D-Wave “Strange Metals” for 2d Bosons and Electrons

MPA Fisher (with Lesik Motrunich)

KITP Higher Tc Conference, 6/25/09

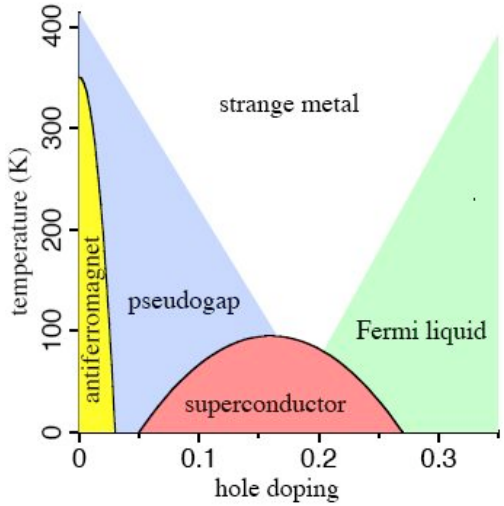
Interest: Non-Fermi liquid phases of itinerant 2d electrons

Long term wish: Construct a theoretical description of the “strange metal phase” in the cuprates

Strange Metal holds the Key

Premise:

- *The* theory of high Tc must **begin** with the Strange metal.
  (low energy physics emerges from high energy, not vice versa)
- Strange metal is a true (T=0) Non-Fermi liquid quantum phase or quantum phase transition
  (not just an "incoherent" finite T crossover)
- Pseudogap and d-wave SC should be understood as "instabilities" of the strange metal
  (akin to low Tc BCS sc emerging from Fermi liquid)
- Symmetry breaking "order" in the pseudogap regime is very beautiful, but (perhaps) a diversion



Strategy: Construct candidate Non-Fermi liquid quantum states as putative strange metals


ORDERVILLE


Fisher's
GOLD &
JEWELRY
ROCKS & GEMS

One attempt - Towards a “D-wave Metal”

(ongoing with Lesik Motrunich)

Underlying is a “D-wave *Bose*-Metal”

Informed/Motivated by the FQHE: Why?

- Worked for Laughlin!
- Have Non-Fermi liquids in QHE (Composite Fermi liquids)
- Have “High Tc pairing” in QHE (Pfaffian, Haldane-Rezayi state)

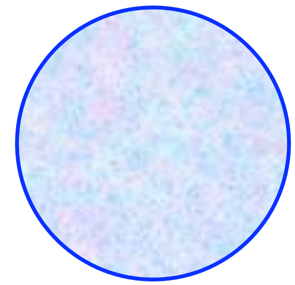
Half-filled Landau band: Composite Fermi Liquid

Physics at nu=1/2 well described by a CFL wavefunction;

$$\Psi_{CFL} = \prod_{i<j}(z_i - z_j)^2 \prod_j e^{-|z_j|^2/4} \cdot \det[e^{i\vec{k}_i \cdot \vec{r}_j}]$$

Product of nu=1/2 Laughlin state for bosons and free Ferm sea

$$\Psi_{CFL} = \Phi^{\nu=1/2}_{Laughlin} \times \Psi_{Fermi-sea}$$



p+ip Paired State at nu=5/2

Physics at nu=5/2 consistent with Moore/Read Pfaffian

$$\Psi_{Moore/Read} = \prod_{j<k}(z_j - z_k)^2 \prod_j e^{-|z_j|^2/4} \cdot \mathrm{Pf}\left(\frac{1}{z_j - z_k}\right)$$

Product of nu=1/2 Laughlin state and BCS p+ip wavefunction

$$\Psi_{Moore/Read} = \Phi^{\nu=1/2}_{Laughlin} \times \Psi^{p_x+ip_y}_{BCS}$$

“Pairing on the scale of the “Fermi energy”

Problems for Hi-Tc; No spin, strong magnetic field

Composite Fermi liquid with spin

$$\Psi_{CFL}^{spin} = \prod_{i<j}(z_i - z_j)^2 \prod_j e^{-|z_j|^2/4} \times \det[e^{i\vec{k}_i \cdot \vec{r}_{j\uparrow}}] \cdot \det[e^{i\vec{k}_i \cdot \vec{r}_{j\downarrow}}]$$

nu=1/2 Bosons times a spinful Fermi sea

$$\Psi_{CFL}^{spin} = \Phi_{Laughlin}^{\nu=1/2} \cdot \Psi_{Fermi-sea}^{spin}$$

Haldane-Rezayi State: d+id singlet pairing of composite fermions

$$\Psi_{HR} = \prod_{i<j}(z_i - z_j)^2 \prod_j e^{-|z_j|^2/4} \times \det[\frac{u_i v_j - v_i u_j}{(z_i - z_j)^2}]$$

