Phases of Kaluza-Klein Black Holes

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0407050 (JHEP) (with **H. Elvang and T. Harmark**)

0509011 (JHEP) (with **V. Niarchos and T. Harmark**)

Pure (Higher D) Gravity Motivations

- Kaluza-Klein black holes are interesting in their own right

  - richer phase structure when compact directions present
  - gravitational phase transitions between different solutions with event horizons (sometimes: topology change)
  - end-point of Gregory-Laflamme instability (new phases)
  - (non)-uniqueness theorems in higher dimensional gravity
  - possible objects in universe/accelerators

Cf. situation in asymptotically flat spaces: black rings etc.

String/Gauge Theory Motivations

- phase structure of Kaluza-Klein black holes related to objects and phenomena in string theory/gauge theory

Harmark,NO

Bostock,Ross/Aharony,Marsano,Minwalla,Wiseman/

$\longrightarrow$ phase structure of near-extremal branes (with circle in transverse space)

•new insights into phase structure of strongly coupled large N theories (non-gravitational theories dual to near-extremal branes, AdS/CFT)

•new phases, phase transitions from e.g. Gregory-Laflamme instability

•continuation to weak coupling

•qualitative/quantitative tests of gauge/gravity correspondence

• new phase of near-extremal NS5-brane (LST)

• correlated stability conjecture (CSC) (Gubser-Mitra)

Outline

- Characterization of Kaluza-Klein black holes
  - defining phase diagram, thermodynamics

- Black holes and strings on cylinders

- Phases with Kaluza-Klein bubbles

- Applications
  - map to phases of charged branes (non- and near-extremal)
  - CSC conjecture for near-extremal smeared branes

Outlook and open problems

Kaluza-Klein black holes

for neutral black holes in asymptotically flat space:
one physical quantity: mass M

consider static solutions of vacuum Einstein equations with:

- event horizon
- asymptoting to $\mathcal{M}^d \times S^1$ ($d \geq 4$)

⇨ Kaluza-Klein black holes

Harmark,NO/Kol,Sorkin,Piran/
Traschen,Fox/Towsend,Zamaklar

two physical quantities:

mass M

tension $\mathcal{T}$

⇨ plot known phases in two-dimensional phase diagram

Mass and tension for KK BHs

consider static solutions of vacuum Einstein equations

- coordinates for $\mathcal{M}^d \times S^1$ ($d \geq 4$)

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2 + dz^2 \qquad z \sim z + L$$

asymptotics
$$g_{tt} \simeq -1 + \frac{c_t}{r^{d-3}} \;, \quad g_{zz} \simeq 1 + \frac{c_z}{r^{d-3}}$$

→ 2 (gauge-invariant) asymptotic quantities

Harmark,NO/Kol,Sorkin,Piran/Traschen,Fox/Towsend,Zamaklar

$$M = \frac{\Omega_{d-2} L}{16\pi G_N}\left[(d-2)c_t - c_z\right] \;, \quad \mathcal{T} = \frac{\Omega_{d-2}}{16\pi G_N}\left[c_t - (d-2)c_z\right]$$

→ mass

→ tension

1st law of thermo $\delta M = T\delta S + \mathcal{T}\delta L$

Smarr formula $TS = \frac{d-2}{d-1}M - \frac{\mathcal{T}L}{d-1}$

using Komar integral (time-translation sym.)

Tension

Harmark,NO/Kol,Sorkin,Piran/
Traschen,Fox/Towsend,Zamaklar

- method of equivalent sources

consider static Newtonian matter with non-zero energy-momentum tensor T_{00}, T_{zz}

- solve Einstein equations in weak gravitational field

- use Hamiltonian approach of Hawking-Horowitz

foliation of space-time along time direction t

mass (energy) wrt ref. background

$$M = -\frac{1}{8\pi G_N} \int_{S_t^\infty} N_t \left(\mathcal{K}_t^{(D-2)} - \mathcal{K}_{t,0}^{(D-2)} \right)$$

generalize by considering foliation along spatial direction z

$$\mathcal{T} = -\frac{1}{8\pi G_N} \int_{\hat{S}_z^\infty} N_z \left(\mathcal{K}_z^{(D-2)} - \mathcal{K}_{z,0}^{(D-2)} \right)$$

