Halometry from Astrometry

Ken Van Tilburg (NYU & IAS)

arXiv:1804.01991

with Anna-Maria Taki, Neal Weiner (NYU)

New Probes for Physics Beyond the Standard Model
KITP, April 9, 2018

time-domain, astrometric, weak gravitational lensing

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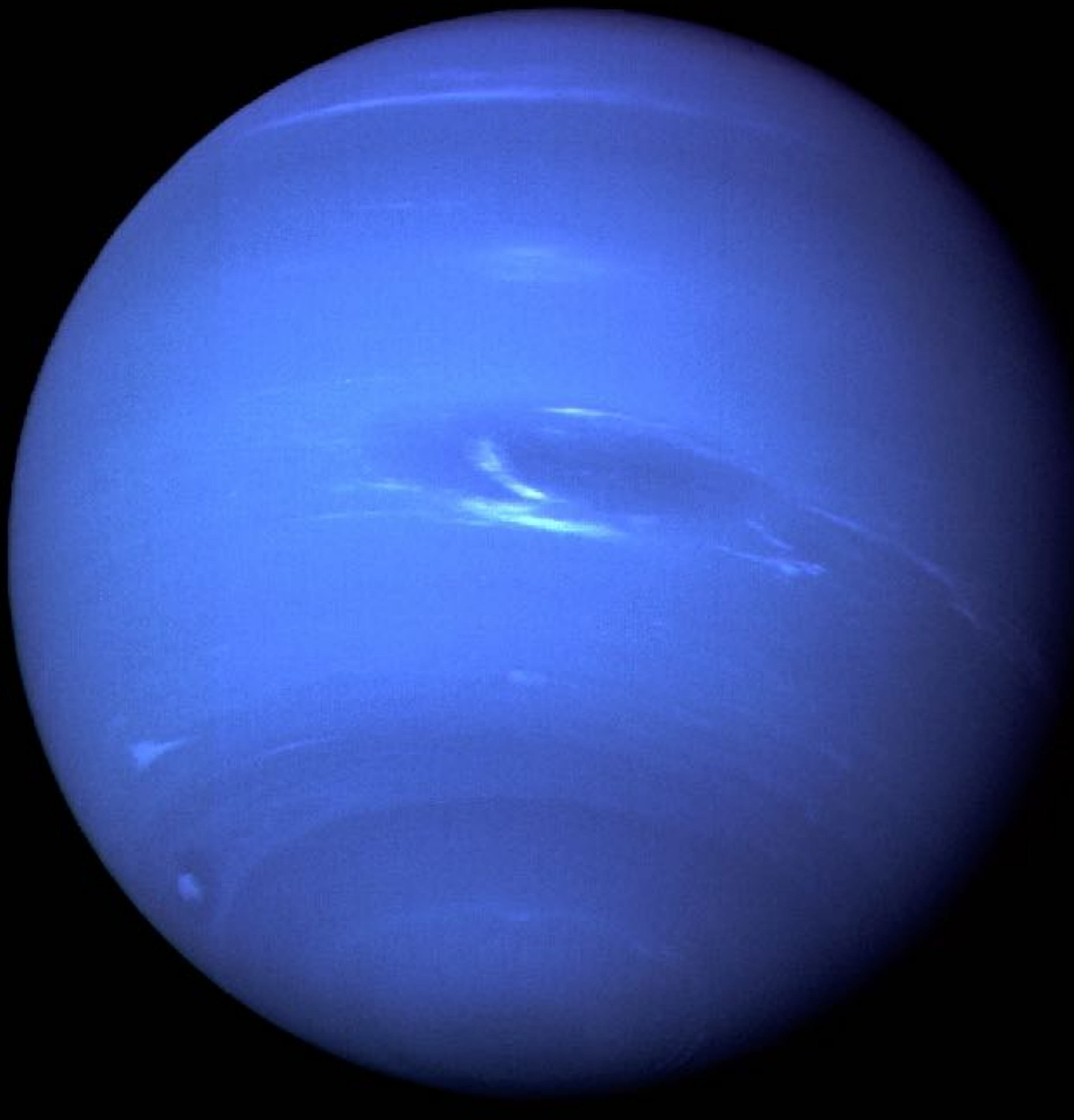
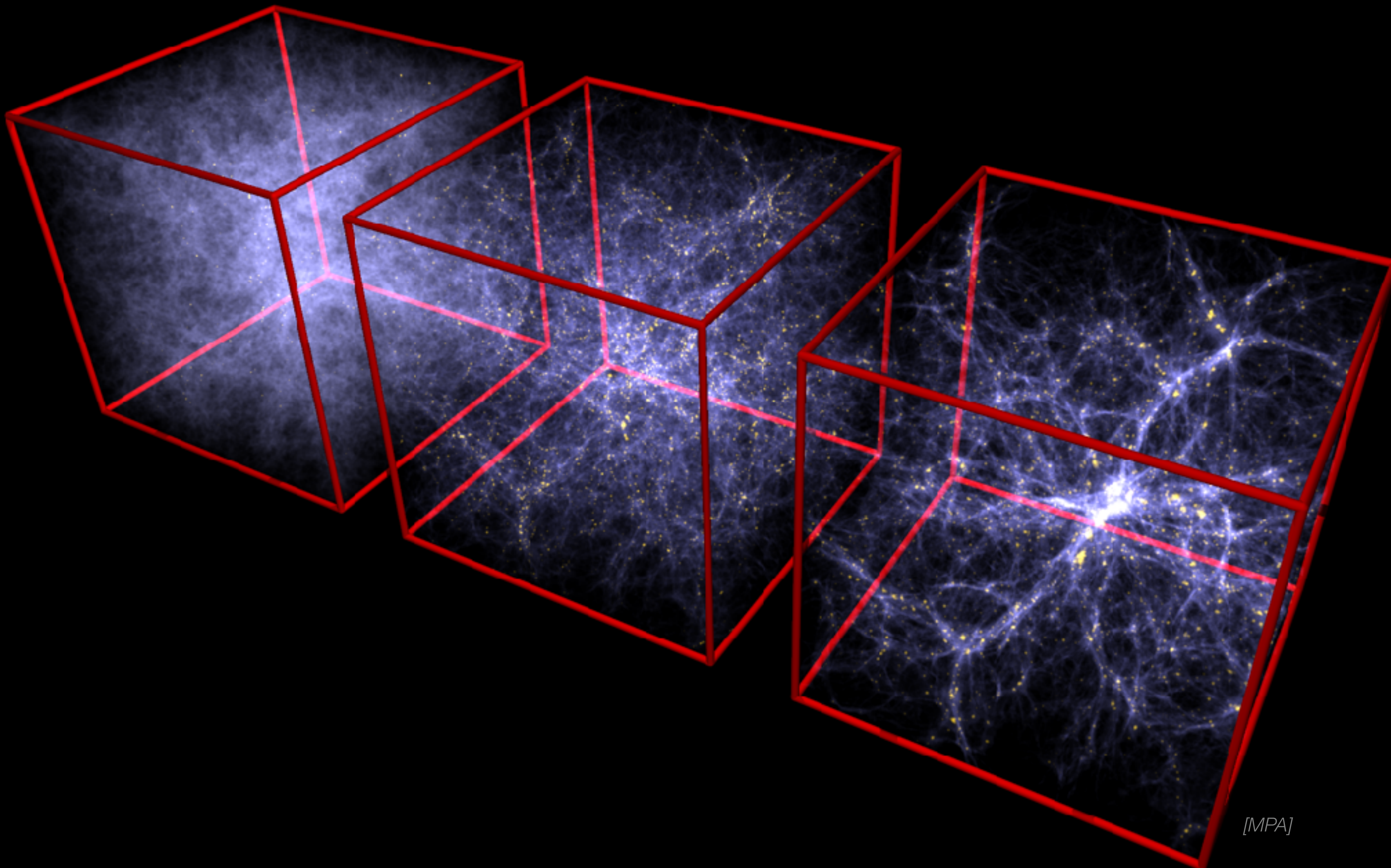
Dark Matter

Not-So-Dark Matter



Dark Matter Density Fluctuations



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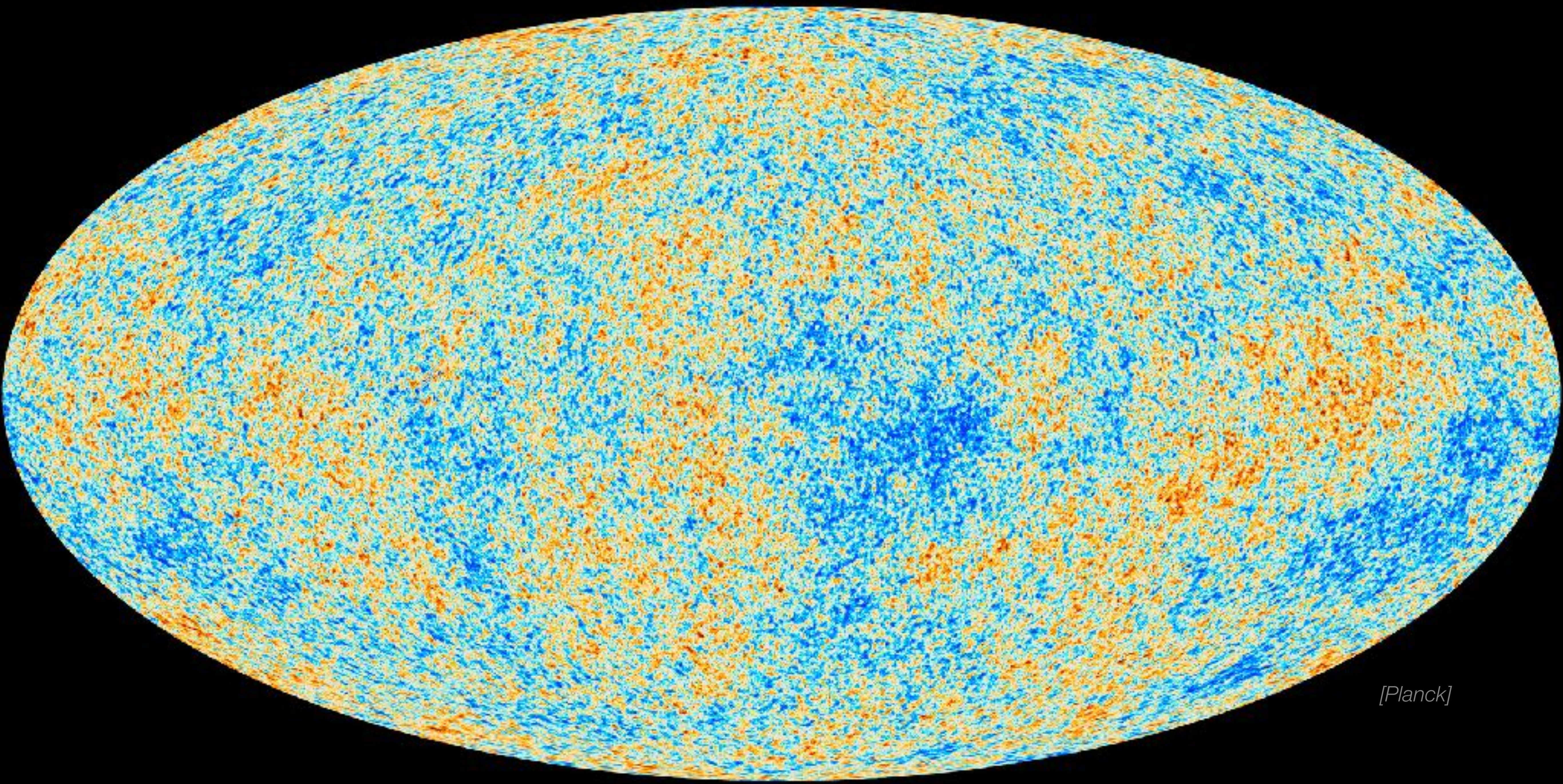
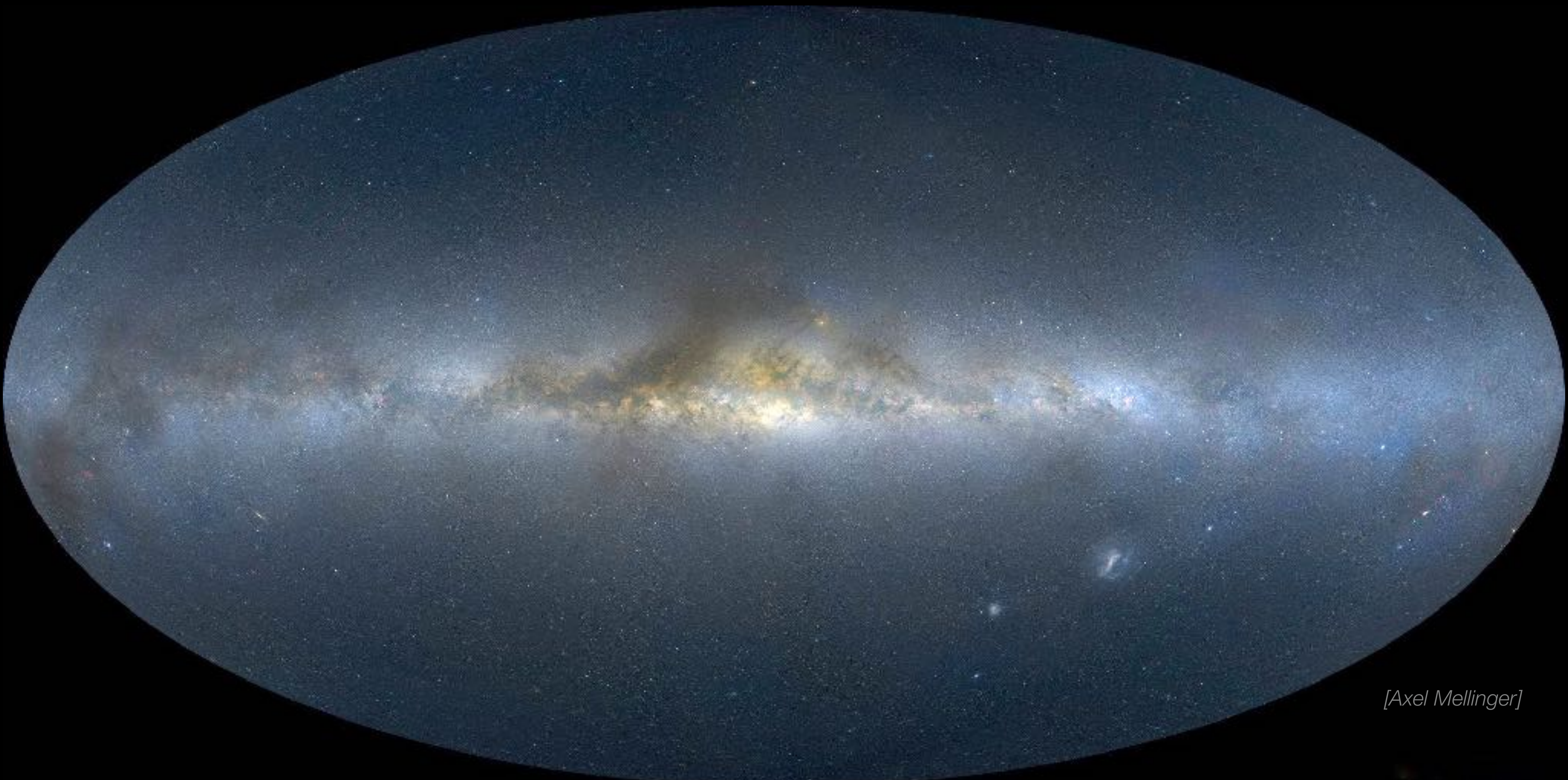


[Planck]

