Hidden Fermi Liquid

The moral: A good low-energy effective theory is worth all of Monte Carlo with Las Vegas thrown in.

Philip Anderson, Princeton

1] Shankar RNG and when it works.

2] The Ansatz

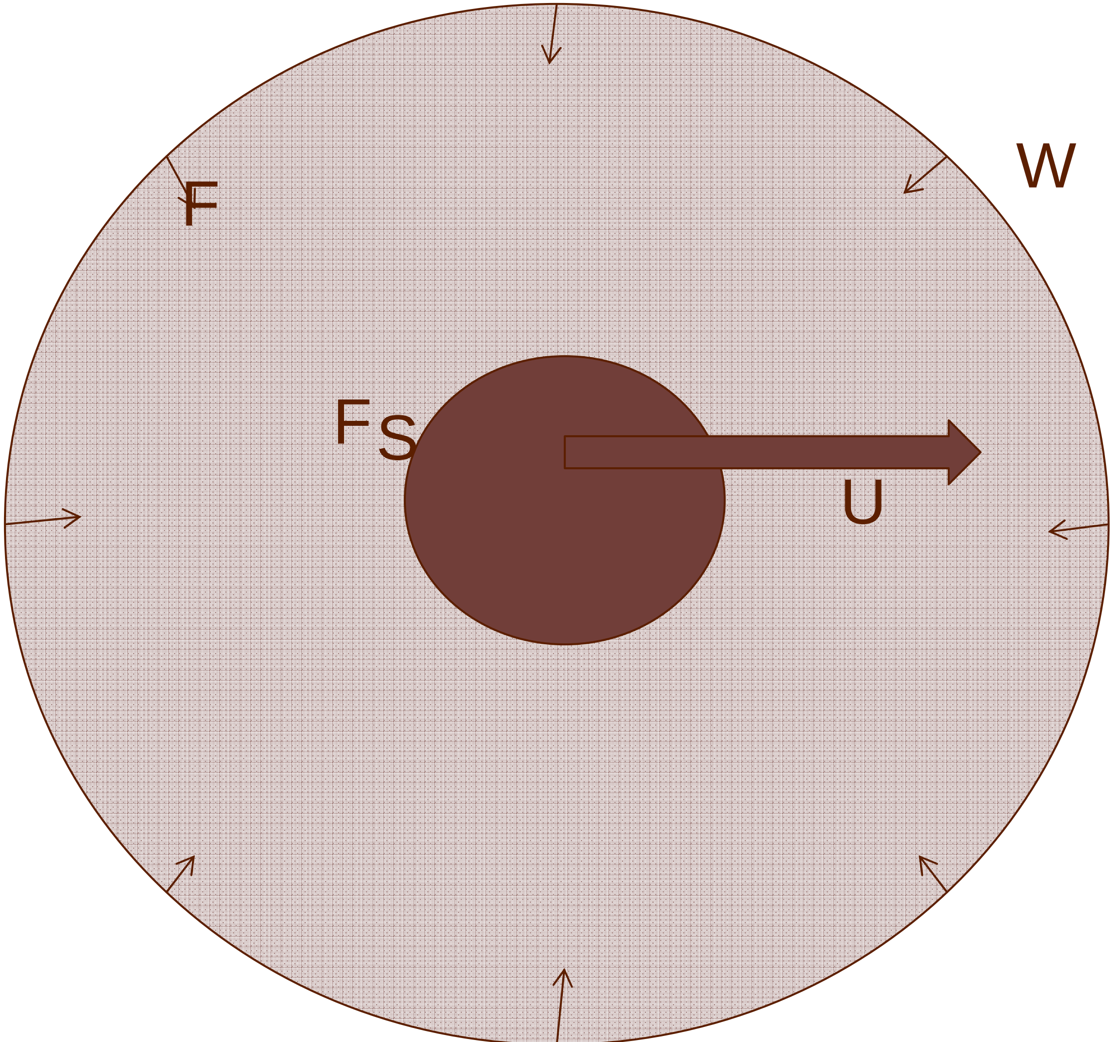
3] The "normal" case: IR power law; ARPES spectra

4] Superconductor: Optimal, coherent tunneling; The Resonance raises its ugly head. Pockets happen!

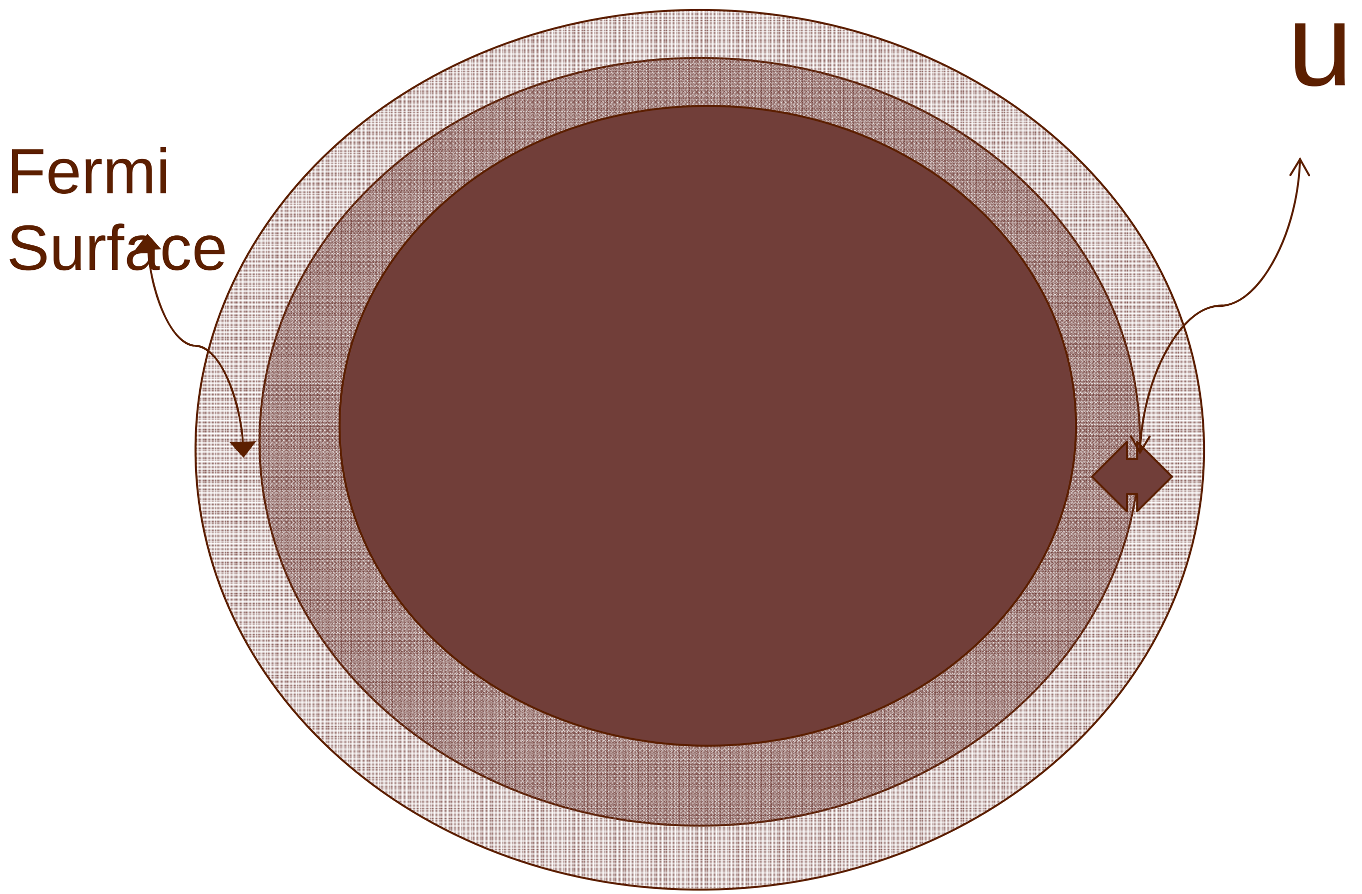
5] Back to "normal" strange metal: Bottleneck, resistivity

KITP Higher Tc Conf, June 23, 2009

Examples of low-energy effective theories: (LEET's): Fermi Liquid theory, Eliashberg theory; these derived by 'poor-man's RNG" of Shankar

