Axion Interferometry

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Strong CP Problem

- Axions are arguably the simplest and most minimal solution to the Strong CP problem
  - Closest competitor is the minimal LR symmetric models
- Solves a problem and can be dark matter
- If it is dark matter, how can we look for it?

Axion dark matter

- Axion dark matter obtains its number abundance through the misalignment mechanism
  - Produces cold dark matter regardless how light the axion is
- The axion is a classical field due to large number abundance

$$a(t) \sim a_0 \cos m_a t$$

- Non-relativistic, $v \sim 10^{-3}$

Axion dark matter

Axion is localized to a distance

$$L \sim \frac{1}{m_a v}$$

So it takes a time

$$\tau \sim \frac{1}{m_a v^2}$$

Until whole axion wave passes by and replaced by a new axion wave

Axion dark matter

- Axion dark matter is a wave with

$$\omega \sim m_a \pm 10^{-6} m_a$$

- It has a quality factor of

$$Q = \frac{1}{\omega\tau} \sim 10^6$$

Looking for the axion

$$\mathcal{L} \supset \frac{a}{4f} F\tilde{F}$$

- Looking for the axion through the coupling to gluons is HARD
    - Very few experiments can reach the QCD axion line
- Instead look for the axion through its coupling with the photon

DISCLAIMER

- QCD Axion
  - Solves the Strong CP problem
  - Couples to photons and gluons and fermion spin
- ALP (Axion like particles)
  - Does NOT solve the Strong CP problem
  - Couples to photons and/or fermion spin
- Axions
  - Can be either
  - Figure it out from context

Effect of photon coupling

$$\mathcal{L} \supset \frac{a}{4f} F\tilde{F}$$

- For circularly polarized light

$$-\omega^2 + k^2 \mp \frac{da}{dt}\frac{k}{f} = 0$$

$$v_{\text{phase}} \approx 1 \pm \frac{\dot{a}}{2kf}$$

Effect of photon coupling

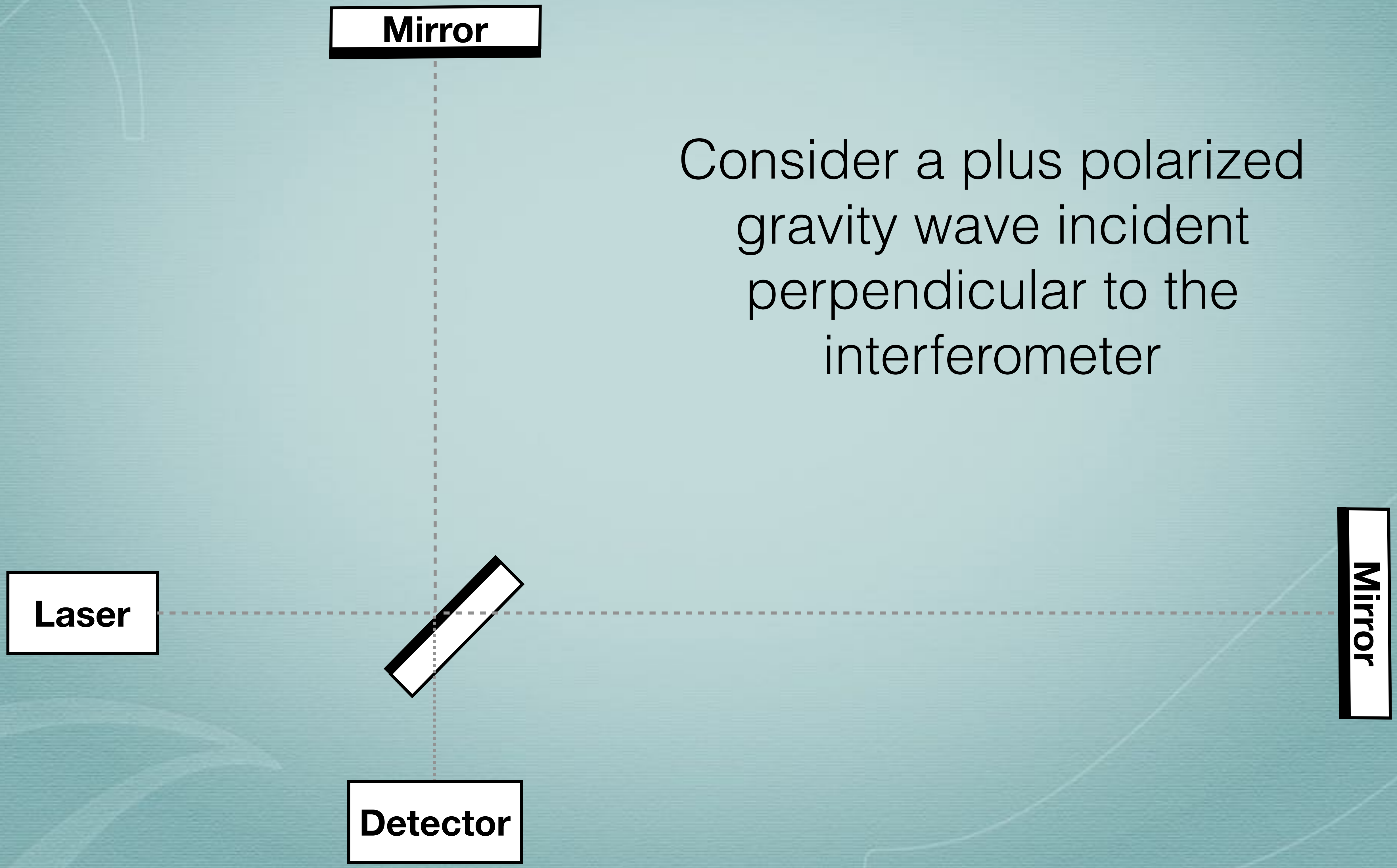
$$v_{\mathrm{phase}} \approx 1 \pm \frac{\dot{a}}{2kf}$$

- Phase velocity of circularly polarized light is different depending on which polarization it is
- Device most sensitive to differences in phase velocities is an interferometer

Axion interferometry

- One-to-one mapping between axion interferometry and gravity wave interferometry
- An axion interferometer can double as a gravity wave detector
- Axion dark matter appears in the same manner as a continuous gravity wave signal with a quality factor of 10^6

Gravity wave interferometry



Consider a plus polarized gravity wave incident perpendicular to the interferometer

No Gravity Wave

$$E = \frac{1}{2}E_0 e^{-iw_L(t-2L_x)} - \frac{1}{2}E_0 e^{-iw_L(t-2L_y)}$$

Relative minus sign due to reflection at beam splitter

$$E^2 = E_0 \sin^2 w(L_x - L_y)$$

Typically adjust L_x and L_y so that the electric field at the detector is small

Gravity wave

Time it takes for the light to go to the mirror and back

$$2L_x = (1 - \frac{1}{2}h_+(t)) \int_{t_0}^{t_0+\tau} dt \qquad h_+ = h_0 \cos \omega_g t$$

$$\tau = 2L_x + h_0 L_x \frac{\sin w_g L_x}{w_g L_x} \cos w_g(t_0 + L_x) = 2L_x + L_x h(t_0 + L_x) \frac{\sin w_g L_x}{w_g L_x}$$

Can get the y result by L_x to L_y and h_0 to $-h_0$

Gravity wave

$$E = -iE_0 e^{-iw_L t + iw2L} \sin(w_L(L_x - L_y) + w_L L h(t - L_x)\frac{\sin w_g L}{w_g L})$$