space-time is asymptotically translationally invariant in z

(μ,n) phase diagram for KK BH's

→ dimensionless physical quantities

$$\mu = \frac{16\pi G_N}{L^{d-2}} M = \frac{\Omega_{d-2}}{L^{d-3}} [(d-2)c_t - c_z] \ , \quad n = \frac{\mathcal{T} L}{M} = \frac{c_t - (d-2)c_z}{(d-2)c_t - c_z}$$

rescaled mass

relative tension

bounds $\mu \geq 0 \ , \quad 0 \leq n \leq d-2$

positive tension

Traschen/Shiromizu,Ida,Tomizawa

gravitational force on test particle at infinity attractive

1st law of thermo $\delta\mu = \mathfrak{t}\,\delta\mathfrak{s}$

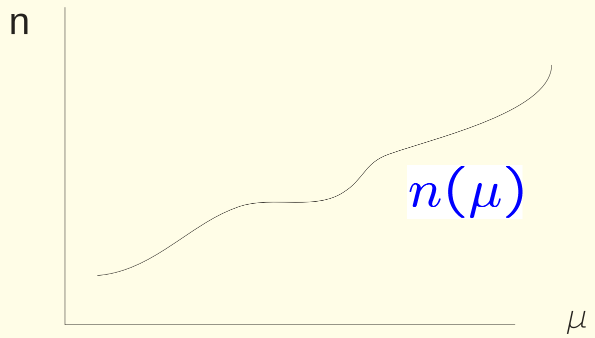
generalized Smarr $\mathfrak{t}\mathfrak{s} = \frac{d-2-n}{d-1}\mu$

⇒ plot phase structure in terms of (μ,n)-phase diagram

Curves in the phase diagram and Intersection Rule

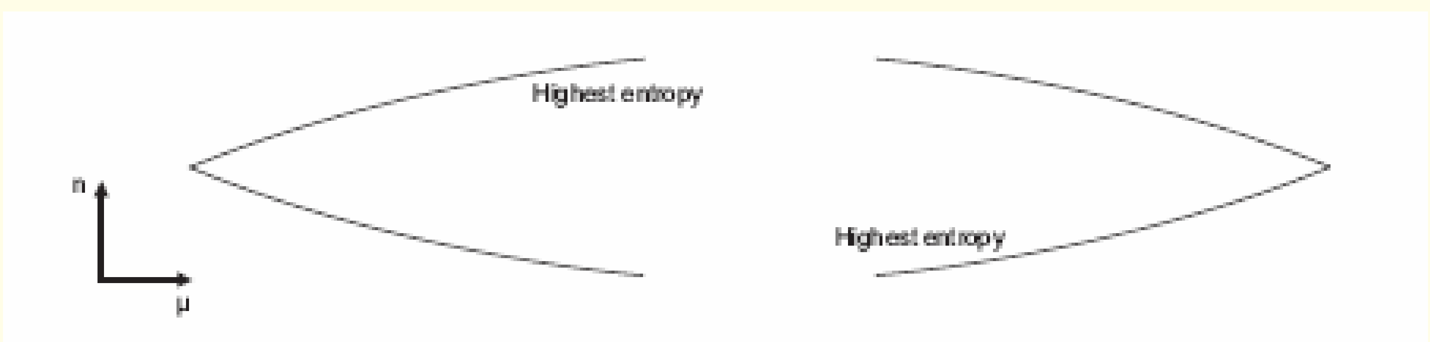
using first law and Smarr formula

$$\frac{\delta \log \mathfrak{s}(\mu)}{\delta \log \mu} = \frac{d-1}{d-2-n(\mu)}$$