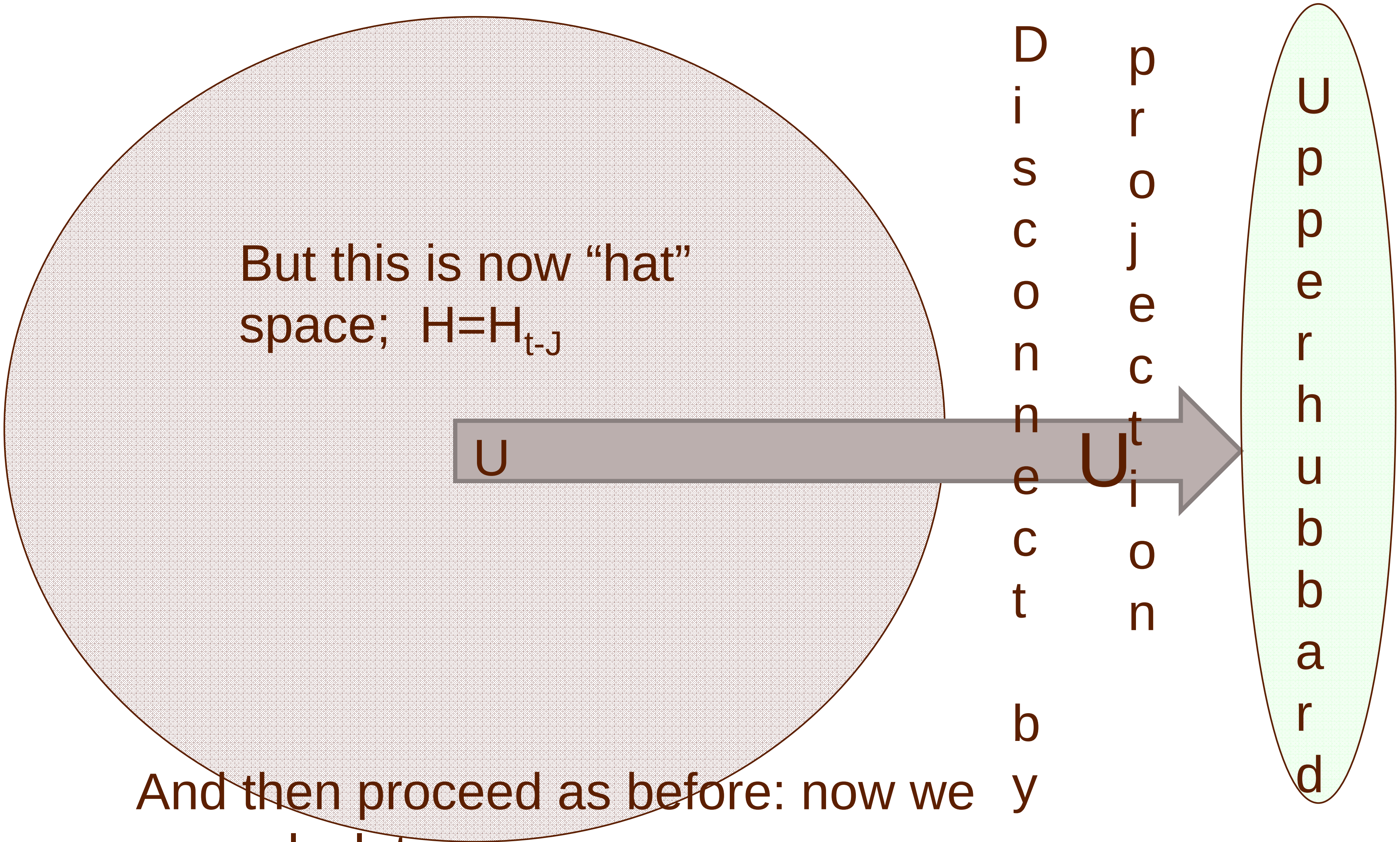


Final stage



And then one can calculate (and understand) things

How does Shankar fail? If U>W



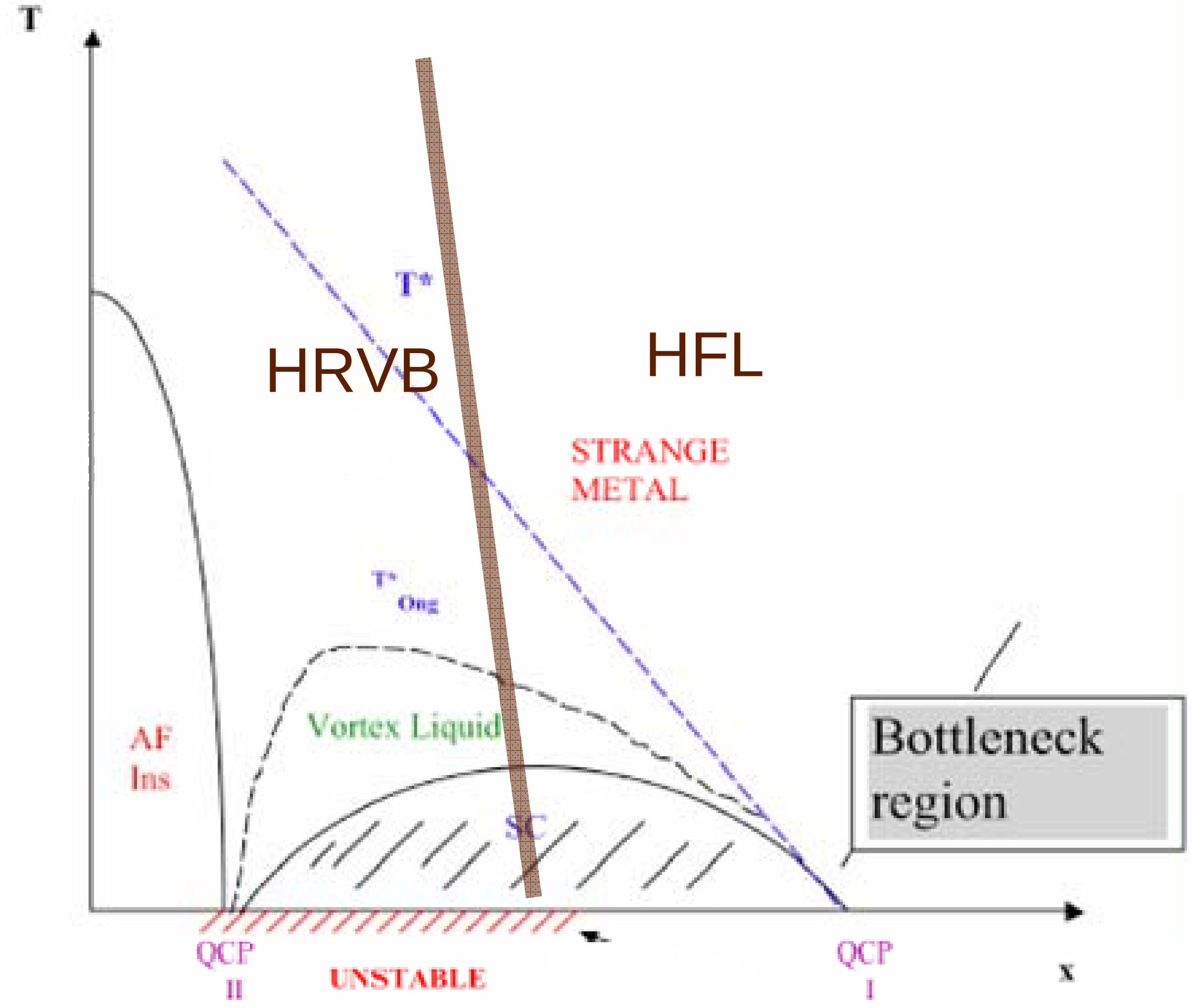
But this is now “hat” space; $H=H_{t-J}$

And then proceed as before: now we can calculate

PA1

Slide 4

PA1 Phil Anderson, 2/28/2009



T
T*
HRVB
HFL
STRANGE
METAL
Bottleneck
region
AF
Ins
Vortex Liquid
SC
QCP
II
UNSTABLE
QCP
I
x

acknowledge: doug scalapino
acp&icam (4 years ago?)
v muthukumar
phil casey
dan dessau and jake koralek
tom timusk
seamus, ali Y
and, long ago, gideon yuval

Doug asked a question: why do quasiparticles work so well (in a sense) for the superconductor?--yet the normal state is not a Fermi liquid?

the answer

because there *is* a Fermi liquid in the problem, undergoing a BCS transition;

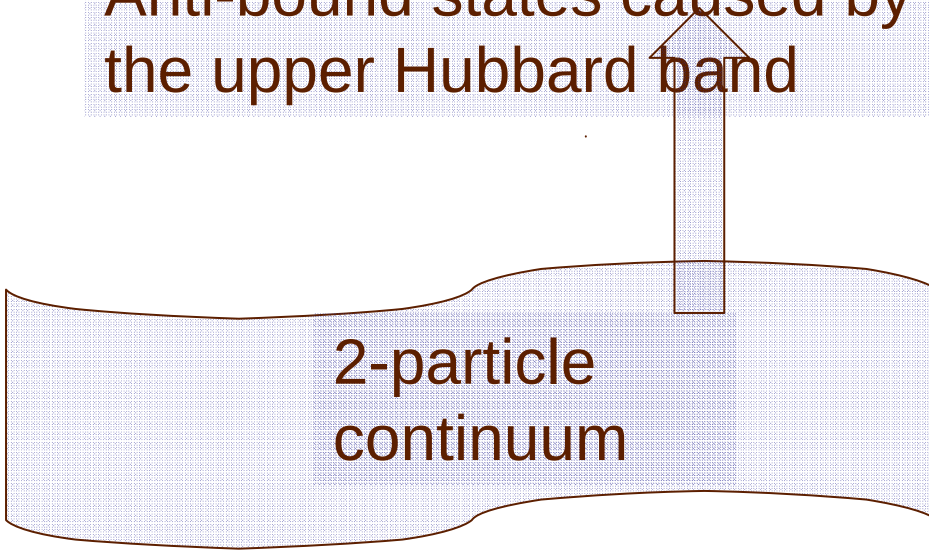
but it's *hidden* because its quasiparticles are not the real physical electrons.

In the "normal" state, the strange metal, the wave-function renormalization connecting the two, Z, is zero;

When the gap opens, Z becomes finite; there is a coherent quasiparticle.

The Problem,again

Anti-bound states caused by U:
the upper Hubbard band



We aren't playing with a full deck!

The solution: projection

Gutzwiller projection is not a mathematical trick, it's a physical fact!

Doing cuprates without G P is like doing QED without renormalizing the electron mass.

IT IS FUNDAMENTAL TO transform to projected Hamiltonian

$$H_{t-J} = e^{iS} H_o e^{-iS} = PtP + J \sum_{i,j} S_i \cdot S_j$$

$$P = \prod_i (1 - n_{i,\uparrow} n_{i,\downarrow})$$

THIS IS NOT OPTIONAL! (EVEN IF PHONONS)

Eigenstates (ground and single-particle excitations) Must be of the form

$\Psi = P\Phi(r_1, r_2 \cdots)$, so we try to find Φ variationally.

We make the obvious Hartree-Fock-BCS Ansatz

$\Phi = \prod_k (u_k + v_k c^*_{k+} c^*_{-k-})\Psi_{vac}$ and determine the coeffi

u and v variationally, thus acquiring a set of GAP EQ

THESE EQUATIONS ARE THOSE FOR THE TRUE SPECTRUM: THEY DETERMINE THERMODYNAMICS, MAGNETIC RESPONSE, ETC

Gutzwiller projected Fermi sea: the ‘hidden’ FL

“Ansatz”: The ***unprojected*** low-energy states of a strongly correlated (that is, with an UHB) Fermi gas can be chosen to be a Fermi liquid. (If no gap) That is, in equations:

$$P = \prod_i (1 - n_{i\uparrow} n_{i\downarrow}); \quad T = \sum_{ij\sigma} t_{ij} c^*_{i\sigma} c_{j\sigma};$$

$$H = PTP; \quad \Psi = P\Phi$$

$$H\Psi = E\Psi \text{ is the same as } H\Phi = E\Phi$$

$$\Phi, \textit{not } \Psi, \text{ is assumed to have Fermi Liquid properties: the hidden FL.}$$

Another possibility: the hidden RVB

Suppose J>>PTP. J will control the “hidden” structure, an RVB, “Fermi surface” is 4 point nodes which expand into Rice-Zhang “pockets” of Fermi surface upon doping.

Yang, Rice, Zhang ‘06 have made a good start at this theory, all indications are it is right (Zhou, ‘09) for well underdoped cases.

NOT a competitor: two different limits! Crossover is challenging!
Keep Tuned! much is happening, but that is not this talk.

To return to the HFL:

So, if Φ_0 is the ground state of this problem,

$c_{k\sigma}\Phi_0, k < k_F$ *and* $c^*_{k\sigma}\Phi_0, k > k_F$ create eigen - excitations with finite amplitude Z, if k is near a *sharp* Fermi surface, determined by Hartree - Fock equations using projected H.

Why? - -Why not?! Shankar's "poor - man's renormalization" seems to apply - - all Fermi systems renorm to FL in shell around FS.

(they also create pieces in the upper Hubbard band, but these are projected away by the Hamilltonian and don't mix.)

Since the two problems are equivalent--$P=P^2$--these are also excitations of the **real** problem.

but we cannot access them directly via real

one-particle operators because $Pc\neq cP$.

The real one-particle operators are

$$\hat{c}_{i\uparrow}=c_{i\uparrow}(1-n_{i\downarrow}n_{i\uparrow})=c_{i\uparrow}c_{i\downarrow}c^*_{i\downarrow}=\sum_{k,k',k''}c_{k\uparrow}c_{k'\downarrow}c^*_{k''\downarrow}$$

and similarly for $c^*P=\hat{c}^*$

These operators will automatically keep us within the lower Hubbard band, so ”all” we need to do is to evaluate the Green’s function of a three-Fermion operator.

This looks like a hopeless mess but it isn't. Because of the strong exclusion principle restrictions on momentum, and to make the creation of real pseudoparticles energetically possible, all have to be near the Fermi surface and travelling in the same direction. Two factorizations are important:

$$\hat{c}_{k\uparrow} \approx \sum_q (c_{k-q\uparrow}\rho_{q\downarrow} + c_{k-q\downarrow}S_q^+) \quad [*]$$

ρ and S^+ are density and spin Tomonaga waves moving in the direction of the Fermi velocity v_F of k. Haldane has shown that these bosons are a valid alternative representation of a Fermi liquid.

