

# ***Physical Properties of $s_{\pm}$ Superconductors as a Model of Iron Pnictides***

*Alexander Golubov*

*University of Twente, The Netherlands*

## In collaboration with

*A. Brinkman*, University of Twente,  
Netherlands

*O.V. Dolgov*, Max Planck Institute, Stuttgart,  
Germany

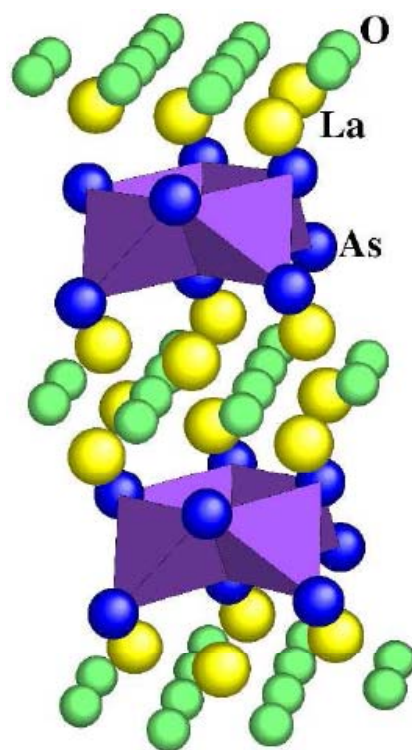
*I.I. Mazin, D. Parker*, Naval Research Lab,  
Washington, USA

*Y. Tanaka*, Nagoya University, Japan

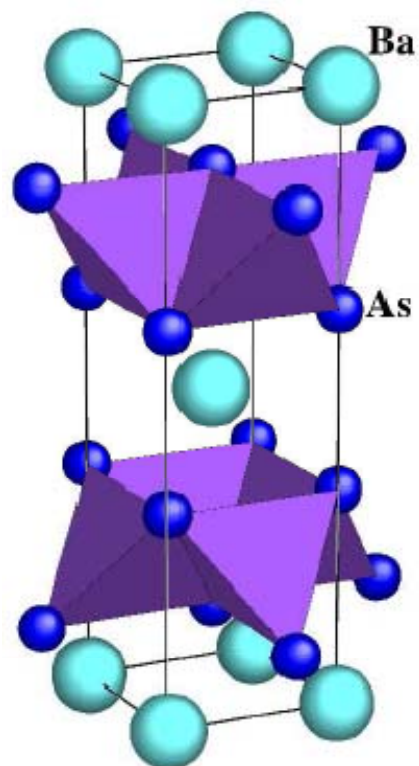
# Outline

- Introduction
- $\mathbf{s}_{\pm}$  pairing and impurity scattering
- Absence of the *Hebel-Slichter* peak in NMR relaxation and  $T$ -dependence of  $1/T_1$
- Electromagnetic response in the  $\mathbf{s}_{\pm}$  model
- Tunneling in the  $\mathbf{s}_{\pm}$  model

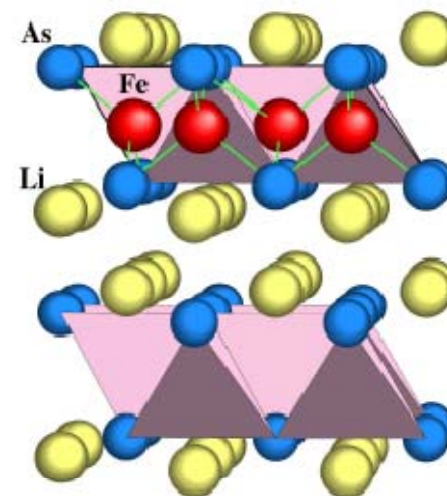
# Crystal structure of FeAs superconductors



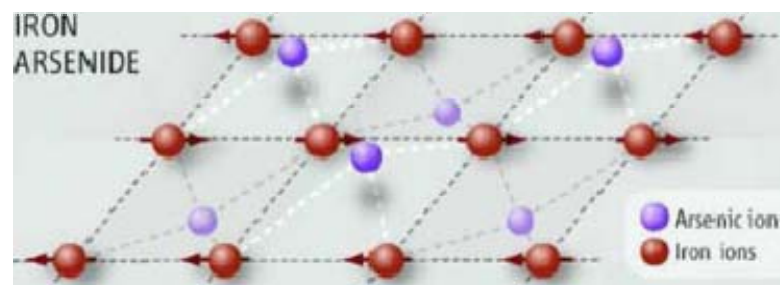
LaOFeAs



BaFe<sub>2</sub>As<sub>2</sub>



LiFeAs



FeAs tetrahedra form two-dimensional layers surrounded by LaO, Ba or Li.  
Fe ions inside tetrahedra form a square lattice.

# Basic Experiments on FeAs

J|A|C|S  
COMMUNICATIONS

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February 2008

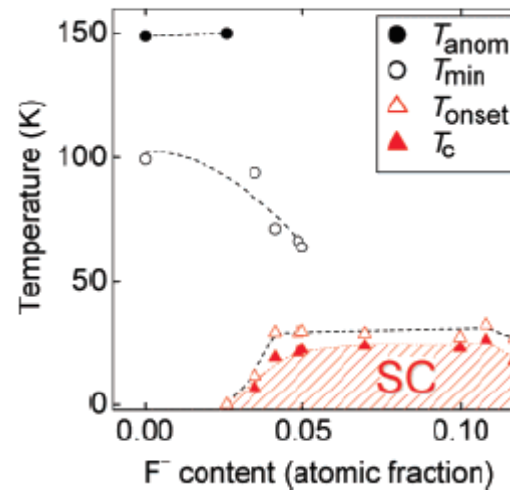
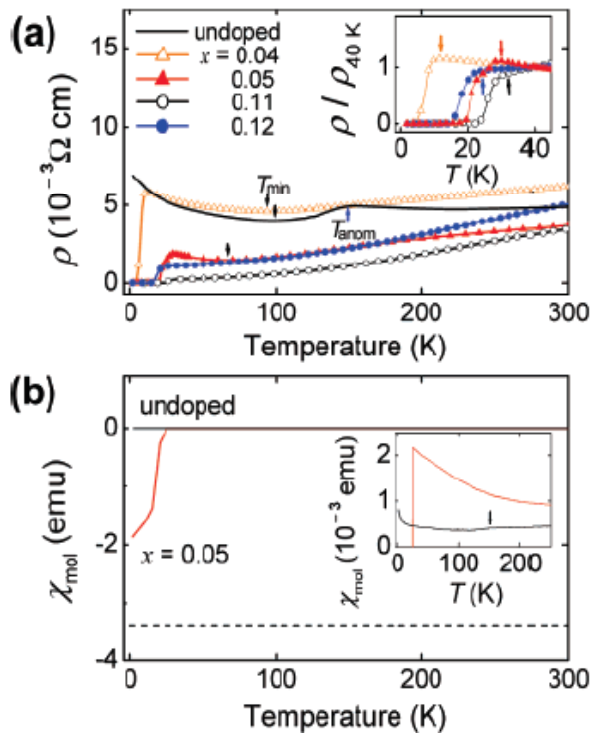
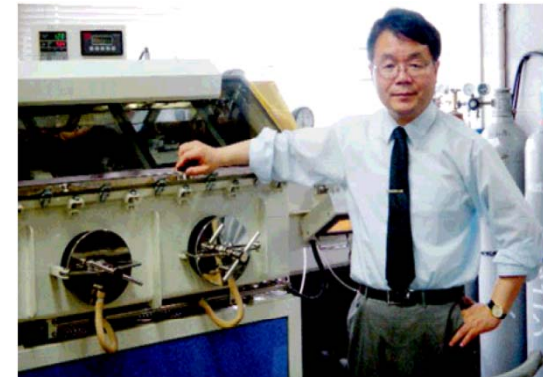
## Iron-Based Layered Superconductor $\text{La}[\text{O}_{1-x}\text{F}_x]\text{FeAs}$ ( $x = 0.05-0.12$ ) with $T_c = 26$ K

Yoichi Kamihara,<sup>\*,†</sup> Takumi Watanabe,<sup>‡</sup> Masahiro Hirano,<sup>†,§</sup> and Hideo Hosono<sup>†,‡,§</sup>

*ERATO-SORST, JST, Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, Materials and Structures Laboratory, Tokyo Institute of Technology, Mail Box R3-1, and Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan*

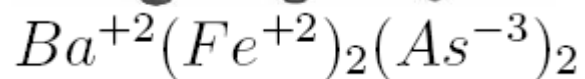
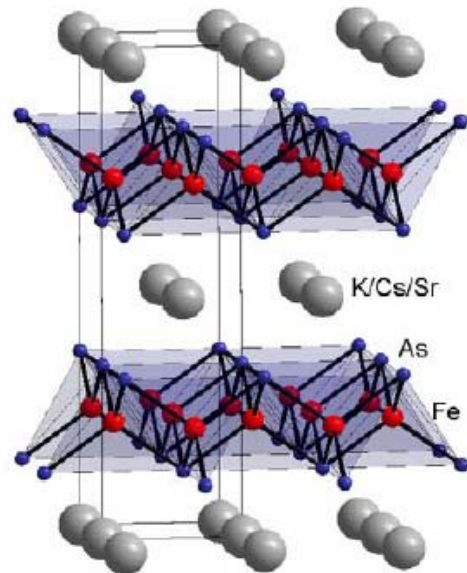
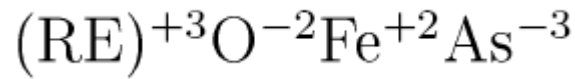
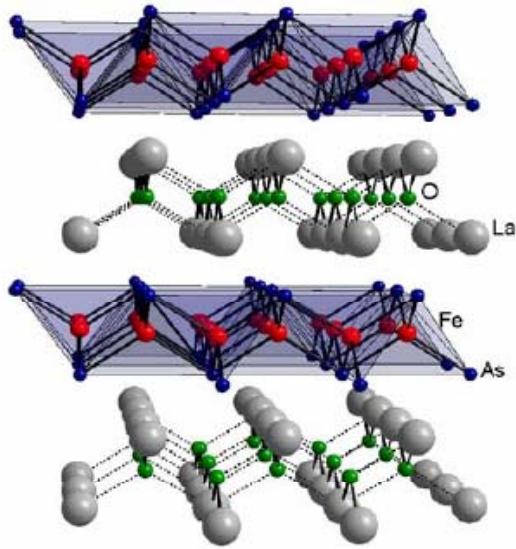
Received January 9, 2008; E-mail: hosono@msl.titech.ac.jp