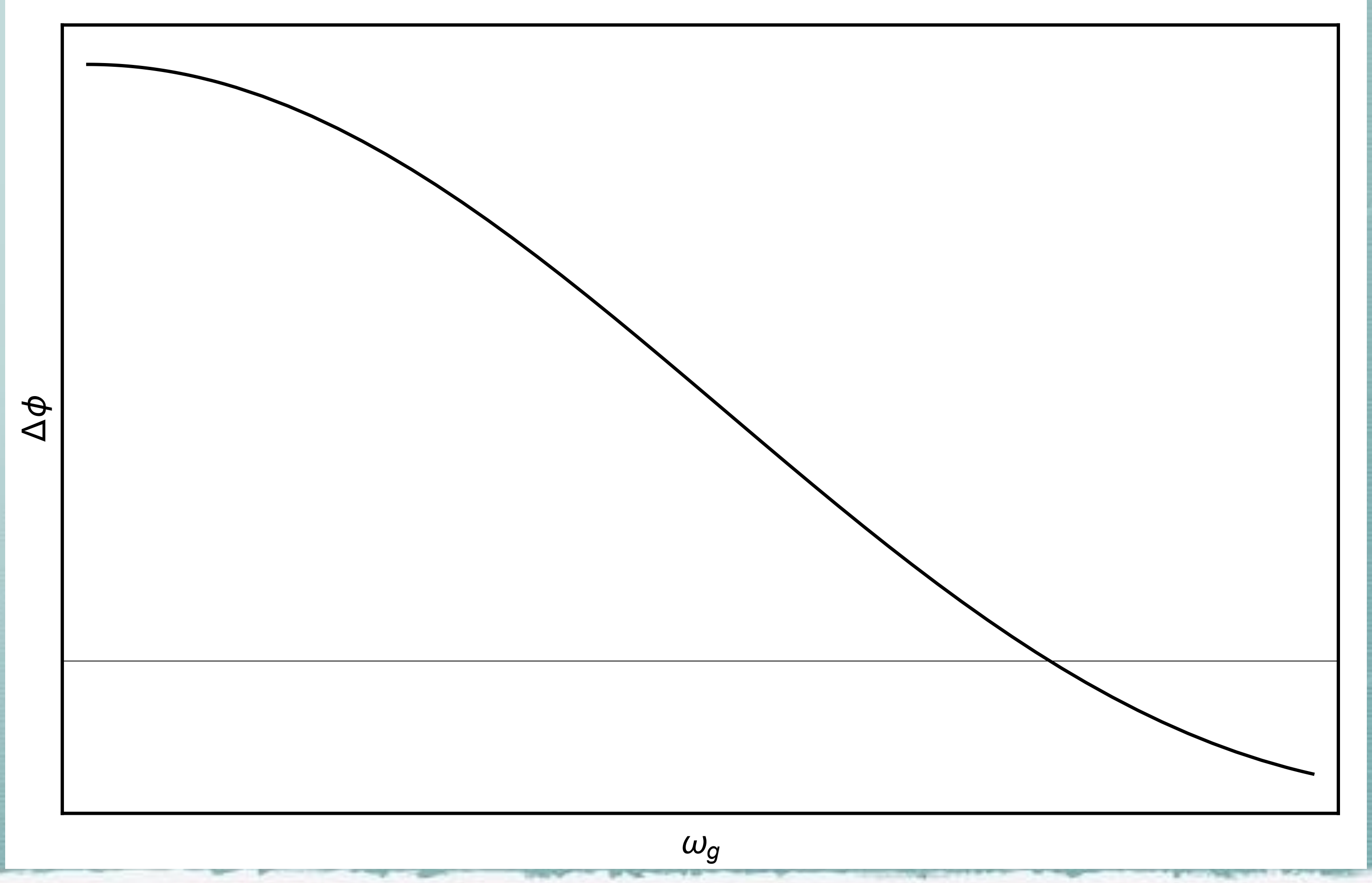
Look for the oscillations in the amplitude of the electric field

$$\Delta\phi = \frac{\omega_L h_0}{\omega_g} \sin\left(\omega_g L\right) \cos\left(\omega_g t + \alpha\right)$$

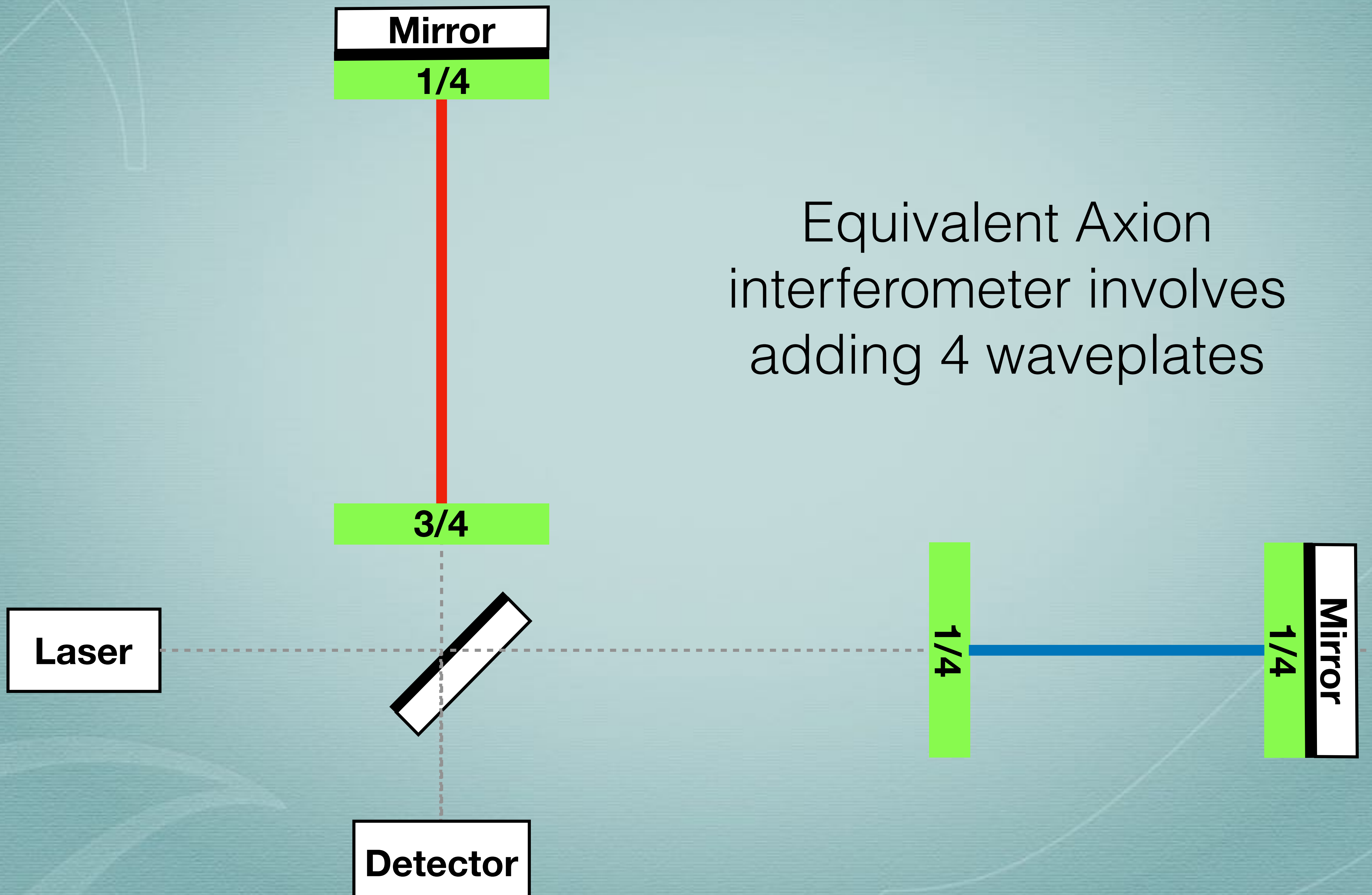
Gravity wave

$$\Delta\phi = \frac{\omega_L h_0}{\omega_g} \sin(\omega_g L) \cos(\omega_g t + \alpha)$$