Dark Matter Density Fluctuations



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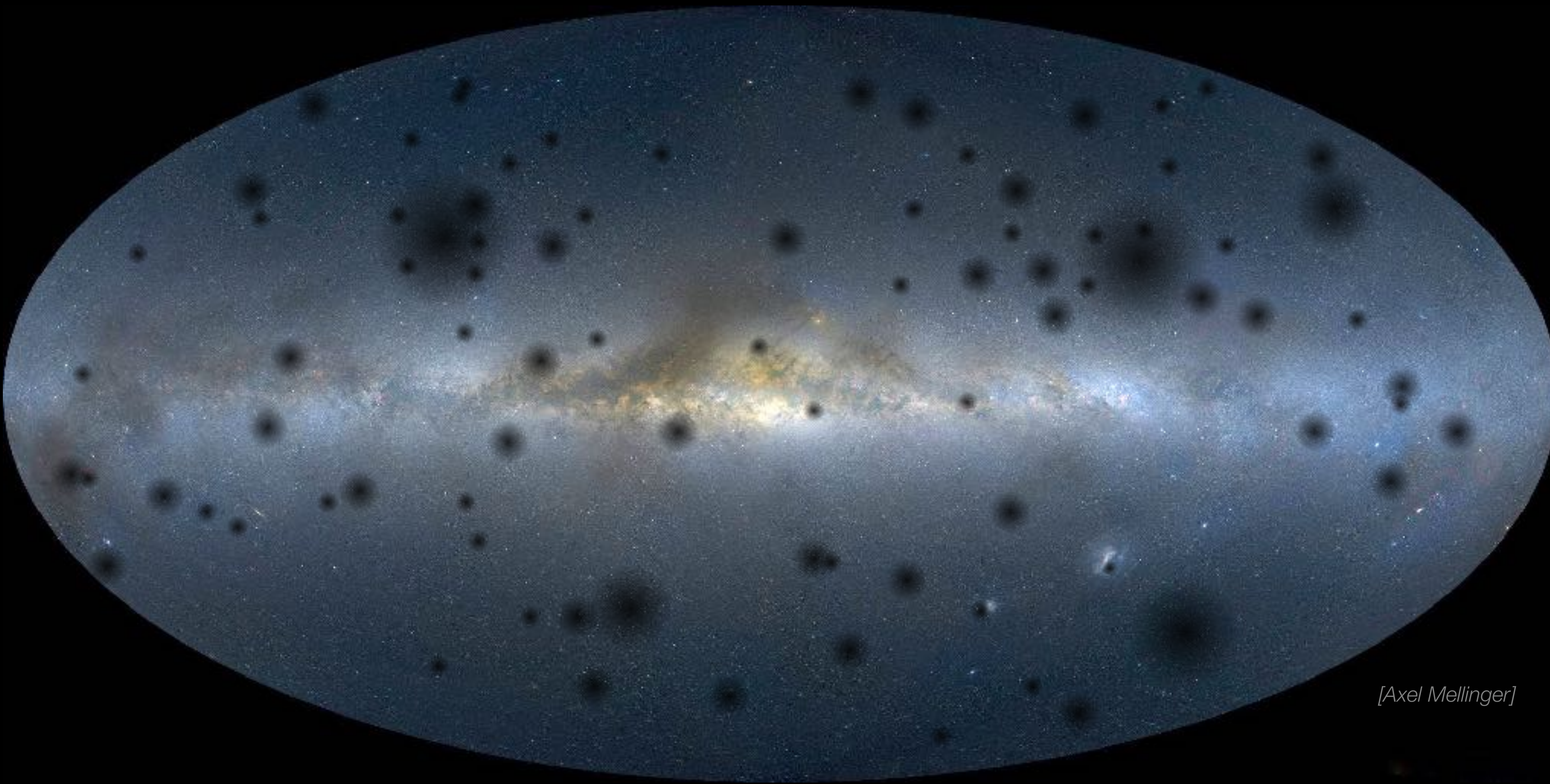
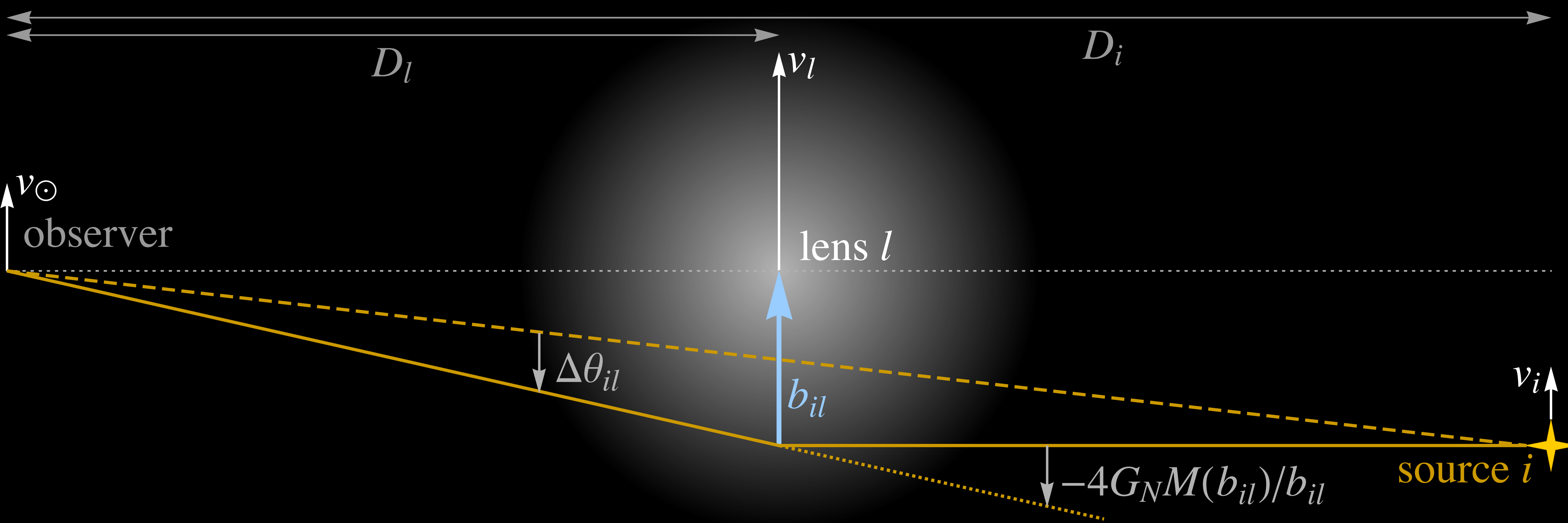


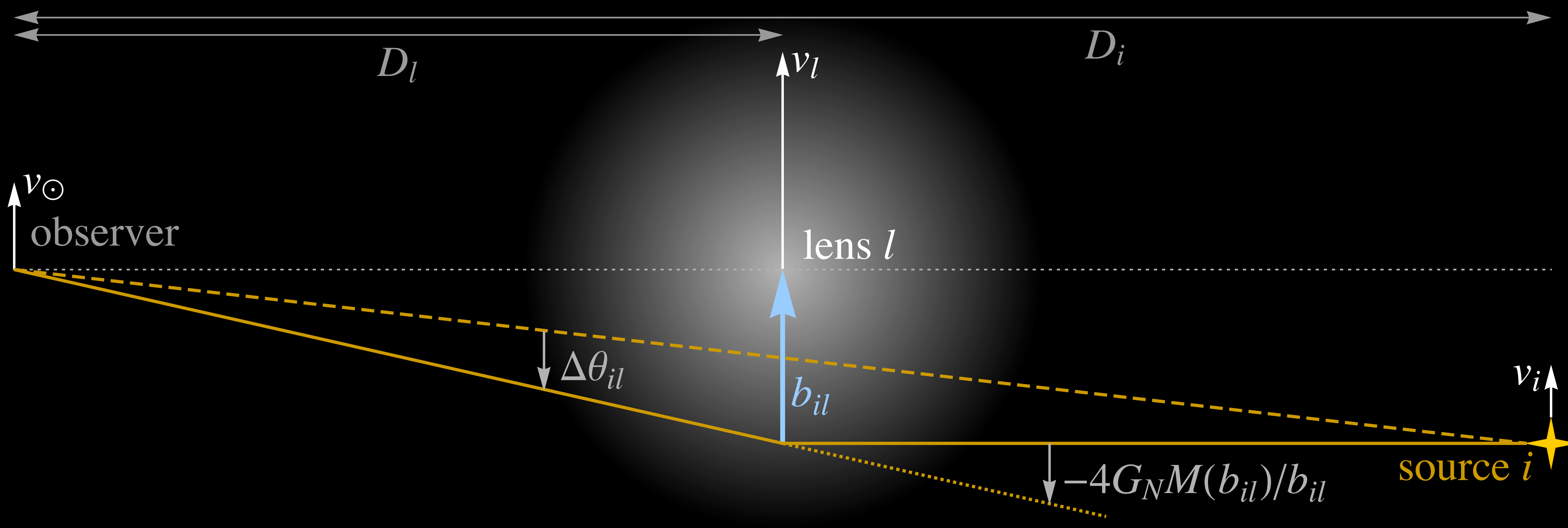
[Axel Mellinger]

Astrometric Weak Gravitational Lensing



$$\Delta\boldsymbol{\theta}_{il} = -\left(1 - \frac{D_l}{D_i}\right)\frac{4G_N M(b_{il})}{b_{il}}\hat{\mathbf{b}}_{il} \approx 4\ \mu\text{as}\left[\frac{M(b_{il}^{\text{typical}})}{10^6\ M_\odot}\right]\left[\frac{10\ \text{kpc}}{b_{il}^{\text{typical}}}\right]$$

Astrometric Weak Gravitational Lensing



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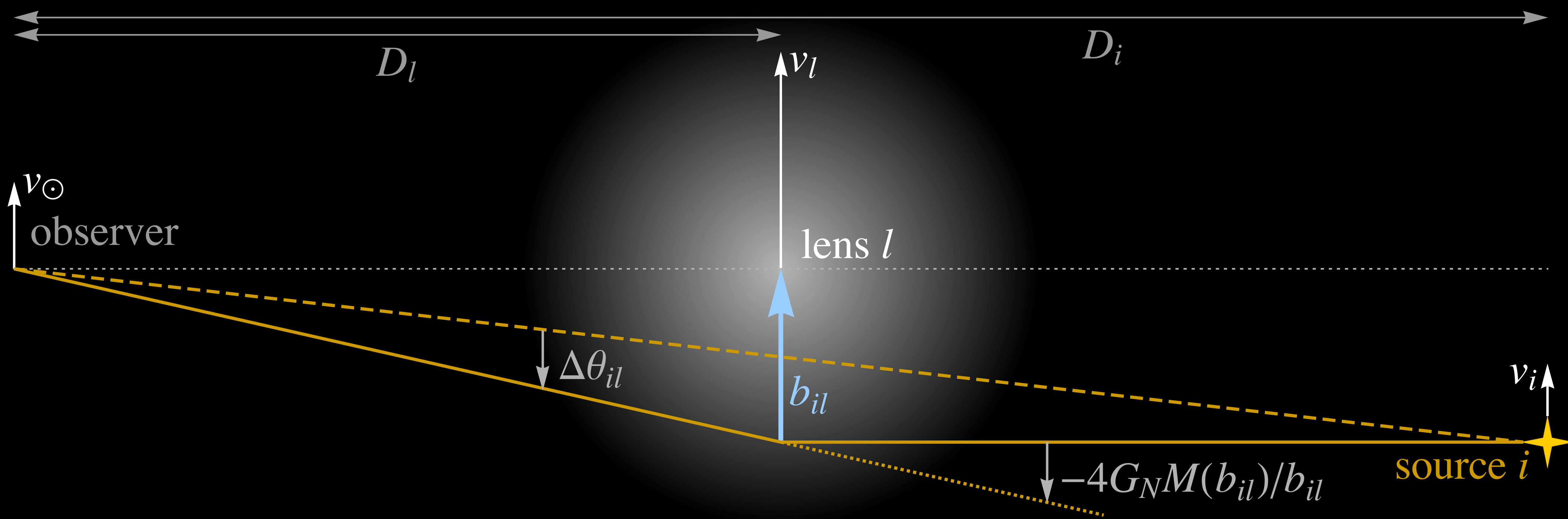
$\boldsymbol{\theta}_i$ unknown a priori

for finite density and size of lens,

$$\lim_{b_{il}\to 0} M(b_{il}) = 0$$

Astrometric Weak Gravitational Lensing



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$\boldsymbol{\theta}_i$ unknown a priori, but:

$$\mathbf{v}_{il} \equiv \dot{\mathbf{b}}_{il} = \mathbf{v}_l - \left(1 - \frac{D_l}{D_i}\right)\mathbf{v}_\odot - \frac{D_l}{D_i}\mathbf{v}_i$$

$\Longrightarrow$ time-domain $\Delta\dot{\boldsymbol{\theta}}_{il}, \Delta\ddot{\boldsymbol{\theta}}_{il}, \ldots$

for finite density and size of lens,

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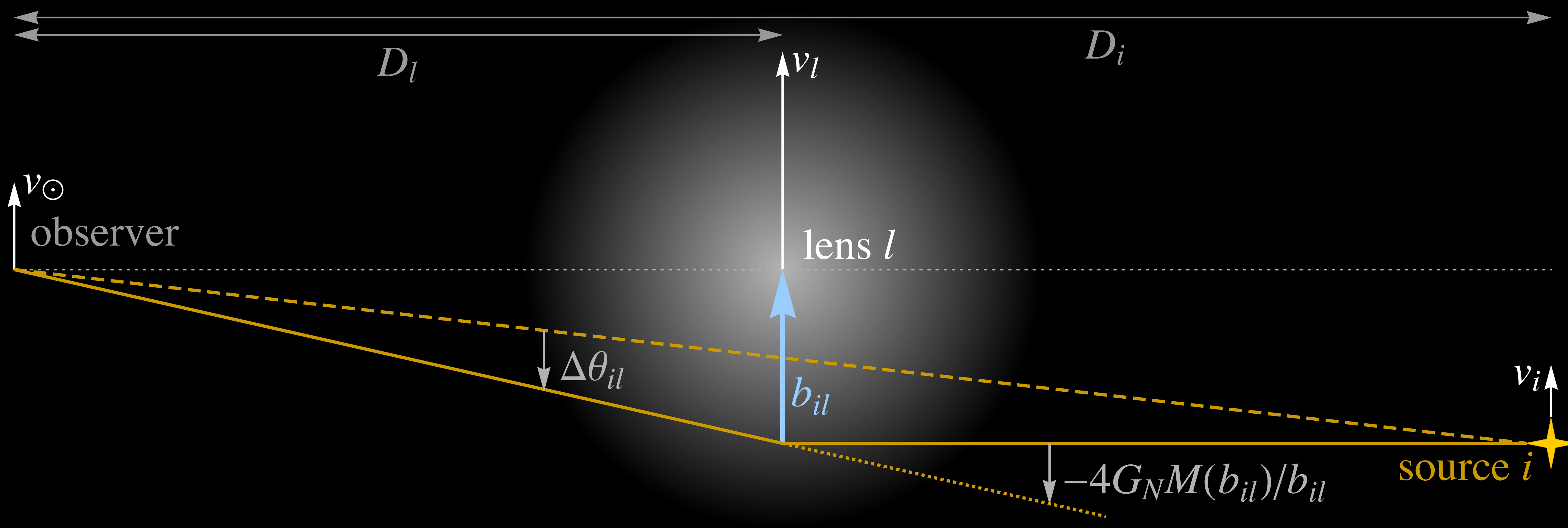
Astrometric Weak Gravitational Lensing



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$\Longrightarrow$ time-domain $\Delta\dot{\boldsymbol{\theta}}_{il}, \Delta\ddot{\boldsymbol{\theta}}_{il}, \ldots$

for finite density and size of lens,

$$\lim_{b_{il}\to 0} M(b_{il}) = 0$$

$\Longrightarrow$ many sources behind 1 lens

Numerical Estimates

Time-domain effects:

$$\Delta\dot{\theta}_{il} \sim \frac{4G_N M(b_{il}) v_{il}}{b_{il}^2} \sim 10^{-3}\ \mu\text{as y}^{-1} \left(\frac{M(b_{il})}{10^6\ M_\odot}\right)\left(\frac{10^2\ \text{pc}}{b_{il}}\right)^2$$

$$\Delta\ddot{\theta}_{il} \sim \frac{4G_N M(b_{il}) v_{il}^2}{b_{il}^3} \sim 4\times 10^{-3}\ \mu\text{as y}^{-2} \left(\frac{M(b_{il})}{M_\odot}\right)\left(\frac{10^{-2}\ \text{pc}}{b_{il}}\right)^3$$

Light source statistics:

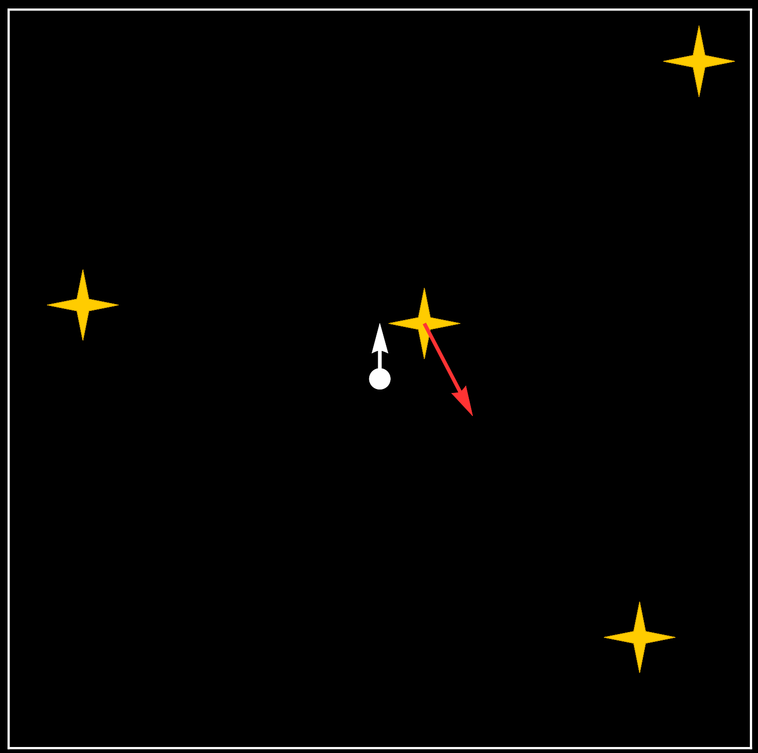
$$\#\text{ of lensed sources} \sim \Sigma_0 \frac{r_s^2}{D_l^2} \sim 10^4 \left(\frac{\Sigma_0}{10^8}\right)\left(\frac{r_s}{10^2\ \text{pc}}\right)^2\left(\frac{10\ \text{kpc}}{D_l}\right)^2$$

$$\left\langle \min_{i,l} b_{il} \right\rangle \simeq \sqrt{\frac{M_l}{\pi \rho_l D_i N_0}} \approx 2\times 10^{-5}\ \text{pc}\sqrt{\left[\frac{5\ \text{kpc}}{D_i}\right]\left[\frac{10^{-2}\rho_{\text{DM},\odot}}{\rho_l}\right]\left[\frac{M_l}{M_\odot}\right]\left[\frac{10^9}{N_0}\right]}$$