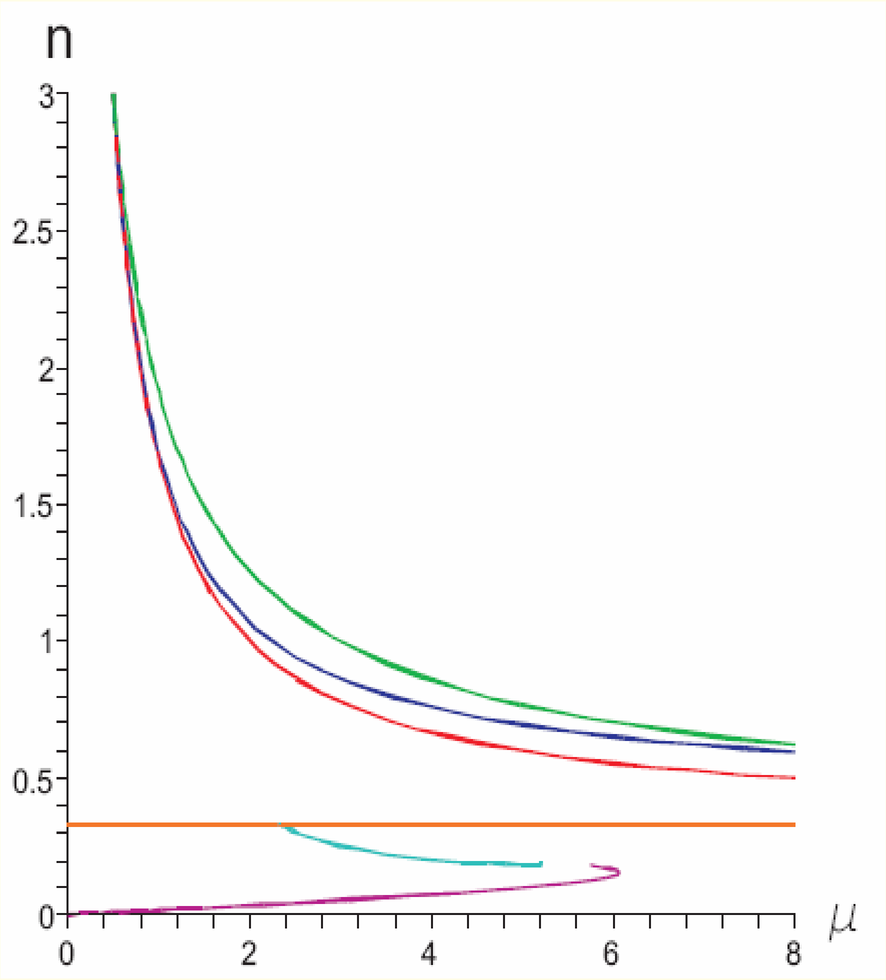


knowledge of curve in phase diagram
can derive entire thermodynamics

two curves intersecting in same solution



Present knowledge of KK black holes



$\frac{n}{d-2}$, $\frac{1}{d-2}$

solutions with Kaluza-Klein bubbles

BH – bub –BH –… - bub

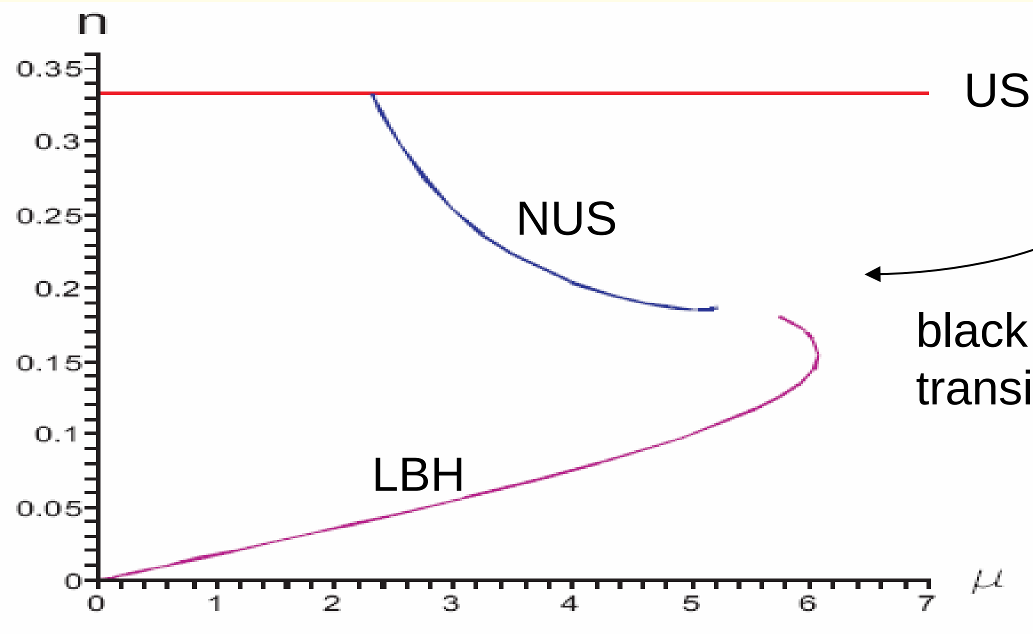
Emparan,Reall/Horowitz,Elvang/ Elvang,Harmark,NO