Product of nu=1/2 bosons and a singlet d+id BCS state

$$\Psi_{HR} = \Phi_{Laughlin}^{\nu=1/2} \times \Psi_{BCS}^{d+id}$$

Problem: CFL breaks T-reversal invariance

Laughlin state breaks time reversal

$$\Psi^{spin}_{CFL} = \Phi^{\nu=1/2}_{Laughlin} \cdot \Psi^{spin}_{Fermi-sea}$$

Drag one particle around another - 2 units of angular momentum

$$\Phi_{\nu=1/2}(z) \sim (z - z_i)^2$$

nu=1/2 Laughlin state for bosons has $d_{x^2-y^2} + i d_{xy}$ two-particle correlations

Strategy to construct a T-invariant non-Fermi liquid:

(a) First construct a wavefunction for hard core bosons with d_{xy} or d_{x2-y2} two-particle correlations $\Phi^{Bose}_{d_{xy}}$

(b) Multiply Boson wavefunction by a filled Fermi sea

Will obtain a "d-wave Metal":

$$\Psi^{Metal}_{d_{xy}} = \Phi^{Bose}_{d_{xy}} \times \det[e^{i\vec{k}_i \cdot \vec{r}_{j\uparrow}}] \cdot \det[e^{i\vec{k}_i \cdot \vec{r}_{j\downarrow}}]$$

Essentially a "Gutzwiller-Plus" approach

Gutzwiller projection of a filled Fermi sea?   Expected to give a Fermi liquid

$$\Psi_{FL} = \mathcal{P}_G \det[e^{i\vec{k}_i \cdot \vec{r}_{j\uparrow}}] \det[e^{i\vec{k}_i \cdot \vec{r}_{j\downarrow}}]$$

Build in additional correlations with Jastrow factor (3-He)?   Still a Fermi liquid

$$\tilde{\Psi}_{FL} = \mathcal{P}_G \; e^{-\sum_{i<j} V(\vec{R}_i - \vec{R}_j)} \det[e^{i\vec{k}_i \cdot \vec{r}_{j\uparrow}}] \det[e^{i\vec{k}_i \cdot \vec{r}_{j\downarrow}}] \qquad \{\vec{R}_i\} = \{\vec{r}_{i\uparrow}, \vec{r}_{j\downarrow}\}$$

Constructing a non-Fermi liquid wavefunction requires modifying the
sign structure of the filled Fermi sea

$$\Psi^{Metal}_{d_{xy}} = \Phi^{Bose}_{d_{xy}}(\vec{R}_i) \times \det[e^{i\vec{k}_i \cdot \vec{r}_{j\uparrow}}] \det[e^{i\vec{k}_i \cdot \vec{r}_{j\downarrow}}]$$

Need a boson wf with non-trivial
sign structure, eg. a D-wave Bose-Metal

Wavefunction for D-wave Bose-Metal (DBM)

Hint: nu=1/2 Laughlin is
a determinant squared

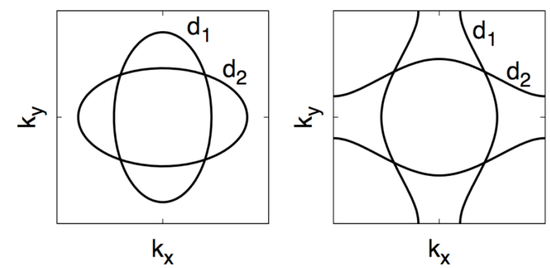
$$\Psi_{\nu=1/2} = [\Psi_{\nu=1}]^2 \qquad \Phi_{\nu=1}(z) \sim (z - z_i)$$

p+ip 2-body

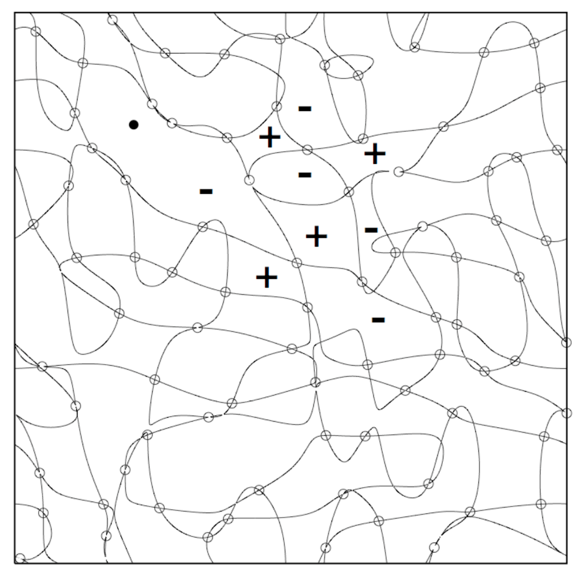
Try squaring Fermi sea wf:
No, "s-wave" with ODLRO

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = (\det e^{i\mathbf{k}_i \cdot \mathbf{r}_j})^2 \ , \quad \text{(S-type)}.$$

``D-wave" Bose-Metal:
Product of 2 different Fermi sea determinants,
elongated in the x or y directions

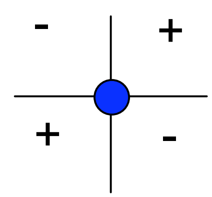


$$\Psi_{D_{xy}}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = (det)_x \times (det)_y$$