1. Optimal Length is as expected $L = \lambda_g / 4$
2. Broadband detector

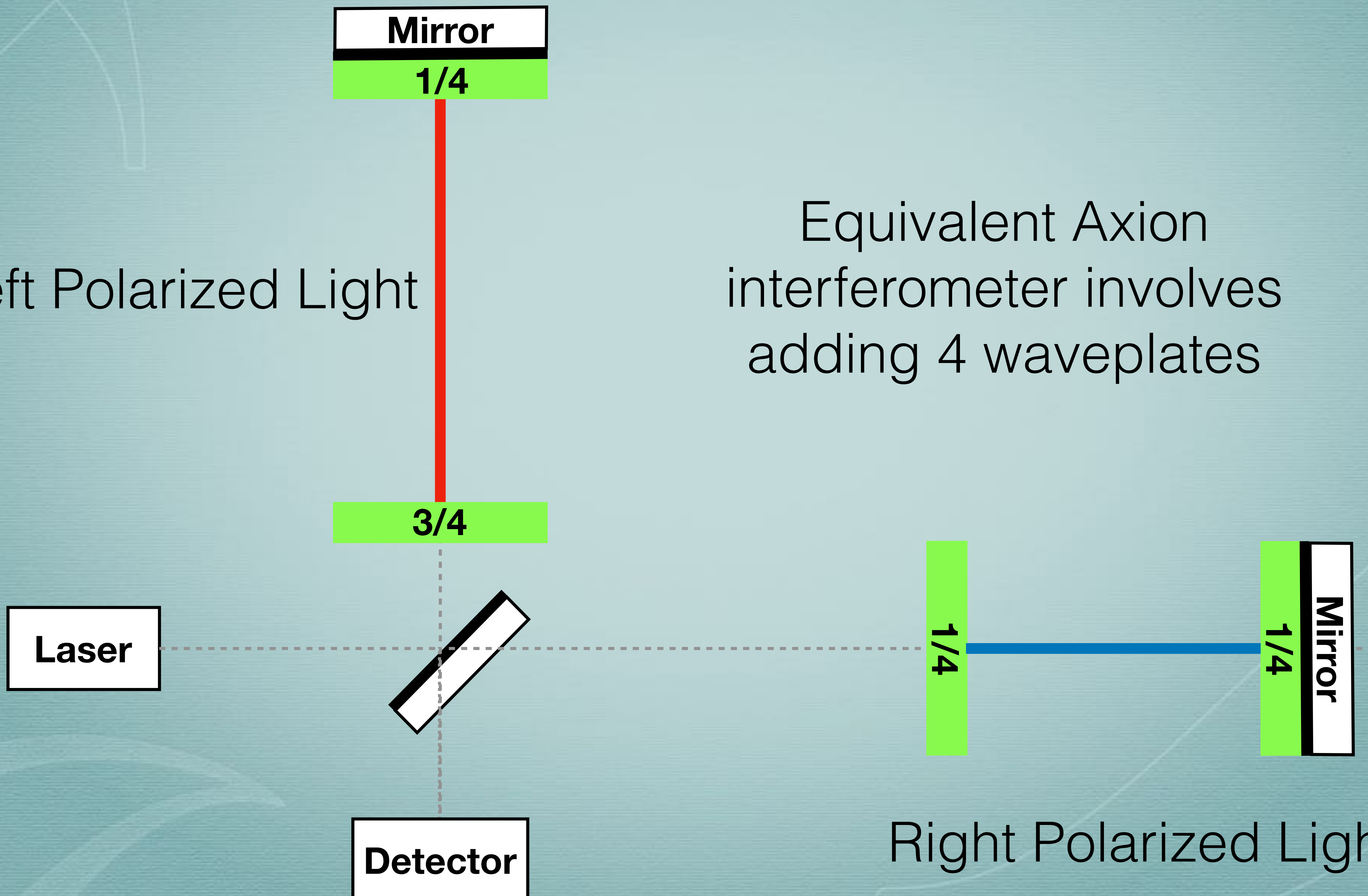


Axion wave interferometry



Equivalent Axion interferometer involves adding 4 waveplates

Axion interferometry



Left Polarized Light

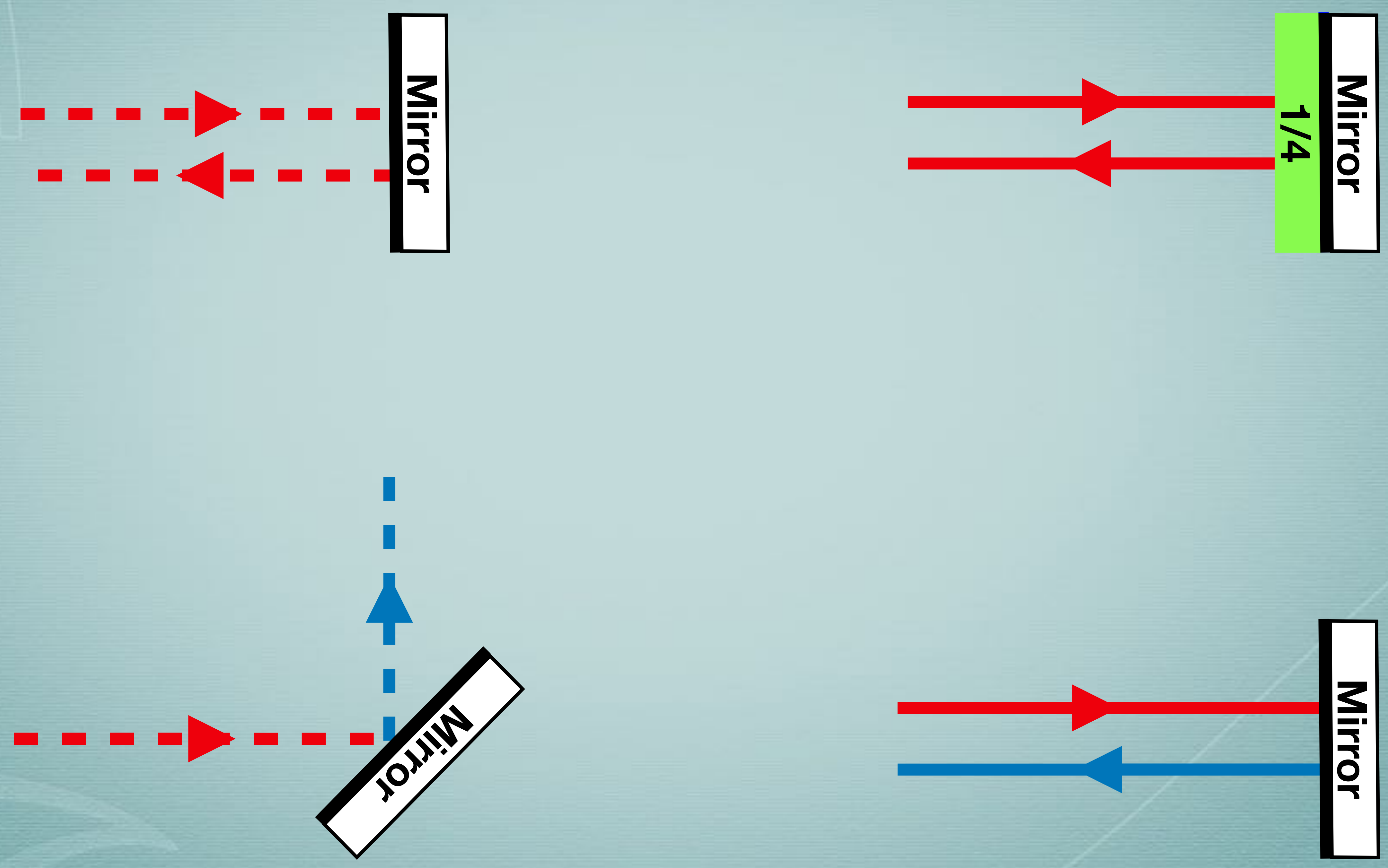
Equivalent Axion interferometer involves adding 4 waveplates

Right Polarized Light

Axion wave

- Only difference is the presence of wave plates
- Needed to maintain polarization

Axion wave



No Axion DM

Exactly the same as a gravity wave interferometer

Experiment doubles as a gravity wave detector

No need to send the legs in different directions otherwise

Axion interferometry

Time it takes for the light to go to the mirror and back

$$2L_x = (1 - \frac{1}{2}\frac{\dot{a}(t)}{\omega_L f}) \int_{t_0}^{t_0+\tau} dt$$

$$\dot{a}(t) = i m_a a(t) = \sqrt{2\rho_{DM}} \cos m_a t$$

$$\tau = 2L_x + L_x \frac{m_a a(t)}{\omega_L f} \frac{\sin m_a L_x}{m_a L_x}$$

Can get the y result by L_x to L_y and a(t) to -a(t)