■ J. AM. CHEM. SOC. 2008, 130, 3296–3297

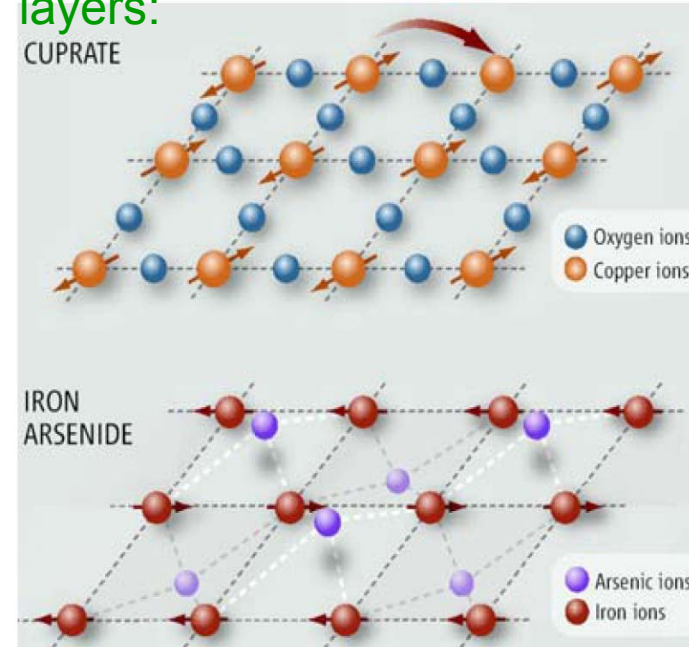


Electron doping!

# Basic crystal structure of FeAs superconductors

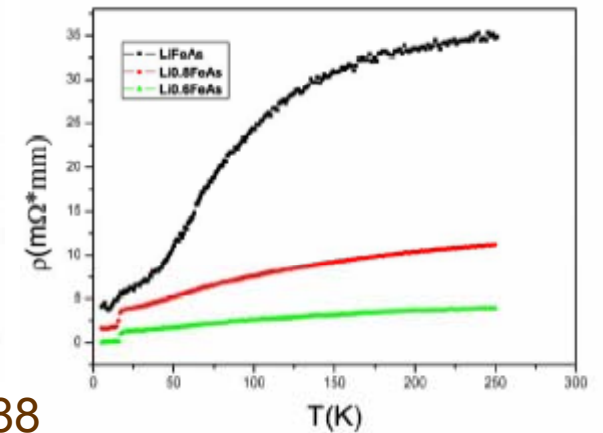
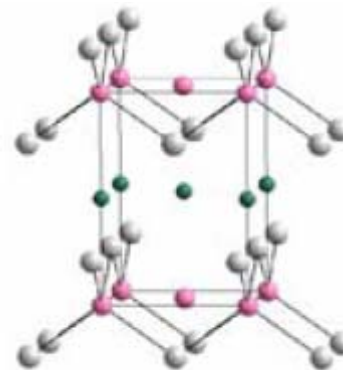


CuO<sub>2</sub> as compared with FeAs layers:



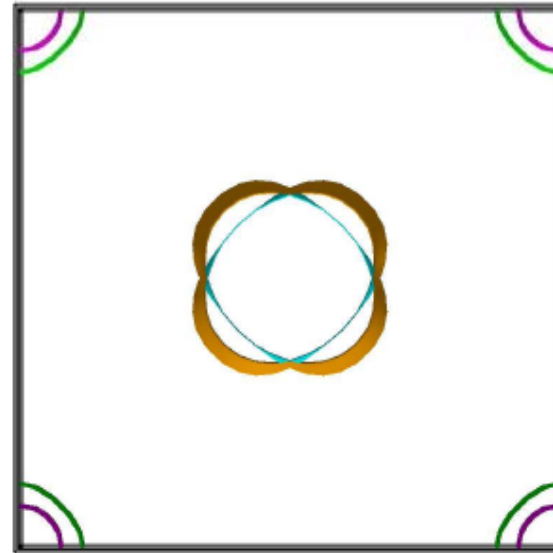
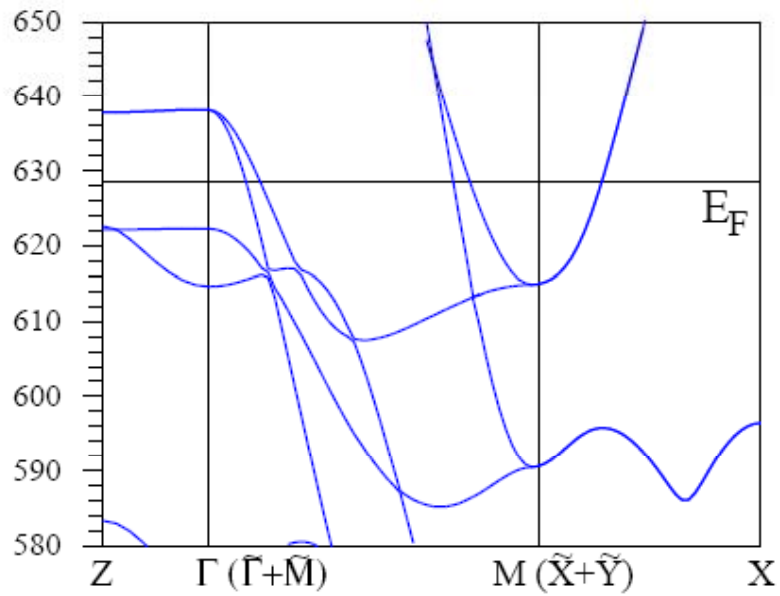
The superconductivity at 18 K in Li<sub>1-x</sub>FeAs compounds

X.C.Wang, Q.Q. Liu, Y.X. Lv, W.B. Gao, L.X. Yang, R.C. Yu, F.Y. Li, C.Q. Jin\*



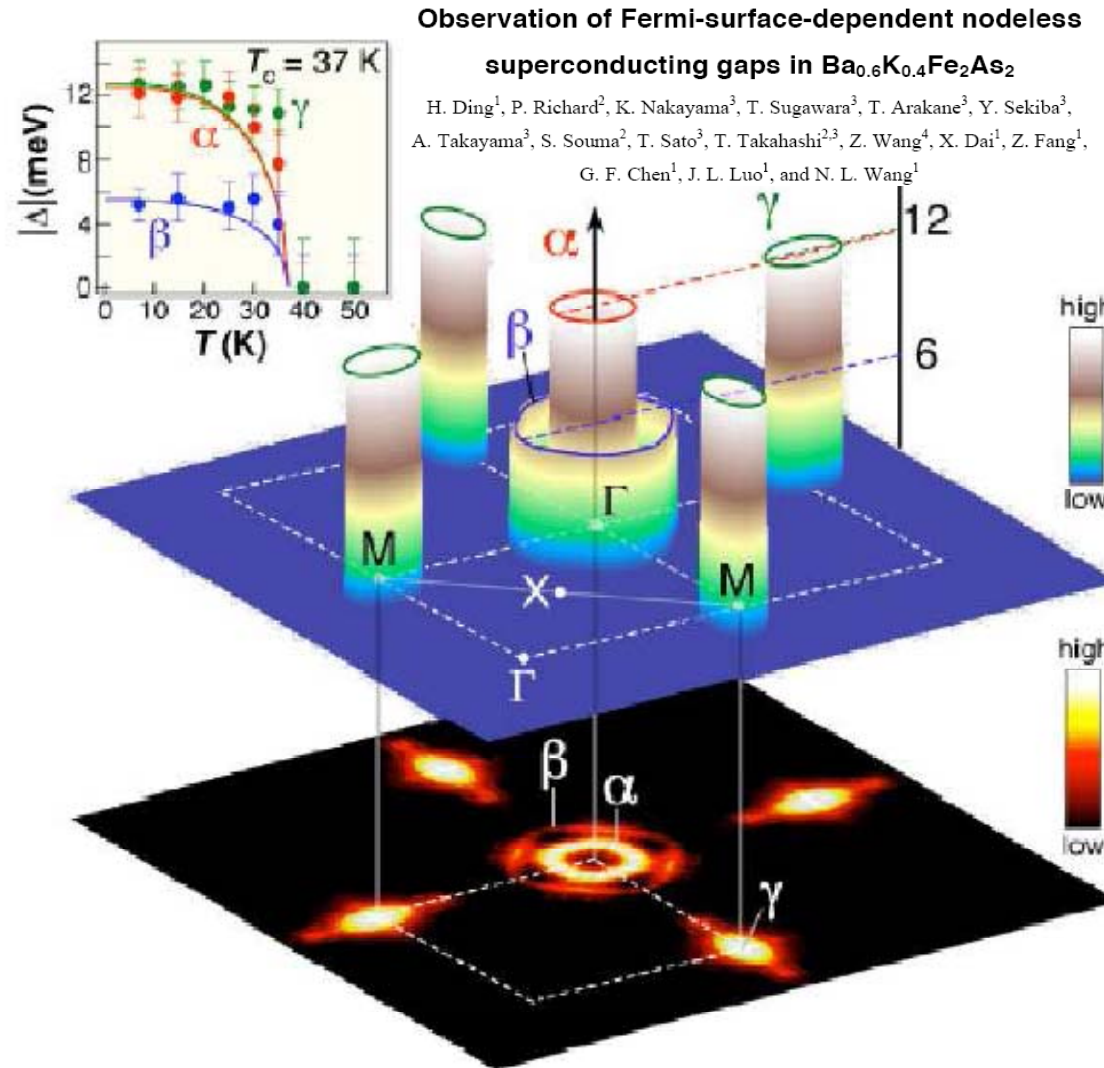
arXiv: 0806.4688

# Band structure





# Superconducting gap – ARPES data



arXiv: 0807.0419

Schematic picture of superconducting gaps in  $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$ . Lower picture represents Fermi surfaces (ARPES intensity), upper insert – temperature dependence of gaps at different Sheets of the Fermi surface.



