THE UNIVERSITY OF TOKYO

"Do Static Black Holes Exist on Randall-Sundrum Branes?

Gary Horowitz"

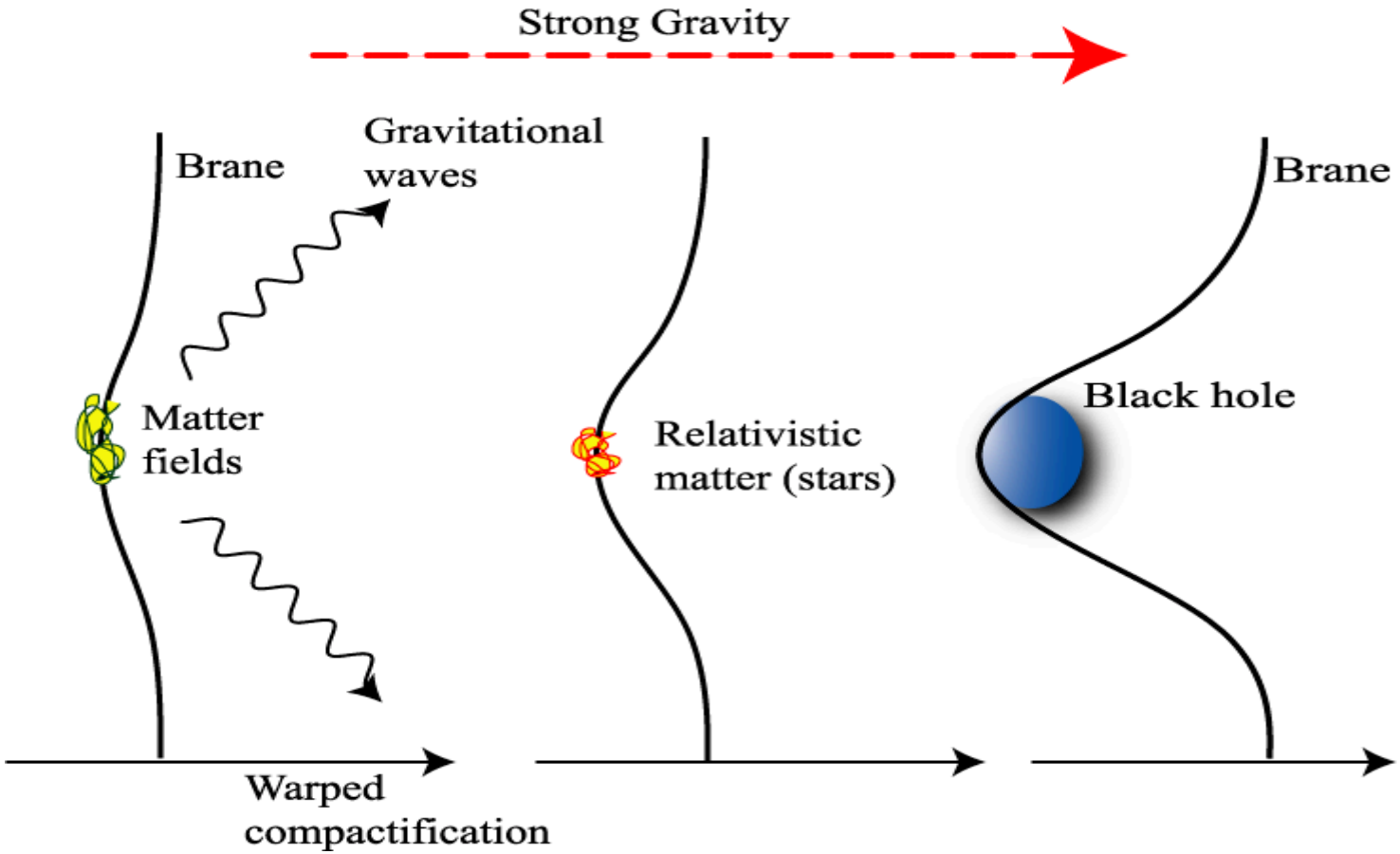
Maybe yes ….