Nodal structure of DBM wavefunction:

$$\Phi_{D_{xy}}(\mathbf{r}) \sim (x - x_i)(y - y_i)$$



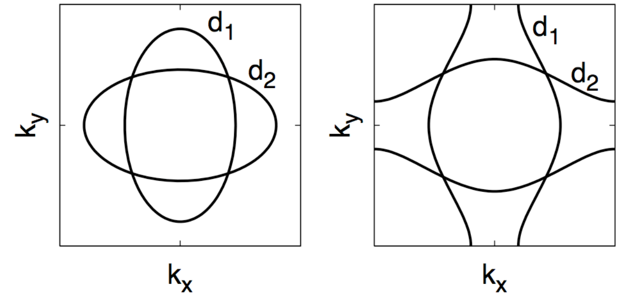
D_{xy} relative
2-particle correlations

(no double occupancy,
Gutzwiller projection unnecessary)

Bose Surfaces in DBM

Slave Fermion decomposition for lattice bosons: $b^\dagger(\mathbf{r}) = d_1^\dagger(\mathbf{r}) d_2^\dagger(\mathbf{r})$

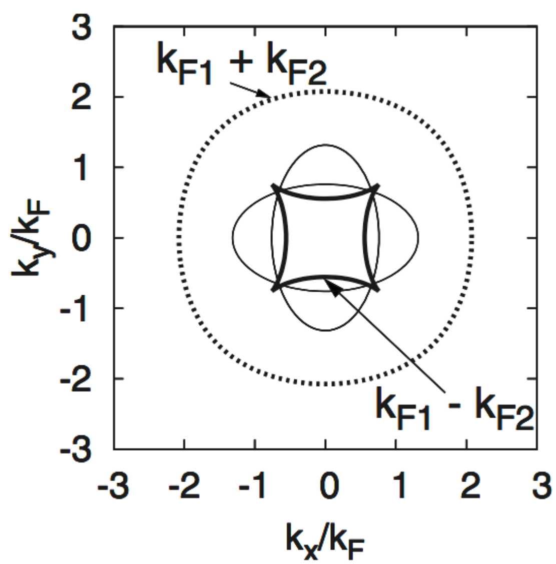
Mean Field Green's functions factorize:
(no gauge fluctuations)



$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau) G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$

Momentum distribution function:

$$\langle b_k^\dagger b_k \rangle$$



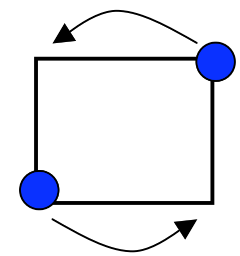
Two singular lines in
momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$

Hamiltonian for DBM?

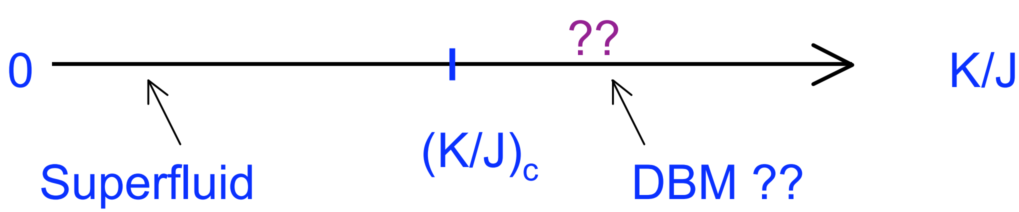
Strong coupling limit of gauge theory: Boson Hamiltonian

$$
\begin{aligned}
H &= H_J + H_4 \ , \\
H_J &= -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mu}} + h.c.) \ , \\
H_4 &= K_4 \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) \ ,
\end{aligned}
$$



boson ring term

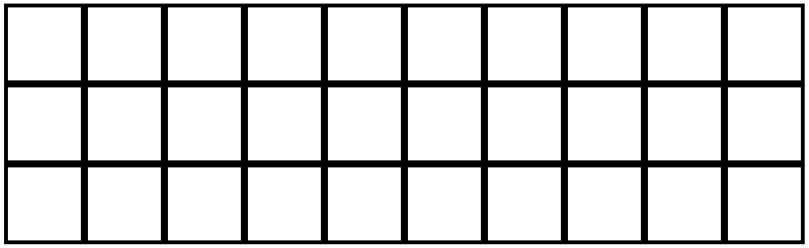
Phase diagram: K/J and density of bosons



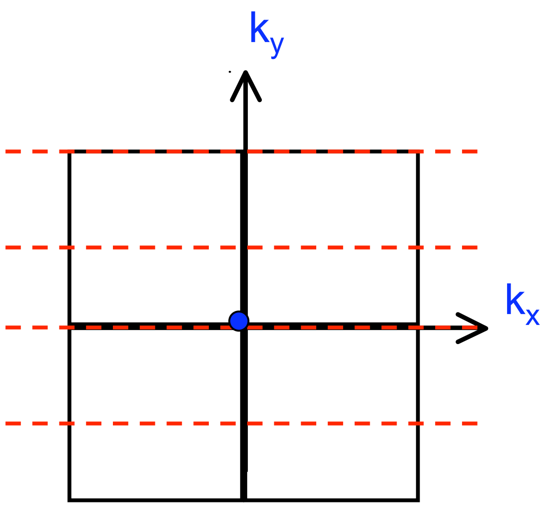
J-K Model has a sign problem - completely intractable