## Spin fluctuations

$$\chi(\mathbf{q}, \omega) = \frac{\chi_0(\mathbf{q}, \omega)}{1 - J(\mathbf{q}, \omega) \chi_0(\mathbf{q}, \omega)}$$

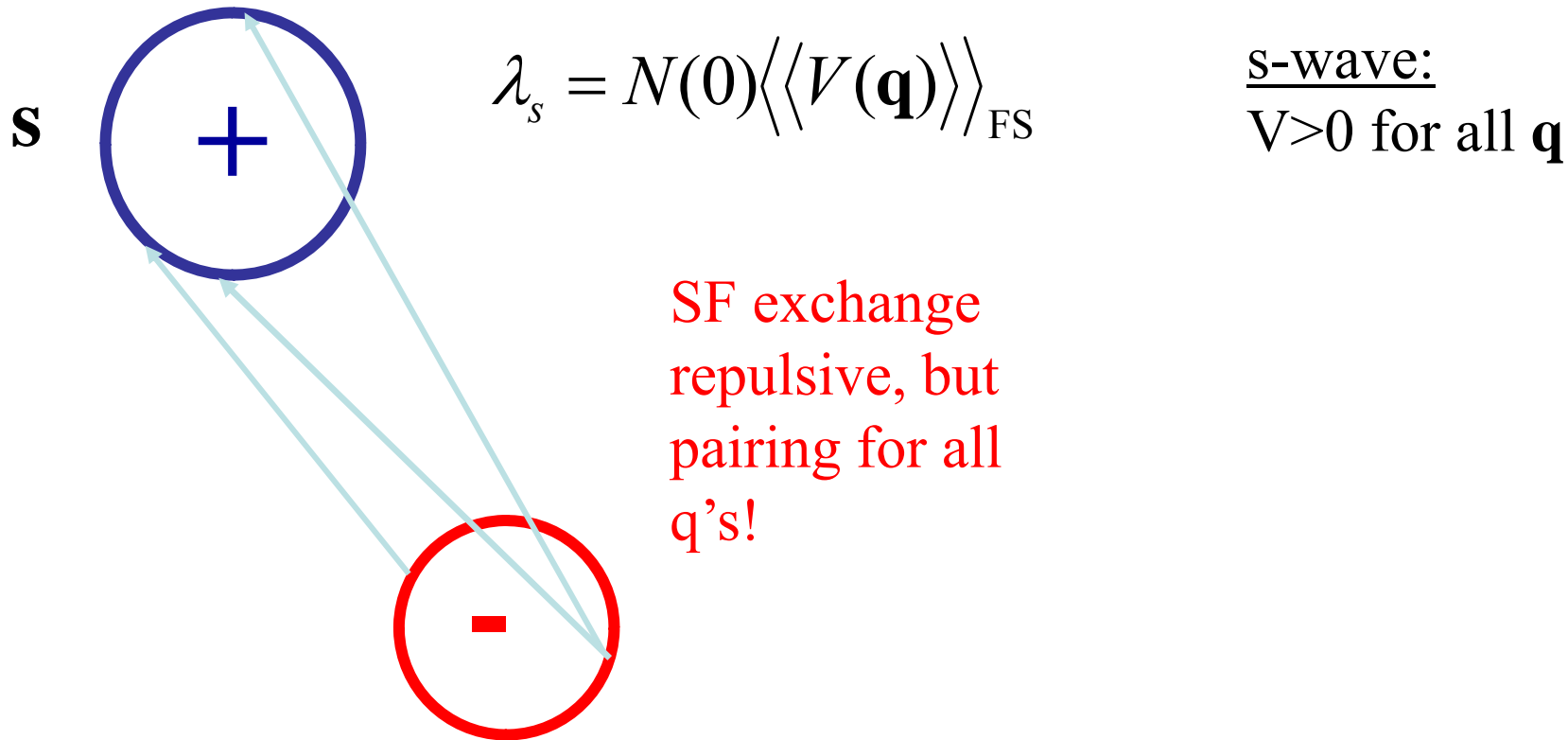
For a Mott-Hubbard system,  
 $J(\mathbf{q}, \omega)$  -magnetic interaction  
is *local in real space*

$$\chi_0(\mathbf{q}, \omega) = \sum_{\mathbf{k}} \frac{f(\varepsilon_{\mathbf{k}+\mathbf{q}}) - f(\varepsilon_{\mathbf{k}})}{(\varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}} - \omega - i\delta)}$$

the s/c physics is just the same as old Berk-Schrieffer physics

- 1) Interaction is repulsive and peaked at  $(\pi, \pi) \rightarrow$  pairing in  $s_{\pm}$  channel
- 2) Mass renormalization expected up to the energy of spin fluctuations

## Repulsive pairing interactions in the $s_{+/-}$ channel:



Simple model: spin fluctuations with spectral function

$$B_{ij}(\omega) = \lambda_{ij} \pi \omega \Omega_{sf} / (\Omega_{sf}^2 + \omega^2)$$

Parameters:  $\Omega_{sf} = 25 \text{ meV}$ ,  $\lambda_{11} = \lambda_{22} = 0.5$ ,  $\lambda_{12} = \lambda_{21} = -2$        $T_c = 27 \text{ K}$

# Some properties of the $s_{+/-}$ state

1. Thermodynamics is exponential in clean limit (weak coupling)  
C/T, penetration depth, NMR  $1/TT_1$ . Singlet K
2. Reversed role of magnetic/nonmagnetic interband impurity scattering  
See next slides
3. Coherence factors: depend on the probing wave vector, BCS at  $q \approx 0$ ,  
anti-BCS at  $q \approx (\pi, \pi)$ .  
- *suppressed Hebel-Slichter peak*  
*suppressed Hebel-Slichter peak*
4. Andreev bound states at finite energies  
See next slides
5. Enhancement of spin susceptibility near  $q \approx (\pi, \pi)$  below  $T_c$   
observed
7. Full gap(s) in tunneling  
Full gaps in ARPES, Andreev conductance ?

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## Experiments: symmetry of the superconducting order parameter

### Fully gapped superconducting state:

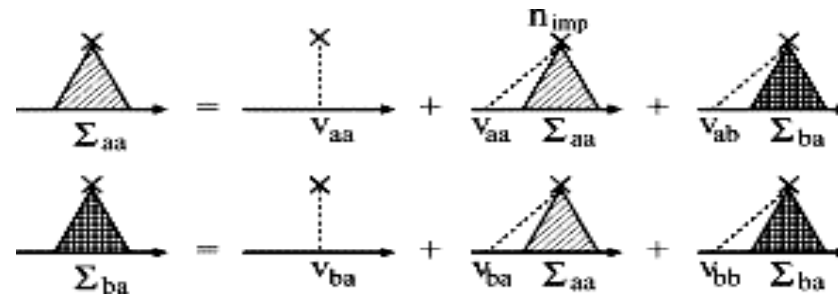
- ❑ **PCAR**: Y.Y. Chen et al., Nature 453, 1224 (2008); K.A. Yates et al., Supercond. Sci. Technol. 21, 092003 (2008); R.S. Gonnelli et al., arXiv:0807.3149.
- ❑ **ARPES**: L. Zhao et al., arXiv:0807.0398; H. Ding et al., EPL 83, 47001 (2008); T. Kondo et al., arXiv:0807.0815.
- ❑ **Penetration depth**: C. Martin et al., arXiv:0807.0876; K. Hashimoto et al., arXiv:0806.3149; L. Malone et al., arXiv:0806.3908

### NMR – Lines of nodes at the FS:

- ❑  **$^{75}\text{As}$  NMR**:  $1/T_1$  with a  $T^3$  behavior below  $T_c$  (line of nodes?)  
Y. Nakai et al., JPSJ 77, 073701 (2008)
  - ❑  **$^{19}\text{F}$  NMR**:  $1/T_1$  can be explained by a two d-wave gap scenario,  $\Delta = \Delta_1 + \Delta_2$ ,  $\Delta_1 = 3.5 k_B T_c$ ,  $\Delta_2 = 1.1 k_B T_c$   
K. Matano et al., EPL 83, 57001 (2008)
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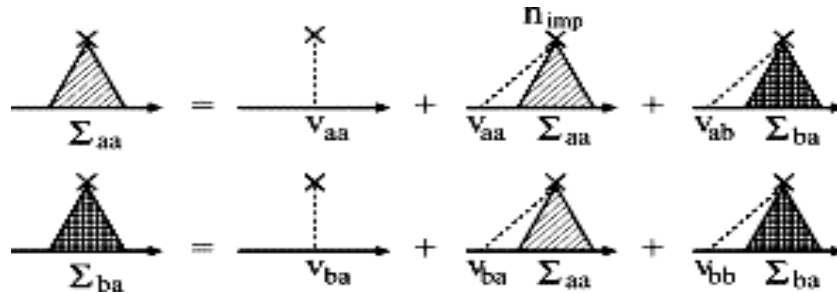
# Effect of impurities

$$\hat{G} = \hat{G}_0 - \hat{\Sigma}_{imp}$$



# Effect of impurities

$$\hat{G} = \hat{G}_0 - \hat{\Sigma}_{imp} \quad \hat{v} = \begin{pmatrix} 0 & v \\ v & 0 \end{pmatrix}$$



