



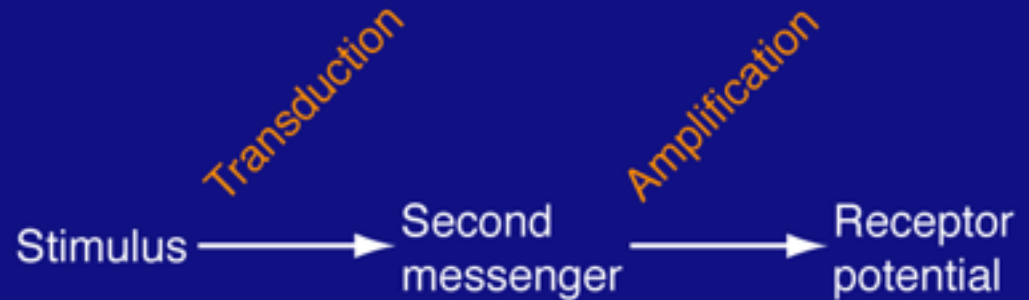
## ENERGETICS OF SENSORY TRANSDUCTION

Vision:

500-nm photon

$E \approx 240 \text{ kJ}\cdot\text{mol}^{-1}$

$\approx 400 \text{ zJ per molecule}$   
( $\sim 100\cdot\text{kT}$ )

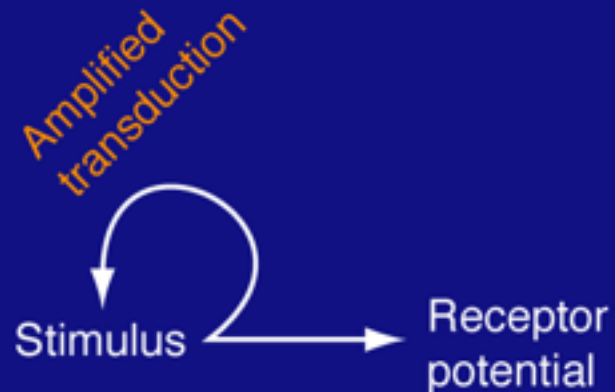


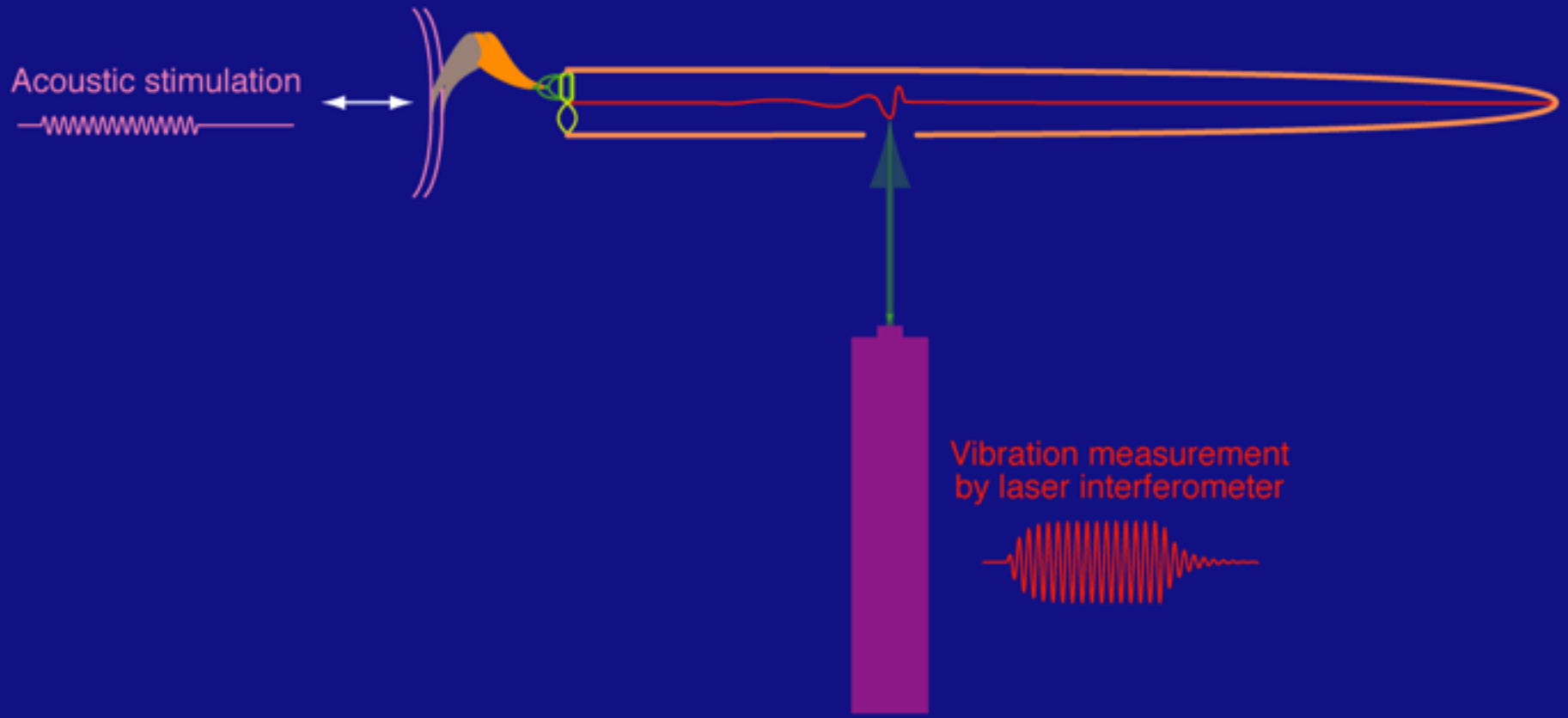
Hearing:

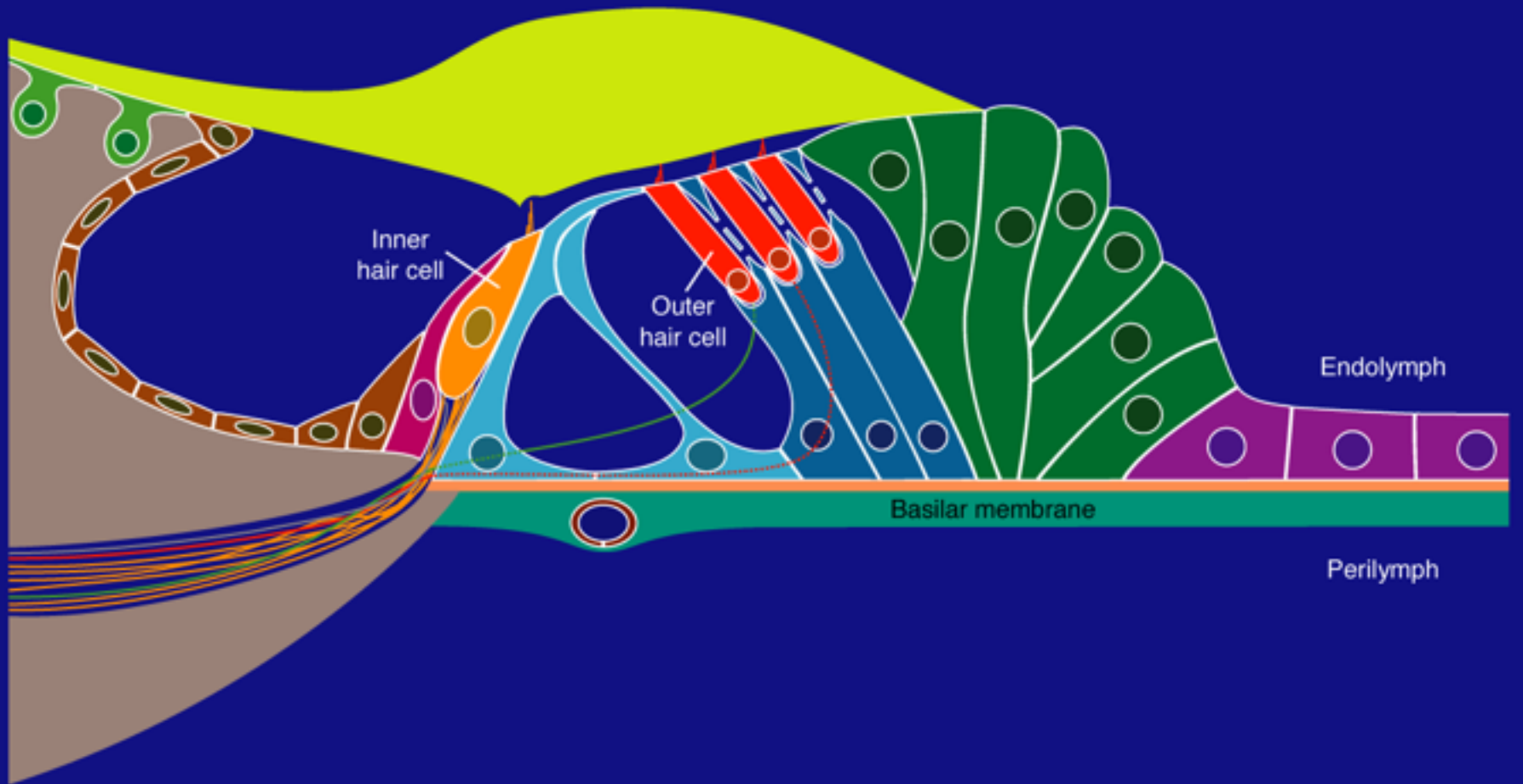
$\pm 0.3\text{-nm}$  vibration

$\Delta G \approx 0.3 \text{ kJ}\cdot\text{mol}^{-1}$  per cycle

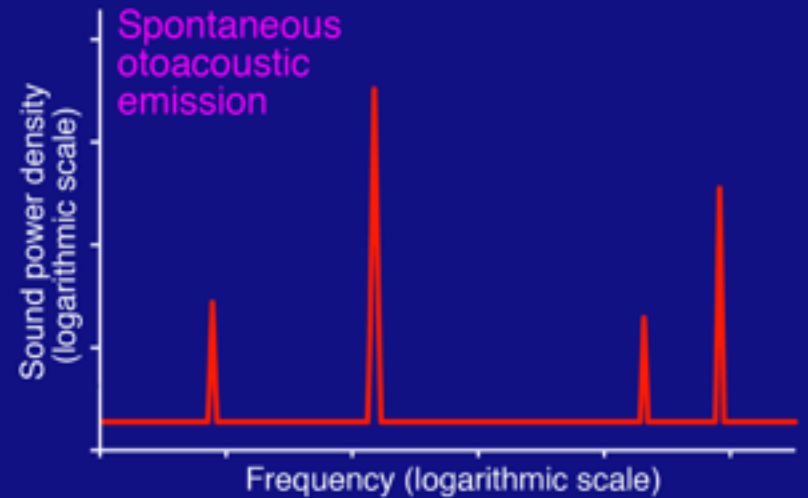
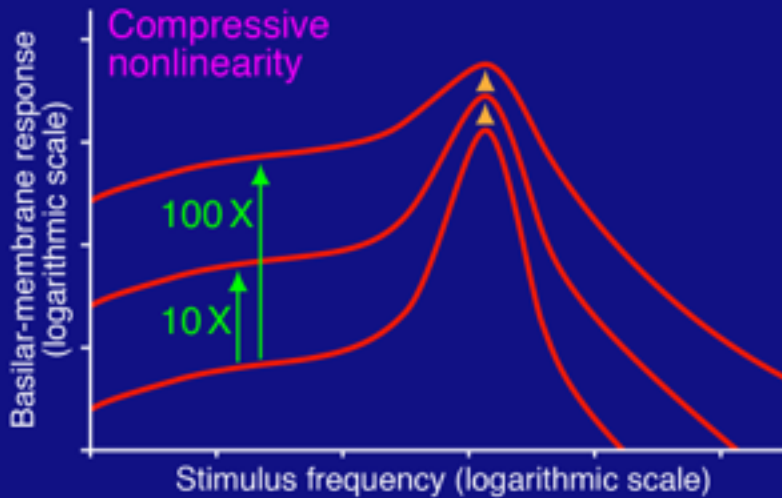
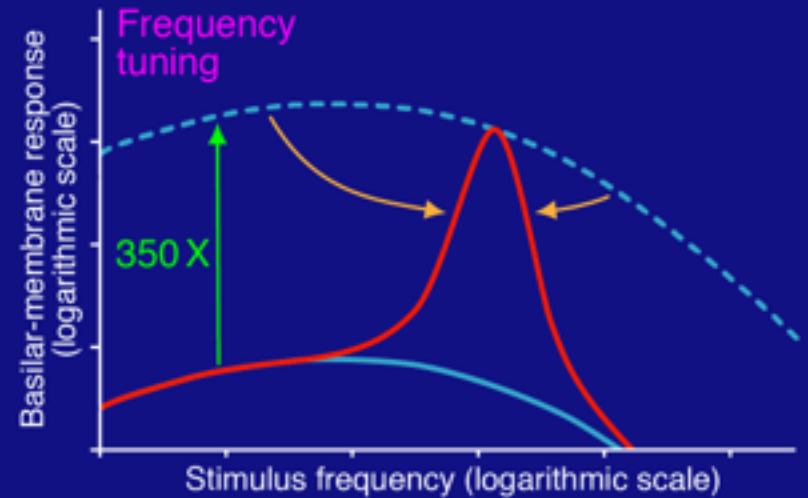
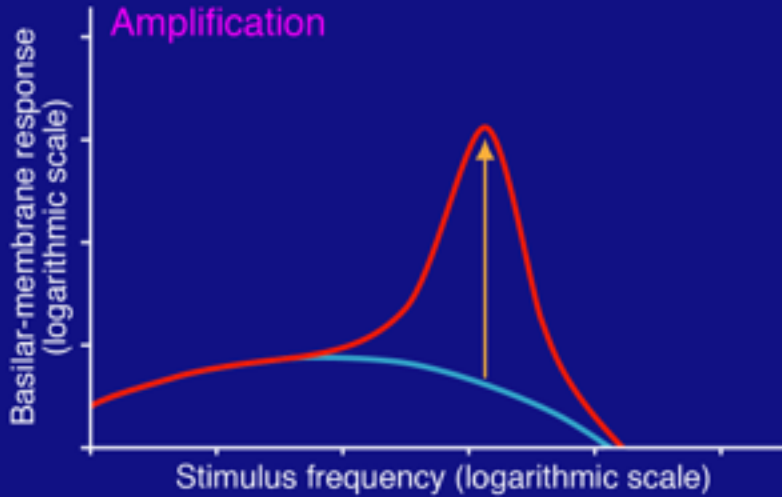
$\approx 0.5 \text{ zJ per molecule per cycle}$   
( $\sim 0.1\cdot\text{kT}$  per cycle)



