

# Dynamics of supertubes

*Samir D. Mathur*

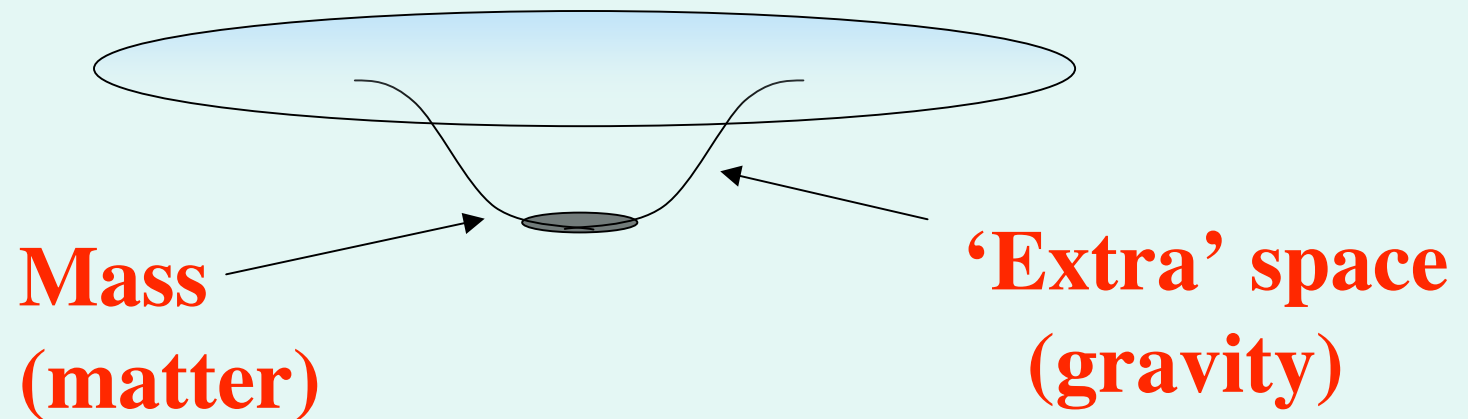
(Work with S. Giusto, O. Lunin, Y. Srivastava)

## **Plan of the talk:**

- 1. Two abstract questions**
- 2. Review of 2-charge systems:  
(black holes)**
- 3. Observing a ‘phase transition  
(matter picture and gravity picture)**
- 4. Conjecture: how to separate bound  
states from unbound states**

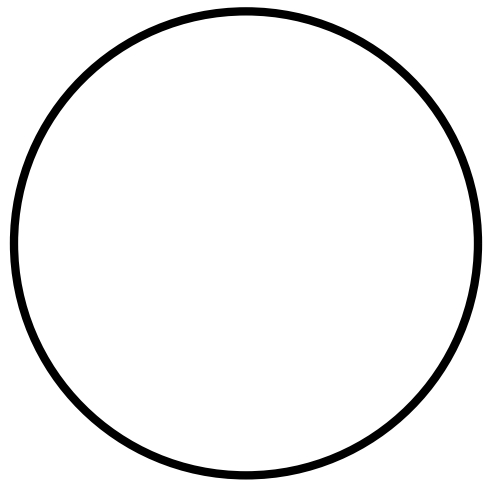
# A naïve confusion ... *What is AdS/CFT duality?*

General relativity:



But in AdS/CFT ‘matter degrees of freedom’ are *DUAL* to supergravity fields ...

## AdS/CFT duality (Maldacena)



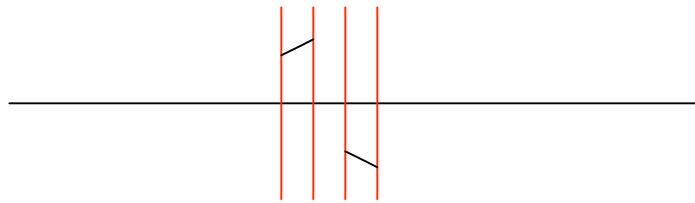
**CFT ‘matter’  
on boundary**

**DUAL**



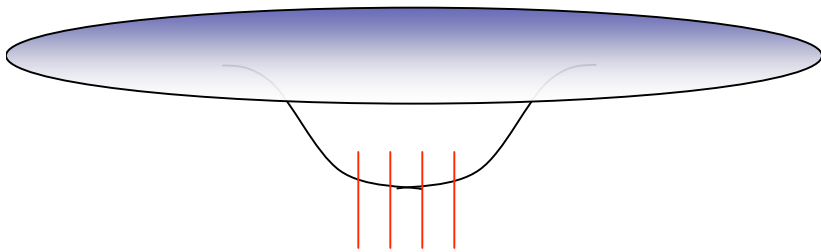
**Gravity (string theory)  
in bulk AdS space**

**But for the black hole problem, we start a bit differently ...**

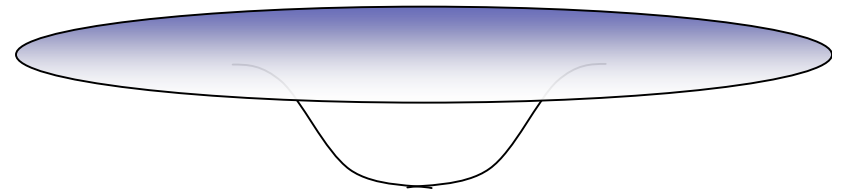


$g \rightarrow 0$  D branes in flat space, give CFT

**At larger  $g$  :**



**OR  
??**



**At what  $g$  can we replace the branes by supergravity?**

## Notation:

Type IIA string theory: **gravitons, NS1, NS5**

Compactify:  $M_{9,1} \rightarrow M_{4,1} \times S^1 \times T^4$

Radius of  $S^1$  is  $R_y$ , Volume of  $T^4$  is  $(2\pi)^4 V$

NS1 wrapped on  $S^1$

Momentum modes  $P$  along  $S^1$

NS5 branes wrapped on  $S^1 \times T^4$



NS1 wrapped on  $S^1$

Momentum modes  $P$  along  $S^1$

NS5 branes wrapped on  $S^1 \times T^4$



***S, T dualities can be used to permute  
NS1, P, NS5 in all the 3! possible ways***

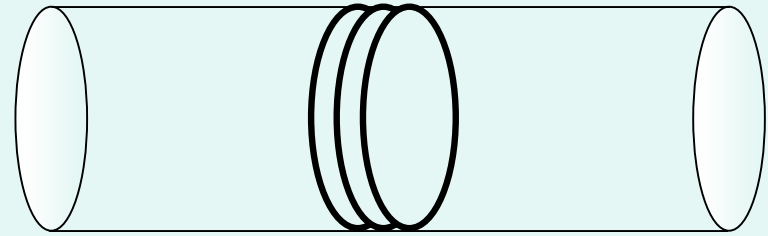
**Also, S duality changes NS1, P, NS5 to D1, P, D5**

A horizontal orange banner with a scroll-like appearance, featuring a vertical strip on the left side and small circular details at the top corners.