Interband impurities

$$\Sigma_{aa}^{(0)} = -i\gamma_a \frac{\sigma \bar{\omega}_{an} \sqrt{\bar{\Delta}_{bn}^2 + \bar{\omega}_{bn}^2} - (\sigma - 1) \bar{\omega}_{bn} \sqrt{\bar{\Delta}_{an}^2 + \bar{\omega}_{an}^2}}{\det a}$$

$$\Sigma_{aa}^{(1)} = \gamma_a \frac{\sigma \bar{\Delta}_{an} \sqrt{\bar{\Delta}_{bn}^2 + \bar{\omega}_{bn}^2} + (\sigma - 1) \bar{\Delta}_{bn} \sqrt{\bar{\Delta}_{an}^2 + \bar{\omega}_{an}^2}}{\det a},$$

where  $\gamma_i = \frac{n_{imp} \sigma}{\pi N_i(0)}$ , and  $\sigma = \frac{\pi^2 N_a(0) N_b(0) u^2}{1 + \pi^2 N_a(0) N_b(0) u^2}$ .

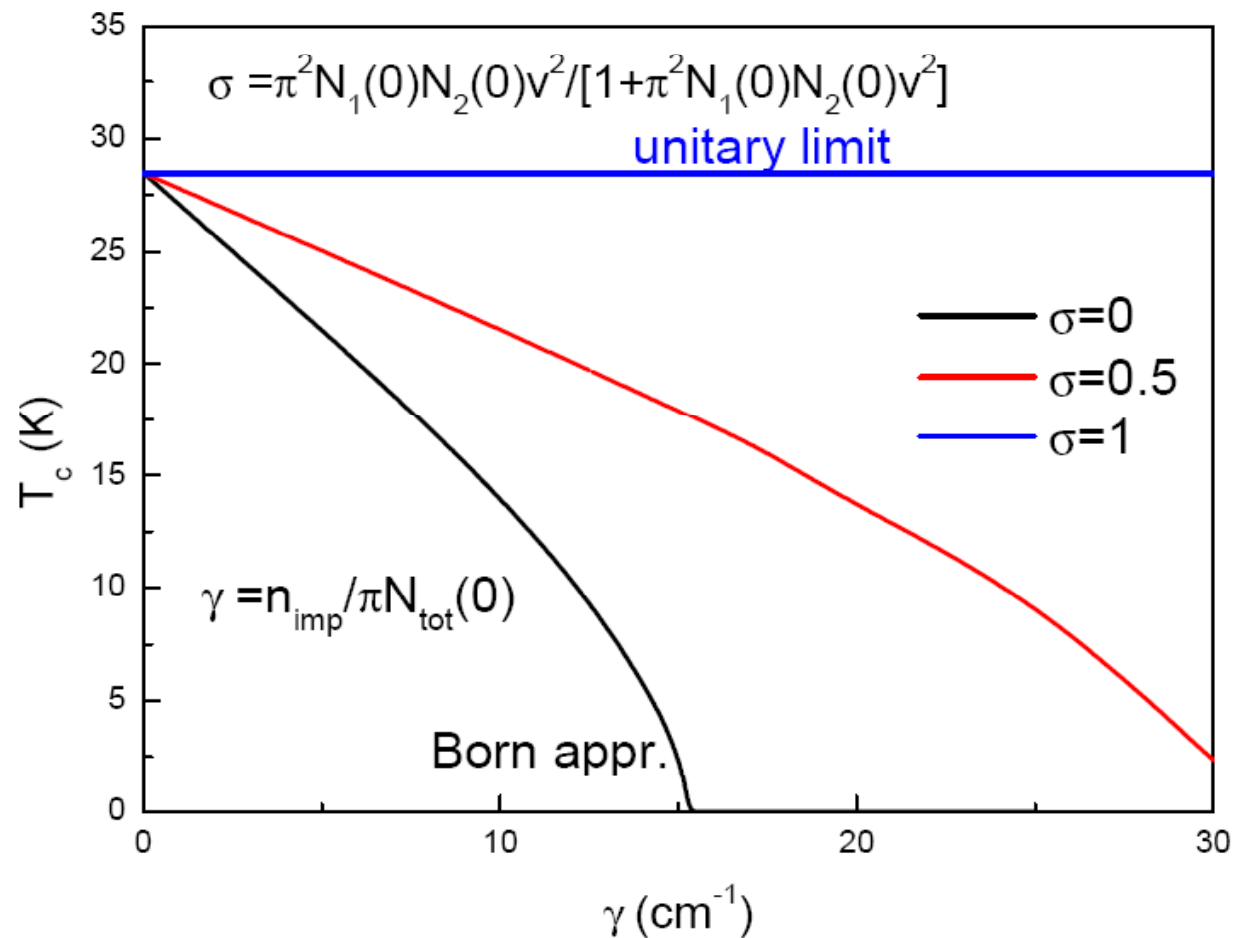
$$\det a = \left[ 2\sigma(\sigma - 1) \left( \sqrt{\bar{\Delta}_{an}^2 + \bar{\omega}_{an}^2} \sqrt{\bar{\Delta}_{bn}^2 + \bar{\omega}_{bn}^2} - \bar{\omega}_{an} \bar{\omega}_{bn} + \bar{\Delta}_{an} \bar{\Delta}_{bn} \right) + \sqrt{\bar{\Delta}_{an}^2 + \bar{\omega}_{an}^2} \sqrt{\bar{\Delta}_{bn}^2 + \bar{\omega}_{bn}^2} \right].$$



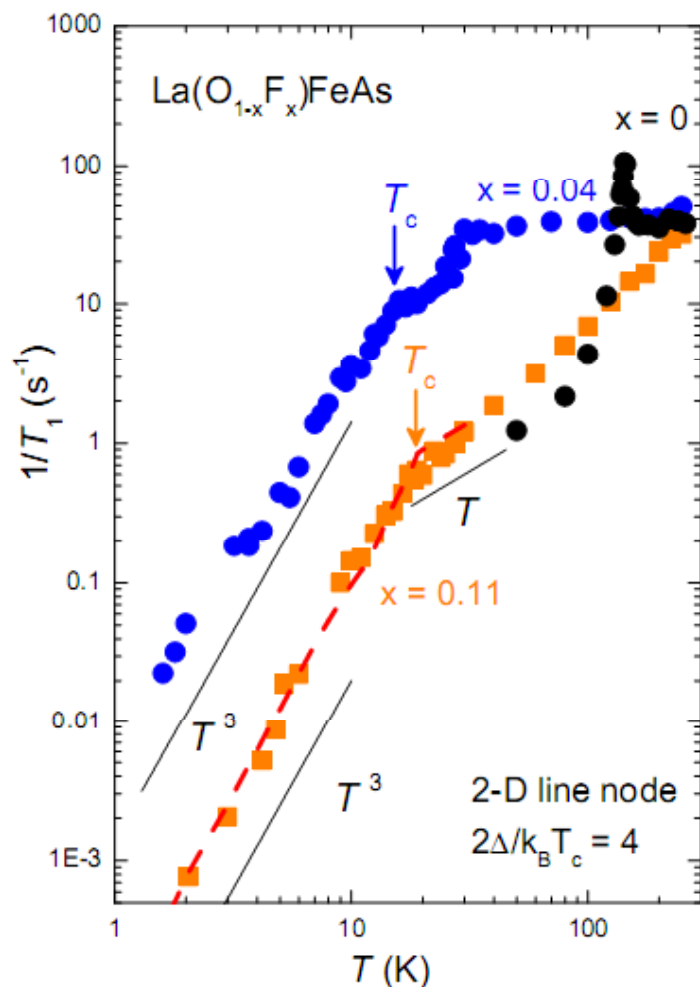
# Robustness of $T_c$ in the presence of impurities

$$\bar{\omega}_{an} = \omega_n + \gamma_{ab} \text{sign} \omega_n$$

$$\bar{\Delta}_{an} = \Delta_{an} + \gamma_{ab} \left[ \sigma \bar{\Delta}_{an} / \bar{\omega}_{an} - (1 - \sigma) \bar{\Delta}_{bn} / \bar{\omega}_{bn} \right] \text{sign} \omega_n$$