solutions without KK bubbles and local SO(d-1)symmetry

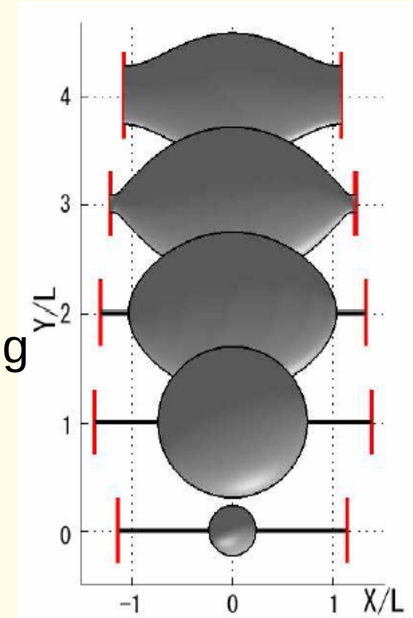
- black holes: S^{d-1}
- black strings: $S^{d-2} \times S^1$

phase diagram for six dimensions

black hole/string region $0 \le n \le \frac{1}{d-2}$

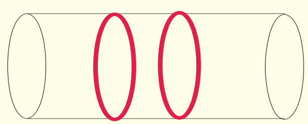


black hole/black string transition



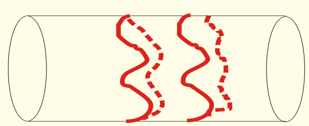
Koduh,Wiseman

•uniform black string (US)



Schwarzschild-Tangherlini(d) $\times$ S^1

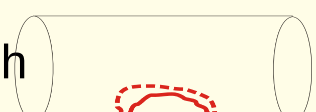
•non-uniform black string (NUS)



emanates from uniform at Gregory-Laflamme point

Gregory,Laflamme/Gubser/Wiseman/Sorkin
motivated in part by: Horowitz,Maeda

•localized black hole branch (LBH)



Schwarzschild (d +1) + $\mathcal{O}$ (μ)

Harmark,NO/Harmar/Kol,Gorbonos/
Sorkin,Kol,Piran/Koduh,Wiseman

Consistent ansatz

- solutions with $0 < n < \frac{1}{d-2}$ → SO(d-1) symmetry   Harmark,NO//Wiseman

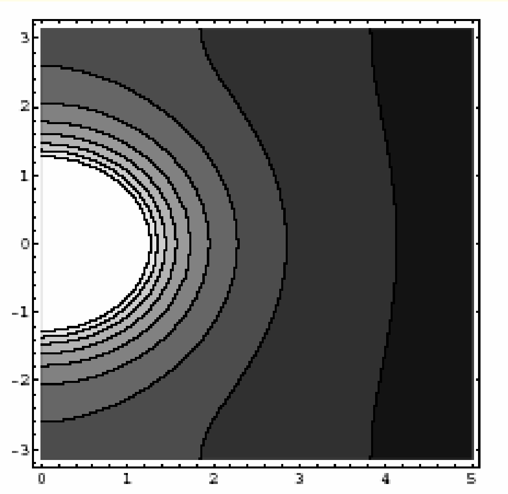
$$ds^2_{d+1} = -f dt^2 + \frac{L^2}{(2\pi)^2}\left[\frac{A}{f}dR^2 + \frac{A}{K^{d-2}}dv^2 + KR^2 d\Omega^2_{d-2}\right] \;,\quad f = 1 - \frac{R_0^{d-3}}{R^{d-3}}$$

•involves two functions instead of three functions
in the conformal ansatz $ds^2 = -e^{2B}dt^2 + e^{2C}(dr^2 + dz^2) + e^{2D} d\Omega^2_{d-2}$