# **Black Holes**



$$S_{micro} = \ln(1) = 0$$

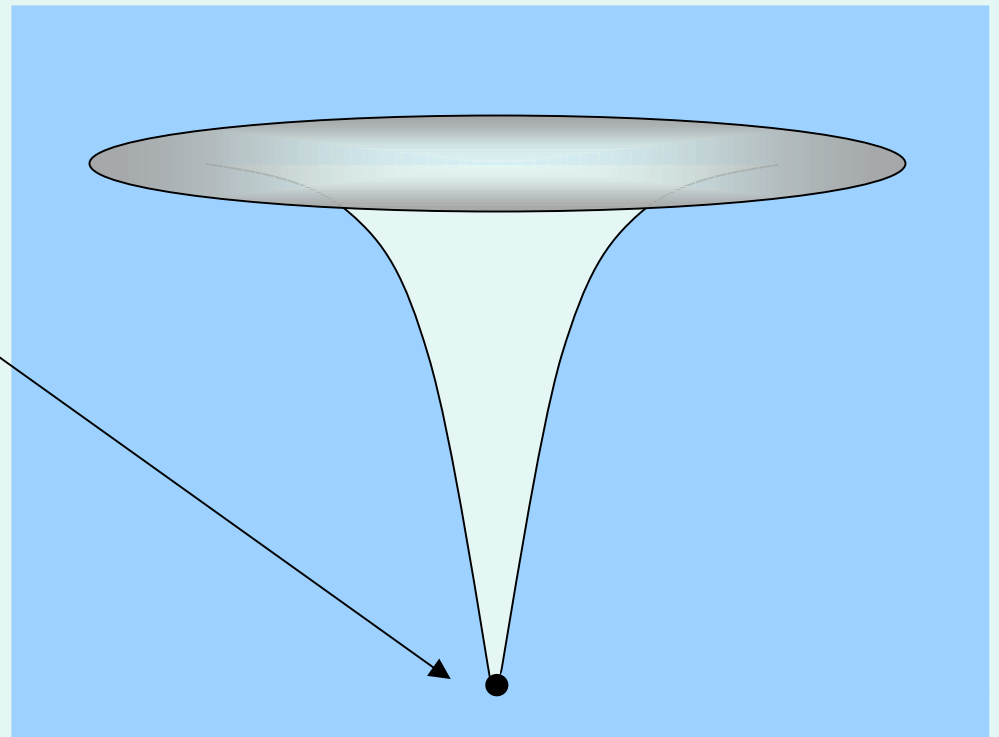


Mass:  $M = n_w 2\pi RT$

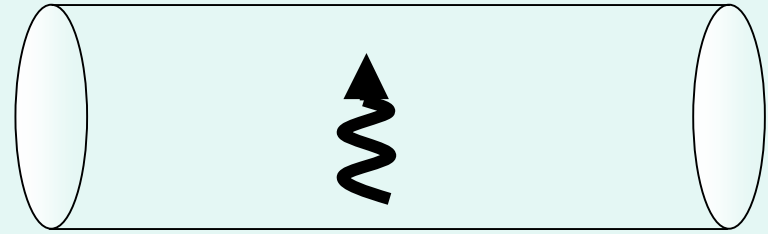
Winding charge:  $Q = n_w$

No Horizon:  $A = 0$

$$S_{bek} = \frac{A}{4G} = 0$$



$$S_{micro} = \ln(1) = 0$$

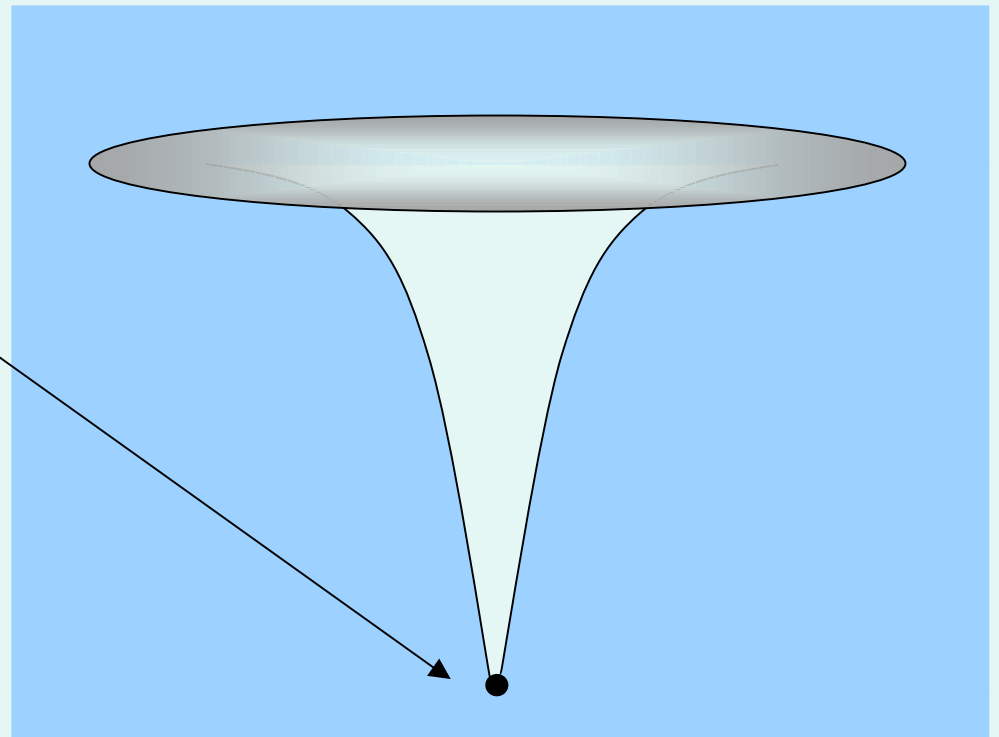


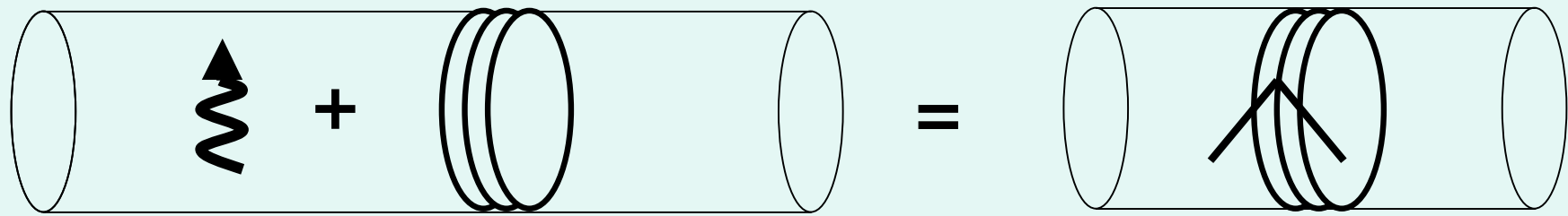
Mass:  $M = n_p 2\pi RT$

Momentum charge :  $Q = n_p$

No Horizon :  $A = 0$

$$S_{bek} = \frac{A}{4G} = 0$$

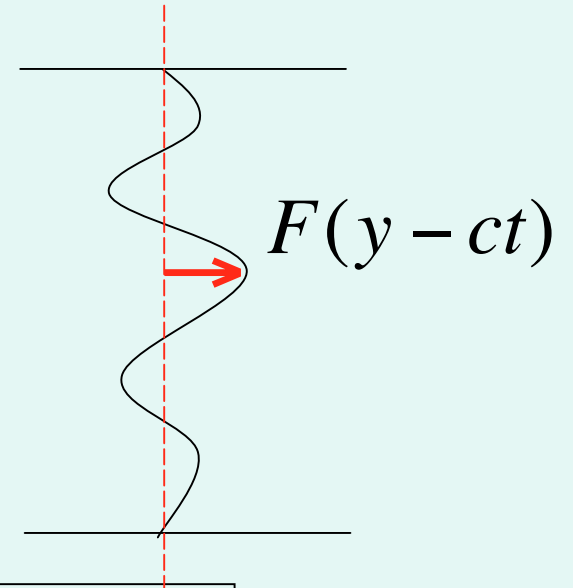




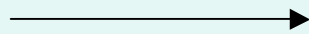
$n_p$

$n_1$

**Traveling waves**



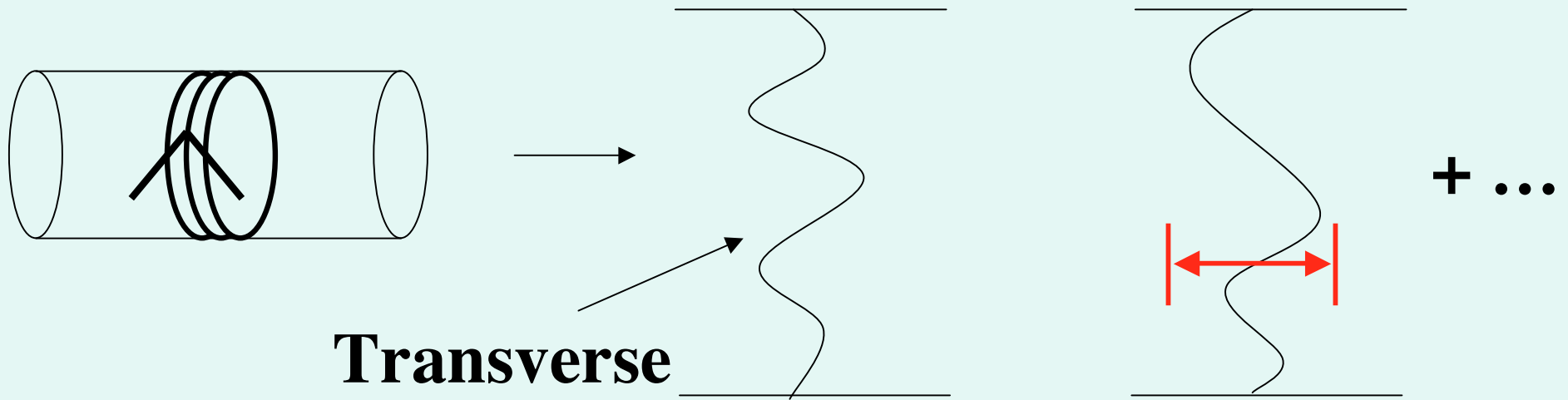
**Many ways to partition the momentum among different harmonics**



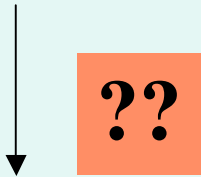
$$e^{2\pi\sqrt{2}\sqrt{n_w n_p}}$$

**states**

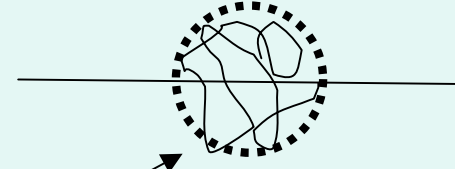
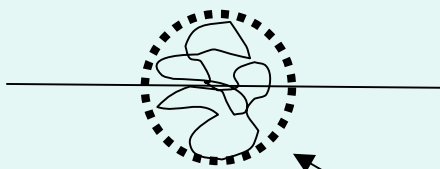
# 'Size' of the bound state



**Transverse vibrations**

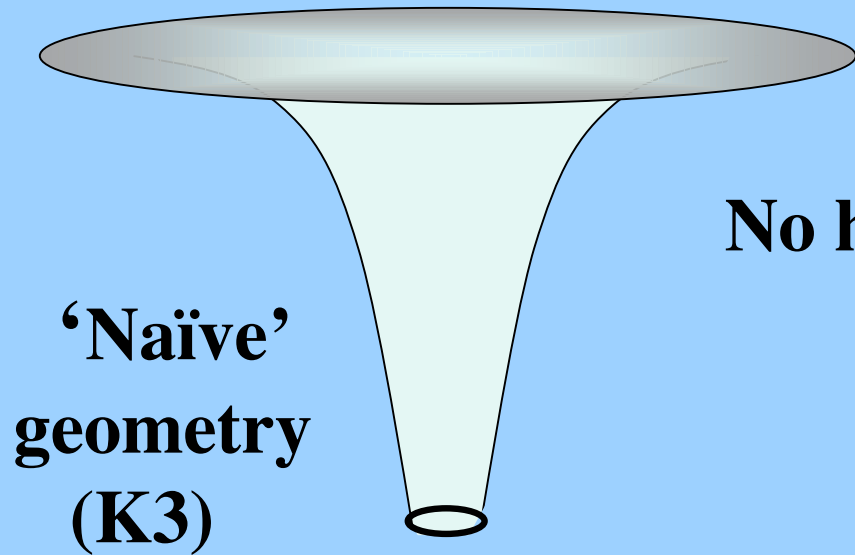
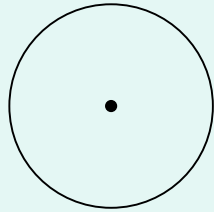