Axion interferometry

$$\tau = 2L_x + L_x \frac{m_a a(t)}{\omega_L f} \frac{\sin m_a L_x}{m_a L_x}$$

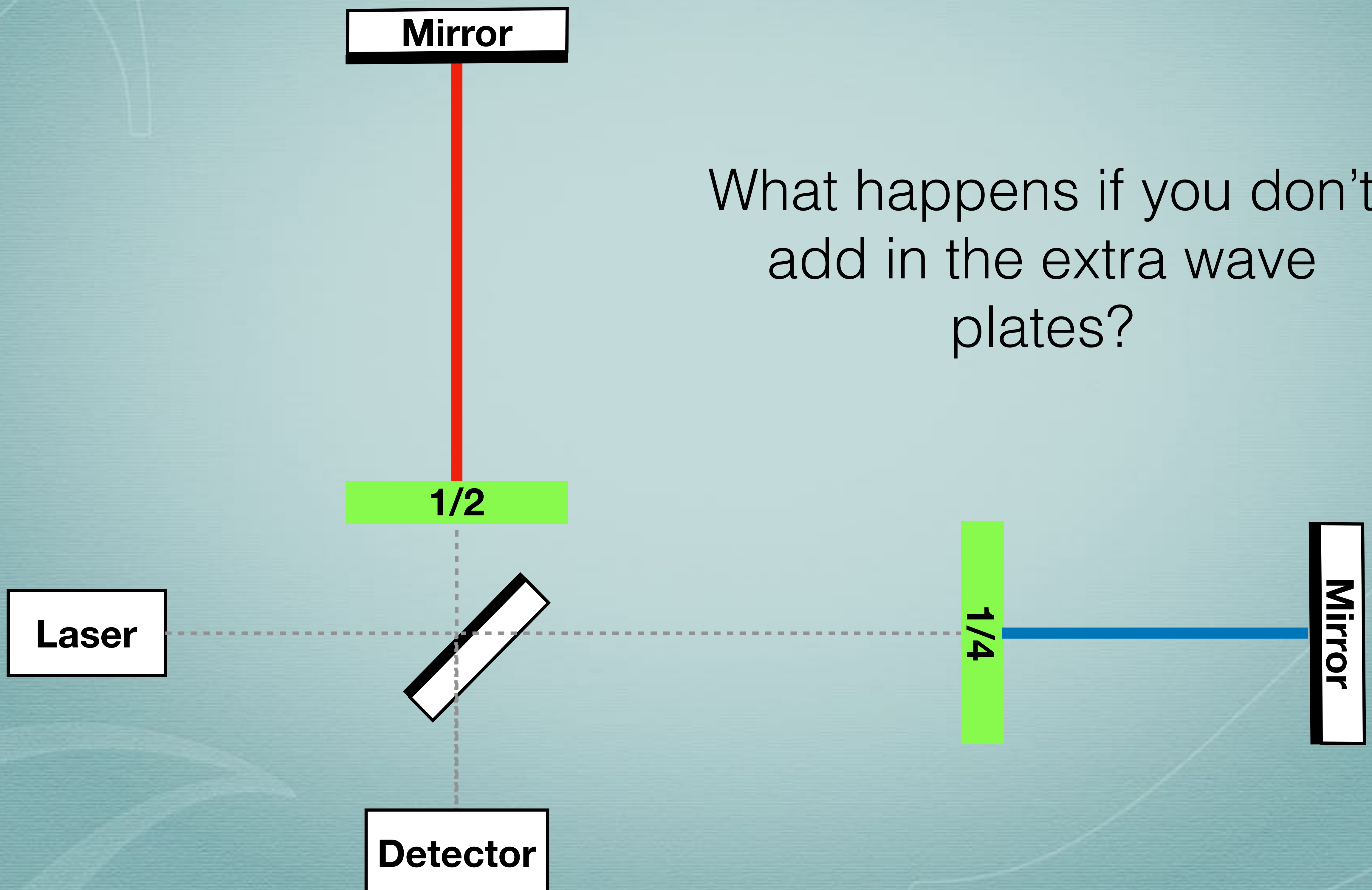
$$\tau = 2L_x + h_0 L_x \frac{\sin w_g L_x}{w_g L_x} \cos w_g (t_0 + L_x) = 2L_x + L_x h(t_0 + L_x) \frac{\sin w_g L_x}{w_g L_x}$$

Axion interferometer equivalent to gravity wave interferometer!

$$h_0 \to \frac{m_a a_0}{f\omega} = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{\omega f} \qquad \omega_g \to m_a$$

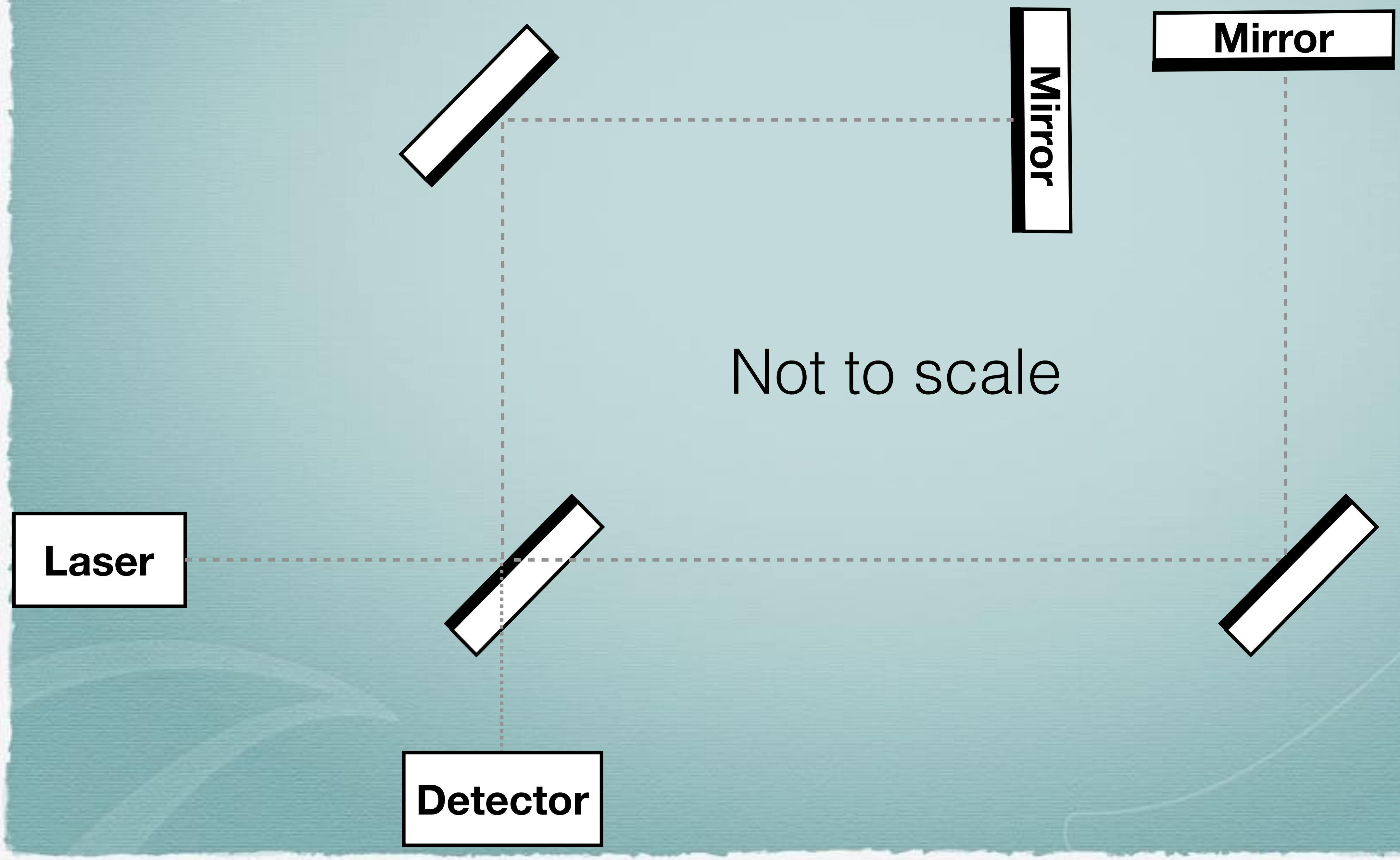
Resonant interferometry

What happens if you don't add in the extra wave plates?