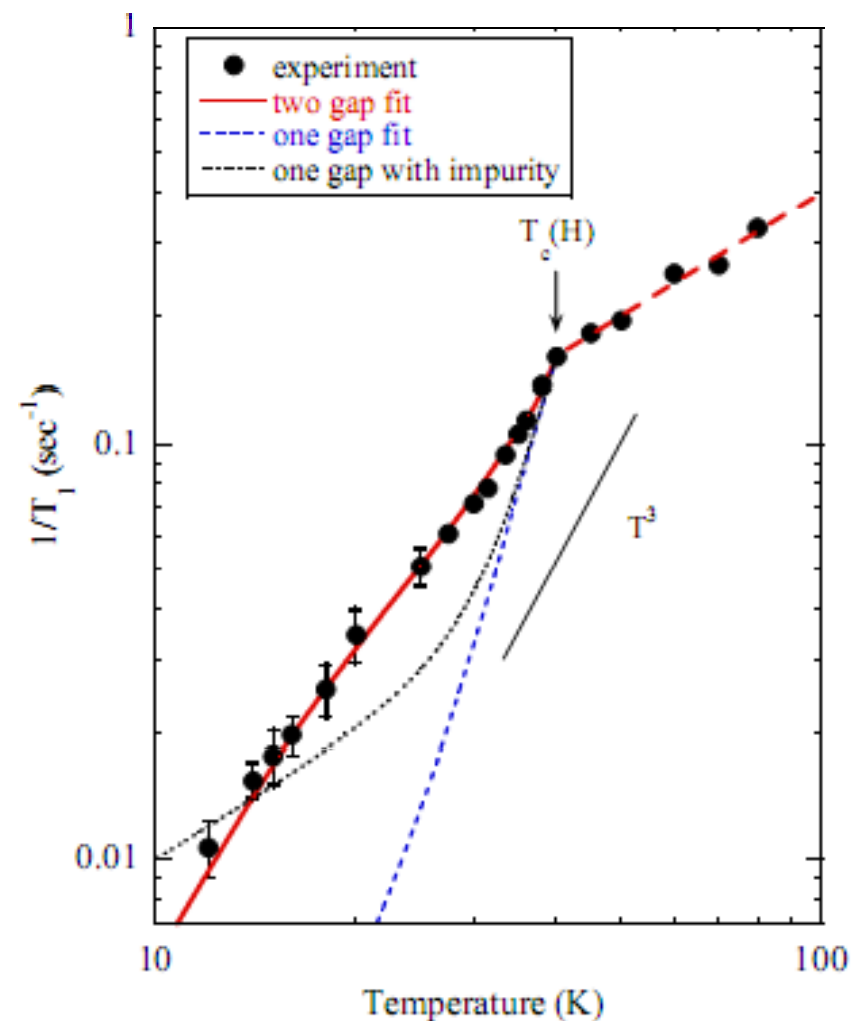


## NMR experiments: superconductivity



<sup>75</sup>As NMR:  $1/T_1$  with a  $T^3$  behavior (line of nodes?)

Y. Nakai et al., JPSJ 77, 073701 (2008)



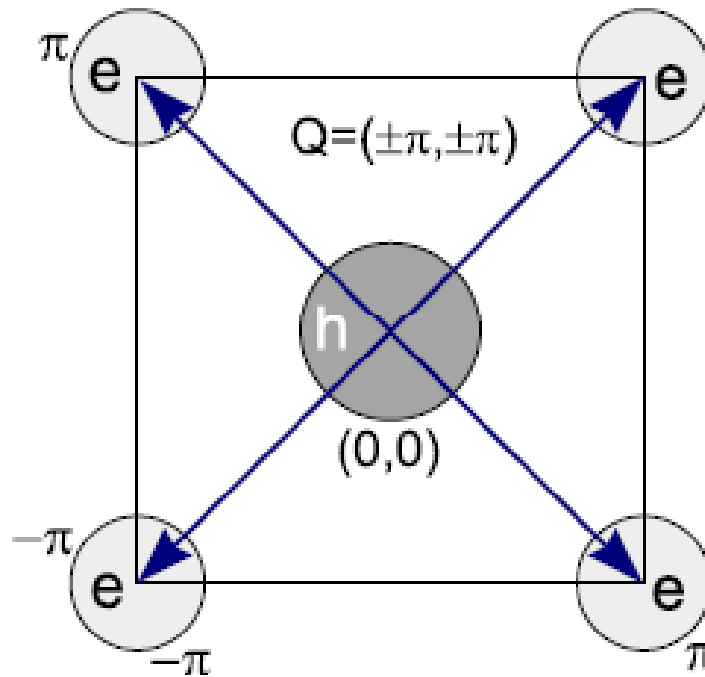
<sup>19</sup>F NMR:  $1/T_1$  can be explained by a two d-wave gap scenario,  
 $\Delta = \Delta_1 + \Delta_2$ ,  $\Delta_1 = 3.5 k_B T_c$ ,  $\Delta_2 = 1.1 k_B T_c$

K. Matano et al., EPL 83, 57001 (2008)

# How to reconcile $s_{\pm}$ model with NMR $1/T_1$ ?

## Use simplest model

Two types of pockets (h and e) at the Fermi surface in the *folded* BZ, reproduce LDA topology



Obtain spin-lattice relaxation rate using interband susceptibility  $\chi_{12}$ :

$$1/T_1 T \propto \lim_{\omega \rightarrow 0} \text{Im} \chi_{12}(\omega) / \omega$$

# Absence of the Hebel-Slichter peak

Weak coupling:  $\frac{1}{T_1 T} \propto \sum_{\mathbf{k}\mathbf{k}'} \left( 1 + \frac{\Delta_1 \Delta_2}{E_{\mathbf{k}} E_{\mathbf{k}'}} \right) \left[ -\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}} \right] \delta(E_{\mathbf{k}} - E_{\mathbf{k}'})$

s-wave

$$\Delta_1 = \Delta_2 = \Delta$$

$$\frac{1}{T_1} \propto \int_{\Delta(T)}^{\infty} dE \frac{E^2 + \Delta^2}{E^2 - \Delta^2} \operatorname{sech}^2\left(\frac{E}{2T}\right)$$

Hebel-Slichter peak

# Absence of the Hebel-Slichter peak

Weak coupling: 
$$\frac{1}{T_1 T} \propto \sum_{\mathbf{k}\mathbf{k}'} \left( 1 + \frac{\Delta_1 \Delta_2}{E_{\mathbf{k}} E_{\mathbf{k}'}} \right) \left[ -\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}} \right] \delta(E_{\mathbf{k}} - E_{\mathbf{k}'})$$

s-wave

$$\Delta_1 = \Delta_2 = \Delta$$

$$\frac{1}{T_1} \propto \int_{\Delta(T)}^{\infty} dE \frac{E^2 + \Delta^2}{E^2 - \Delta^2} \operatorname{sech}^2\left(\frac{E}{2T}\right)$$

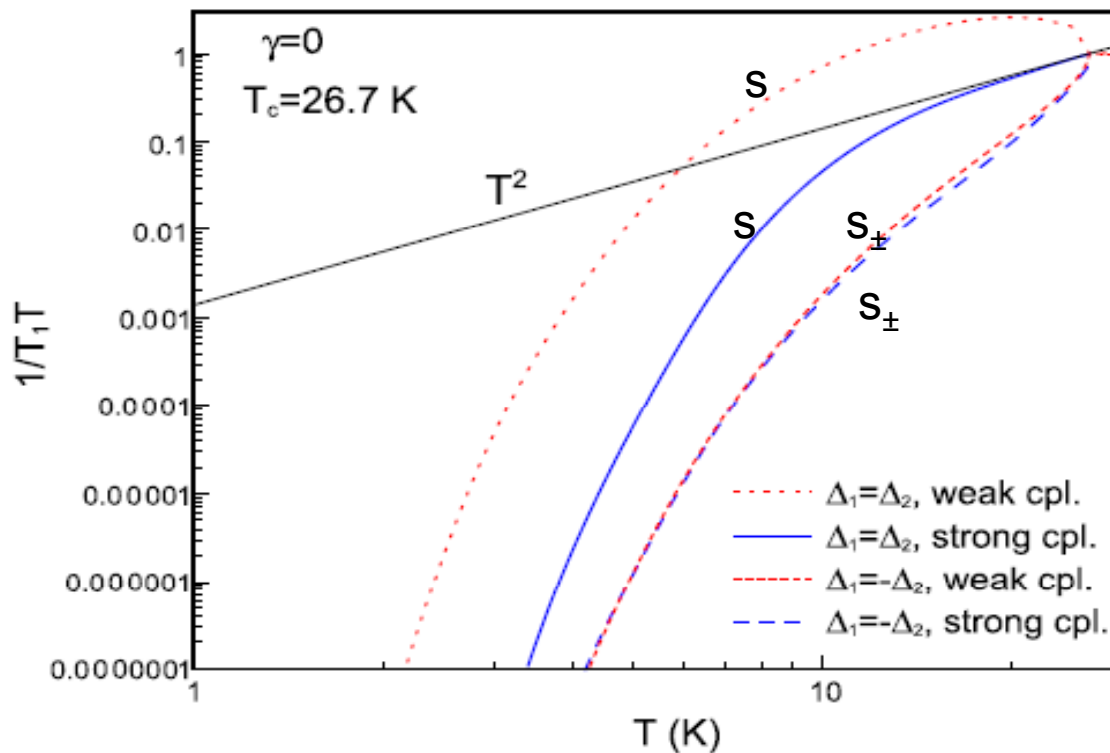
Hebel-Slichter peak

simplest  $s_{\pm}$ -wave

$$\Delta_1 = -\Delta_2 = \Delta$$

$$\frac{1}{T_1} \propto \int_{\Delta(T)}^{\infty} dE \frac{E^2 - \Delta^2}{E^2 - \Delta^2} \operatorname{sech}^2\left(\frac{E}{2T}\right) = \int_{\Delta(T)}^{\infty} dE \operatorname{sech}^2\left(\frac{E}{2T}\right)$$

No Hebel-Slichter peak!



## Strong-coupling limit

$$\frac{1}{T_1 T} \propto \int_0^\infty d\omega \left( -\frac{\partial f(\omega)}{\partial \omega} \right) \{ [\text{Re}g_1^Z(\omega) + \text{Re}g_2^Z(\omega)]^2 + [\text{Re}g_1^\Delta(\omega) + \text{Re}g_2^\Delta(\omega)]^2 \}.$$

Here  $g_i^Z(\omega) = n_i(\omega) Z_i(\omega) \omega / D_i(\omega)$

$g_i^\Delta(\omega) = n_i(\omega) \phi_i(\omega) / D_i(\omega)$

$D_i(\omega) = \sqrt{[Z_i(\omega)\omega]^2 - \phi_i^2(\omega)}$ .