**NO !**



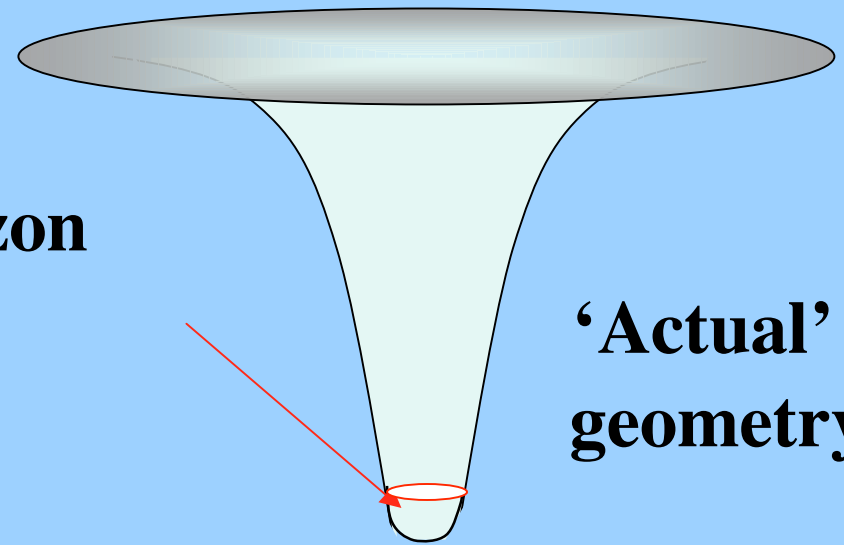
**Fuzzballs**

# Geometry created by the fuzzball



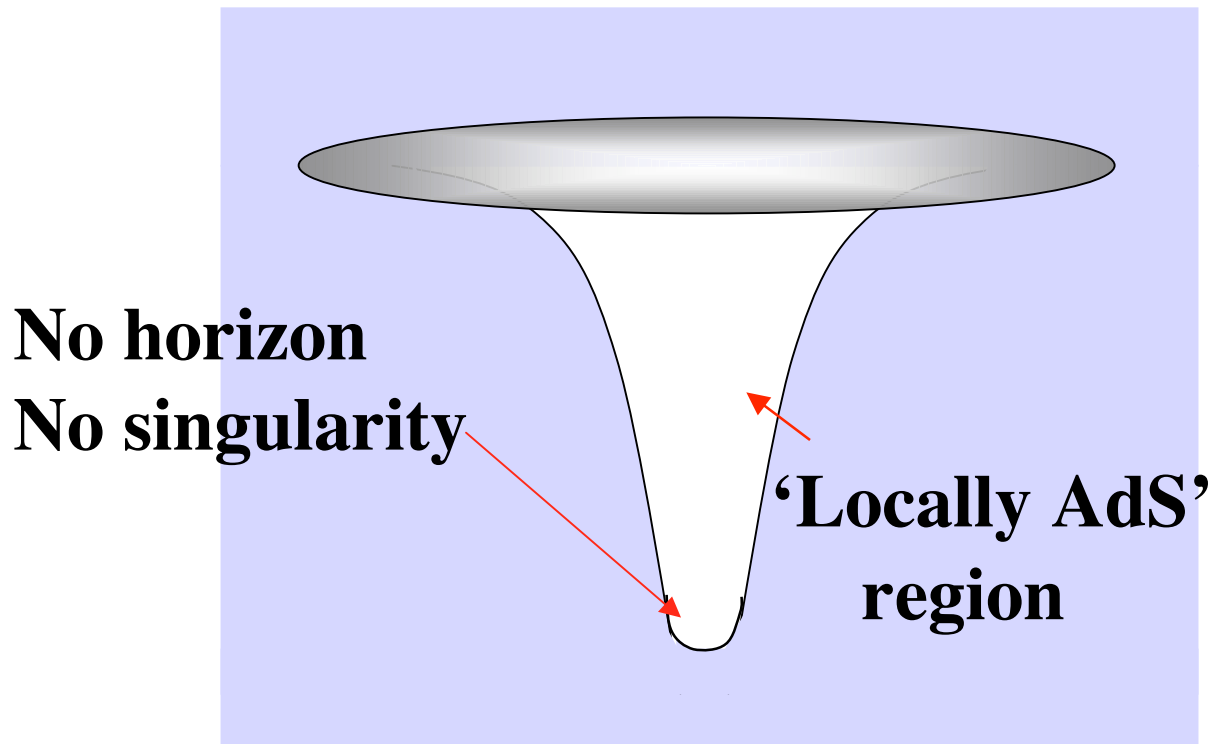
**'Naive'  
geometry  
(K3)**

**No horizon**



**'Actual'  
geometry**

**Dualize NS1-P  $\longrightarrow$  NS1-NS5**

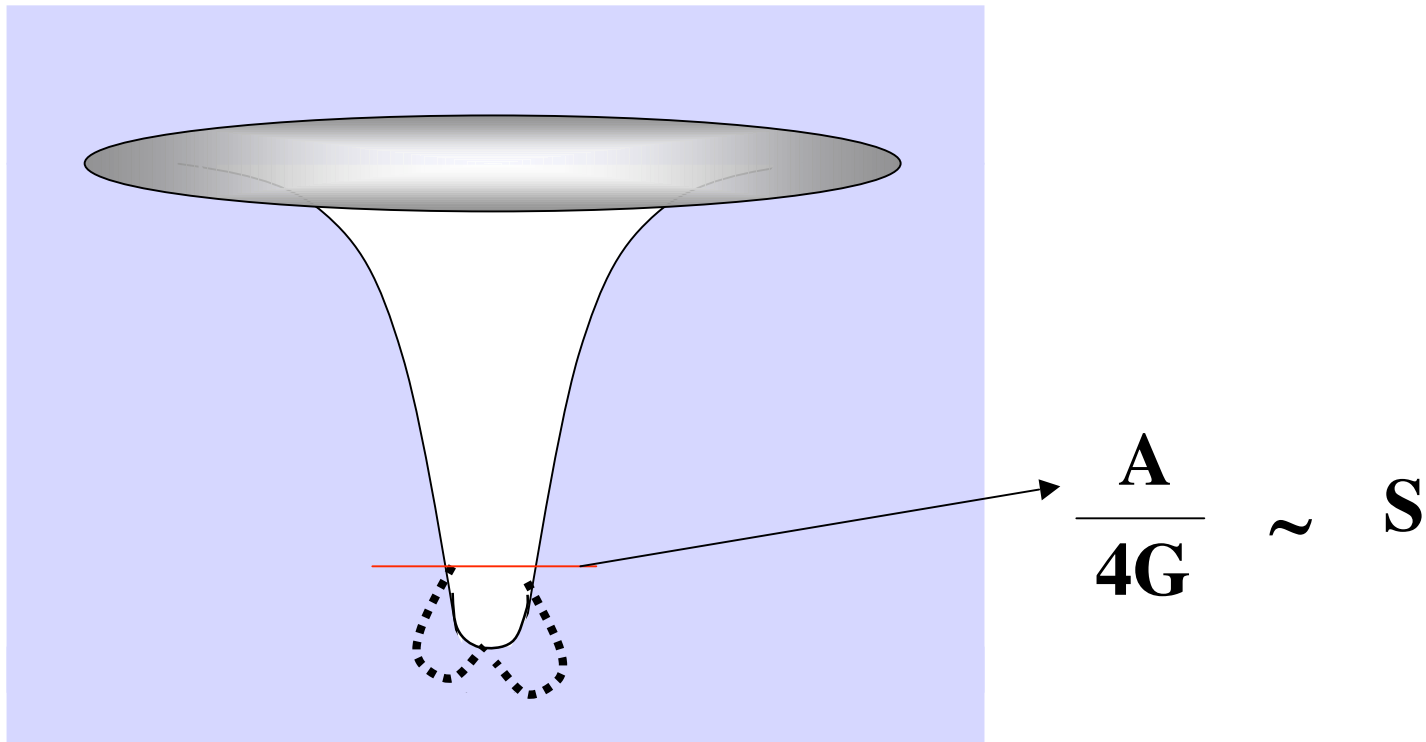


**First such metric:**  
**Balasubramanian+**  
**De Boer+ Keski-Vakkuri**  
**+ Ross; Maldacena+Maoz**

**General metrics: Lunin+SDM**

**Also,**  
**'Supergravity supertubes'**  
**Emparan+Mateos+Townsend**

**We have used coupling  $g$  such that the throat is much 'deeper' than its 'width'. In this case we see that there are no branes at the end of the throat.**



**The ‘size’ of the typical fuzzball is such that the area of its surface yields a Bekenstein type relation**

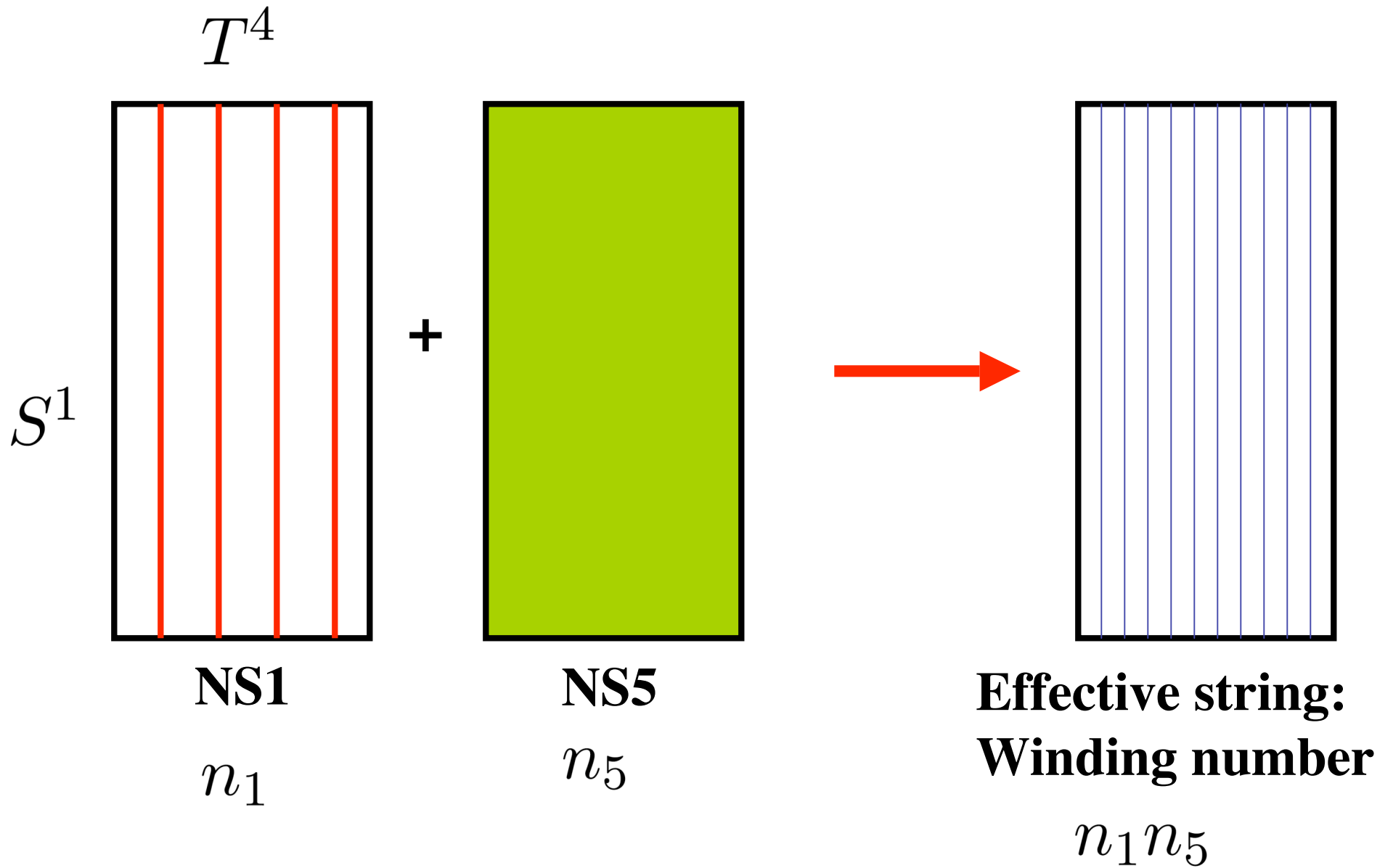
**And in this domain of  $g$  (where the throat is much deeper than its width) we also get a good illustration of AdS/CFT duality ...**

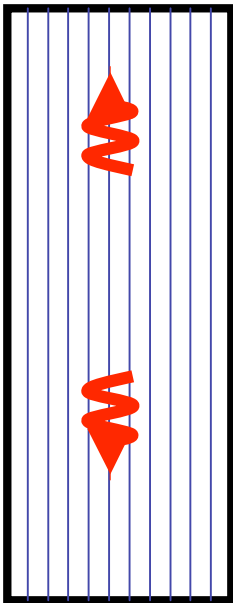
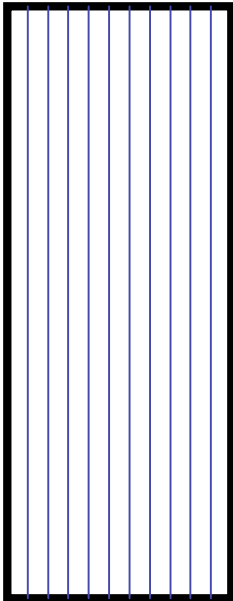
**To see this, consider the NS1-NS5 system (which has an AdS type region)**

**First we must understand a little bit about the NS1-NS5 CFT ...**



# Bound state of NS1 and NS5

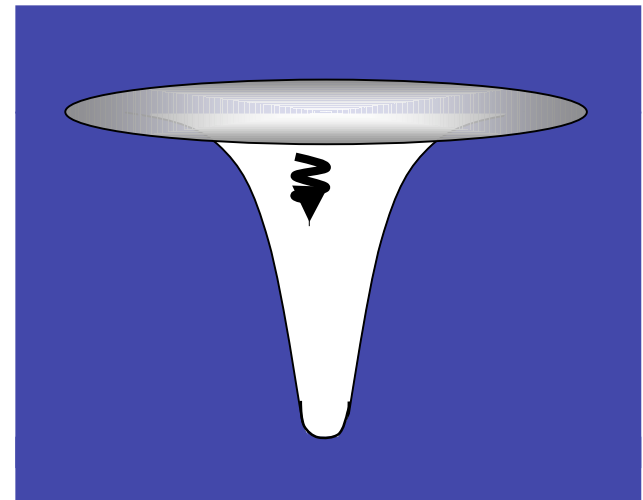
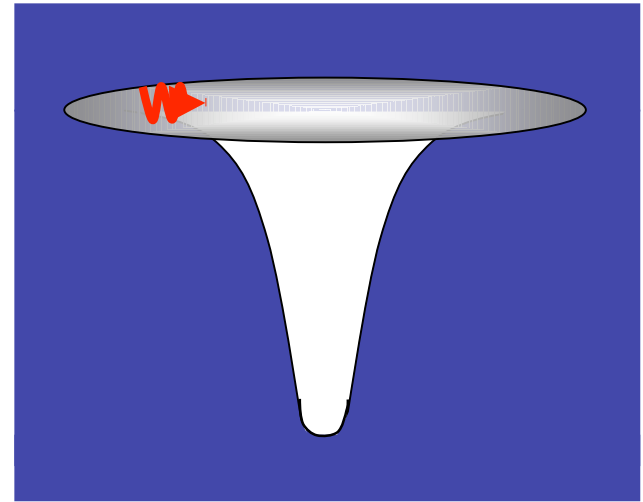




**Probability  
for absorption**

$P_{CFT}$

$P_{SUGRA}$

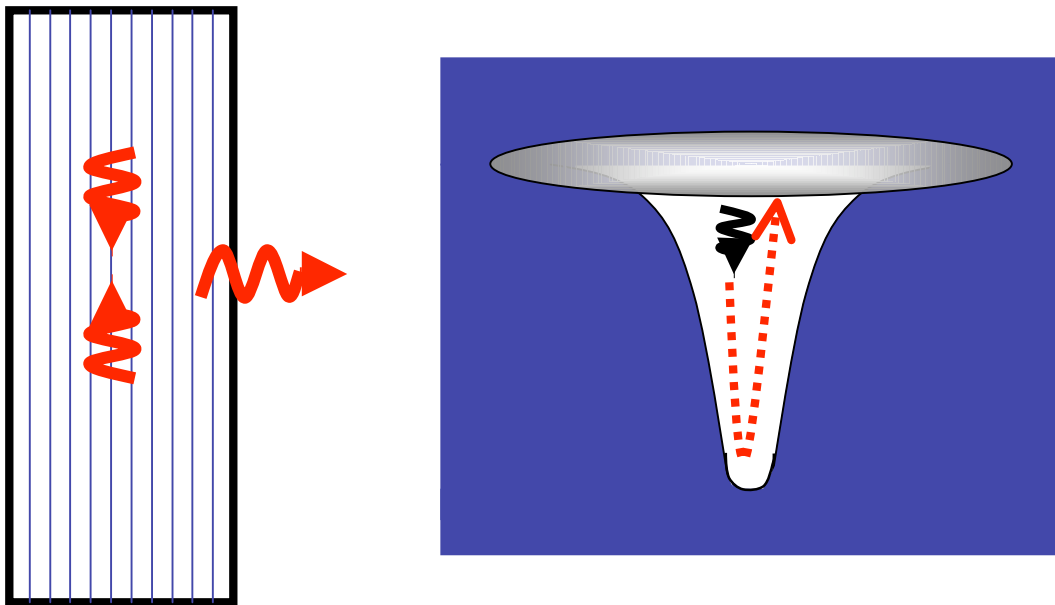


**We find**

$$P_{CFT} = P_{SUGRA}$$

$$P_l = 4\pi^2 \left( \frac{\bar{Q}'_1 \bar{Q}'_5 \omega'^4}{16} \right)^{l+1} \left[ \frac{1}{(l+1)!!} \right]^2$$