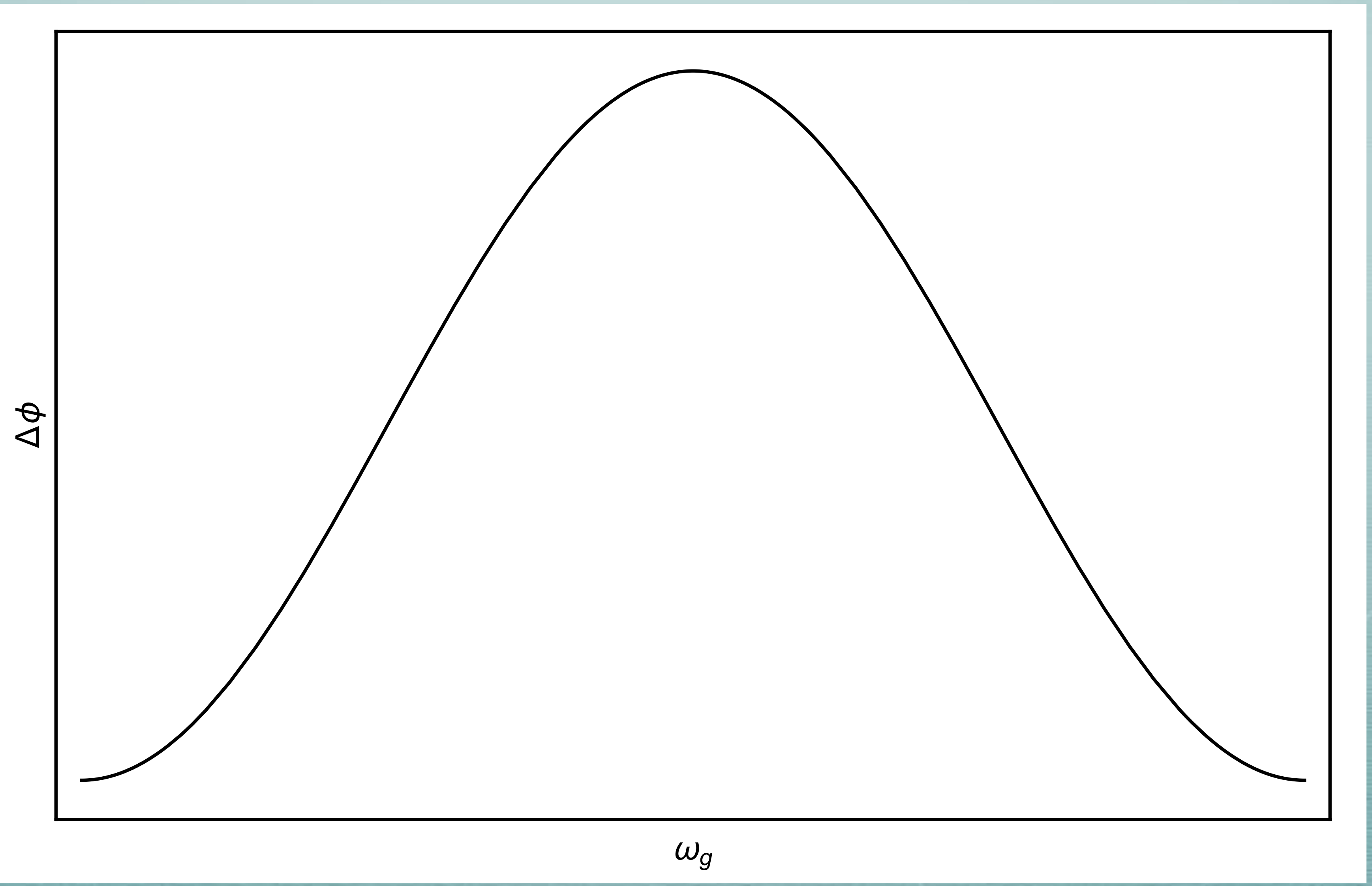
Resonant interferometry

Resonant Detector instead!

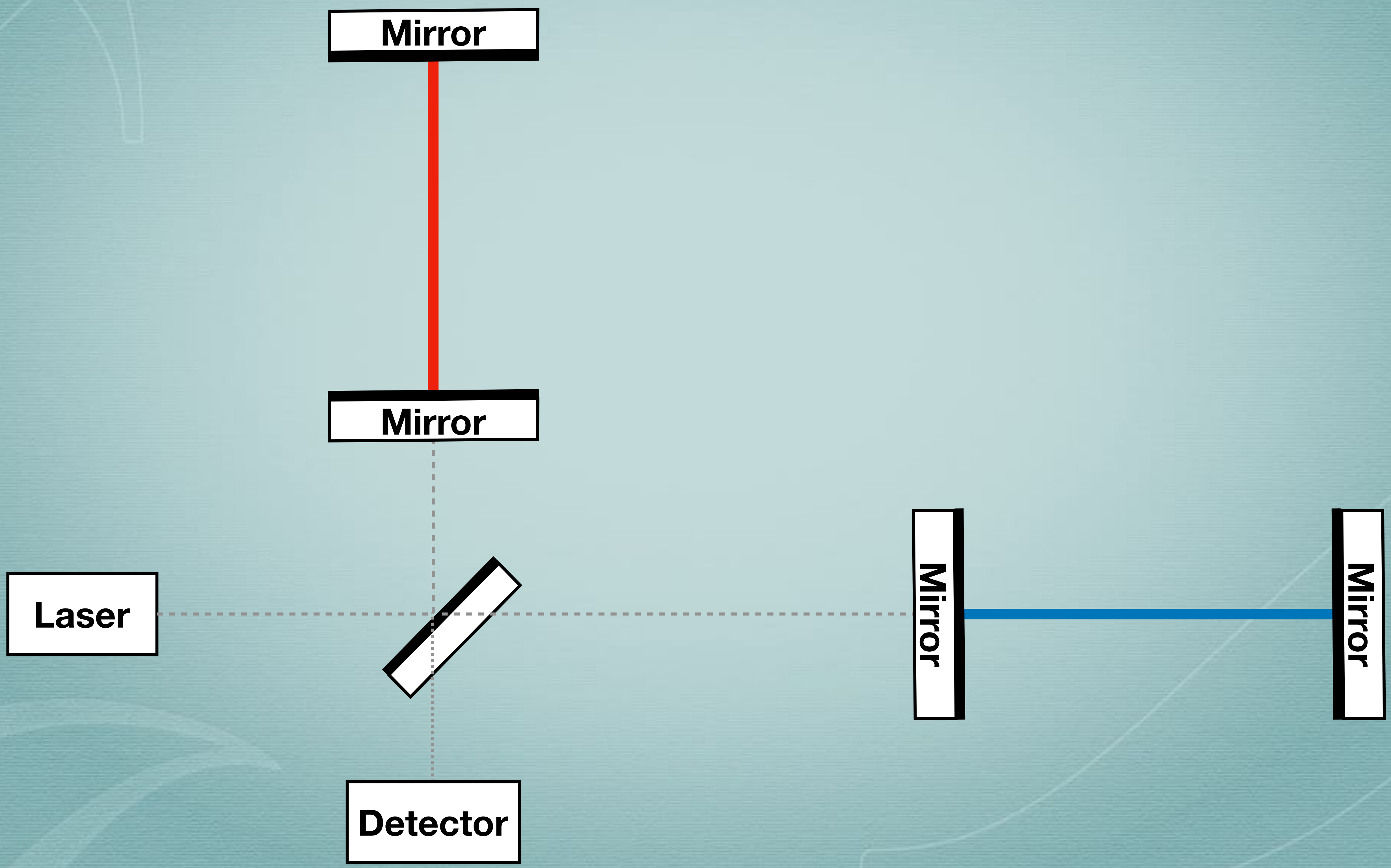


Resonant interferometry

1. Optimal Length is as expected $L = \lambda_g/2$
2. Resonant detector

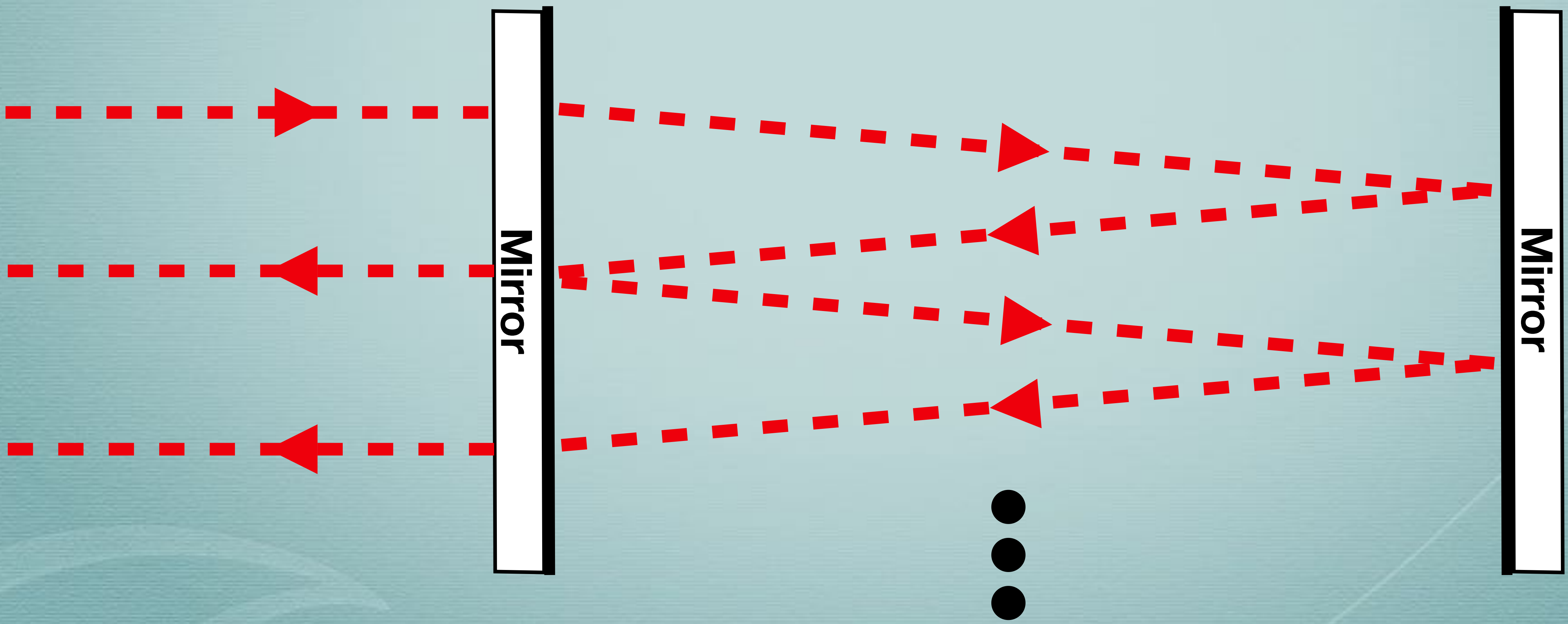


Fabry-Perot



Fabry-Perot

Fabry-Perot Cavity



Fabry-Perot

The phase accumulated over a single round trip is

$$\Delta\phi = \frac{w_L h_0}{w_g} \sin w_g L$$

$$E = E_0 e^{-i w_L t + i \Delta\phi \cos(w_g t + \alpha)}$$

Fabry-Perot

The phase accumulated over a single round trip is

$$\Delta\phi = \frac{w_L h_0}{w_g} \sin w_g L$$

$$E = E_0 e^{-iw_L t + i\Delta\phi \cos(w_g t + \alpha)}$$

$$\approx E_0 \left(e^{-iw_L t} + \frac{i}{2}\Delta\phi e^{i\alpha} e^{-i(w_L - w_g)t} + \frac{i}{2}\Delta\phi e^{-i\alpha} e^{-i(w_L + w_g)t} \right)$$