Signal Observables

Signal Observables

Outlier acceleration $\mathcal{O}_\alpha$



Signal Observables

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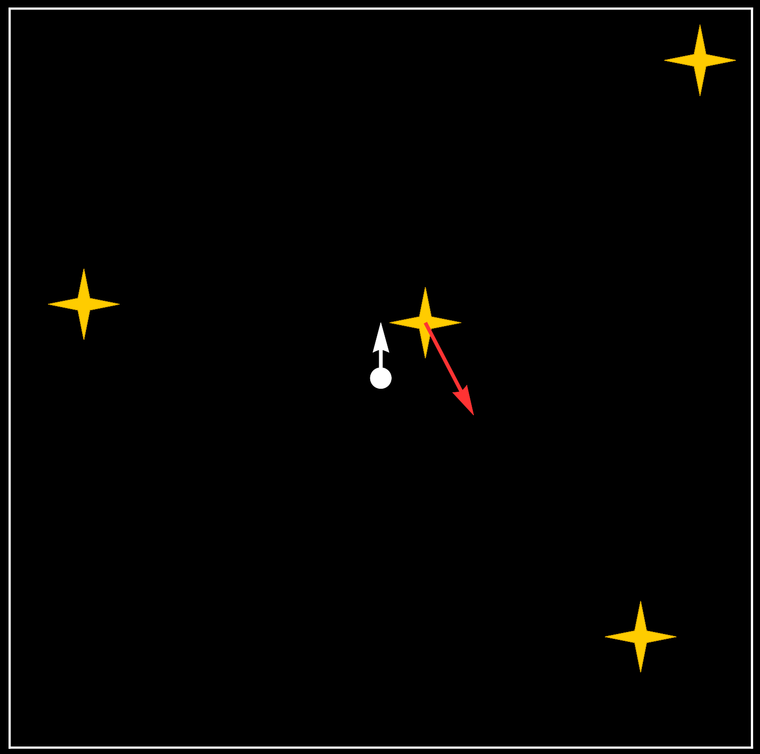
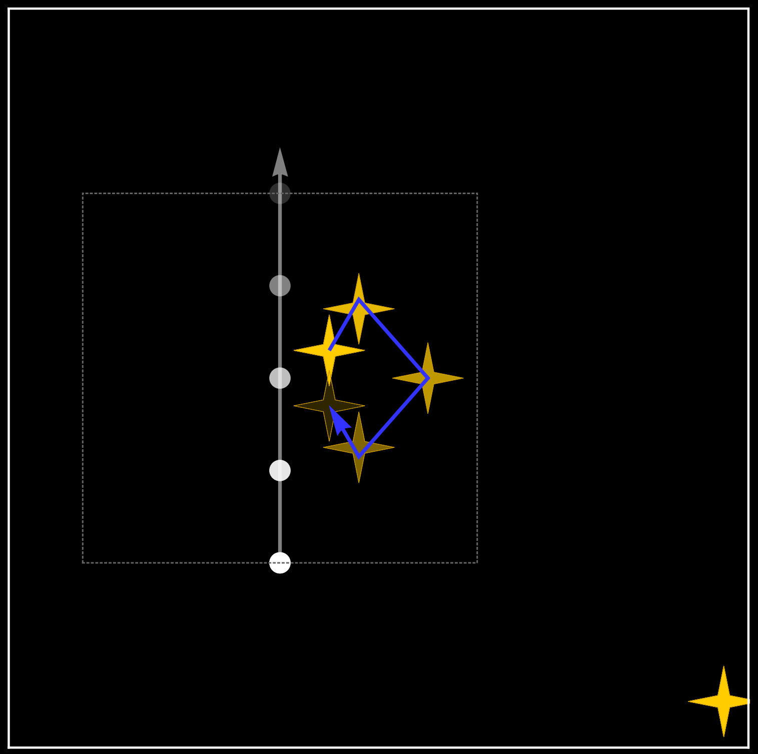


Mono−blip $\mathcal{B}_l^{\mathrm{mono}}$



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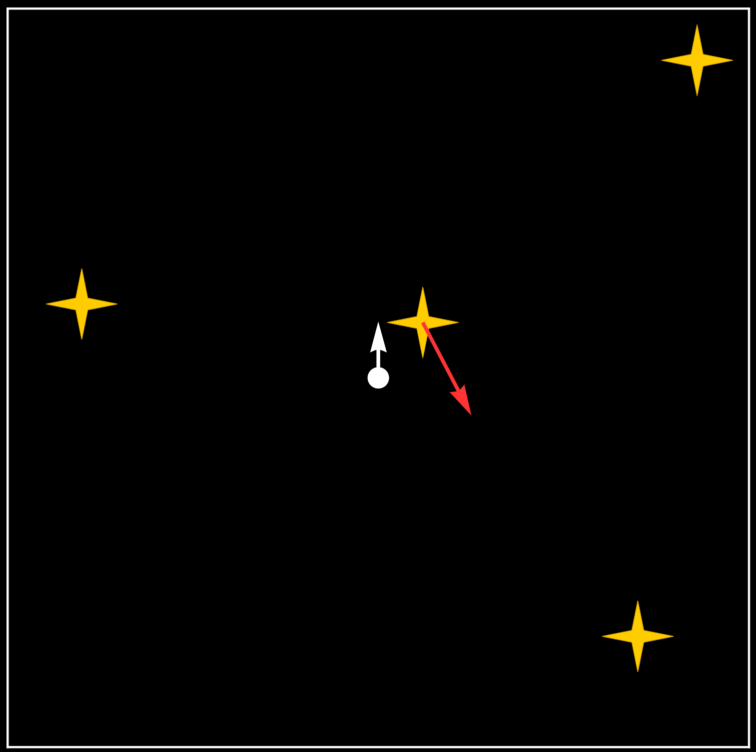

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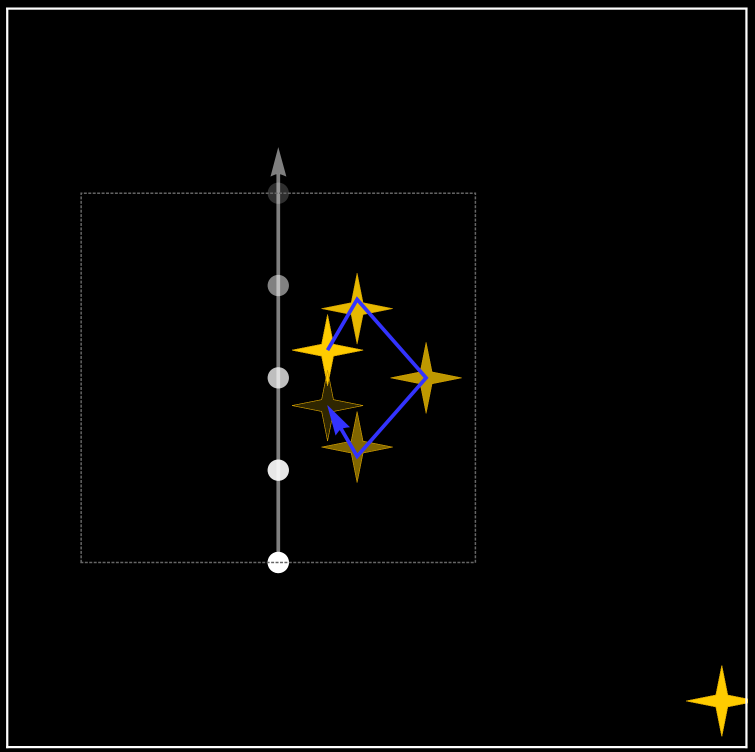
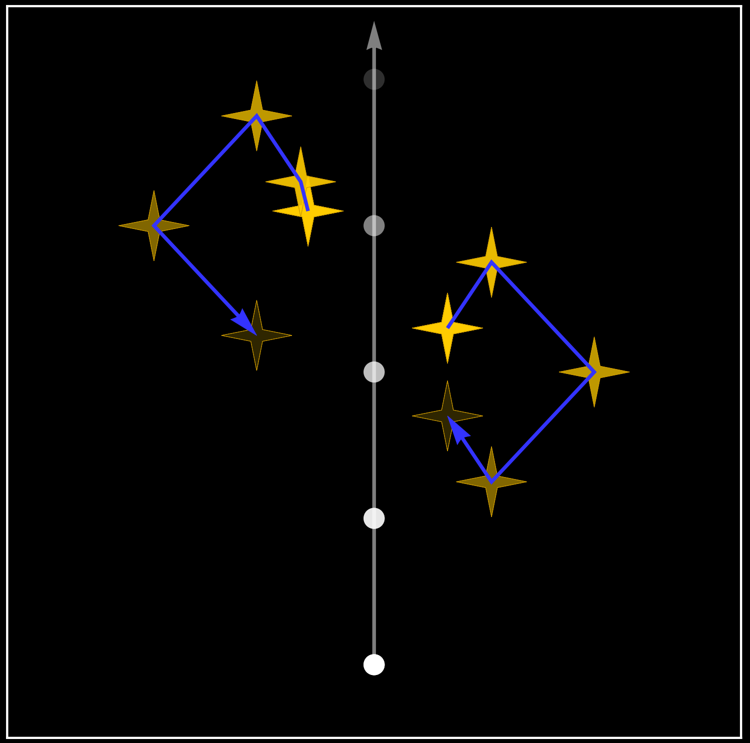
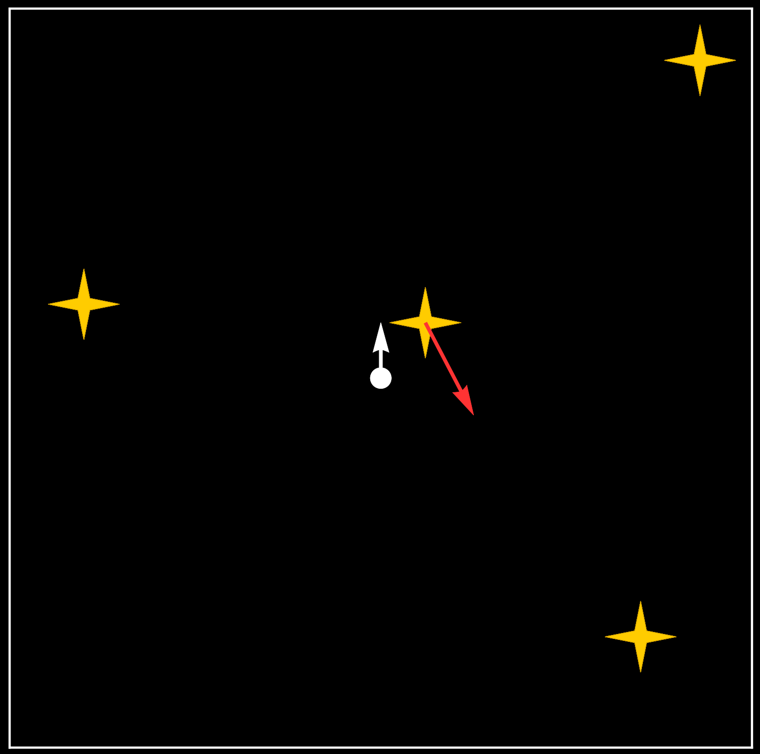


Multi−blip $\mathcal{B}_l^{\mathrm{multi}}$



Signal Observables

Outlier acceleration $\mathcal{O}_\alpha$



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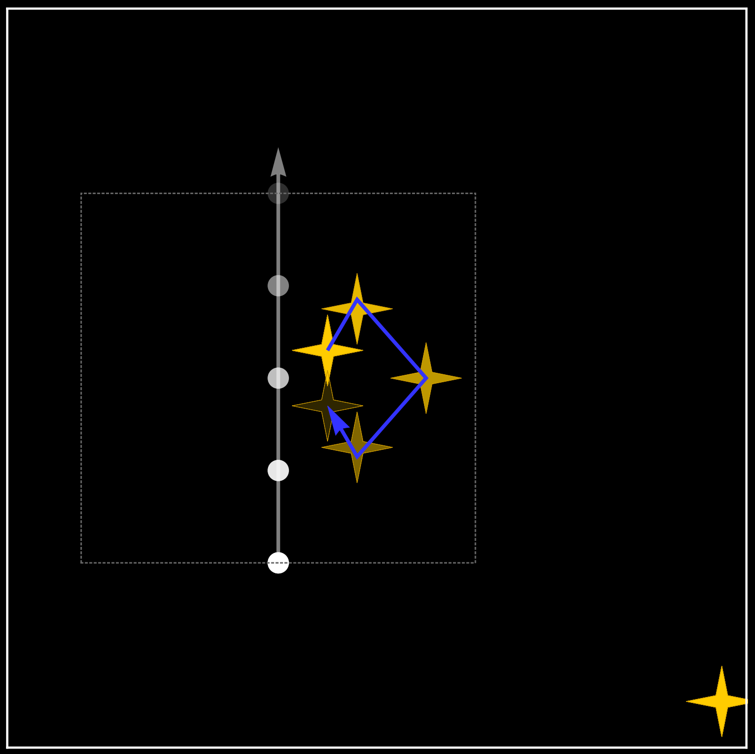
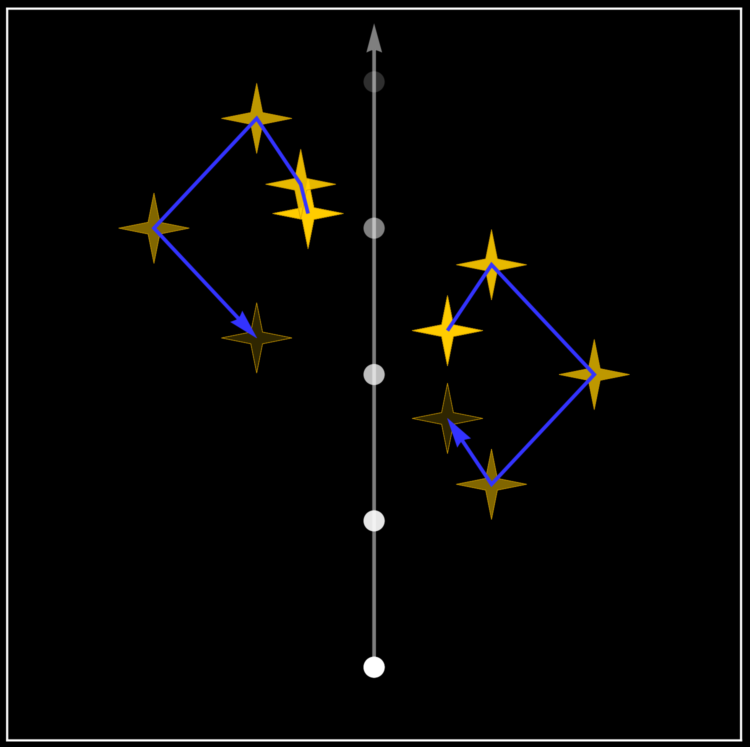
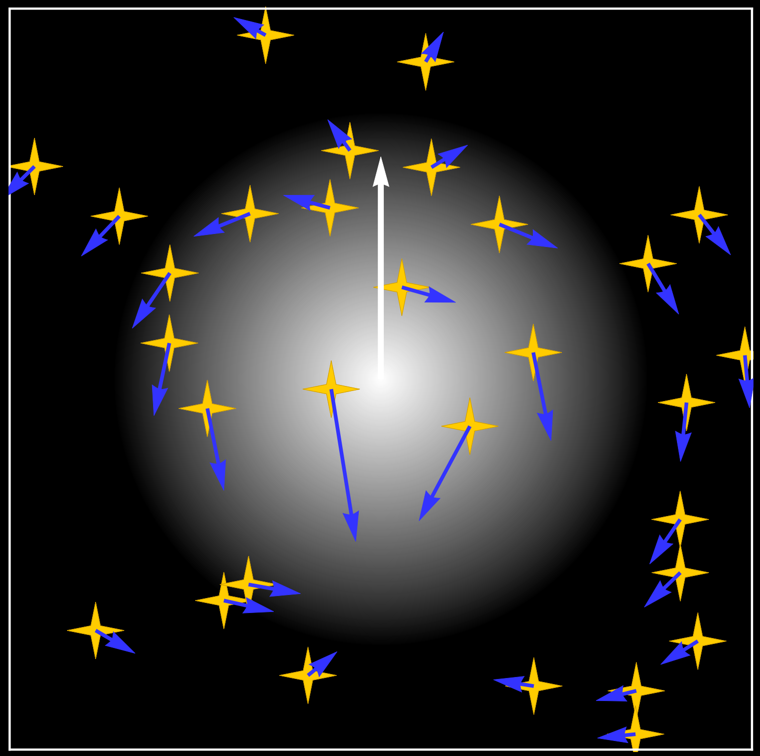