Ladders to the Rescue

Transverse y-components of momentum become quantized

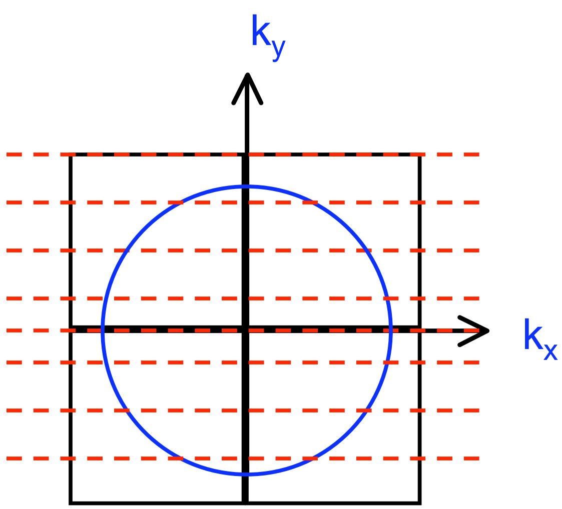


Put Bose superfluid on n-leg ladder



Single gapless 1d mode

Put Fermi Liquid on n-leg ladder



Many gapless 1d modes, one for each Fermi point

Signature of 2d Fermi surface present on ladders

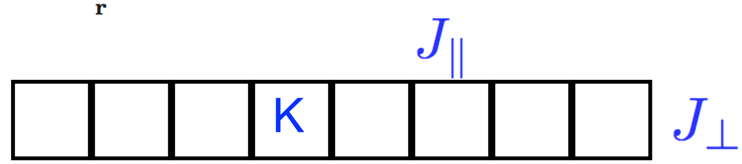
Expectation: Signature of Bose surface in DBM present on n-leg ladders!!

DBM on the 2-Leg Ladder

- Exact Diagonalization (2 x 18)
- Variational Monte Carlo
- DMRG (2 x 50)

(E. Gull, D. Sheng, S. Trebst,
O. Motrunich and MPAF)

$$
\begin{aligned}
H &= H_J + H_4 \ , \\
H_J &= -J \sum_{\mathbf{r};\ \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mu}} + h.c.) \ , \\
H_4 &= K_4 \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) \ ,
\end{aligned}
$$

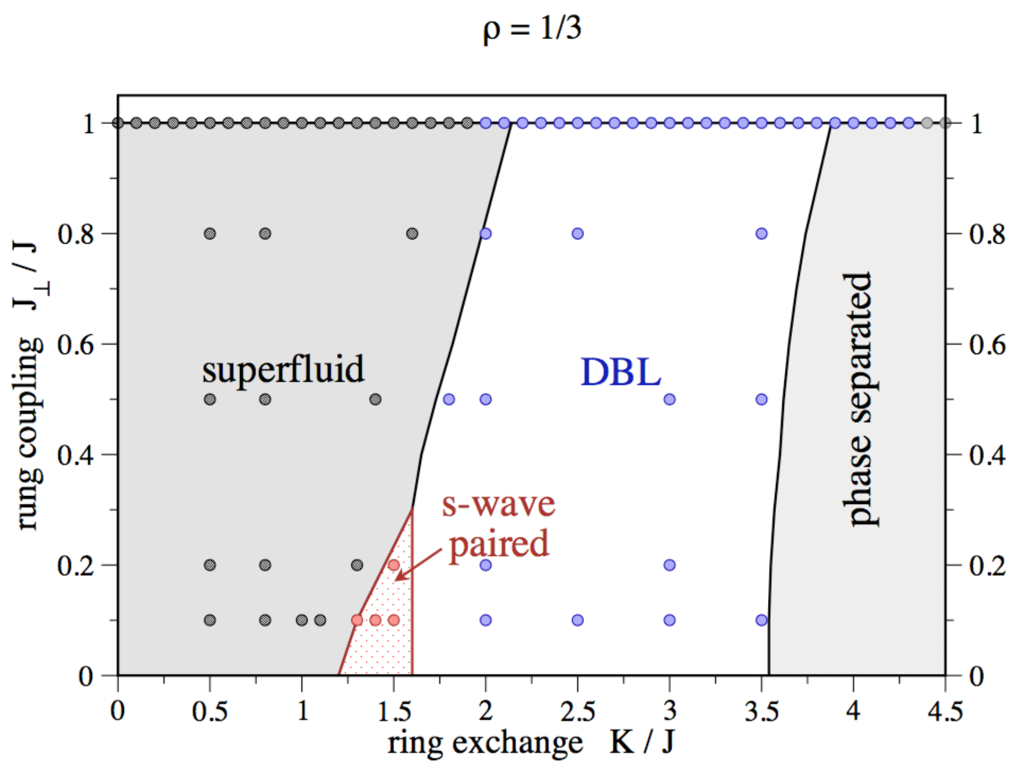


Correlation Functions:

