two short (true) stories

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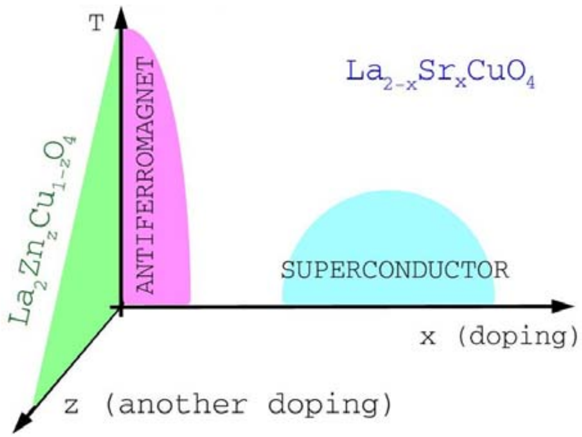
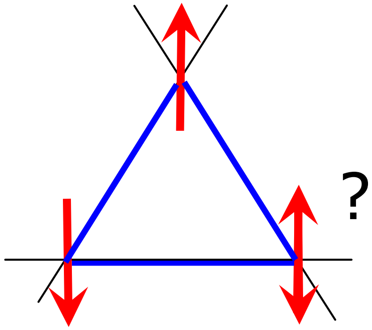
lower T_N / higher T_c

lower T_N

☑ dilution (Zn-doped La_2CuO_4)



☑ frustration (excitation spectrum)



dilution of the square-lattice AF

SC

Shiu Liu

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SC, Y. C. Chen, and A. H. Castro Neto,
PRB **65**, 104407 (2002); PRL, **87**, 067209 (2001)

C.-W. Liu, S. Liu, Y.-J. Kao, **SC**, and A. Sandvik,
PRL **102**, 167201 (2009) + more to come

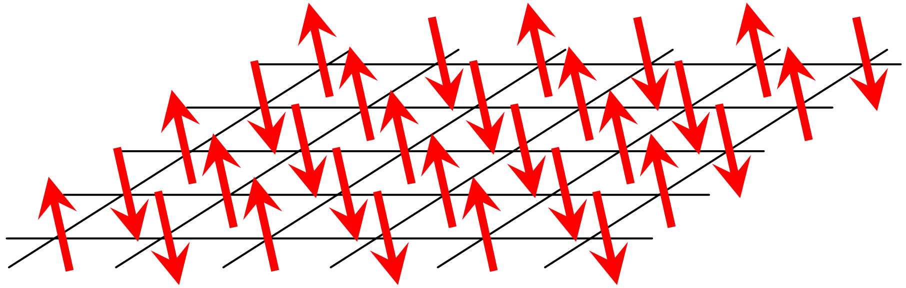
main idea

in oxides, dilution can induce/enhance frustrating interactions

- ☑ experimental hints from Zn-doped La_2CuO_4
- ☑ 3-band versus 1-band, role of oxygen and central idea
- ☑ quantitative discussion

La_2CuO_4 - Cu + Zn

$$\mathbf{H} = J \sum_{<ij>} \mathbf{S}_i \cdot \mathbf{S}_j$$



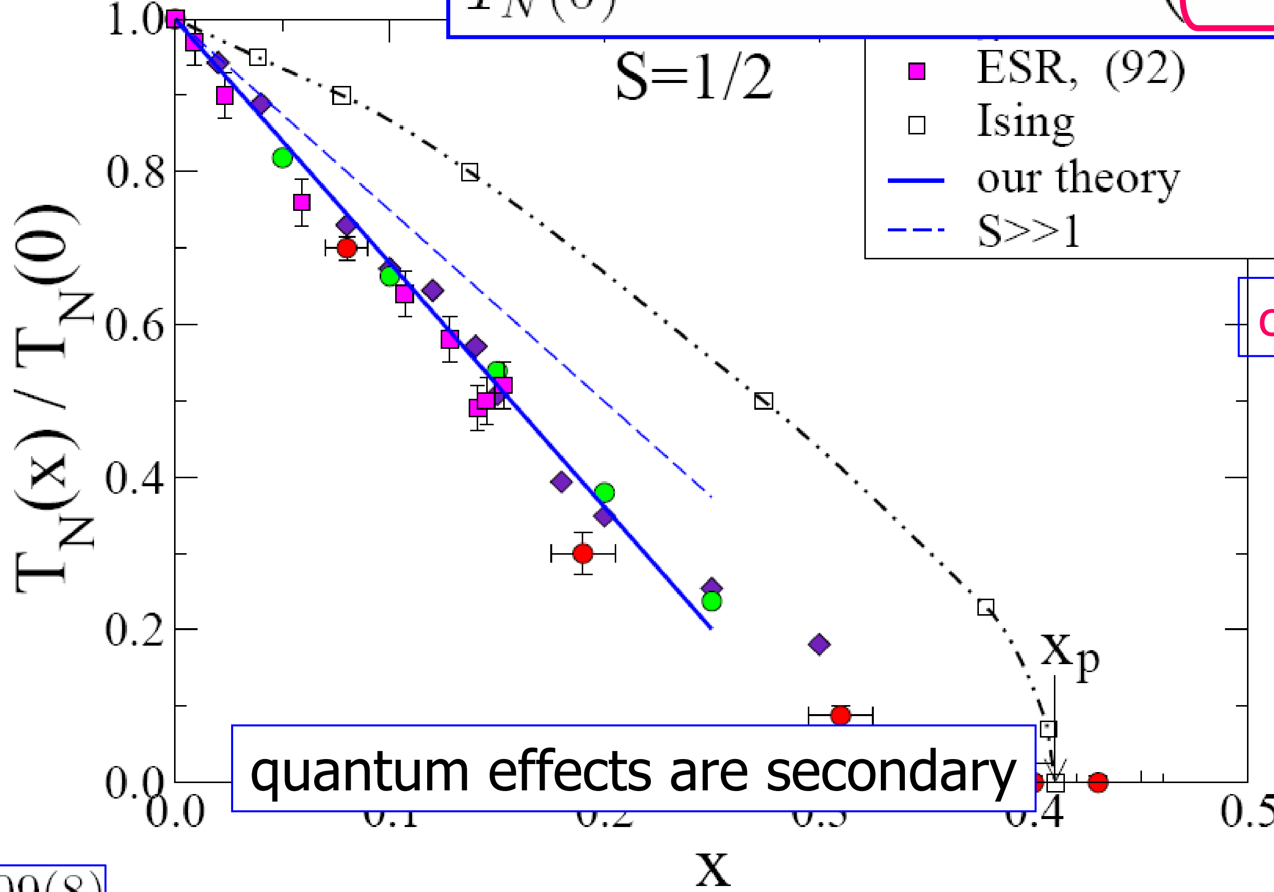
what can a substitution of Cu to Zn lead to?

→ breaks AF-bonds around Zn

$$\mathbf{H} = J \sum_{<ij>} p_i p_j \mathbf{S}_i \cdot \mathbf{S}_j$$

from the Past ..., I

$$\frac{T_N(x)}{T_N(0)} \simeq 1 - A_s\, x = 1 - x\Bigg(\underbrace{\pi - \frac{2}{\pi}}_{\text{classical}} + \underbrace{\frac{B}{S-\Delta}}_{\text{quantum}}\Bigg)$$



quantum effects are secondary

$B = 0.209(8)$

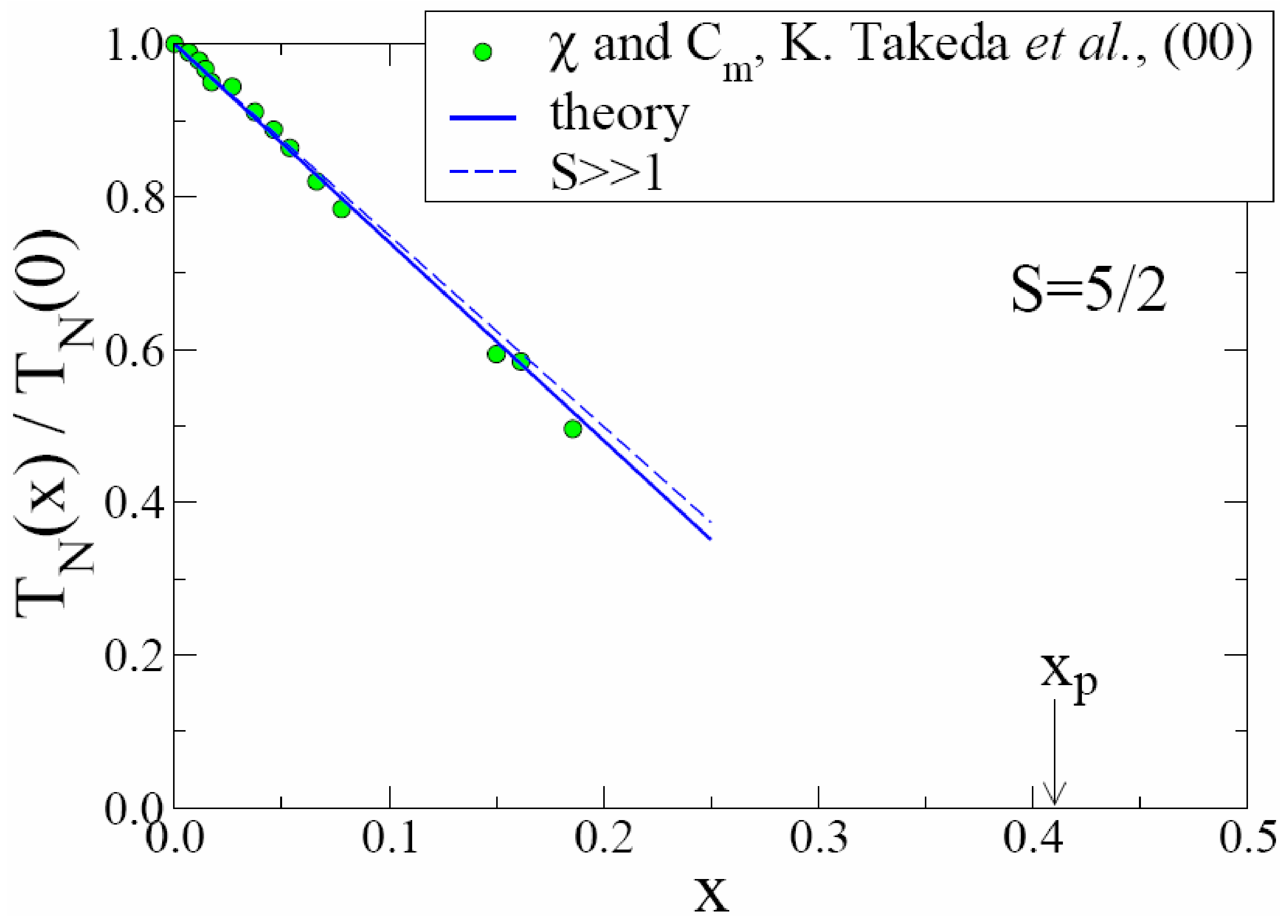
$\Delta = \sum_{\mathbf{k}} v_{\mathbf{k}}^2 \simeq 0.1966$

$A_{1/2} \approx 3.2$

2D spin-1/2 , copper deuteroformate tetradeuterate (CFTD)

A. R. McGurn, J. Phys. C **12**, 3523 (1979); $A_s = \pi$

from the Past ..., II



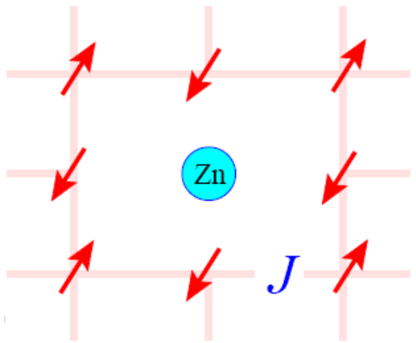
$Mn_{1-x}Zn(Mg,Cd)_x(HCO_2)_2 2(NH_2)_2CO$

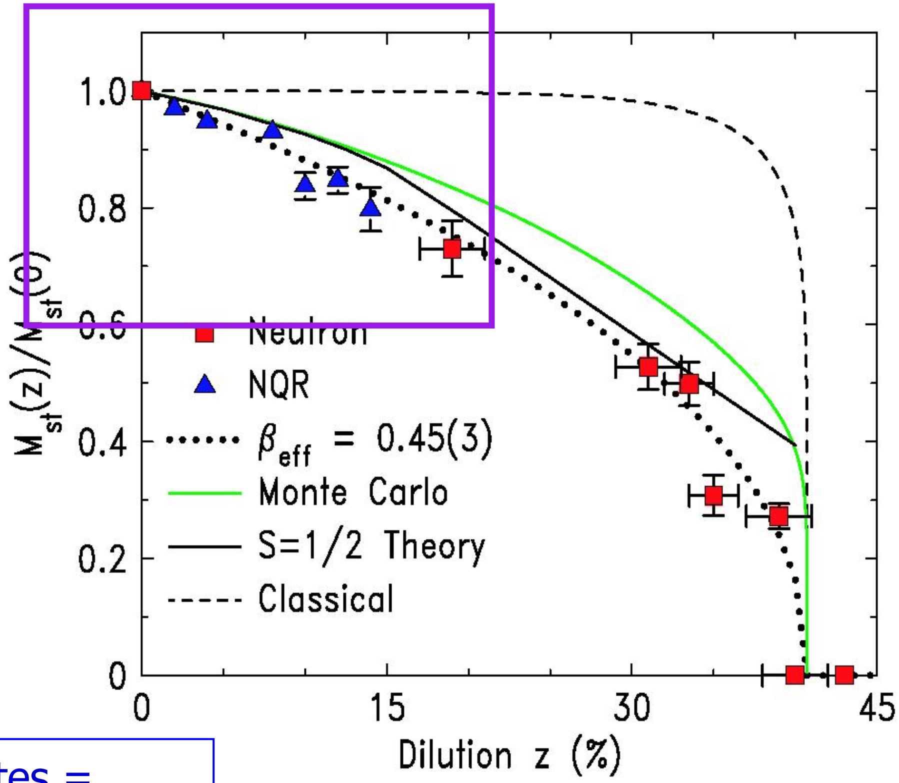
$A_{5/2} \approx 2.6$

K. Takeda etal., J. Phys. Soc. Jpn. **69**, 3696 (2000)

La_2CuO_4 - Cu + Zn

$$\mathbf{H} = J \sum_{<ij>} p_i p_j \mathbf{S}_i \cdot \mathbf{S}_j$$

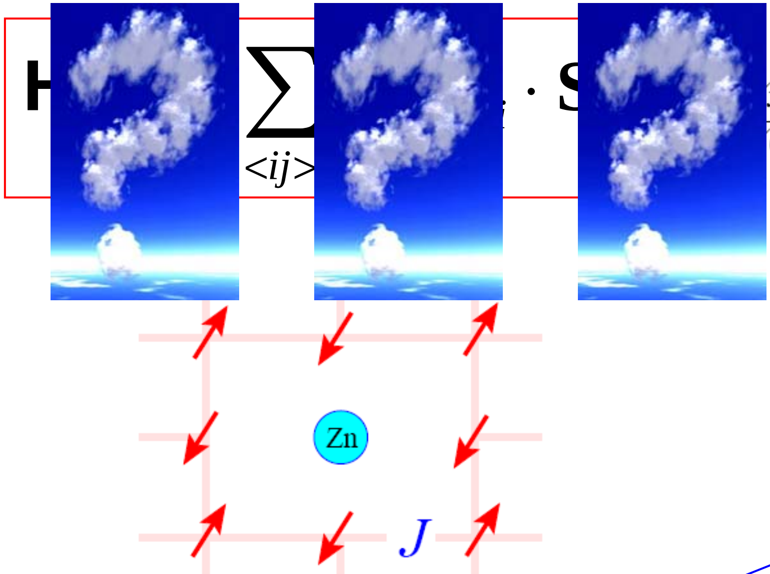




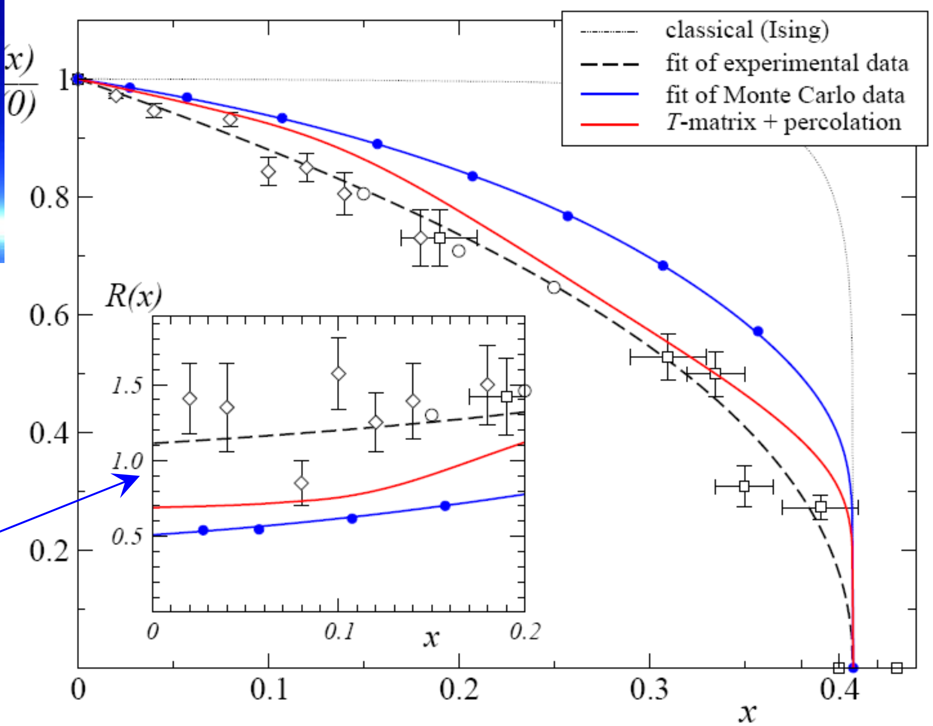
M(z) normalized to # of magnetic sites =
effect of classical dilution is taken into account

P. Carretta, A. Rigamonti, and R. Sala, PRB **55**, 3734 (1997);
O. P. Vajk, P. K. Mang, M. Greven, P. M. Gehring, J. W. Lynn
Science **295**, 1691 (2002); PRL **89**, 177202 (2002); Solid State Comm. **126**, 93 (2003);
A. L. Chernyshev, Y. C. Chen, and A. H. Castro Neto, PRL **87**, 067209 (2001);
PRB **65**, 104407 (2002); A. W. Sandvik, PRB **66**, 024418 (2002).

La_2CuO_4 - Cu + Zn



$$R(x) = \frac{1}{x}\left(1 - \frac{M(x)}{M(0)}\right)$$



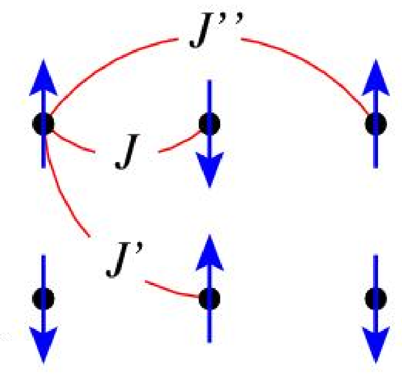
slope of M(x)/M(0) v.s. x is the # of ordered moments suppressed per impurity = measure of Zn-induced quantum fluctuations

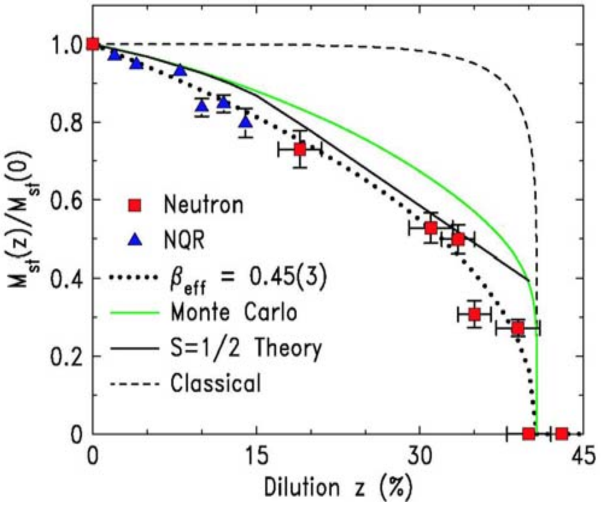
large discrepancy of th. and exp. slopes -- factor ≈ 2.0 !
dilution-only picture is incomplete

C.-W. Liu, S. Liu, Y.-J. Kao, **SC**, and A. Sandvik, PRL **102**, 167201 (2009).

explanation?

☑ ~~further neighbour/cyclic interactions in the host AF?~~





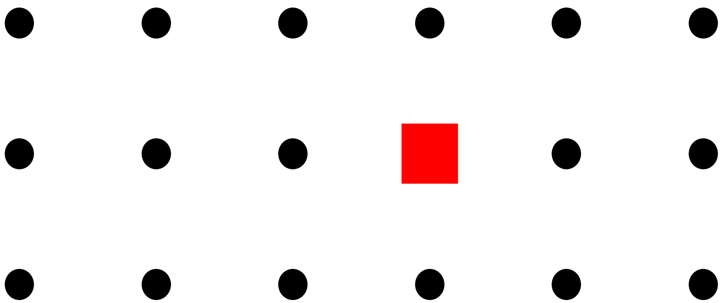
$$M(x,J') \approx M(0,0)\left(1 - x\cdot R(0) - A\frac{J'}{J}\right)$$

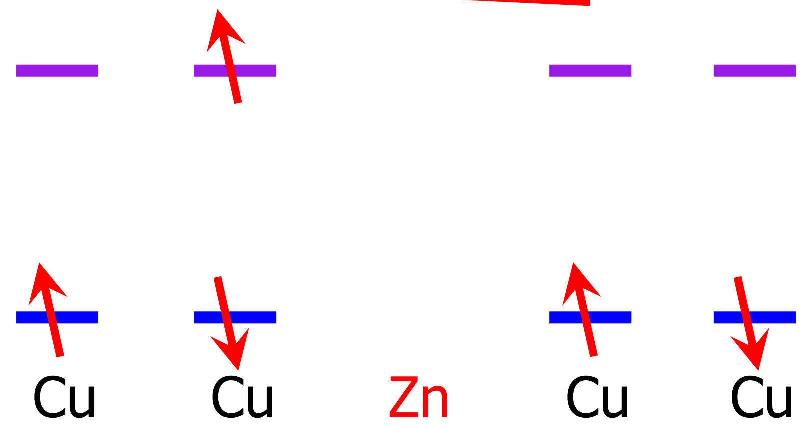
$$\boxed{\frac{M(x,J')}{M(0,J')} \approx 1 - x\cdot R(0)\left(1 + A\frac{J'}{J}\right)}$$

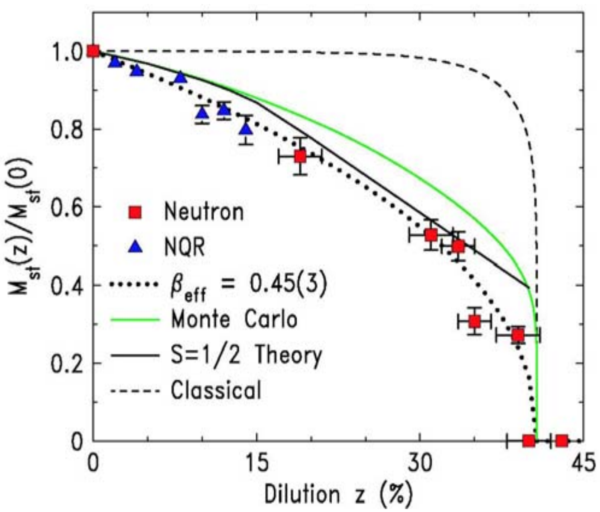
- decreases M(0)
- *J', J''* are small enough to cause a strong effect
- breaking "bad" bonds?

explanation?