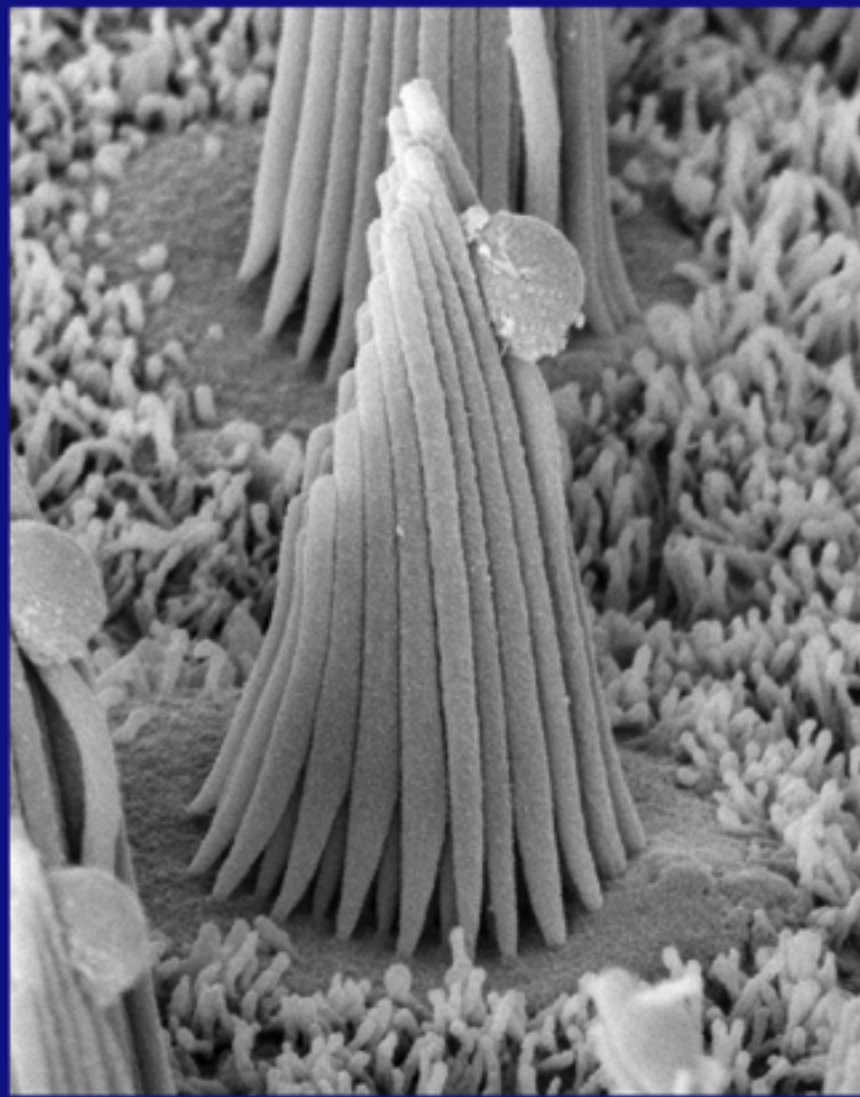


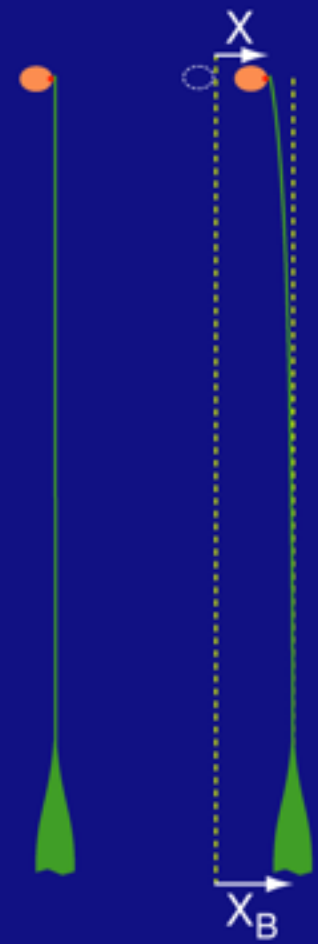
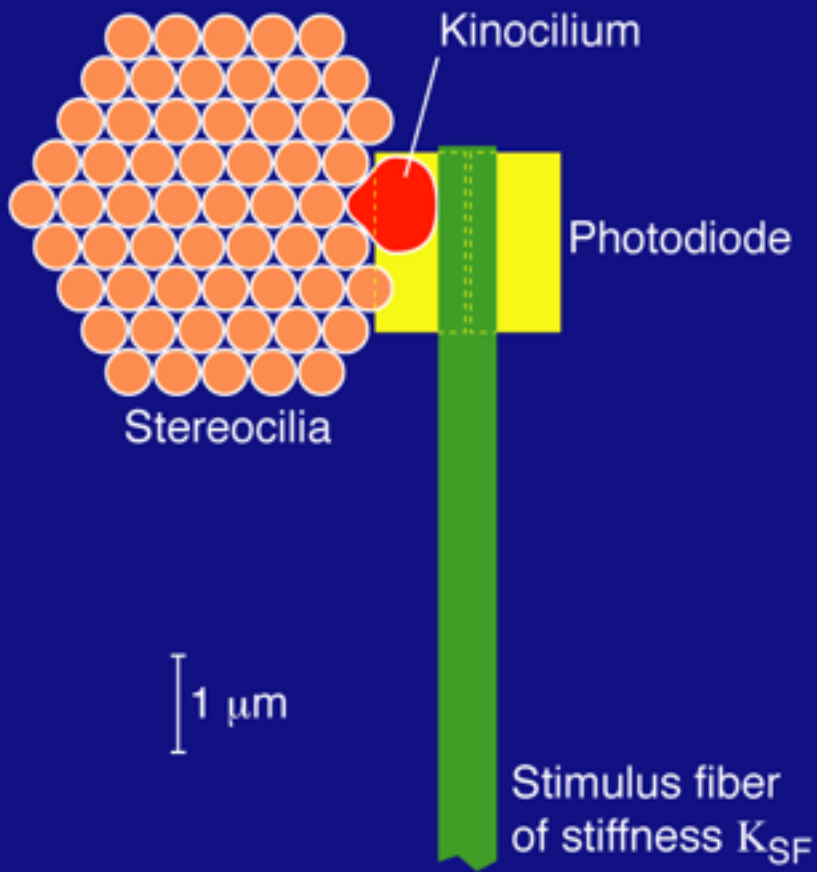


# THE ACTIVE PROCESS OF HAIR CELLS



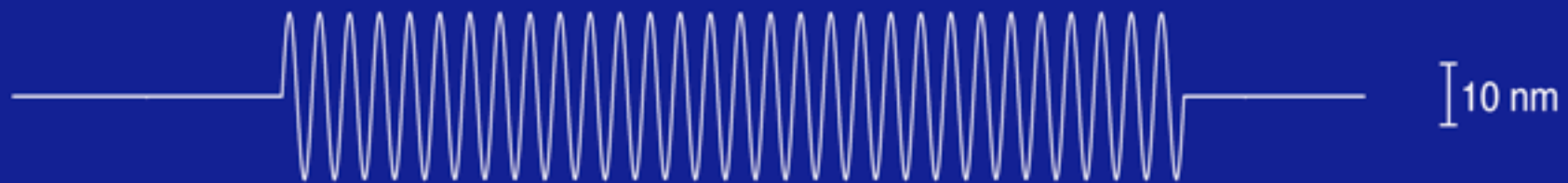
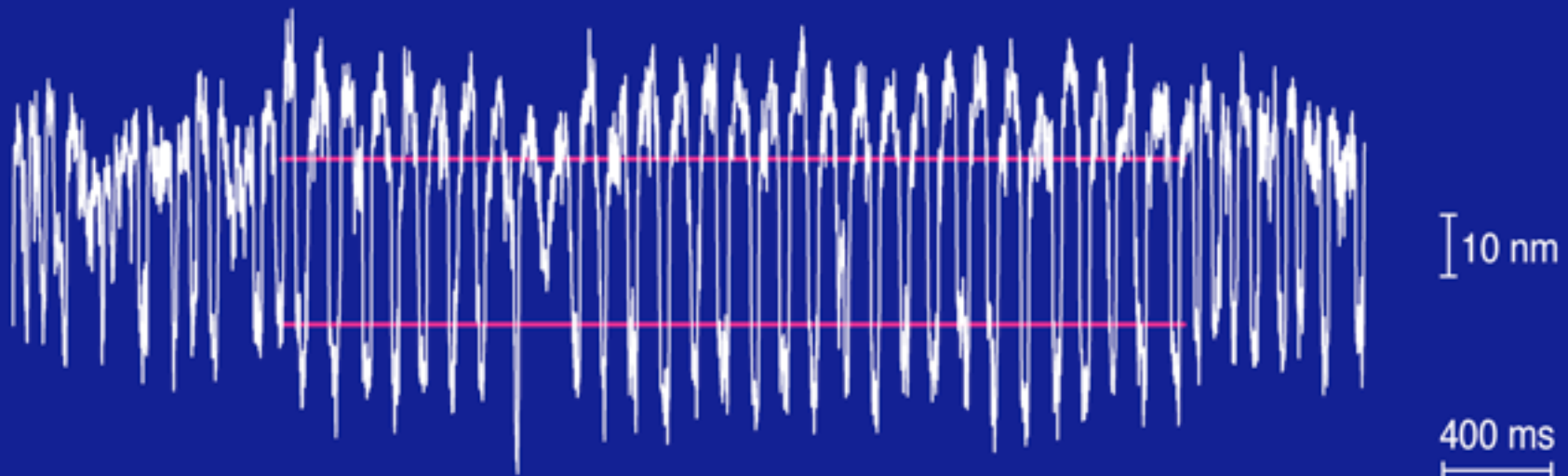
MANIFESTATIONS OF THE EAR'S ACTIVE PROCESS  
BY INDIVIDUAL HAIR BUNDLES





$$F = K_{SF}(X_B - X)$$





At any time,

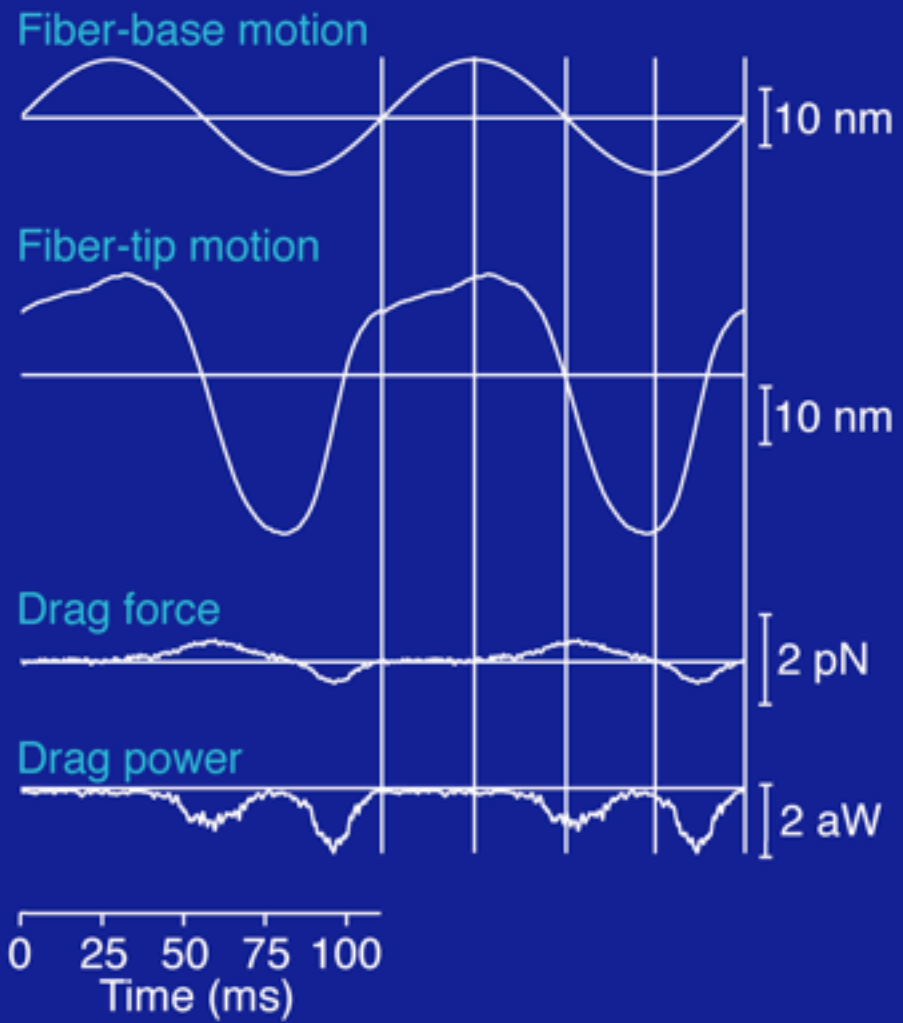
$$F_{\text{EXTERNAL}} + F_{\text{INERTIAL}} + F_{\text{DRAG}} + F_{\text{ELASTIC}} + F_{\text{ACTIVE}} = 0$$

$$F_{\text{ACTIVE}} = -K_{\text{SF}}(X_{\text{B}} - X) - (m_{\text{HB}} + m_{\text{SF}}) \cdot \frac{d^2X}{dt^2} - (\xi_{\text{HB}} + \xi_{\text{SF}}) \cdot \frac{dX}{dt} - K_{\text{HB}}X$$