1) Momentum Distribution function $n(k_x, k_y) = \langle b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \rangle; \quad k_y = 0, \pi$

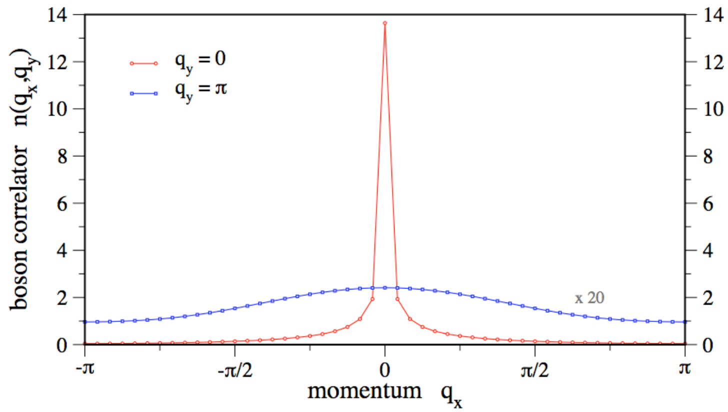
2) Density-density structure factor $\mathcal{D}(\mathbf{k}) = \sum_{\mathbf{r}} e^{i\mathbf{k}\cdot\mathbf{r}} \langle n_{\mathbf{r}} n_{\mathbf{0}} \rangle \qquad n_{\mathbf{r}} = b^\dagger_{\mathbf{r}} b_{\mathbf{r}}$

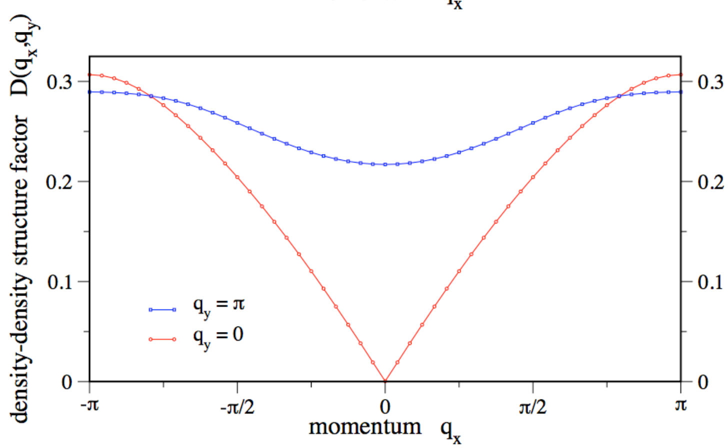
Phase Diagram for 2-leg ladder

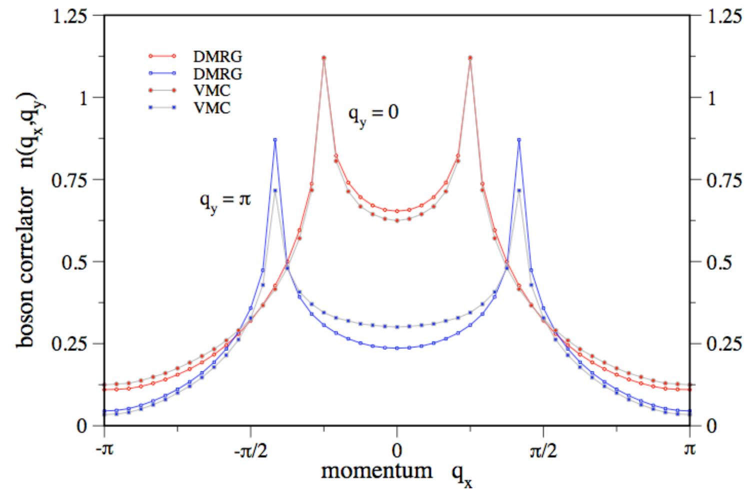


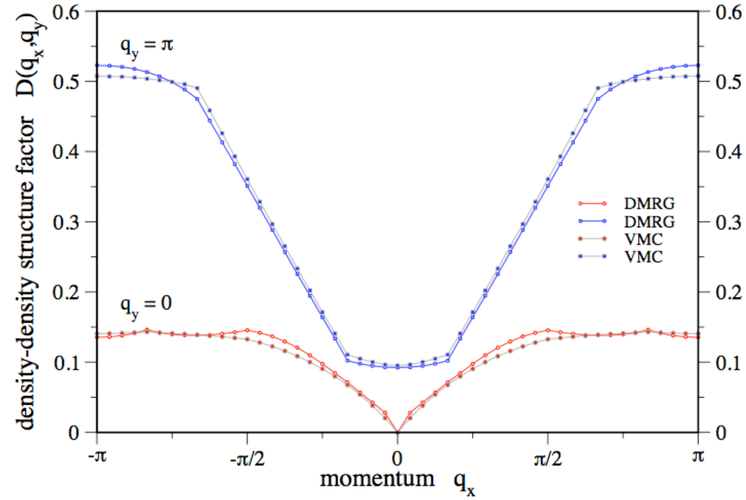
D-wave Bose-Metal occupies large region of phase diagram

Superfluid versus DBM correlators









Superfluid - “condensed”
at zero momentum

D-wave Bose-Metal; Singular
“Bose points” at $q_y = 0, \pi$

“D-Wave Metal”

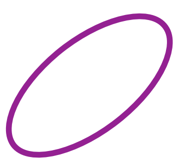
Itinerant non-Fermi liquid phase of 2d electrons?

