole}} = 2g^8 st A^{\text{tree}}(N_c^3 V^{(\mathrm{A})} + 12N_c(V^{(\mathrm{A})} + 3V^{(\mathrm{B})})) \times (u\mathrm{Tr}[T^{a_1}T^{a_2}T^{a_3}T^{a_4}] \; + \; \text{perms})$$

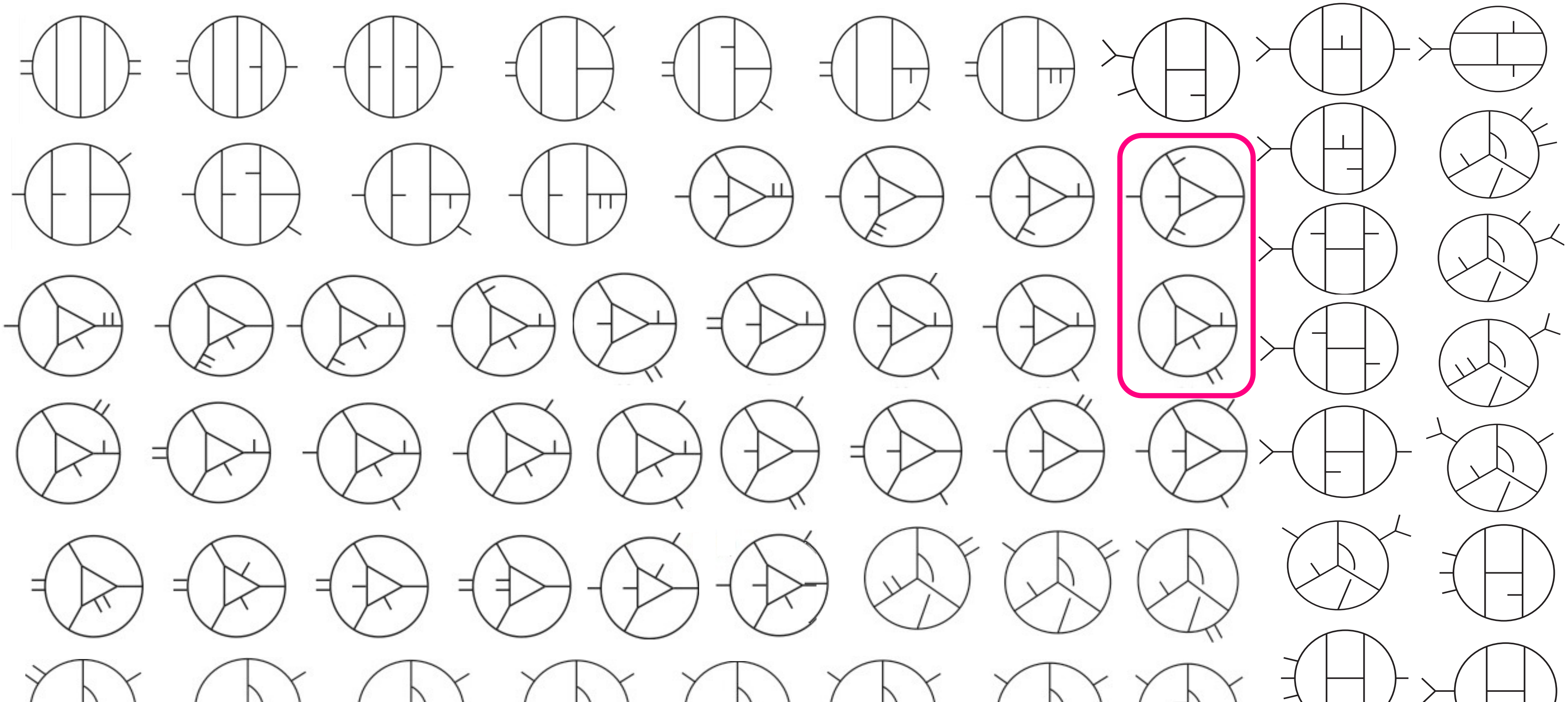
$$\mathcal{M}^{(3)}\Big|_{\text{pole}} = 10\left(\frac{\kappa}{2}\right)^8 (stu)^2 M^{\text{tree}}(V^{(\mathrm{A})} + 3V^{(\mathrm{B})})$$