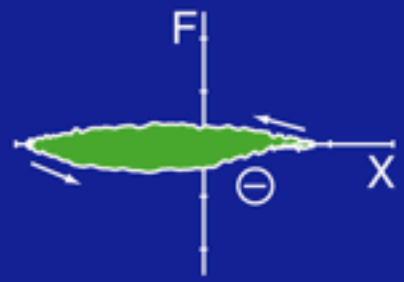
Over an average cycle,

$$\bar{W}_{\text{EXTERNAL}} + \cancel{\bar{W}_{\text{INERTIAL}}} + \bar{W}_{\text{DRAG}} + \cancel{\bar{W}_{\text{ELASTIC}}} + \bar{W}_{\text{ACTIVE}} = 0$$

$$\bar{W}_{\text{ACTIVE}} = -K_{\text{SF}} \oint (X_{\text{B}} - X) \cdot dX - (\xi_{\text{HB}} + \xi_{\text{SF}}) \oint \frac{dX}{dt} \cdot dX$$

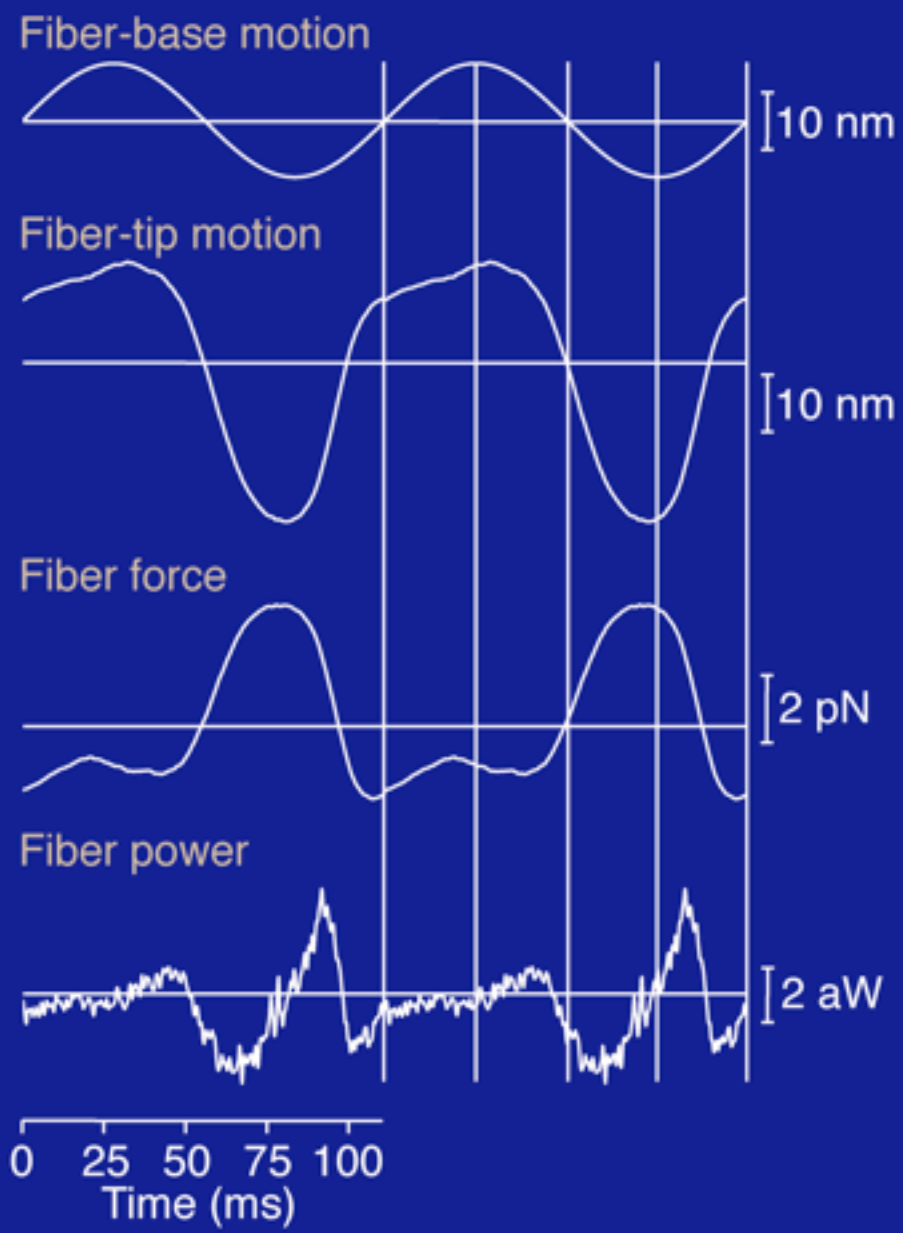


Work done on bundle by drag (-39 zJ)



10 nm

1 pN



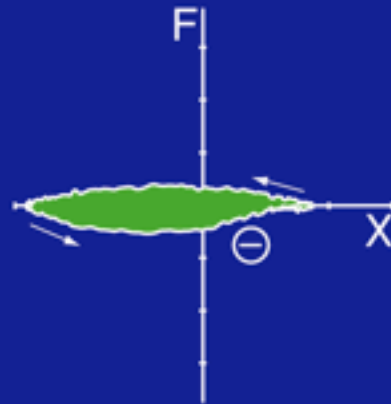
Work done on bundle by fiber (-40 zJ)



10 nm

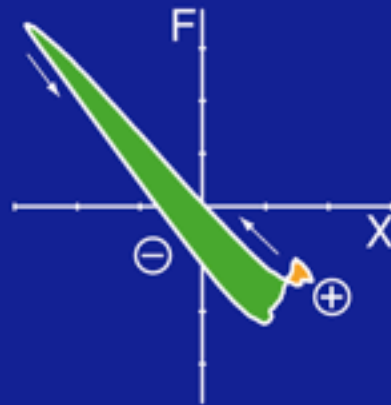
1 pN

Work done on bundle by drag (-39 zJ)



$$\bar{W}_{\text{ACTIVE}} = -\bar{W}_{\text{DRAG}} - \bar{W}_{\text{FIBER}}$$

Work done on bundle by fiber (-40 zJ)



$$\bar{W}_{\text{ACTIVE}} = +79 \text{ zJ}$$

10 nm

1 pN

## Outset of experiment



Work done  
on bundle  
by drag

Work done  
on bundle  
by fiber

$$\bar{W}_{\text{ACTIVE}} = -\bar{W}_{\text{DRAG}} - \bar{W}_{\text{FIBER}} = +48 \text{ zJ}$$

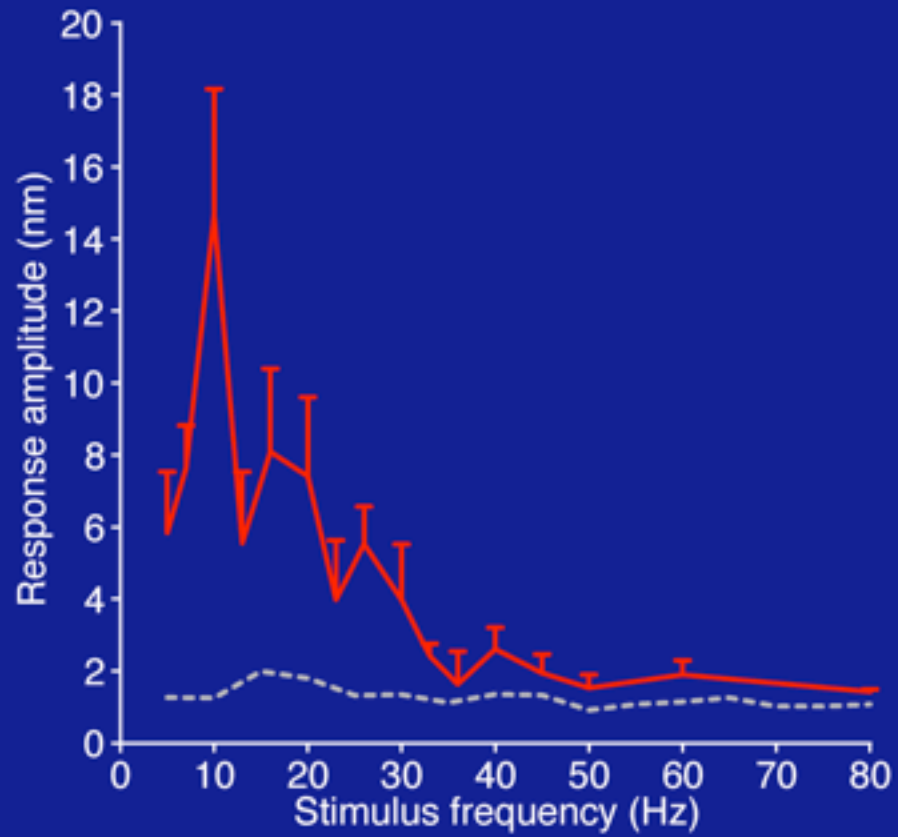
10 nm

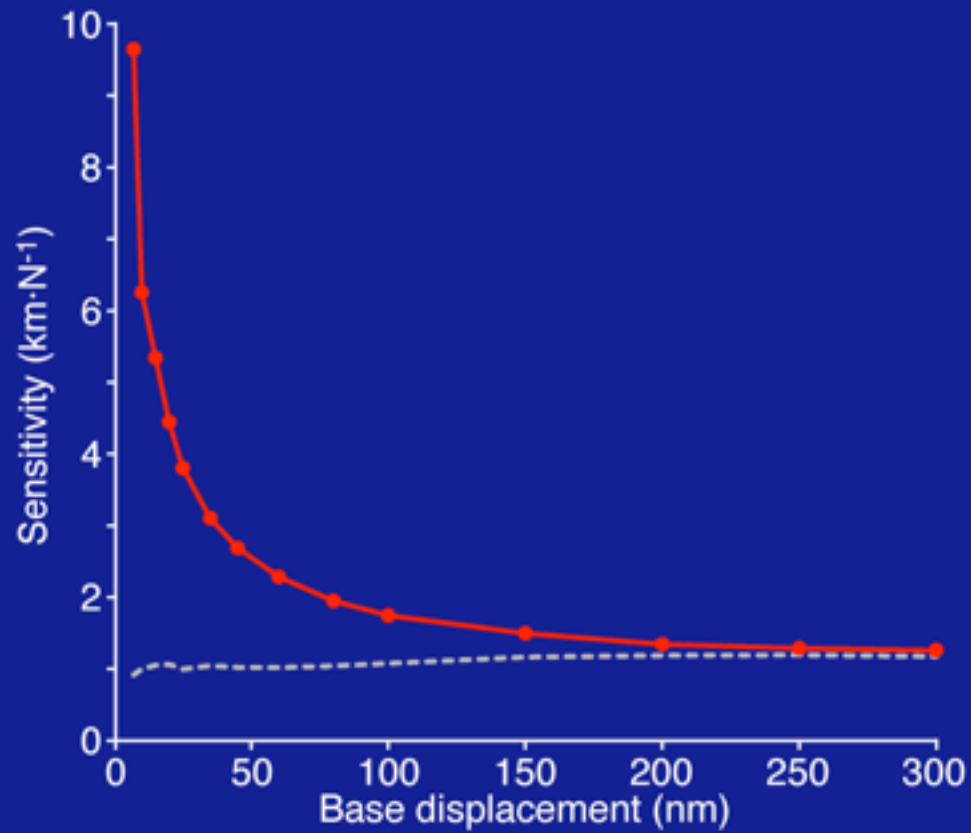
## After hair-bundle fatigue



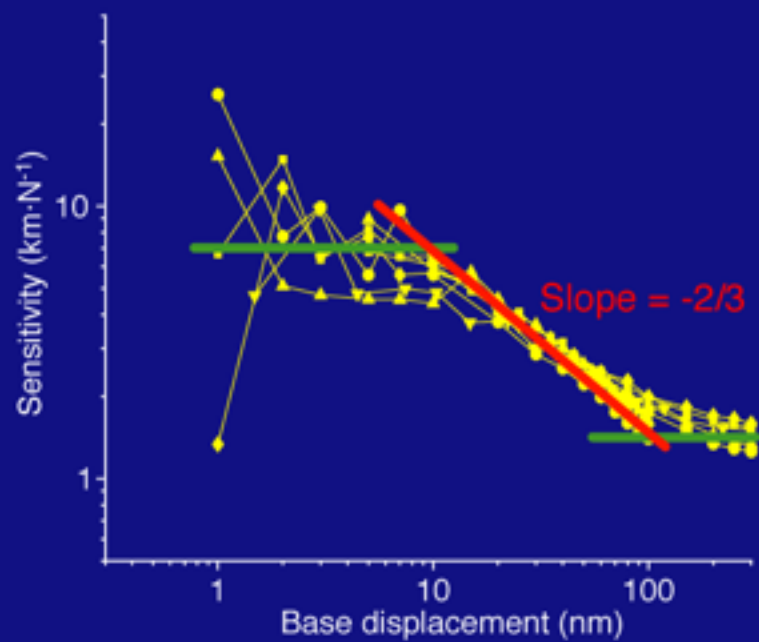
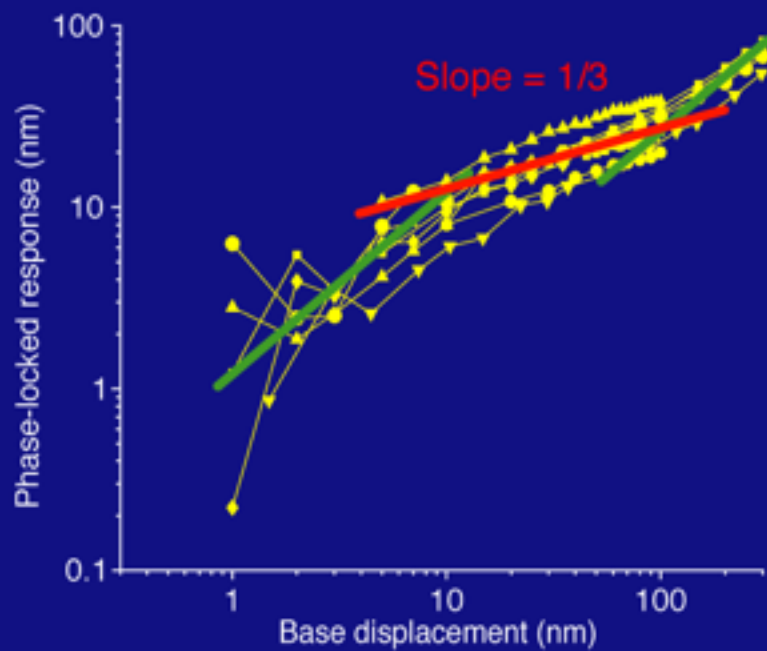
$$\bar{W}_{\text{ACTIVE}} = -\bar{W}_{\text{DRAG}} - \bar{W}_{\text{FIBER}} = -1 \text{ zJ}$$