☑ ~~would anything be different in the t-t′-…-U Hubbard model?~~







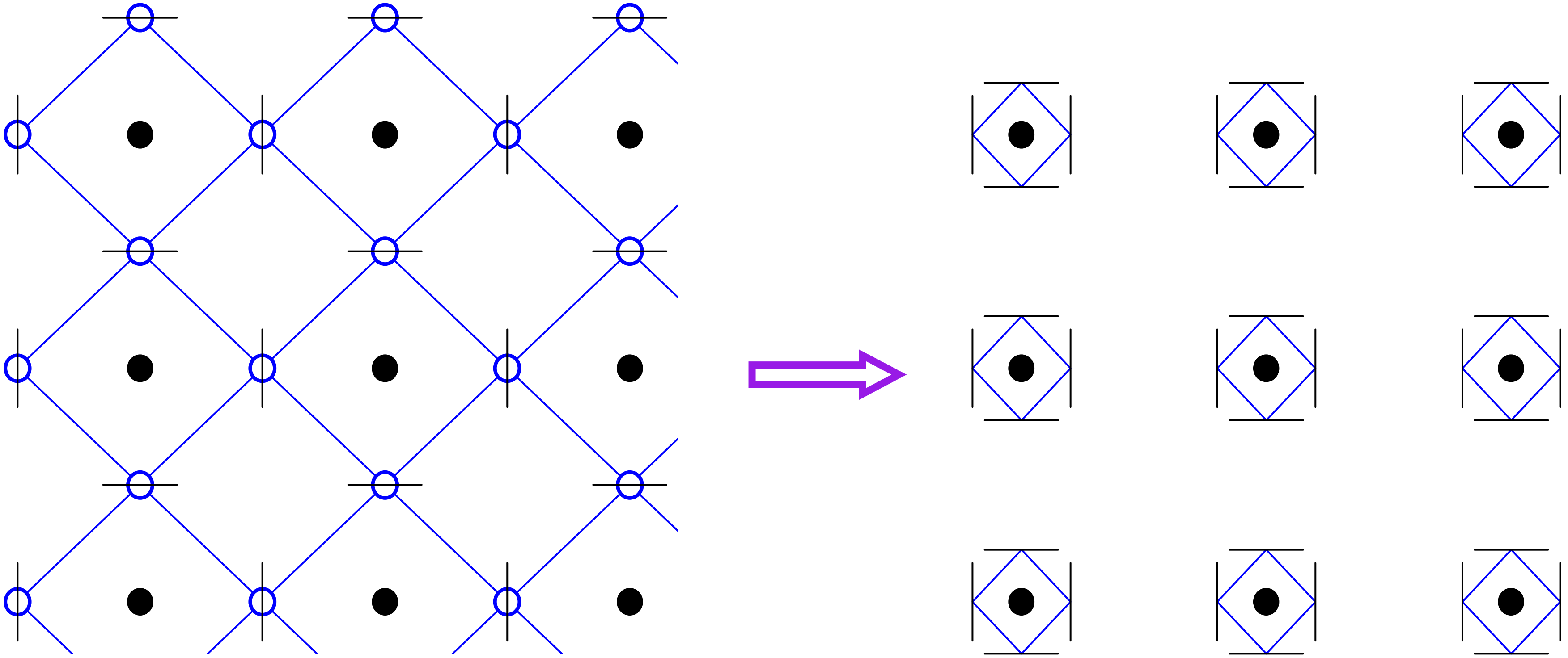
- starting from *t-t′-..-U* we are going to get the same *J-J′-J″..* effective model + dilution

- need something else

J.-Y. P. Delannoy *etal.*, PRB **79**, 224414 (2009).

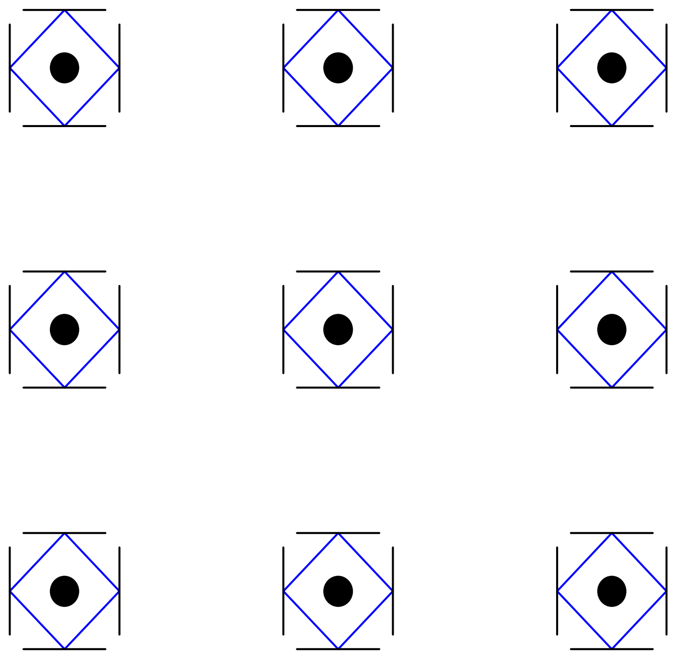
explanation?

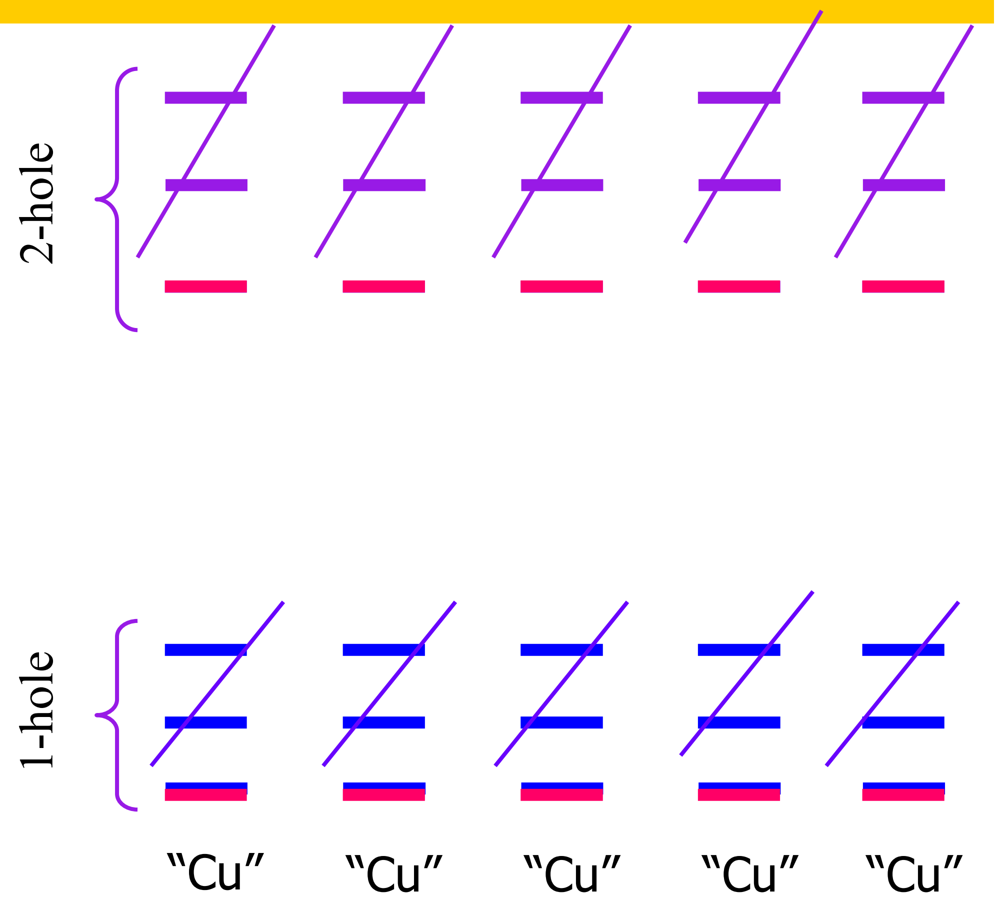
- ☑ 3-band Hubbard model??



multi-orbital *t-t′..-U-U′-..* model
(no approximations here)

3-band

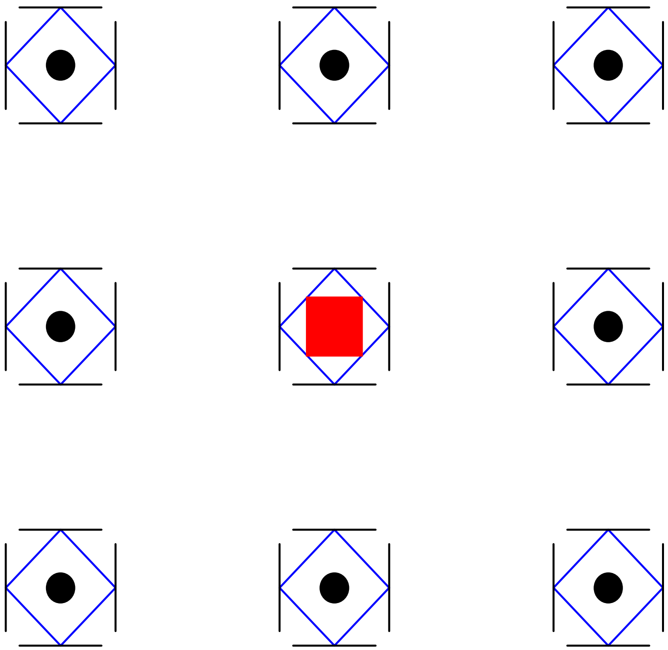


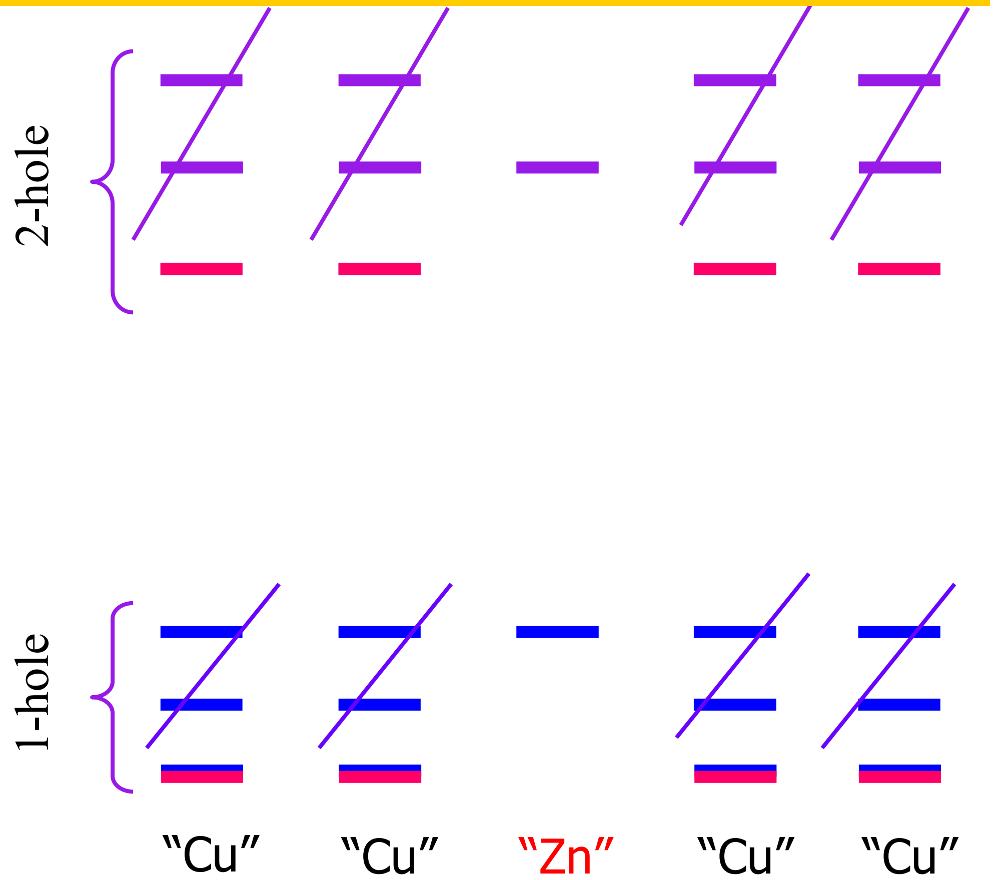


multi-orbital *t-t′..-U-U′-..* model

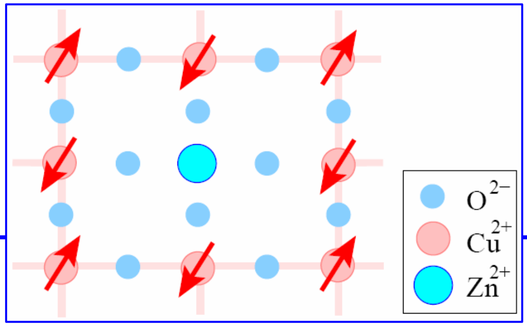
- at half-filling starting from the multi-orbital *t-t′-..-U-U′..* we will get the same *J-J′-J″..* low-energy effective model (no Zn yet)

Zn++

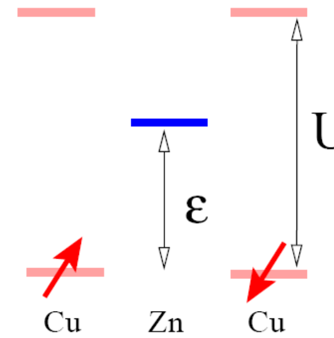




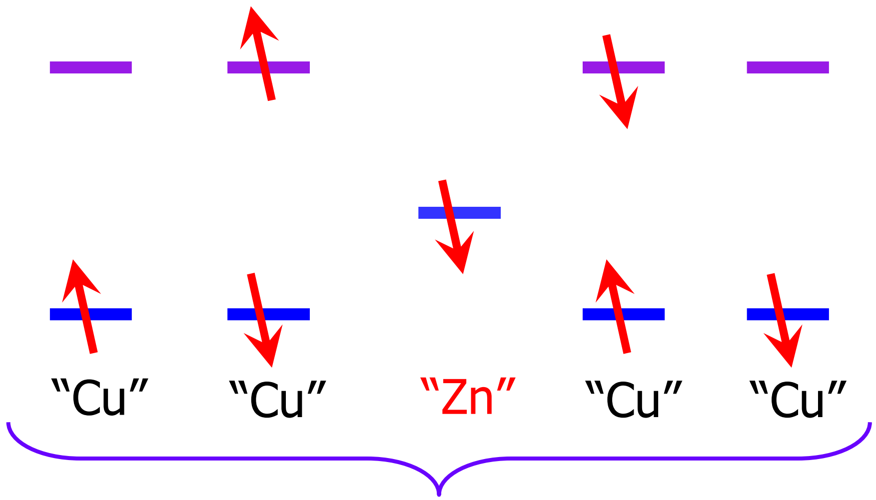
multi-orbital *t-t′..-U-U′-..* model



t-ε-U model



at half-filling we have this

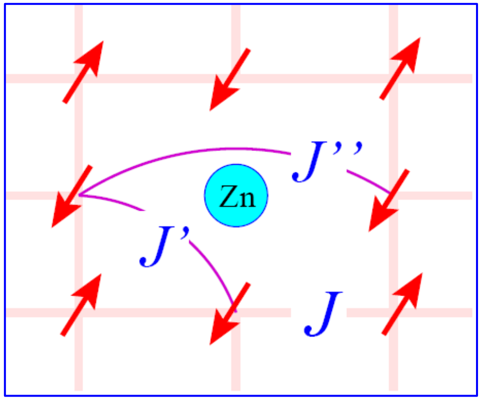


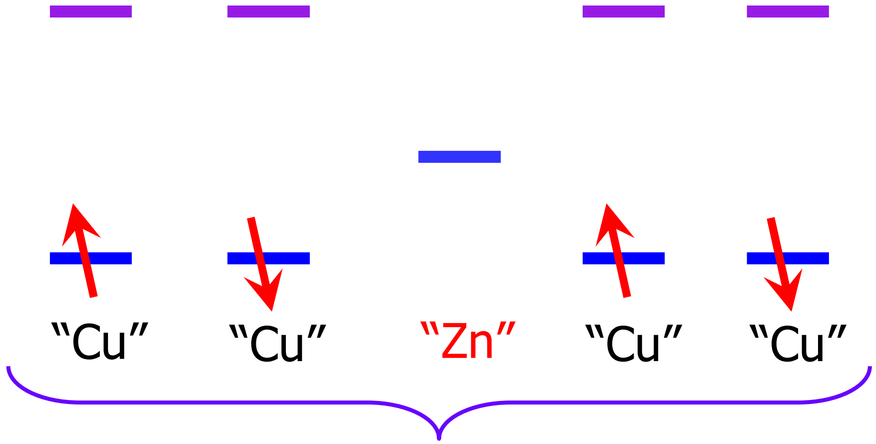
$$H = -t \sum_{ij,\sigma} c^\dagger_{i,\sigma} c_{j,\sigma} + \varepsilon \sum_{l,\sigma} n^{Zn}_{l,\sigma} + U \sum_{i} n_{i,\uparrow} n_{j,\downarrow}$$

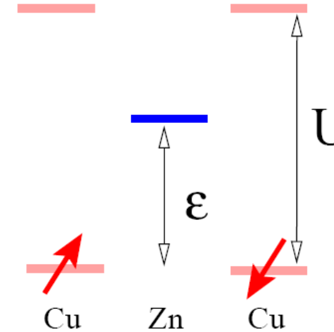
t-ε-U → ?

C.-W. Liu, S. Liu, Y.-J. Kao, **SC**, and A. Sandvik, PRL **102**, 167201 (2009).

t-ε-U model leads to frustration







$$\mathcal{H} = J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j + J'_{Zn}\sum_{\langle ij\rangle'}\mathbf{S}_i\cdot\mathbf{S}_j + J''_{Zn}\sum_{\langle ij\rangle''}\mathbf{S}_i\cdot\mathbf{S}_j$$

dilution + frustration, or J-J'_{Zn}-J''_{Zn} model

C.-W. Liu, S. Liu, Y.-J. Kao, **SC**, and A. Sandvik, PRL **102**, 167201 (2009).

very naïve estimations

$J = 4t^2/U$

$J'_{Zn} = 4t^4/\varepsilon^2(U\text{-}\varepsilon)+4t^4/\varepsilon^3$

higher order, but $\varepsilon < U$

$J'_{Zn}/\,J = (t^2U/\varepsilon^3)\,(1+\varepsilon/(U\text{-}\varepsilon))$

taking $\varepsilon = U/2$ yields:

$J'_{Zn}/\,J = 16\,(t^2/U^2)$

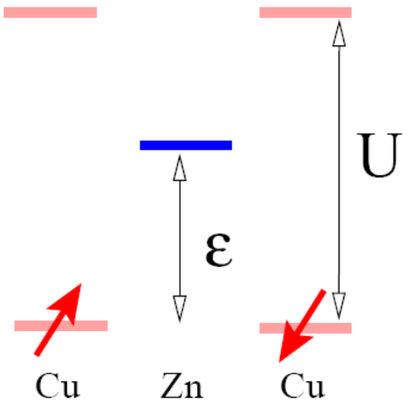
if $U/t=10$:

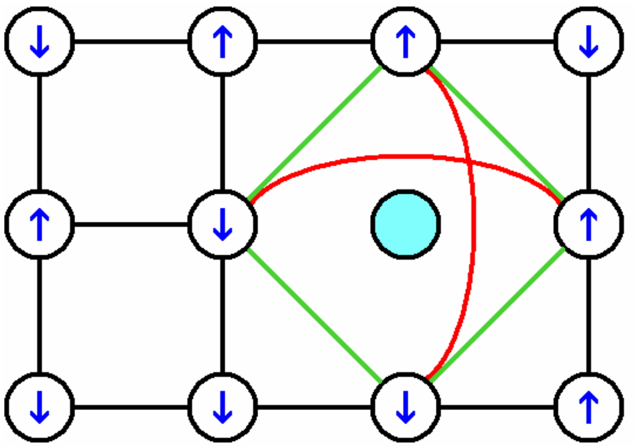
$\boldsymbol{J'_{Zn}\,/\,J \sim 0.1\text{-}0.2\,!}$

we have 6 such bonds ➔

$J'_{tot} = 4\,J'_{Zn} + 2\,J''_{Zn} \sim J$

per impurity ➔ strong effect





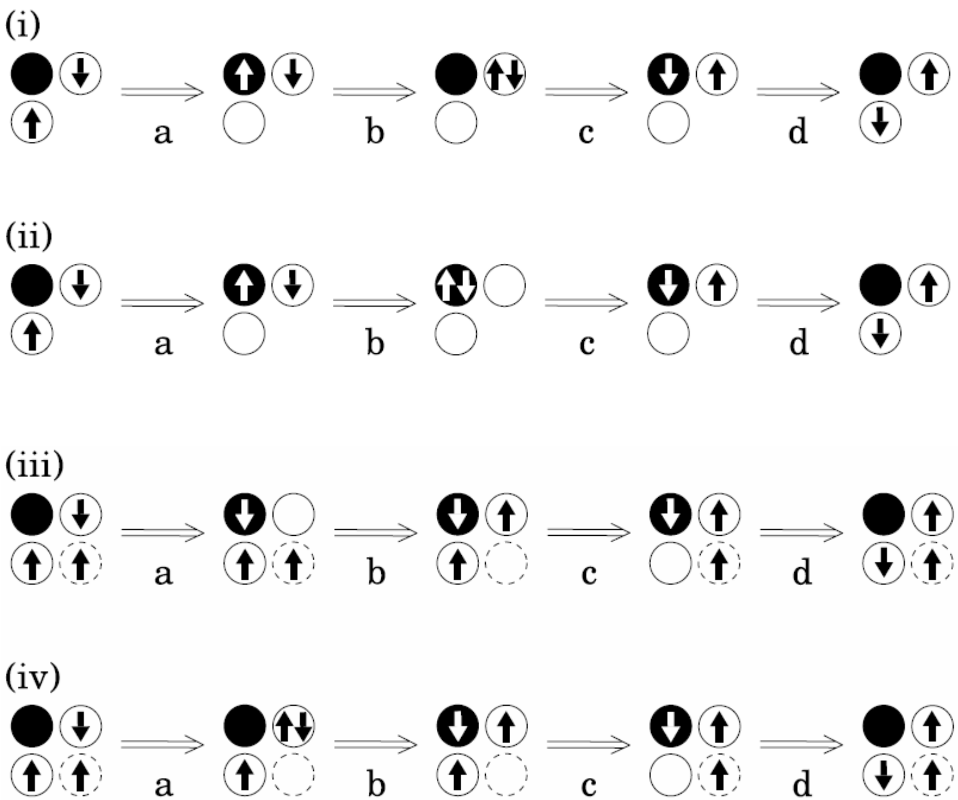
two questions

→ whether the less-naïve (full 3-band) calculations uphold this picture (proper derivation of the t-ε-U model)