**(Das + SDM 1996, Maldacena+Strominger 1996)**



**Re-emission possible  
after some time:**

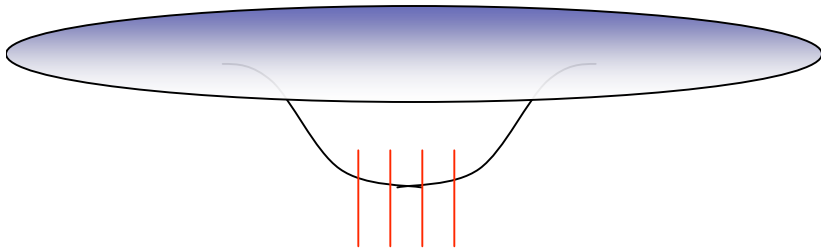
$$\Delta T_{CFT} = \Delta T_{SUGRA}$$

**(Lunin + SDM 2001)**

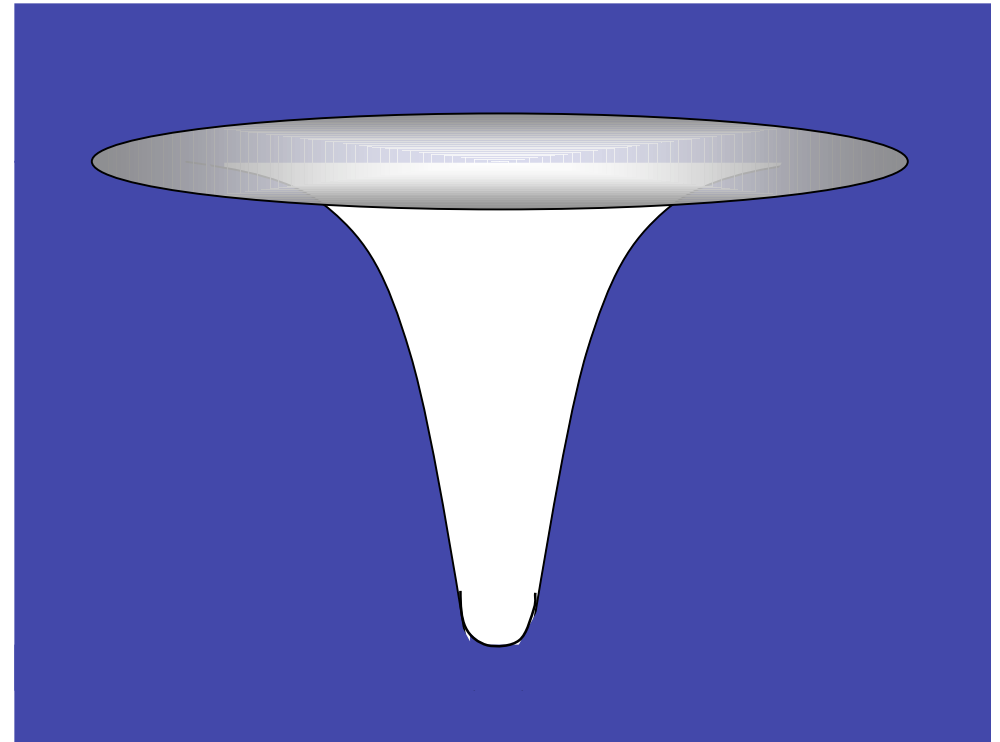


## **Two Questions**

**(A):**



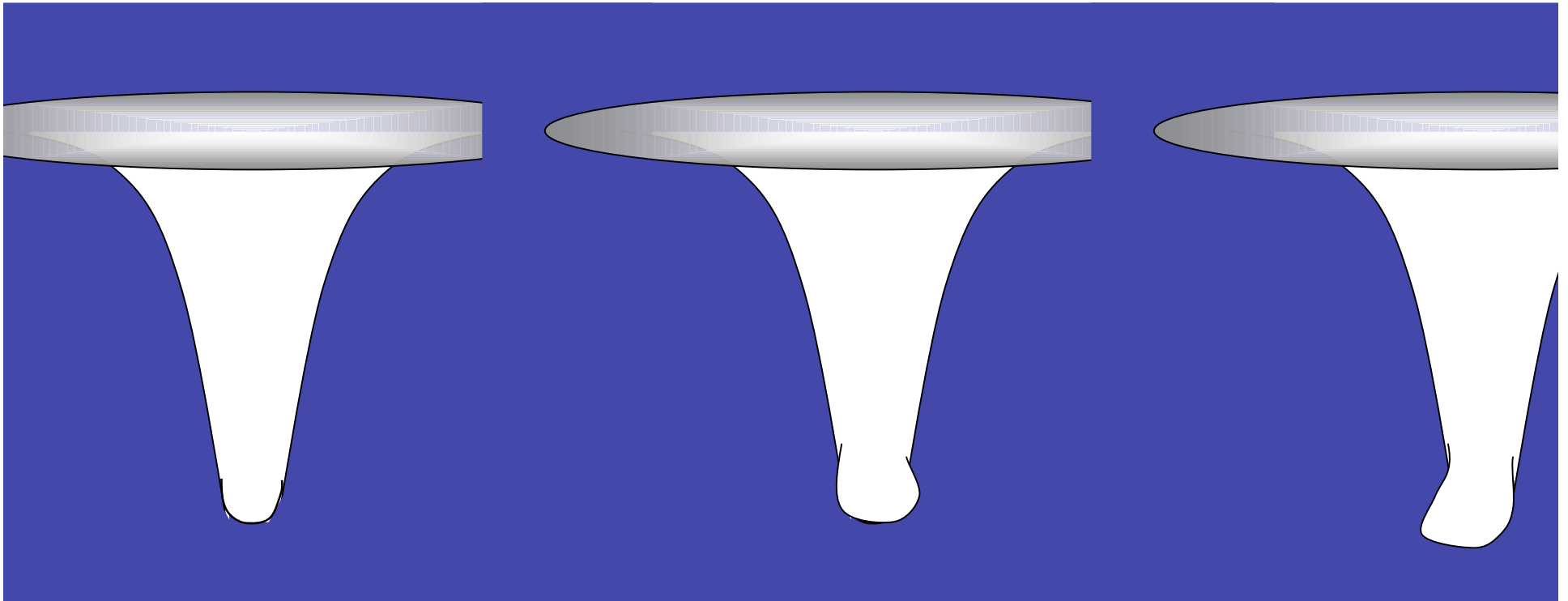
**Small  $g$ , 'see' the branes,  
Expect brane dynamics ..**



**Larger  $g$ , branes 'disappear',  
Can be replaced by  
Supergravity fields ....**

**Does the dynamical behavior change with  $g$ ?  
At what  $g$  does the change occur?**

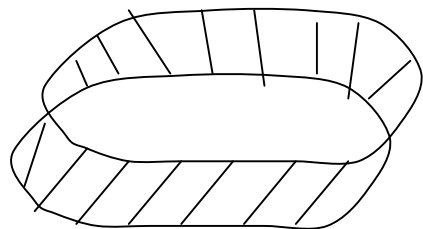
**(B):**



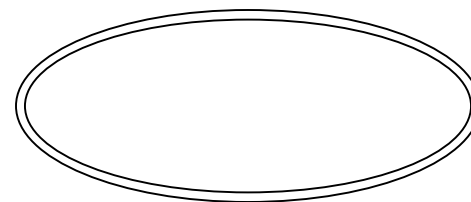
**We have a continuous family of smooth supergravity solutions, all with the same mass, charge**

**Is the low energy dynamics a ‘drift’ over a moduli space?**

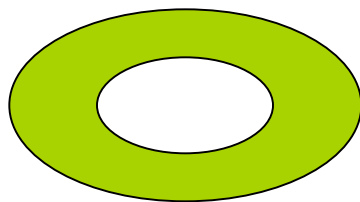
# Summary of result for (A)



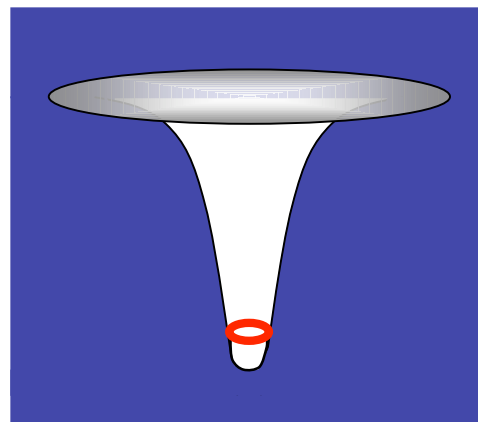
$$g = 0$$



$$g \ll g_c$$

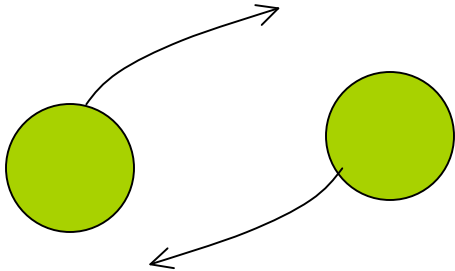


$$g \sim g_c$$

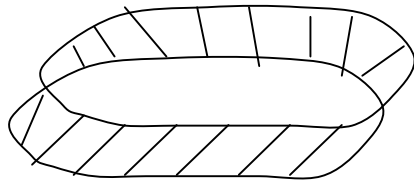


$$g \gg g_c$$

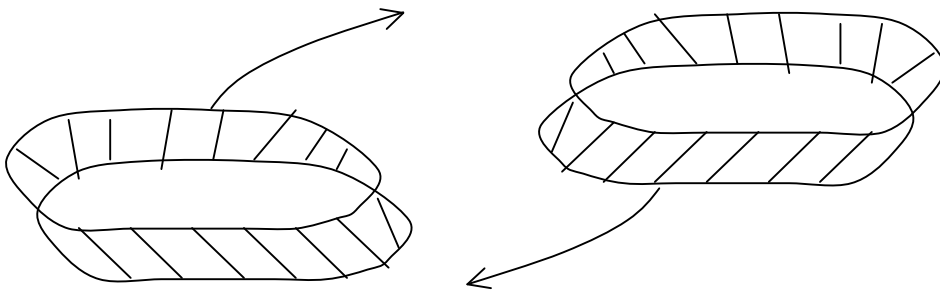
## Summary of result for (B):



**Unbound state, drift on moduli space**



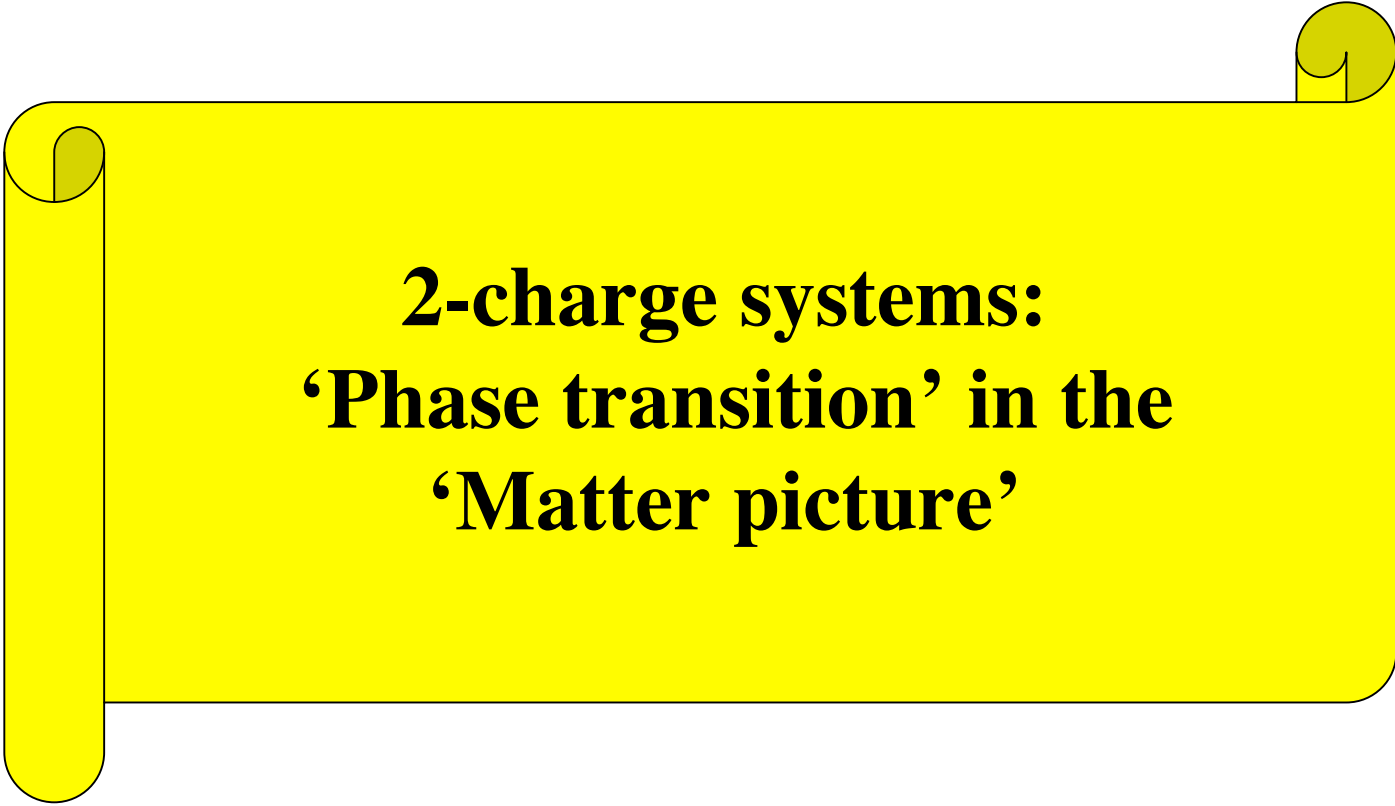
**Bound state: 'quasi-oscillations',  
NO drift on moduli space**



**Unbound states:  
Expect drift mode for  
center of mass degree  
of freedom ....**

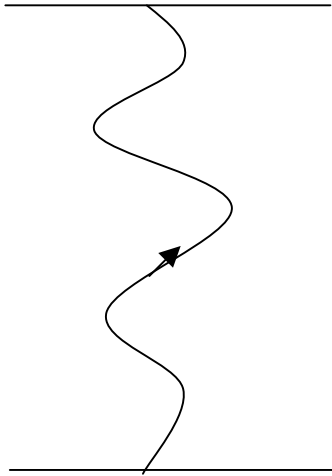
**Can use to distinguish bound from unbound states ...**



A yellow scroll graphic with a black outline, featuring a vertical strip on the left side and a small circular element at the top right corner. The text is centered within the scroll.