$$V^{(\mathrm{A})} + 3V^{(\mathrm{B})} = \frac{\zeta_3}{6}$$




naïve calculation → $\sim 10^{21}$ terms

# 4-loops: 85 diagrams, 2 masters



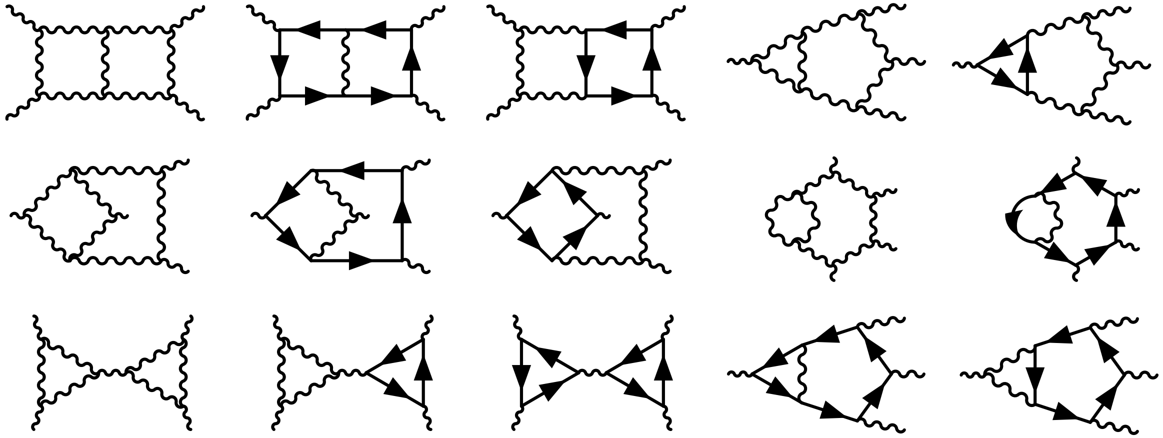
$$
\begin{aligned}
N_{18} = \; & \tfrac{1}{4}(6u^2\tau_{25} + u(2s(5\tau_{25}+2\tau_{26}) - \tau_{15}(7\tau_{16}+6t)) \\
& + t(\tau_{15}\tau_{26} - \tau_{25}(\tau_{16}+7\tau_{26})) + s(4\tau_{15}(t-\tau_{26}) + 6\tau_{36}(\tau_{35}-\tau_{45}) \\
& - \tau_{16}(4t+5\tau_{25}) - \tau_{46}(5\tau_{35}+\tau_{45})) + 2s^2(t+\tau_{26}-\tau_{35}+\tau_{36}+\tau_{56}))\,, \\
N_{28} = \; & \tfrac{1}{4}(s(2\tau_{15}t + \tau_{16}(2t-5\tau_{25}+\tau_{35}) + 5\tau_{35}(\tau_{26}+\tau_{36}) + 2t(2\tau_{46}-\tau_{56}) - 10u\tau_{25} \\
& - 4s^2\tau_{25} - 6u(\tau_{46}(t-\tau_{25}+\tau_{45}) + \tau_{25}\tau_{26}) - t(\tau_{15}(4\tau_{36}+5\tau_{46}) + 5\tau_{25}\tau_{36}))\,.
\end{aligned}
$$

# E.g. Complete *N*=2 SQCD 2-loop calculation

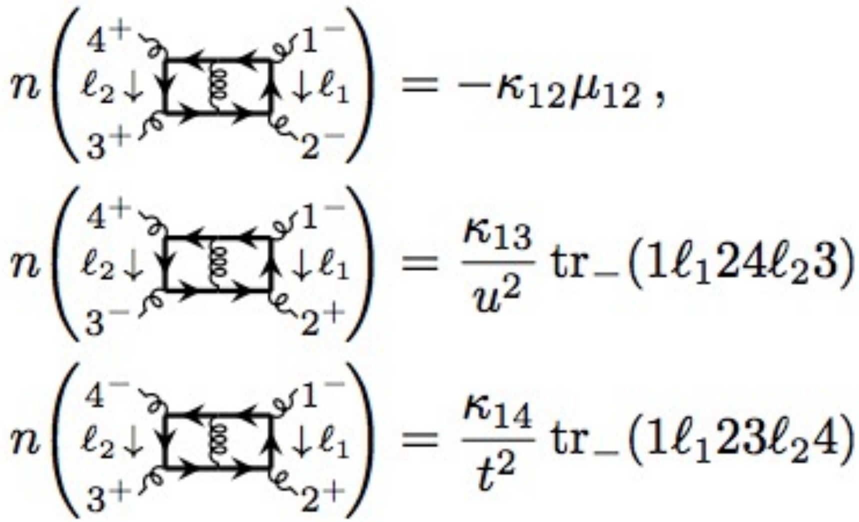
Integrand computed using color-kinematics duality and supersymmetric decomposition