Effect of gravity waves is to create side bands (light with slightly different frequencies)

Fabry-Perot

What comes out of a Fabry-Perot Cavity is an infinite sum of light that has bounced around many times

$$\Delta\phi_x = h_0 w_L L \frac{2F}{\pi} \frac{1}{\sqrt{1 + (\frac{f_g}{f_p})^2}}$$

An enhanced sensitivity over the standard interferometer by Finesse ~ number of times light bounces around before escaping

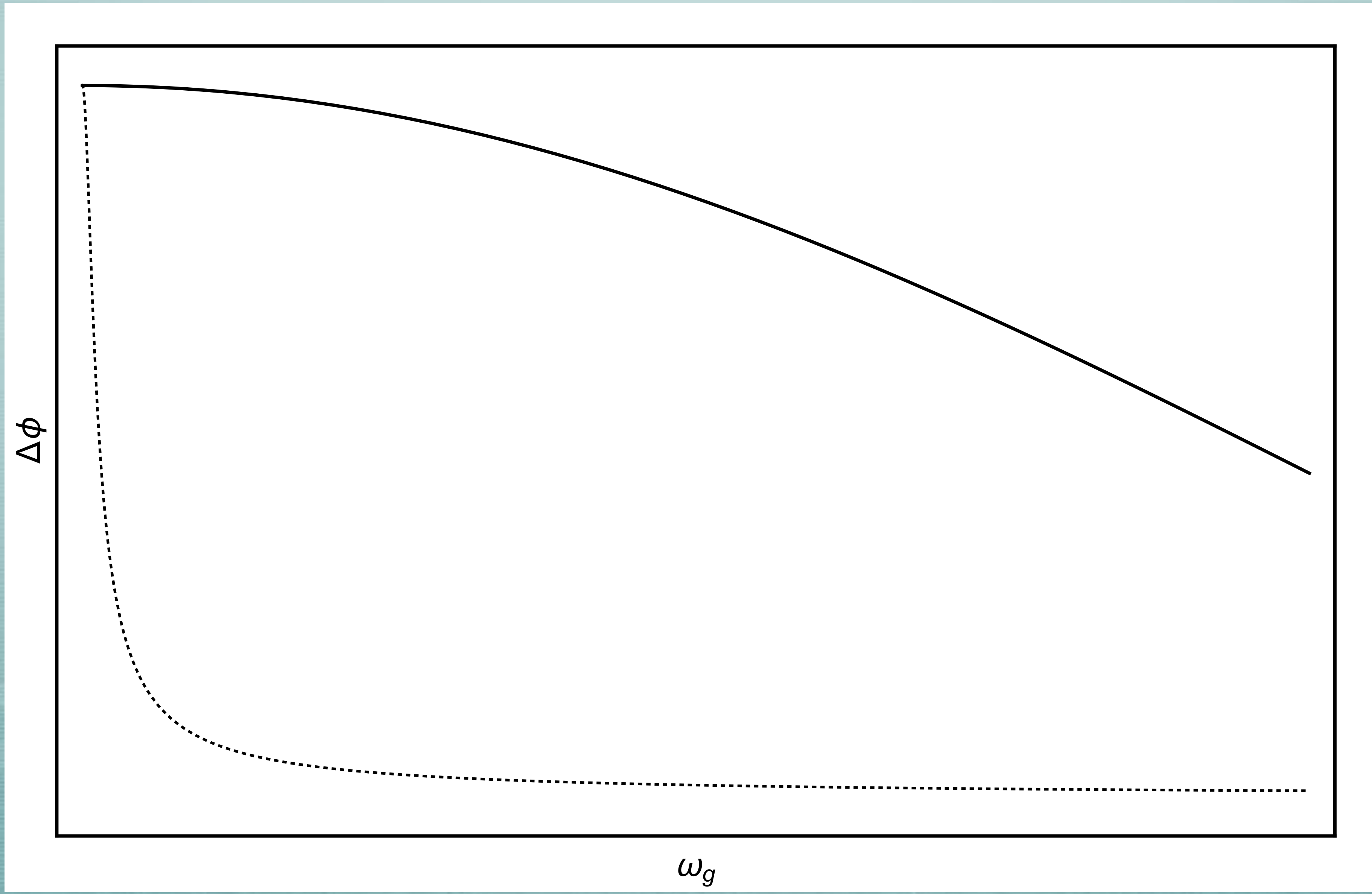
Fabry-Perot

$$\Delta\phi = h_0 \omega_L L \frac{2F}{\pi} \qquad \Delta\phi = h_0 \omega_L L$$

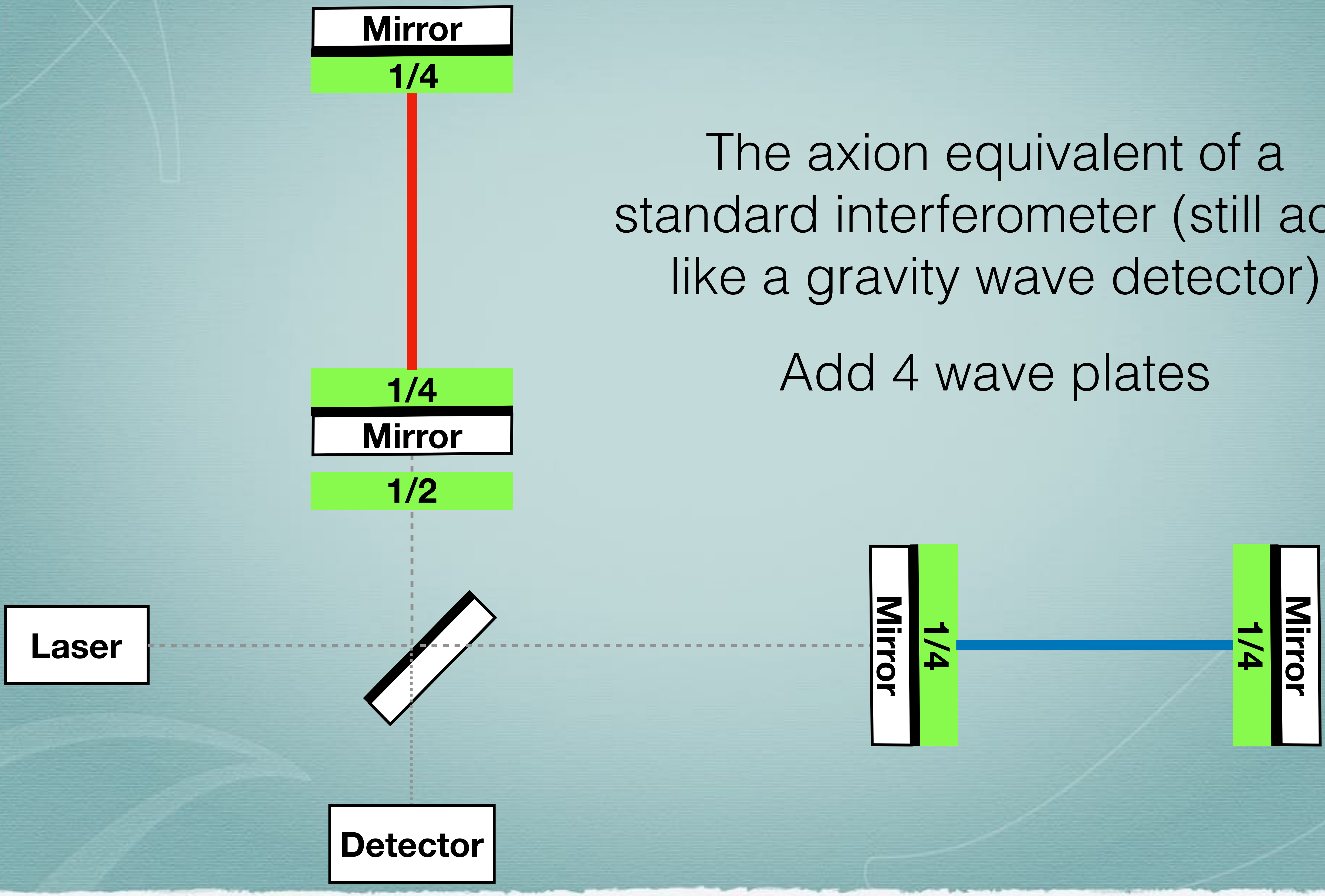
- For low frequencies Fabry Perot Cavity better by a factor of Finesse
- Get an interferometer whose arm length is effectively longer