•$A(R,v)$ determined in terms of $K(R,v)$

• boundary condition $K(R,v) = 1 - \chi \frac{R_0^{d-3}}{R^{d-3}} + \mathcal{O}(R^{-2(d-3)})$

•(R,v) coordinates: interpolate between spherical - cylindrical



lines of constant R are equipotential surfaces
of extremal p-branes with transverse space
$R^{d-1} \times S^1$

(equivalently: charged particle on circle)

Thermodynamics of the ansatz

$$\mu = \frac{(d-3)\Omega_{d-2}}{(2\pi)^{d-3}} R_0^{d-3}\left[\frac{d-2}{d-3} - \chi\right] , \quad n = \frac{1-(d-2)(d-3)\chi}{d-2-(d-3)\chi}$$

$$\mathfrak{t} = \frac{d-3}{2\sqrt{A_h}R_0} , \quad \mathfrak{s} = \frac{4\pi\Omega_{d-2}}{(2\pi)^{d-2}}\sqrt{A_h}R_0^{d-2}$$

$R = R_0$ is a Killing horizon

$A_h \equiv A(R,v)|_{R=R_0}$ constant on horizon (can show explicitly from EOM)

→ constant surface gravity

- 1st law of thermodynamics explicitly checked for the ansatz:

  - static and Ricci-flat perturbation of the ansatz
  - variation of Smarr relation

Black holes and strings on cylinders

- uniform black string branch

$$ds^2 = -\left(1 - \frac{r_0^{d-3}}{r^{d-3}}\right) dt^2 + \left(1 - \frac{r_0^{d-3}}{r^{d-3}}\right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2 + dz^2$$

- non-uniform black string branch

Wiseman

d=5: branch numerically known

Sorkin

$d \leq 50$: departure from Gregory-Laflamme point (numeric)

$$n(\mu) = \frac{1}{d-2} - \gamma(\mu - \mu_{\rm GL}) + \mathcal{O}((\mu - \mu_{\rm GL})^2) \,, \quad 0 \leq \mu - \mu_{\rm GL} \ll 1$$

- localized black hole branch

Kudoh,Wiseman

d=4,5: branch numerically known

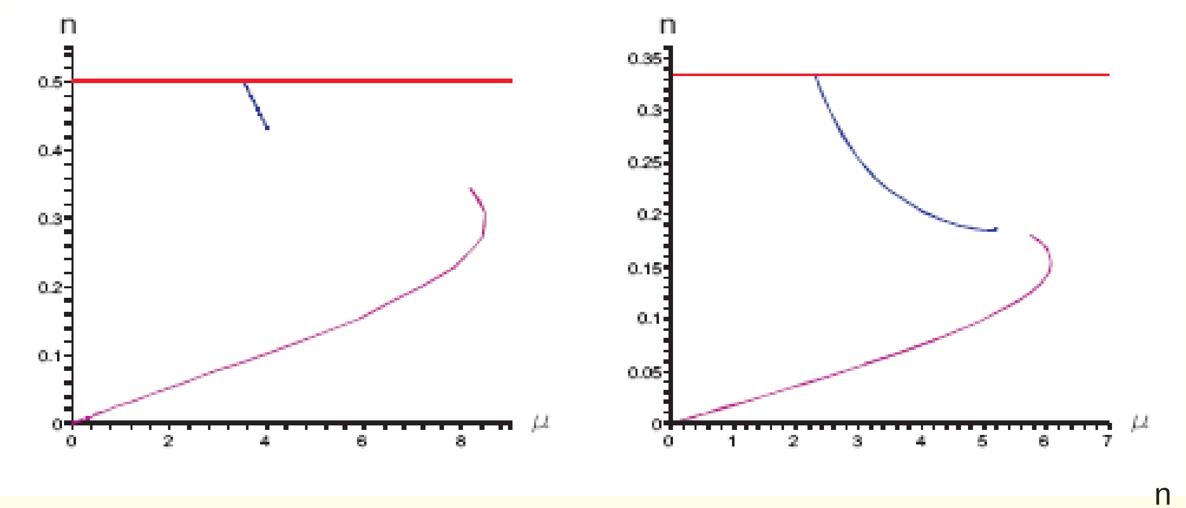
all d : (analytic) first correction to the metric known