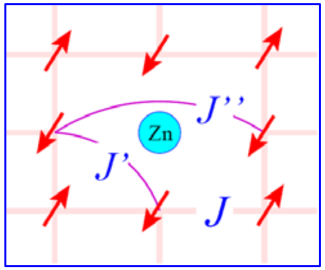
→ whether the frustration effect of J'_{Zn} , J''_{Zn} is substantial enough

details of the processes …

$$\boxed{\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J'_{Zn} \sum_{\langle ij \rangle'} \mathbf{S}_i \cdot \mathbf{S}_j + J''_{Zn} \sum_{\langle ij \rangle''} \mathbf{S}_i \cdot \mathbf{S}_j}$$



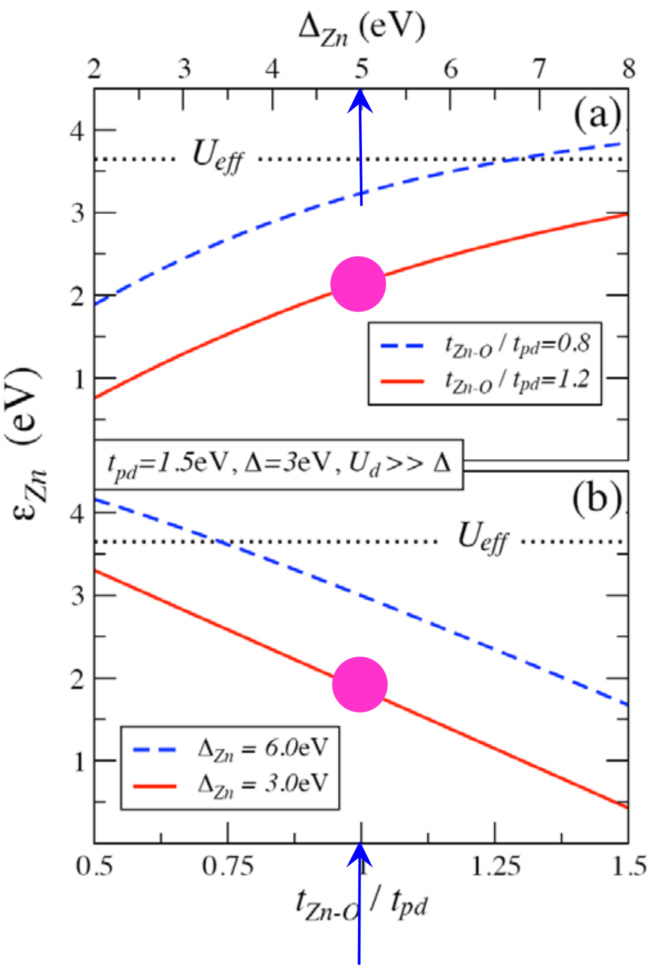
➔ superexchange-type processes generate the same J'_{Zn} and J''_{Zn}

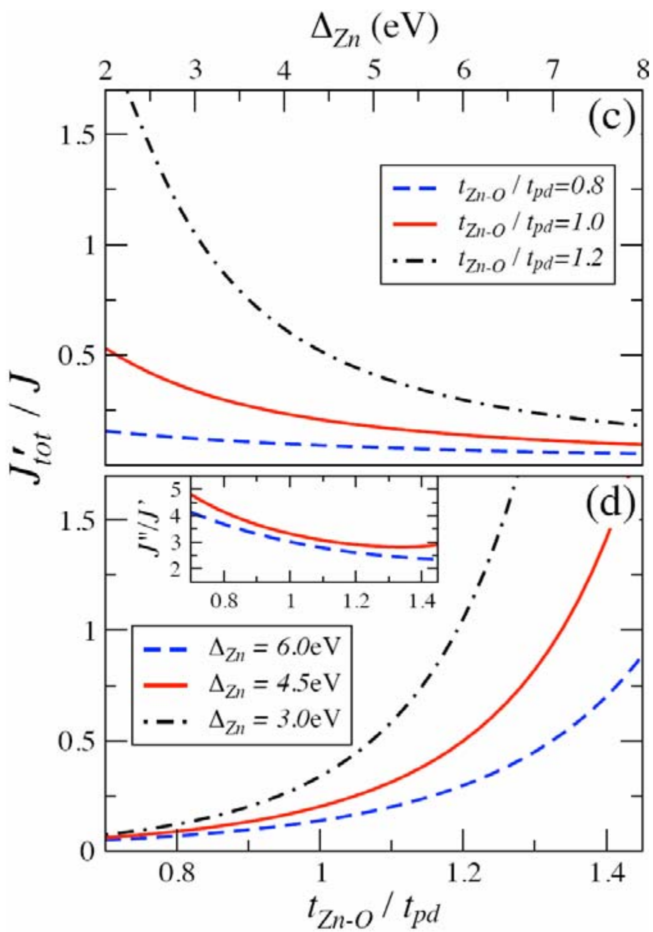


➔ cyclic-type processes reduce J'_{Zn}, no contribution to J''_{Zn}

$\boxed{\text{counterintuitively} \quad J''_{Zn} > J'_{Zn}}$

3-band parameters being varied …





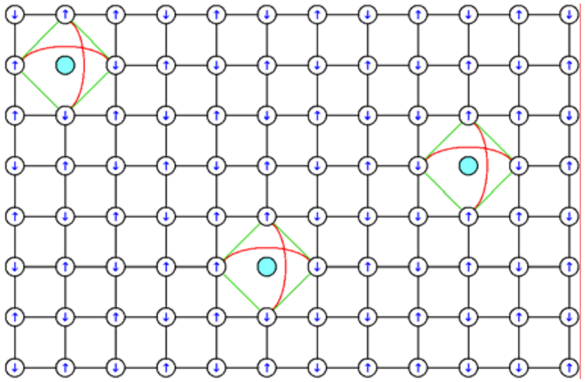
estimate of the total frustrating effect: $J'_{tot} \sim (0.2\text{-}1.0)\, J$,
individual J''_{Zn} and $J'_{Zn} \sim$ 3-15% of J, with the ratio $J''_{Zn} / J'_{Zn} \approx 2 \sim 4$

C.-W. Liu, S. Liu, Y.-J. Kao, **SC**, and A. Sandvik, PRL **102**, 167201 (2009).

frustration effect in J-J'_{Zn}-J''_{Zn} model

$$\boxed{\mathcal{H} = J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j + J'_{Zn}\sum_{\langle ij\rangle'}\mathbf{S}_i\cdot\mathbf{S}_j + J''_{Zn}\sum_{\langle ij\rangle''}\mathbf{S}_i\cdot\mathbf{S}_j}$$

fix the ratio $J''_{Zn}/J'_{Zn} = 2$, vary J''_{Zn}/J



reduction of magnetization

$$\boxed{\delta M(z) = -\sum_{\mathbf{k}}\int_{-1}^{0}\frac{d\omega}{\omega_{\mathbf{k}}}\left[A^{11}_{\mathbf{k}}(\omega) - \gamma_{\mathbf{k}}A^{12}_{\mathbf{k}}(\omega)\right]}$$

$$A^{ij}_{\mathbf{k}}(\omega) = -\frac{1}{\pi}\mathrm{Im}\,\hat{G}^{ij}_{\mathbf{k}}(\omega)$$

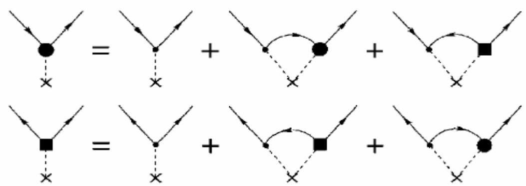
spin-wave approximation

$$\mathcal{H}_0 = \sum_{\mathbf{k}}\omega_{\mathbf{k}}(\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}} + \beta^\dagger_{\mathbf{k}}\beta_{\mathbf{k}})$$

$$\mathcal{H}_{imp} = -\sum_{l,\mathbf{k},\mathbf{k}'} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{R}_l}\hat{\mathcal{A}}^\dagger_{\mathbf{k}}\hat{\mathcal{V}}^l_{\mathbf{k},\mathbf{k}'}\hat{\mathcal{A}}_{\mathbf{k}'}$$

$$\hat{\mathcal{A}}_{\mathbf{k}} = \begin{bmatrix}\alpha_{\mathbf{k}}\\ \beta^\dagger_{-\mathbf{k}}\end{bmatrix},\quad \hat{\mathcal{A}}^\dagger_{\mathbf{k}} = [\alpha^\dagger_{\mathbf{k}},\ \beta_{-\mathbf{k}}]$$

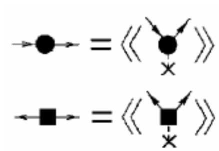
T-matrix method



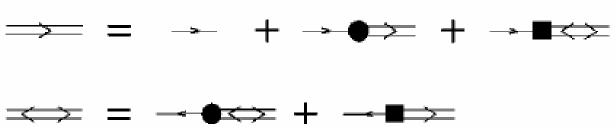
$$\hat{T}^{l,\mu}_{\mathbf{k},\mathbf{k}'}(\omega) = -\hat{\mathcal{V}}^{l,\mu}_{\mathbf{k},\mathbf{k}'} - \sum_{\mathbf{q}}\hat{\mathcal{V}}^{l,\mu}_{\mathbf{k},\mathbf{q}}\hat{G}^0_{\mathbf{q}}(\omega)\hat{T}^{l,\mu}_{\mathbf{q},\mathbf{k}'}(\omega)$$

$$G^{0,11}_{\mathbf{q}}(\omega) = G^{0,22}_{\mathbf{q}}(-\omega) = \frac{1}{\omega - \omega_{\mathbf{q}} + i0}$$

$$G^{0,12}_{\mathbf{q}}(\omega) = G^{0,21}_{\mathbf{q}}(\omega) = 0$$



$$\hat{\Sigma}_{\mu,\mathbf{k}}(\omega) = z\delta_{\mathbf{k}-\mathbf{k}'}\left[\hat{T}^{A,\mu}_{\mathbf{k},\mathbf{k}'}(\omega) + \hat{T}^{B,\mu}_{\mathbf{k},\mathbf{k}'}(\omega)\right]$$

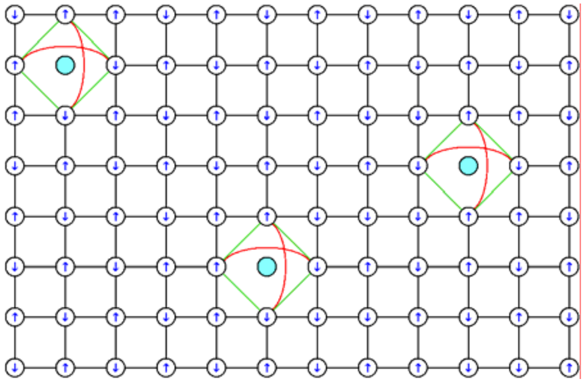


$$\hat{G}_{\mathbf{k}}(\omega) = \begin{pmatrix} -\omega - \omega_{\mathbf{k}} - \Sigma^{22}_{\mathbf{k}}(\omega) & \Sigma^{12}_{\mathbf{k}}(\omega) \\ \Sigma^{21}_{\mathbf{k}}(\omega) & \omega - \omega_{\mathbf{k}} - \Sigma^{11}_{\mathbf{k}}(\omega)\end{pmatrix}\frac{1}{[\omega - \omega_{\mathbf{k}} - \Sigma^{11}_{\mathbf{k}}(\omega)][-\omega - \omega_{\mathbf{k}} - \Sigma^{22}_{\mathbf{k}}(\omega)] - [\Sigma^{12}_{\mathbf{k}}(\omega)]^2}$$

frustration effect in J-J'_{Zn}-J''_{Zn} model

$$\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J'_{Zn} \sum_{\langle ij \rangle'} \mathbf{S}_i \cdot \mathbf{S}_j + J''_{Zn} \sum_{\langle ij \rangle''} \mathbf{S}_i \cdot \mathbf{S}_j$$

fix the ratio $J''_{Zn} / J'_{Zn} = 2$, vary J''_{Zn} / J



reduction of magnetization

$$\delta M(z) = -\sum_{\mathbf{k}} \int_{-1}^{0} \frac{d\omega}{\omega_{\mathbf{k}}} [A^{11}_{\mathbf{k}}(\omega) - \gamma_{\mathbf{k}} A^{12}_{\mathbf{k}}(\omega)]$$

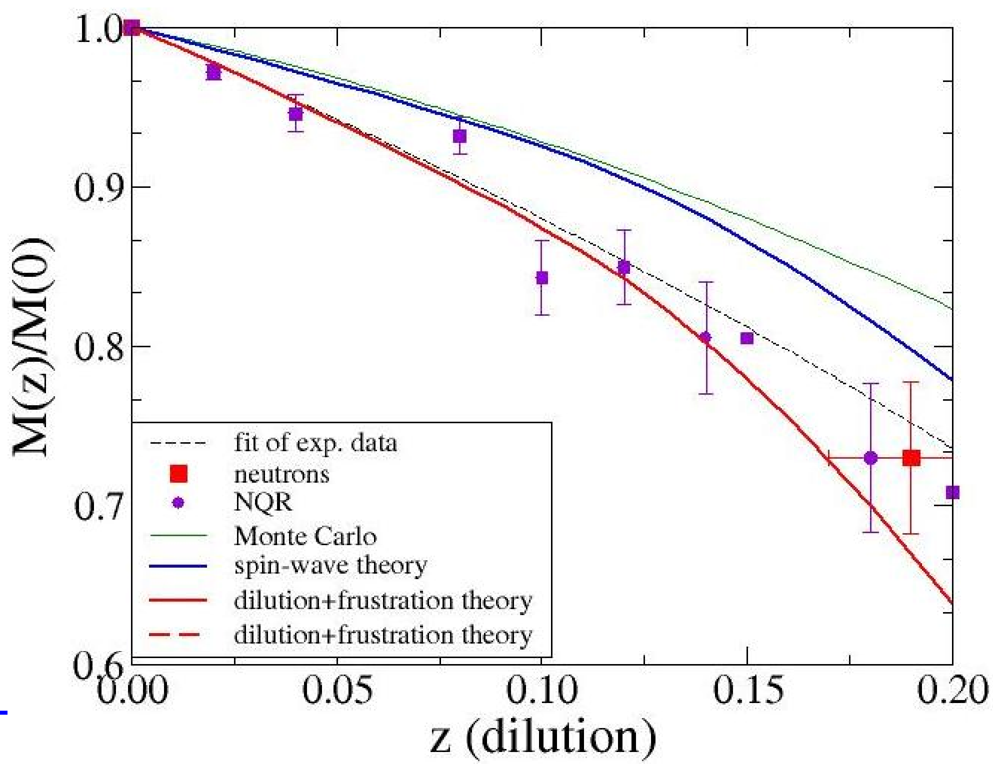
both T-matrix and QMC:

J''_{Zn} is twice as effective as J'_{Zn} of the same strength in suppressing order

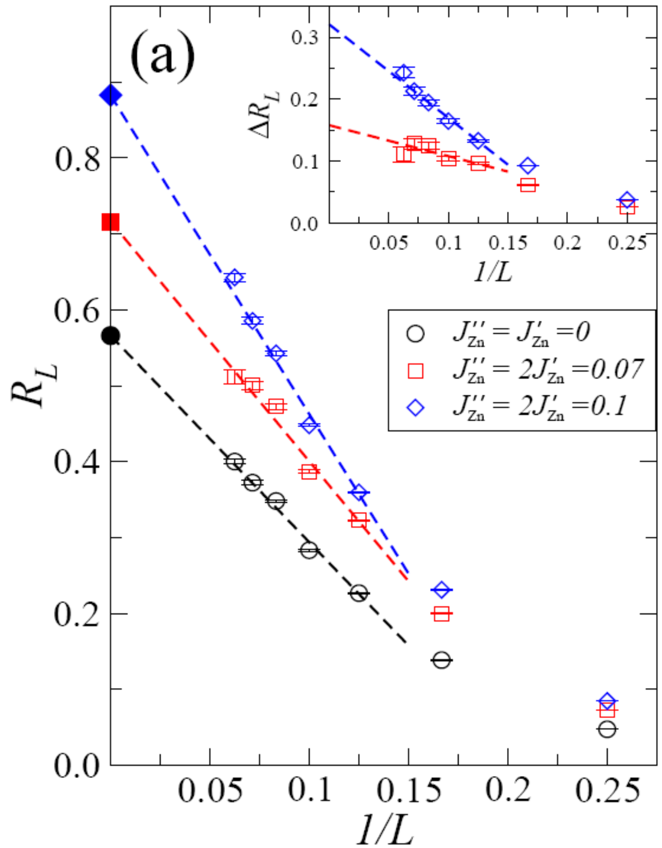
T-matrix :

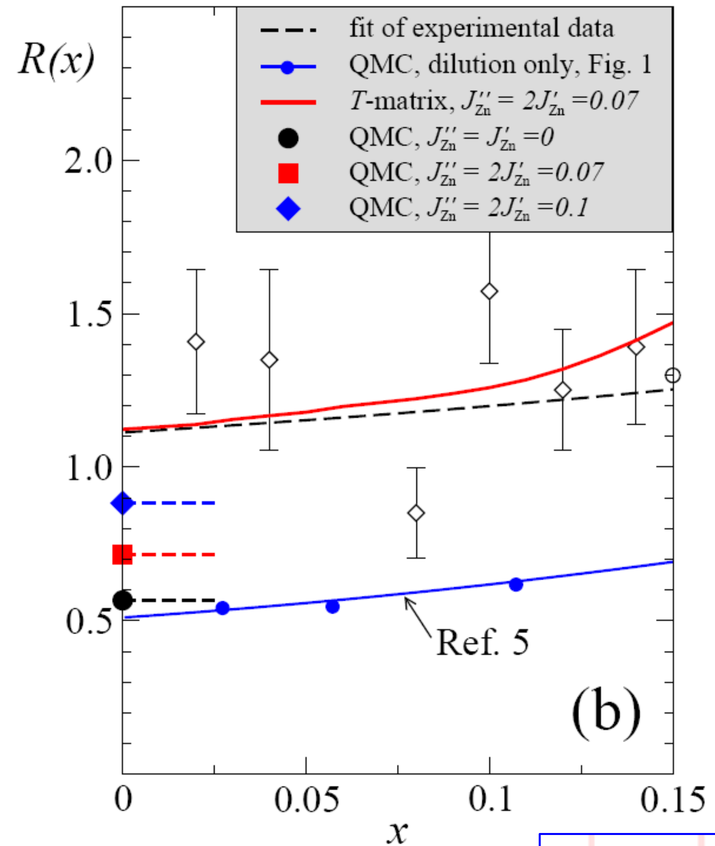
$J''_{Zn} = 2 J'_{Zn} = 0.07$ fits the data

$(J'_{tot} = 0.28 J)$

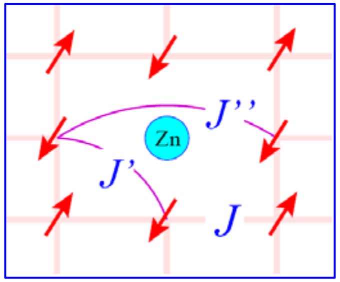


QMC can "beat" the sign problem





1/L- extrapolation yields *R(0)* -- *x=0* slope

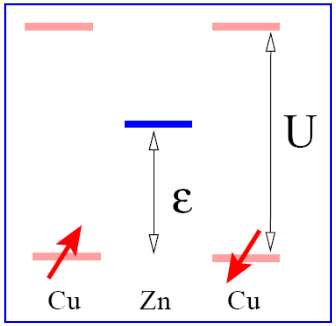


QMC supports T-matrix, implies somewhat larger frustration:
$J''_{Zn} = 2\,J'_{Zn} \geq 0.1$ to fit the data;
still modest $J'_{tot} \geq 0.4\,J$, well within the 3-band estimates

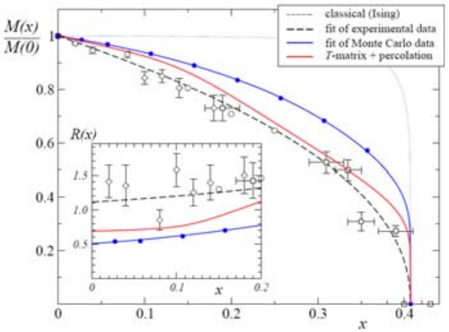
C.-W. Liu, S. Liu, Y.-J. Kao, **SC**, and A. Sandvik, PRL **102**, 167201 (2009).

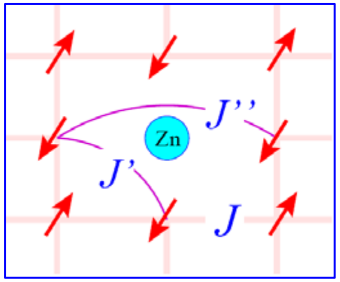
conclusions

- **interesting effect:** a non-frustrated AF doped with spinless impurities must have frustrations around them
- explains the discrepancy of theory-experiment
- should apply to other oxides, halides, fluorides …



- **remaining questions:**
- near the percolation threshold? Glass?
- hole-doped case near the impurity. More pair breaking?