$\phi_i(\omega) = Z_i(\omega) \Delta_i(\omega)$  ← complex order parameter  
 EEliashberg equations:

$Z_i(\omega)$  is the mass renormalization  
 $n_i(\omega)$  is a partial density of states

$$\phi_i(\omega) = \sum_j \int_{-\infty}^{\infty} dz K_{ij}^\Delta(z, \omega) \text{Re}g_j^\Delta(z) + i\gamma \frac{g_1^\Delta(\omega) - g_2^\Delta(\omega)}{2\mathcal{D}}$$

$$(Z_i(\omega) - 1)\omega = \sum_j \int_{-\infty}^{\infty} dz K_{ij}^Z(z, \omega) \text{Re}g_j^Z(z) + i\gamma \frac{g_1^Z(\omega) + g_2^Z(\omega)}{2\mathcal{D}}$$

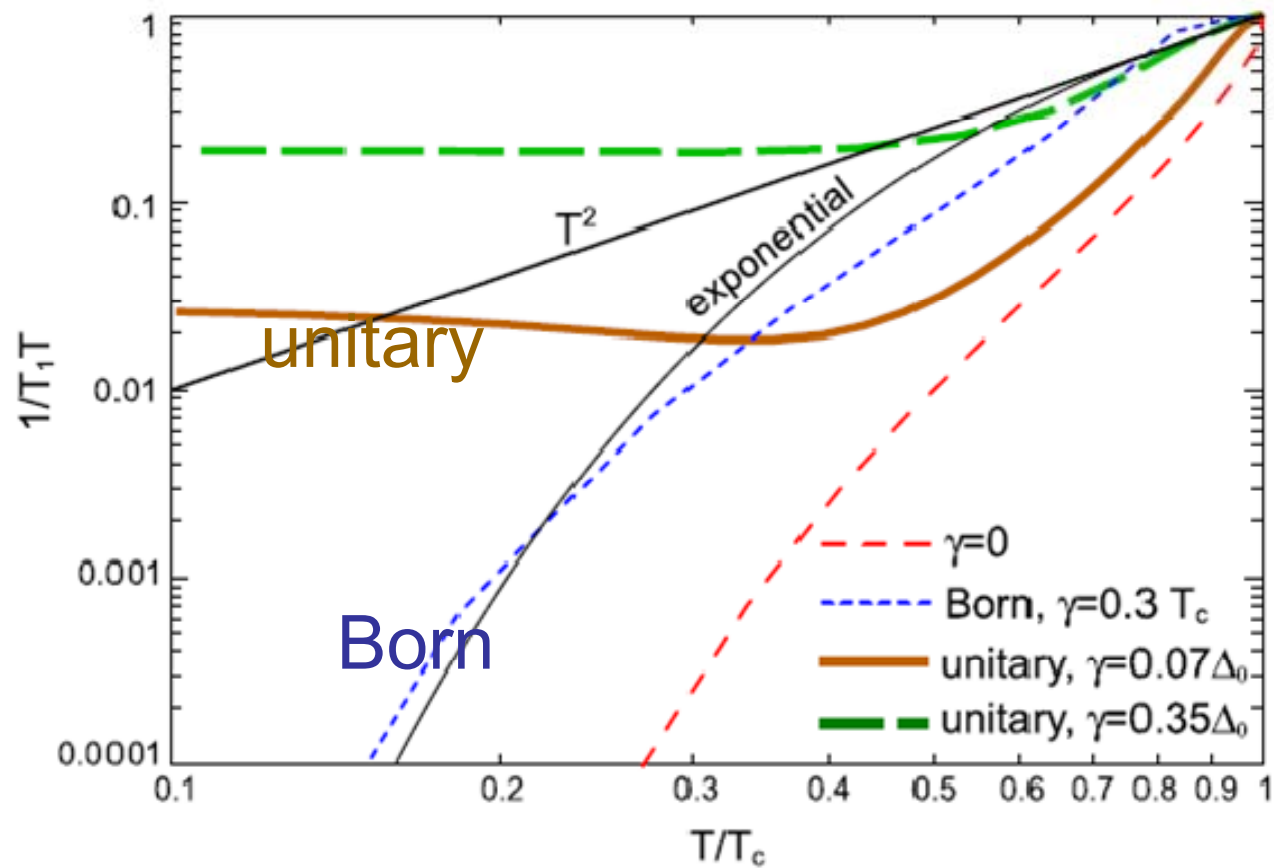
$$\sigma = \frac{[\pi N(0)v]^2}{1 + [\pi N(0)v]^2}$$

$$\mathcal{D} = 1 - \sigma + \sigma \{ [g_1^Z(\omega) + g_2^Z(\omega)]^2 - [g_1^\Delta(\omega) - g_2^\Delta(\omega)]^2 \}$$

$\gamma = 2c\sigma/N(0)$  is the normal-state scattering rate,  
 $c$  is the impurity concentration,  $v$  is the impurity potential,  $\sigma$  is the impurity strength ( $\sigma \rightarrow 0$ : Born limit,  $\sigma = 1$ : unitary limit)

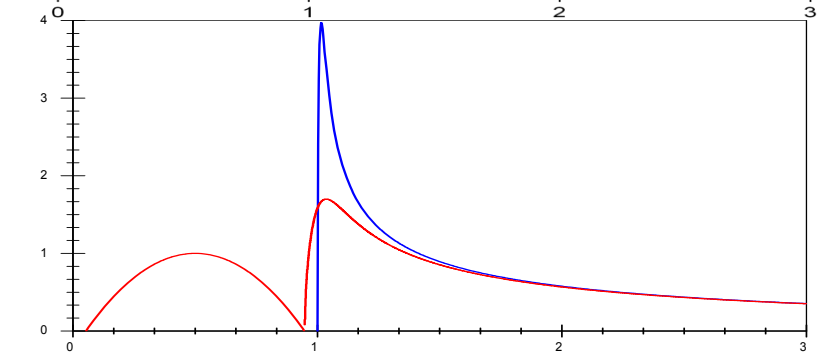
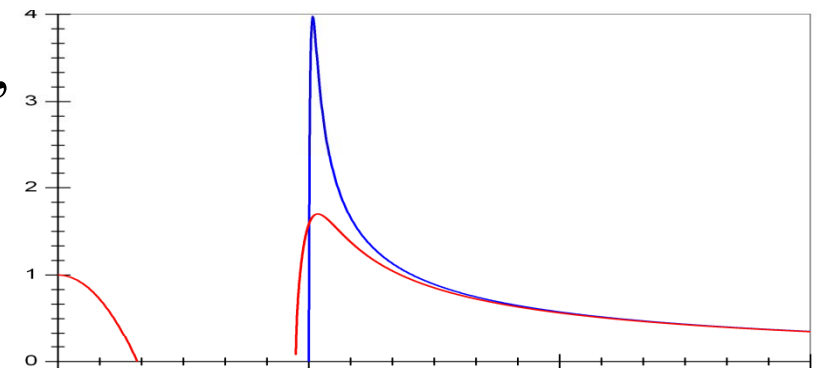
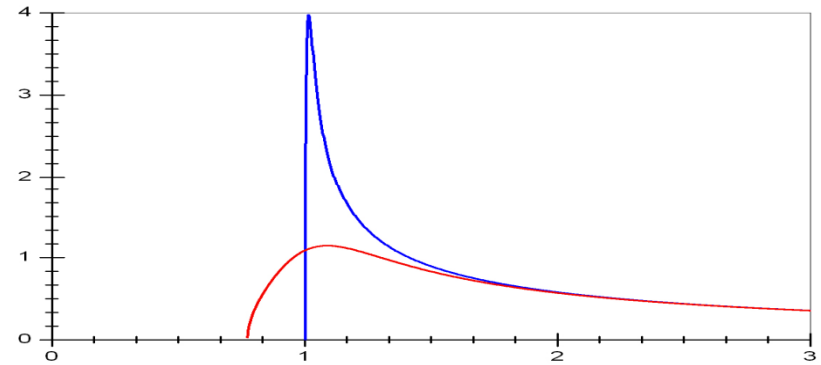


The dependence of  $1/T_1$  is not reproduced in both Born and unitary limits:

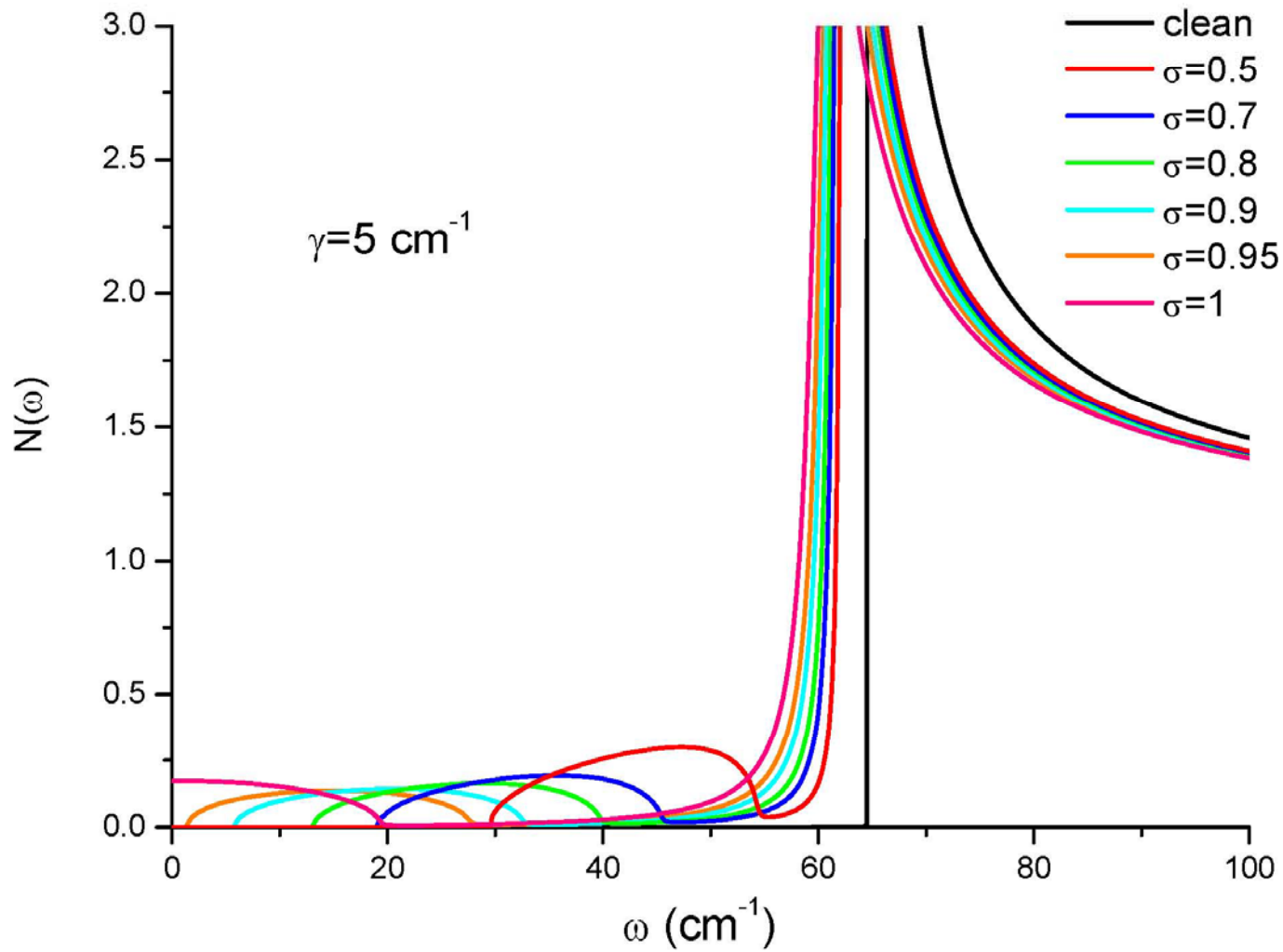


# Impurity scattering: intermediate regime

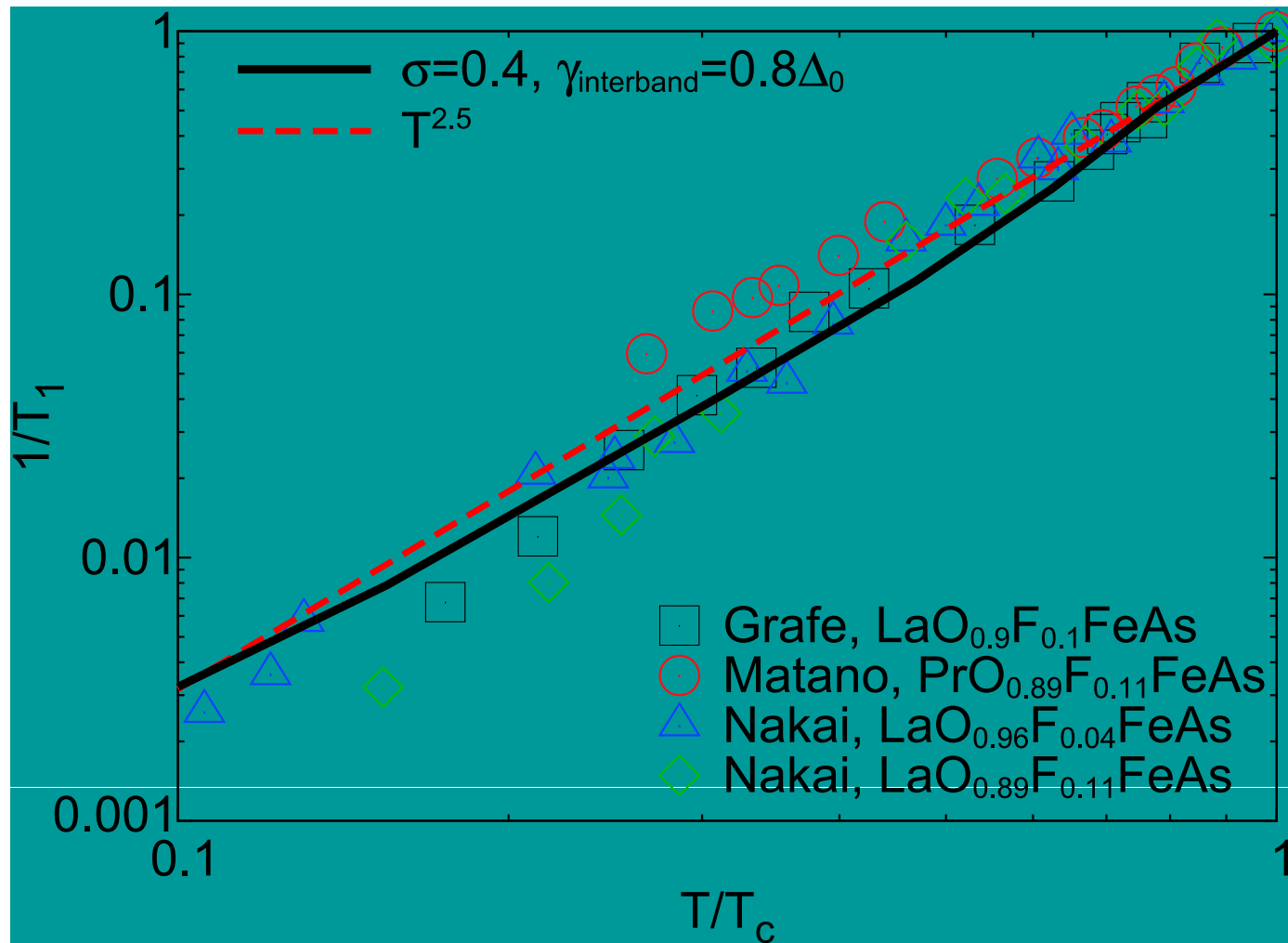
1. Nonmagnetic impurities are pair breaking
2. Born limit: no coherence peak, exponential at low T
3. Unitary limit: weak  $T_c$  suppression, zero-energy bound state
4. Intermediate limit: finite energy bound state, simulates power law



# Impurity scattering: densities of states

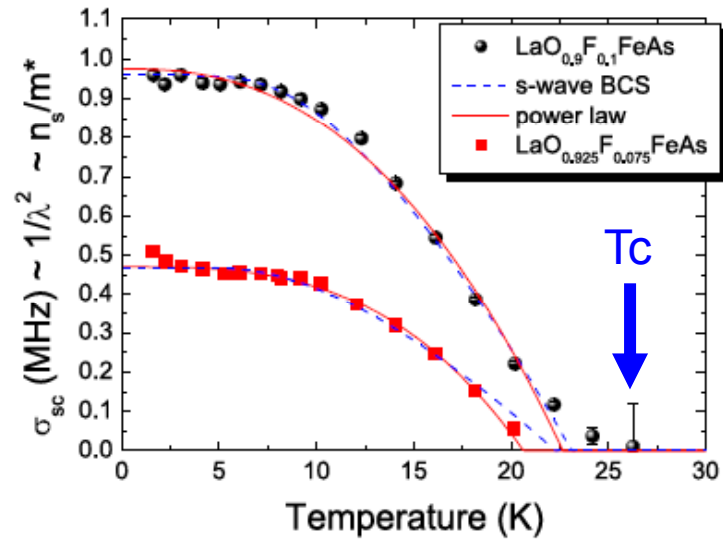


# NMR: possible explanation of low-T behavior

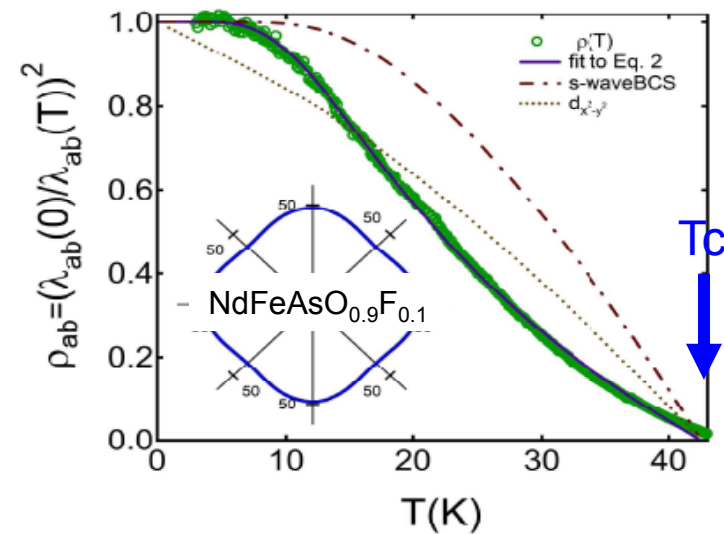


# Superfluid density $n_s$ : experiment

$\mu$ SR: Luetkens, arXiv:0804.3115



$\mu$ SR: Martin et al, arXiv:0807.0876



# Superfluid density: the model

$$1/\lambda_{L,\alpha\beta}^2(T) \equiv (\omega_{p,\alpha\beta}^{sf}(T)/c)^2 = \sum_{i=\sigma,\pi} \left( \frac{\omega_{p,i}^{\alpha\beta}}{c} \right)^2 \pi T \sum_{n=-\infty}^{\infty} \frac{\tilde{\Delta}_i^2(n)}{[\tilde{\omega}_i^2(n) + \tilde{\Delta}_i^2(n)]^{3/2}}$$