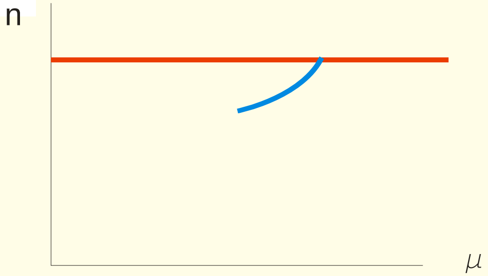
Harmark/Kol,Gorbonos

$$n(\mu) = \frac{(d-2)\zeta(d-2)}{2(d-1)\Omega_{d-1}} \mu + \mathcal{O}(\mu^2)$$

(recently: 2^{nd} order correction for d=4 Karasik et al)

(lower part of) phase diagrams in 5D, 6D + beyond





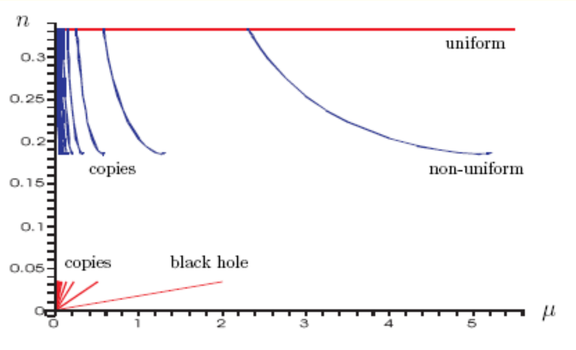
• critical dimension D=14: Sorkin

slope of non-uniform branch reverses

• for solutions that vary in circle direction

Horowitz/Harmark,NO

can generate copies by
copying k times on circle



Comparison of entropies

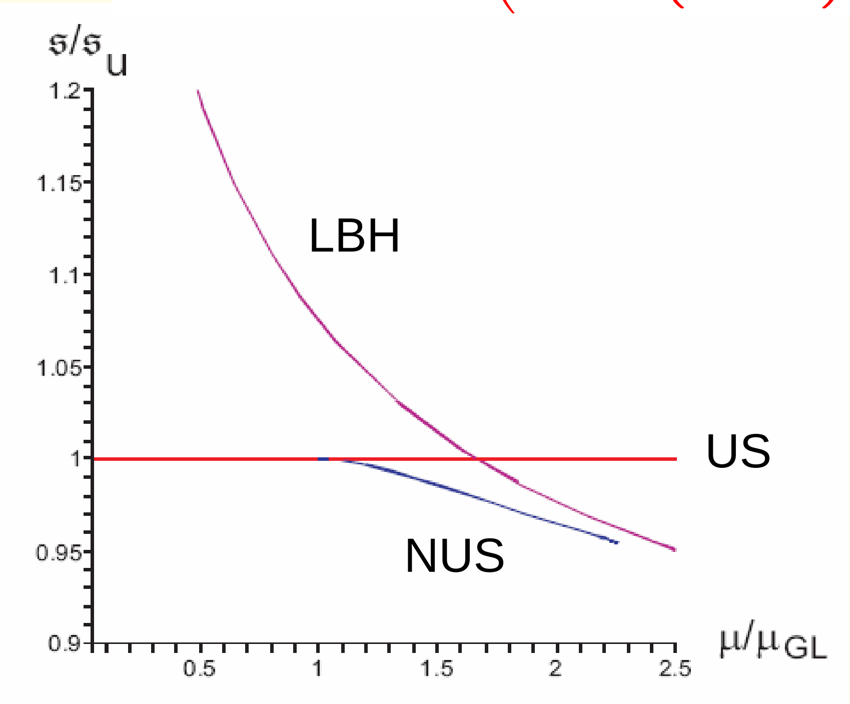
entropy

US $\mathfrak{s}_{\mathrm{u}}(\mu) \propto \mu^{\frac{d-2}{d-3}}$

NUS $\frac{\mathfrak{s}_{\mathrm{nu}}(\mu)}{\mathfrak{s}_{\mathrm{u}}(\mu)} - 1 \propto \frac{\gamma}{\mu_{\mathrm{GL}}}(\mu - \mu_{\mathrm{GL}})^2 + \mathcal{O}((\mu - \mu_{\mathrm{GL}})^3)$