C.-W. Liu, S. Liu, Y.-J. Kao, **SC**, and A. Sandvik, PRL **102**, 167201 (2009).

spin excitations in non-collinear quantum antiferromagnets

frustrated, excited, and breaking apart
the Bright Side of Life of spin waves (in 2D)

Sasha Chernyshev and Mike Zhitomirsky

UCIrvine
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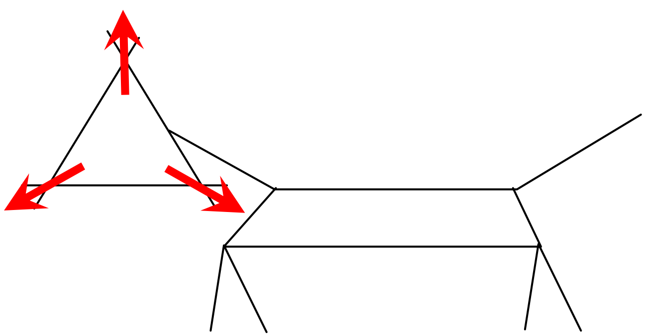
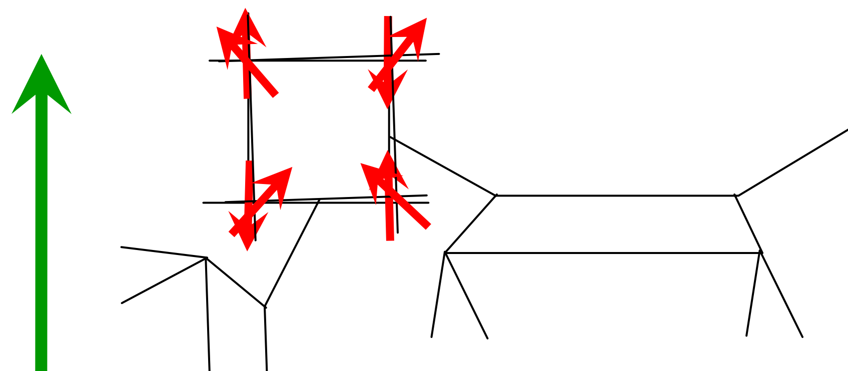
cea Grenoble

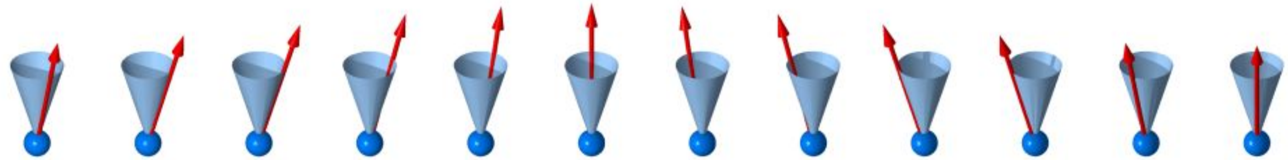




SC and **MZ**, PRL **97**, 207202 (2006);
PRB **79**, 144416 (2009)

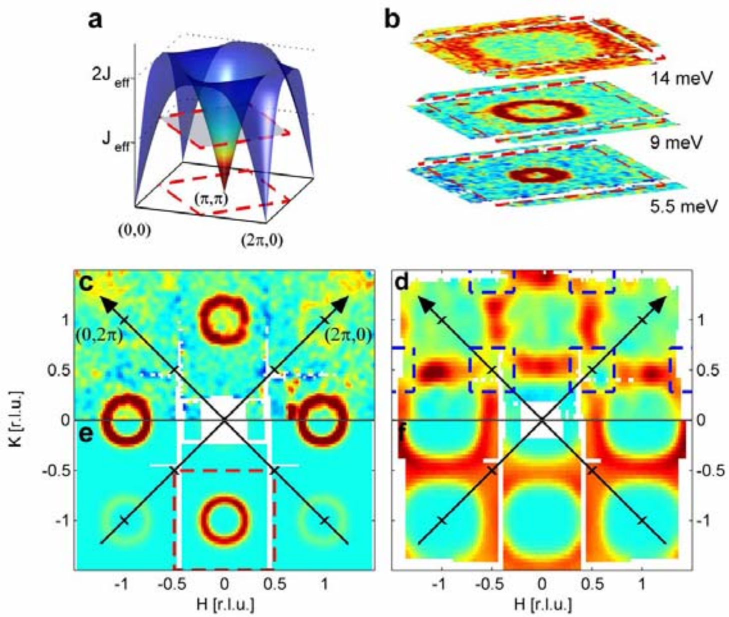
MZ and **SC**, PRL **82**, 4536 (1999);
M. Mourigal, **MZ**, and **SC**, (unpublished)

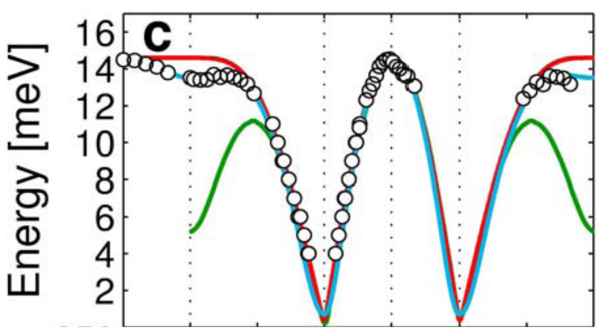
from the Past ...



- spin waves, well defined elementary excitations in *D>1*
- *1930, Felix Bloch*
- theory: *Holstein, Primakoff, Dyson, …*
- *1957,* experiment, INS: *Brockhouse, …*

➔ sharp peaks





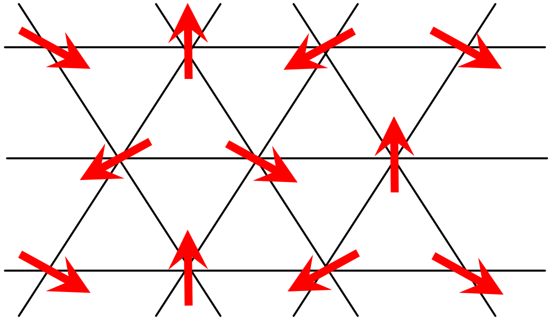
2D spin-1/2 quantum antiferromagnet copper deuteroformate tetradeuterate (CFTD)

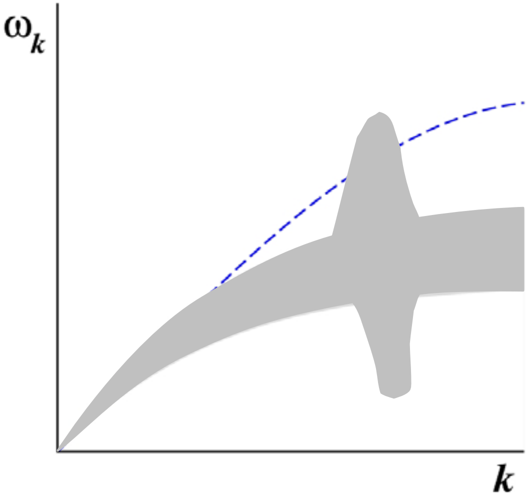
N. B. Christensen etal., PNAS **104**, 15264 (2007)

2-point summary

spin waves in **non-collinear** AFs:

- may decay even at *T=0* (yield broad peaks)
- may have singularities in their spectrum

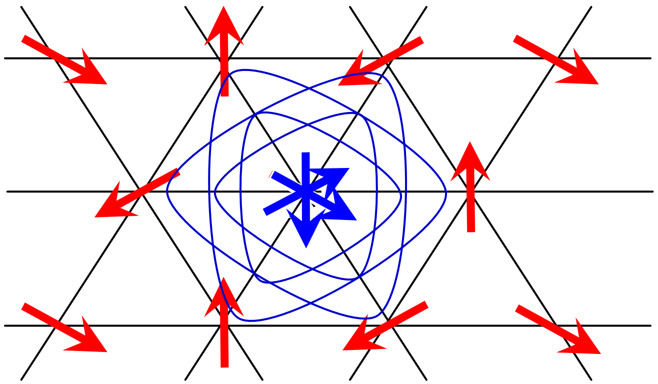


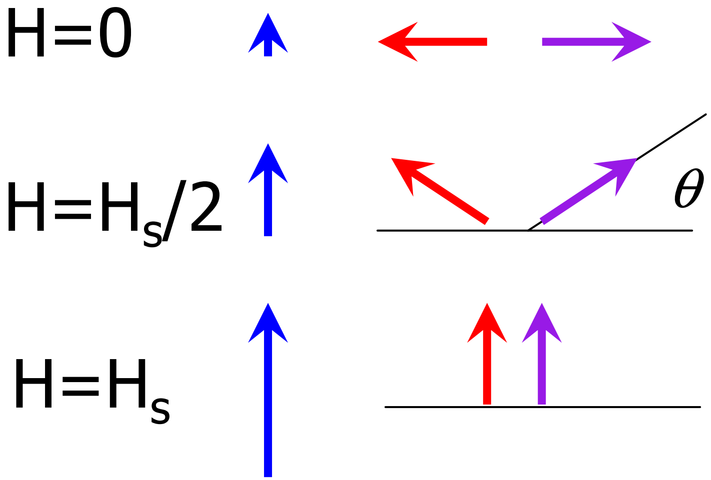


sequence of events …

frustration (competing interactions) → **non-collinearity** → transverse-longitudinal coupling → 3-boson terms → decays

triangular = non-collinear order

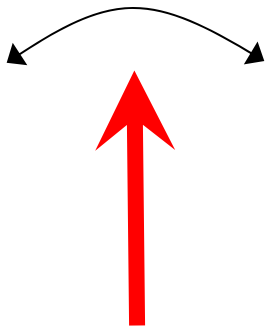




☑ compromise between competing interactions

sequence of events …

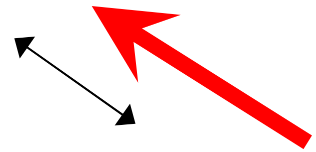
frustration (competing interactions) → **non-collinearity**
→ $\boxed{\text{transverse-longitudinal coupling}}$ → 3-boson terms → decays



transverse (T)

"one-magnon"

$$S^{-} \approx \sqrt{2S}\, a^{\dagger}$$

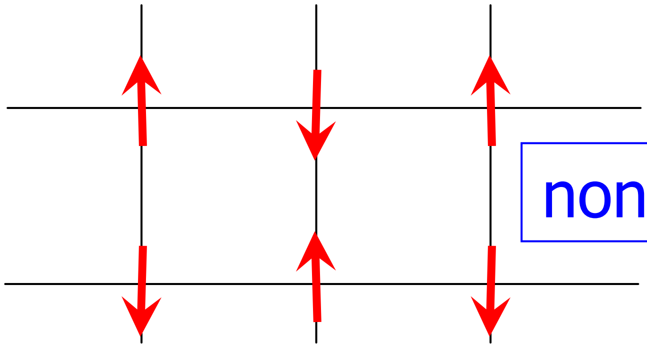


longitudinal (L)

"two-magnon"

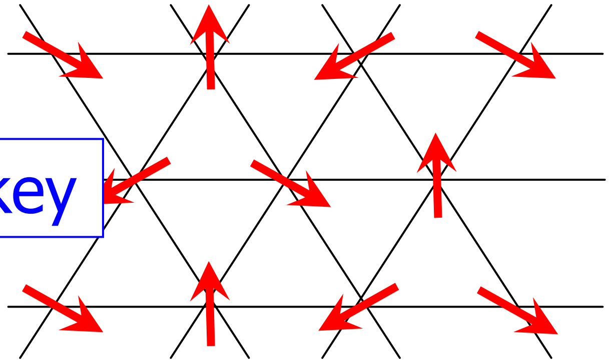
$$S^{z} = S - a^{\dagger} a$$

only T-T or L-L



non-collinearity is the key

strong T-L



remaining $U(1)$ symmetry forbids $S^{z}S^{\pm}$ terms

1-magnon ←→ 2-magnon

frustration (competing interactions) → **non-collinearity**
→ transverse-longitudinal coupling → 3-boson terms → decays

$$\begin{aligned}\hat{\mathcal{H}} &= \frac{1}{2}J\sum_{i,\boldsymbol{\delta}}\Big\{\frac{1}{4}[1+\cos(\mathbf{Q}\cdot\boldsymbol{\delta})](S_i^+S_j^- + S_i^-S_j^+) \\ &\quad + S_i^zS_j^z\cos(\mathbf{Q}\cdot\boldsymbol{\delta}) - S_i^z(S_j^+ + S_j^-)\sin(\mathbf{Q}\cdot\boldsymbol{\delta}) \\ &\quad - \frac{1}{4}[1-\cos(\mathbf{Q}\cdot\boldsymbol{\delta})](S_i^+S_j^+ + S_i^-S_j^-)\Big\}\end{aligned}$$

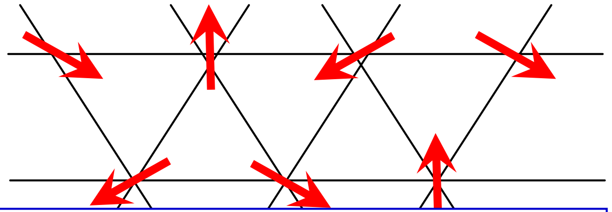
cubic vertex $V_{\mathbf{k},\mathbf{q}} \sim \sin\theta$ for collinear structures

$$\Downarrow$$

coupling of magnon with 2-magnon continuum

$$\hat{\mathcal{H}} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k},\mathbf{q}} V_{\mathbf{k},\mathbf{q}}\left(b_{\mathbf{k}-\mathbf{q}}^\dagger b_{\mathbf{q}}^\dagger b_{\mathbf{k}} + \text{h.c.}\right) + \ldots$$

$$\Downarrow$$



- three-boson terms are necessary for the decays
- "kinematic" conditions (*E*- and *k*- conservations) make it sufficient

two key points

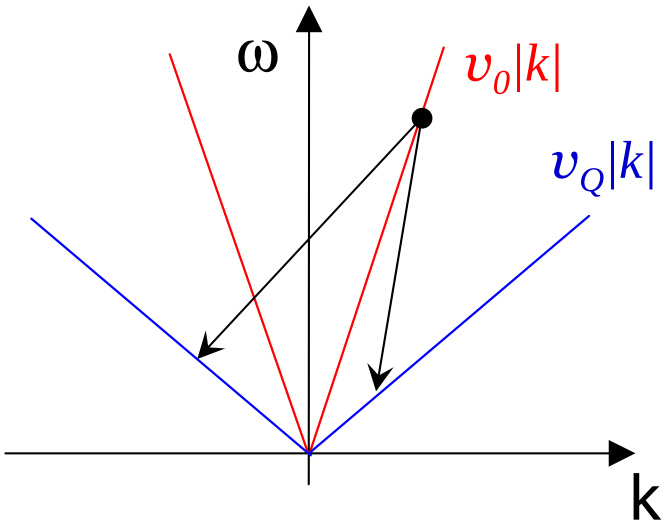
→ three-boson interactions

(can be phonons, bosons in ^{4}He, triplets in spin-gap systems)

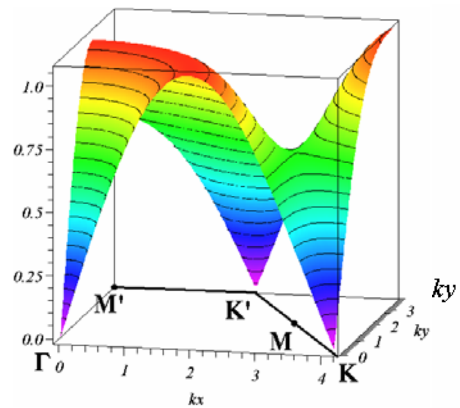
→ kinematic conditions

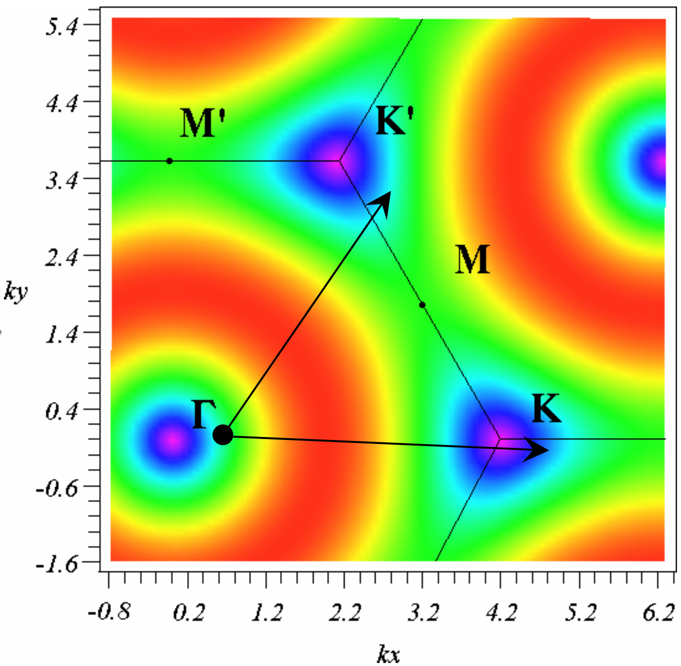
$$\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{k}-\mathbf{q}}$$

how common is “sufficient”?



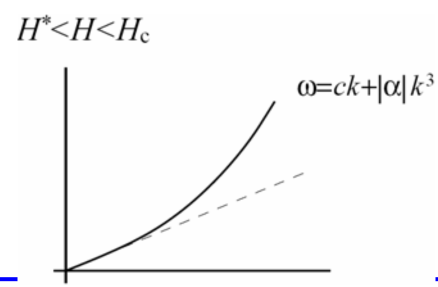
$$\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{k}-\mathbf{q}}$$

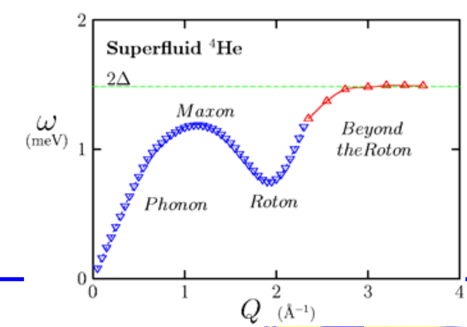




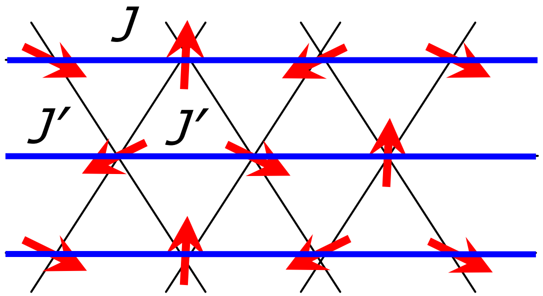
for a **spiral-like state** (more than one type of Goldstone mode)
kinematic conditions are <u>always</u> fulfilled for some k because $v_0 > v_Q$

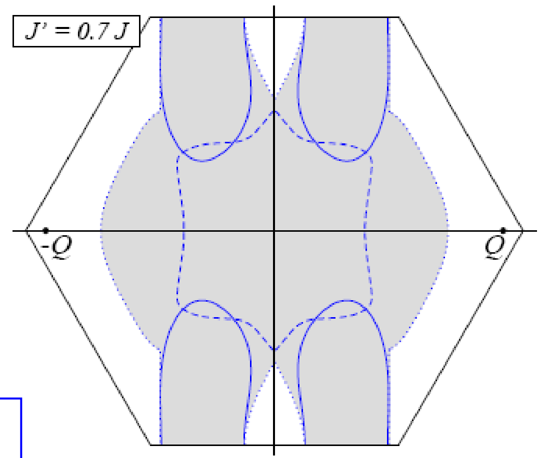
other kinematic cases exist too …

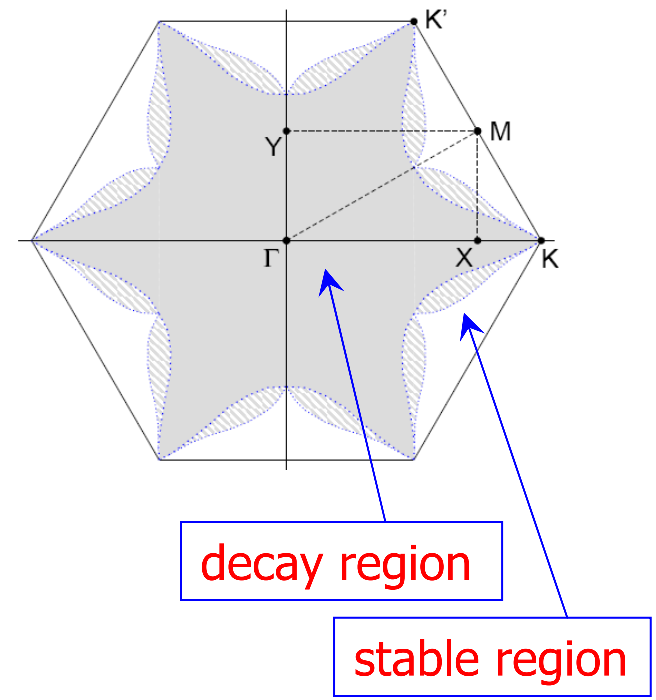




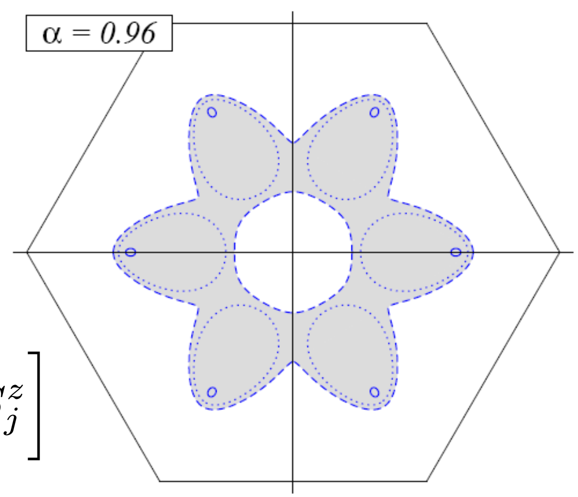
decays **are** common







- large decay regions
- various models
- sustain *1/S*

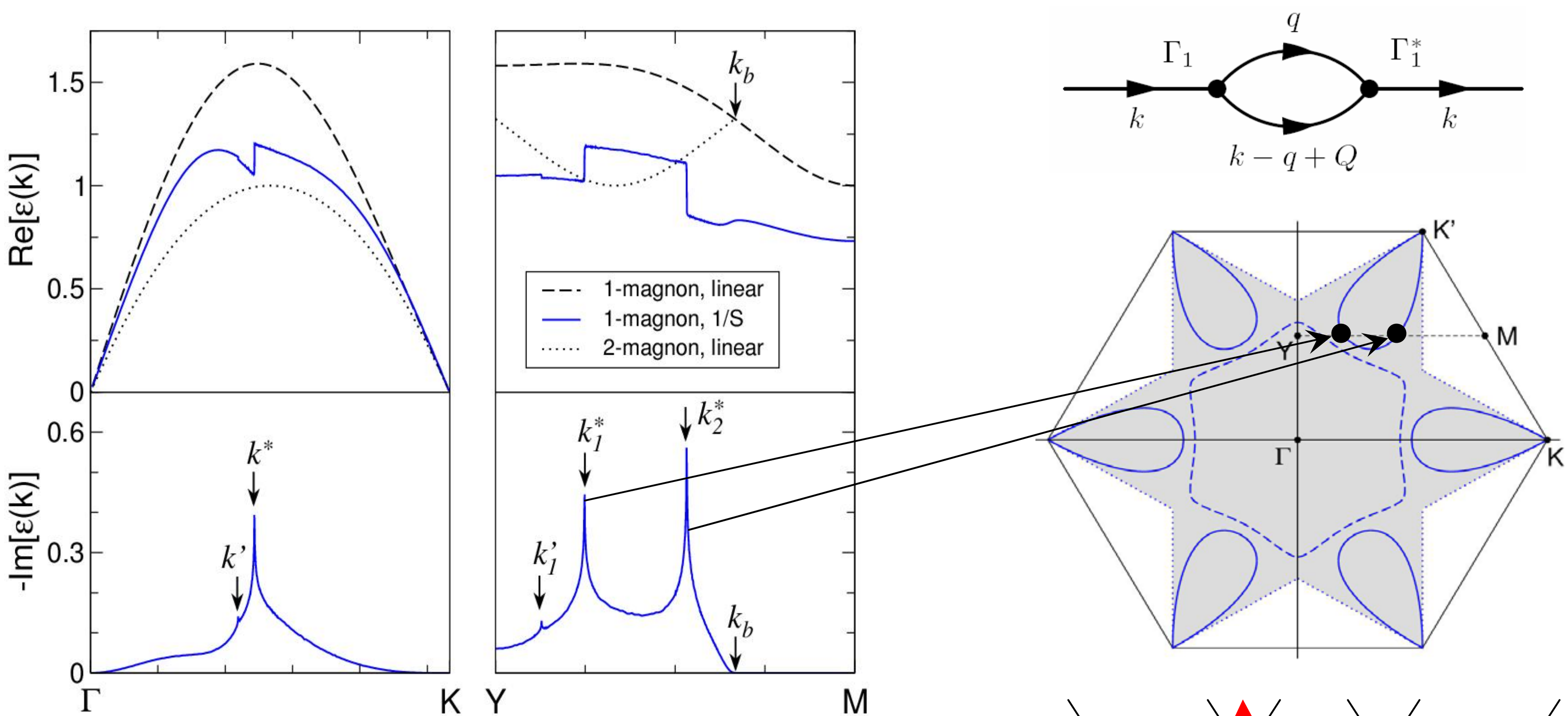


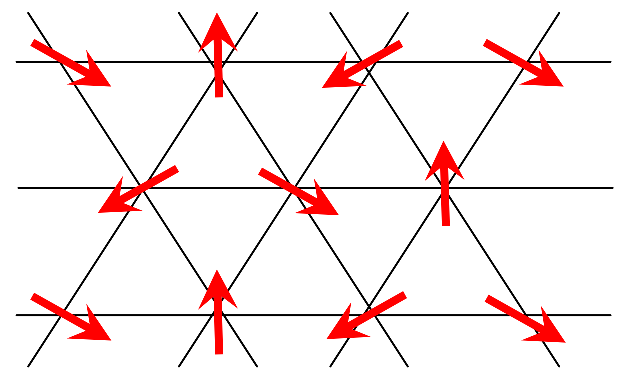
$$\hat{\mathcal{H}} = J \sum_{\langle ij \rangle} \left[S_i^x S_j^x + S_i^y S_j^y + \alpha S_i^z S_j^z \right]$$

Kagome′ jarosite too …

SC and **MZ**, PRB **79**, 144416 (2009)

singularities in *1/S* theory, I





singularities in decay rates, what are they?