(HK, Tanaka & Nakamura *PRD '03*, HK. *PTP'03, PRD'04*)

Hideaki Kudoh

Localized matter, Brane bending and Localized BH

5D dimensional Randall-Sundrum braneworld model (RS II)



$$\Lambda = -\frac{6}{\ell^2}$$

$$S = \int dx^5 \sqrt{g}(R^{(5)} - 2\Lambda) + \int dx^4 \sqrt{g^{(4)}}\,(\sigma + L_{\rm matter})$$

$$\longrightarrow \quad ds^2 = dy^2 + e^{-2y/\ell}\eta_{ij}dx^i dx^j$$

Basic evidence for the conjecture

T.Tanaka '02 Emparan, Fabbri and Kaloper '02

- Break down of Birkhoff's theorem on the brane

  [Bruni, Germani, Maartens '01]

  – Spherical collapse *on the brane* will radiate GWs and not static.
  – But, it is reasonable to think that the exterior metric will be static at late times due to the emission.

- 4D braneworld BH solutions (EHM)

  [Emparan,Horowitz,Myers]

  – Interpretation in terms of the dual 2 + 1 CFT.

  – At least, there are several evidence for the validity of ads/cft in the braneworld

  [Duff & Liu '00, Hawking etc. '00, Tanaka '04]

- Lack of large BHs in numerical investigations

Many works & arguments

- 4D Exact solution (Emparan, Horowitz and Myers '00)
- There are so many works (analytic & numerical method)
  - Shiromizu & Shibata ('00), Dadhich, et. Al. ('00), Chamblin et. Al. ('00)
  - ・・・・・・・・・・・・
  - Kodama ('02), Casadio & Mazzacurati ('02), Kafinas et. al. ('02) and etc. ('03)

    Nobody has succeeded to construct the solution.

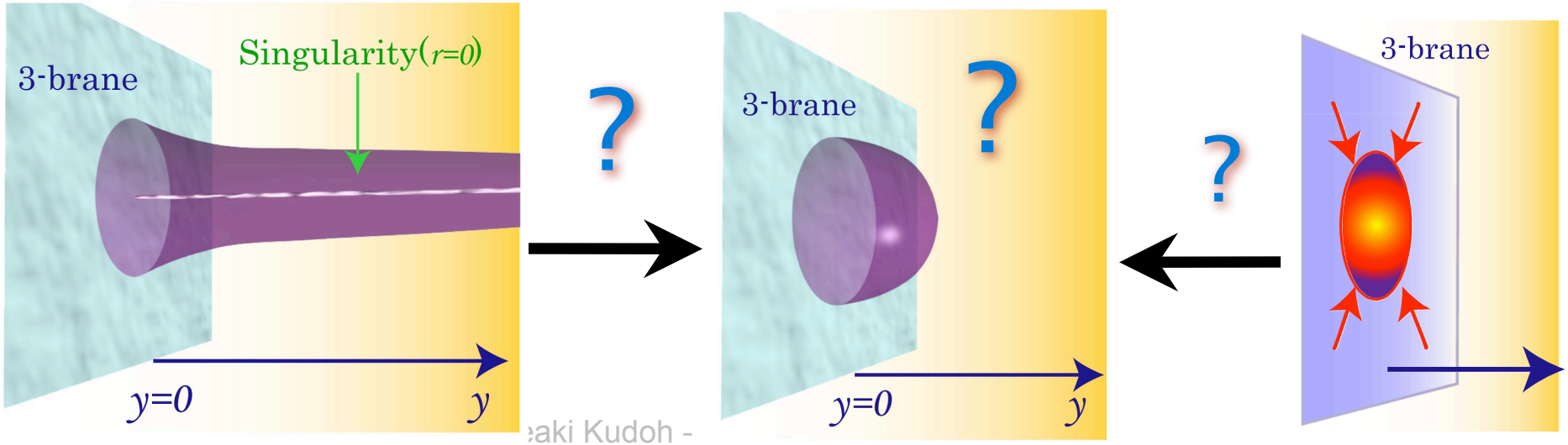
Karasik etc. gr-qc/0404015

- A conjecture (Chamblin, Hawking & Reall '00)

Black string is unstable and has a naked singularity

Localized B.H.