1 pN



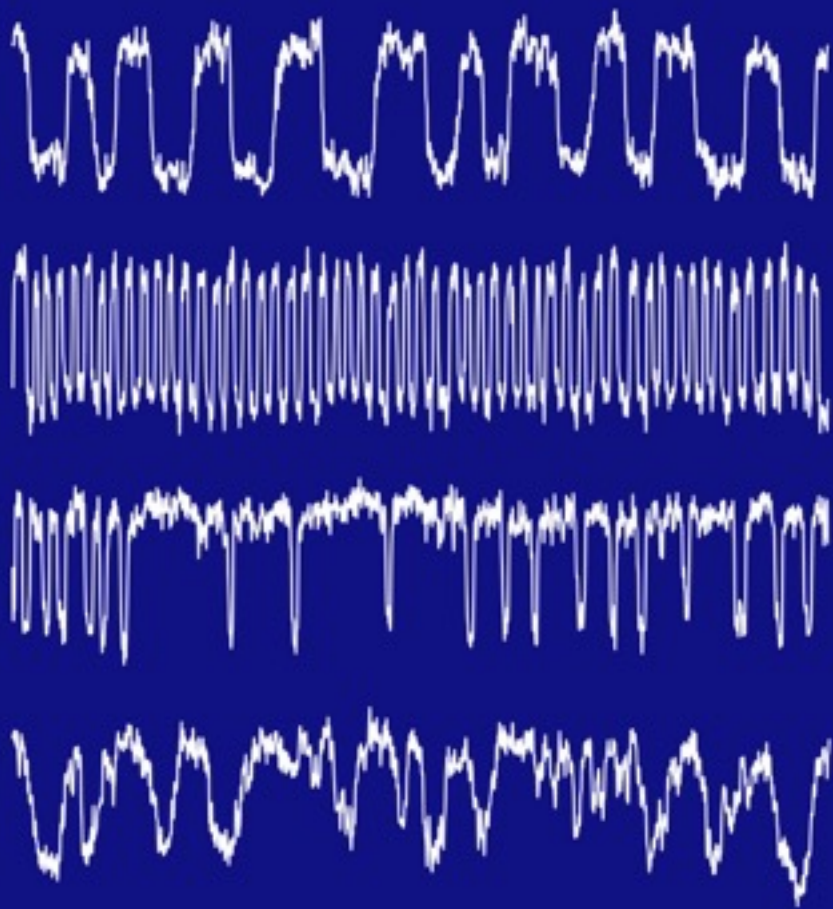






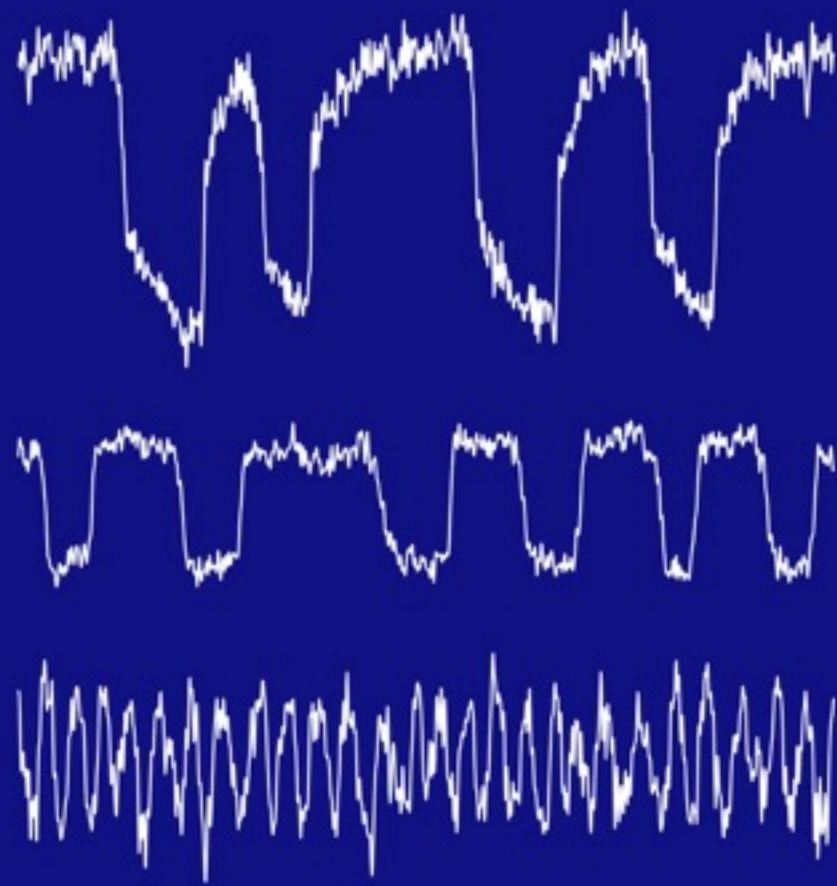






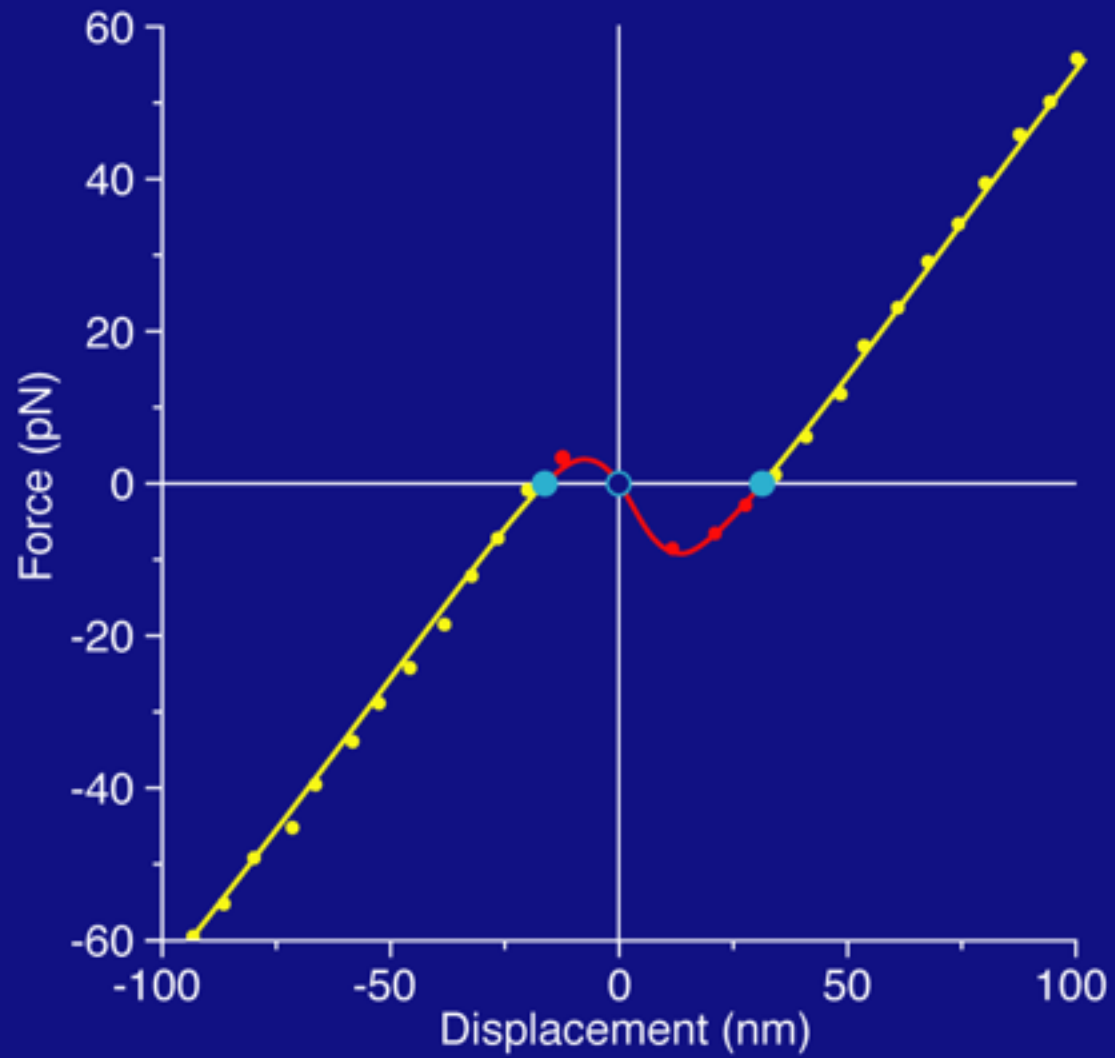
10 nm

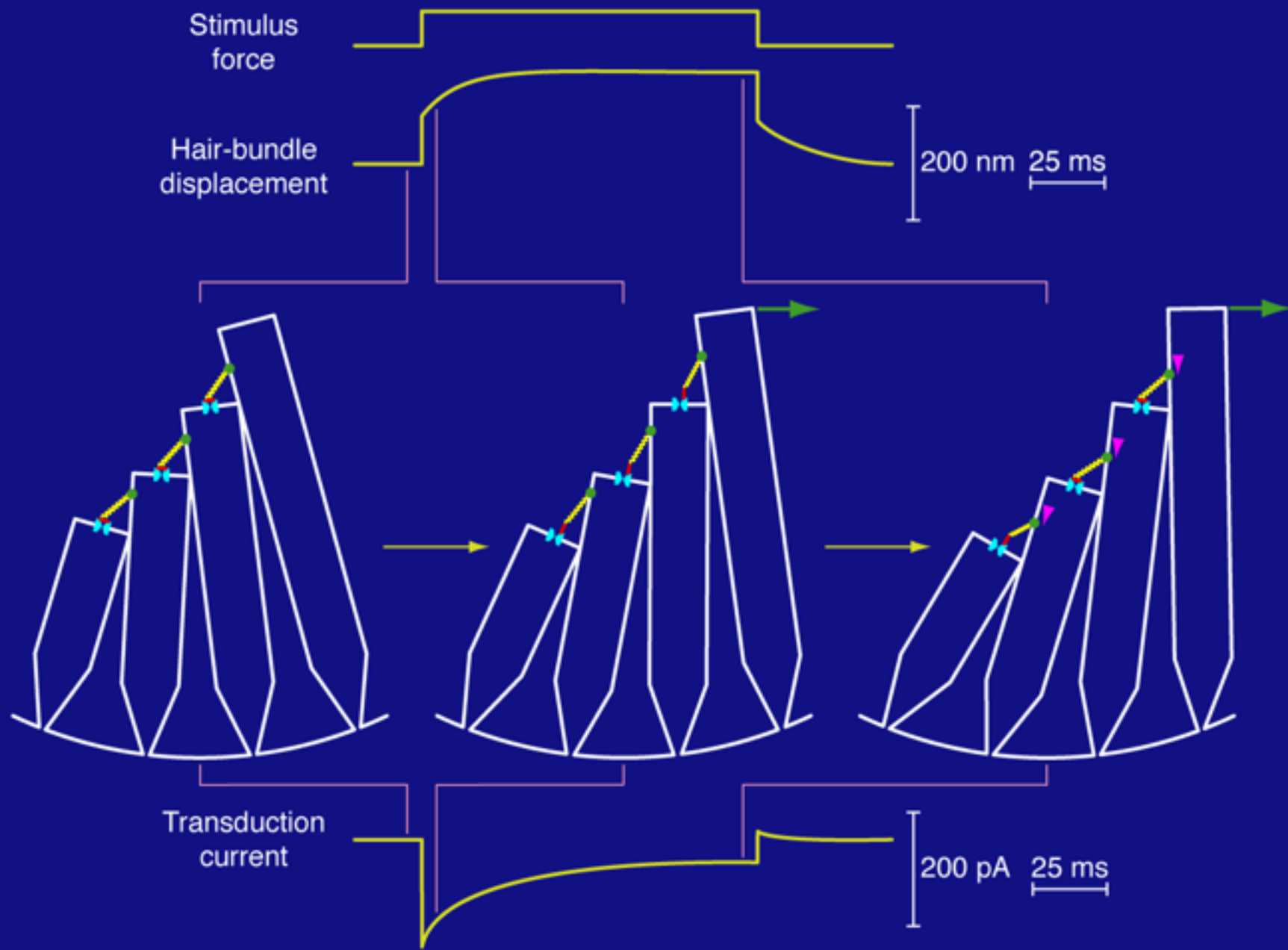
200 ms

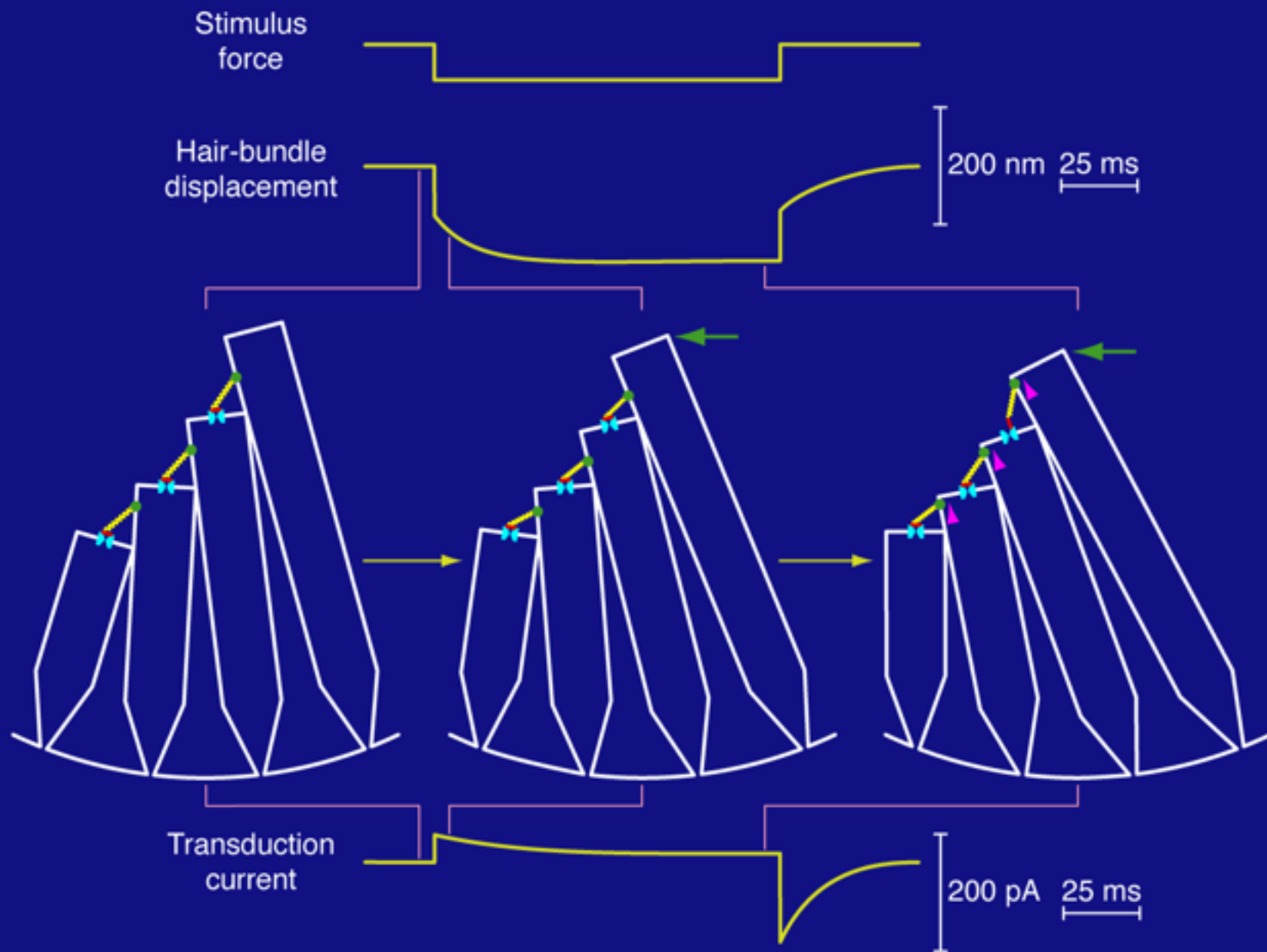


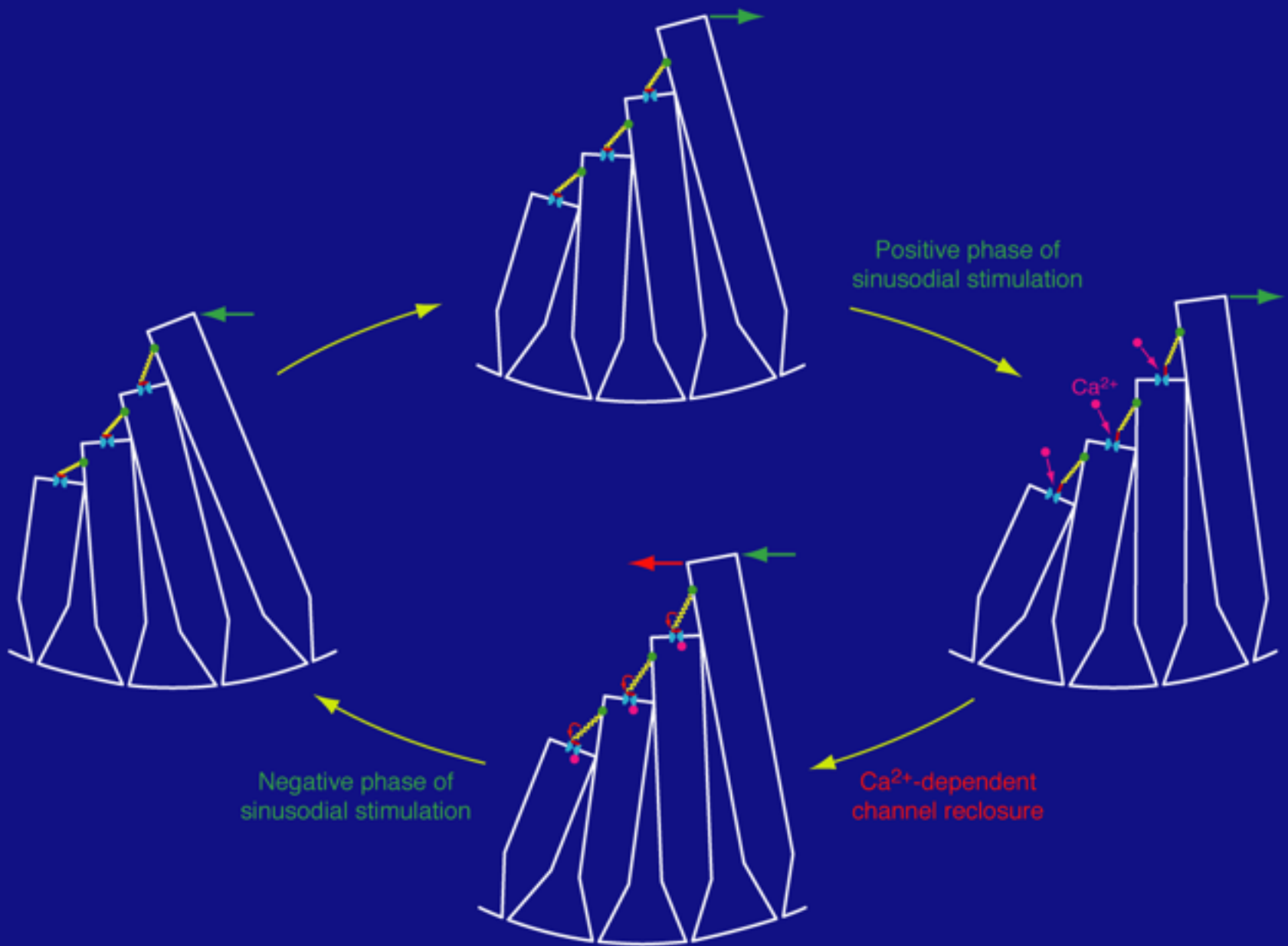
10 nm

50 ms











# DYNAMICAL DESCRIPTION OF HAIR – BUNDLE MOTION AND ADAPTATION

Hair-bundle velocity

$$\dot{x} = \left( \frac{1}{\xi_{HB} + \xi_{EXT}} \right) \left[ A(x-a) - B(x-a)^3 - (\kappa_{SP} + \kappa_{EXT})x + F_{EXT} \right]$$

Drag coefficients

Channel-gating force

Elastic forces

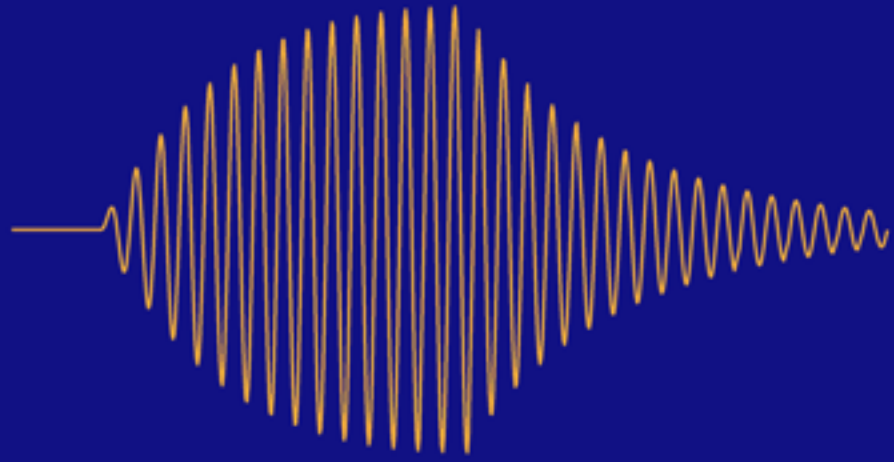
External force

Adaptation rate

$$\dot{a} = \frac{1}{\tau} (Cx - a)$$

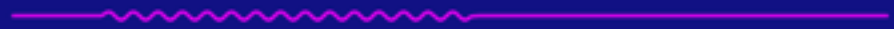
Adaptation time constant

Displacement response with amplification



0.2 nm  
1 ms

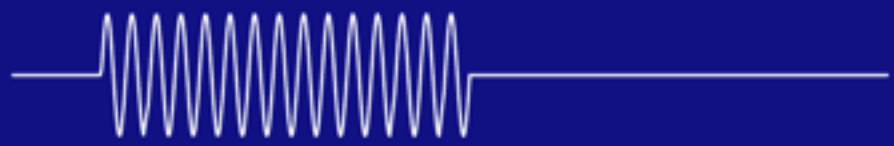
Displacement response with channel gating



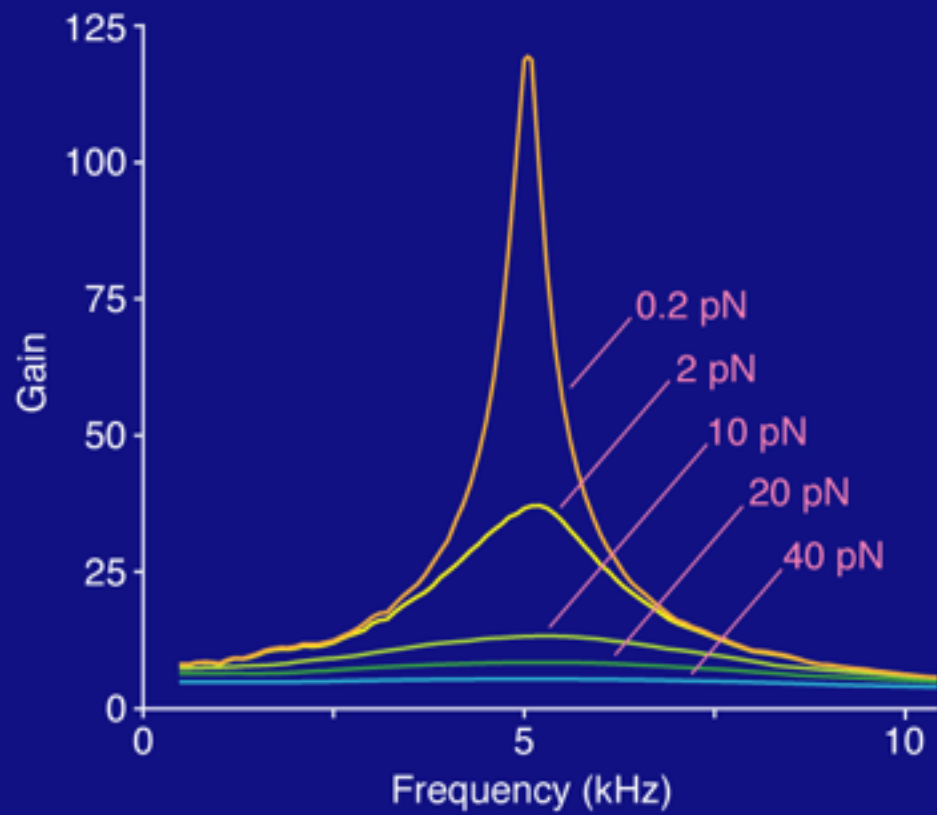
Displacement response of passive hair bundle



Stimulus force

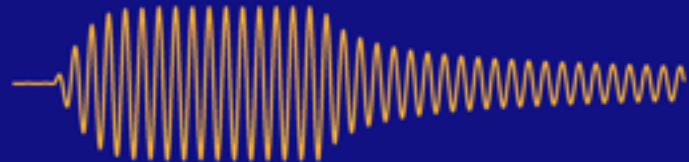