Green's functions of the HFL

To get spectra we have to calculate Green's functions of the "hat" operators, for tunnelling

$$G(i,t) = \langle \hat{c}^*_{i\sigma}(t)\hat{c}_{i\sigma}(0)\rangle \text{ and for ARPES}$$

$$G([r_i - r_j],t) = \langle \hat{c}^*_{\sigma}(r_i,t)\hat{c}_{\sigma}(r_j,0)\rangle$$

(+ the irrelevant part that goes into the upper Hubbard band)

The averages denoted by <> are ground state at T=0, or thermal at finite T. These are surprisingly easy because they factorize, using [*] and Fermi liquid rules (spins independent of each other), into

$G_{free}G^*(t)$

The effect on tunneling spectra was evaluated in Nature Phys 2, 626. At absolute zero G*(t) is the x-ray line problem of Doniach-Sunjic, and is t^{-p}, with $p=2(1-x)^2/8$--the 2 for the 2 channels in [*]. The final result is a power-law Fermi surface singularity:

$$dI/dV \propto \omega^p \quad \text{at finite T} \propto \mathrm{Re}(\mathrm{AT} - \mathrm{i}\omega)^\mathrm{p}$$

Where does this power law come from? Friedel's theorem, basically: in order to change the number of electrons locally, you have to shift the phase of the whole electron gas and, eventually, push electrons out through the boundary. Via the "orthogonality catastrophe" this causes power law corrections to wave function overlaps. (Nozieres-de Dominicis, 1969).

To get EDC's (I e Green's functions) we rely on Huygens' principle:

HuygensÕ Principle

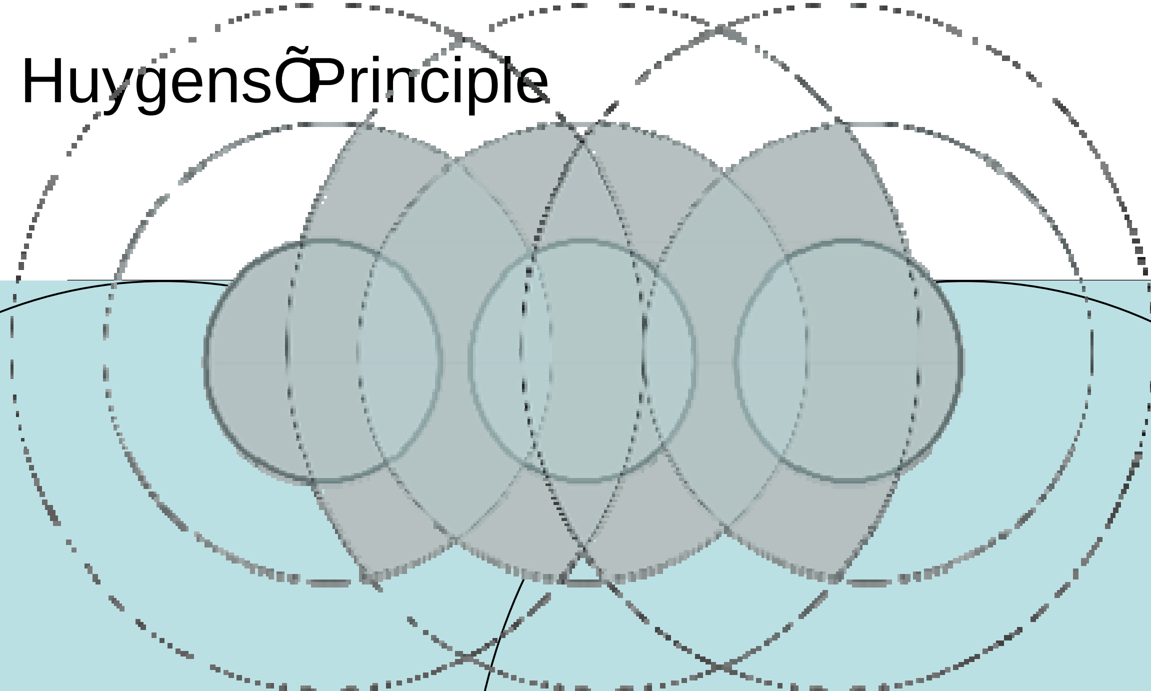


Figure 3. Illustrating Huygens' Principle, that a Plane Wave may be Constructed as a Superposition of Spherical Wavelets

That is, $G^*(t)$ is common to all, so the Green's function in r,t space is $G_0(r-v_F t)G^*(t)$.

To calculate the IR conductivity we use the simple bubble diagram with no vertex correction, and take $\omega \gg T$--both valid approximations. Since early work of Schlesinger and Collins it has been known that σ is a power law:

$$\sigma(\omega) \propto (i\omega)^{-1+2p}$$

(in the Nature Phys ref the exponent is wrong --stupid mistake by me.

Many measurements since 1989--Timusk latest

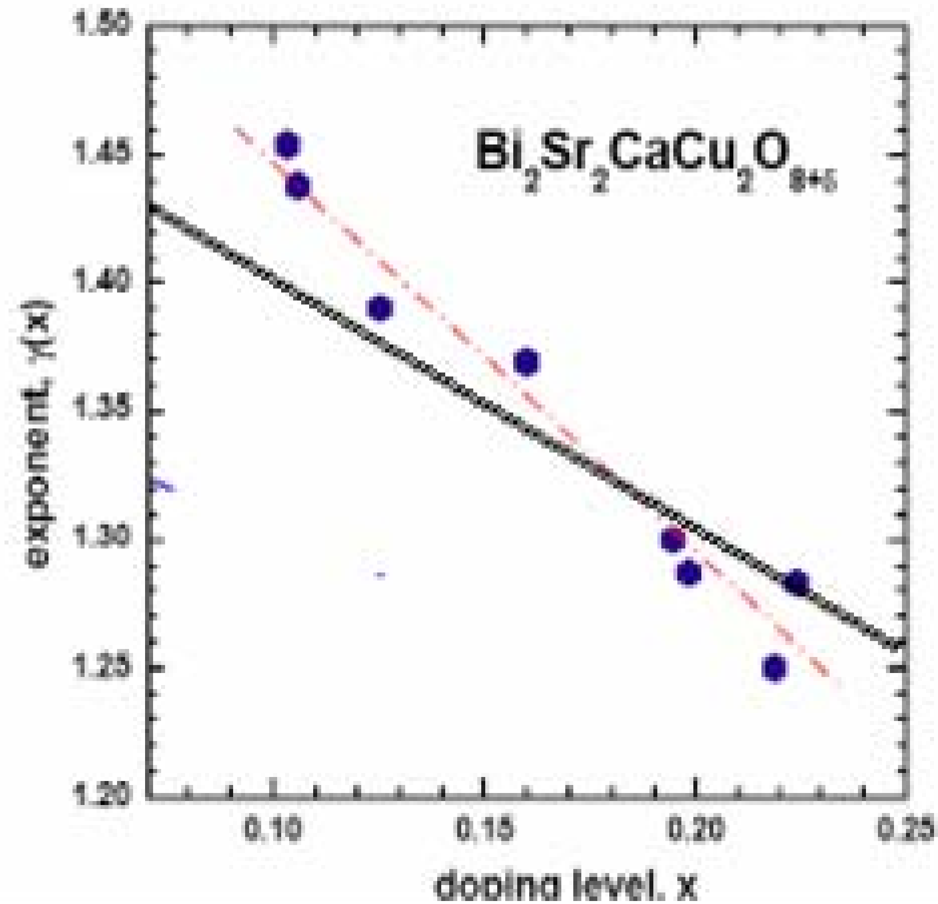
PA2

MFL=1+ε, ε<<1

DOES NOT FIT

P may be protected?

van der Marel: $\sigma(\omega) = C'(-i\omega)^{\gamma-2}$



PA2 power law may be protected!

Phil Anderson, 2/28/2009

finite temperature--a kluge

At finite T, Yuval observed that the integral becomes periodic in imaginary time with period 2π/T

$$t^{-p} \Rightarrow [\sinh(\pi T t)/T]^{-p}$$

This we approximate as

$G^*(t) \propto t^{-p} e^{-\Gamma t}$ Here we take $\Gamma = \mathrm{AT} + \mathrm{C}(\mathrm{k} - \mathrm{k_F})^2$

A is close to unity (pπ is a guess) but C is arbitrary --

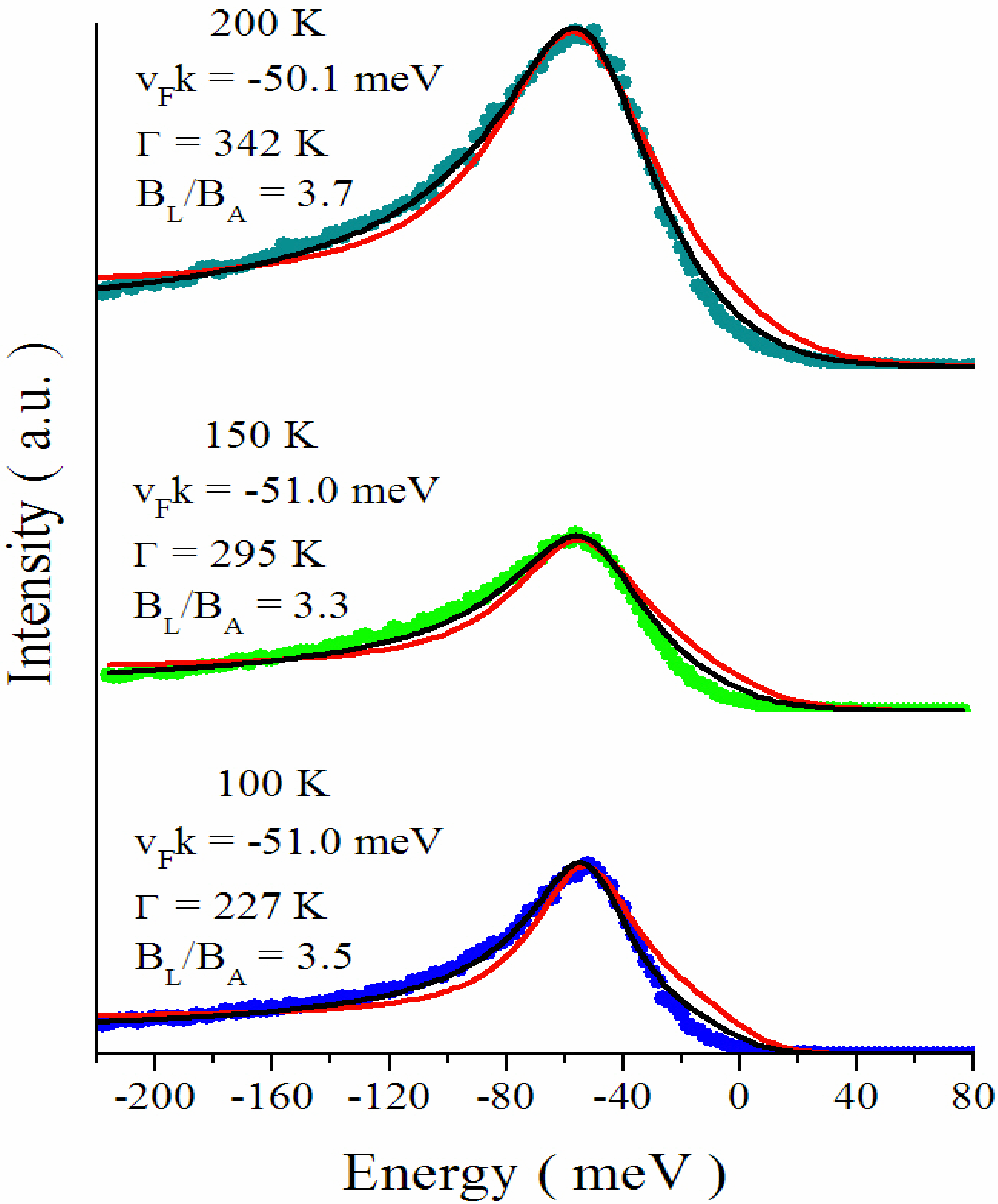
a way of adding in the umklapp scattering rate in the HFL