Fabry-Perot

Better sensitivity at low frequency but not as broadband as before



Axion Interferometer



The axion equivalent of a standard interferometer (still acts like a gravity wave detector)

Add 4 wave plates

Axion Interferometer

Same Mapping as before

Otherwise identical to Gravity wave detector

Noise

- An interferometer counts the number of photons arriving at the detector a second
- How the number of photons a second changes tells us about a time varying phase
- Main sources of noise
  - Shot Noise
  - Radiation Pressure

Shot Noise

In some time T, there are an average number of photons that arrive

$$N_\gamma = \frac{PT}{w_L}$$

The number is given by Poisson statistics

$$\Delta P = \frac{\sqrt{N_\gamma} w}{T} = \sqrt{\frac{P w_L}{T}}$$

Shot Noise

aLIGO sits slightly off the dark spot

$$P = E_0^2 \sin^2 \phi_0 + \Delta\phi_x \qquad \phi_0 = w_L(L_x - L_y)$$

This is so that when a signal arrives

$$\Delta P = P_0 \Delta\phi_x \sin 2\phi_0$$

Linear piece would vanish is sitting on dark spot

Shot Noise

$$\frac{S}{N} = \frac{\Delta P_{GW}}{\Delta P} = \frac{P_0 \Delta\phi_x \sin 2\phi_0}{\sin\phi_0}\sqrt{\frac{T}{P_0 w_L}} = \frac{h_0}{S_n^{1/2}}\sqrt{T}$$

$$S_n^{1/2} = \frac{1}{4L\mathcal{F}}\sqrt{\frac{\pi\lambda}{P_0}}\frac{1}{\sqrt{1+(\frac{f_g}{f_p})^2}}$$

Shot Noise is constant at low frequencies

Shot Noise increases at high frequencies

Radiation pressure

- When a photon hits beam splitter 50/50 chance of going up or down
    - Sometimes more photons go up than down
- Thus the force on the mirrors are not always the same
    - Position of the mirrors will fluctuate
    - Frequency of restoring force small compared to frequency of gravity wave so mirror is effectively a freely falling object
- The fact that L_x and L_y vary in time induces a background for gravity wave detection

Radiation pressure

Via similar calculation to before

$$S^{1/2}_{\text{radiation}} = \frac{16\mathcal{F}}{MLm_a^2}\sqrt{\frac{P}{\pi\lambda}}\frac{m_a L}{\sin m_a L}$$

Radiation pressure relevant at low frequencies

SNR

Thus the final SNR is

$$\mathrm{SNR} = \frac{h_0}{S_{SQL}^{1/2}} (T\tau)^{\frac{1}{4}}$$

Errors added in quadrature $S_{SQL} = S_{\mathrm{shot}} + S_{\mathrm{radiation}}$

SNR only grows like $T^{1/2}$ until approximation that signal is a sin wave breaks down

SNR

$$\mathrm{SNR} = \frac{h_0}{S_{SQL}^{1/2}} \left(T\tau\right)^{\frac{1}{4}}$$

Qualitatively : Add these units of time in quadrature to get $T^{1/4}$ growth

Signal Processing

- There is an optimal way to look for a signal called matched filtering
- Given a hypothetical data stream

$$s(t) = h(t) + n(t)$$

- The noise obeys

$$\langle n(t) \rangle = 0 \qquad \langle n(f) n(f') \rangle = \delta(f - f') \frac{1}{2} S_n(f)$$

Signal Processing

- Define a signal we are interested in

$$\hat{s} = \frac{1}{T}\int_0^T dt s(t) K(t)$$

- Average signal and background are

$$S = \int_{-\infty}^{\infty} dt \langle s(t)\rangle K(t) = \int_{-\infty}^{\infty} df h(f) K(f)$$

$$N^2 = \langle \hat{n}^2 \rangle = \int_{-\infty}^{\infty} dt dt' K(t) K(t') \langle n(t) n(t')\rangle = \int_{-\infty}^{\infty} df \frac{1}{2} S_n(f) K(f)^2$$

Signal Processing

- Define a dot product

$$A \cdot B = \int_{-\infty}^{\infty} df \frac{A(f)B(f)}{1/2 S_n(f)} = 4 \int_0^{\infty} df \frac{A(f)B(f)}{S_n(f)}$$