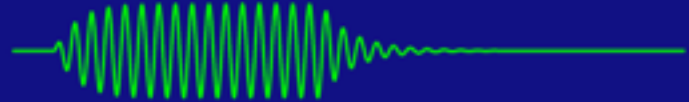
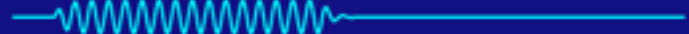
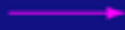


0.2 pN

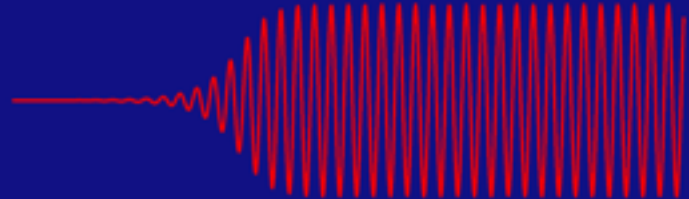


Input

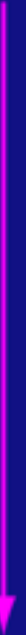
Output



Hopf  
bifurcation



Control parameter



Equation for a generic Hopf bifurcation with stimulation by a force  $F e^{i\omega t}$ :

$$\frac{dz}{dt} = (\mu + i\omega_0)z - |z|^2 z + F e^{i\omega t}$$

$z$ , displacement (complex variable:  $z = x + iy$ )

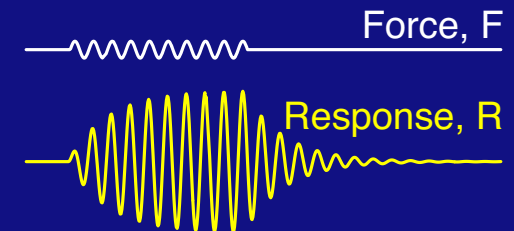
$\omega_0$ , natural (characteristic) frequency

$\mu$ , control parameter

During stimulation near bifurcation ( $\mu \approx 0$ ) and near resonance ( $\omega \approx \omega_0$ ),

response  $R \approx F^{1/3}$ , **amplification with compressive nonlinearity**

sensitivity  $S = \frac{R}{F} \approx F^{-2/3}$



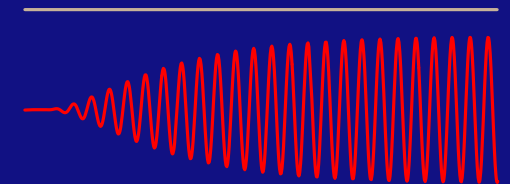
In the absence of stimulation,

$\mu < 0$ ,

$z = 0$ , a quiescent system

$\mu > 0$ ,

$z = \sqrt{\mu} e^{i\omega_0 t} = \sqrt{\mu} [\cos(\omega_0 t) + i \sin(\omega_0 t)]$ , **a stable limit cycle**