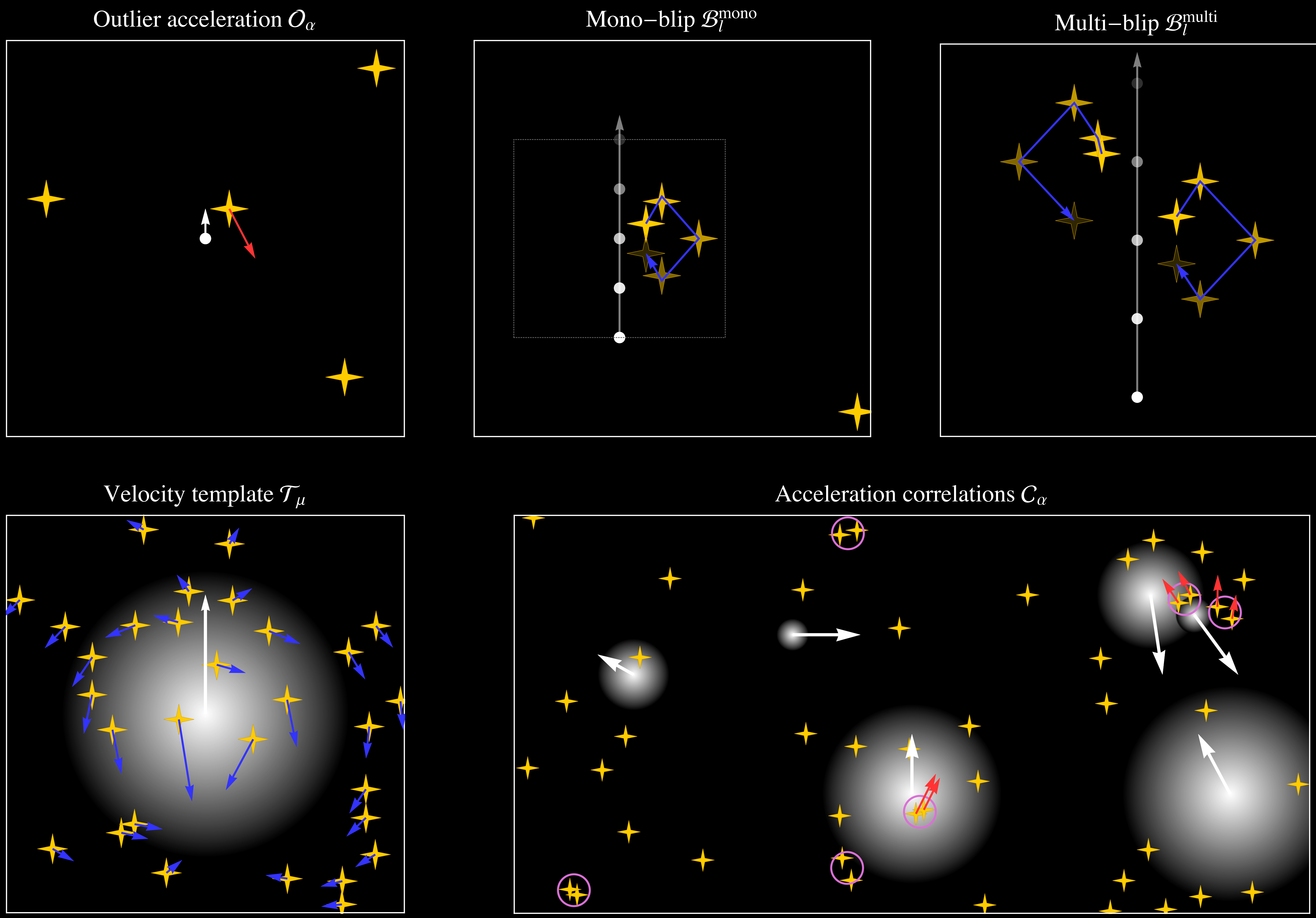
Multi−blip $\mathcal{B}_l^{\mathrm{multi}}$



Velocity template $\mathcal{T}_\mu$

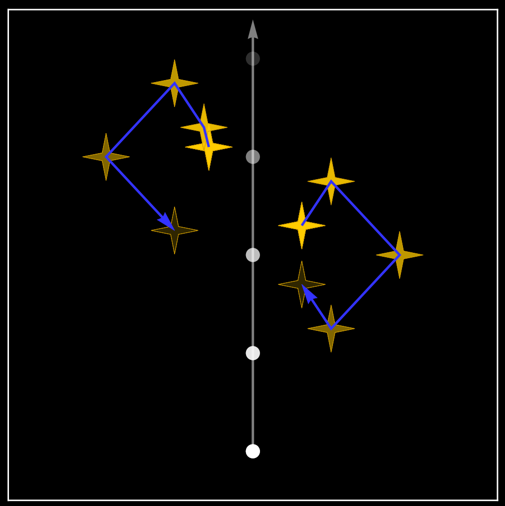


Signal Observables



Multi-Source Observables

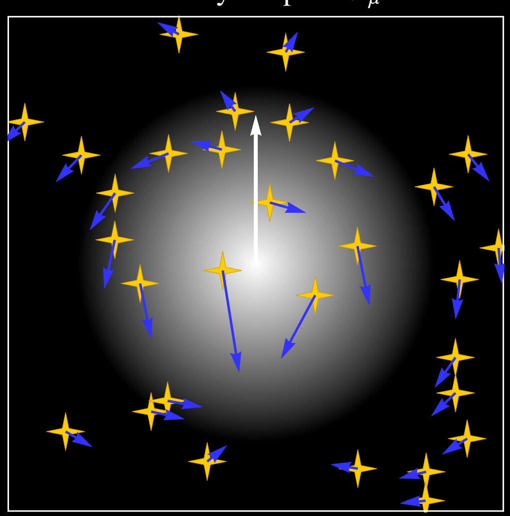
Multi−blip $\mathcal{B}_l^{\mathrm{multi}}$



blips:

$$\mathcal{B}\left[\mathbf{x}_l(t)\right] \equiv \sum_{i\in\square} \frac{1}{\sigma^2_{\delta\theta,i}} \sum_n \delta\boldsymbol{\theta}_i\left(t_n\right) \cdot \Delta\boldsymbol{\theta}_{il}\left[\mathbf{x}_l(t_n)\right]$$

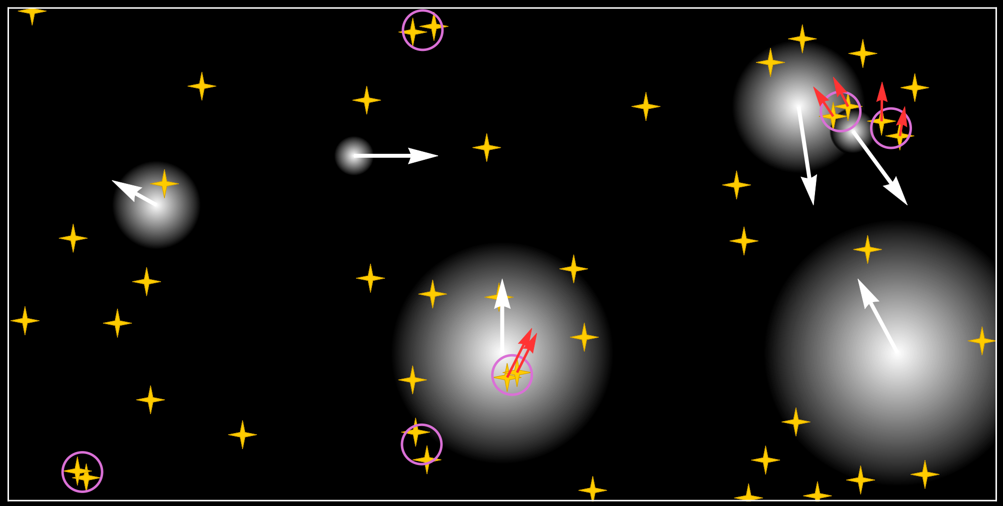
Velocity template $\mathcal{T}_\mu$



template velocity:

$$\mathcal{T}_\mu\left[\boldsymbol{\theta}_t, \beta_t, \hat{\mathbf{v}}_t\right] = \sum_i \frac{\dot{\boldsymbol{\theta}}_i}{\sigma^2_{\mu,i}} \cdot \boldsymbol{\mu}_t\left[\frac{\beta_{it}}{\beta_t}, \hat{\boldsymbol{\beta}}_{it}, \hat{\mathbf{v}}_t\right]$$

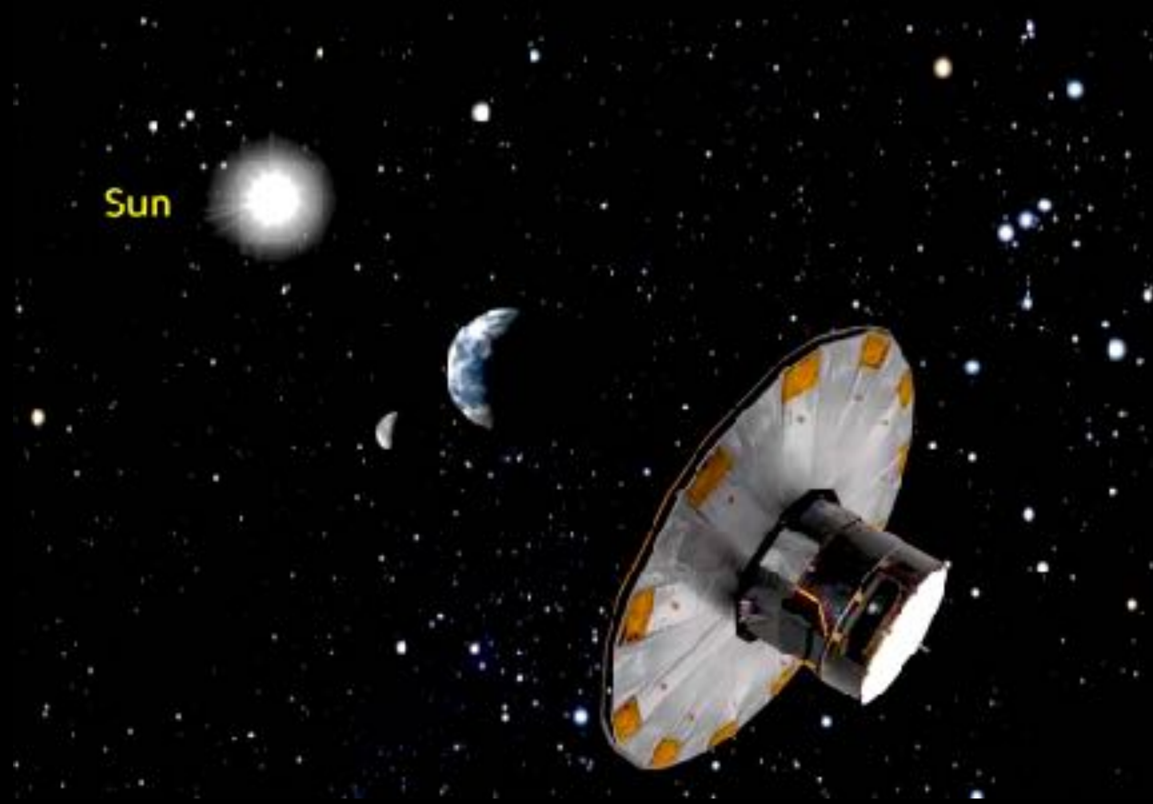
Acceleration correlations C_α



correlated acceleration:

$$\mathcal{C}_\alpha[\beta_-, \beta_+, \delta] = \frac{1}{2} \sum_{i\neq j} \frac{\ddot{\boldsymbol{\theta}}_i \cdot \ddot{\boldsymbol{\theta}}_j}{\sigma^2_{\alpha,i}\sigma^2_{\alpha,j}} \frac{B[\beta_{ij}; \beta_-, \beta_+]}{\beta_{ij}^\delta}$$