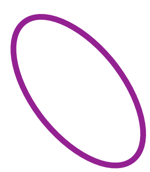
Gauge theory
(parton) construction

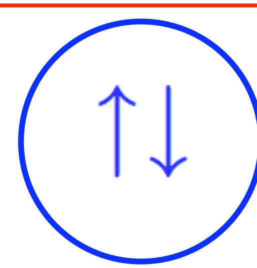
$$c_\alpha^\dagger(\mathbf{r}) = b^\dagger(\mathbf{r}) f_\alpha^\dagger(\mathbf{r}) = d_x^\dagger(\mathbf{r}) d_y^\dagger(\mathbf{r}) f_\alpha^\dagger(\mathbf{r})$$

Wavefunction; Product of determinants $\{\vec{R}_i\} = \{\vec{r}_{i\uparrow}, \vec{r}_{j\downarrow}\}$

$$\Psi^{Metal}_{d_{x^2-y^2}} = \det_{x+y}[e^{i\vec{K}_i\cdot\vec{R}_j}] \cdot \det_{x-y}[e^{i\vec{K}_i\cdot\vec{R}_j}] \times \det[e^{i\vec{k}_i\cdot\vec{r}_{j\uparrow}}] \cdot \det[e^{i\vec{k}_i\cdot\vec{r}_{j\downarrow}}]$$

$x+y$ 

$x-y$ 



Filled Fermi sea

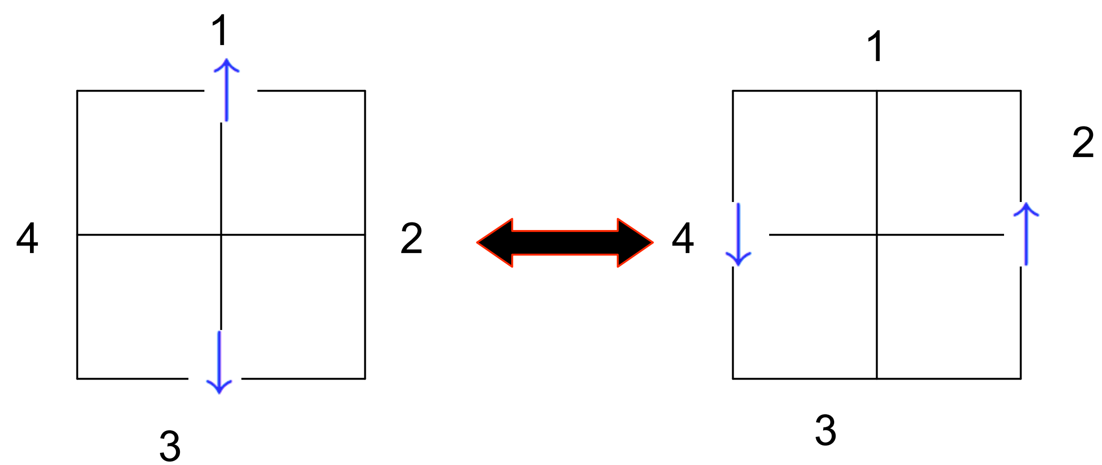
Hamiltonian for D-wave Metal?

t-K "Ring" Hamiltonian (no double occupancy constraint)

$$\mathcal{H}_{tK} = -t\sum_{\langle ij\rangle}[c^\dagger_{i\alpha}c_{j\alpha} + h.c.] + K\sum_{\langle 1234\rangle}[\mathcal{S}^\dagger_{13}\mathcal{S}_{24} + h.c.]$$

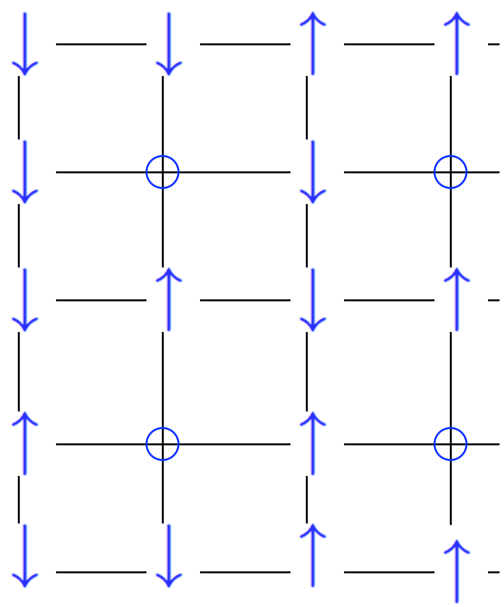
$$\mathcal{S}^\dagger_{ij} = \frac{1}{\sqrt{2}}[c^\dagger_{i\uparrow}c^\dagger_{j\downarrow} - c^\dagger_{i\downarrow}c^\dagger_{j\uparrow}]$$