## ADVANTAGES OF OPERATION NEAR A HOPF BIFURCATION

### Principal virtues

- amplification
- frequency tuning
- compressive nonlinearity

- 100-fold to 1000-fold gain in humans
- 0.2 % frequency discrimination in humans
- 1,000,000-fold amplitude range in humans

### Simplicity

- two-dimensional
- codimension-1
- local

- minimal requirement for resonant tuning and oscillation
- adjustment of only one parameter required to traverse bifurcation
- bifurcation about a fixed point

### Entrainment

- easy phase-locking to periodic stimuli

### Frequency constancy

- frequency of resonant responsiveness near that of spontaneous oscillation

### Epiphenomena

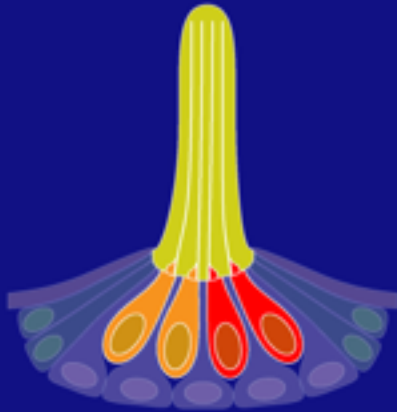
- combination tones
- spontaneous otoacoustic emissions (SOAEs)

- generation of responses at nonharmonic frequencies, *e.g.*  $2f_1 - f_2$
- unprovoked production of one or more pure tones

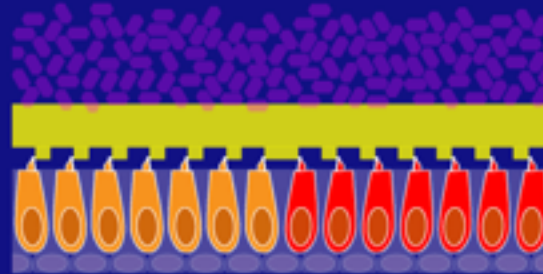
Defining characteristics of the cochlear active process

# THE MECHANICAL LOADS OF HAIR BUNDLES

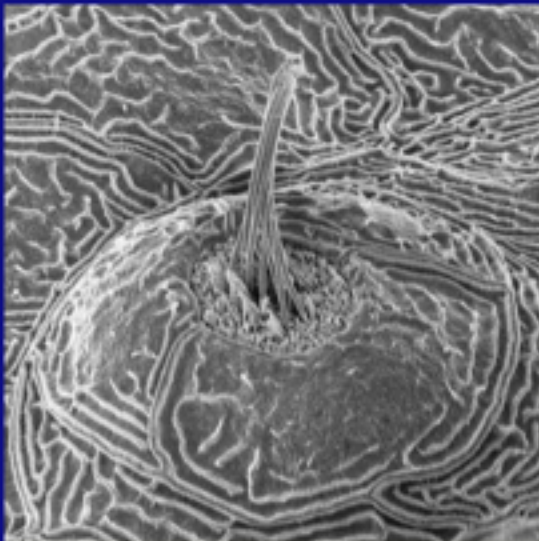
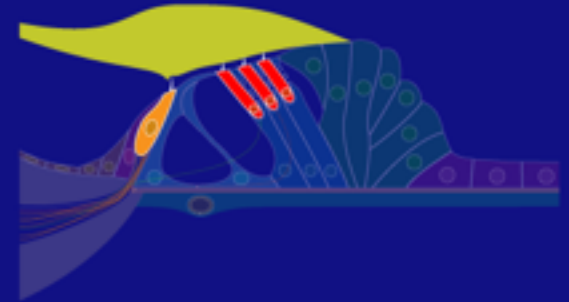
Cupular organs  
lateral-line organs and  
semicircular canals



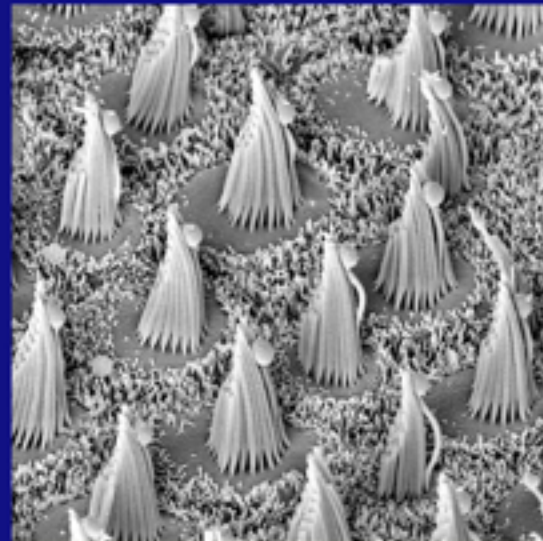
Otolith organs  
utricle and saccule



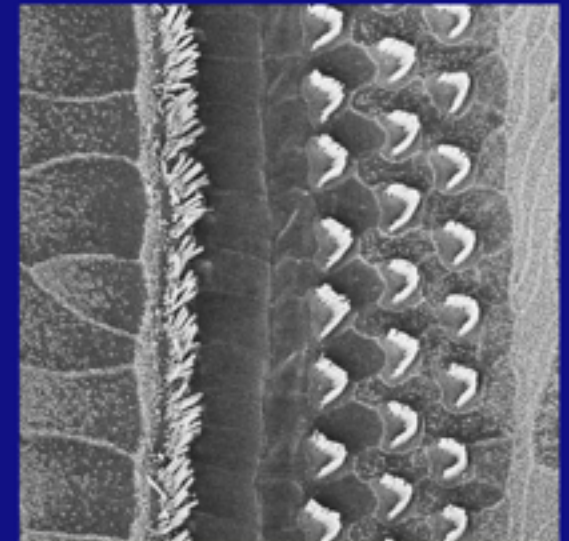
Tectorial organs  
cochleas and basilar  
papillas



Zebrafish's lateral-line organ



Bullfrog's sacculus



Mouse's cochlea

# DYNAMICAL DESCRIPTION OF HAIR – BUNDLE MOTION AND ADAPTATION

Experimentally  
accessible parameters

Hair-bundle  
velocity

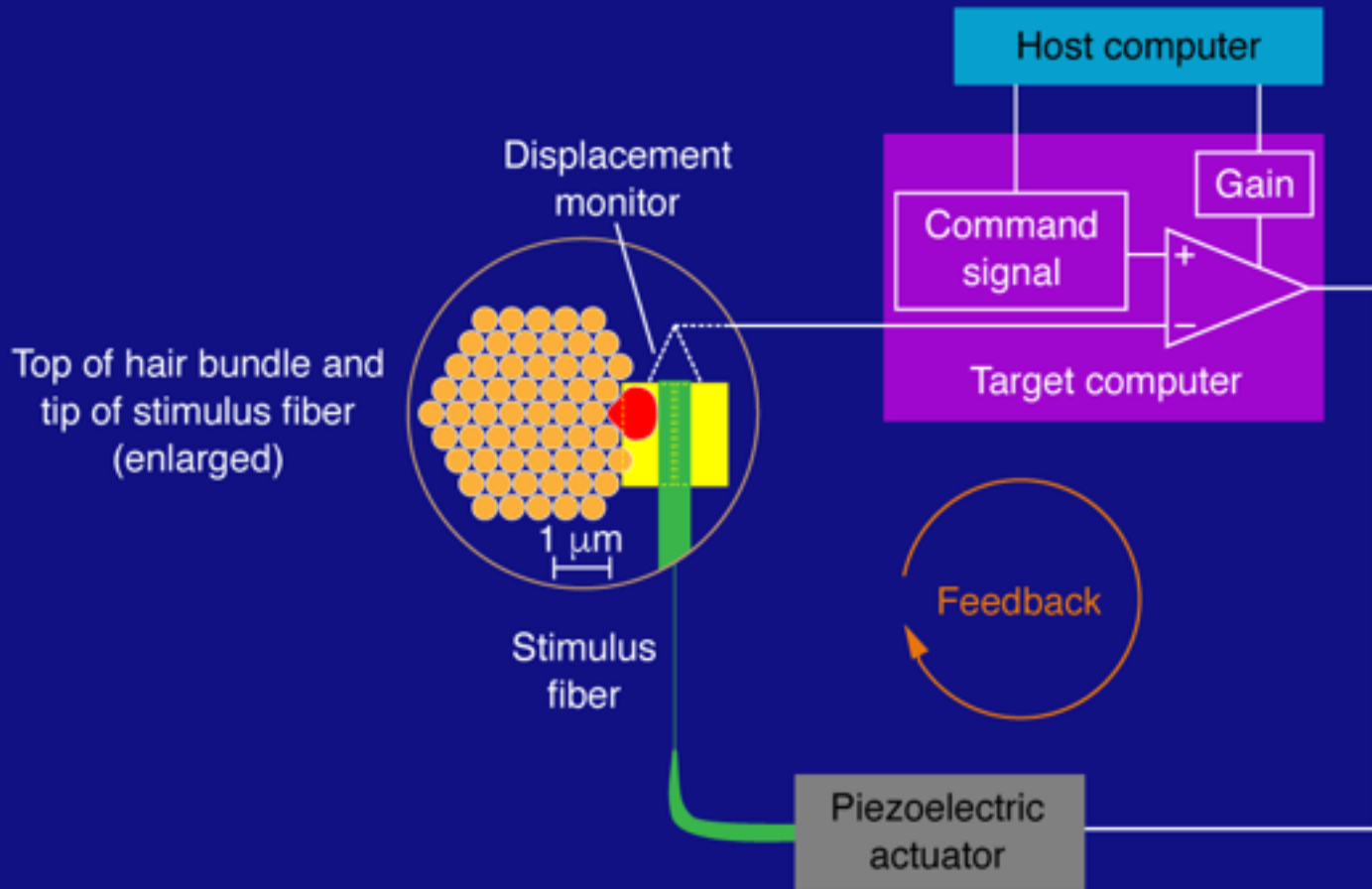
$$\dot{x} = \left( \frac{1}{\xi_{HB} + \xi_{EXT}} \right) \left[ A(x-a) - B(x-a)^3 - (\kappa_{SP} + \kappa_{EXT})x + F_{EXT} \right]$$

Adaptation  
rate

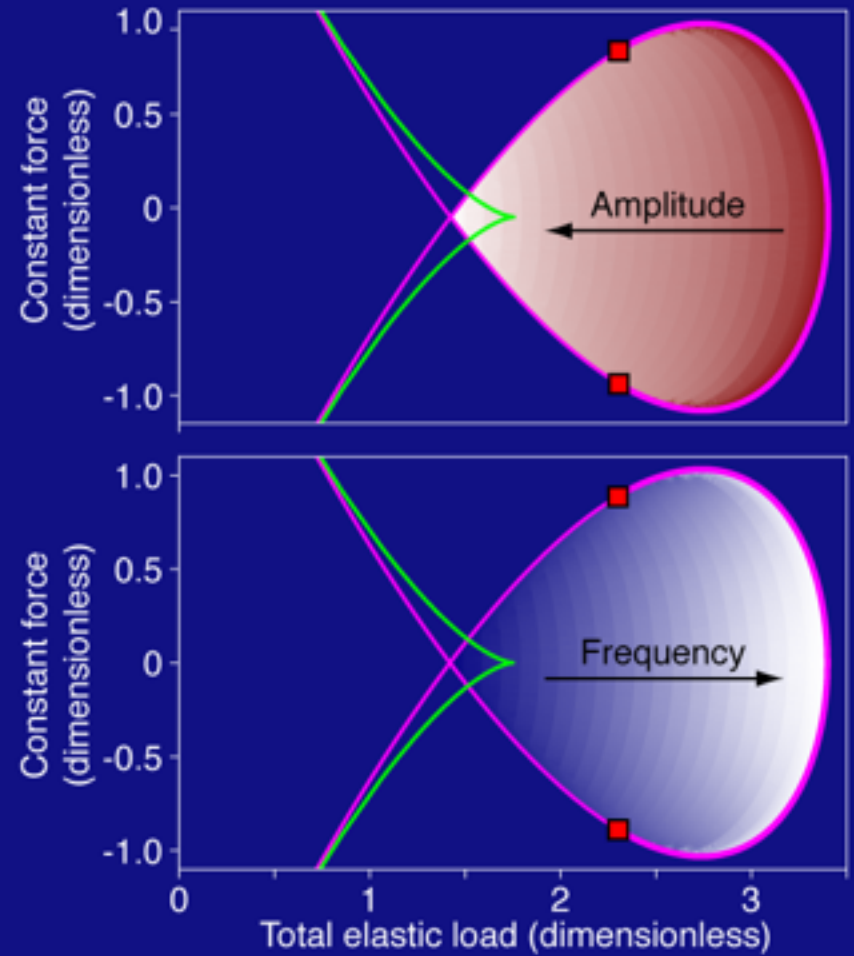
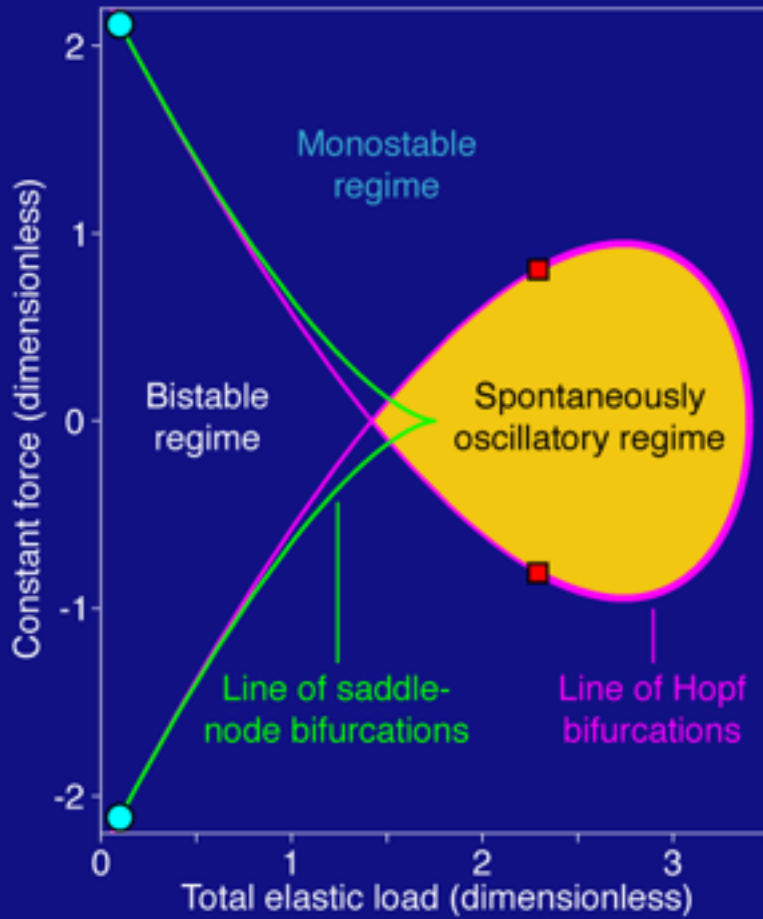
$$\dot{a} = \frac{1}{\tau} (Cx - a)$$