Gravitational Collapse

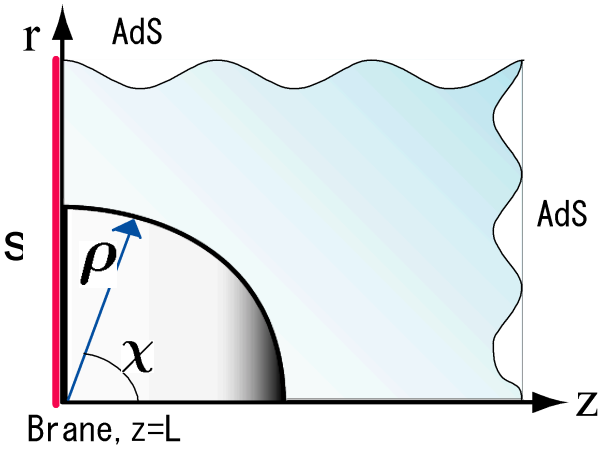


Formulation as a boundary value problem

- General metric form (static, axial sym.)

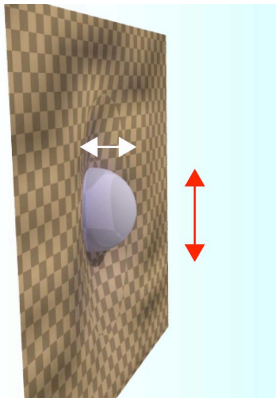
$$ds^2 = \frac{\ell^2}{z^2}\left(-e^{2\alpha}dt^2 + e^{2\beta}(dr^2+dz^2) + r^2 e^{2\gamma} d\Omega^2\right)$$

  - well defined (elliptic) boundary value problems



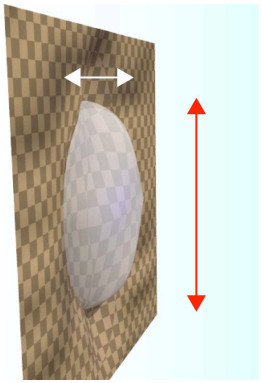
- Expectation of the characteristics

$\rho_h \ll \ell$



$$\Lambda = -\frac{6}{\ell^2}$$

$\rho_h \gg \ell$

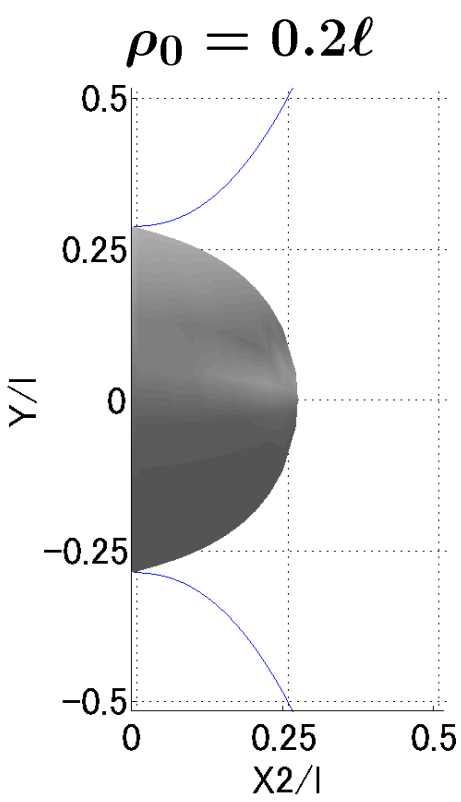


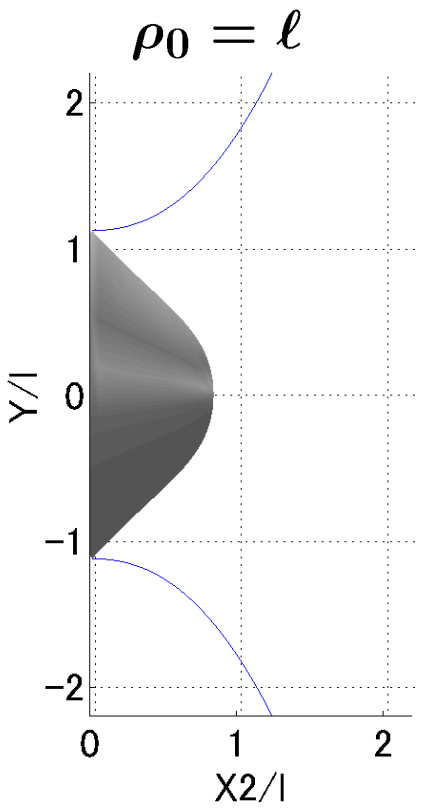
In the small horizon limit, the Schwarzschild BH will be an approximate solution.