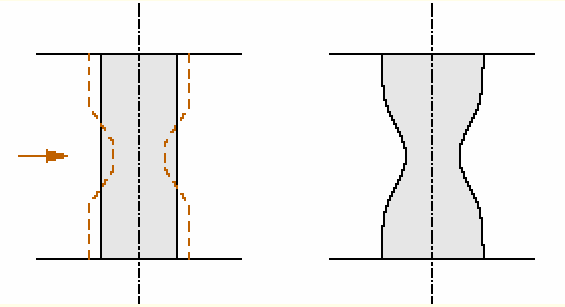
$\gamma \begin{cases} > 0 \ , d < 12 \\ < 0 \ , d \geq 13 \end{cases}$

LBH $\mathfrak{s}_{\mathrm{bh}}(\mu) \propto \mu^{\frac{d-1}{d-2}} \left(1 + \frac{\zeta(d-2)}{2(d-2)\Omega_{d-1}} \mu + \mathcal{O}(\mu^2) \right)$



Endpoint of decay of uniform black string

•no pinching of horizon ? Horowitz,Maeda
(in finite affine affine parameter on event horizon)



GL instability

•numerical study of classical evolution indicates
pinches in infinite affine param.

Choptuik et al/Garfinkle,Lehner,Pretorius

Gregory,Laflamme

• corresponds to finite advanced time

Marolf

• presumably decays to localized black hole
(violation of Cosmic Censorship)

•different behavior for $d \geq 13$
(non-uniform strings with higher entropy than
uniform string exist)

Large dimension behavior

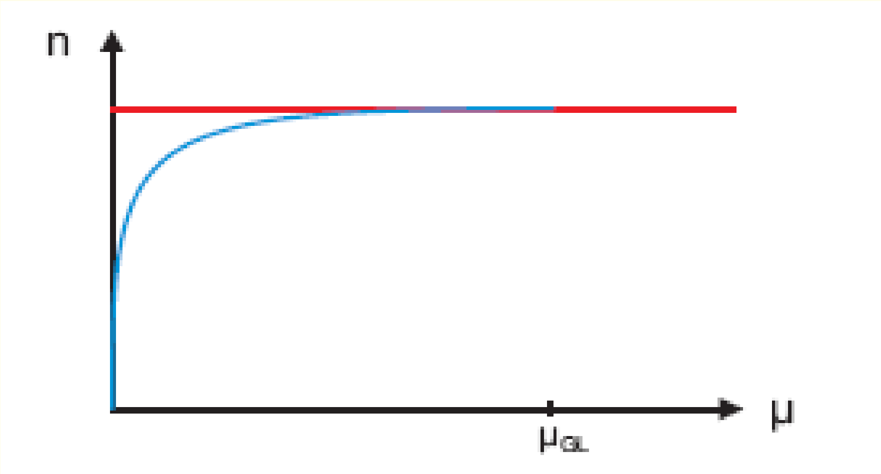
- large d behavior of GL mode and mass is known (numerically + analytically)

Sorkin/Kol,Sorkin

$$\log \mu_{\mathrm{GL}} \simeq d \log \sqrt{\frac{e}{2\pi}}$$

- slope of localized BH branch becomes infinitely steep

$$\frac{n}{n_{\mathrm{GL}}} \simeq d^{d/2} \frac{\mu}{\mu_{\mathrm{GL}}}$$



Upper part of phase diagram

black holes/strings in region $0 \leq n \leq \frac{1}{d-2}$

what about region ? $\frac{1}{d-2} \leq n \leq d-2$

⇨ contains solutions with Kaluza-Klein bubbles Witten

↘ minimal S^2 surfaces in space-time (``bubble of nothing'')

static Kaluza-Klein bubble

$$ds^2 = -dt^2 + \left(1 - \frac{R^{d-3}}{r^{d-3}}\right) dz^2 + \frac{1}{1 - \frac{R^{d-3}}{r^{d-3}}} dr^2 + r^2 \Omega_{d-2}^2$$