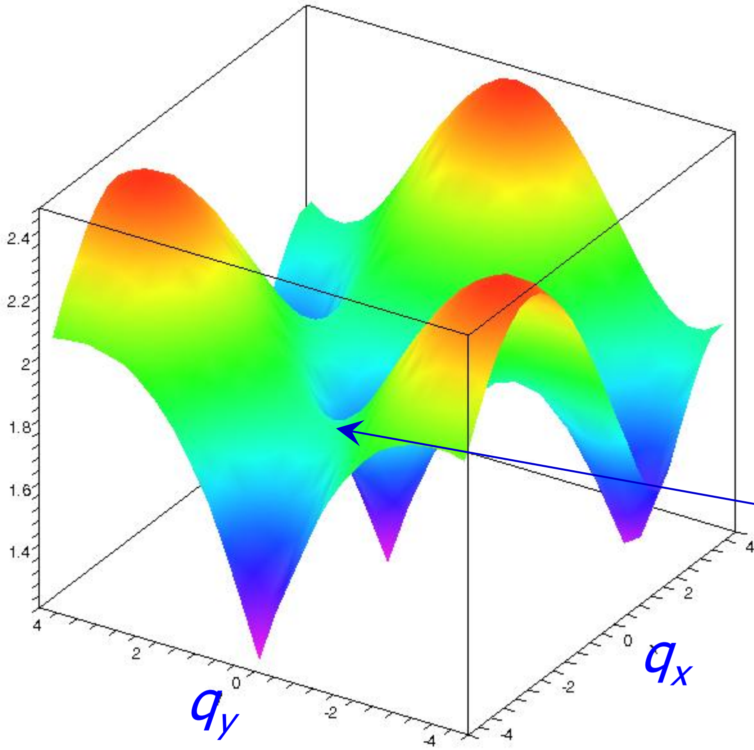
small *k* large *k*

$$\Gamma_{\mathbf{k}} \propto k^2 \ll \varepsilon_{\mathbf{k}} \propto k$$

$$\frac{2\Gamma_{\mathbf{k}}}{\varepsilon_{\mathbf{k}}} \approx 0.3$$

SC and **MZ**, PRB **79**, 144416 (2009)

singularities in *1/S* theory, II

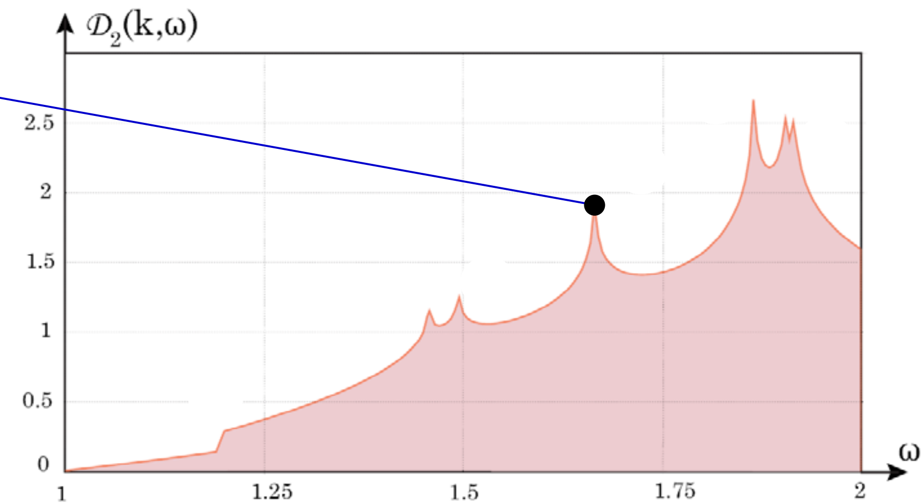


2-magnon continuum at some *k*

energy of the two-particle continuum

$$E_{\mathbf{k},\mathbf{q}} = \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{k}-\mathbf{q}}$$

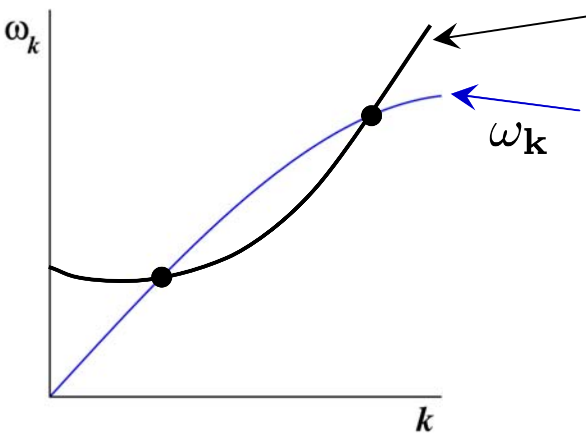
van Hove singularities = saddle points
2D → *log(Δω)*



2-magnon continuum DoS at some *k*

SC and **MZ**, PRB **79**, 144416 (2009)

singularities in *1/S* theory, III

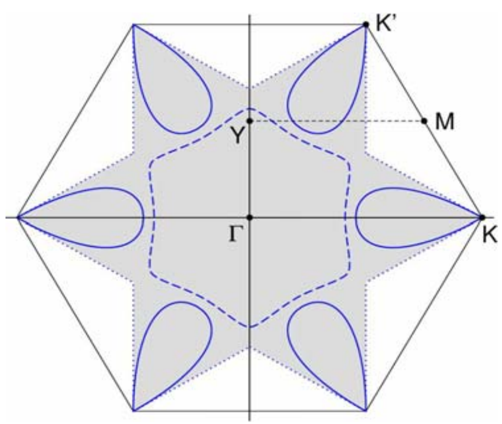


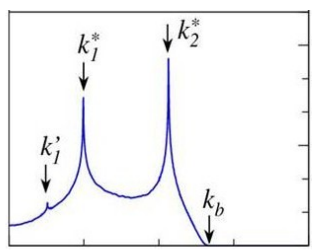
van Hove singularity vs *k*

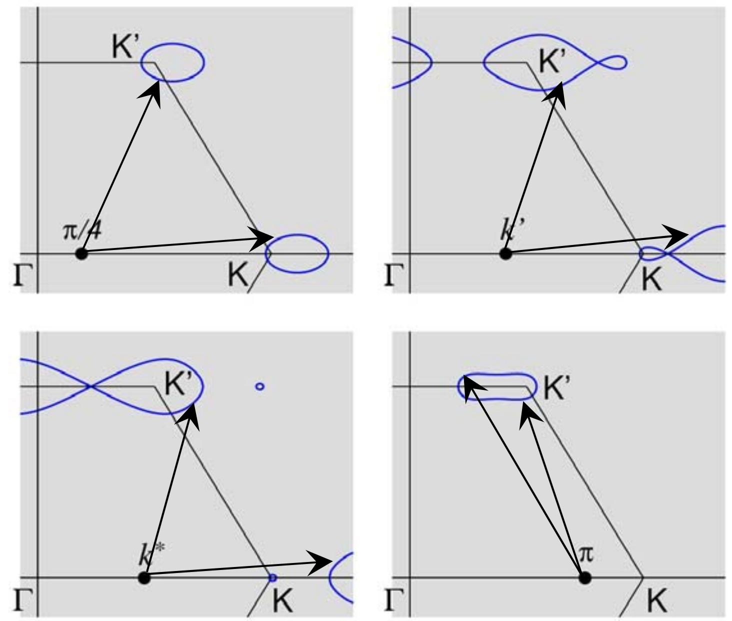
magnon energy

≡ topological transitions in the **decay surface**

intersection yields singularites

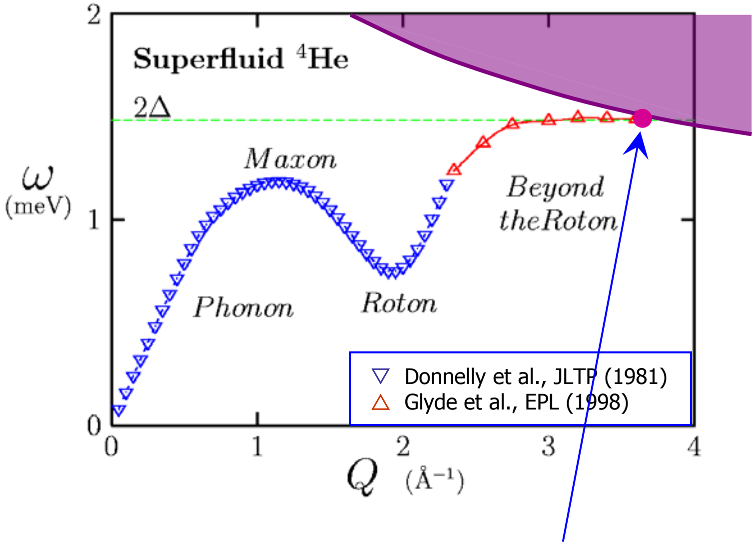






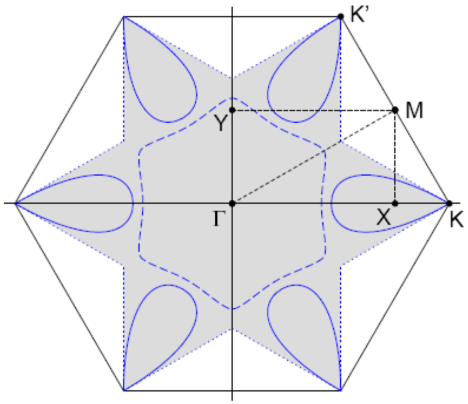
SC and **MZ**, PRB **79**, 144416 (2009)

“new” type of singularity



termination point

different type of threshold points



saddle points of the 2-magnon continuum crosses single-particle branch → singularity

minimum of the 2-particle continuum crosses single-particle branch → singularity

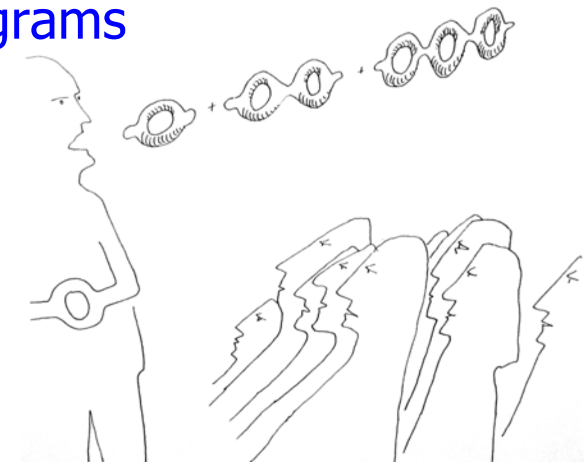
L. P. Pitaevskii, Sov. Phys. JETP **36**, 830 (1959)

regularization of singularities

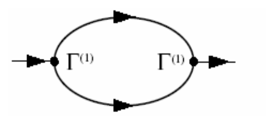
- singularity means breakdown of the *1/S* expansion
- singularities needs to be regularized:
  - method #1 (Pitaevskii):

  -- summation of an infinite series of loop diagrams
  - method #2 (us):

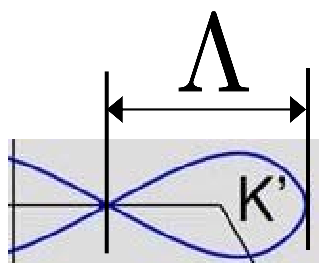
  -- "off-shell"

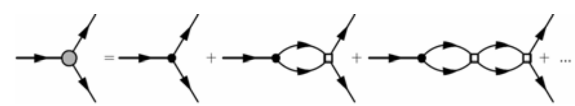


near the singularity (one-loop):



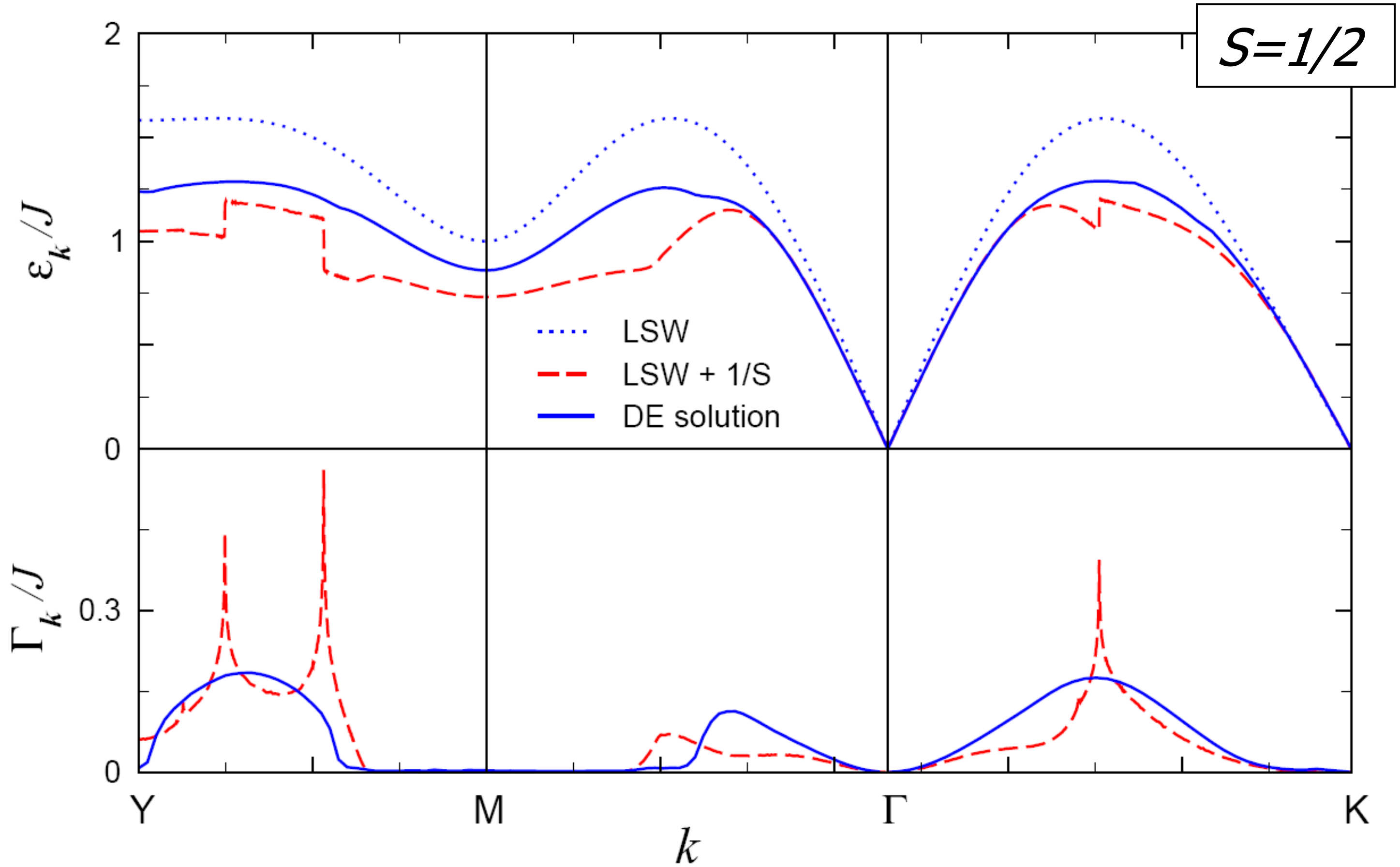
$$\mathrm{Im}\{\Sigma_k\} \propto -\ln|\Lambda/\Delta k|$$
$$\mathrm{Re}\{\Sigma_k\} \propto \mathrm{sgn}(\Delta k)$$





leads to **log-enhancement** of the decay rate: $\Gamma_{\mathbf{k}} \propto \ln(S\Lambda)$

off-shell Dyson's equation

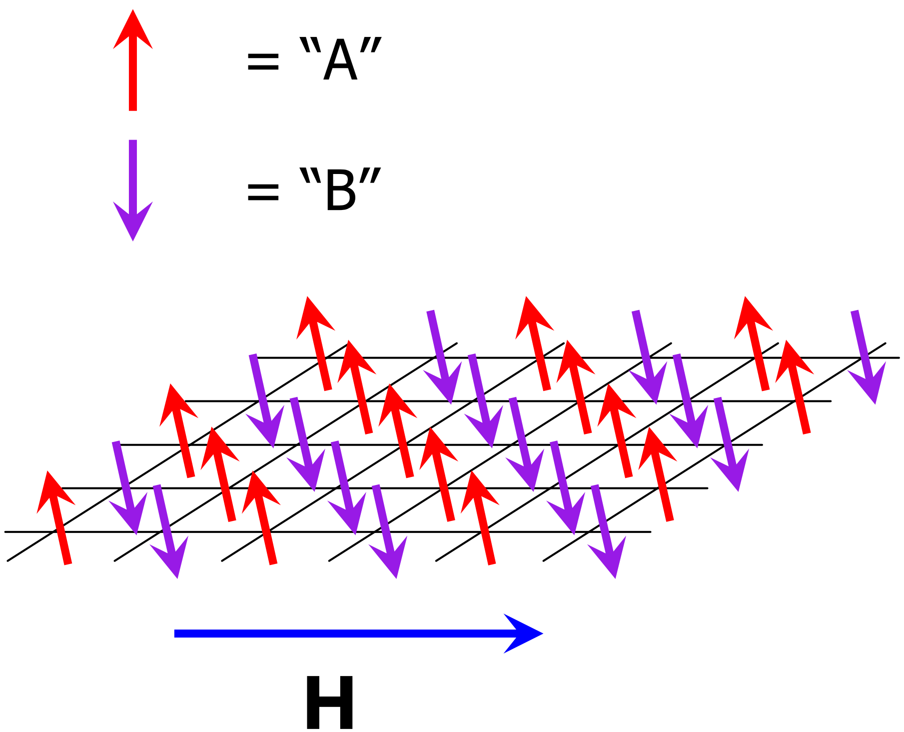


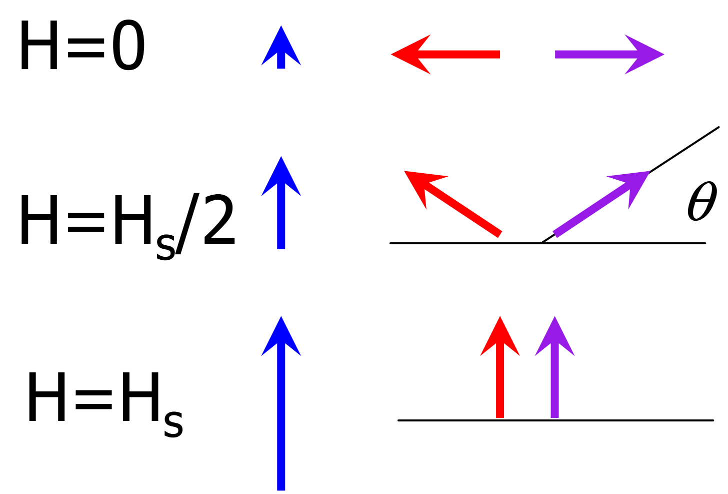
- singularities removed, no jumps, $\Gamma_{\boldsymbol{k}}$ remains substantial

SC and **MZ**, PRB **79**, 144416 (2009)

Heisenberg + field

$$\mathbf{H} \quad = J\sum_{<ij>}\mathbf{S}_i\cdot\mathbf{S}_j-\sum_i\mathbf{H}\cdot\mathbf{S}_i$$



