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Challenges for phase transitions in GR

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Outline

- General
- BH ⟷ B Str (BH = black hole, B Str = black string)
- Ring
- Numerical relativity

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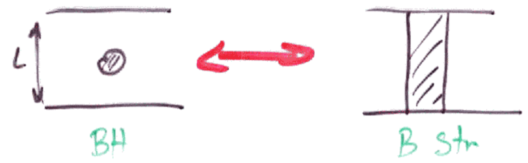
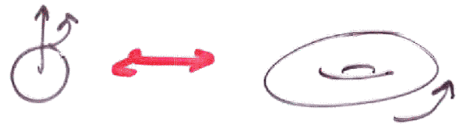
General

"GR ≈ BH + cosmology + waves"

concentrate on ↑ in $d>4$.

The two examples of non-uniqueness:

- 

- 5d 

- no-hair (no-memory) vs. uniqueness

no-hair = small no. of (conserved?) asymp. parameters

uniqueness = a unique sol'n given asymp. parameters (rather than a discrete set)

$d>4$ non-cons'd param., but still macroscopic only

violated

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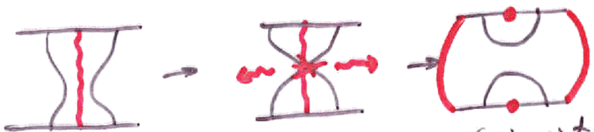
Central challenges

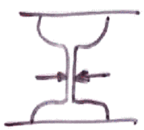
- Cosmic Censorship — its importance



(naive picture)

- Generic
- Exposes short scale (Planck) physics ?
- thunderbolts (≡ shock waves).

- A mathematical alternative



$r_{min} \searrow 0$

for example

$r_{min} \sim \exp(-t)$

- Critical dimensions
- Topology change

} to follow

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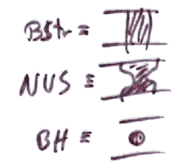
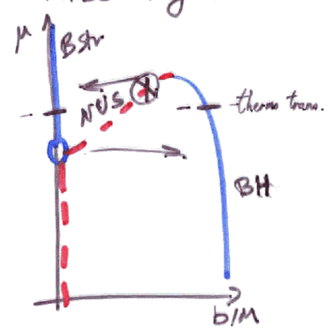
BK Phys. Rep.

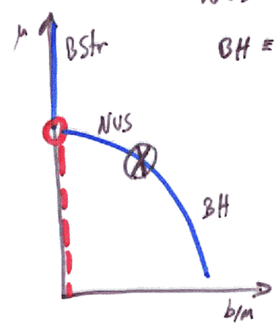
BH - B Str



Phase diagram



$D < D^*$



$D > D^*$

BK hep-th/0206220 - prediction

confirmation (numeric)

Kudoh, Wiseman,

BK, E. Sorkin & T. Piran

small BH

Harmark & Obers

BK & D. Gorbonos

Chu-Goldberger-Rothstein

$\mu \equiv$ dim'less mass

b : $g_{00} \simeq 1 + \frac{b}{r^{d-3}}$

$D^* = 13.5$ Sorkin μ-can

$= 12.5$ canonical Kudoh

- near concensus
- no cone in time evolution expected

≡ "merger" transition (or pinching)

subject of next talk

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back to

* **Critical dimension**

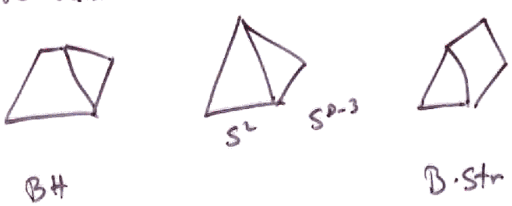
- 1^{st} order vs. 2^{nd} order, $D^* = 13.5, 12.5$
- $D^* = 10$ for cone stability

why D^*??

* **Topology change**

- not only for the horizon, but for space-time
- local model : cone over $S^2 \times S^{D-3}$

the transition :



Also

- Why does a BH have exactly one negative mode? (mathematical reason)
- text book proof for critical GL / GPY

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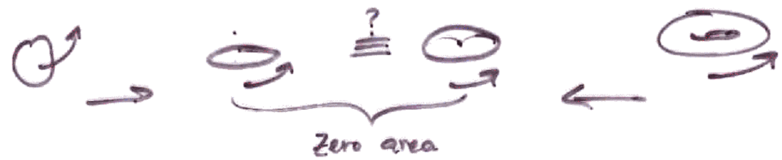
Ring

- Understanding the singularity structure at transition :



- Stability : • BPS are stable
  • original unstable
  • onset of instability

also

- Completing the soln space
  - more than 7 param
  - $d > 5$?
  - original w. $J_2 \neq 0$.

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Numerical Relativity

- Central problem: BH merger ?? – is running out...
- Time evolution of instability

  Adaptive grid and/or improved gauge?

  → to approach singularity

- $D \geq 14$ and "short evolution"

- stability analysis
  - NUS
  - large BH
  - Ring ...
- local cone structure – DSS – next talk.