**2-charge systems:  
'Phase transition' in the  
'Matter picture'**

## A question:

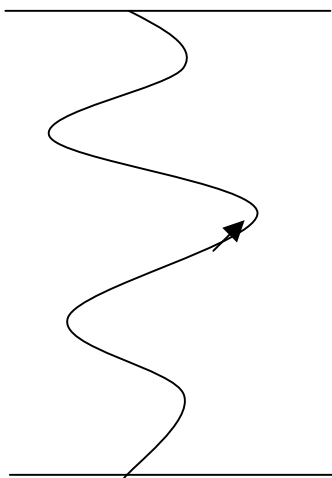


**NS1-P extremal state**

**Add energy  $\Delta E$**

**What happens?**

**$g=0$ , free string, we just get more excitations**



**Since total charges don't change,  
we can call this excitation**

$$P\bar{P} \quad \overline{NS1NS1}$$

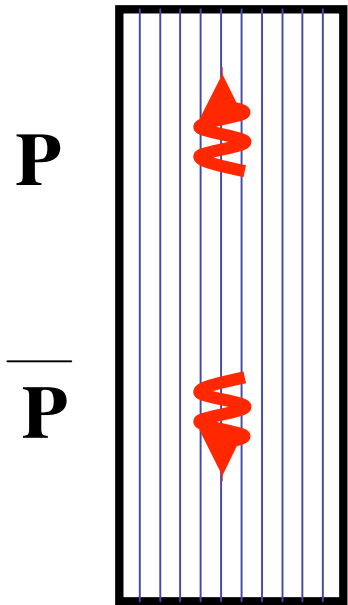
## Minimum energy of excitation

$$M^2 = \left(\frac{R_y n_1}{\alpha'} + \frac{n_p}{R_y}\right)^2 + \frac{4}{\alpha'} N_L = \left(\frac{R_y n_1}{\alpha'} - \frac{n_p}{R_y}\right)^2 + \frac{4}{\alpha'} N_R$$

$$\delta N_L = \delta N_R = 1, \quad 2M \Delta M = \frac{4}{\alpha'}$$

$$\Delta E = \Delta M = \frac{2}{\alpha' M}$$

**This is for  $g=0$  ... but it cannot be true for all  $g$**



NS1-NS5

$$NS1 \ NS5 + \Delta E \rightarrow P \ \bar{P}$$

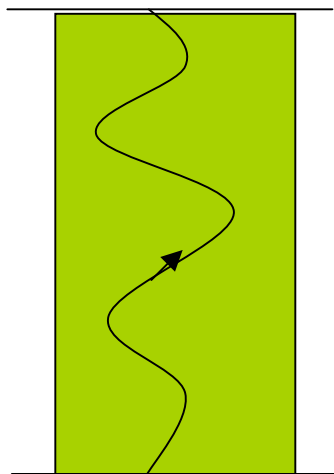
By duality we can permute NS1, NS5, P

$$P \ NS1 + \Delta E \rightarrow NS5 \ \overline{NS5}$$

Minimum energy of excitation:

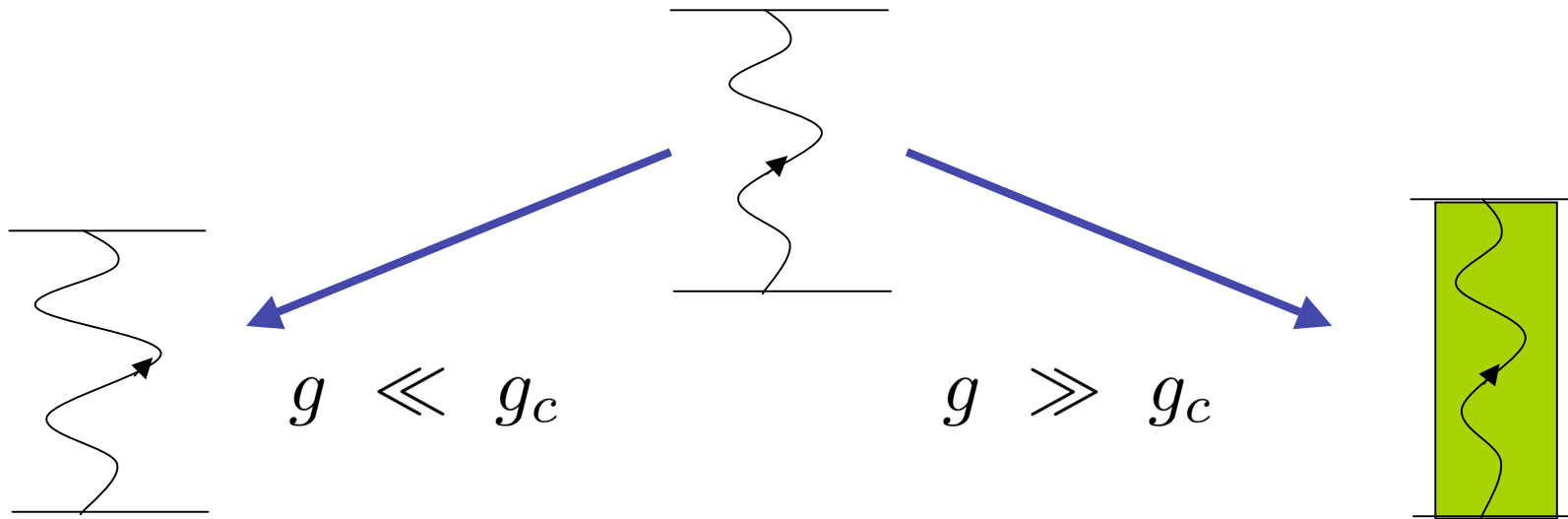
$$\Delta E = 2m_5 = 2 \frac{V R_y}{g^2 \alpha'^3}$$

Heavy at small g, but light at large g



NS1-P

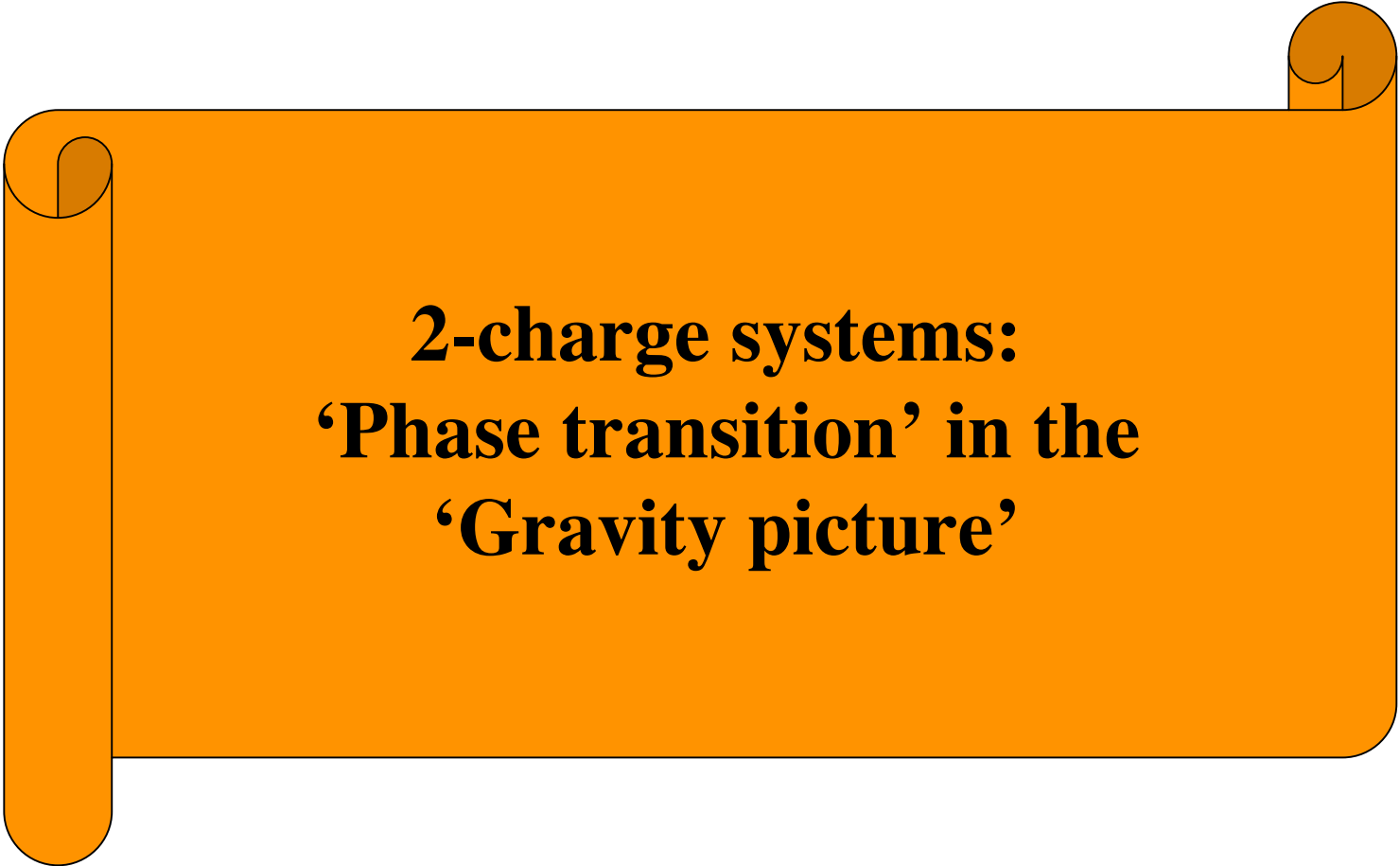
# Phase transition - microscopic (matter) picture



$$\Delta E = \frac{2}{\alpha' M}$$

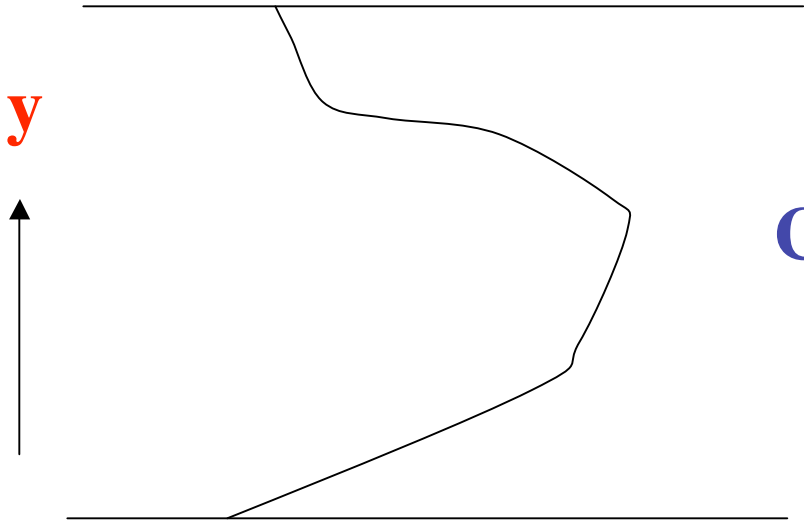
$$\Delta E = 2 \frac{V R_y}{g^2 \alpha'^3}$$

$$g_c = \sqrt{\frac{V R_y M}{\alpha'^2}}$$

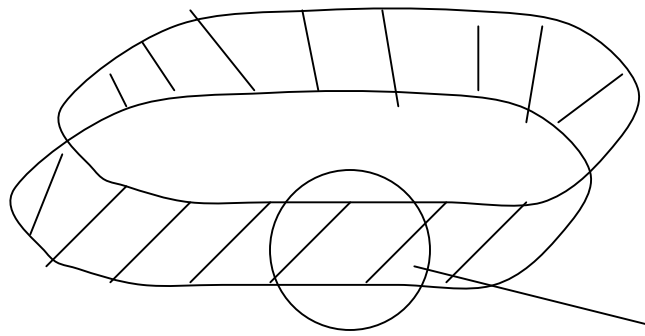
An orange scroll graphic with a black outline, featuring a vertical strip on the left side and a small circular detail at the top right corner. The text is centered on the scroll.