Electron singlet pair "rotation" term

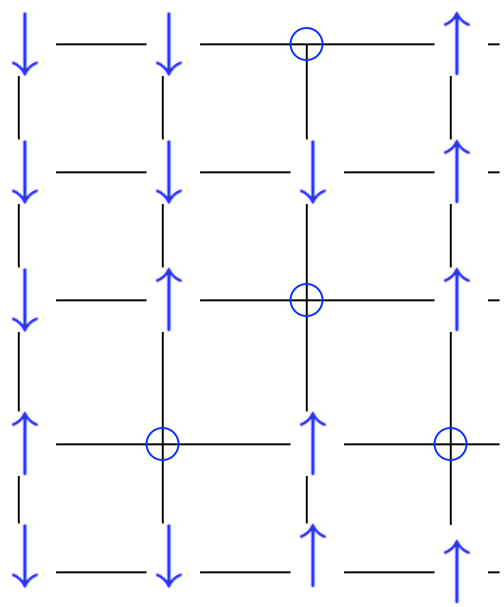


- Ring term will be generated when projecting into a single band model
- Ring term operates when two doped holes are nearby
- Ring term induces 2-particle singlet d-wave correlations (for K>0)

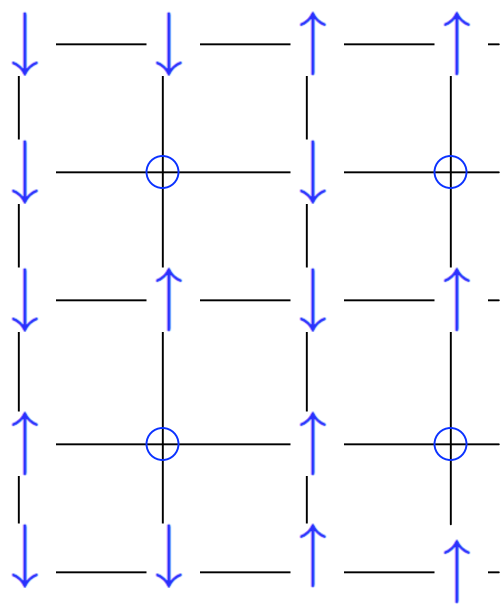
Doping near optimal



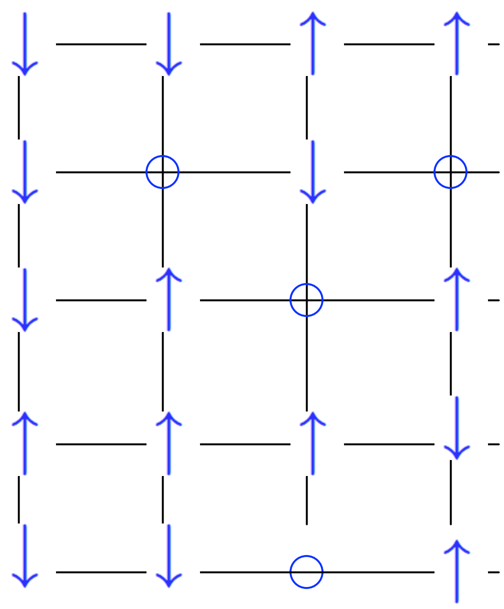
Doping near optimal



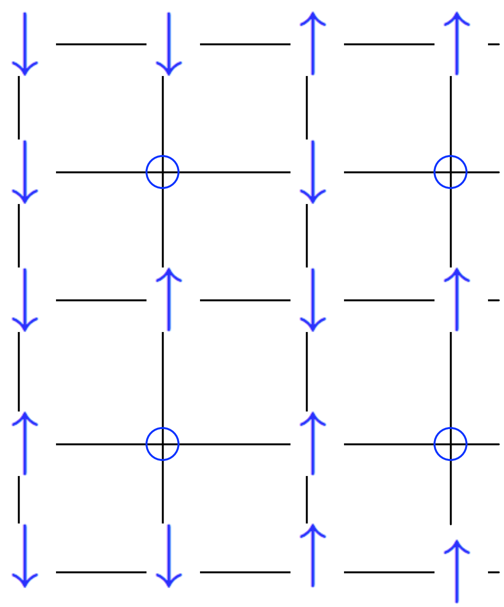
Doping near optimal



Doping near optimal



Doping near optimal



Phase diagram of electron t-K Hamiltonian?

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c^\dagger_{i\alpha} c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}^\dagger_{13} \mathcal{S}_{24} + h.c.]$$

Doping and K/t ??

Fermi liquid for K<<t ?
D-wave metal for K ~ t ?
D-wave superconductor ?

Future: Put t-K Hamiltonian on a 2-leg ladder and attack with DMRG,…

Lessons, if any there be?

Theorists:

- Strange Metal is begging to be understood!
- Order is beautiful but perhaps peripheral
- Without order wavefunctions can be useful,
  and sign structure is important
- t-J model is not the be all end all

Lessons, if any there be?