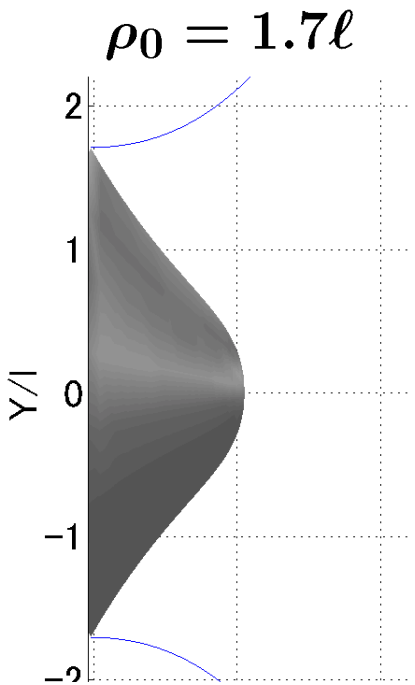
Pancake like (?) due to the warp factor of AdS

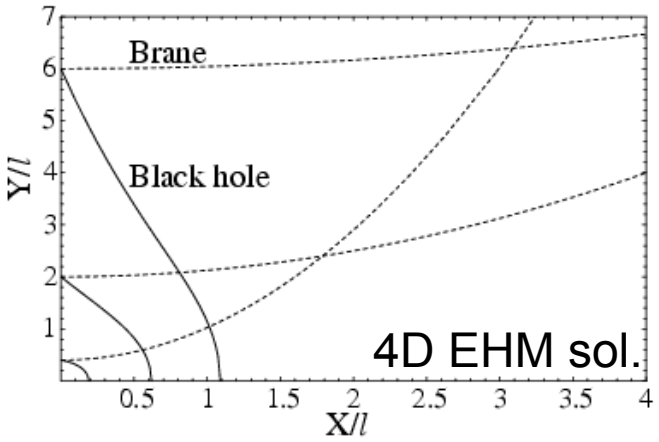
Localized BHs embedded into a Euclidean space

$$dE^2 = dX^2 + dY^2 + Y^2 d\Omega^2$$





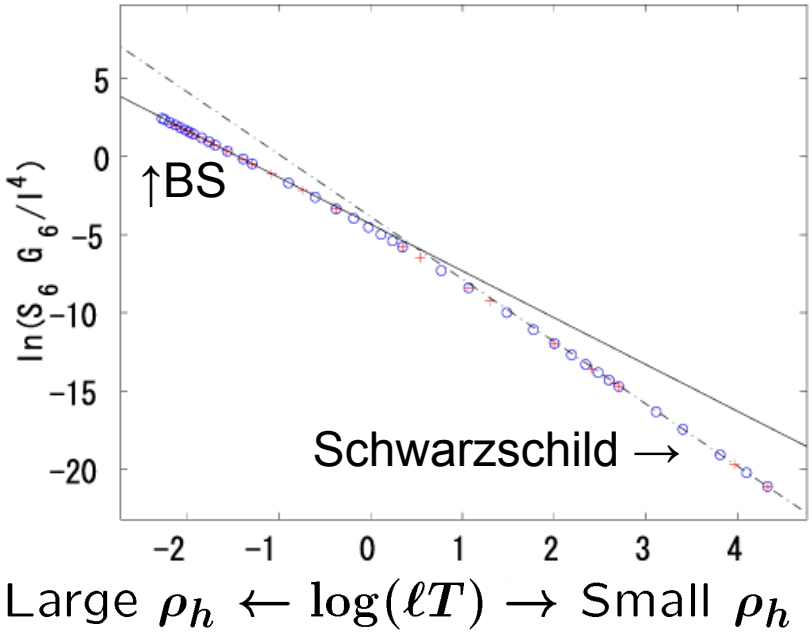




- ✓ ρ_0 is approximately equal to the proper circumferential r
- ✓ No naked singularity in the bulk and on the brane.
- ✓ Intrinsic Riemann curvature *R(4) l2* of the horizon b