**2-charge systems:  
'Phase transition' in the  
'Gravity picture'**

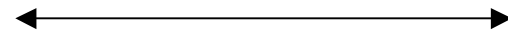
**NS1-P**



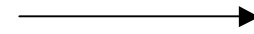
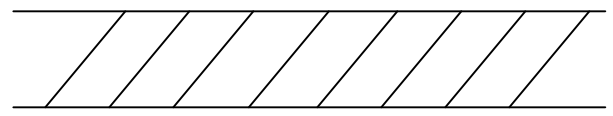
**Covering space**



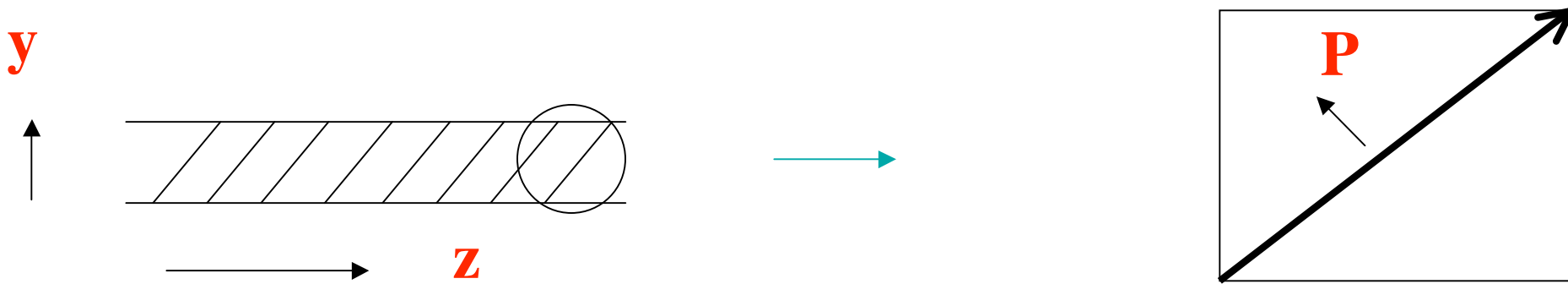
**Actual space**



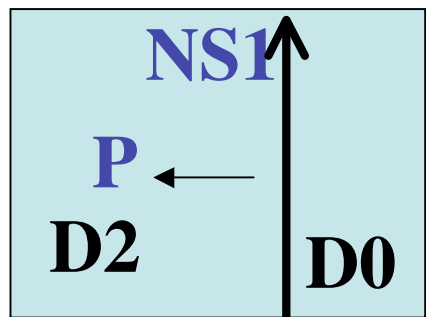
**Non-compact directions**



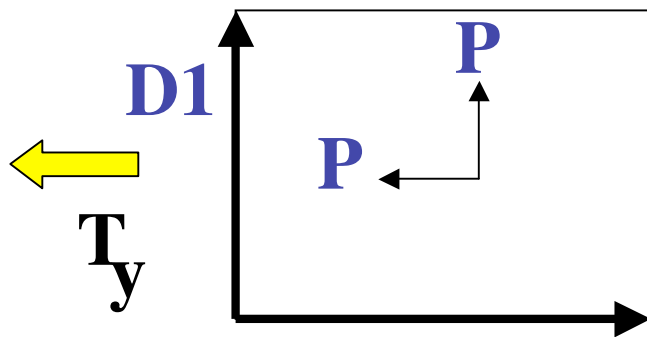
**$z$**



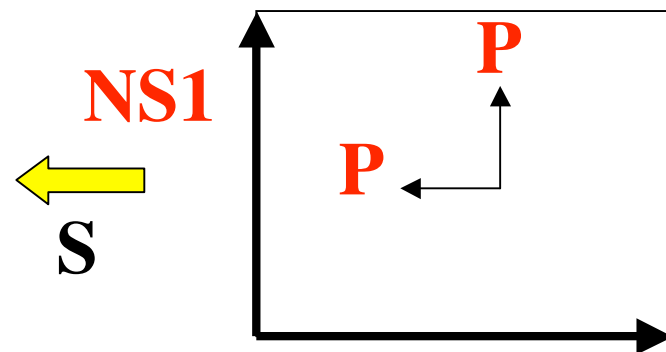
||



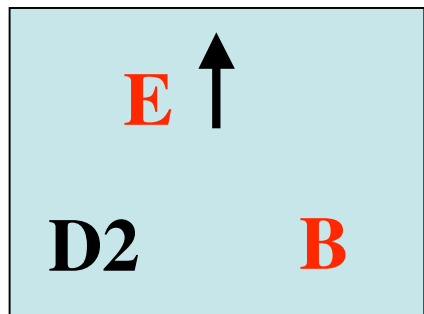
||



$D1$

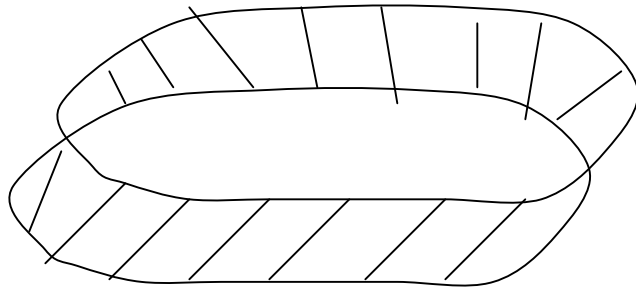


$NS1$



$D2$

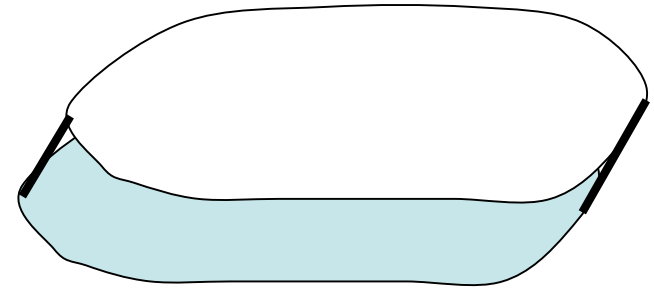




**Dipole charge NS1**  
**True charges, NS1, P**

**Arbitrary shape, arbitrary  
slope of lines at any point**

**Polyakov action for  
A free string**



**Dipole charge D2**  
**True charges NS1, D0**

**E=1, arbitrary shape,  
arbitrary B at any point**

**DBI action for D2**  
$$\mathcal{L} = -T_2 \sqrt{-\det(g + F)}$$

**We wish to solve the DBI equations for general motion of the supertube.**

**Marolf + Palmer 2004** studied small perturbations around supertube with maximal angular momentum **J**

The perturbations exhibited *oscillatory* behavior

There is only one supertube with this J, so we could not have found any ‘drift over moduli space’ in this case

*We need to look at generic supertubes.*

**The DBI equations look hard to solve. But we can solve the NSI-P system (free string), and then dualize ...**

**Polyakov action:**

**Coordinates on world sheet**       $\chi^0 \equiv \hat{\tau}, \chi^1 \equiv \hat{\sigma}$

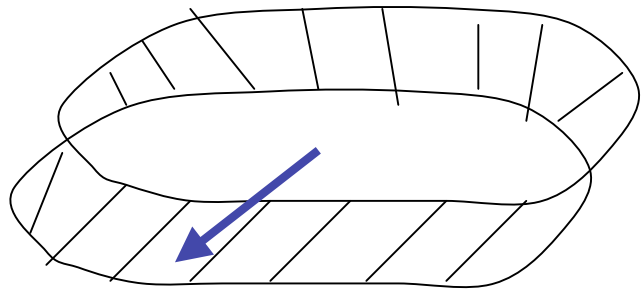
**Let**       $\chi^+ = \chi^0 + \chi^1, \quad \chi^- = \chi^0 - \chi^1$

**The solution separates into left and right movers:**

$$X^\mu = X_+^\mu(\chi^+) + X_-^\mu(\chi^-)$$

**With the constraints**

$$\frac{\partial X^\mu}{\partial \chi^+} \frac{\partial X_\mu}{\partial \chi^+} = 0, \quad \frac{\partial X^\mu}{\partial \chi^-} \frac{\partial X_\mu}{\partial \chi^-} = 0$$



**y** Choose a gauge:

**↑** 
$$X^0 = \hat{a} + \hat{b}\hat{\tau} = \hat{a} + \hat{b}\frac{1}{2}(\chi^+ + \chi^-)$$

**Solve the constraints for y**

$$\partial_+ y_+ = S_+, \quad \partial_- y_- = -S_-$$

**where**

$$S_+ = \sqrt{\frac{\hat{b}^2}{4} - \partial_+ X_+^i \partial_+ X_+^i}, \quad S_- = \sqrt{\frac{\hat{b}^2}{4} - \partial_- X_-^i \partial_- X_-^i}$$

**We can now dualize this to get the solution for the D2 Supertube ...**

## Solution of DBI action for D2:

$$X^i = X_+^i(\chi^+) + X_-^i(\chi^-), \quad A_y = y_+(\chi^+) + y_-(\chi^-)$$

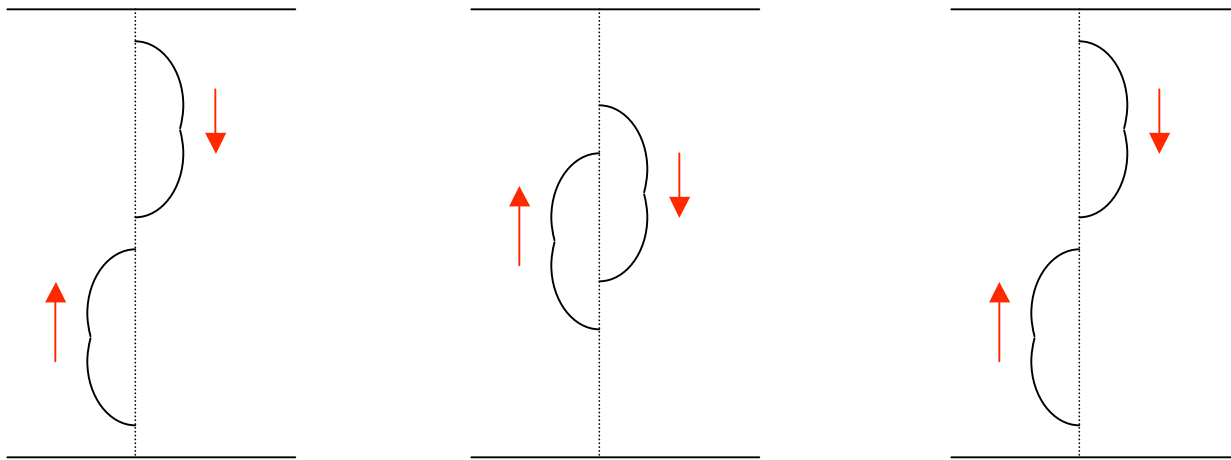
$$E = \partial_{\hat{\tau}} A_y = \partial_+ y_+ + \partial_- y_- = S_+ - S_-$$

$$B = -\partial_{\hat{\sigma}} A_y = -\partial_+ y_+ + \partial_- y_- = -(S_+ + S_-)$$

We can also look at the small oscillations around the equilibrium configuration, to get a better picture of this dynamics ...