HJ, Kälin, Mogull ('17)

- two-loop SQCD amplitude
- color-kinematics manifest
- planar + non-planar
- $N_f$ massless quarks
- integrand valid in $D \leq 6$



e.g. simple SQCD numerators




trace-rep. from
1811.09604
Kälin, Mogull, Ochirov

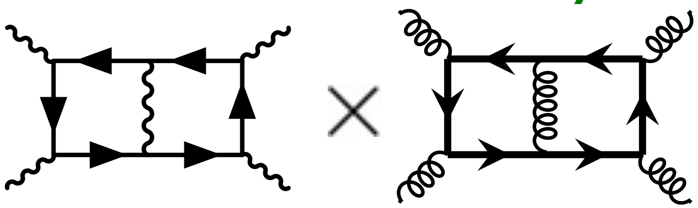
# $\mathcal{N}$=2 SUGRA double copies

Recall that double copy works
if one side obeys C-K duality

HJ, Ochirov;
Chiodaroli, Gunaydin, HJ, Roiban;
Ben-Shahar, Chiodaroli;
Mogull, Kälin, HJ

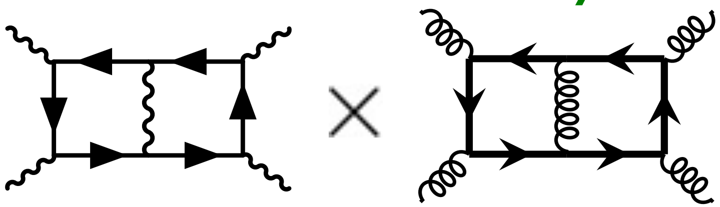
$$(\text{2-loop } \mathcal{N} = 2 \text{ SQCD}) \otimes (D\text{-dim. QCD Feynman rules})$$

*N*=2 SQCD × 4D Feynman



→ *N*=2 Luciani Model (1978)

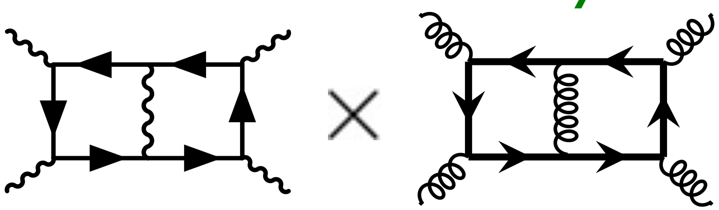
*N*=2 SQCD × 5D Feynman



→ *N*=2 Generic non-Jordan family

Günaydin, Sierra, Townsend (1986)