- Maximizing SNR corresponds to choosing an optimal vector

$$u = \tfrac{1}{2} S_n(f) K(f) \qquad SNR = \frac{u \cdot h}{\sqrt{u \cdot u}}$$

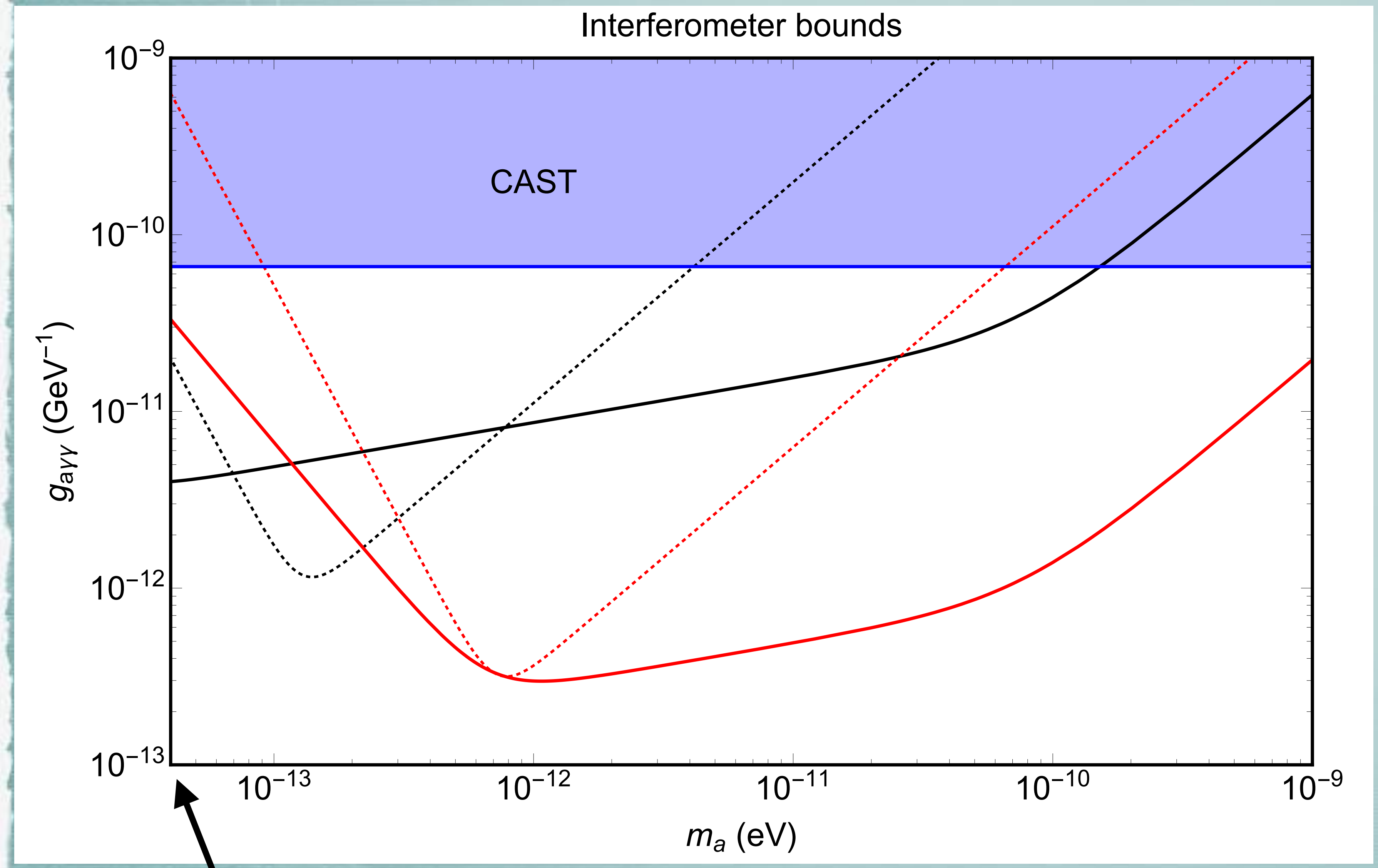
Signal Processing

- Clearly the best way to maximize the signal is to choose u proportional to h

$$SNR^2 = h \cdot h = 4 \int_0^\infty df \, \frac{h(f)^2}{S_n(f)}$$

- Gives the general formula for calculating SNR called waveform matching

Axion Interferometer



40 m arm Length

10 kg mirror

Red : 1 MW power

Black : 1 kW power

Dotted : F = 10^6

Solid : F = 10^2

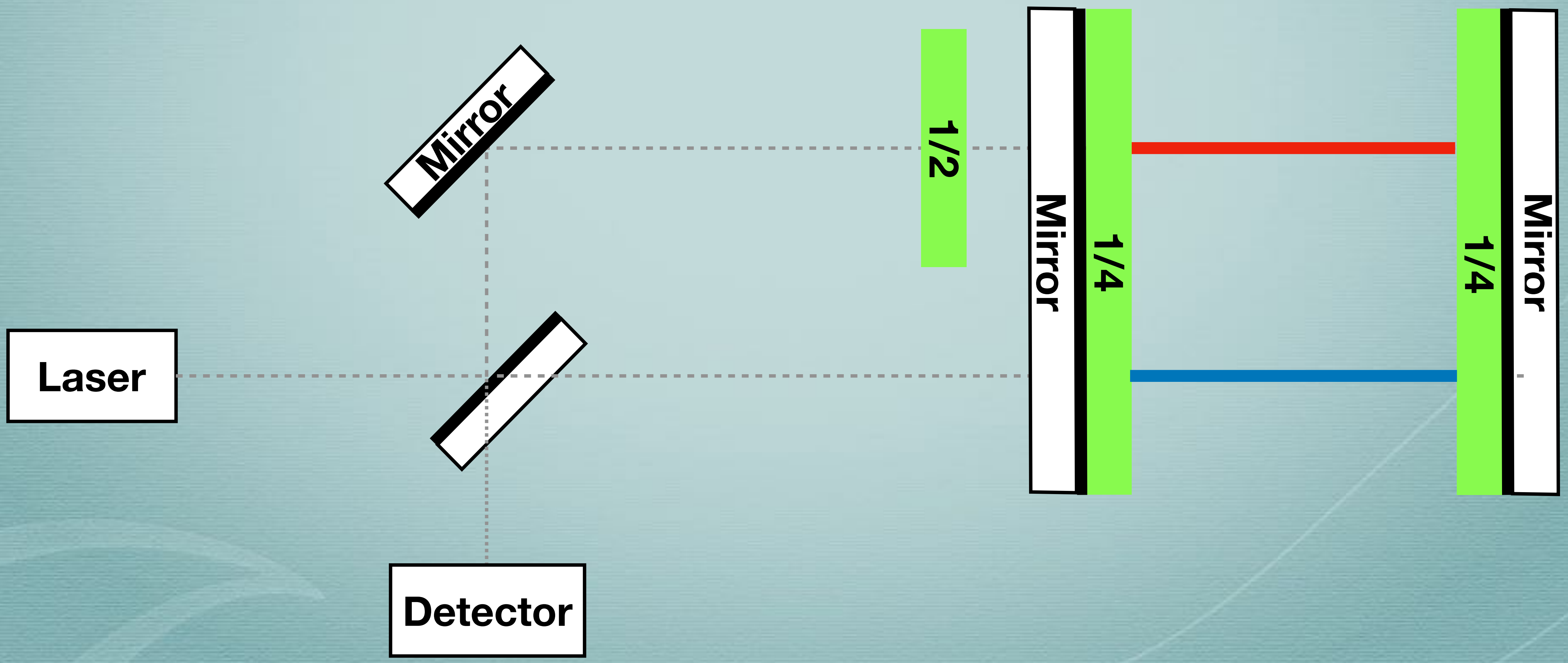
Seismic Noise becomes an issue

Axion Interferometer

- If detector is dedicated to an axion search and not gravity wave search, can do better!
- Radiation pressure can be mitigated if same mirror is used for both arms!

Axion Interferometer

Radiation Pressure replaced by Radiation Torque



Radiation Torque

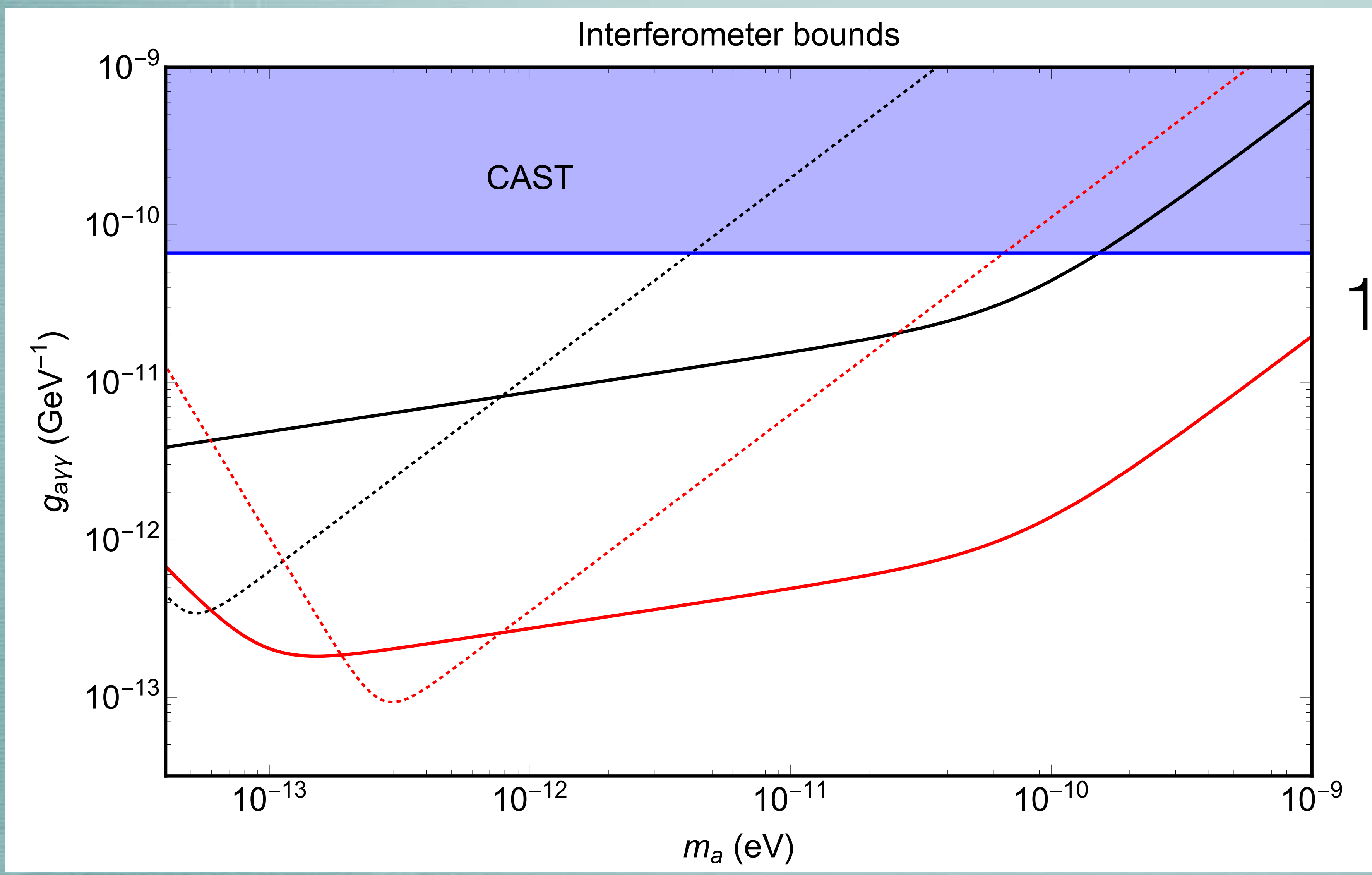
Via similar calculation to before

$$S_{\text{torque}}^{1/2} = \frac{Mr^2}{I} S_{\text{rad}}^{1/2} = \frac{16r^2\mathcal{F}}{ILm_a^2}\sqrt{\frac{P}{\pi\lambda}}\frac{m_a L}{\sin m_a L}$$

Mirrors can only be made so heavy

Geometry is much easier to manage

Axion Interferometer



10 kg mirror

10 cm diameter

1 cm between beams

Red : 1 MW power

Black : 1 kW power

Dotted : F = 10^6

Solid : F = 10^2

Conclusion

- Axion dark matter changes the phase velocity of circularly polarized light
- Can look for this effect in an interferometer
- Can extend bounds by up to 2-3 orders of magnitude over some range of parameters
- Do not need the newest fanciest technology
    - Need to make sure that birefringent backgrounds are under control!