**But it can already be seen that all motion is ‘periodic’, not a ‘drift over moduli space’ ...**

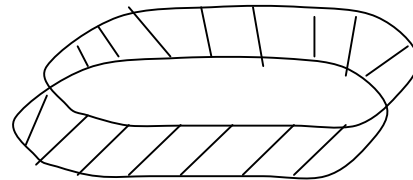
# NS1-P: Waves travel around the string and come back



## Time period

$$\Delta t = \alpha' \pi E = \frac{1}{2} \frac{E}{m_d} = \frac{\text{Total energy of supertube}}{2 \times \text{dipole mass per unit length}}$$

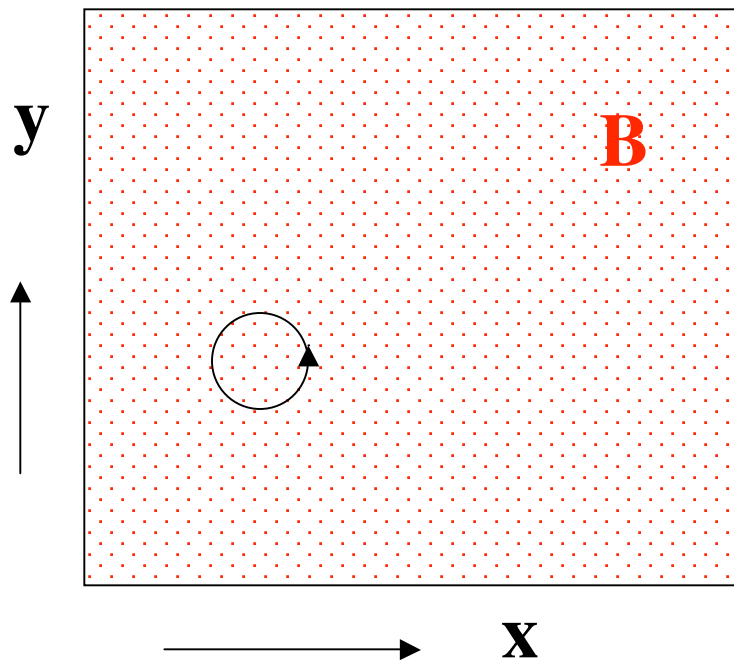
$$m_d = \frac{1}{2\pi\alpha'}$$



→ NS1

We had a family of degenerate configurations, but the system did not ‘drift’ along this family ...

## An example: particle in a magnetic field



Compare: ‘drift’

$$\ddot{x} = \frac{e}{m}\dot{y}, \quad \ddot{y} = -\frac{e}{m}\dot{x}$$

‘Quasi-oscillations’

$$v \sim \epsilon, \quad \Delta t \sim 1, \quad \Delta x \sim \epsilon$$

$$v \sim \epsilon, \quad \Delta t \sim \frac{1}{\epsilon}, \quad \Delta x \sim 1$$

**A little work shows that the supertube equations of motion are exactly like the equations for the particle in a magnetic field**

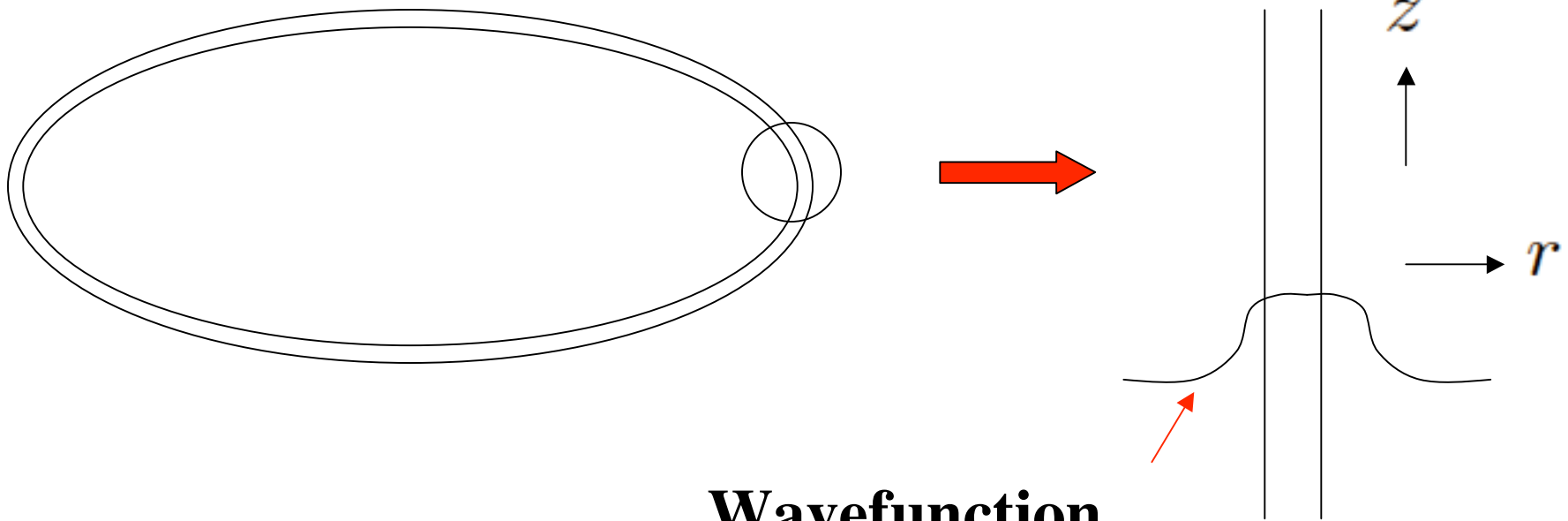
**Thus the supertube has ‘quasi-oscillations’ instead of usual oscillation zero modes which would give ‘drift along moduli space’**

**But this was all at  $g=0$  .... What happens as we increase  $g$  ?**



**Small but nonzero coupling  $g$ : Supertube is a ‘thin ring’**

*Ring thickness shows range of gravitational field*



**Wavefunction  
for gravity fields**

## Metric around the ‘thin tube’

$$ds_{string}^2 = H^{-1} [-2dt dv + \tilde{K} dv^2 + 2A dv dz] + dz^2 + dx_i dx_i + dz_a dz_a$$

$$B = (H^{-1} - 1) dt \wedge dv + H^{-1} A dv \wedge dz$$

$$e^{2\Phi} = H^{-1}$$

$$H = 1 + \frac{Q_1}{r}, \quad \tilde{K} = 1 + K = 1 + \frac{Q_p}{r}, \quad A = \frac{\sqrt{Q_1 Q_p}}{r}$$

## Perturbation: String oscillation in a torus direction

$$ds_{string}^2 \rightarrow ds_{string}^2 + 2 \mathcal{A}^{(1)} dz_{\bar{a}}, \quad B \rightarrow B + \mathcal{A}^{(2)} \wedge dz_{\bar{a}}$$

**Let**  $\mathcal{A}^{\pm} = \mathcal{A}^{(1)} \pm \mathcal{A}^{(2)}$

$$\mathcal{A}_v^- = (\tilde{\alpha} + \tilde{\beta}) H^{-1} (Q_1 - Q_p) e^{ikz - i\omega t} \frac{e^{-|\tilde{k}|r}}{r}$$

$$\mathcal{A}_t^- = -2(\tilde{\alpha} + \tilde{\beta}) H^{-1} Q_1 e^{ikz - i\omega t} \frac{e^{-|\tilde{k}|r}}{r}$$

$$\mathcal{A}_z^- = -2(\tilde{\alpha} + \tilde{\beta}) H^{-1} \sqrt{Q_1 Q_p} e^{ikz - i\omega t} \frac{e^{-|\tilde{k}|r}}{r}$$

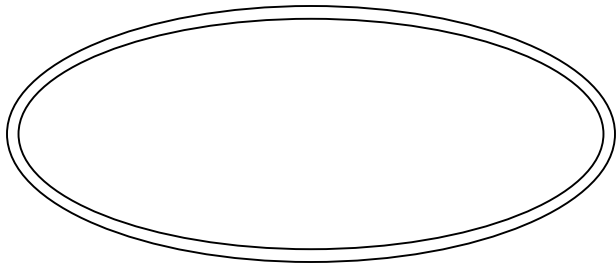
$$\omega = -k \frac{2\sqrt{Q_1 Q_p}}{Q_1 + Q_p}$$

$$\tilde{k}^2 = k^2 - \omega^2 = k^2 \left( \frac{Q_1 - Q_p}{Q_1 + Q_p} \right)^2$$

# Period of oscillations

**Speed of wave  
along tube**

$$v = \frac{\omega}{|k|} = 2 \frac{\sqrt{Q_1 Q_p}}{Q_1 + Q_p}$$

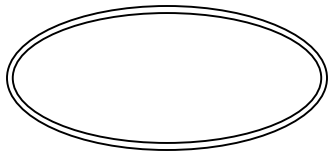


$$\begin{aligned} \Delta t &= \int_0^{L_z} \frac{dz}{v} = \int_0^{L_z} dz \frac{Q_1 + Q_p}{2\sqrt{Q_1 Q_p}} = \frac{1}{2} \int_0^{L_z} dz \left[ \sqrt{\frac{Q_1}{Q_p}} + \sqrt{\frac{Q_p}{Q_1}} \right] \\ &= \frac{1}{2T} (M_{NS1} + M_P) \end{aligned}$$

**This agrees with the period found for the  $g=0$  supertube**

But far away from the tube ...

$$\square \Psi = 0$$



$$\Psi = e^{-i\omega t} \mathcal{R}(\bar{r}) Y^{(l)}(\theta, \phi, \psi)$$



$$\mathcal{R} = \frac{r_+ e^{i\omega \bar{r}} + r_- e^{-i\omega \bar{r}}}{\bar{r}^{3/2}} (1 + O(\bar{r}^{-1}))$$

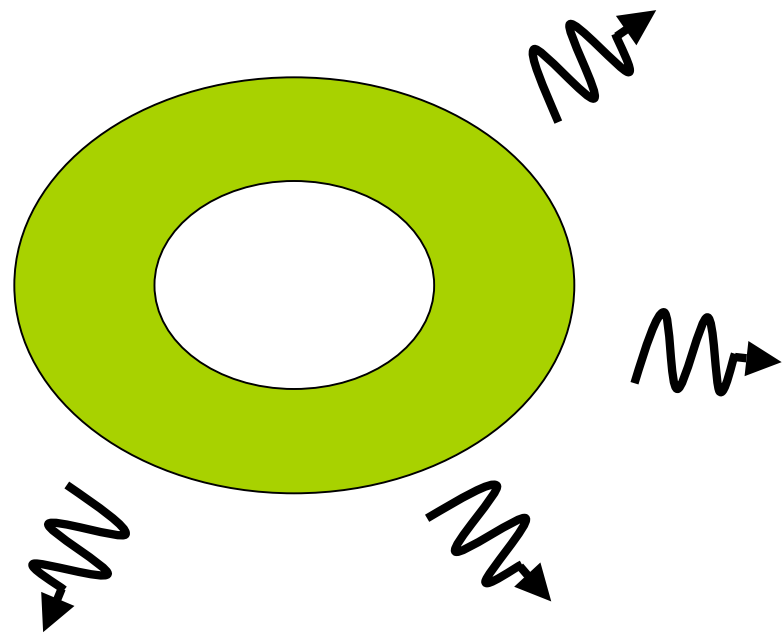
$$e^{-i\omega t}, \quad \omega^2 > 0$$

Oscillation mode embedded in the continuum 

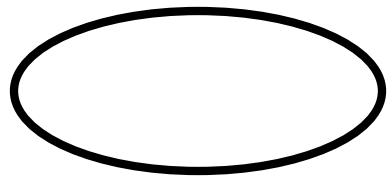
Mixes with the radiation modes of the gravitational field

Energy of oscillation leaks off to infinity,  
but only slowly since amplitude is small in radiation zone

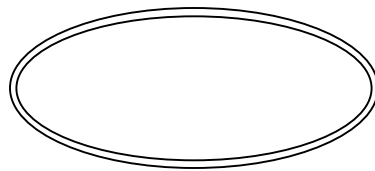
## At still larger values of the coupling $g$ ...