*N*=2 SQCD × 6D Feynman

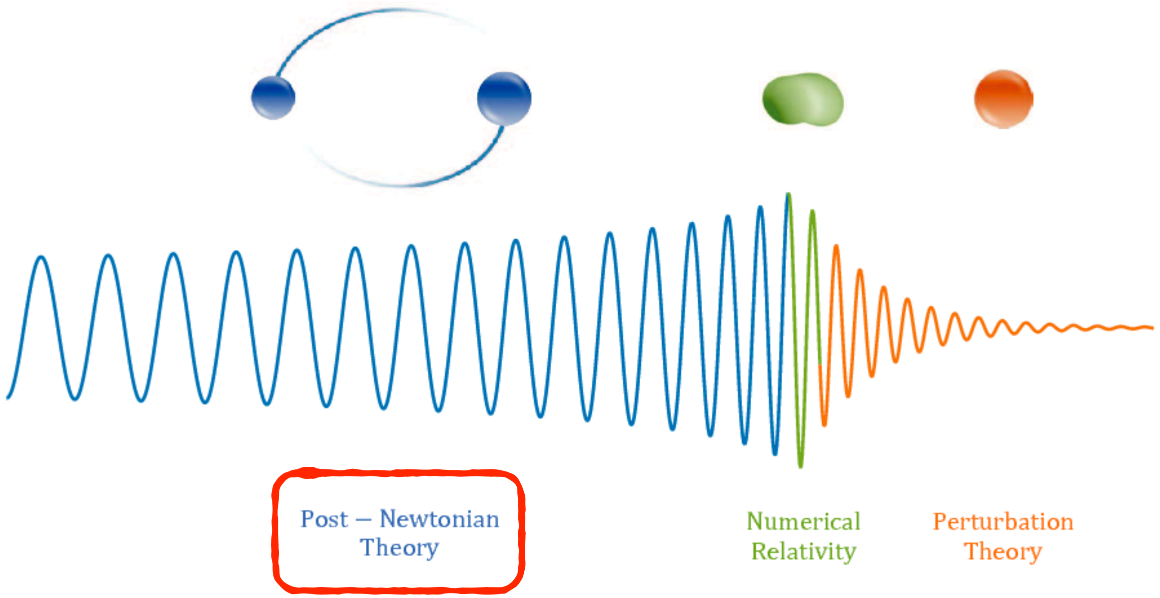


→ *N*=2 Generic Jordan family

Günaydin, Sierra, Townsend (1984)
(self-dual tensors in 6D)

# Double copy and black hole amplitudes

# Double copy and gravitational waves




**Explicit PM calculations done using double copy:**

Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove ('18)
Bern, Cheung, Roiban, Shen, Solon, Zeng ('19)+ Ruf, Parra-Martinez ('21)
Brandhuber, Chen, Travaglini, Wen (21)

**Some methods developed for PM calc. using double copy:**

Bjerrum-Bohr, Cristofoli, Damgaard, Gomez+Brown;
Cristofoli, Gonzo, Kosower, O'Connell;
Maybee, O'Connell, Vines; Luna, Nicholson, O'Connell, White; …

# AHH amplitudes $\leftrightarrow$ Kerr BH?

Arkani-Hamed, Huang, Huang ('17) wrote down natural higher-spin ampl's:

Gauge th 3pt:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}}, \qquad A(1\phi^s, 2\bar{\phi}^s, 3A^-) = \frac{m}{x}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Gravity 3pt:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+) = im^2x^2\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}}, \qquad M(1\phi^s, 2\bar{\phi}^s, 3h^-) = i\frac{m^2}{x^2}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Shown to reproduce Kerr by: Guevara, Ochirov, Vines ('18)

---

Gravity Compton ampl.
via BCFW recursion ?

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+, 4h^+) = i\frac{\langle \mathbf{12}\rangle^{2s}[34]^4}{m^{2s-4}s_{12}t_{13}t_{14}}$$

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-, 4h^+) = i\frac{[4|p_1|3\rangle^{4-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{s_{12}t_{13}t_{14}}.$$

spurious pole for $s > 2$

# What EFTs give the AHH amplitudes ?

Rewrite the 3pt AHH amplitudes on covariant form → identify theory

Chiodaroli, HJ, Pichini;
HJ, Ochirov

1) introduce generating series, e.g.

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A^+) = \frac{mx}{1 - \frac{\langle \mathbf{12} \rangle^2}{m^2}}$$

2) rewrite covariantly (for both helicity sectors):

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A) = A_{\phi\phi A} + \frac{A_{WWA} - (\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 A_{\phi\phi A}}{(1 + \boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 + \frac{2}{m^2} \boldsymbol{\varepsilon}_1 \cdot p_2 \, \boldsymbol{\varepsilon}_2 \cdot p_1}$$

$$A_{\phi\phi A} \equiv i\sqrt{2}\, \varepsilon_3 \cdot p_1 \,, \quad A_{WWA} \equiv i\sqrt{2}\, (\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2 \, \varepsilon_3 \cdot p_2 + \boldsymbol{\varepsilon}_2 \cdot \varepsilon_3 \, \boldsymbol{\varepsilon}_1 \cdot p_3 + \varepsilon_3 \cdot \boldsymbol{\varepsilon}_1 \, \boldsymbol{\varepsilon}_2 \cdot p_1)$$