Precision Astrometry



Space-based optical surveys

current: *Gaia*

future: *Theia*

per-epoch precision $\sigma_{\delta\theta} \sim 10\ \mu\text{as}$

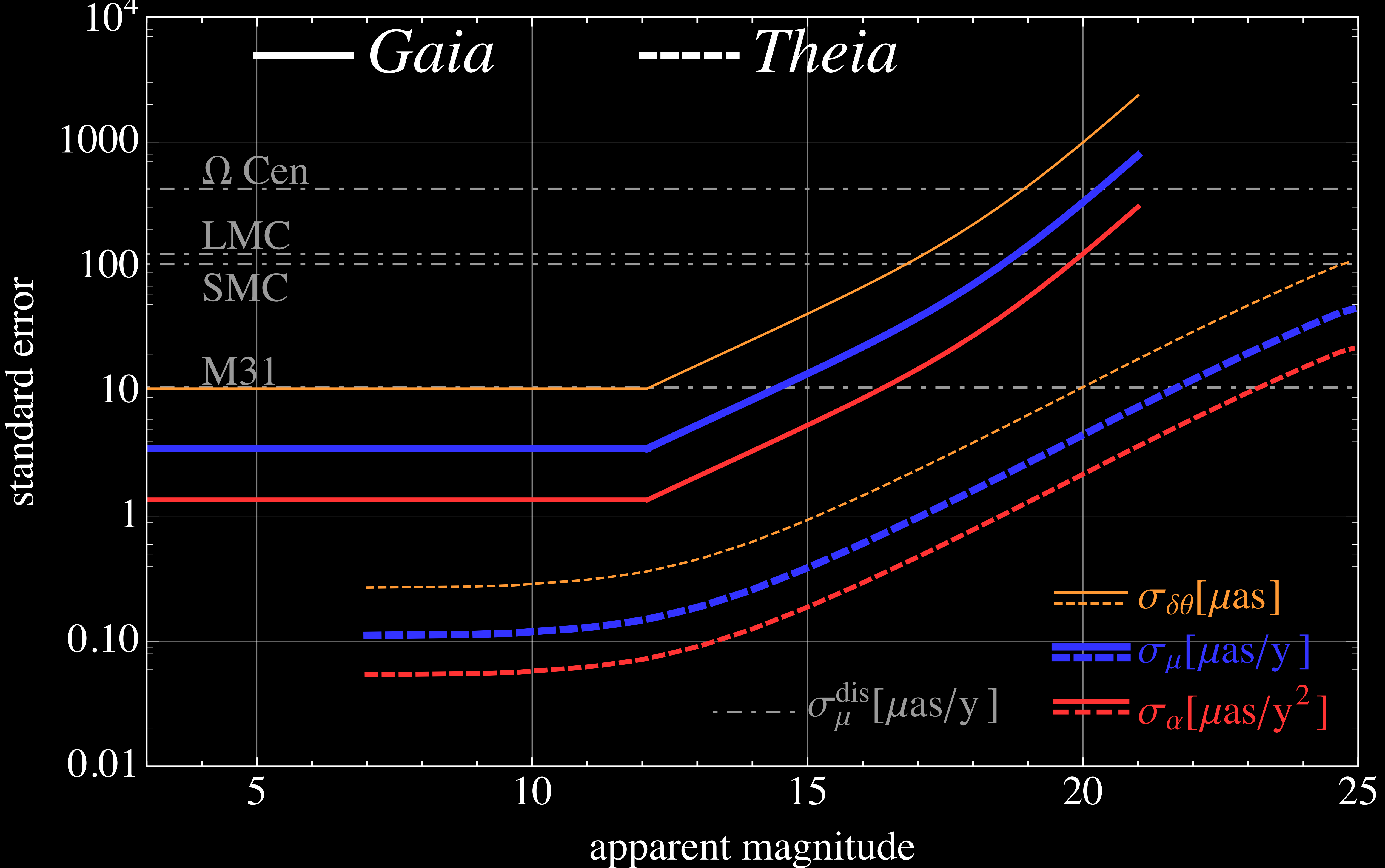
full-sky catalogs $N_0 \gtrsim 10^9$

Ground-based radio surveys

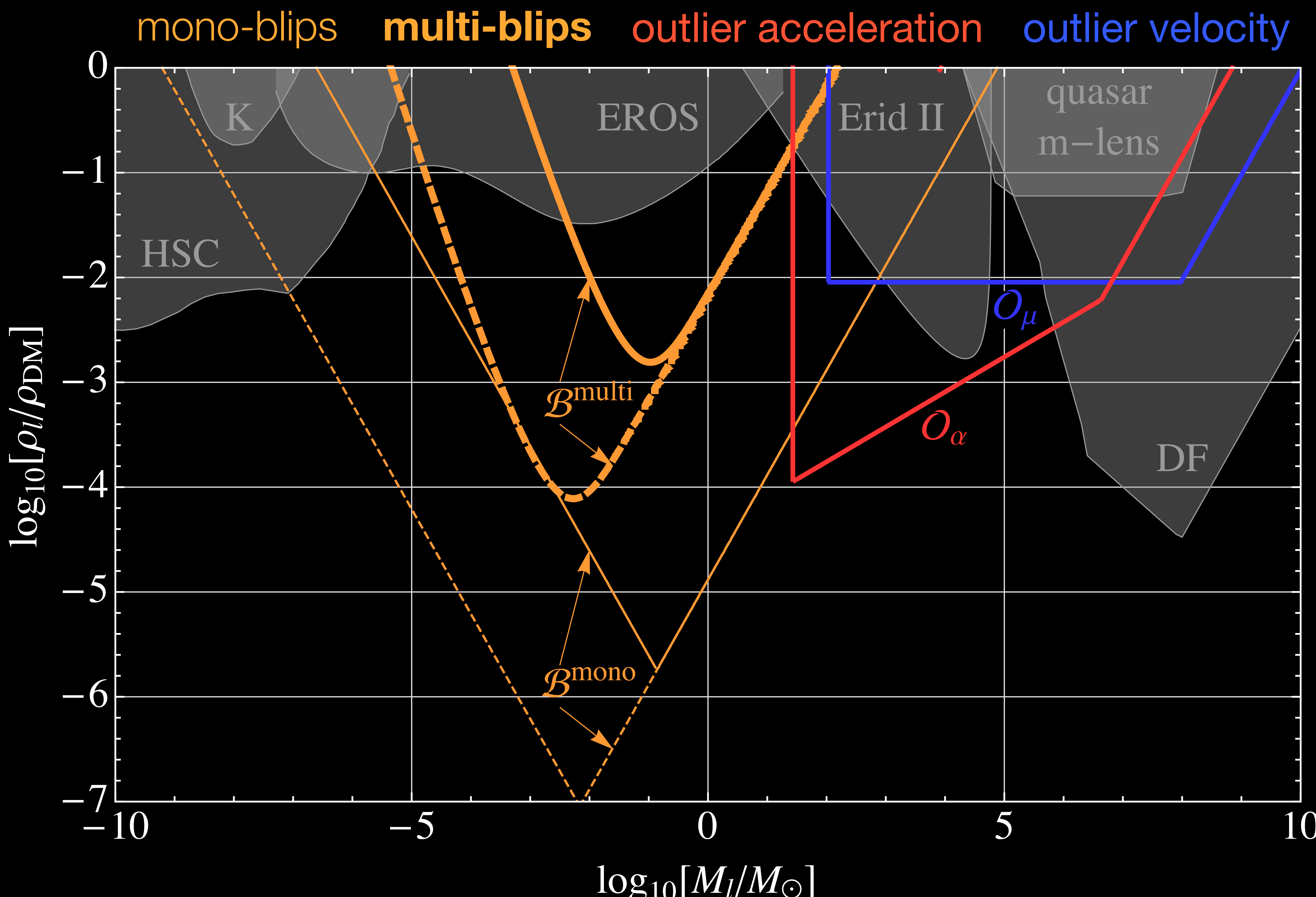
current: VLA

future: SKA

Stochastic Noise

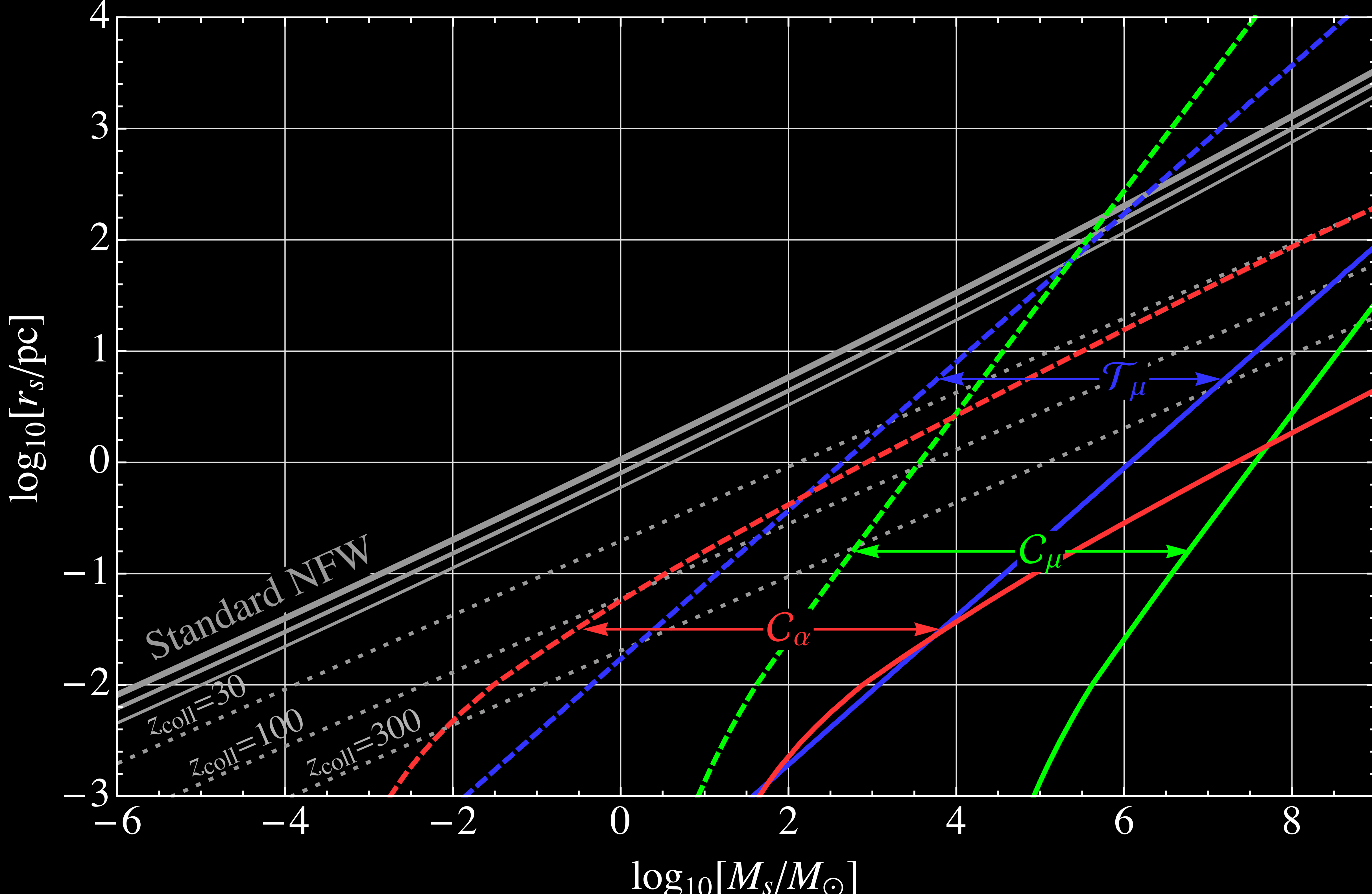


Compact Objects



NFW Subhalos

correlated acceleration template velocity correlated velocity



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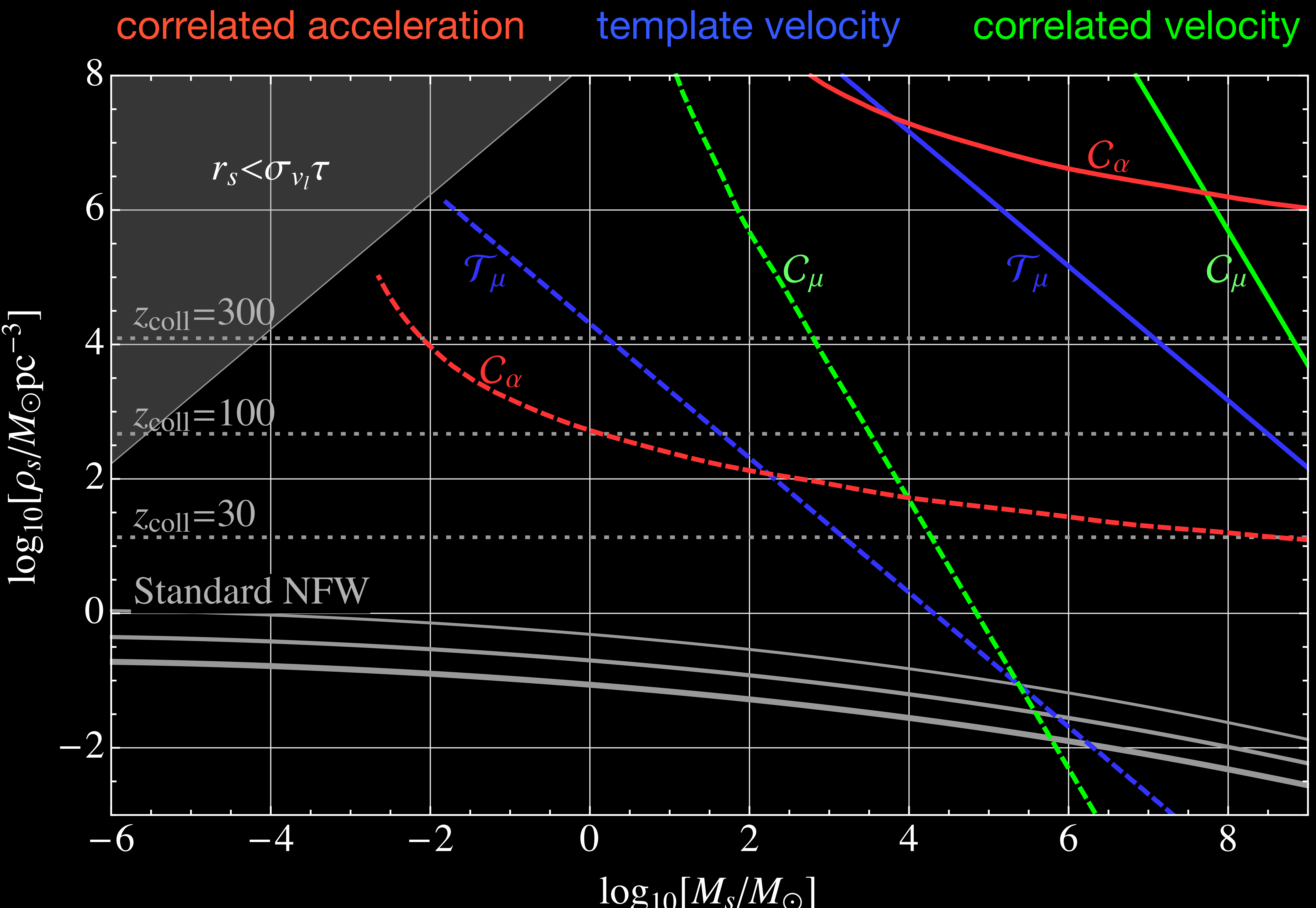
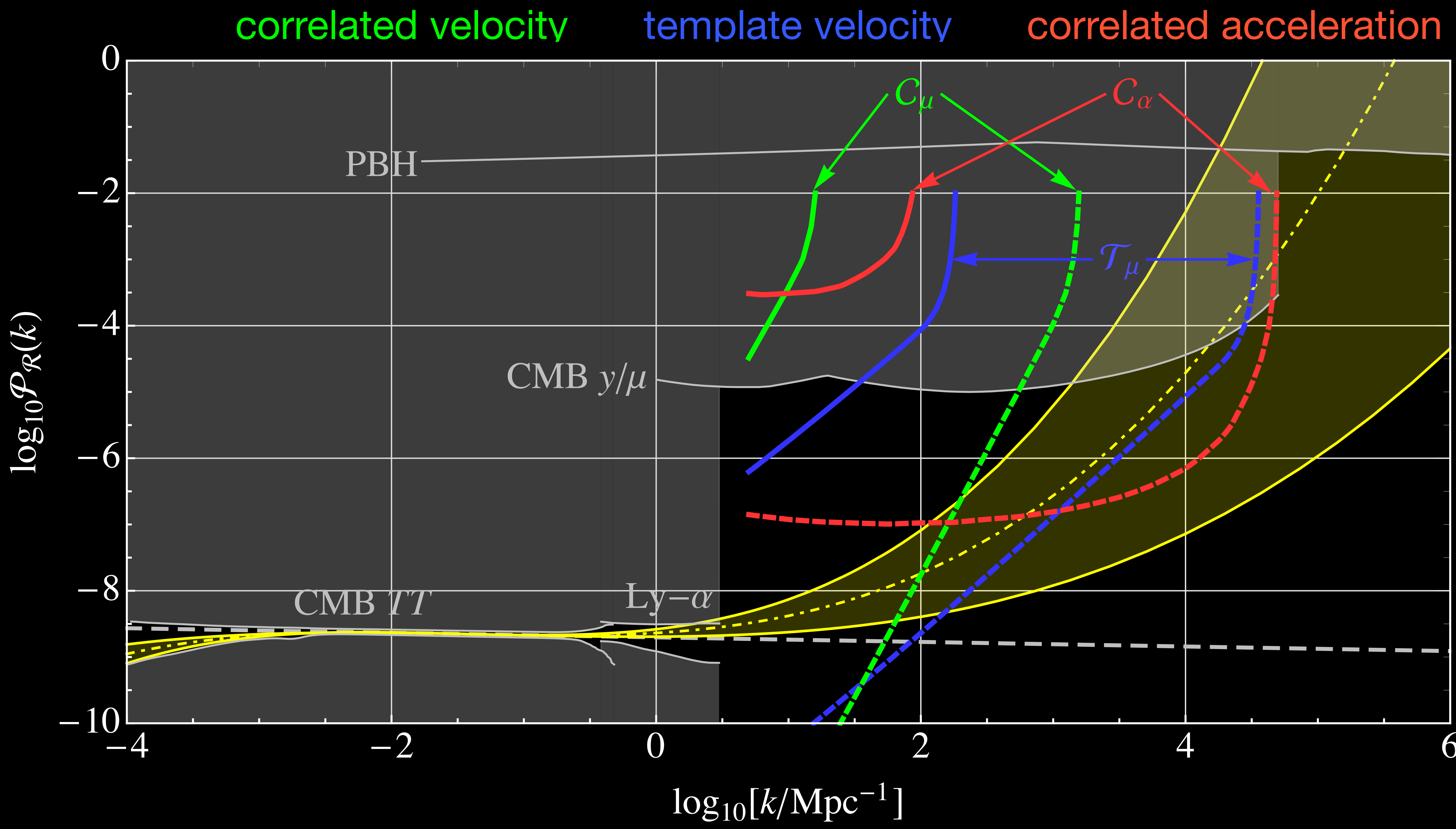
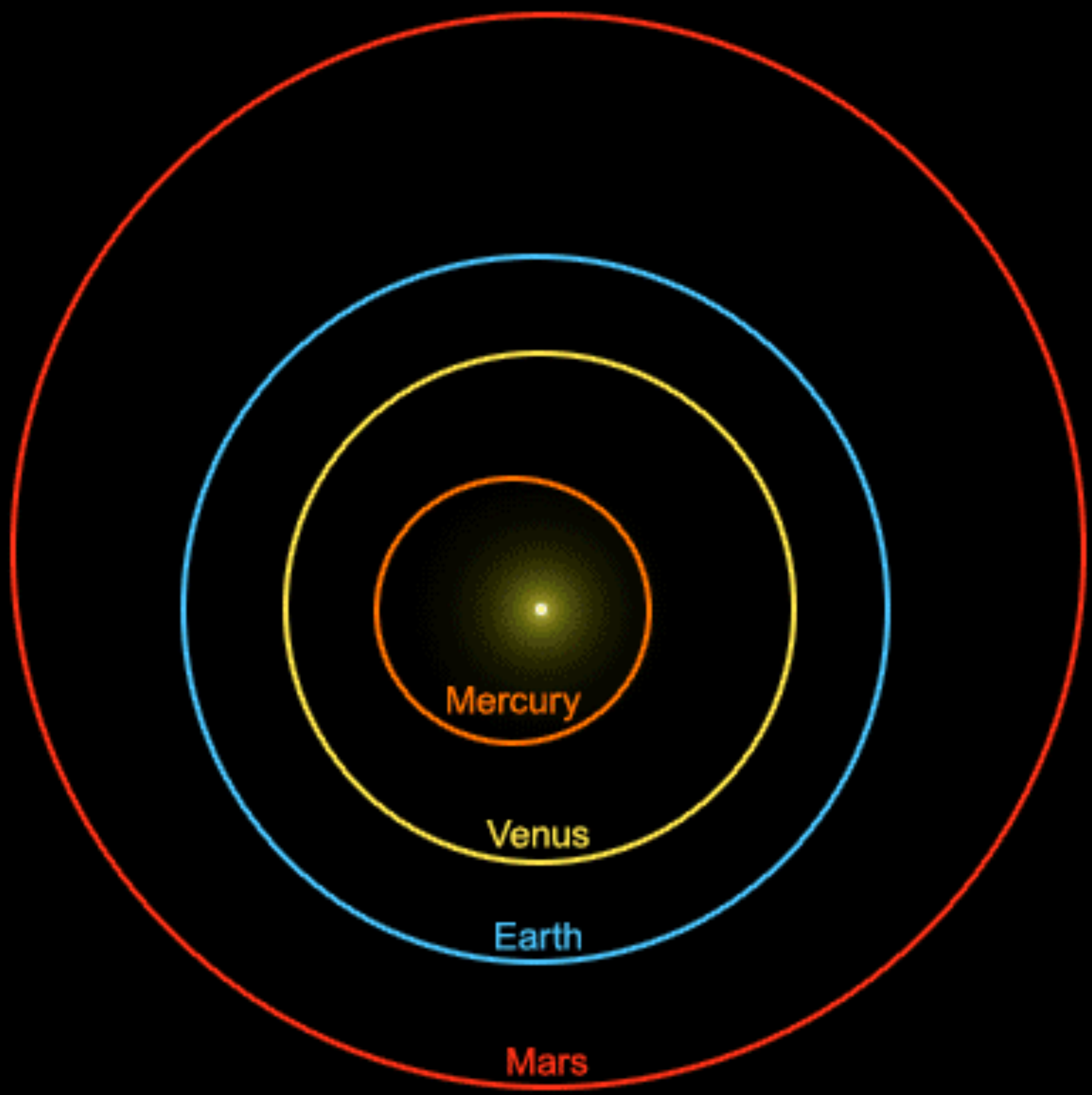