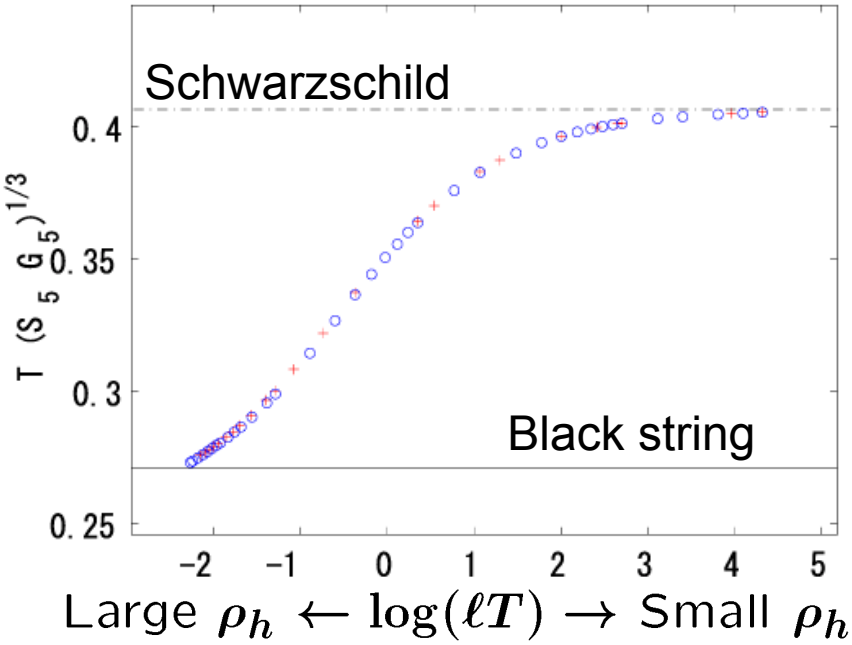
Qualitative Comparison with Sch. BH and BS

$\ln(S_6/\ell^4)$ (total area)



Large $\rho_h \leftarrow \log(\ell T) \rightarrow$ Small ρ_h

$T \ln(S_5/\ell^3)$ (area on the brane)



Large $\rho_h \leftarrow \log(\ell T) \rightarrow$ Small ρ_h

large localized BH ~ black string which is capped off at the end

(The two reference solutions are not physically acceptable solutions)

The LBH has larger entropy than those of equal mass BS and Schw. BH

(Thermodynamic) mass for large LBHs are approximately given by

$$\mathcal{M} \approx \frac{1}{4}\left(\frac{3}{\sqrt{\pi}}S\right)^{2/3}\left(\frac{\ell}{G}\right)^{1/3}$$

How large?

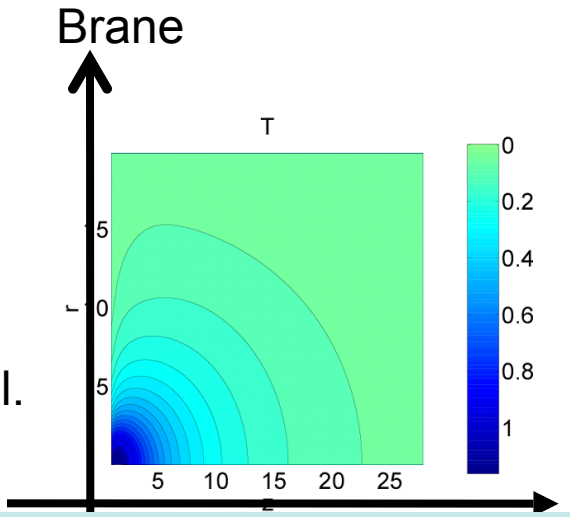
Black holes are not so large

- $R_h / l \sim 2$
  - For larger BHs, convergence of numerical cal. is lost.
  - Does this indicate something new effects?

Relativistic stars on the brane (Wiseman '02)

- For small stars (up to $R / l << 1$)
  - O.K. [existence of upper mass limit, etc]

- For large stars (up to $R / l \sim 3$)
  - 4D Effective theory description seems to work well.
  - Upper mass limit ?
  - It is hard to get much larger stars



Taking into account the full nonlinearity of BH, "$R_h / l \sim 2$" is a moderate size that we could tackle at the time, with "primitive" numerics

Summary

Braneworld BHs

- 4D : the EHM solutions are only known non-trivial exact solutions.
- Numerical Localized BHs
  - Transition of the thermodynamic properties of LBHs

    $T < l$: ~ Schwarzschild BH

    $T > l$: ~ black string (which is capped off at the end)
  - The "transition" temperature is given by

$$T = 0.9 \times (1/2\pi l) \text{ for } (R_h = l)$$

Other braneworld ?

- DGP (Dvali,Gabadadze,Porrati) model

  There are several arguments … but no explicit solution.
- RS I model
  - BH on a IR brane