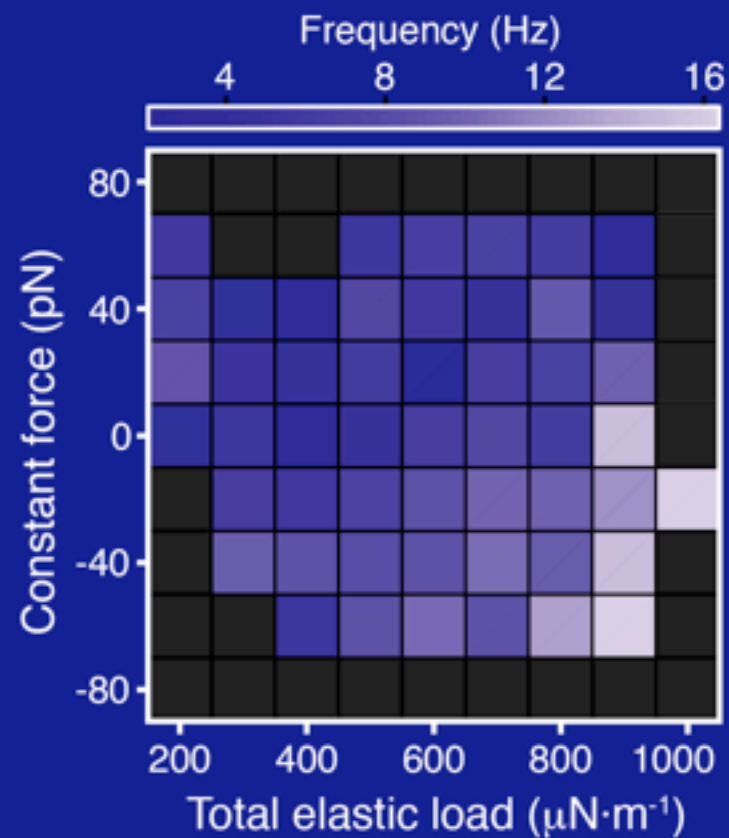
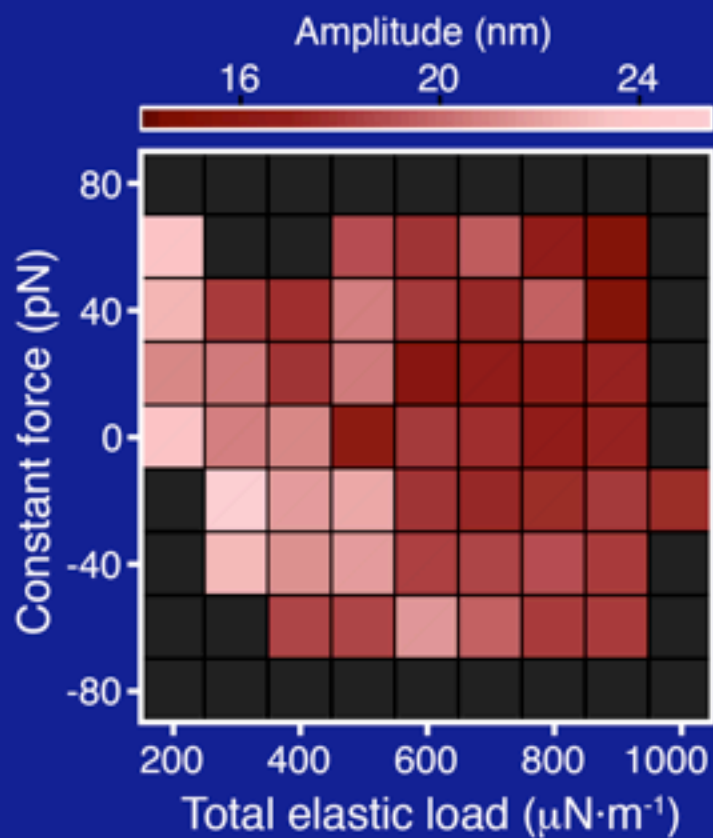
# MECHANICAL – LOAD CLAMP



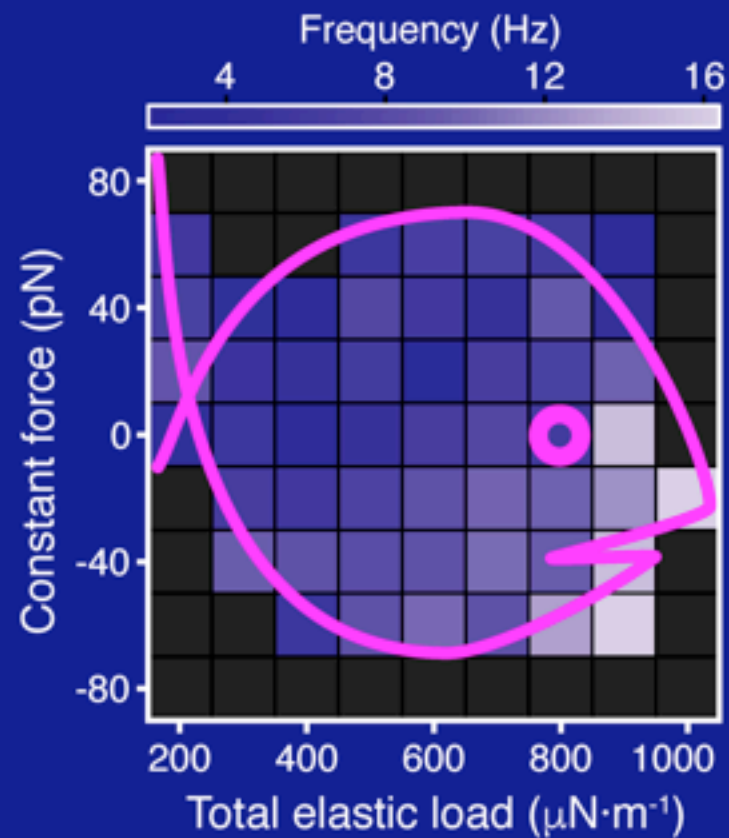
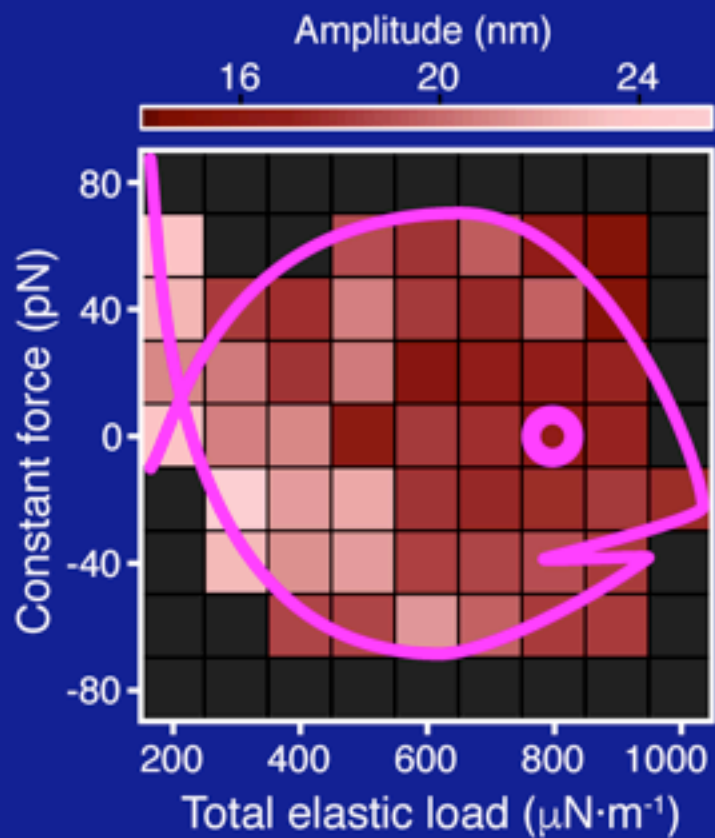
# THEORETICAL STATE DIAGRAM



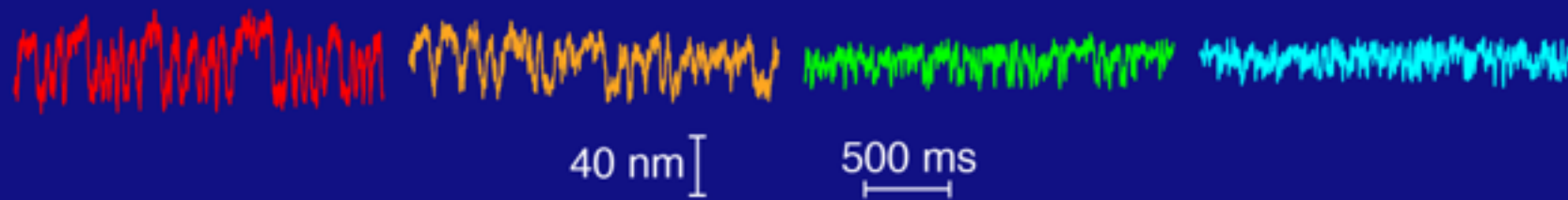
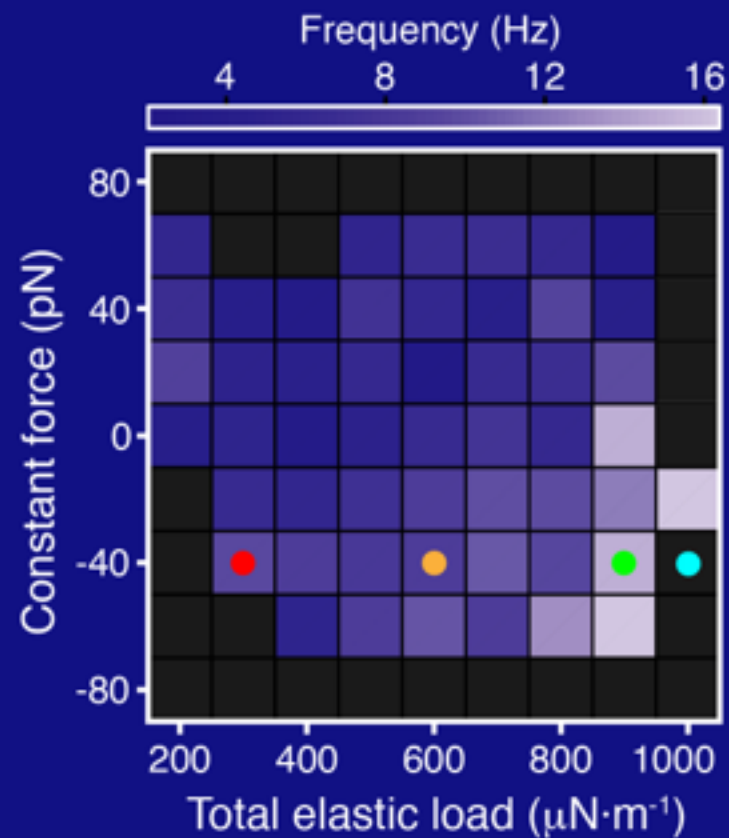
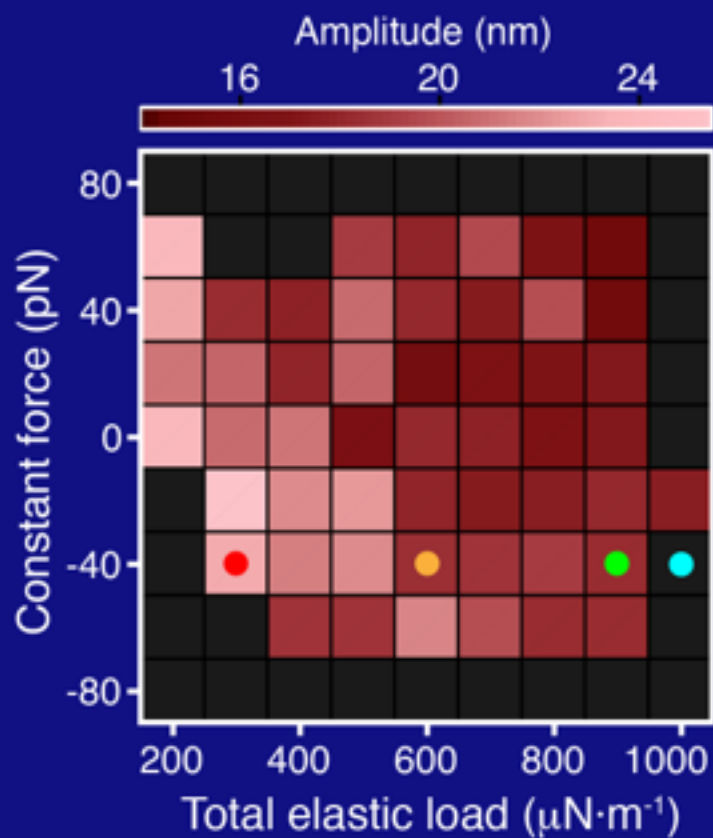
# EXPERIMENTAL STATE DIAGRAM