only 1 arbitrary fitting parameter!! --C--

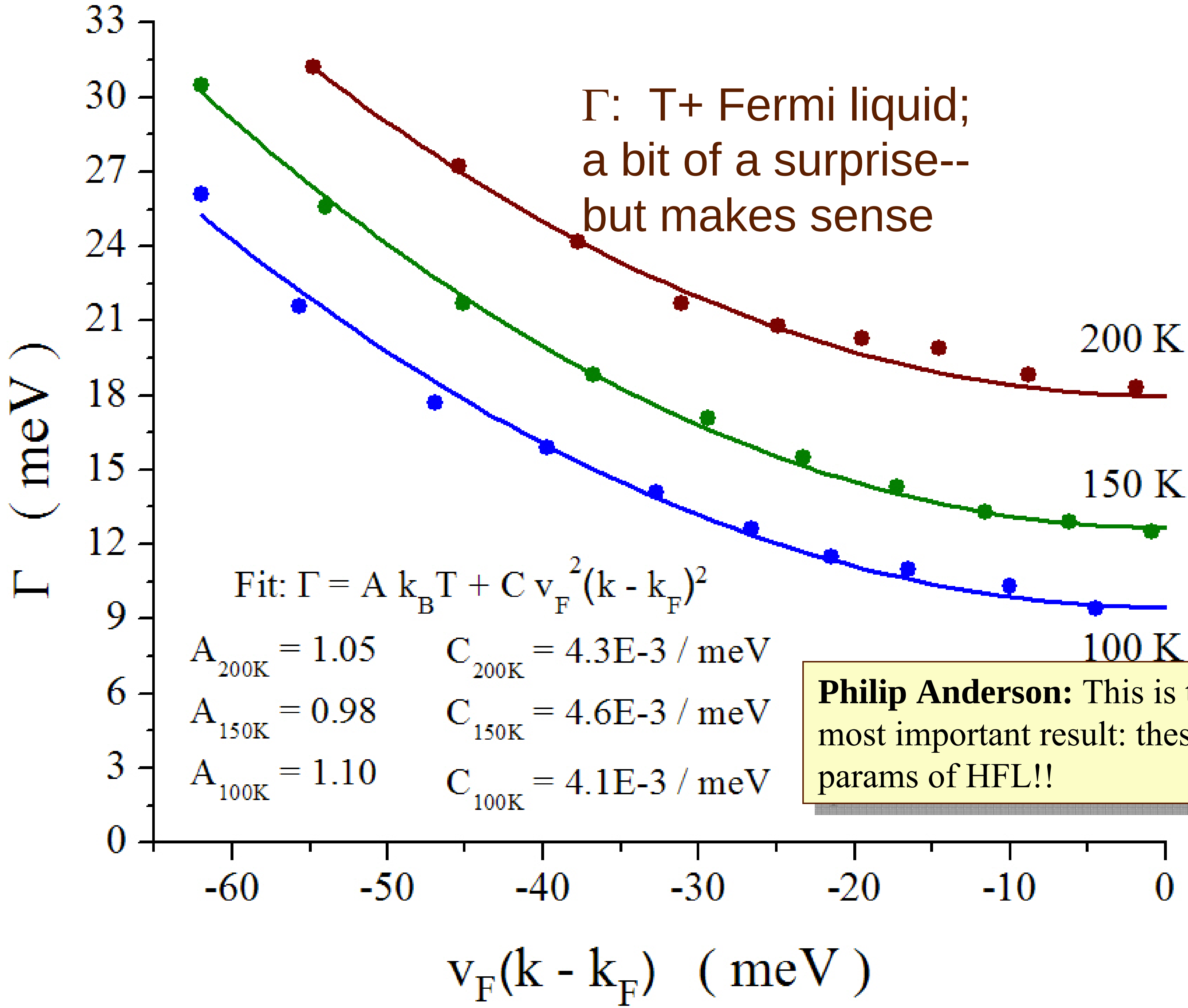
It is now easy to Fourier transform to get a “Doniach-Sunjic” line shape for the EDC. Fitting to Dessau and Korelek’s experiments (an example next slide) Casey could get the parameter values in the following slide.

(the $[k-k_F]^2$ dependence came from this fit, and was a pleasant surprise). In the fits red= Lorentz + arbitrary background, black =Casey-PWA, points =laser ARPES by Dessau et al.

C can be used to estimate T^2 relaxation of the HFL: it agrees well with the I/T^2 Hall effect relaxation time (a long-standing puzzle)



200 K
v_F k = -50.1 meV
Γ = 342 K
B_L/B_A = 3.7
150 K
v_F k = -51.0 meV
Γ = 295 K
B_L/B_A = 3.3
100 K
v_F k = -51.0 meV
Γ = 227 K
B_L/B_A = 3.5
Intensity (a.u.)
-200
-160
-120
-80
-40
0
40
80
Energy (meV)

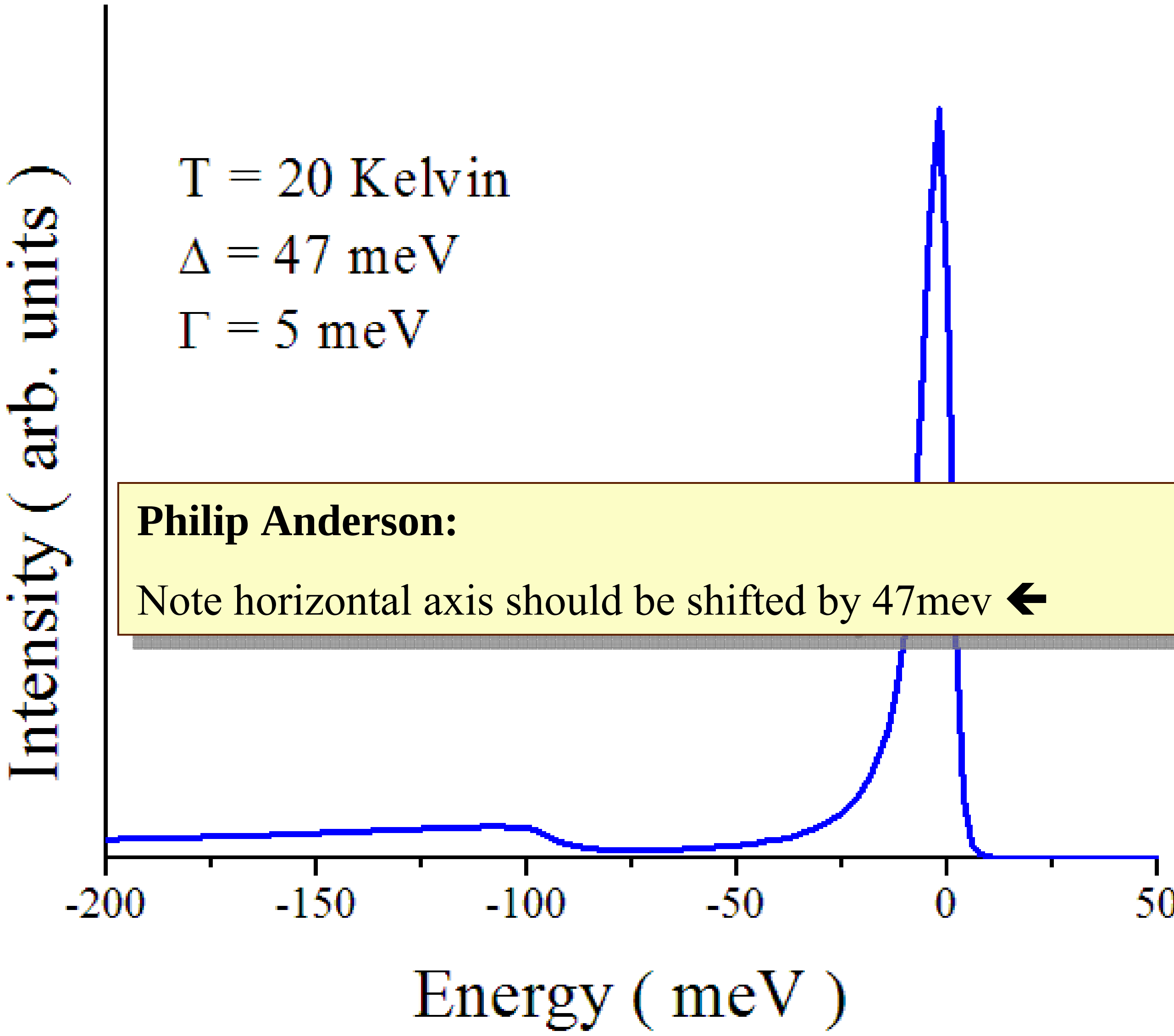


Philip Anderson: This is the most important result: these are params of HFL!!

what about superconductor?

the reason for the power-law decay of G*(t) is the infrared catastrophe. But with a gap, IR catastrophe goes away. To calculate line-shape,we need to do Doniach-Sunjic in a superconductor. Fortunately, there is a crib: Yanjun Ma, P R 1985. (Prola says citations=0!!) Phil Casey calculated a typical EDC and it looks like the slide:

Photoemission Spectrum for SC state



Philip Anderson:

Note horizontal axis should be shifted by 47mev ←

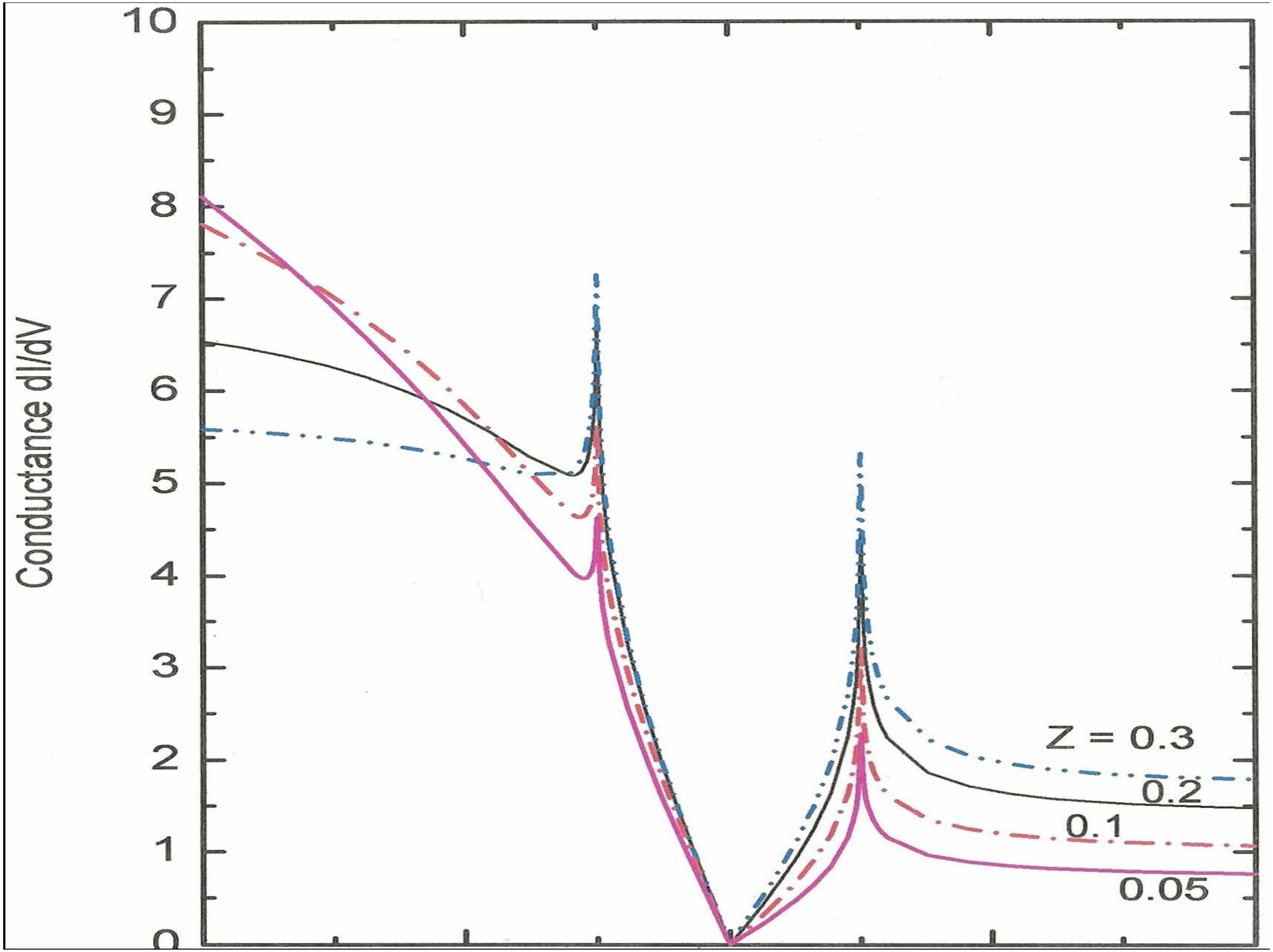
Ref: Y. Ma, *PRB* **32**, 1472 (1985).

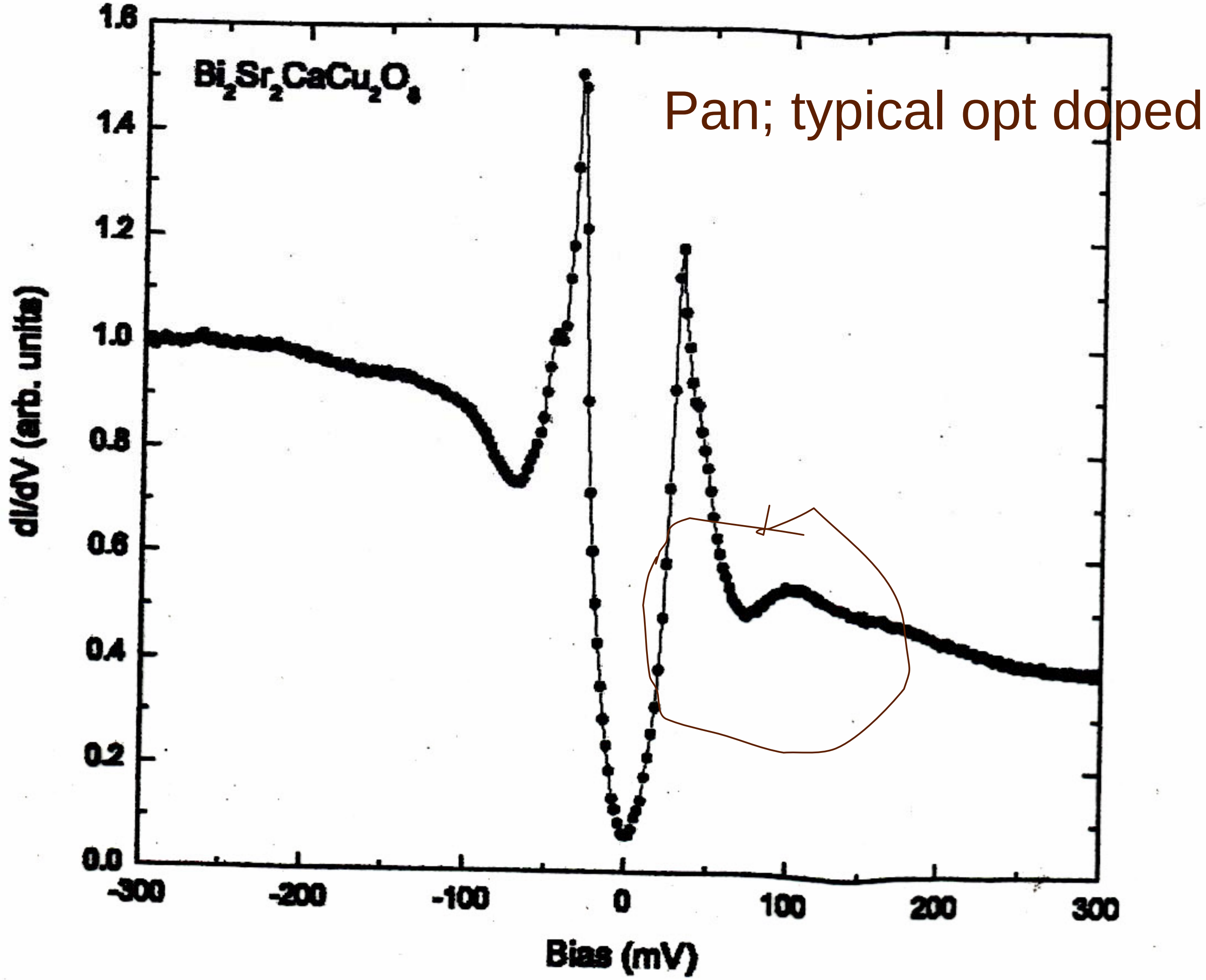
Near optimal doping, therefore, it is fairly accurate to calculate using only the coherent spectrum (PWA and Ong, 2004) and get a good simulacrum of tunnelling spectrum **complete with universal asymmetry!**

WITHOUT PROJECTION CAN"T EXPLAIN ASYMMETRY IN POINT CONTACT TUNNELLING

(WANNIER'S THEOREM)

MEAN FIELD CALCULATION OF SPECTRUM





$Bi_2Sr_2CaCu_2O_8$
dI/dV (arb. units)
Bias (mV)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-300
-200
-100
0
100
200
300
Pan; typical opt doped

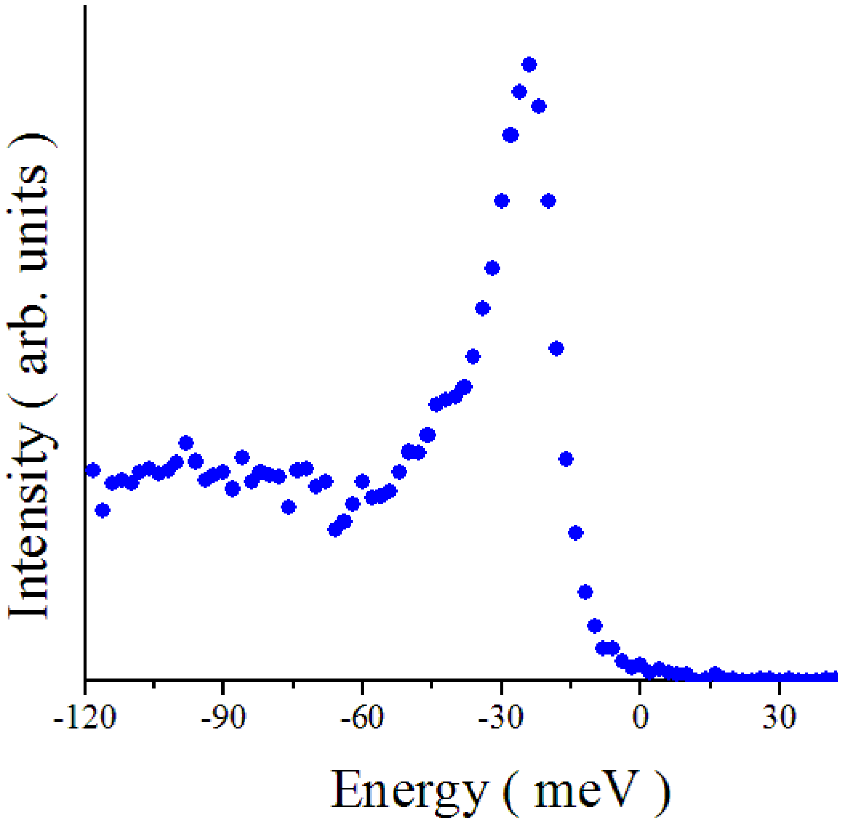
Dessau’s typical results --shown on slide--are more like our prediction than previous attempts, but still not very good: a] in the real data, the peaks are ragged rather than broadened--this must be gap inhomogeneity, as emphasized by Yazdani.
b]There’s much too big a background, at too low energy.
What is new?
D-wave superconductivity greatly enhances the spin susceptibility and lowers the energy of spin fluctuations in the general region of (π,π) (because of coherence factors). The system is starting to see the AF instability. (note that d-wave and AF help each other, not compete)
Again, the trick is to factorize the “hat” operator and thence the Green’s function

$$\hat{c}_{i\sigma} = c_{i\sigma} c_{i-\sigma} c^*{}_{i-\sigma} = c_{i-\sigma} S_i^- \text{ or } = \mathrm{c}_{i\sigma}(1 - n_{i-\sigma})$$

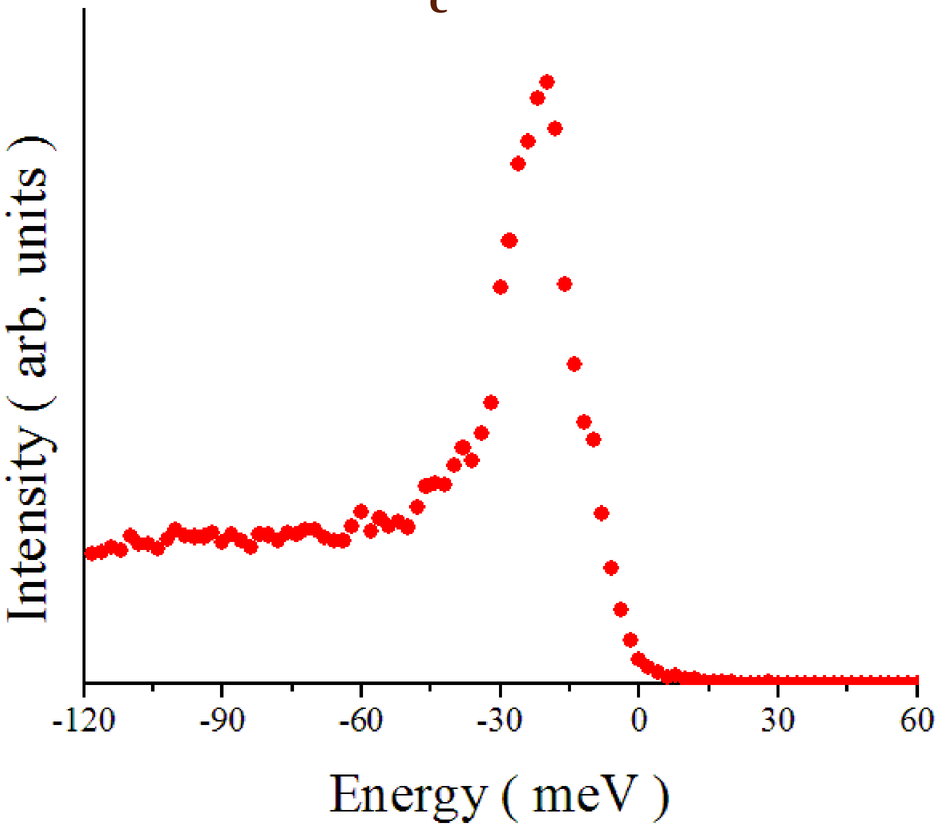
Laser-ARPES off-nodal EDCs, T = 20 Kelvin