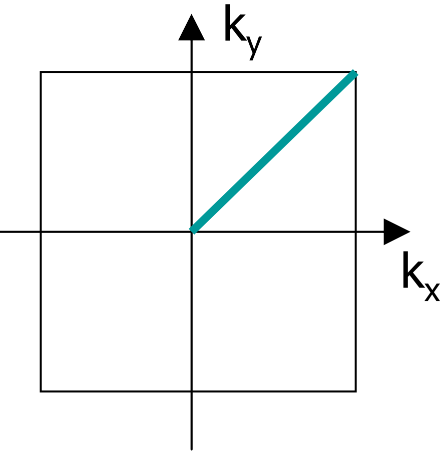


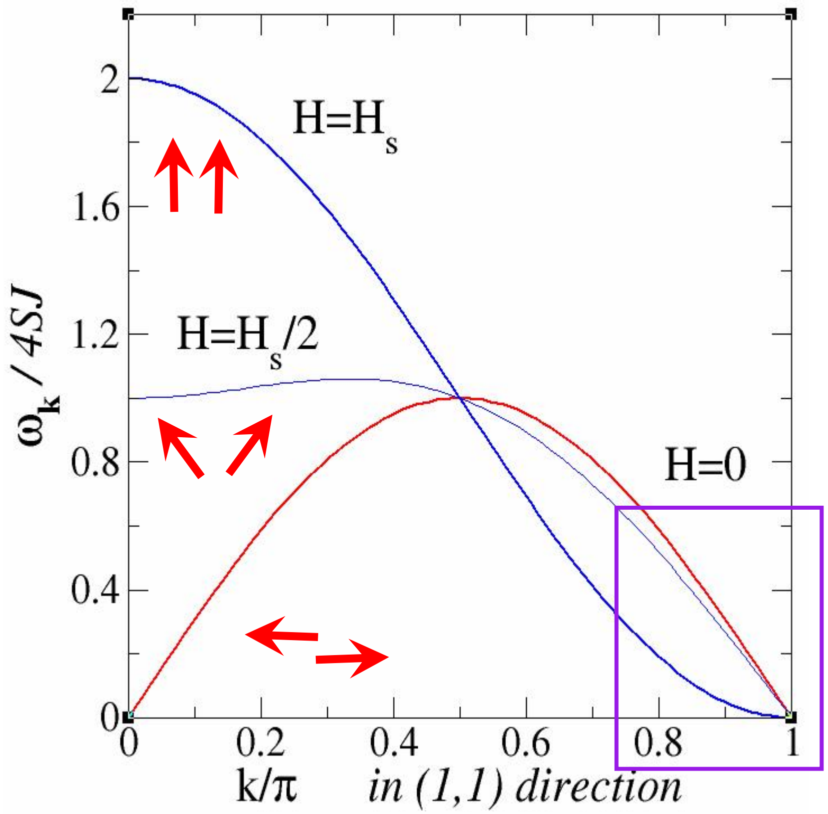
$H_s = 8\,S\,J$

$H/H_s = \sin\theta$

field induces non-collinearity

spectrum in a field, I

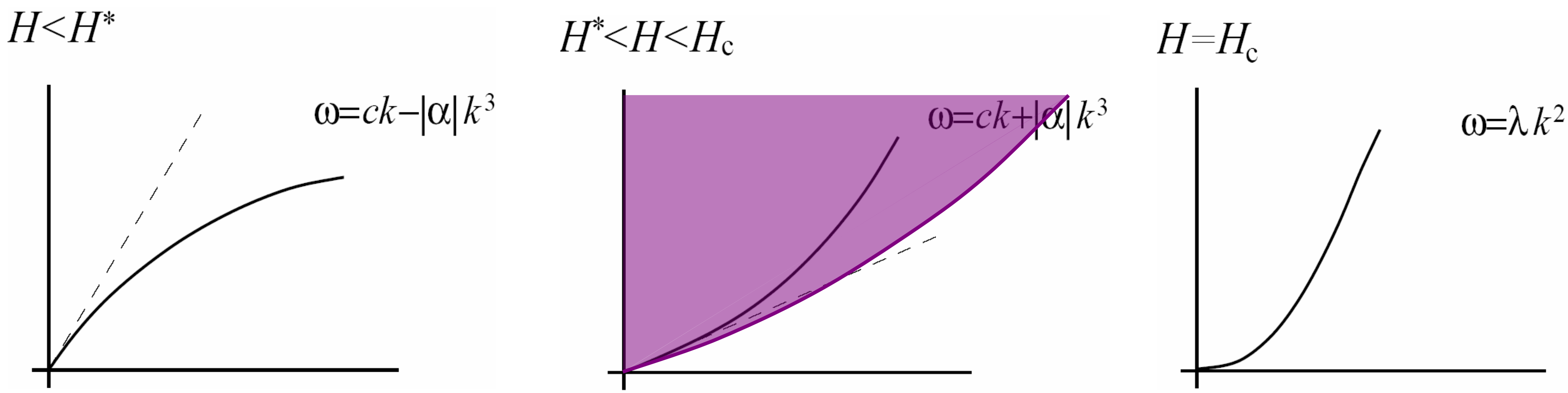


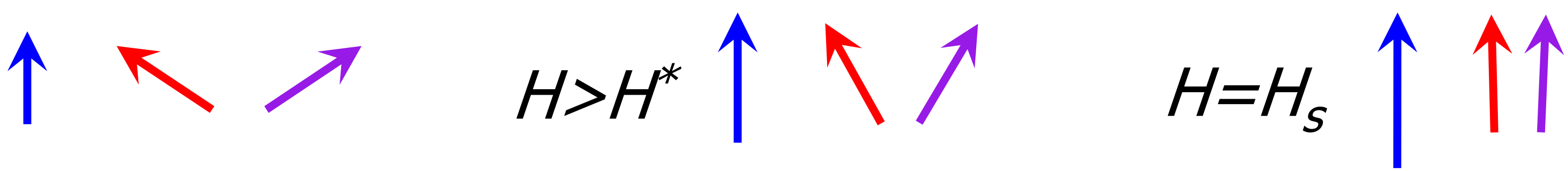


field induces a gap at (0,0) = "uniform precession mode"
with a simple field dependence: $\Delta_{00} = H$

M. E. Zhitomirsky and **SC**, PRL **82**, 4536 (1999).

spectrum in a field, II





$H<H^*$ convex

$H>H^*$ concave

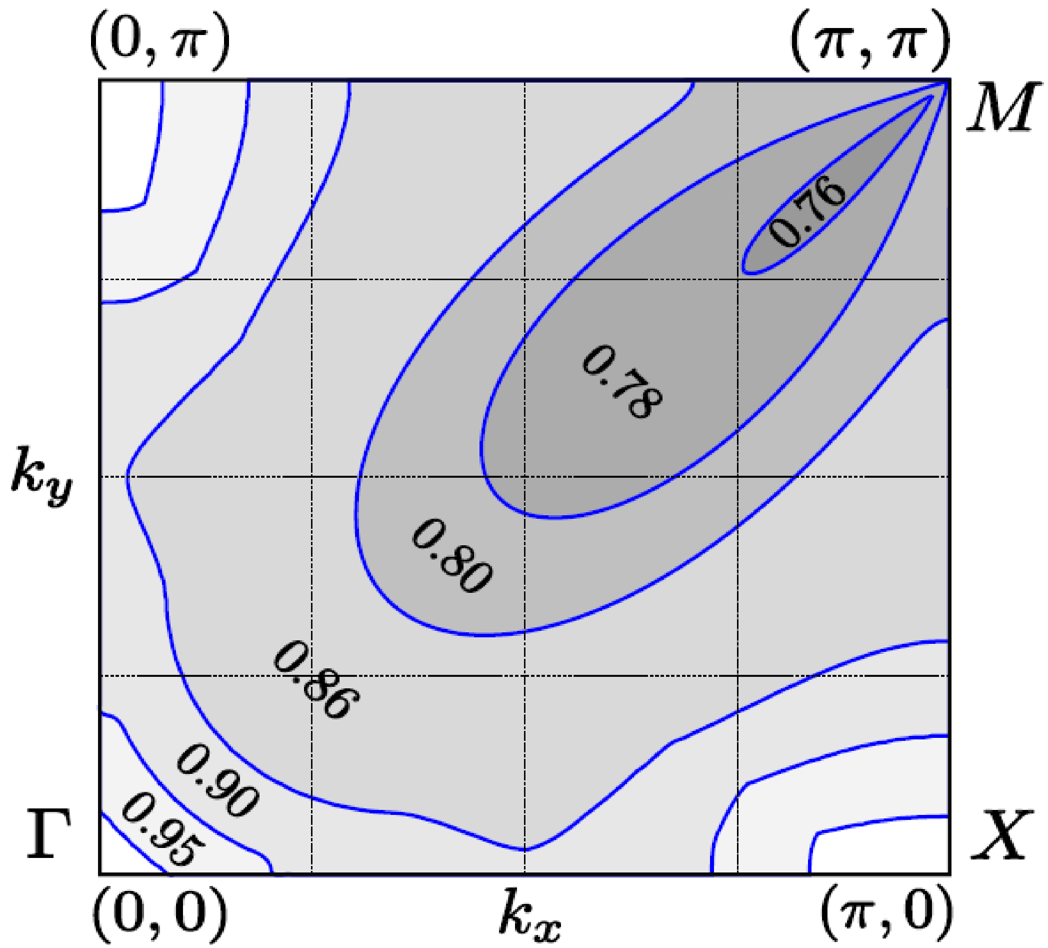
$H=H_s$

two-particle continuum energy: $E^{(2)}_{\mathbf{k}}(\mathbf{q}) = \omega(\mathbf{q}) + \omega(\mathbf{k-q})$

$$\mathbf{H} = \Sigma_{\mathbf{k}}\, \omega(\mathbf{k})\, a_{\mathbf{k}}^{+} a_{\mathbf{k}} + \Sigma_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} (a_{\mathbf{k}} a^{+}_{\mathbf{k}'} a^{+}_{\mathbf{k}-\mathbf{k}'} + \mathrm{H.c.}) + \ldots$$

M. E. Zhitomirsky and **SC**, PRL **82**, 4536 (1999).

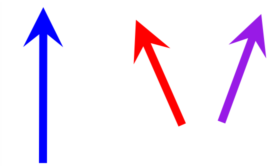
decay regions in a field

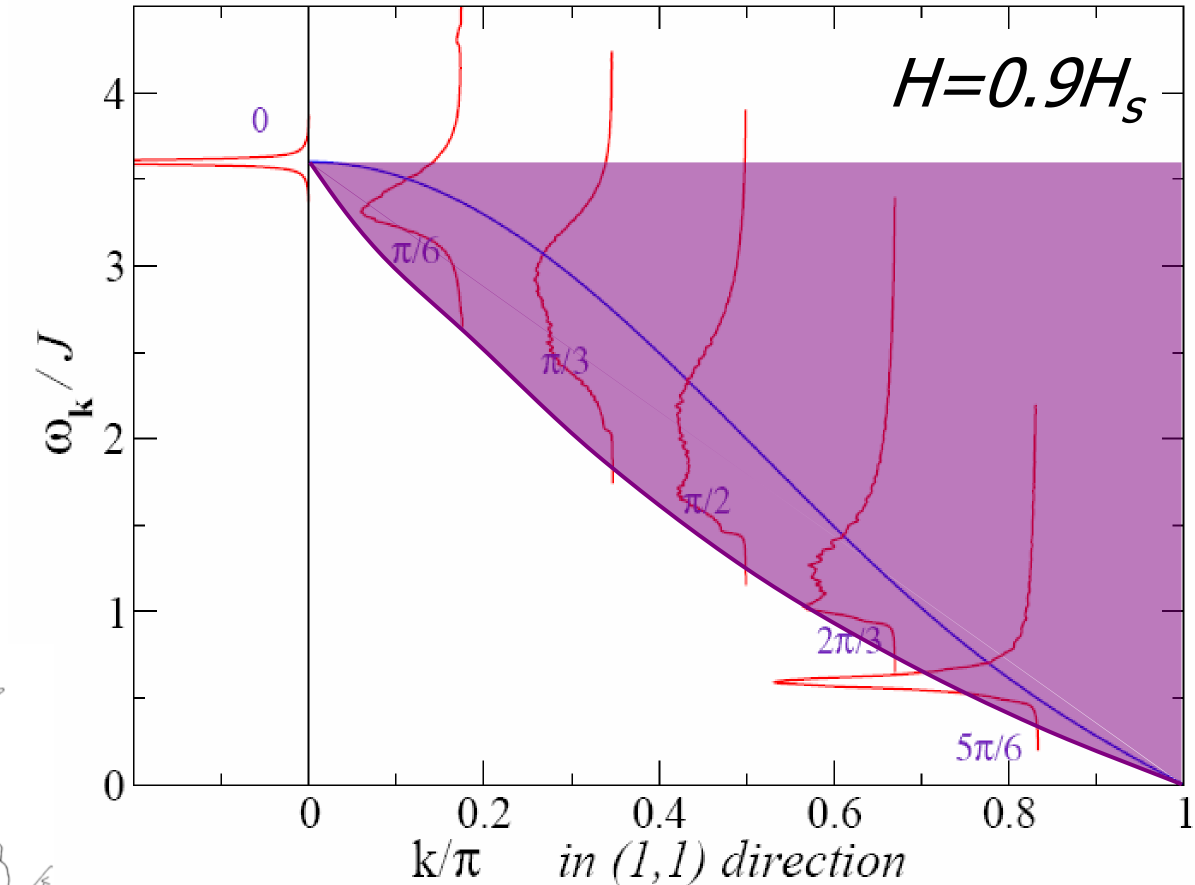


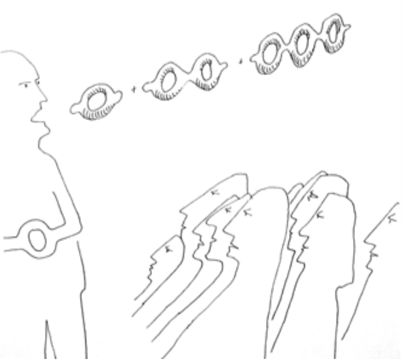
M. Mourigal, M. E. Zhitomirsky, and **SC**, unpublished

broad peaks in a field

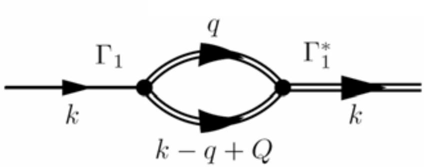
"cascading" decay … 2-particle continuum: $E^{(2)}_{\mathbf{k}}(\mathbf{q}) = \omega(\mathbf{q}) + \omega(\mathbf{k-q})$





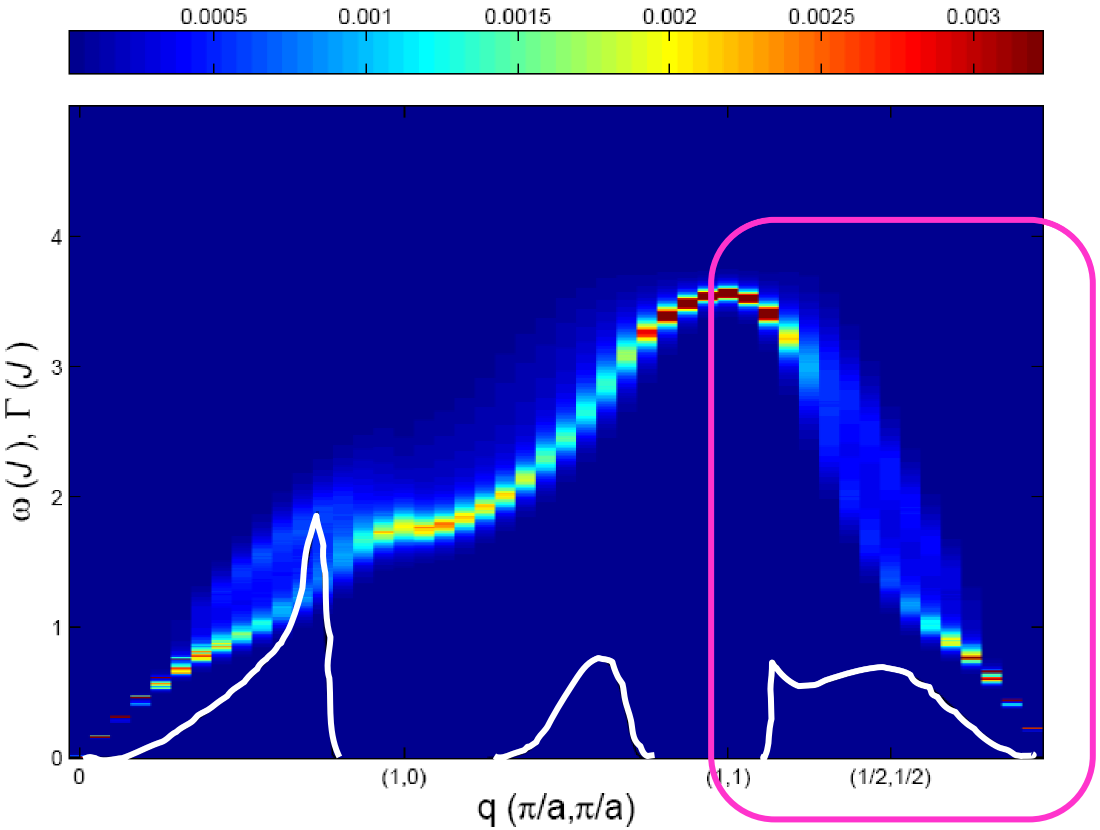


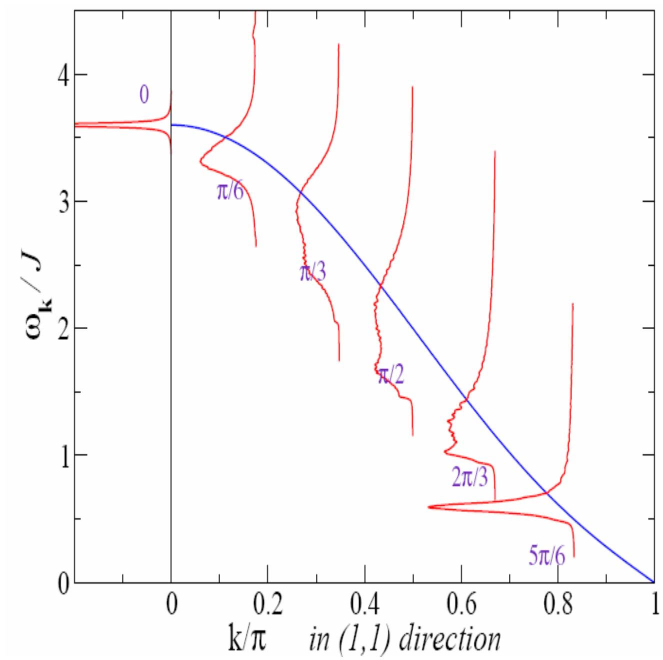
self-consistent Born approximation



M. E. Zhitomirsky and **SC**, PRL **82**, 4536 (1999).

numerical confirmation

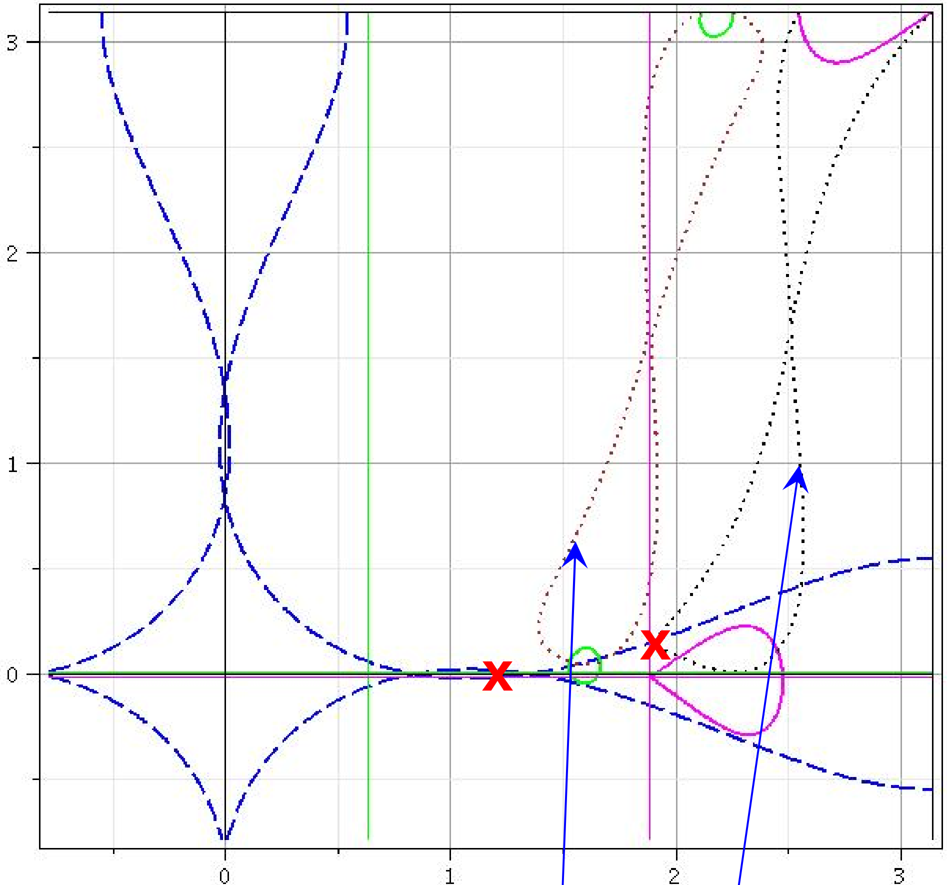


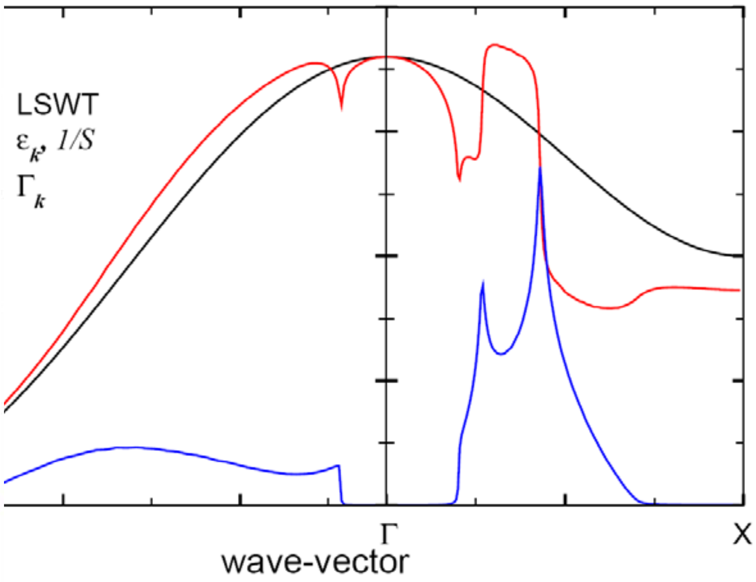


O. F. Syljuåsen, Phys. Rev. B **78**, 180413 (2008)
A. Lüscher and A. M. Läuchli, Phys. Rev. B **79**, 195102 (2009)

MZ and **SC**, PRL **82**, 4536 (1999).