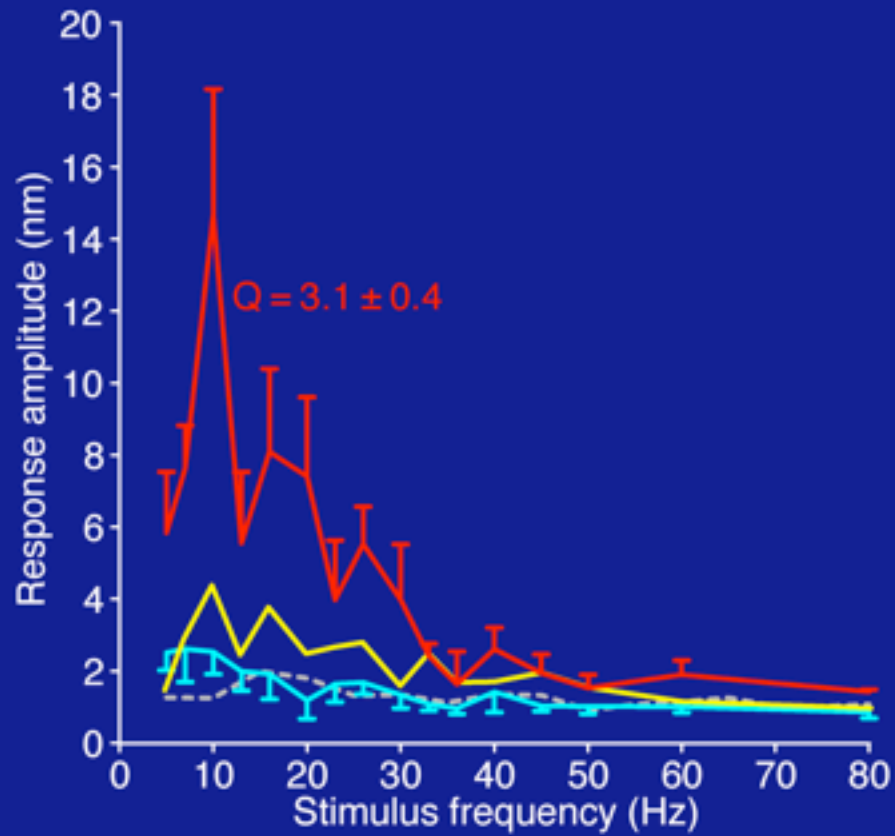


# EXPERIMENTAL STATE DIAGRAM



# EXPERIMENTAL STATE DIAGRAM

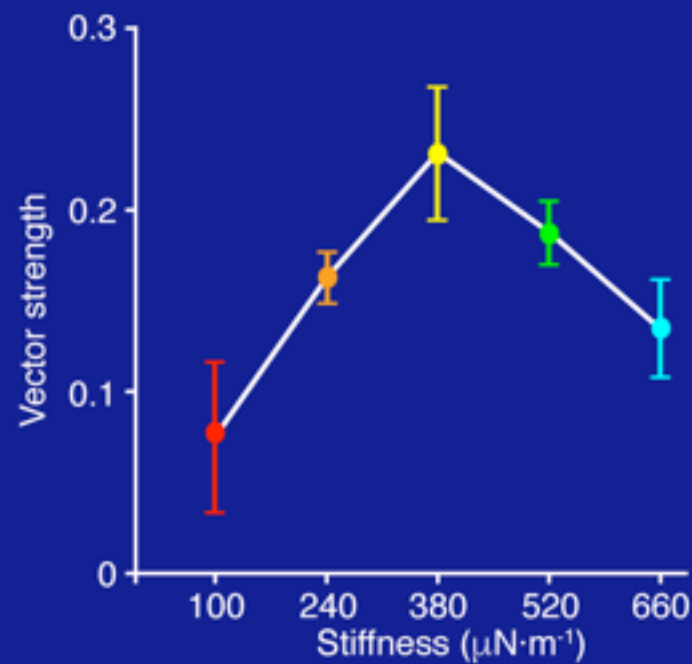
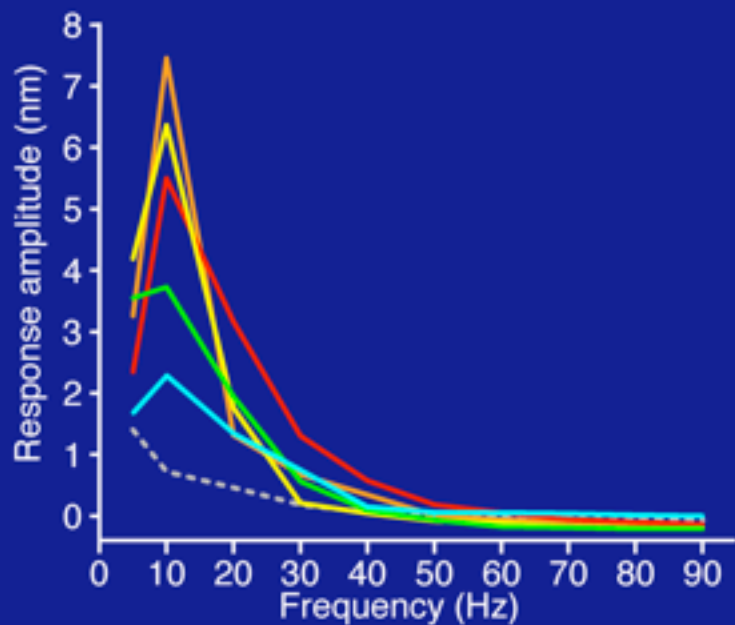




Oscillatory regime      Quiescent regime

Stiffness ( $\mu\text{N}\cdot\text{m}^{-1}$ ):    300      400      500

Bifurcation

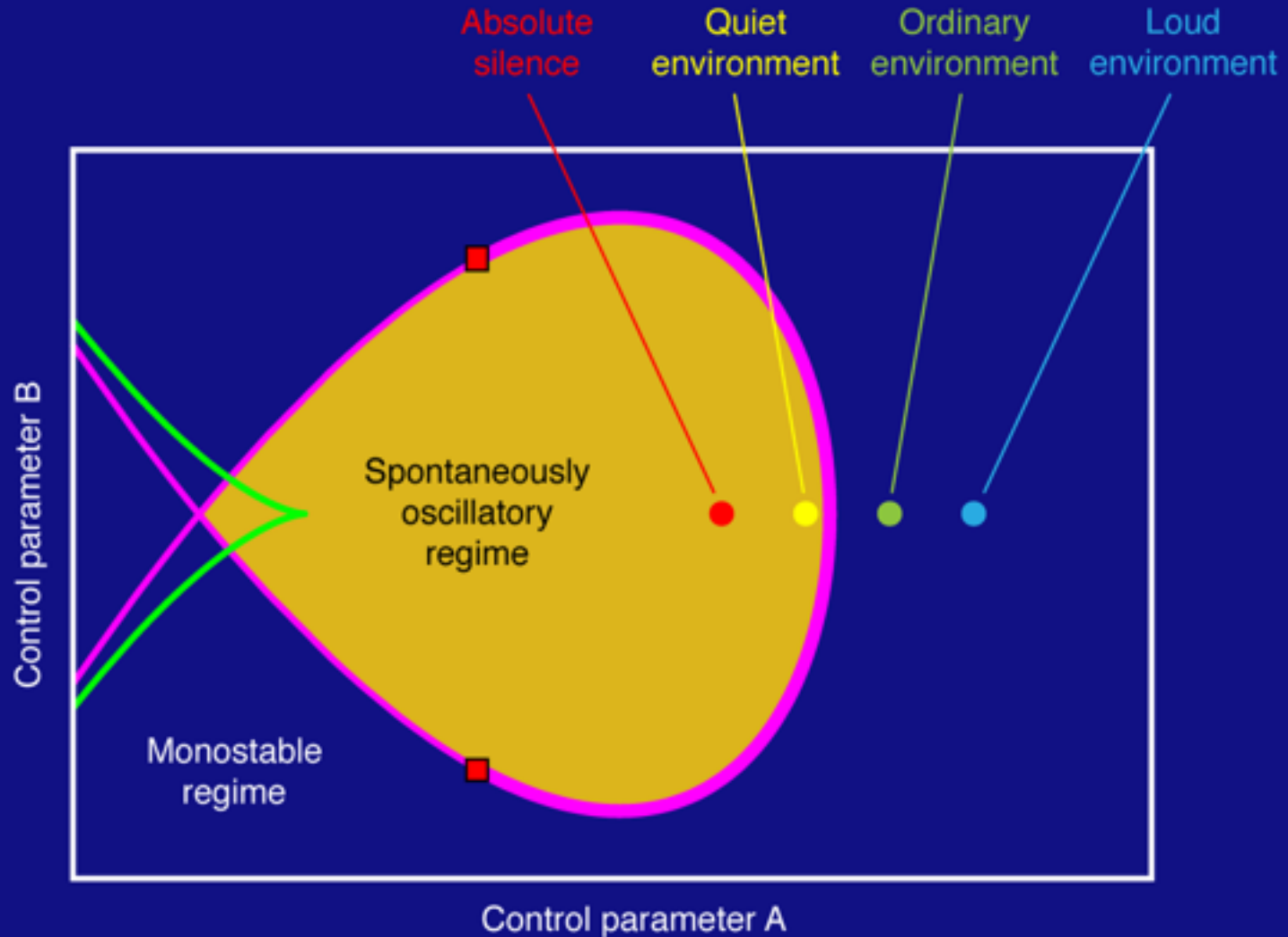


Oscillatory regime      Quiescent regime

Stiffness ( $\mu\text{N}\cdot\text{m}^{-1}$ ): 100 240 380 520 660

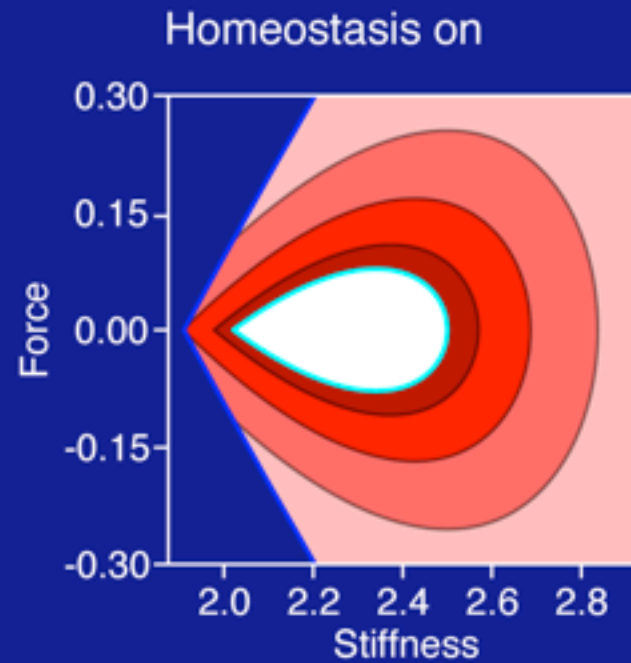
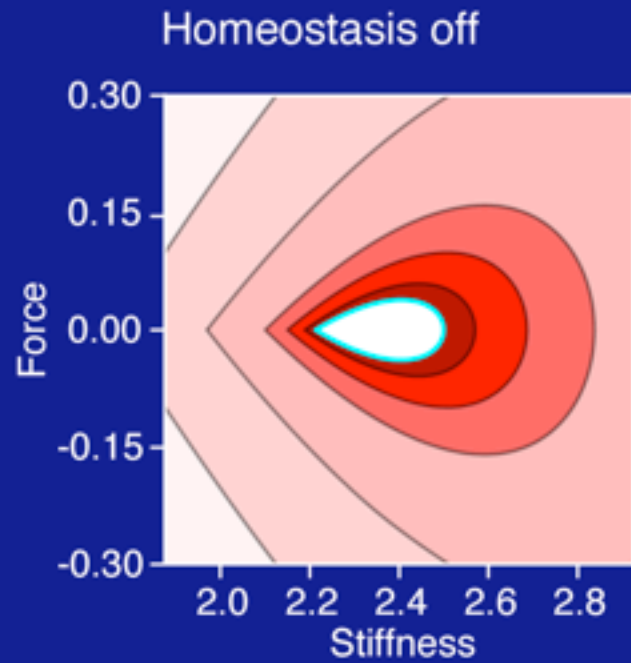
↑  
Bifurcation

# SELF – TUNED CRITICALITY





## HOMEOSTATIC REGULATION OF HAIR – BUNDLE SENSITIVITY



## ADVANTAGES OF HAIR CELLS FOR BIOPHYSICAL INVESTIGATION

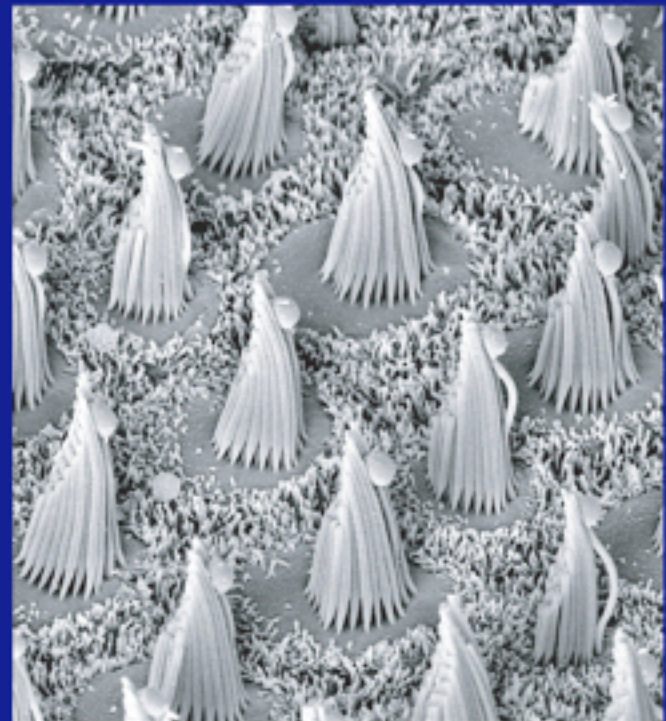
Stereotyped cells with high reproducibility

Experimental accessibility

Ability to measure and control conjugate variables  
displacement and force of hair bundle  
voltage and current across cellular membrane

Few relevant degrees of freedom  
hair-bundle motion along axis of symmetry  
state of adaptation

Evolutionary pressures on system  
threshold near level of thermal noise  
energetic efficiency



# Geological regime

# Representative contemporary species