J.D. Koralek & D.S. Dessau, *et al.*

OP Bi2212



OD Bi2212
T_c = 65 K



$$G(0,t) = \left\langle 0 \mid \hat{c}_{i,\uparrow}{}^*(t) c_{i,\uparrow}(0) \mid 0 \right\rangle$$

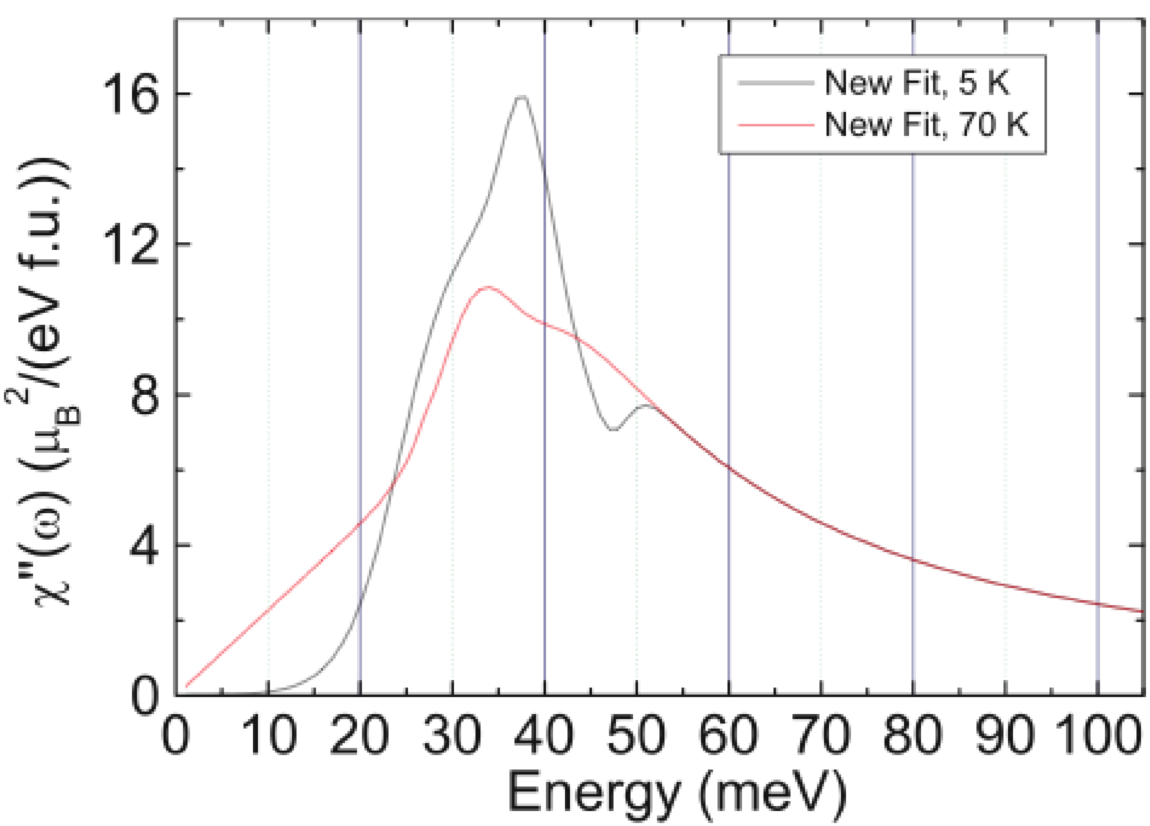
$$= G_{coherent} + G_{inc,\, density} + G_0(0,t) \left\langle 0 \mid S_i^+(t) S_i^-(0) \mid 0 \right\rangle$$

The last term is the contribution from the resonance.

Since Go is sharply peaked in energy, the shape of the background is that of the susceptibility--see slides (work of Phil Casey) and the ‘hump” in optimally doped BISCO

$$\int dt e^{i\omega t} \left\langle 0 \mid S_i^+(t) S_i^-(0) \mid 0 \right\rangle = \chi_i''(\omega) = N^{-1} \sum_k \chi''(k,\omega)$$

ARPES is a much harder problem--but clearly, as observed, there will be a big increase in background for states which can scatter at (π,π). But--crossover to HRVB?


New Fit, 5 K
New Fit, 70 K
χ''(ω) (μB²/(eV f.u.))
Energy (meV)
0
4
8
12
16
0
10
20
30
40
50
60
70
80
90
100

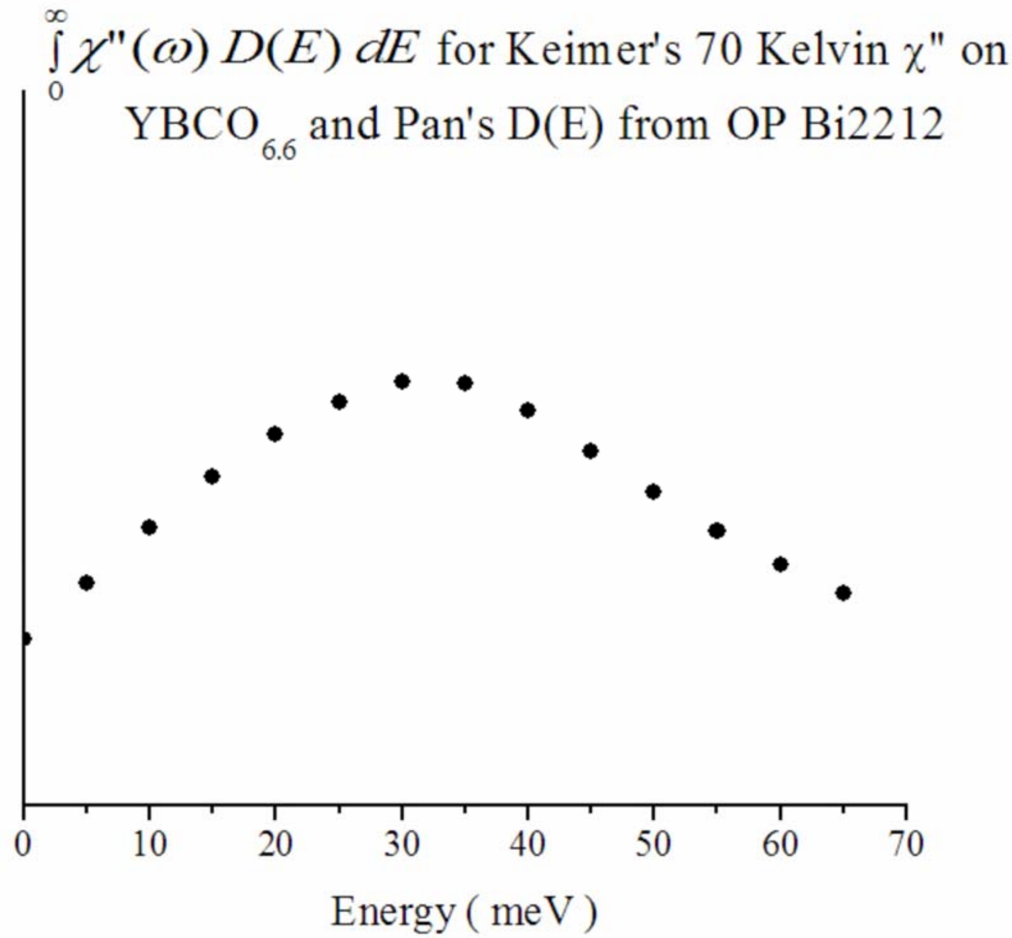


Figure 1. "Hump" in tunneling spectrum estimated from Hinkov data. The energy is measured from the coherence peak which should be imagined to be added in around E=0 with a total area about twice that of the hump.

Resistivity in the strange metal: the bottleneck effect

There are two *different* dissipative processes for accelerated electrons. One may be thought of as the decay of quasiparticles--which are what the electric field sees--into pseudoparticles, which are the true excitation spectrum. The second is the scattering rate of the pseudoparticles, which are a simple Fermi liquid with T^2 dependence. These two processes act in series to dissipate the momentum to the lattice. This means that the slowest one controls the rate of dissipation, not the fastest. That is, it's an anti-Matthiessen's rule: the conductivities add, not the resistivities! This is the BOTTLENECK EFFECT.

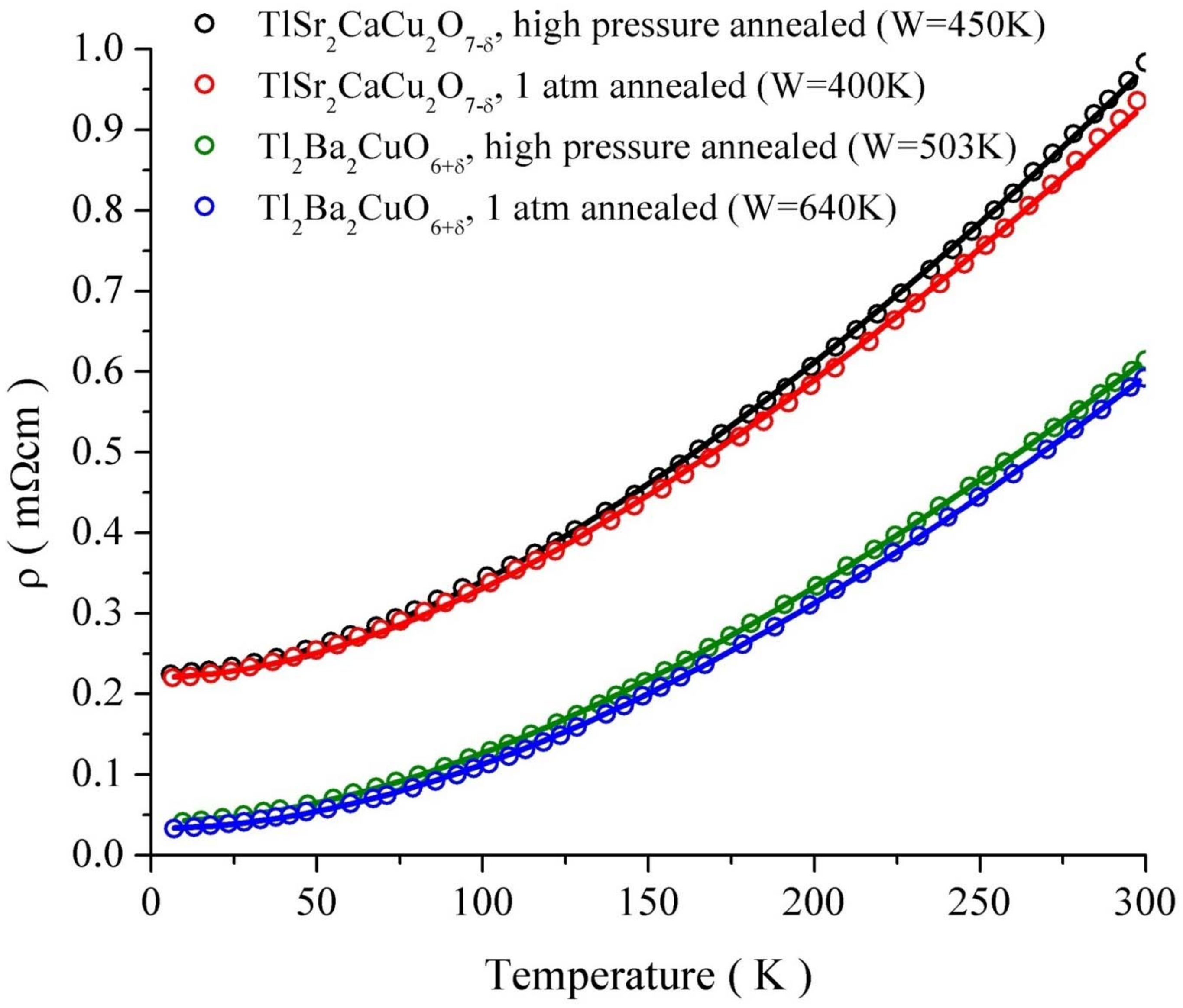
$\rho = \mathrm{Const} \times (1/T + T_0/T^2)^{-1}$