Primordial Power Spectrum



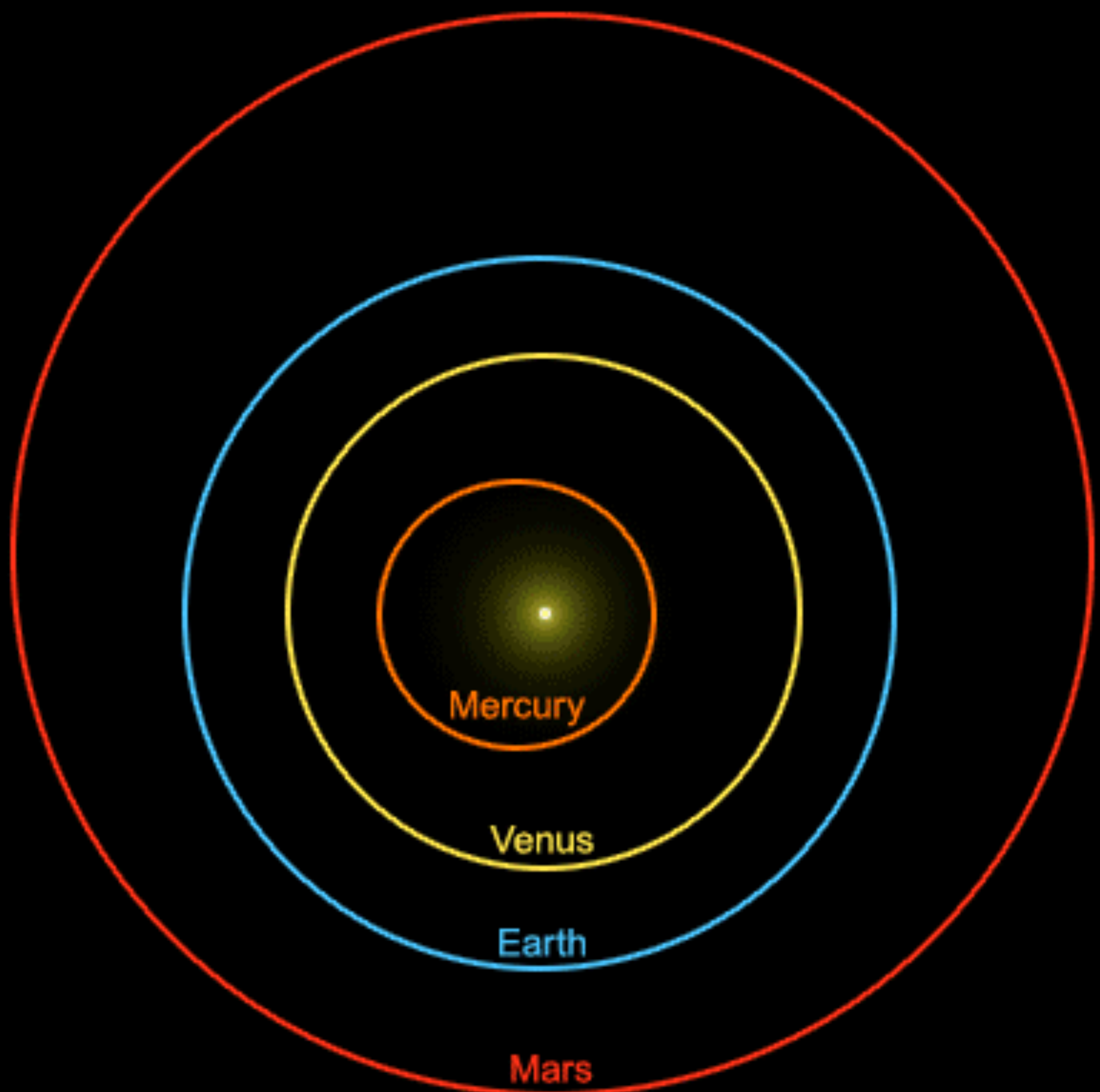
Planet Nine



Terrestrial Planets

[nagualdesign]

Planet Nine



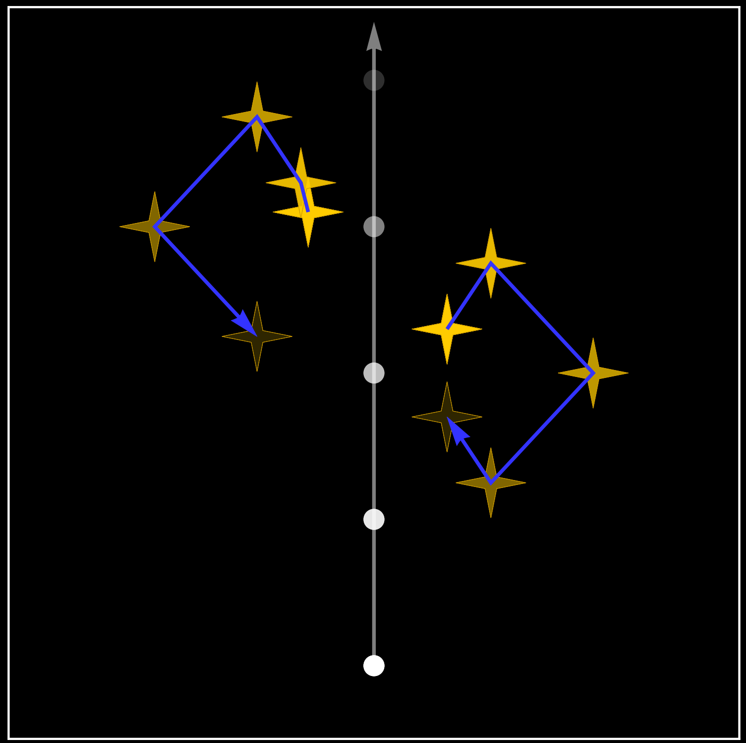
Terrestrial Planets

[nagualdesign]

Planet Nine

reflected sunlight : $P \propto \frac{M^{2/3}}{R^4}$

Multi−blip $\mathcal{B}_l^{\text{multi}}$



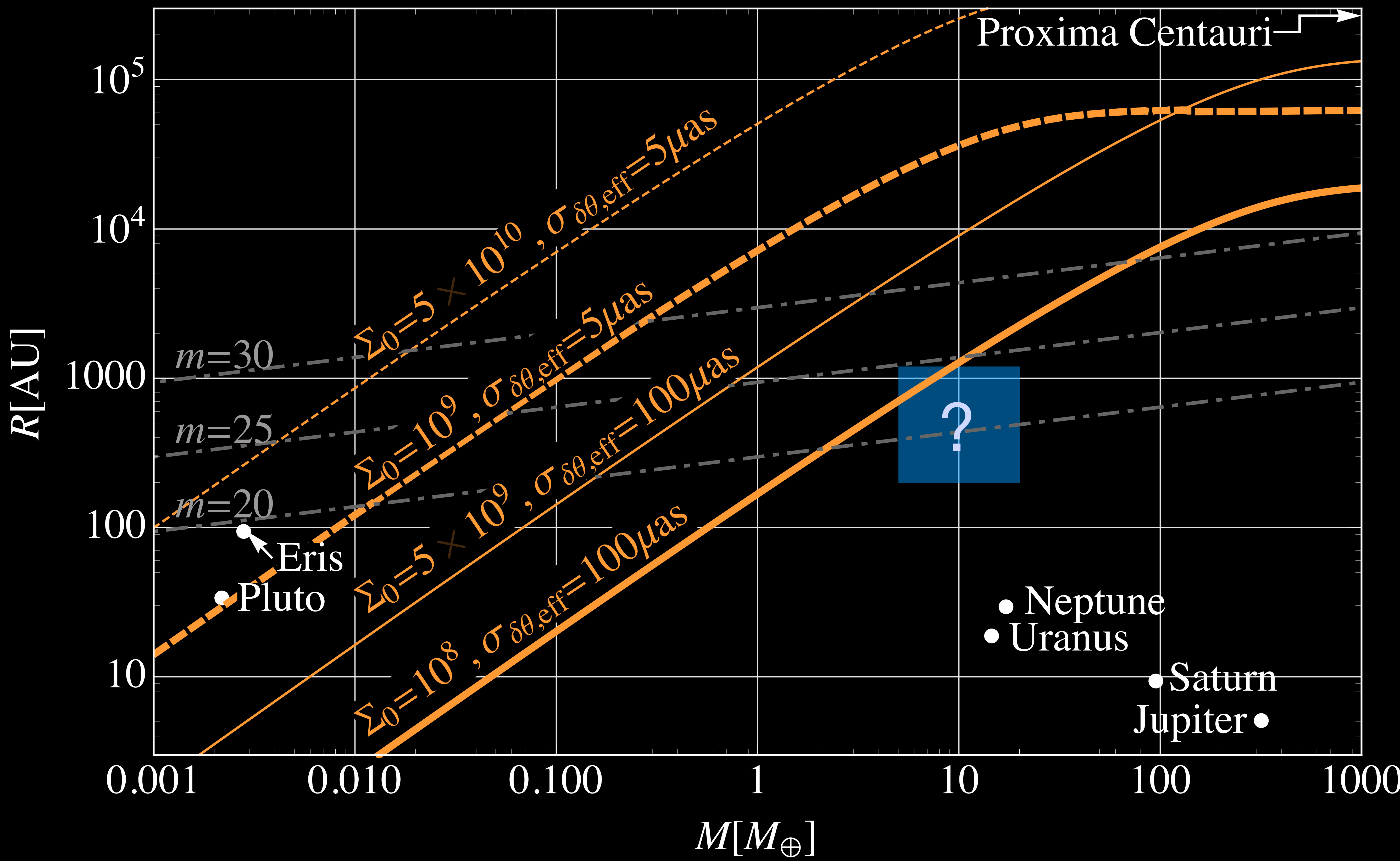
$$\Delta\theta_{il} = \frac{4GM_l}{b_{il}^*} \approx 0.024\ \mu\text{as} \left[\frac{M_l}{M_\oplus}\right] \left[\frac{\text{AU}}{b_{il}}\right]$$

$$N_0^{\mathcal{B}} \simeq \frac{4\Sigma_0 \text{AU}^2}{R_l^2} \approx 400 \left[\frac{\Sigma_0}{10^8}\right] \left[\frac{1000\ \text{AU}}{R_l}\right]^2$$

$$\text{SNR}_{\mathcal{B}_l}^{\text{multi}} \simeq \frac{4G_N M_l}{R_l} \frac{\sqrt{f_{\text{rep}}\tau}}{\sigma_{\delta\theta,\text{eff}}} \sqrt{2\pi\Sigma_0 \ln\left[\frac{4\Sigma_0 \text{AU}^2}{R_l^2}\right]}$$

Planet Nine

Multi−blip searches for outer Solar System planets



Systematic Noise

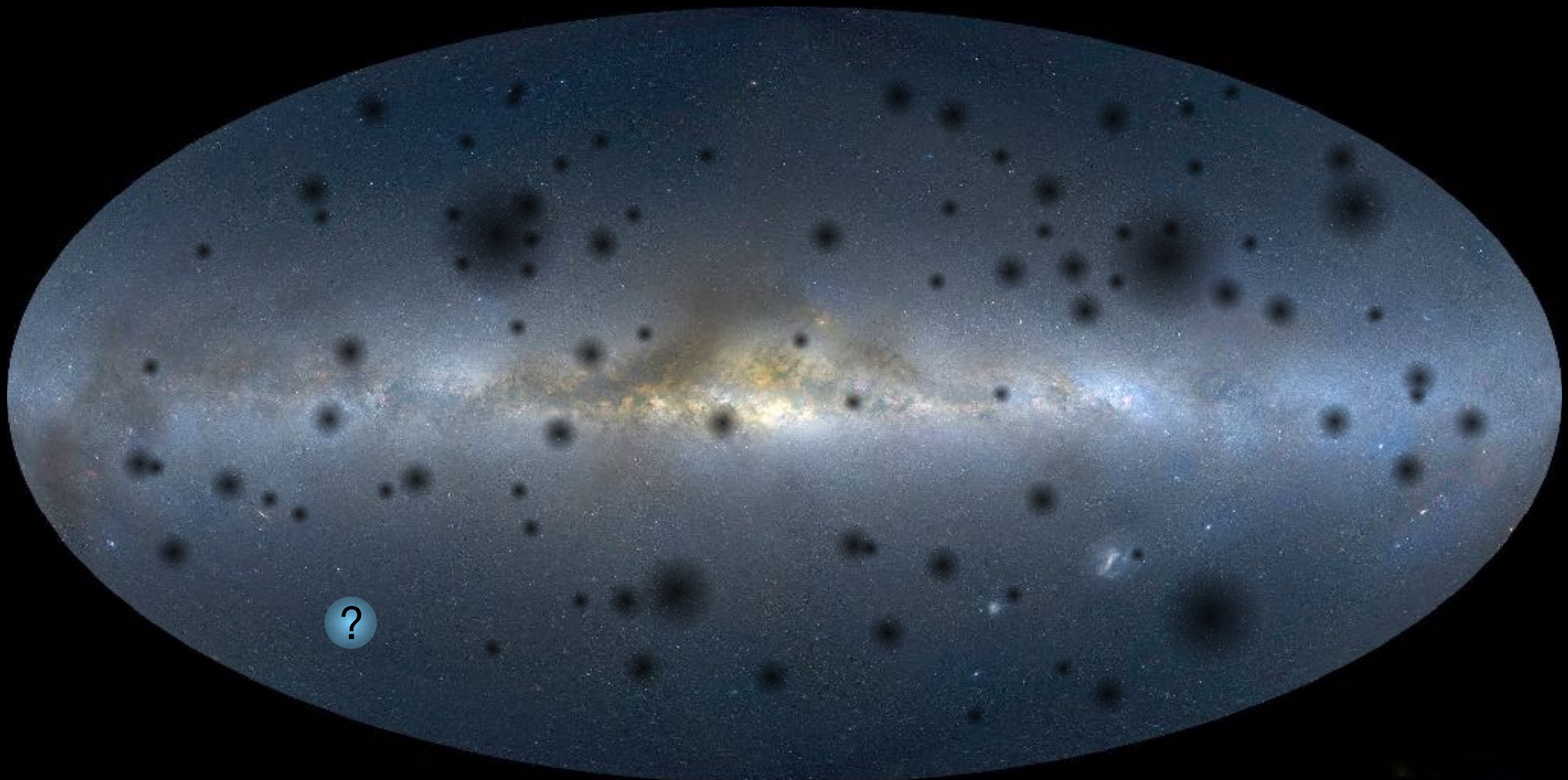
Observable	Systematic	Discrimination
outlier velocities	hypervelocity stars	nearby proper motions line-of-sight velocity
outlier accelerations	wide binary companions	disk vs Magellanic Clouds
blips	baryonic lensing	disk vs Magellanic Clouds mass determination follow-up optical studies
template velocities	rotational velocity streams	template checks extra-galactic sources
correlated velocities	rotational velocity streams	angular scaling extra-galactic sources
correlated accelerations	MW disk + halo attraction local anomalies	line-of-sight distance angular scaling

Future Outlook

modern astrometric surveys: precision + statistics

time-domain, astrometric, weak gravitational lensing

halometry = measure motions and spectrum of dark objects in Galactic Halo



Backup

Universality Classes

$$\rho(r) = \frac{2^{3-\gamma}\rho_s}{\left(\frac{r}{r_s}\right)^{\gamma}\left(1+\frac{r}{r_s}\right)^{3-\gamma}}$$

$$\Delta\dot{\theta}_{il} \sim \frac{4G_N M_s v_{il}}{b_{il}^2} \min\left[1, \left(\frac{b_{il}}{r_s}\right)^{\min[2,3-\gamma]}\right]$$

$$\Delta\ddot{\theta}_{il} \sim \frac{4G_N M_s v_{il}^2}{b_{il}^3} \min\left[1, \left(\frac{b_{il}}{r_s}\right)^{\min[2,3-\gamma]}\right]$$

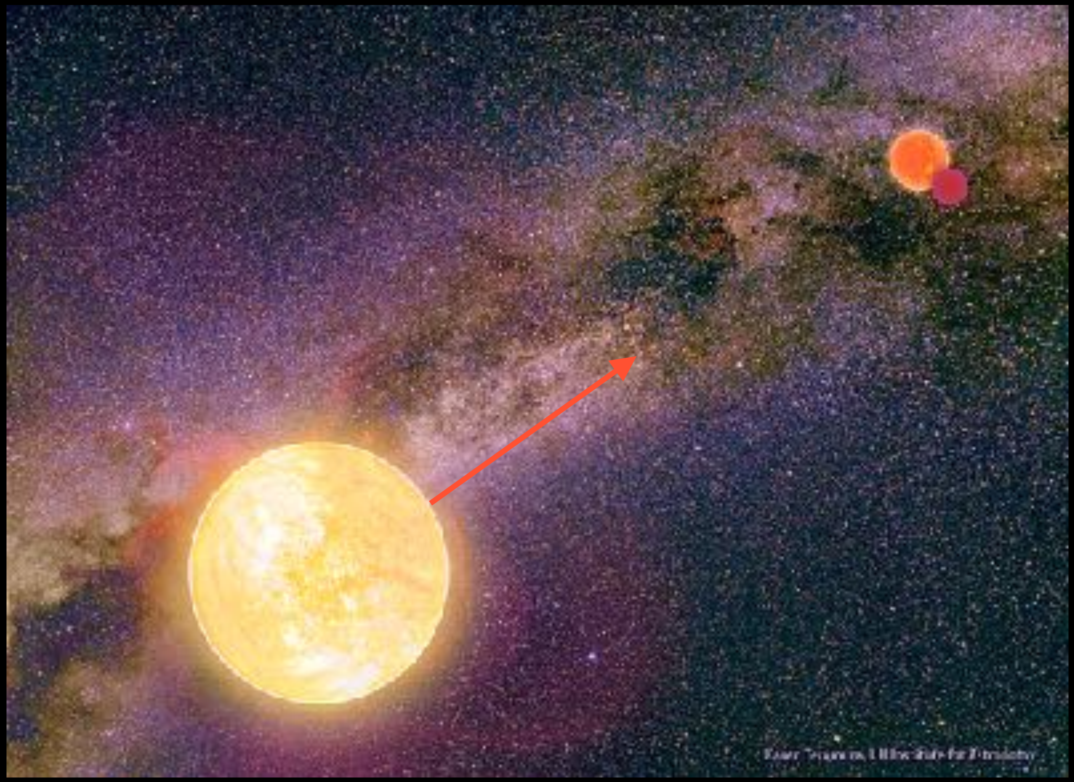
"soft lens": $\gamma < 2$ "hard lens": $\gamma \geq 2$