to avoid conical singularity: z periodic with $L = \frac{4\pi R}{d-3}$

⇨ static KK bubble is static solution of pure gravity asymptoting to KK space

specific point in the phase diagram with $(\mu, n) = (\mu_b, d-2)$

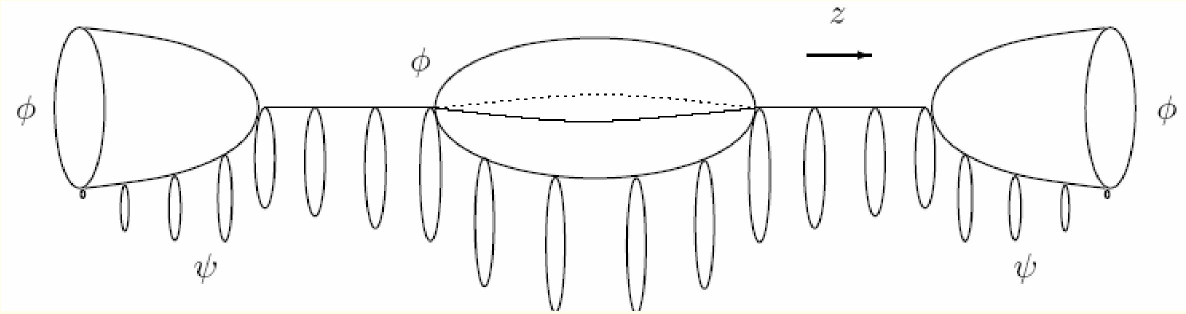
other instance of critical dimension: for D $\leq$ 10 $\mu_b < \mu_{GL}$

Bubble-black hole sequences

Emparan,Reall/Elvang,Horowitz/Elvang,Harmark,NO

- found explicitly using generalized Weyl ansatz in five and six dimensions (with compact circle)
- exact, regular and static solutions of vacuum Einstein equations
- new topologies (black ring $S^2 \times S^1$, black tuboid $S^2 \times S^1 \times S^1$)
- infinite non-uniqueness

black hole – bubble – black hole- bubble – black hole



S^3 $S^1 \times I$ $S^2 \times S^1$ $S^1 \times I$ S^3

⇨ bubbles play double role:

- holding the black holes apart
- support S^1 ‘s on horizon against gravitational collapse

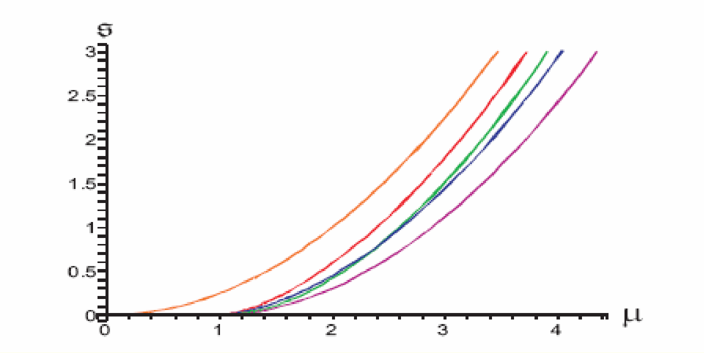
Some features of bubble-black hole sequences

- (p,q) bubble-black hole sequences have q parameters (size of each black hole) $\longrightarrow$ large degree of non-uniqueness

- map between 5D and 6D case

- can associate temperature to each disconnected horizon component
  -special 1-parameter class of solutions exists with all temperatures equal (minimizes entropy)

- entropy of (1,1) solution always less than uniform string of same mass



- stability ?
  -static bubble is unstable (GPY negative mode): expands/collapses

Brill,Horowitz/Corley,Jacobson/Sarbach,Lehner

Open issues

- Numerically compute non-uniform and localized branches for other dimensions

- Find analytic form of these branches ?
  - perturbatively e.g. with 2nd order correction to localized BHs)

- Examine classical stability of the branches

- Generalize bubble-black hole sequences to higher dimensions
  (difficult: generalized Weyl ansatz cannot be used anymore)

- Understand appearance of critical dimensions (cf. also Hovdebo,Myers)
  - more generally: dependence of phase diagram on dimension

- Further examine time evolution of string branch (horizon pinching) and possible violation of Cosmic Censorship Hypothesis

- Generalization to higher compact spaces, e.g. 2-torus
  (cf. lumpy rotating black holes of Emparan,Myers)
  + adding angular momentum

- better understanding of topology changing black hole/string transition
  merger point $\Longleftrightarrow$ Choptuik scaling (Kol)