singularities in a field

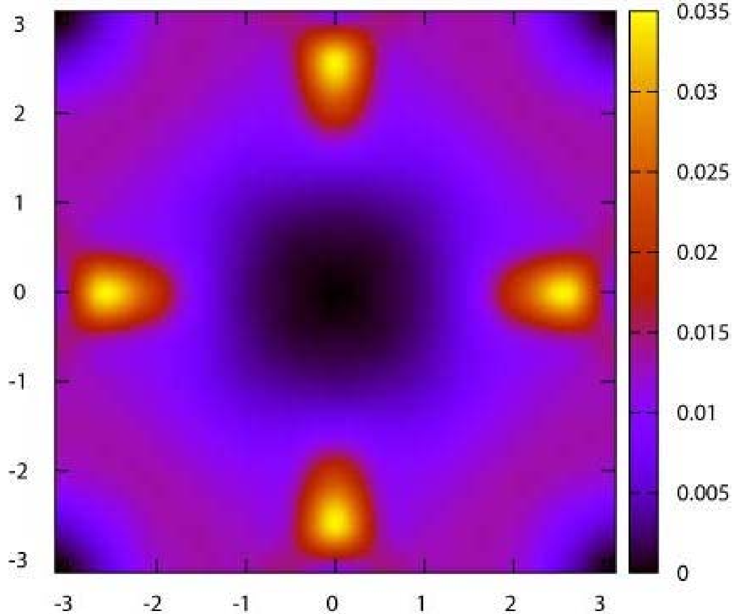




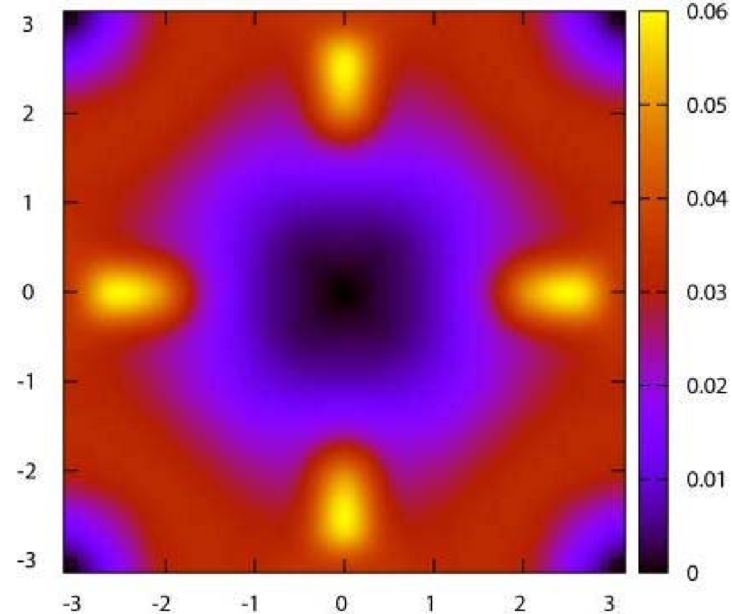
decay surfaces (contours)

M. Mourigal, M. E. Zhitomirsky, and **SC**, unpublished

decay rate intensity maps

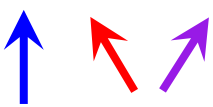


S=5/2, H=0.98H$_s$



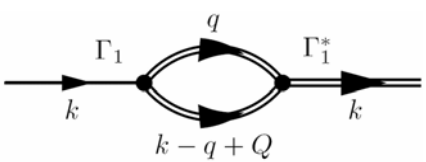
S=1, H=0.9H$_s$

*H>H**



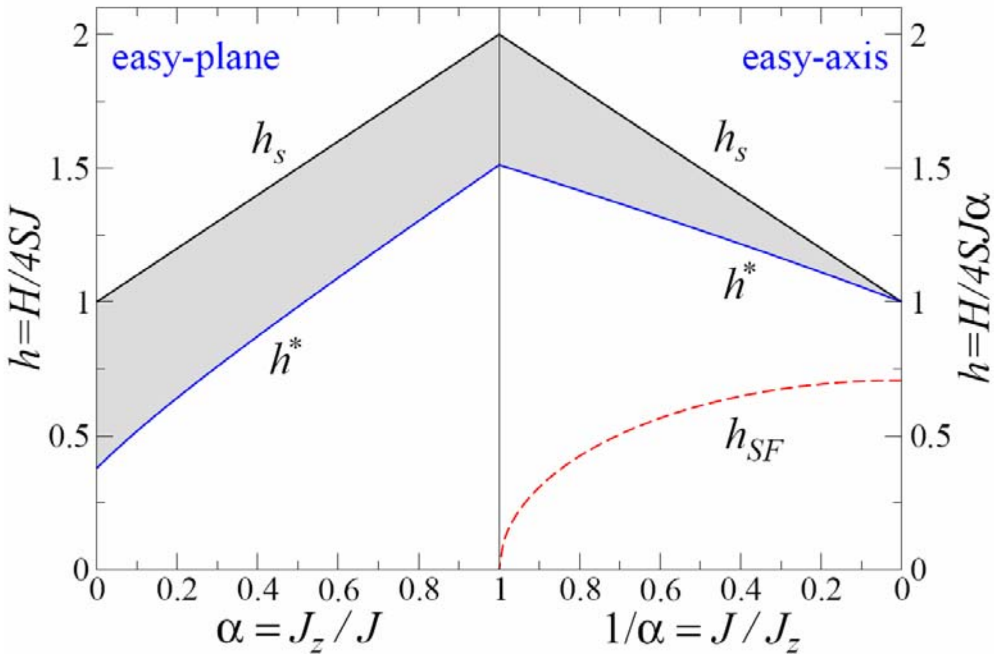
- remnants of singularities $\Gamma_{\mathbf{k}} \propto \ln(S\Lambda)$
- can be used for "fingerprinting" spin-wave decays

self-consistent Born approximation

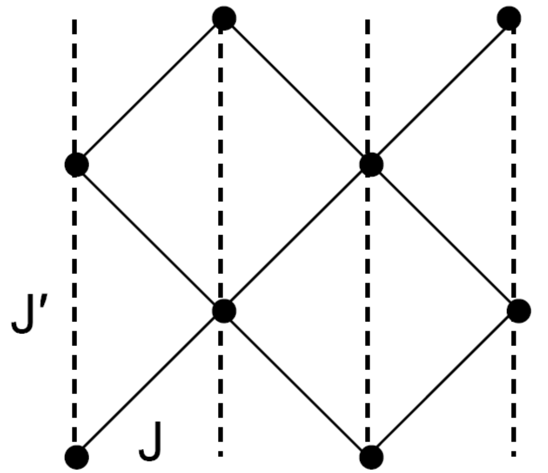


M. Mourigal, M. E. Zhitomirsky, and **SC**, unpublished

models, models …

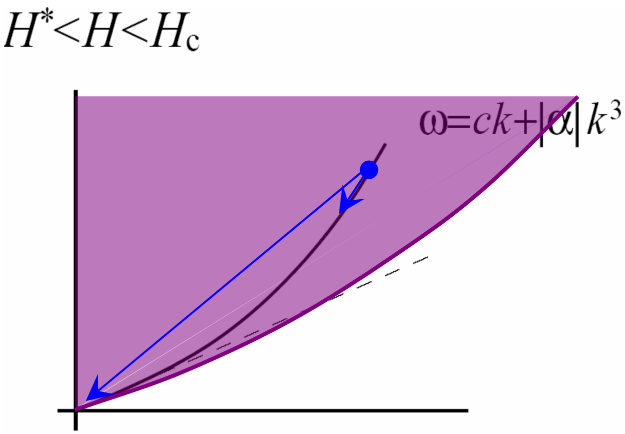


$Cu(tn)Cl_2$



M. Mourigal, M. E. Zhitomirsky, and **SC**, unpublished

“principal” decay types

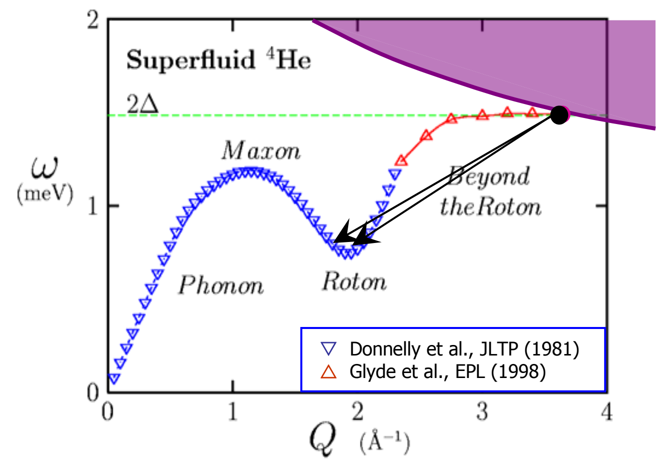


square lattice AF (and others), field-induced, *[may, or may not require external field]*

concave spectrum

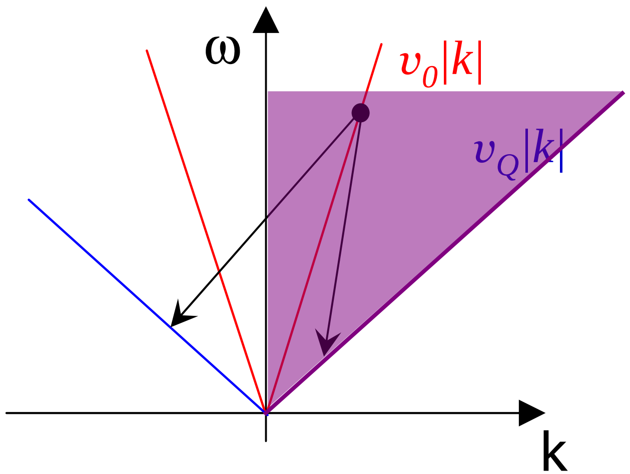
Bose-gas with repulsion and ^{4}He

short-wavelength cases



square+H, triangular, *J-J′*, spin-gap SL (and others)

termination point in ^{4}He

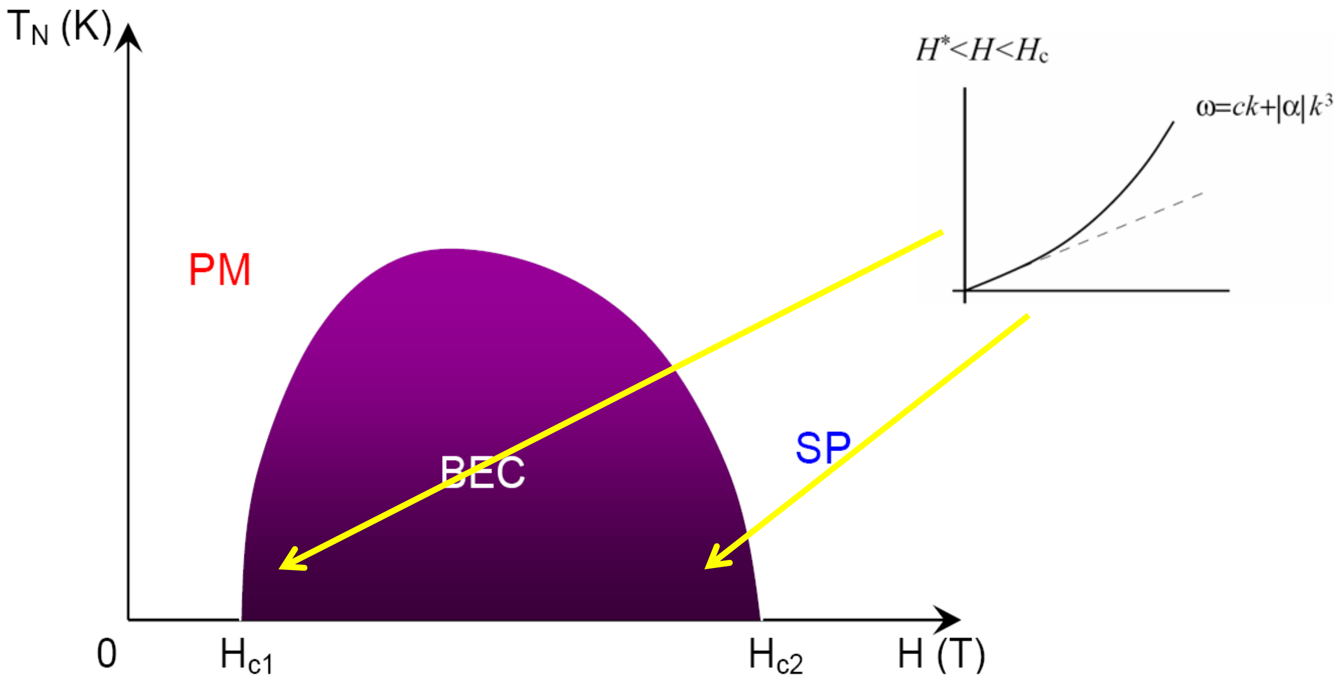


case of two species of excitations

triangular lattice AF (and others), *[zero field]*

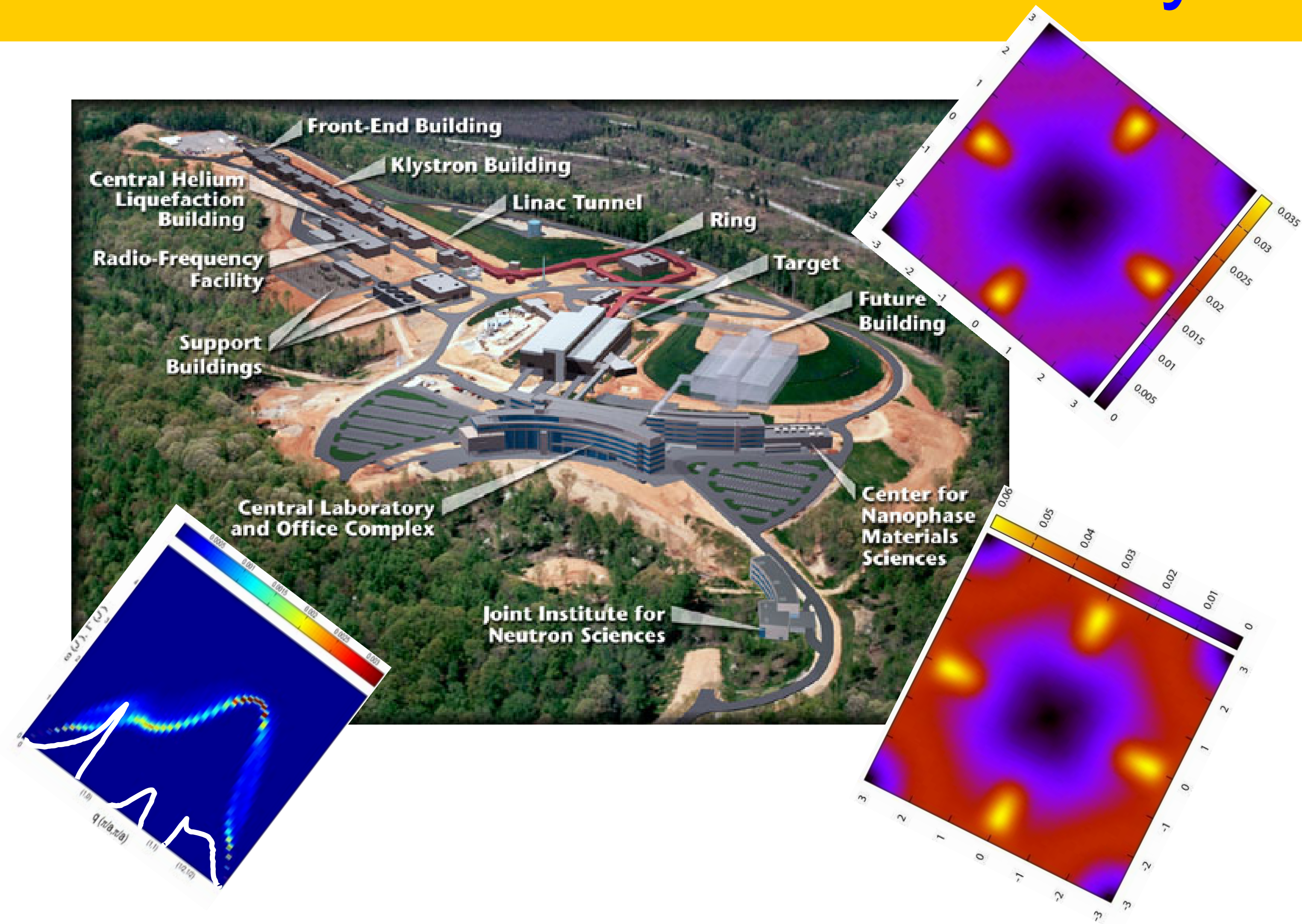
phonons in crystals

other systems/experiments in a field



- ☑ BEC in a field = XY AF = square in a field
- ☑ quasi-1D, stronger singularities?
- ☑ thermal conductivity

to the Future with decays …



*SNS=Spalation Neutron Source

conclusions

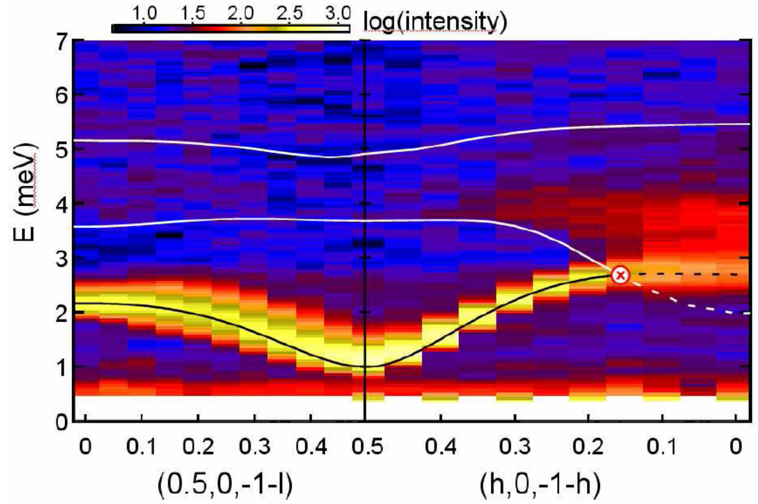
- **decays** and **singularities** in the bosonic spectra are generic effects
- enhanced by lower D
- remnants of **singularities** can be used for fingerprinting
- finite-*T* ?

I. Aivazovsky, "*The Ninth Wave*" (1850)

spin-gap systems, triplet decays

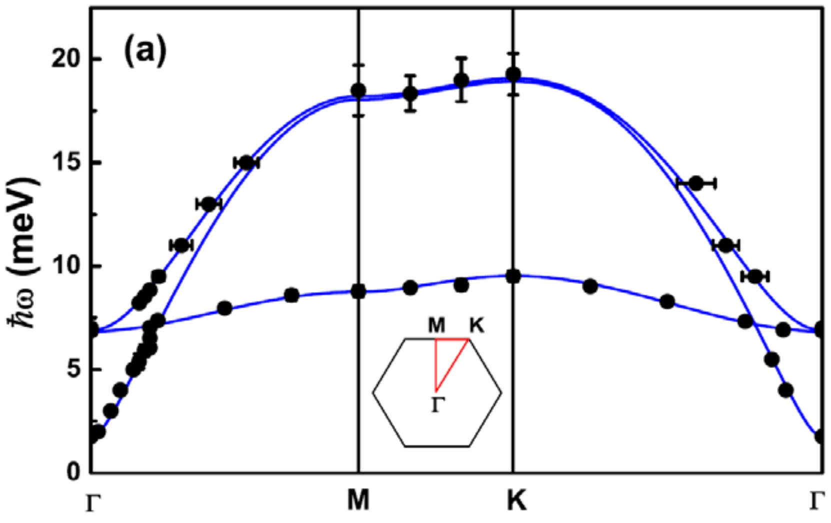
$$\hat{V}_3 = \frac{1}{2}\sum_{\mathbf{k},\mathbf{q}} \Gamma(\mathbf{k},\mathbf{q})\,\epsilon^{\alpha\beta\gamma} t^\dagger_{\mathbf{k}\alpha} t^\dagger_{\mathbf{q}\beta} t_{\mathbf{k}+\mathbf{q}\,\gamma}$$

M. Stone *etal.,* Nature **440,** 187-190 (2006).

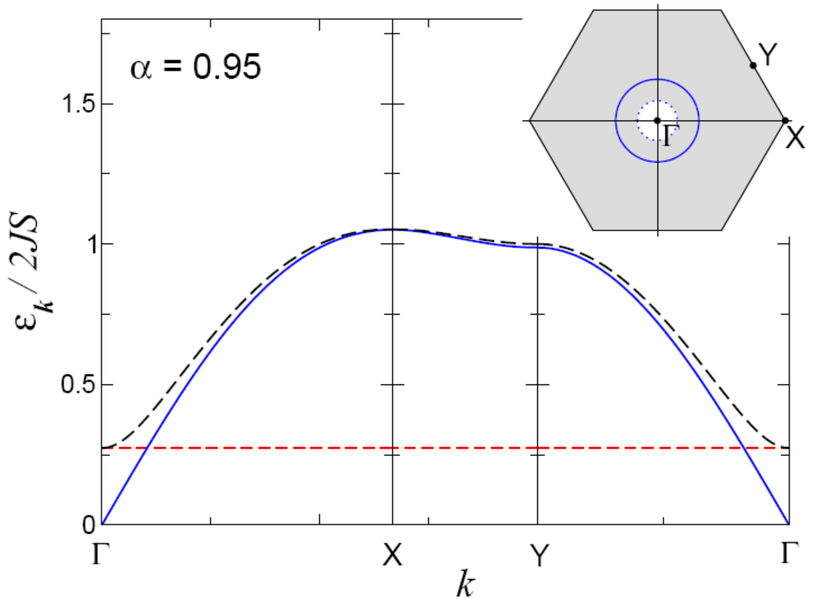


A. Kolezhuk and S. Sachdev, Phys. Rev. Lett. **96**, 087203 (2006),
M. E. Zhitomirsky, Phys. Rev. B **73**, 100404 (2006)

Kagome´ lattice AF potassium jarosite, S=5/2



$KFe_3(OH)_6(SO_4)_2$



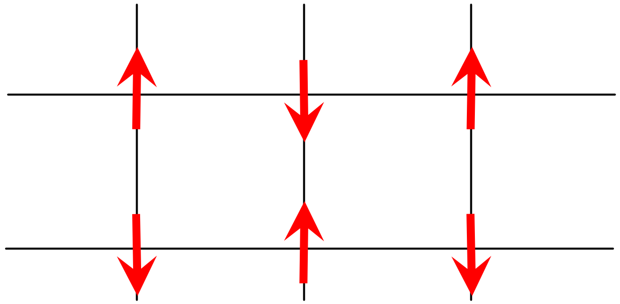
La$_2$CuO$_4$ ($J = 1500$ K, $H_s = 4500$ T), CFDT ($J = 72$ K, $H_s = 220$ T),

Rb$_2$MnF$_4$ ($J = 7.6$ K, $H_s = 103$ T), CuPyNO ($J = 1.9$ K, $H_s = 5.3$ T),

(5Cap)$_2$CuCl$_4$ ($J = 1.25$ K, $H_s = 3.6$ T)