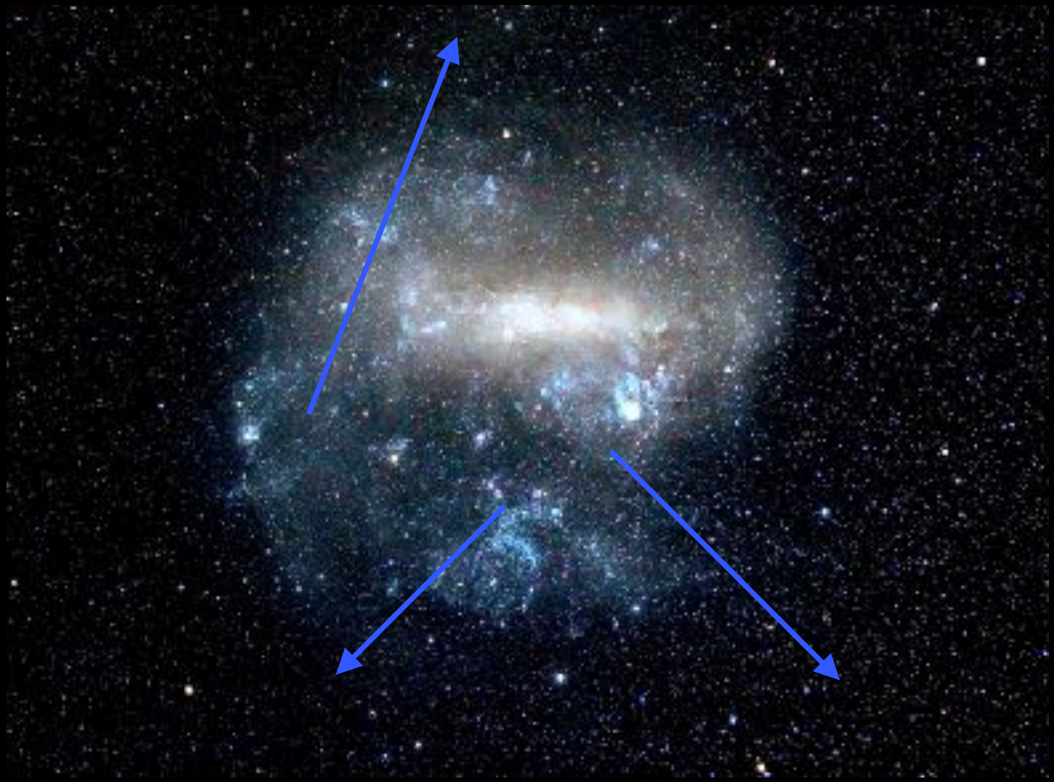
Systematics: outliers

accelerations: disk vs Magellanic Clouds:

$$\ddot{\theta}_{ic} = \frac{G_N^{1/3}}{D_i} \frac{m_c}{m_i^{2/3}} \left(\frac{2\pi}{T}\right)^{4/3}$$

$$\approx 0.18\ \mu\text{as y}^{-2} \left[\frac{10\ \text{kpc}}{D_i}\right] \left[\frac{m_c}{10^{-3} M_\odot}\right] \left[\frac{M_\odot}{m_i}\right]^{2/3} \left[\frac{10\ \text{y}}{T}\right]^{4/3}$$



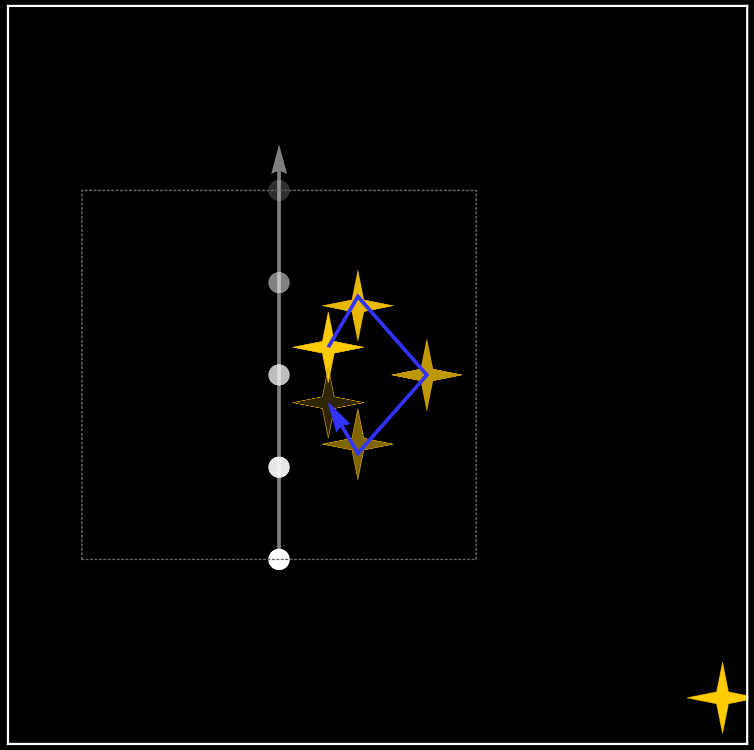


velocities: nearby proper motions + line-of-sight velocity

Systematics: blips

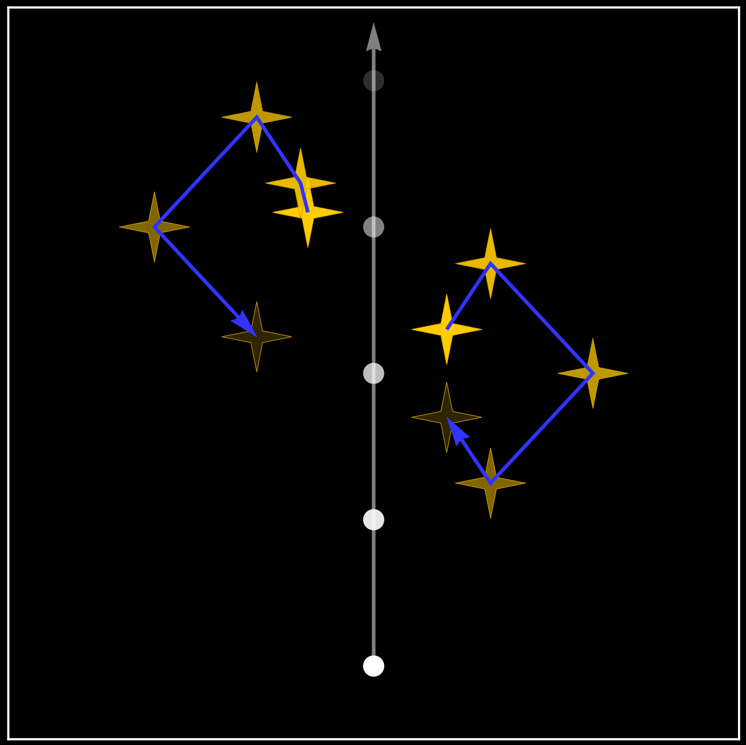
mono-blips: disk vs Magellanic Clouds
mass determination

Mono−blip $\mathcal{B}_l^{\mathrm{mono}}$



multi-blips: follow-up optical studies

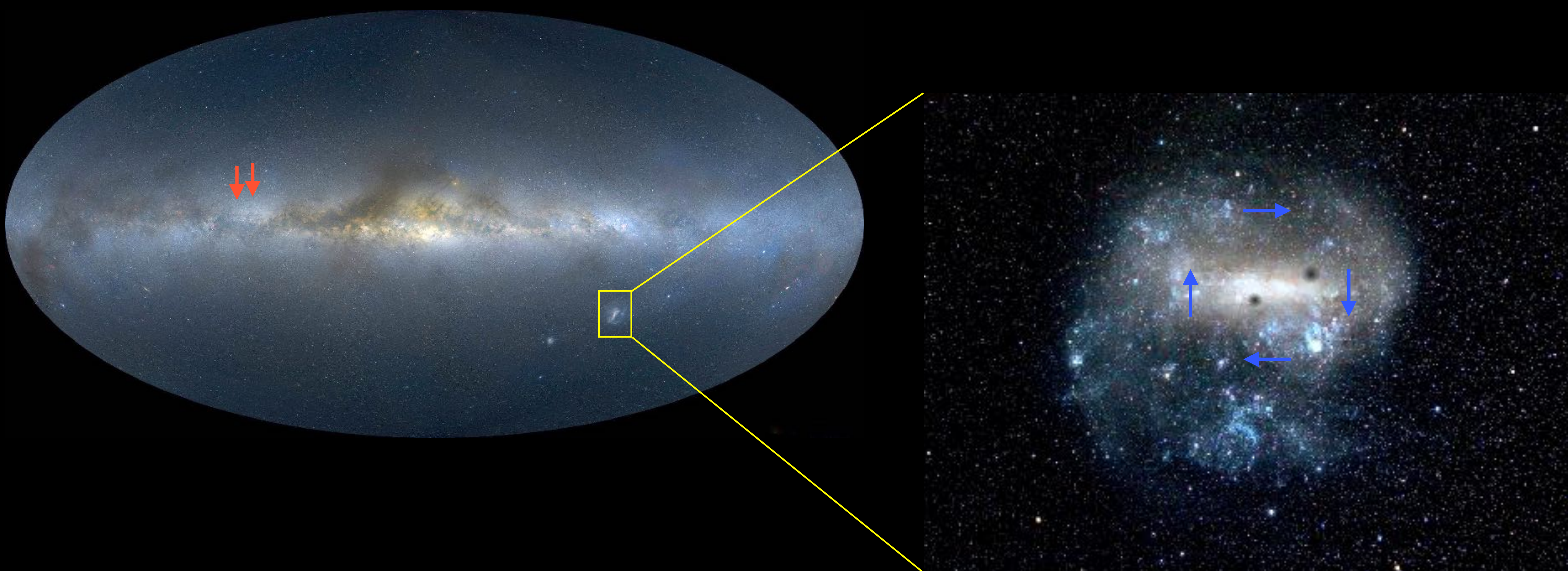
Multi−blip $\mathcal{B}_l^{\mathrm{multi}}$



Systematics: correlations

accelerations: $\alpha_d[D_i, R, z] \simeq \frac{G_N \Sigma_{d,0}}{2D_i} e^{-\frac{R}{R_d}} \left[1 - e^{-\frac{|z|}{z_d}}\right]$

$$\approx 2 \times 10^{-6} \mu\text{as y}^{-2} \left[\frac{10 \text{ kpc}}{D_i}\right] \left[\frac{e^{-\frac{R}{R_d}}}{e^{-\frac{R_\odot}{R_d}}}\right] \left[1 - e^{-\frac{|z|}{z_d}}\right]$$



velocities: scale separation + template checks + extra-galactic sources

Look-Elsewhere Effect

$$\mathrm{SNR}^{\mathrm{global}} \simeq \frac{\mathrm{SNR}^{\mathrm{local}}}{\sqrt{1+\ln N_{\mathrm{trial}}}}$$

$$\sqrt{1+\ln N_{\mathrm{trial}}} \begin{cases} \lesssim 4.7 & (\mathcal{O}_\mu, \mathcal{O}_\alpha) \\ \approx 5.4 & (\mathcal{B}^{\mathrm{mono}}) \\ \approx 4.1 & (\mathcal{B}^{\mathrm{multi}}) \\ \lesssim 3.5 & (\mathcal{T}_\mu) \\ = 1 & (\mathcal{C}_\mu, \mathcal{C}_\alpha) \end{cases}$$

SNR: mono-blip

$$\left\langle \min_{i,l} b_{il}^* \right\rangle = \frac{M_l}{\sigma_{v_l} \tau \rho_l D_i \Sigma_0 \Delta\Omega} \lesssim \sigma_{v_l} \tau$$

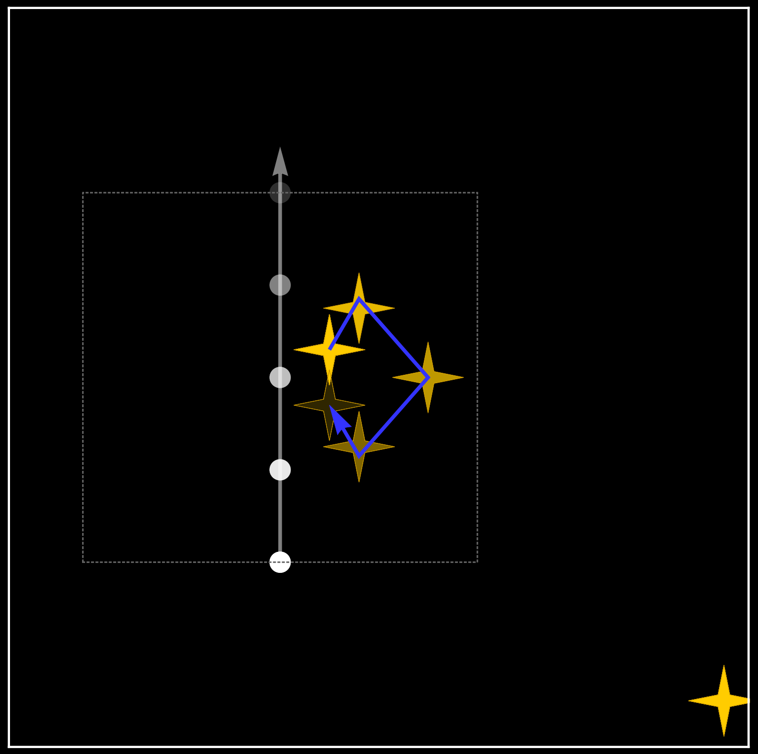
$$\mathrm{SNR}_{\mathcal{B}}^{\mathrm{mono}} \simeq \frac{4 G_N M_l}{\sigma_{\delta\theta,\mathrm{eff}}} \sqrt{\frac{\pi f_{\mathrm{rep}}}{\sigma_{v_l}} \frac{1}{\langle \min_{i,l} b_{il}^* \rangle}}$$

SNR: mono-blip

$$\left\langle \min_{i,l} b^*_{il} \right\rangle = \frac{M_l}{\sigma_{v_l} \tau \rho_l D_i \Sigma_0 \Delta\Omega} \lesssim \sigma_{v_l} \tau$$

$$\mathrm{SNR}^{\mathrm{mono}}_{\mathcal{B}} \simeq \frac{4 G_N M_l}{\sigma_{\delta\theta,\mathrm{eff}}} \sqrt{\frac{\pi f_{\mathrm{rep}}}{\sigma_{v_l}} \frac{1}{\langle \min_{i,l} b^*_{il} \rangle}}$$

Mono−blip $\mathcal{B}^{\mathrm{mono}}_l$



SNR: velocity template

$$\mathrm{SNR}_{\mathcal{T}_\mu}\left[D_l, v_{il}\right] = C_1 \left(1 - \frac{D_l}{D_i}\right) \frac{4 G_N M_s v_{il}}{r_s D_l} \sqrt{\frac{\Sigma_0}{\sigma^2_{\mu,\mathrm{eff}}}}$$

$$\left\langle \min_l D_l \right\rangle = \left(\frac{3}{n_l \Delta\Omega}\right)^{1/3} \approx 18\ \mathrm{kpc} \left[\frac{M_s}{10^7 M_\odot} \frac{1}{\Omega_{\mathrm{sub}}} \frac{0.01}{\Delta\Omega}\right]^{1/3}$$

$$\left\langle \max_l \mathrm{SNR}_{\mathcal{T}_\mu} \right\rangle \simeq \frac{\pi^{1/2} C_1}{2^{1/2} 3^{1/3}} \frac{4 G_N M_s \sigma_{v_l}}{r_s} \left(n_l \Delta\Omega\right)^{1/3} \sqrt{\frac{\Sigma_0}{\sigma^2_{\mu,\mathrm{eff}}}}$$

$$\approx 0.4\ \Omega_{\mathrm{sub}}^{1/3} \left[\frac{M_s}{10^7 M_\odot}\right]^{2/3} \left[\frac{10\ \mathrm{pc}}{r_s}\right] \left[\frac{N_0}{10^7}\right]^{1/2} \left[\frac{0.01}{\Delta\Omega}\right]^{1/6} \left[\frac{200\ \mu\mathrm{as\ y}^{-1}}{\sigma_{\mu,\mathrm{eff}}}\right]$$

SNR: correlations

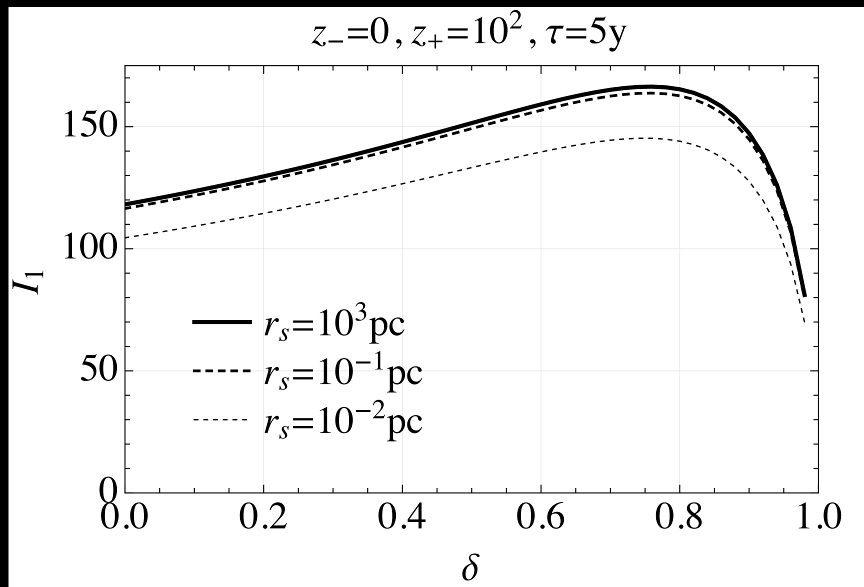
$$\mathrm{SNR}_{\mathcal{C}_\mu} = \sqrt{\frac{\pi}{2}}\sqrt{\Sigma_0^2\Delta\Omega}\,(n_l r_s^3)\left(\frac{4G_N M_s v_{il}}{r_s^2\sigma_{\mu,\mathrm{eff}}}\right)^2 I_1\left[\frac{\beta_-}{r_s/D_{\mathrm{max}}},\frac{\beta_+}{r_s/D_{\mathrm{max}}},\delta,\epsilon_s\right]$$