$= T^2/(T+T_0)$

This magic formula fits lots of early data—like a glove!

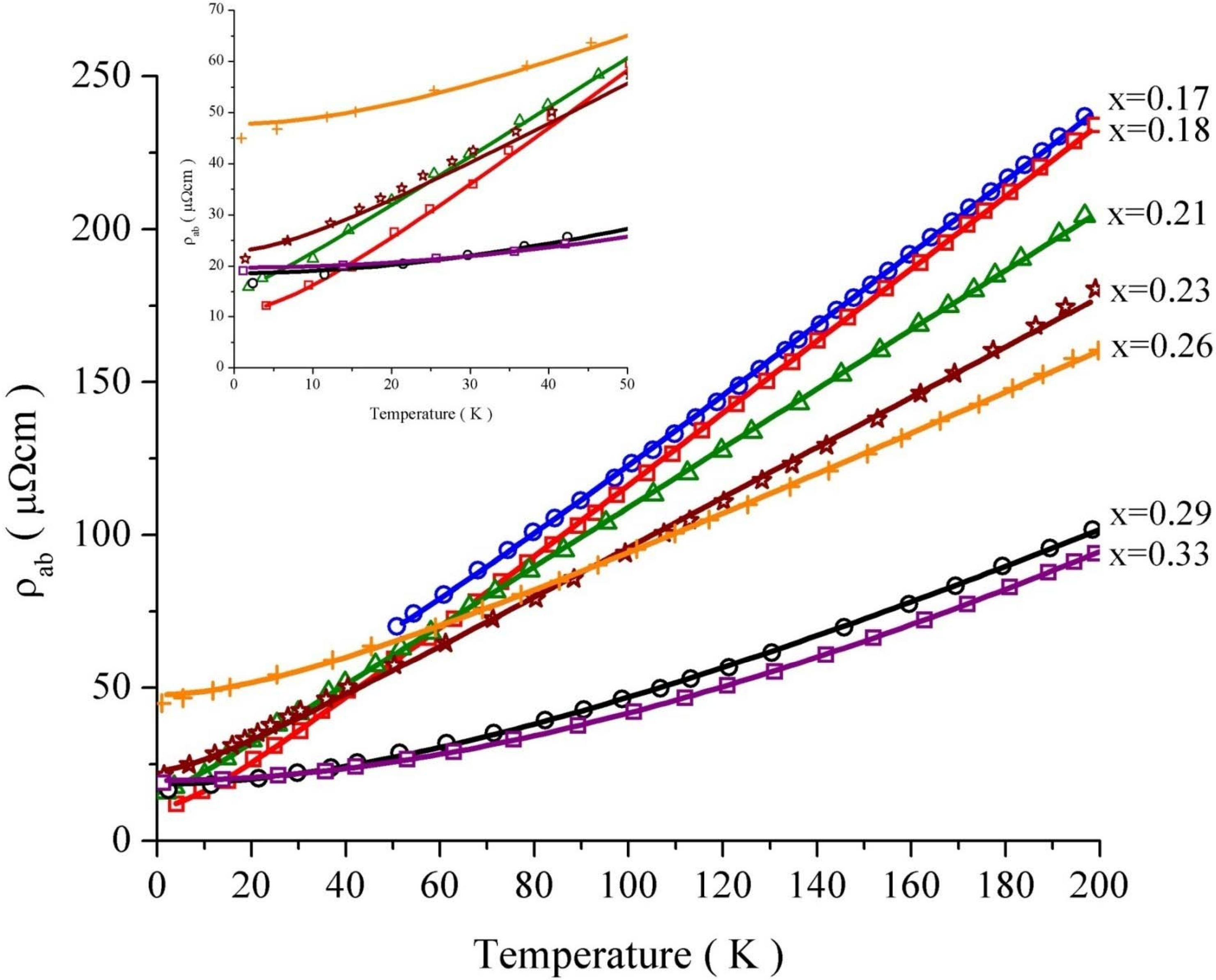
Note it is T^2 at low T, linear with negative intercept at high—puzzling characteristics from the beginning.

Strange Metal ρ(T) comparison to polycrystalline Tl- cuprates



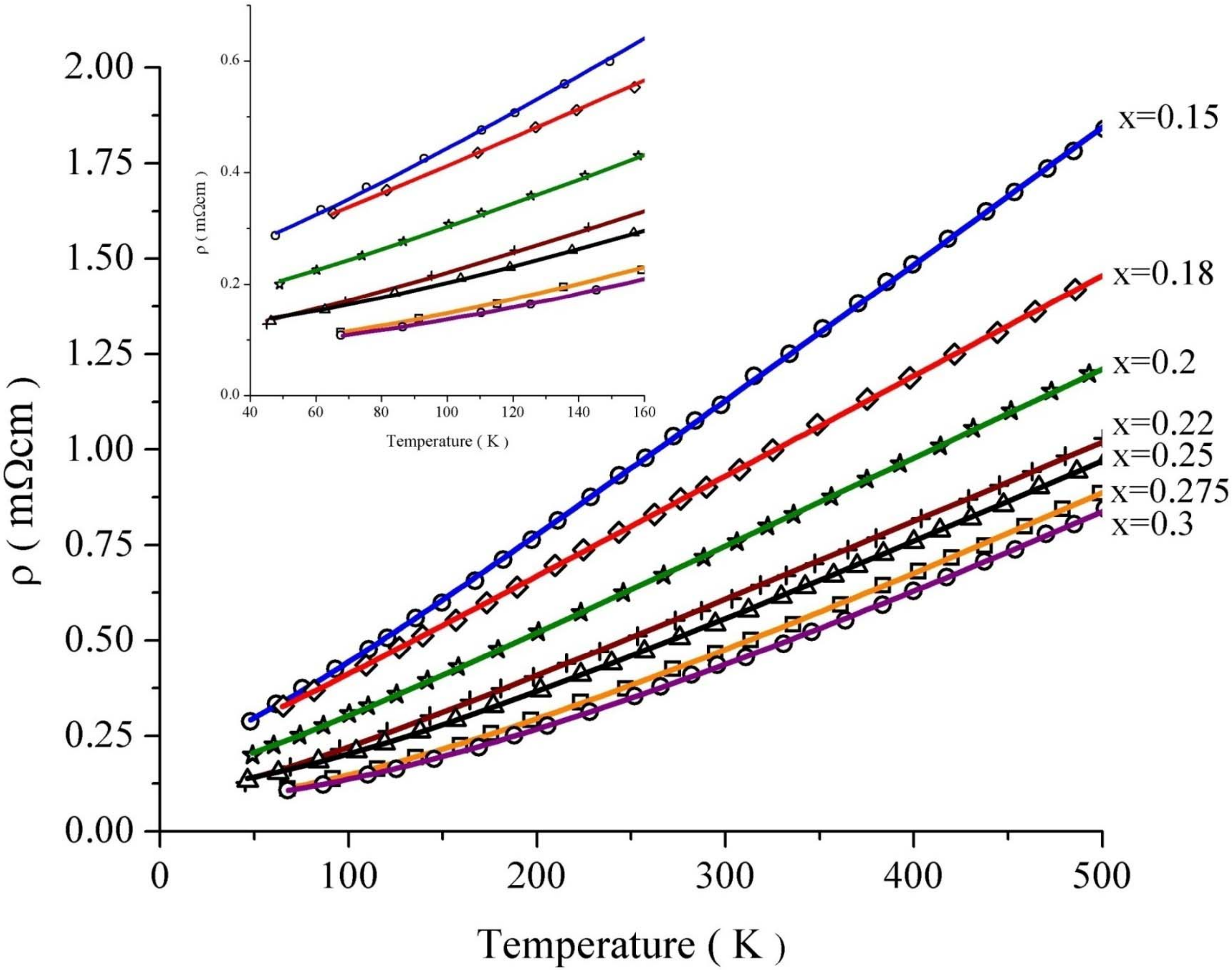
Data from: Y. Kubo *et al.*, PRB **50** 21 16033 (1994).