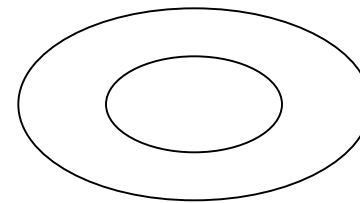
**No oscillation mode at all,  
Energy radiates away on same  
timescale as period of oscillation**



**$g=0$ , periodic  
oscillations**

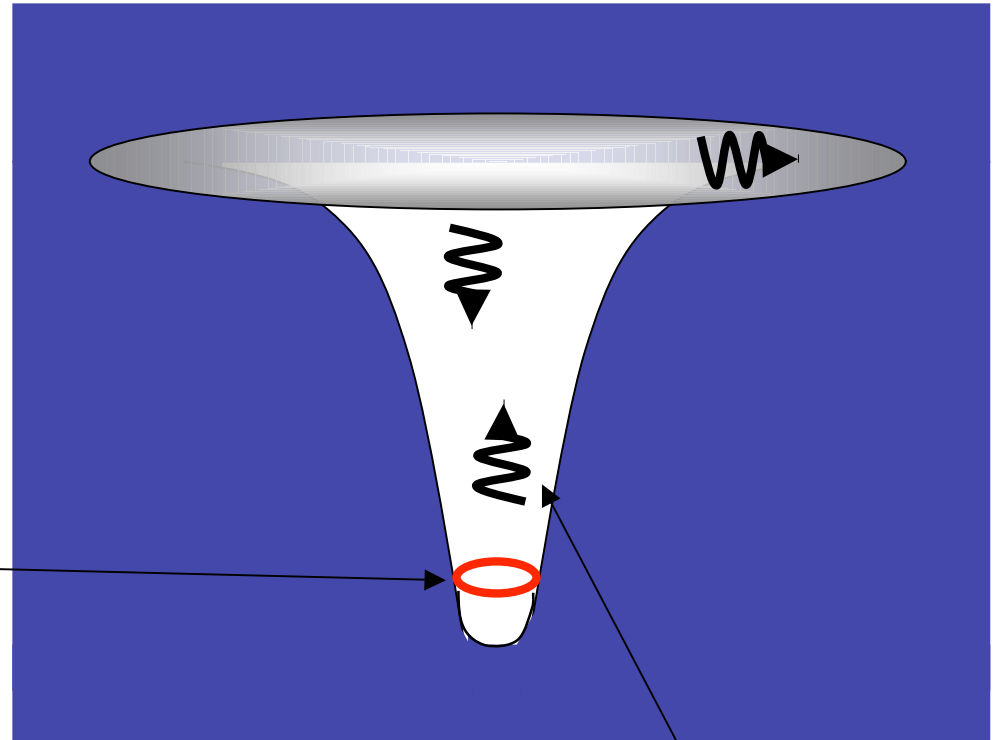
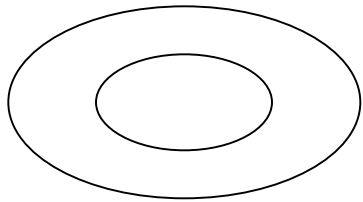


**small  $g$ , long  
lived oscillations**



**larger  $g$ , no  
oscillatory behavior**

**Let us increase  $g$  still further ...**



**Location of  
original supertube**

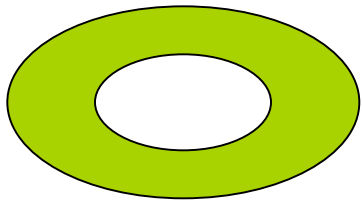
**Supergravity quanta  
bounce up and down the  
throat for long times,  
Slow leakage to infinity**

*So we again find long lived  
excitations ...*

$$\beta = \frac{(\bar{Q}_1 \bar{Q}_p)^{\frac{1}{4}}}{a}$$

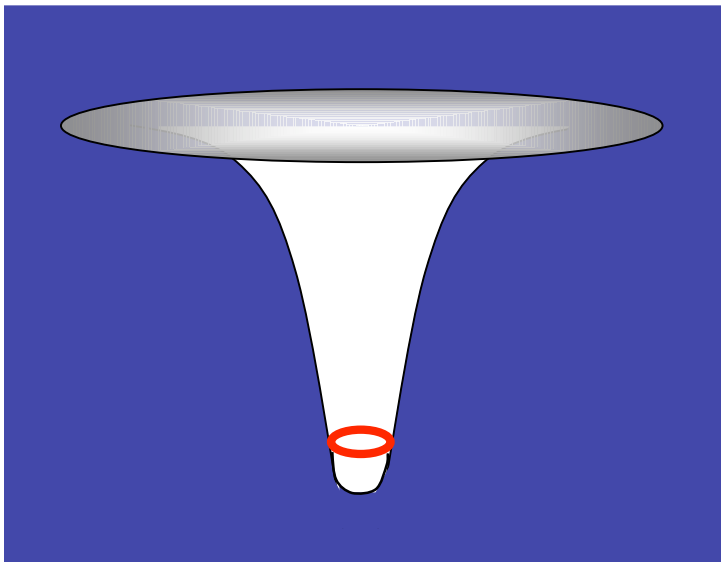
**a = radius of ring**  
 **$\bar{Q}_1, \bar{Q}_p$  = charge radii**

$$\alpha \equiv \frac{\Delta t_{escape}}{\Delta t_{osc}} \sim \beta^4$$



**Ring thickness comparable to ring radius**

$$\beta \sim 1, \quad \alpha \sim 1$$



$$\beta \gg 1, \quad \alpha \gg 1$$

**So we get long lived excitations**



Putting in the numbers ...

$$\beta = \frac{(\bar{Q}_1 \bar{Q}_p)^{\frac{1}{4}}}{a}$$

$$\bar{Q}_1 = \frac{g^2 \alpha'^3 n_1}{V}, \quad \bar{Q}_p = \frac{g^2 \alpha'^4 n_p}{V R_y^2}, \quad a = \sqrt{n_1 n_p \alpha'}$$

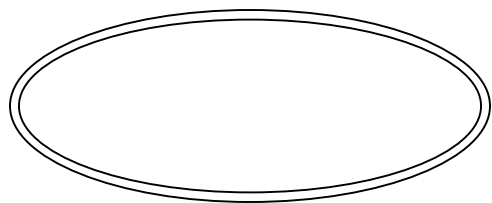
We find

$\beta \sim 1$  for

$$g \sim g_c = \sqrt{\frac{M V R_y}{\alpha'^2}}$$

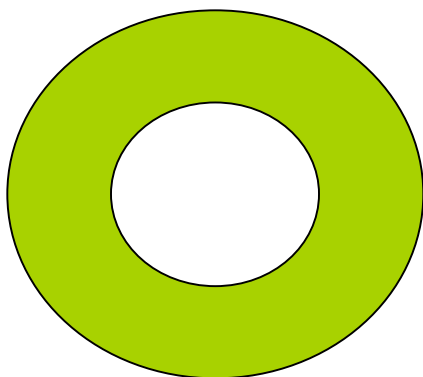
(M is the total mass of the NS1-P state)

# Supergravity 'phase transition'



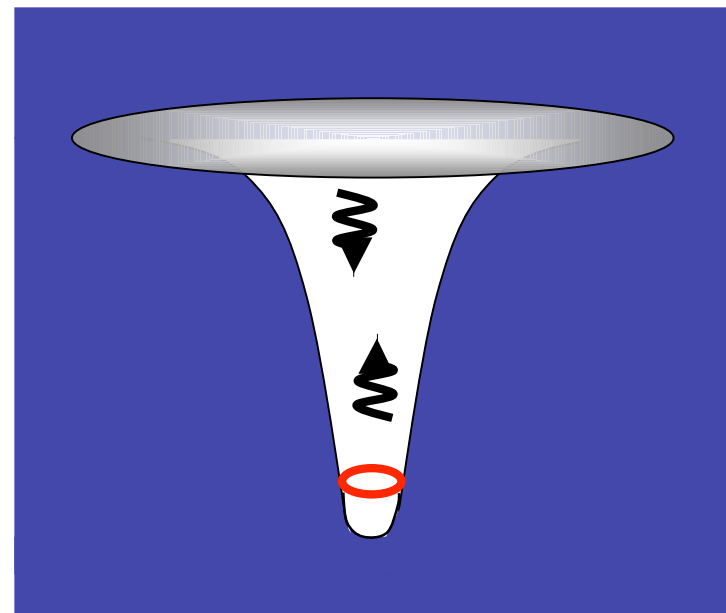
$$g \ll g_c$$

**Long lived oscillations of supertube 'matter', gravity not involved**



$$g \sim g_c$$

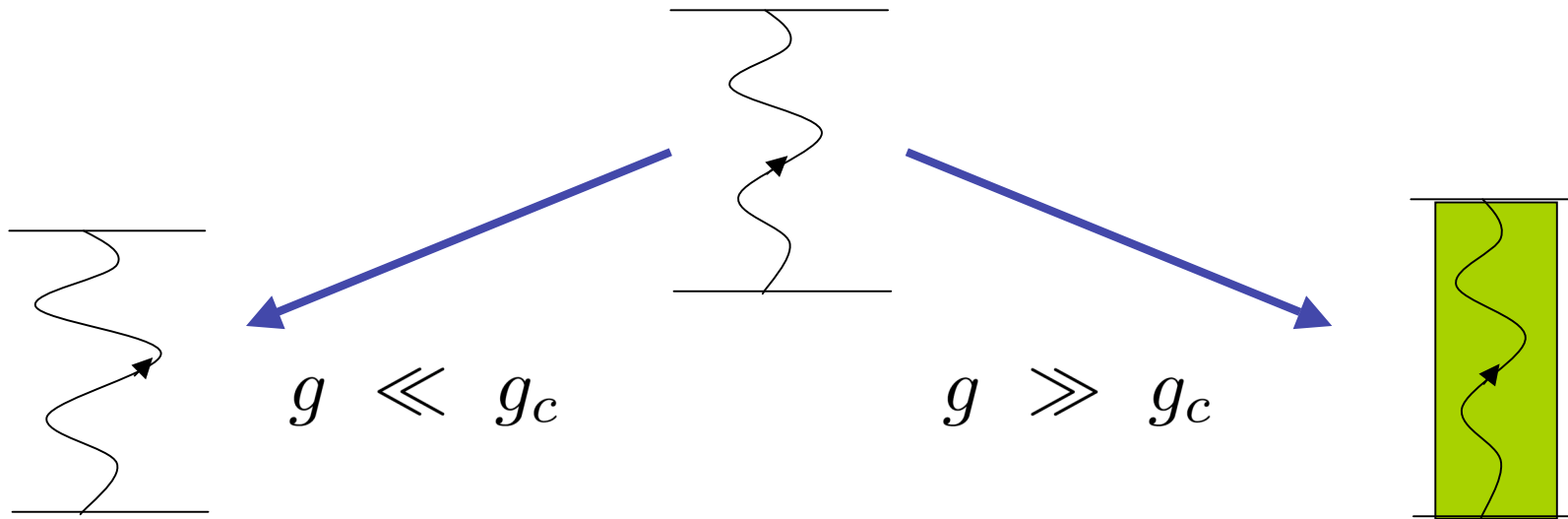
**Oscillations disappear, merge with continuum of Supergravity modes**



$$g \gg g_c$$

**Long lived excitations, 'captured from Supergravity modes of the neighborhood**

# Phase transition - microscopic (matter) picture

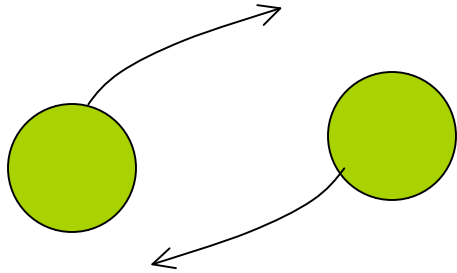


$$\Delta E = \frac{2}{\alpha' M}$$

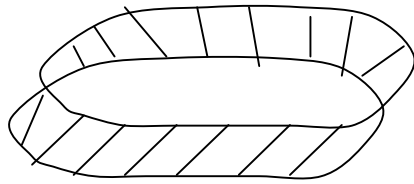
$$\Delta E = 2 \frac{V R_y}{g^2 \alpha'^3}$$

$$g_c = \sqrt{\frac{V R_y M}{\alpha'^2}}$$

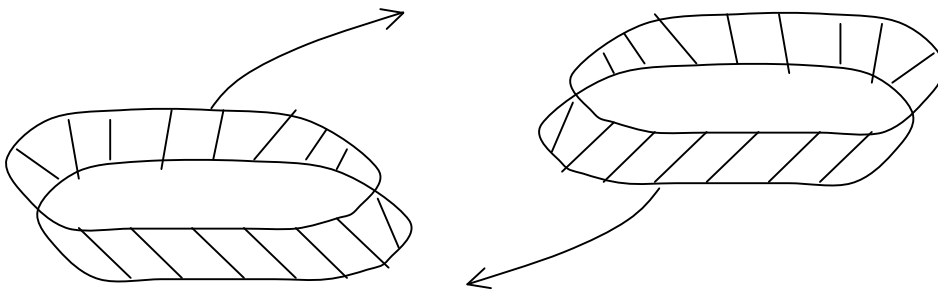
# What about the results on black hole moduli space?



**Unbound state, drift on moduli space**



**Bound state: quasi-oscillations**



**Expect drift mode for  
center of mass degree  
of freedom ....**

**Conjecture:** Bound states have no ‘drift’ modes,  
while unbound states do

*Use of the conjecture:*

**We have made all BPS 2-charge bound states.**

**We do not know how to make all BPS 3-charge bound states, but if these could be constructed then we would essentially solve all black hole paradoxes.**

**There is a way to write down all 3-charge BPS states, but this includes bound and unbound states**

**(Gauntlett,+Gutowski+Hull+ Pakis+ Reall 2002,  
Gutowski+ Martelli+ Reall 2003, Bena+Warner 2004)  
Bena+Warner 2005, Berglund+Gimon+Levi 2005**

**Conjecture:** Look at this set of BPS states, select those that have no drift modes ... these should be the microstates of the 3-charge black hole

If these all look like ‘fuzzballs’, and the degeneracy is correct, then we would resolve the information paradox