$s = 0$ & $s = 1/2$ minimally coupled scalar & fermion

$s = 1$ W-boson $\quad$ $s = 3/2$ charged/massive gravitino

# EFTs for AHH 3pt gravity amplitudes?

Are related to the gauge th. ones via KLT

$$M(1\phi^s, 2\bar{\phi}^s, 3h^\pm) = iA(1\phi^{s_\mathrm{L}}, 2\bar{\phi}^{s_\mathrm{L}}, 3A^\pm)A(1\phi^{s_\mathrm{R}}, 2\bar{\phi}^{s_\mathrm{R}}, 3A^\pm)$$

Works for any decomposition: $s = s_\mathrm{L} + s_\mathrm{R}$

Preferred decomposition $s = 1 + (s-1)$ give fewest derivatives :

$$\sum_{2s=0}^{\infty} M(1\phi^s, 2\bar{\phi}^s, 3h) = M_{0\oplus 1/2} + A_{WWA}\Big(A_{0\oplus 1/2} + \frac{A_{1\oplus 3/2} - (\varepsilon_1\cdot\varepsilon_2)^2 A_{0\oplus 1/2}}{(1+\varepsilon_1\cdot\varepsilon_2)^2 + \frac{2}{m^2}\varepsilon_1\cdot p_2\,\varepsilon_2\cdot p_1}\Big)$$

From double-copy structure, we can infer:

$s=0,\ s=1/2,\ s=1,\ s=3/2$ minimally-coupled matter (Proca th, massive gravitino)

$s=2$ Kaluza-Klein graviton

Also works for Compton, and higher-point amplitudes (Lagrangians known)

# Summary of EFTs

The AHH ampl's for $s \leq 2$ admit double copies to any multiplicity

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \mathrm{scalar}) = (\mathrm{GR} + \mathrm{scalar})$$

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \mathrm{fermion}) = (\mathrm{GR} + \mathrm{fermion})$$

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \text{W-boson}) = (\mathrm{GR} + \mathrm{Proca})$$

$$(\mathrm{YM} + \text{W-boson}) \otimes (\mathrm{YM} + \mathrm{fermion}) = (\mathrm{GR} + \text{massive gravitino})$$

$$(\mathrm{YM} + \text{W-boson}) \otimes (\mathrm{YM} + \text{W-boson}) = (\mathrm{GR} + \text{massive KK graviton})$$

Lagrangians unique: have no non-minimal terms beyond cubic order in fields

Can be used for $(S^\mu)^{\leq 4}$ PM/PN calculations.

Compton $(S^\mu)^4$ yet to be confirmed via other methods (BHPT, worldline).

# What special about the EFTs ?

The $s \leq 1$ gauge theories and $s \leq 2$ gravities admit a massless limit

and all states that carries vector indices acquires a gauge symmetry

$s = 1$ (YM + W-boson) $\rightarrow$ non-abelian gauge symmetry

$s = 3/2$ (GR + massive gravitino) $\rightarrow$ supersymmetry

$s = 2$ (GR +massive KK graviton) $\rightarrow$ General covariance

(Note: we only study amplitudes with 2 massive states, and *n*-2 massless in which case the enlarged theories consistently truncate)

# Summary & Outlook

- Color-kinematics duality lies at the root of gravity:
  → makes perturbative GR more manageable!
  → allows for simpler classification of gravity theories
  → kinematic algebra is a well-hidden gem of YM (and GR)
  → useful for PM calculations

- Explored amplitudes for massive spinning matter → Kerr BH ?
  → Double copy works well up to spin-2 (KK graviton)
  → AHH amplitudes seems to be originating from theories with SSB ?
  → Paolo Pichini can give more details on higher-spin results

- Not discussed: String theories exhibit novel double copy structures.

$$\text{string tree ampl} = \text{String} \otimes \text{QFT}$$

  Azevedo, Chiodaroli, HJ, Schlotterer ('18)

- Not discussed: C/K duality in AdS space (Herderschee, Roiban, Teng; [...])
- Not discussed: Classical double copies of BH solutions (O'Connell et al. [...])

The topic of double copy & CK duality has grown significantly in the last few years, you will likely hear more about it in this KITP program!