Strange Metal ρ(T) comparison to single-crystal $La_{2-x}Sr_xCuO_4$



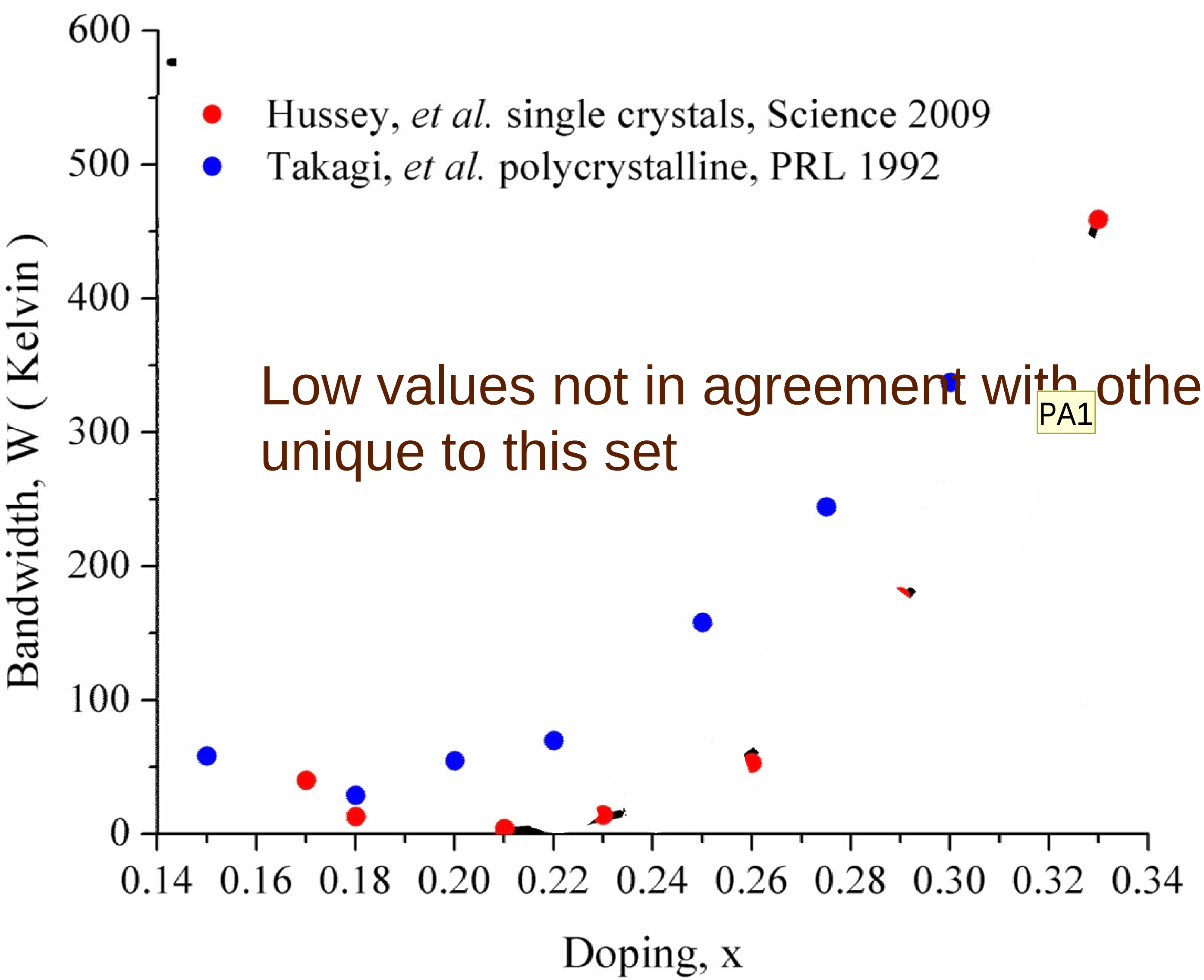
Data from: N.E. Hussey, *et al. Science* **323** 603-607 (2009).

Strange Metal ρ(T) comparison to polycrystalline $La_{2-x}Sr_xCuO_4$



Data from: H. Takagi, *et al. PRL* **69** 2975-2978 (1992).

Measure of the Hidden Fermi Liquid contribution to the Strange Metal Resistivity



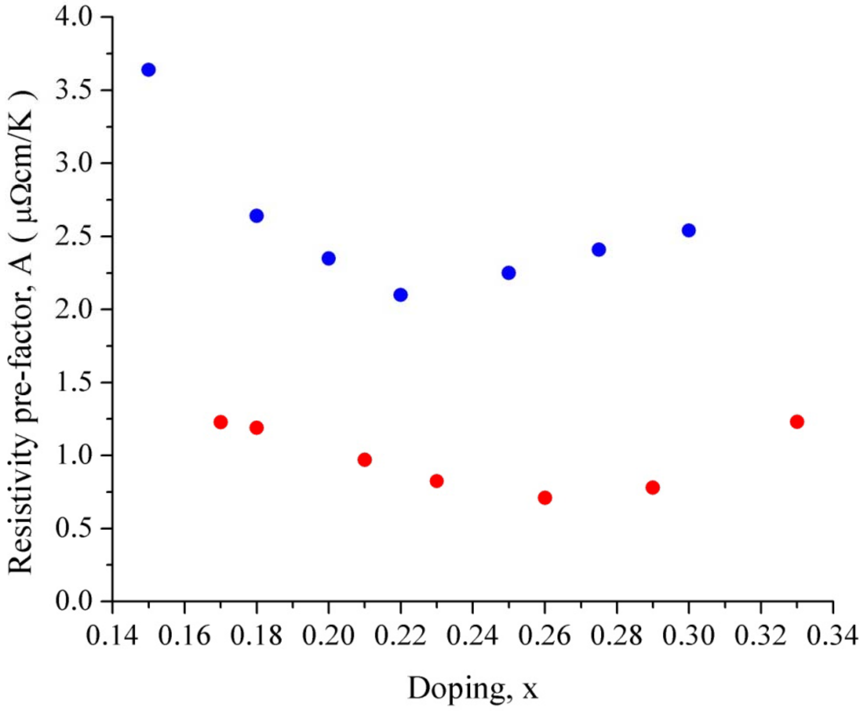
Low values not in agreement with other data—unique to this set

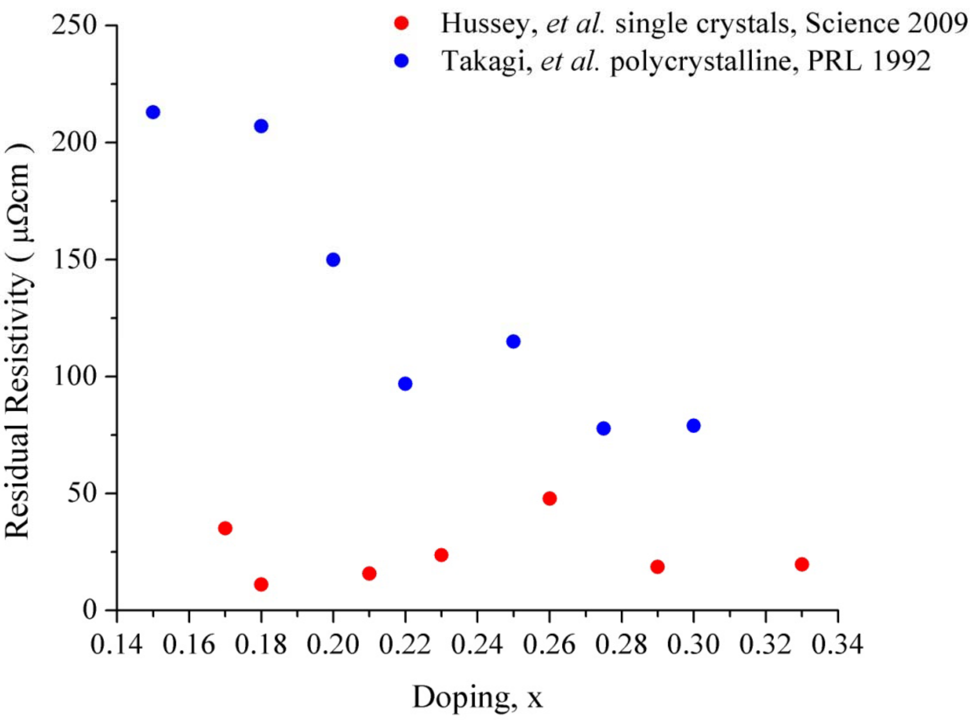
PA1

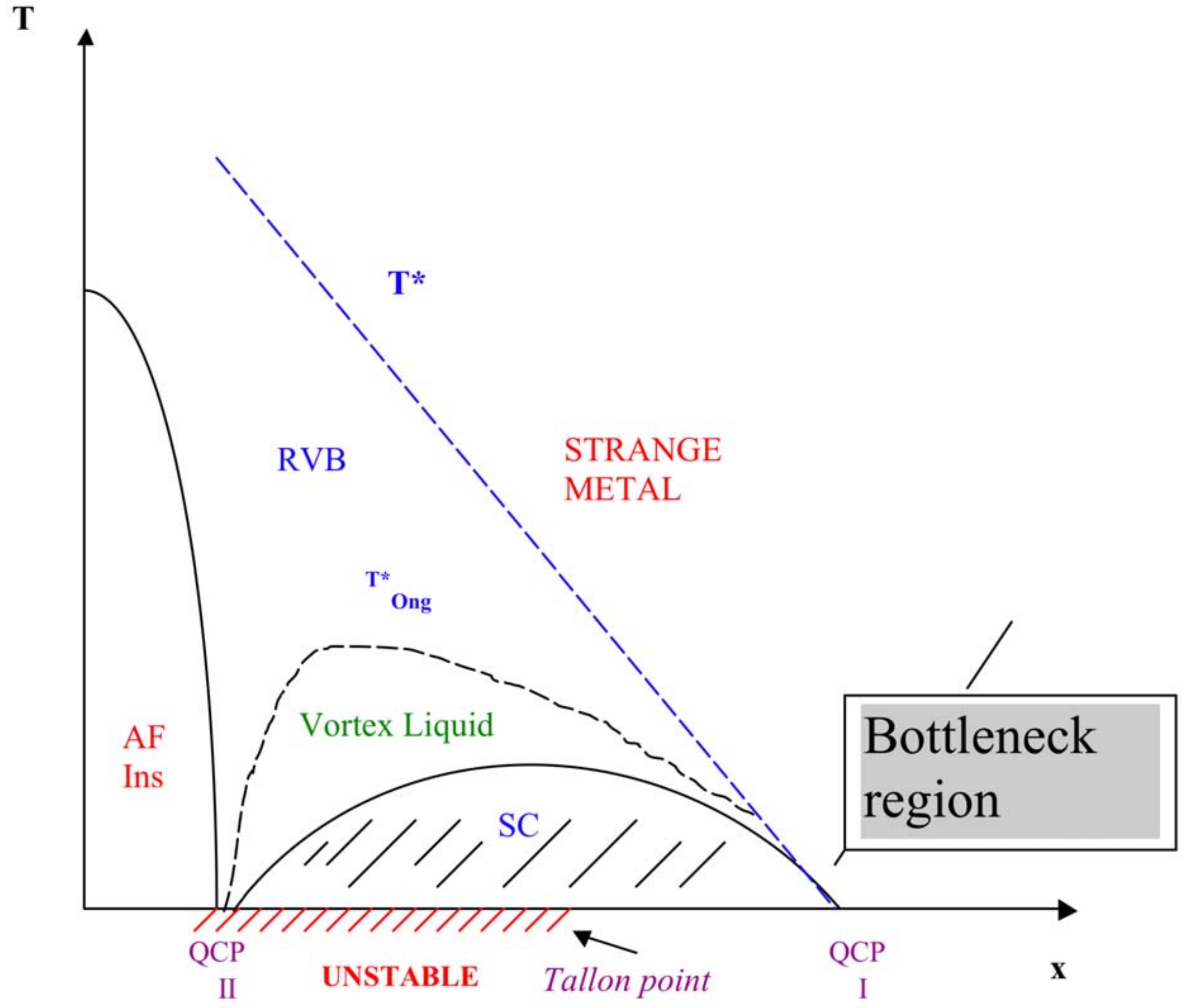
PA1 note low values of W for superconducting samples. further investgation finds confusing, sample-dependent results--in fact, best data fits already-determined T-squared parameters quite well.

Phil Anderson, 6/17/2009

Strange Metal ρ(T) Pre-factor and Residual Resistivity






T
T*
RVB
STRANGE
METAL
T*Ong
Vortex Liquid
AF
Ins
SC
Bottleneck
region
QCP
II
UNSTABLE
Tallon point
QCP
I
x

remarks and conclusions

conventional perturbation theory WON"T WORK: analytic structure is cuts, not poles.
When we go superconducting gapping of Tomonagons allows
real QP's--but tail still not integrable!
Manipulations of diagram theory NOT legit and lead to mistakes(Scalapino)
It appears we now have a systematic, controlled formalism for Gutzwiller projection *which works and is useful---please give it a try!!.*
Same formalism can work for HRVB

important!!!

It also appears laser ARPES is fantastically accurate (recent data from Zhou in China confirms Dessau)