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Choptuik scaling and the merger transition

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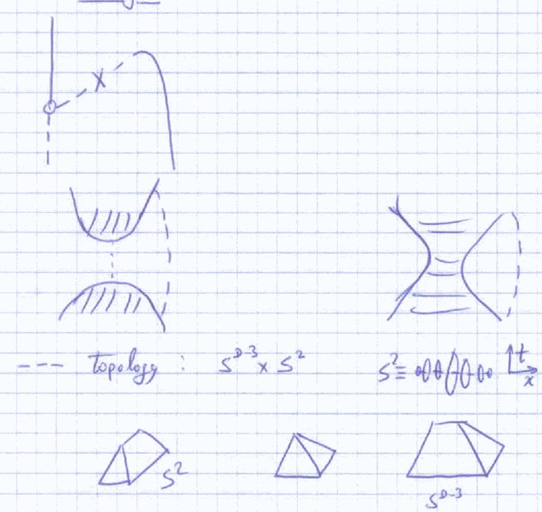
Outline

- Review merger transition
- Review Choptuik scaling
- Motivation for relation
- Action
- Boundary conditions
- Stability
- Implication and follow-up (discussion)

Merger



Symmetries $SO(D-2)_\Omega \times U(1)_t \times Z_{2\,z}$

Most general metric (w/o losing EOM)

$$ds^2 = e^{2A}dt^2 + ds^2_{(r,z)} + e^{2C}d\Omega^2_{S^{D-3}}$$

– 5 func. of (r,z)

double cone sol^n

$$\left(ds^2 = d\rho^2 + \frac{\rho^2}{3}\left[d\Omega^2_{S^2} + d\Omega^2_{S^2}\right] \qquad 5d \right)$$

$$ds^2 = d\rho^2 + \frac{\rho^2}{D-2}\left[d\Omega^2_{S^2} + (D-4)\, d\Omega^2_{S^{D-3}}\right]$$

Linearized deformation

$$ds^2 = d\rho^2 + \frac{\rho^2}{D-2}\left[e^{2\epsilon/2}d\Omega_2^2 + (D-4)\, e^{-2\epsilon/(D-3)}d\Omega^2_{D-3}\right]$$

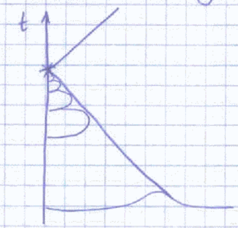
$$\epsilon = \rho^{S_{1,2}}$$

$$S_{1,2} = \frac{D-2}{2}\left(-1 \pm i\sqrt{\frac{8}{D-2} - 1}\right)$$

$$D^* = 10$$

Non-linear analysis of phase space is available. 2 smoothed cones are the only solutions (up to scaling) which are smooth at tip.

Choptuik scaling –



$$S \sim \int R + (\partial\phi)^2$$

universal contents: γ, Δ

$$M_{BH} \simeq (p - p_*)^\gamma$$

$$\gamma \simeq 0.374$$

Z_* – the critical metric

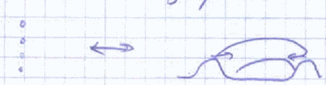
DSS

with log – period Δ. $\Delta = 3.45$

so $e^\Delta \simeq 30$ is the "similarity ratio"

Motivation for connection

- Initially look different: BH creation vs BH↔BStr, time evolution vs. static
- Features: essentially 2d, scaling
- D-branes in imaginary time Gaiotto, Itzhaki, Rastelli



imploding shell

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Action

- both 2d (r,z) (r,t)
  (different sig.)
- Matter content $g_{\mu\nu}^{(D)} \updownarrow t$ $g_{\mu\nu}^{(d)}, \phi$
  $g_{\mu\nu}^{(d)}, g_{H} \sim e^{2\phi}$
- Action $S \sim e^{2\phi}\left(R + (\partial\phi)^2\right)$

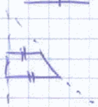
Weyl rescaling $\rightarrow R + (\partial\phi)^2$ – exactly the same

Except for analytic continuation
indep of x sufficient
$\cos x \rightarrow \cosh x$
In general Z_2 $x \rightarrow -x$ sym. suffices

$\Rightarrow$ Same Eq. of motion. What about "$g_{\mu\nu}(x) \in \mathbb{R} \Rightarrow g_{\mu\nu}(ix) \in \mathbb{R}$"

b.c.

Choptuik Merger



scaling in radial direction

axis $r=0$ reflection $r \neq 0$ (e^c)

horizon smooth $g_H \rightarrow 0$

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Stability

is reversed. Toy example $\mathcal{L} = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2$ $x=0$ stable soln.

$t \rightarrow it \Rightarrow \mathcal{L} = -\frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2$

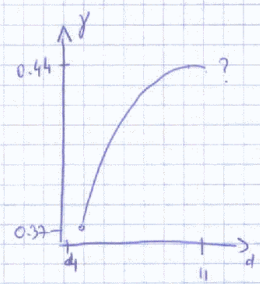
$x=0$ unstable

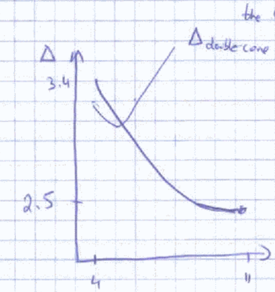
Therefore, even though analytic continuation is also a soln it is better not to.

Wise words of the referee: "long on buzzword and short on physics"

Implications

- Is the critical merger soln CSS or DSS?
- Find scaling const for merger γ, Δ?
  (Estimate? $s = -\frac{1}{\gamma} \pm \frac{2\pi i}{\Delta}$)
- Study Choptuik for various D — Sorkin-Oren; similar results by the Canadian grp.





- Also CFT $\not\!\!\!\!\!\quad$ Second order P.T.