Theorists:

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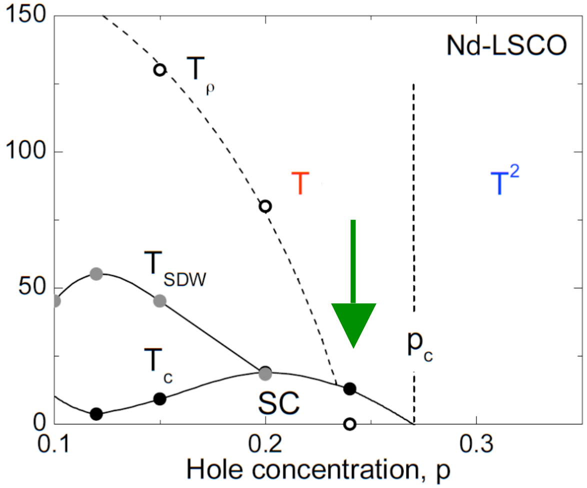
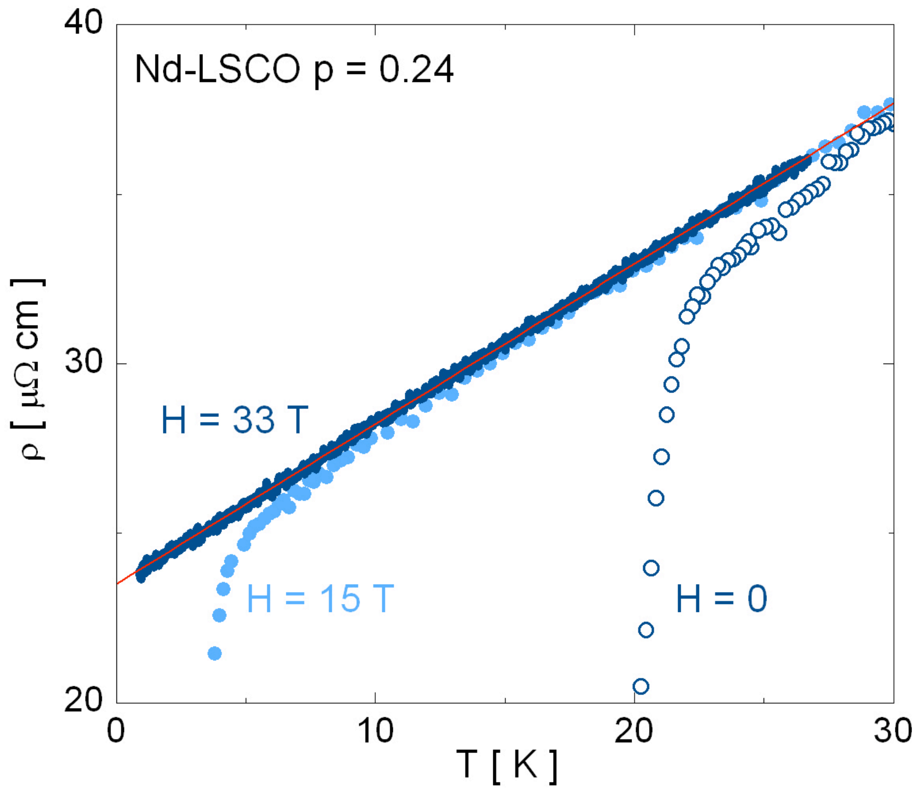
Experimentalists:

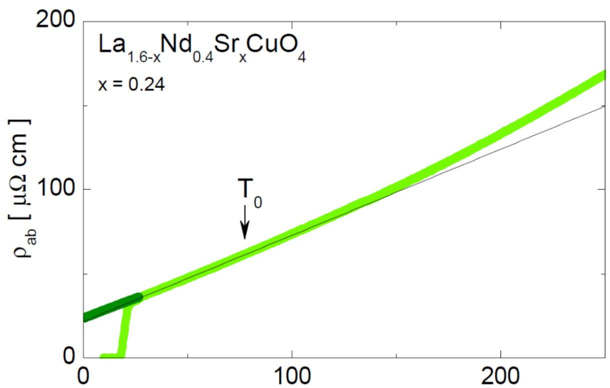
- ***Don't listen to the theorists!***

Linear resistivity (L. Taillefer) Nd-LSCO

Nd-LSCO







Daou et al., Nature Physics (2009)

Summary & Outlook

- Established existence of the DBL Phase on 2-leg ladder
  - J-K ring Hamiltonian on 2-leg ladder has DBL ground state
  - Gutzwiller Variational wavefunction "close" to exact ground state
  - Presence of (one) Bose surface revealed
  - Amperean rule in Gauge Theory formulation accounts for location of Bose surface

- Properties of 2d DBL:
  - 2d uncondensed quantum fluid of itinerant bosons
  - Gapless strongly interacting excitations
  - Metallic type transport
  - Local d-wave pair correlations
  - "Bose Surfaces" which satisfy a Luttinger sum-rule

Future:

- Numerics on n-leg ladders for DBL
- Properties, instabilities and energetics of the D-Wave Metal
- D-Wave Metal on the n-leg ladder?
- Non-relativistic critical spin liquids on n-leg ladders (eg. Hubbard on triangular ladder)