Cu(tn)Cl$_2$

square = collinear



$$\mathcal{H} = J \sum_{\langle i,j \rangle} \left(-S_i^z S_j^z + \frac{1}{2}\left(S_i^+ S_j^- + S_i^- S_j^+ \right) \right)$$

$$+$$

$$S^z \Rightarrow S - b^\dagger b, \quad S_i^+$$

$$\Downarrow$$

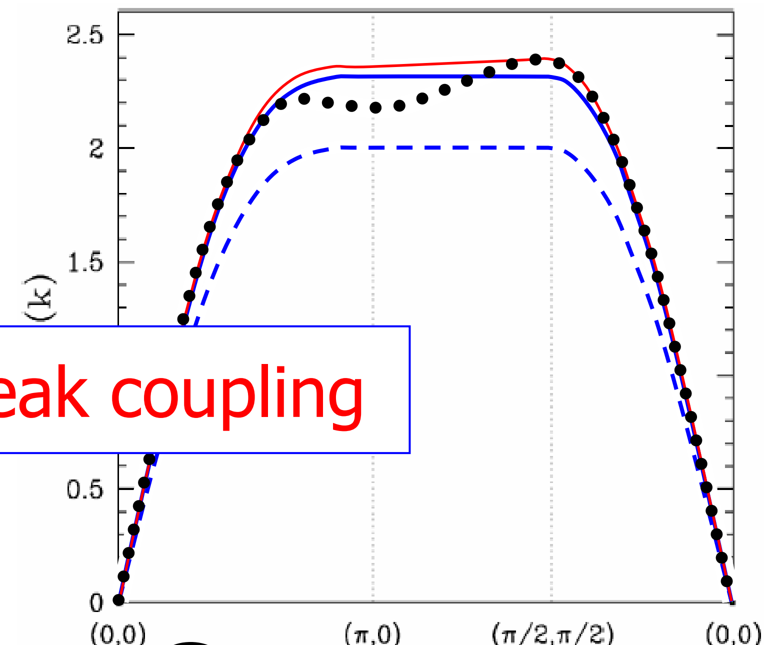
$$\mathcal{H} \Rightarrow E_0 + \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \left(b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \frac{1}{2} \right)$$
$$+ \frac{J}{2} \sum_{i,j} \left[-\frac{1}{2} b_i^\dagger b_i \langle b_j^\dagger b_j \rangle + \ldots \right.$$

no coupling with 2-magnon continuum

in non-collinear case

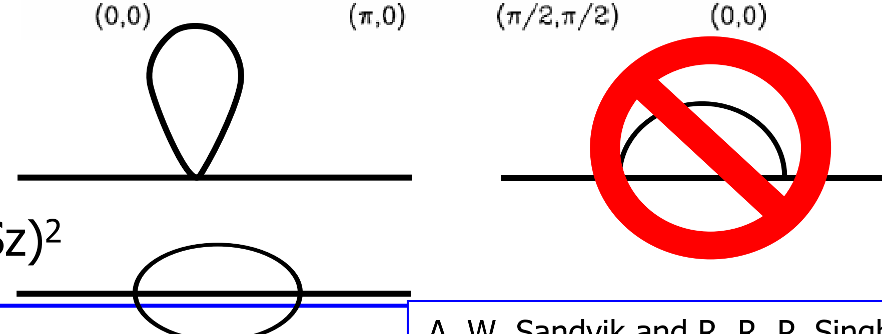
$$\Downarrow$$

weak renormalization of the SW spectrum



collinear = weak coupling

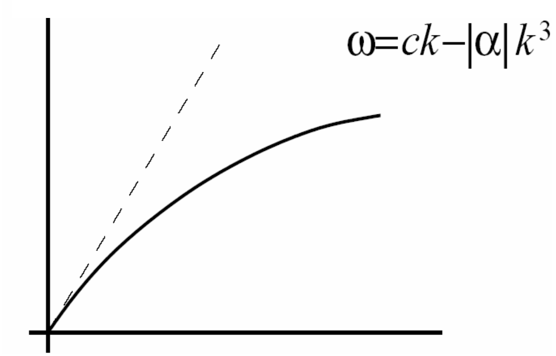
1/2Sz

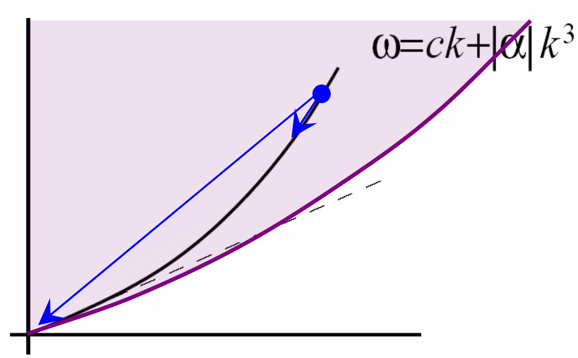


1/(2Sz)^2

A. W. Sandvik and R. R. P. Singh, PRL **86**, 528 (2001)

decays 101, kinematic conditions





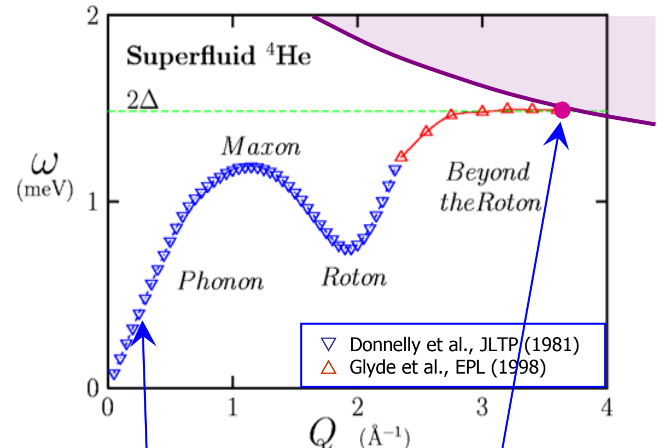
convex = decay free

concave = prone to decays

two-particle continuum energy: $E_{\mathbf{k},\mathbf{q}} = \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{k}-\mathbf{q}}$

Bose-gas with repulsion and ^{4}He

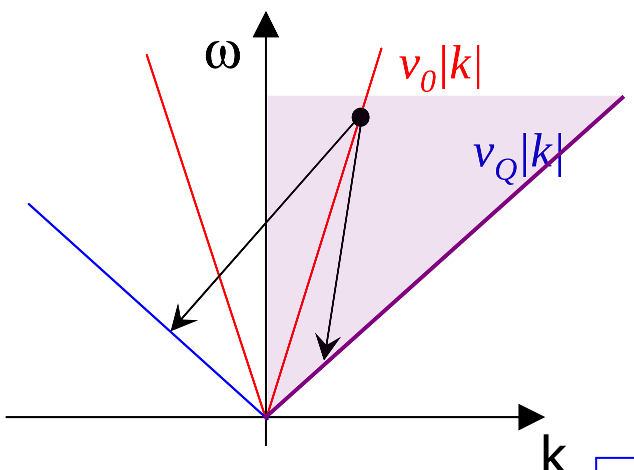
short-wavelength case



termination point

sound attenuation

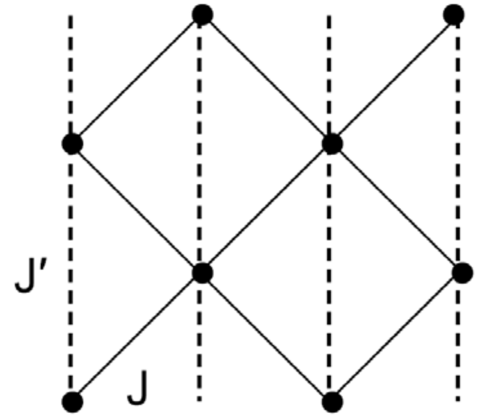
L. P. Pitaevskii, Sov. Phys. JETP **36**, 830 (1959)

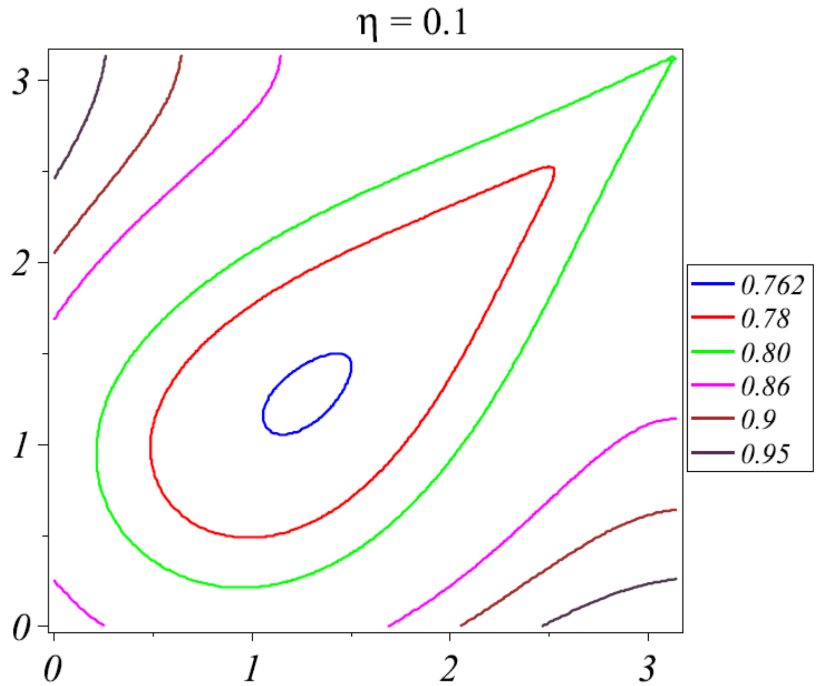


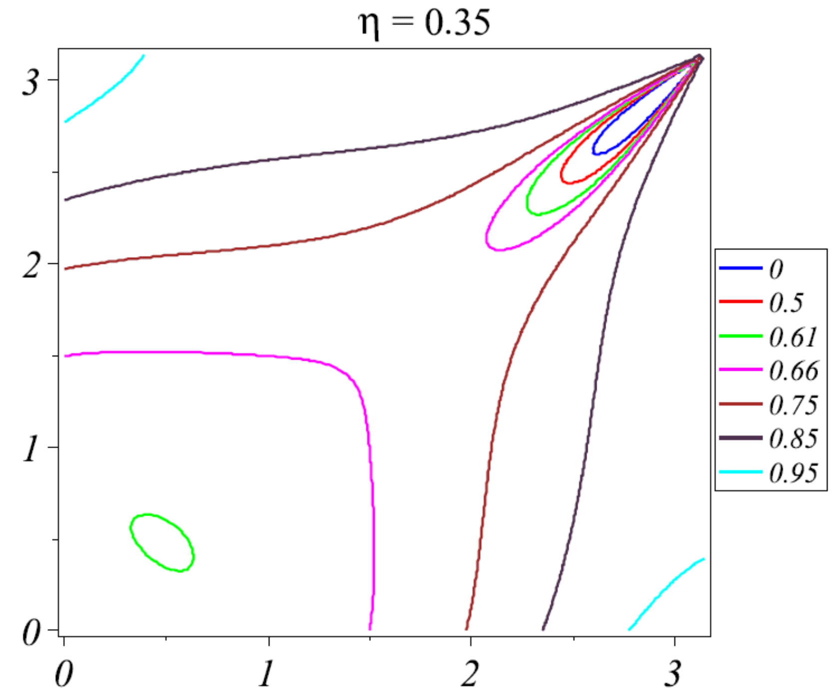
case of two excitations

anharmonic (3-boson) terms are present in crystals

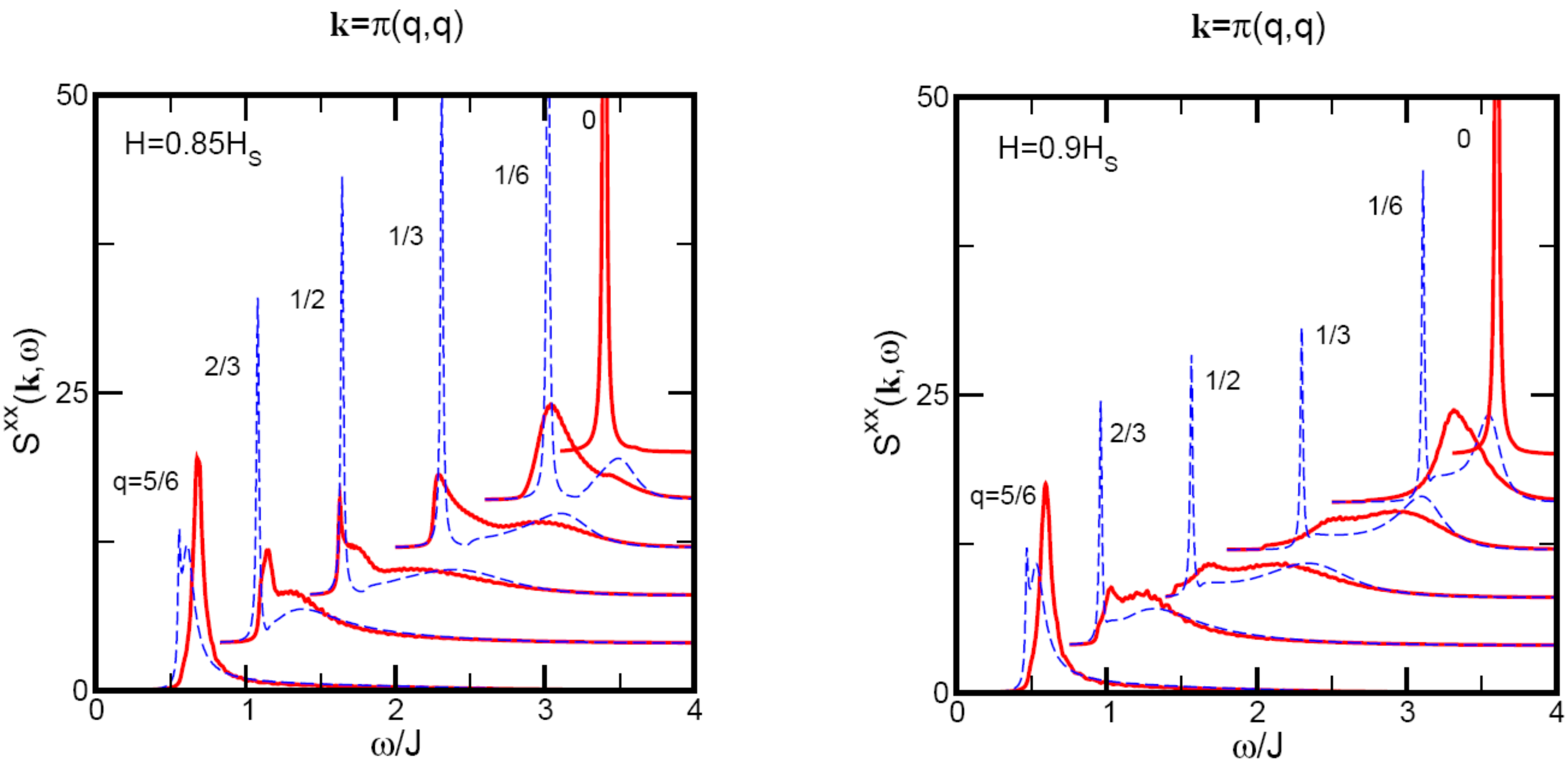
3-boson terms are present in ^{4}He

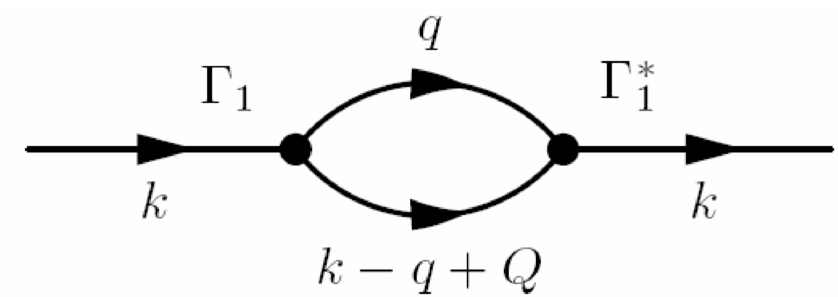


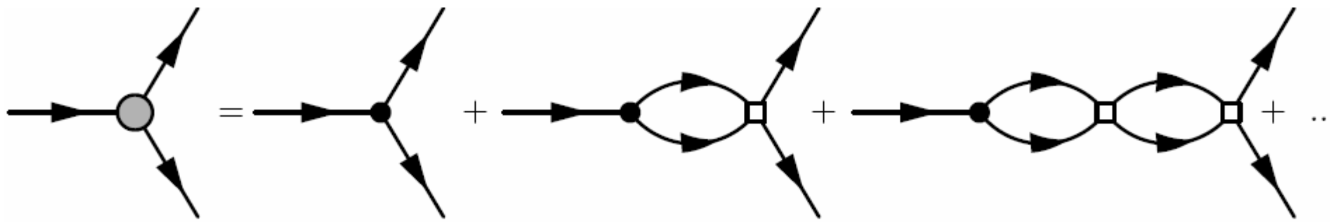


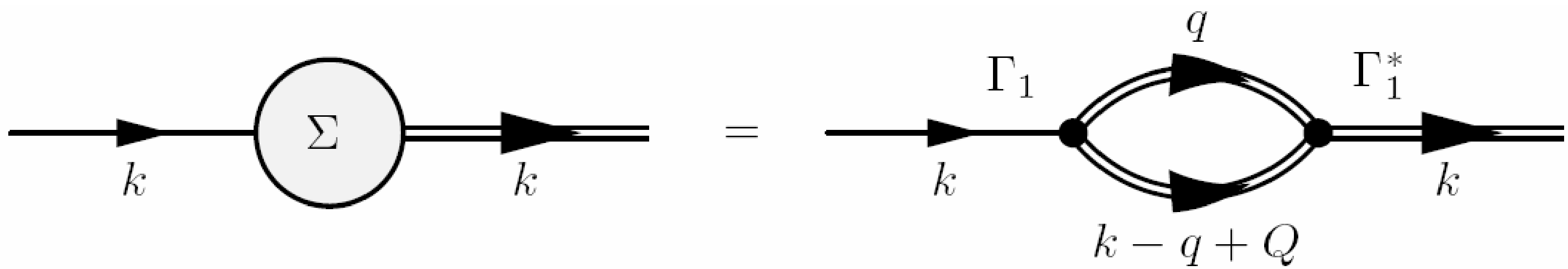


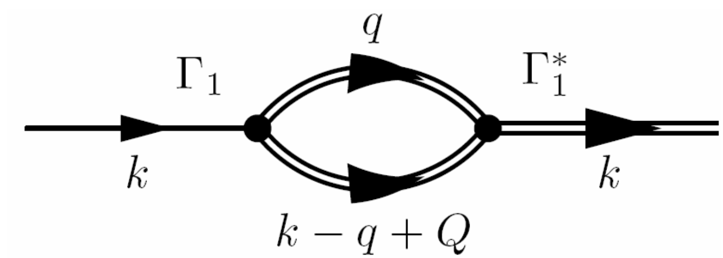
M. Mourigal, M. E. Zhitomirsky, and **SC**, unpublished

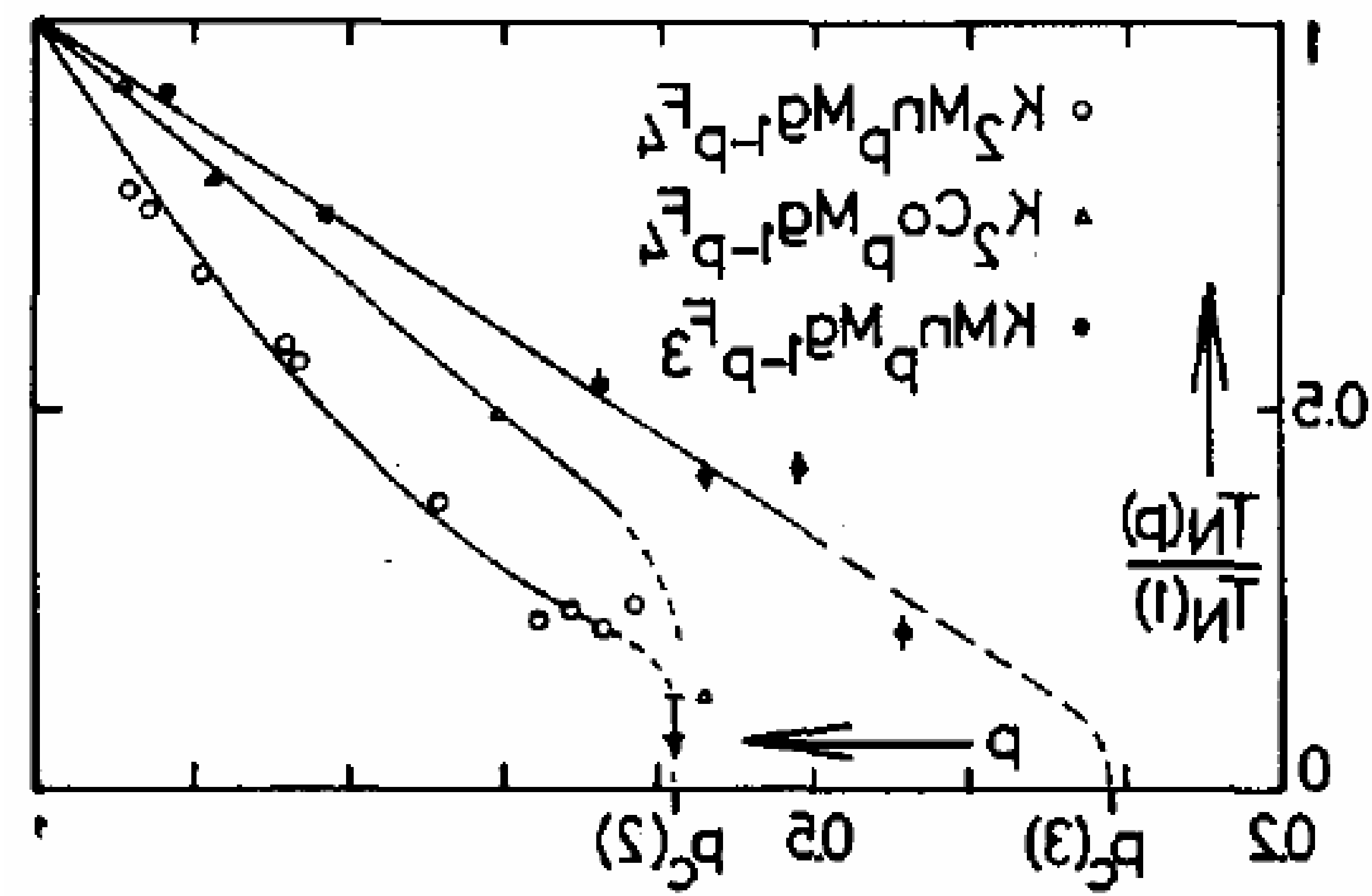

k=π(q,q)
H=0.85H_S
q=5/6
2/3
1/2
1/3
1/6
0
S^xx(k,ω)
ω/J
k=π(q,q)
H=0.9H_S
q=5/6
2/3
1/2
1/3
1/6
0
S^xx(k,ω)
ω/J


q
Γ_1
Γ_1^*
k
k
$k - q + Q$


$=$
$+$
$+$
$+ \ \dots$


Σ
k
k
$=$
q
Γ_1
Γ_1^*
k
k
$k - q + Q$


q
Γ_1
Γ_1^*
k
k
$k - q + Q$



$K_2Mn_pMg_{1-p}F_4$
$K_2Co_pMg_{1-p}F_4$
$KMn_pMg_{1-p}F_3$
$\frac{T_N(p)}{T_N(1)}$
1
0.5
0
p
$p_c(2)$
0.5
$p_c(3)$
0.2