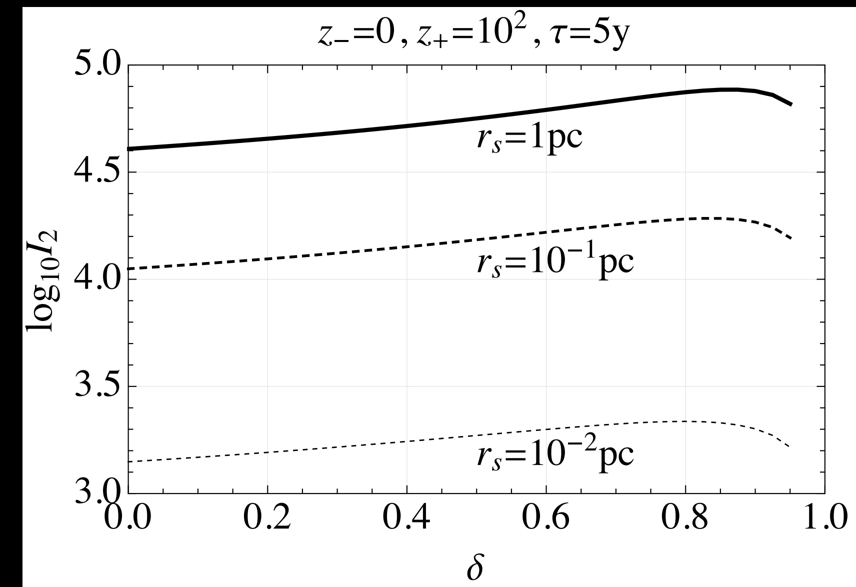
$$I_1\left[z_-,z_+,\delta,\epsilon_s\right] \equiv \sqrt{\frac{2-2\delta}{z_+^{2-2\delta}-z_-^{2-2\delta}}}\int_{z_-}^{z_+}\mathrm{d}z\, z^{1-\delta}\int_0^1\mathrm{d}y\int_{x>zy+\epsilon_s}\mathrm{d}^2x\;|\mathbf{G}_1[x,\hat{\mathbf{x}},\hat{\mathbf{v}}]|^2$$

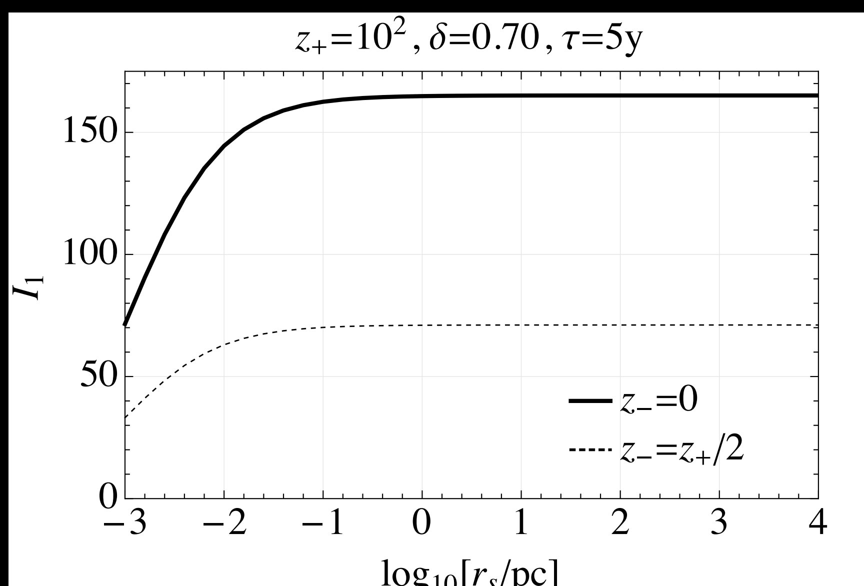
$$\mathrm{SNR}_{\mathcal{C}_\alpha} = \sqrt{\frac{\pi}{2}}\sqrt{\Sigma_0^2\Delta\Omega}\,(n_l r_s^3)\left(\frac{4G_N M_s v_{il}^2}{r_s^3\sigma_{\alpha,\mathrm{eff}}}\right)^2 I_2\left[\frac{\beta_-}{r_s/D_i},\frac{\beta_+}{r_s/D_i},\delta,\epsilon_s\right]$$

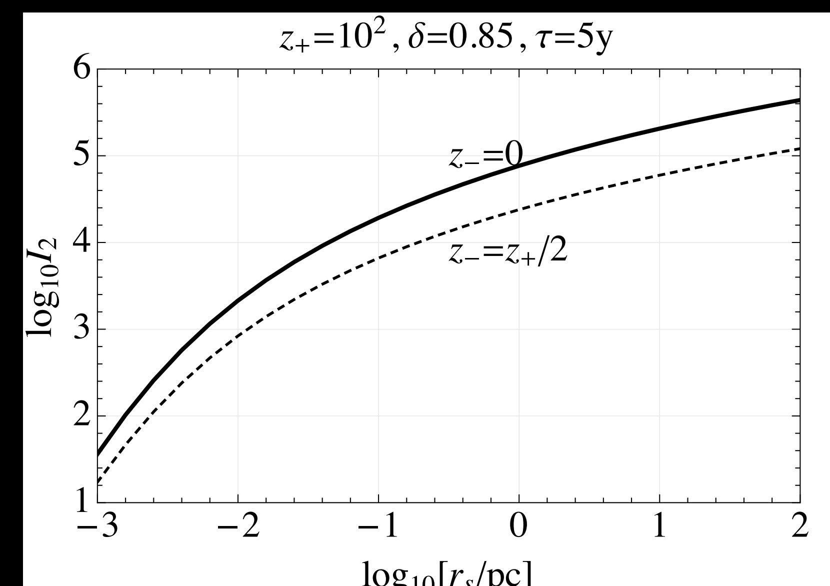
$$I_2\left[z_-,z_+,\delta,\epsilon_s\right] \equiv \sqrt{\frac{2-2\delta}{z_+^{2-2\delta}-z_-^{2-2\delta}}}\int_{z_-}^{z_+}\mathrm{d}z\, z^{1-\delta}\int_0^1\mathrm{d}y\,(1-y)^2\int_{x>zy+\epsilon_s}\mathrm{d}^2x\;|\mathbf{G}_2[x,\hat{\mathbf{x}},\hat{\mathbf{v}}]|^2$$

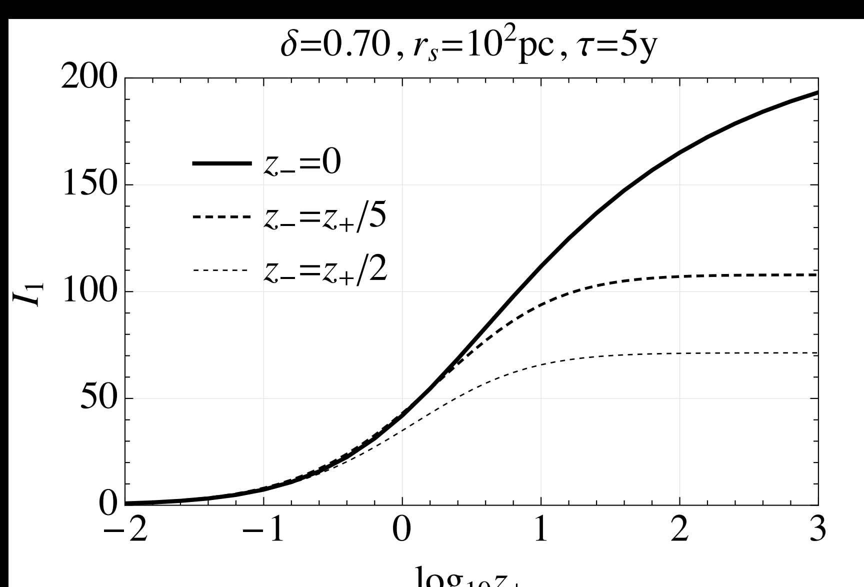
SNR: correlations

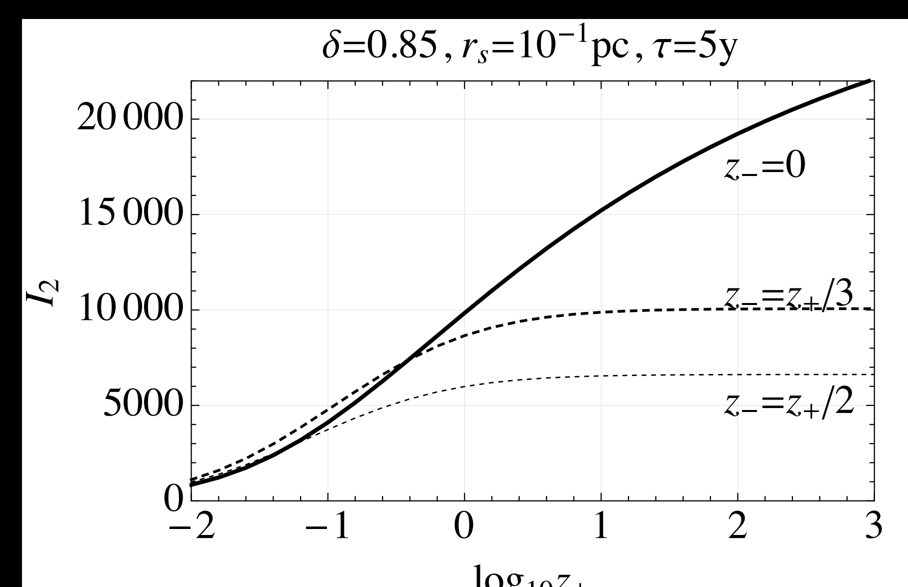




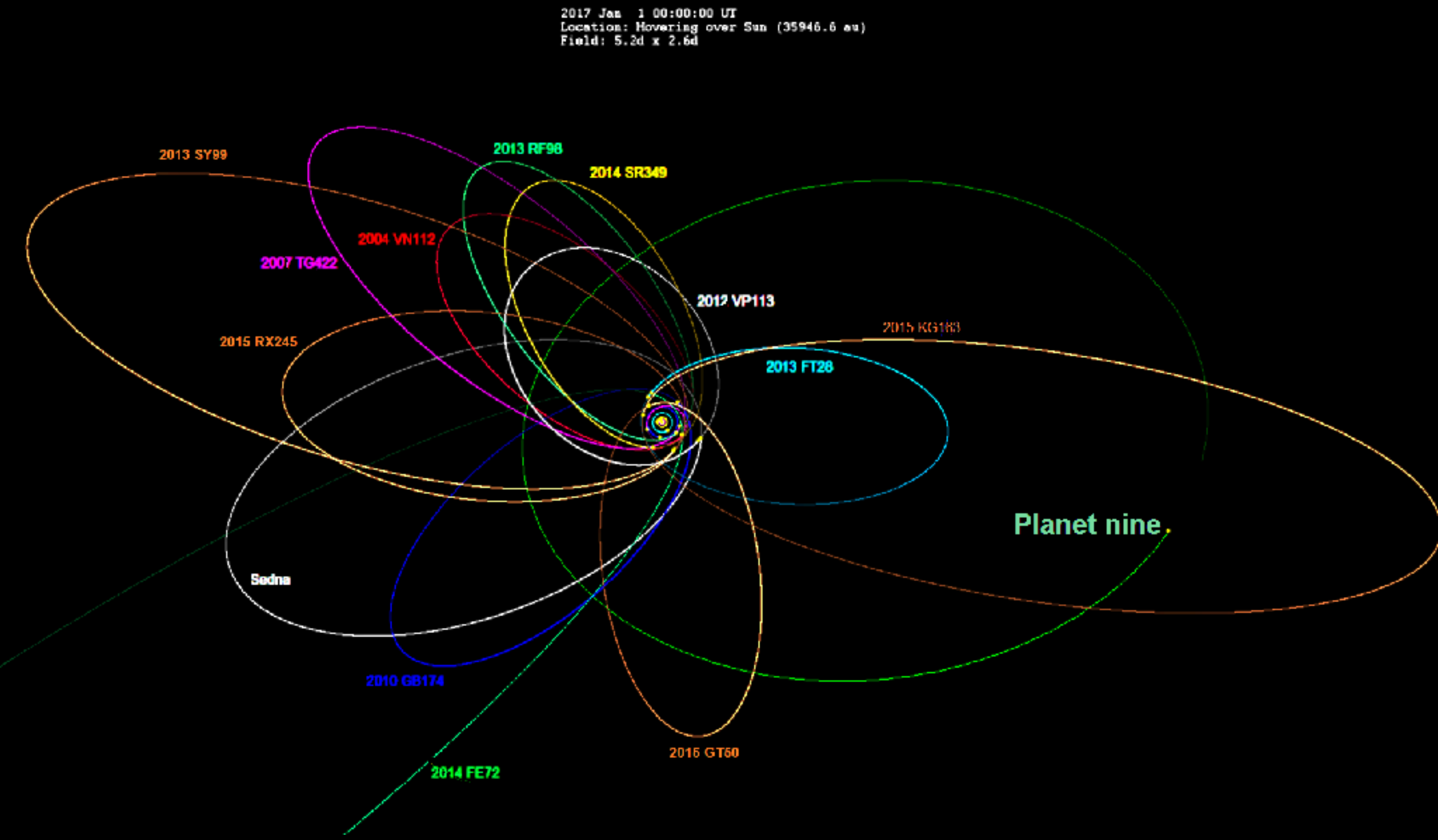








Planet Nine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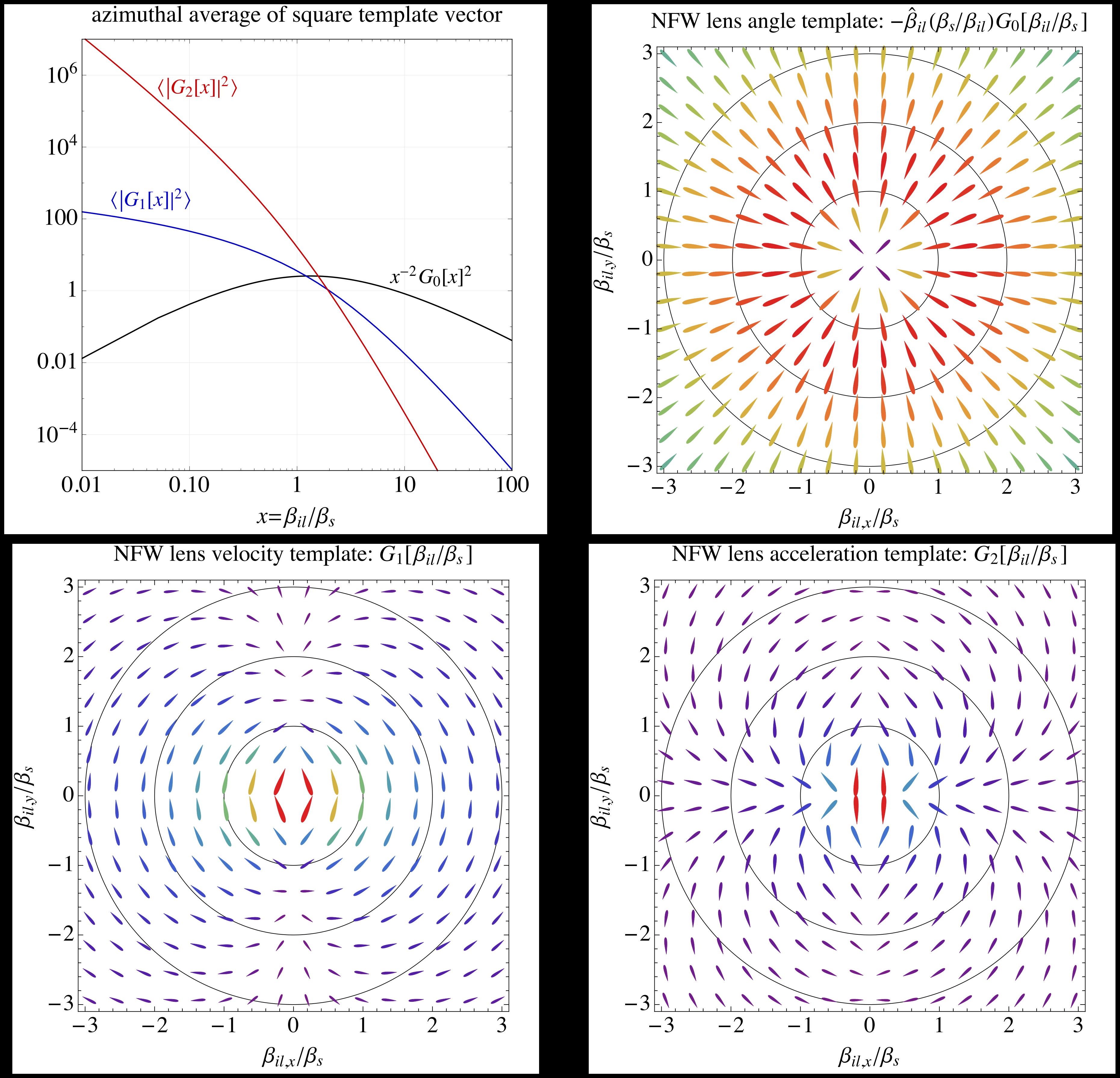
2013 RF98
2014 SR349
2004 VN112
2007 TG422
2012 VP113
2015 KG163
2015 RX245
2013 FT28
Planet nine
Sedna
2010 GB174
2015 GT50
2014 FE72

NFW templates



Tidal stripping