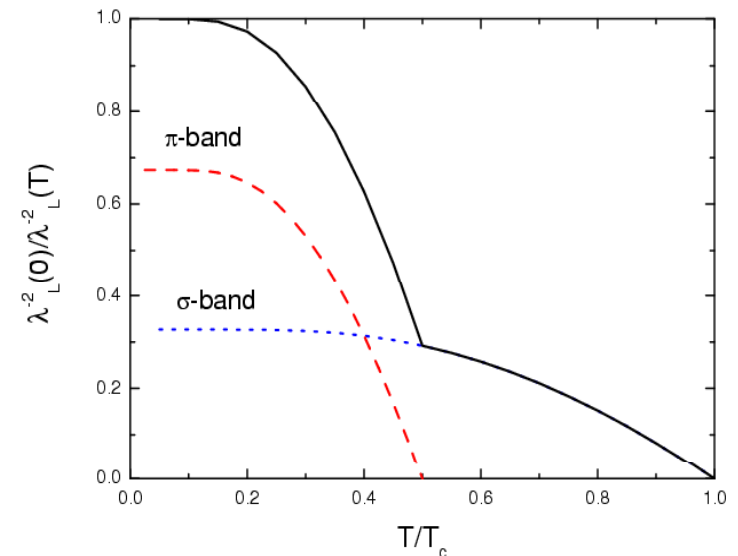
where  $\tilde{\omega}(n) = \omega_n Z(\omega_n)$  and  $\tilde{\Delta}(\omega_n) = \Delta(\omega_n) Z(\omega_n)$  are the solutions of the Eliashberg equations.

## Effects of impurities

$$\Delta_i \rightarrow \Delta_i^0 + \sum_j \gamma_{ij} \Delta_j / 2 \sqrt{\omega_n^2 + \Delta_j^2},$$

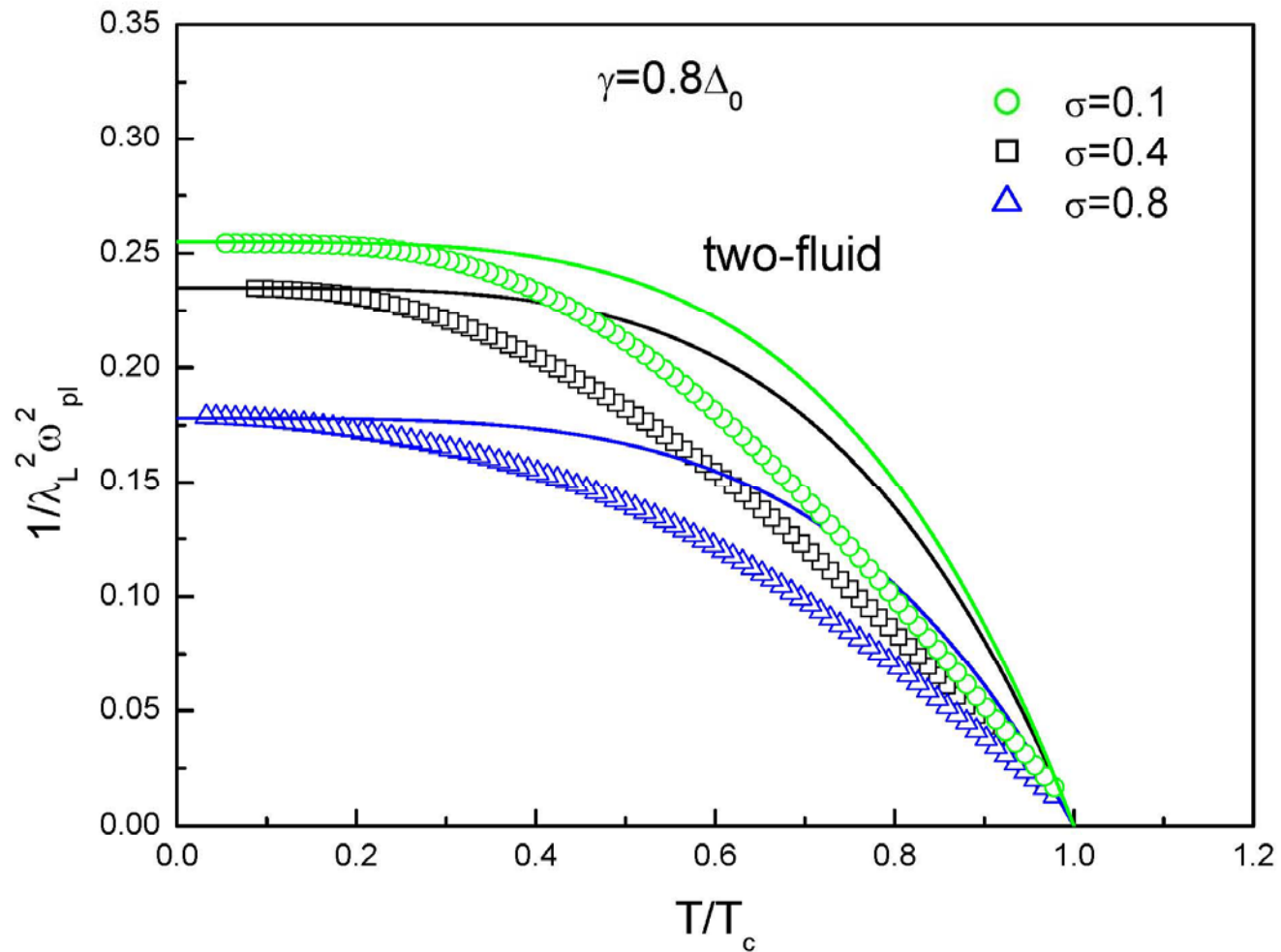
$$Z(\omega_n) \rightarrow Z^0(\omega_n) + \sum_j \gamma_{ij} / 2 \sqrt{\omega_n^2 + \Delta_j^2}$$

The case of weakly coupled bands (MgB2)

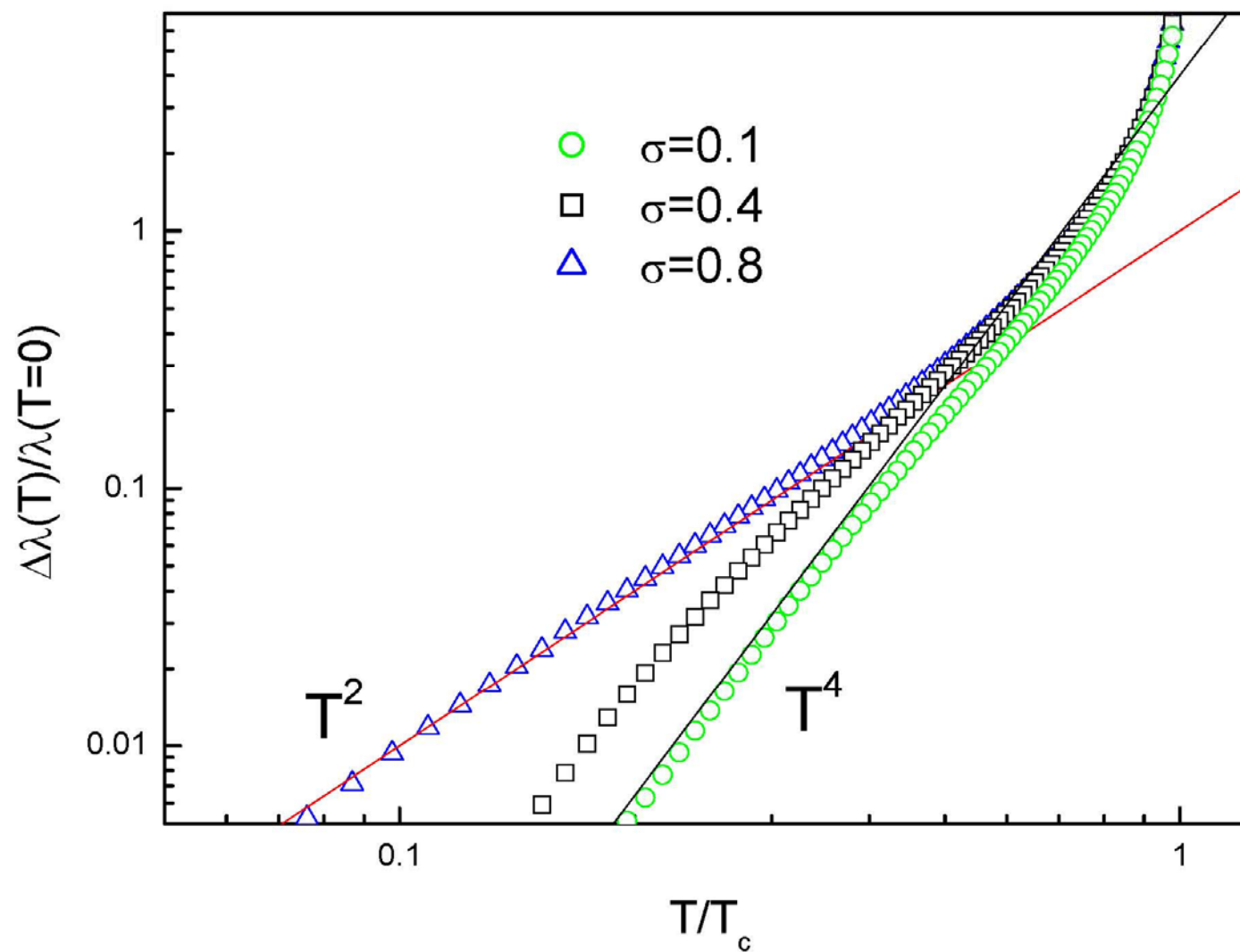




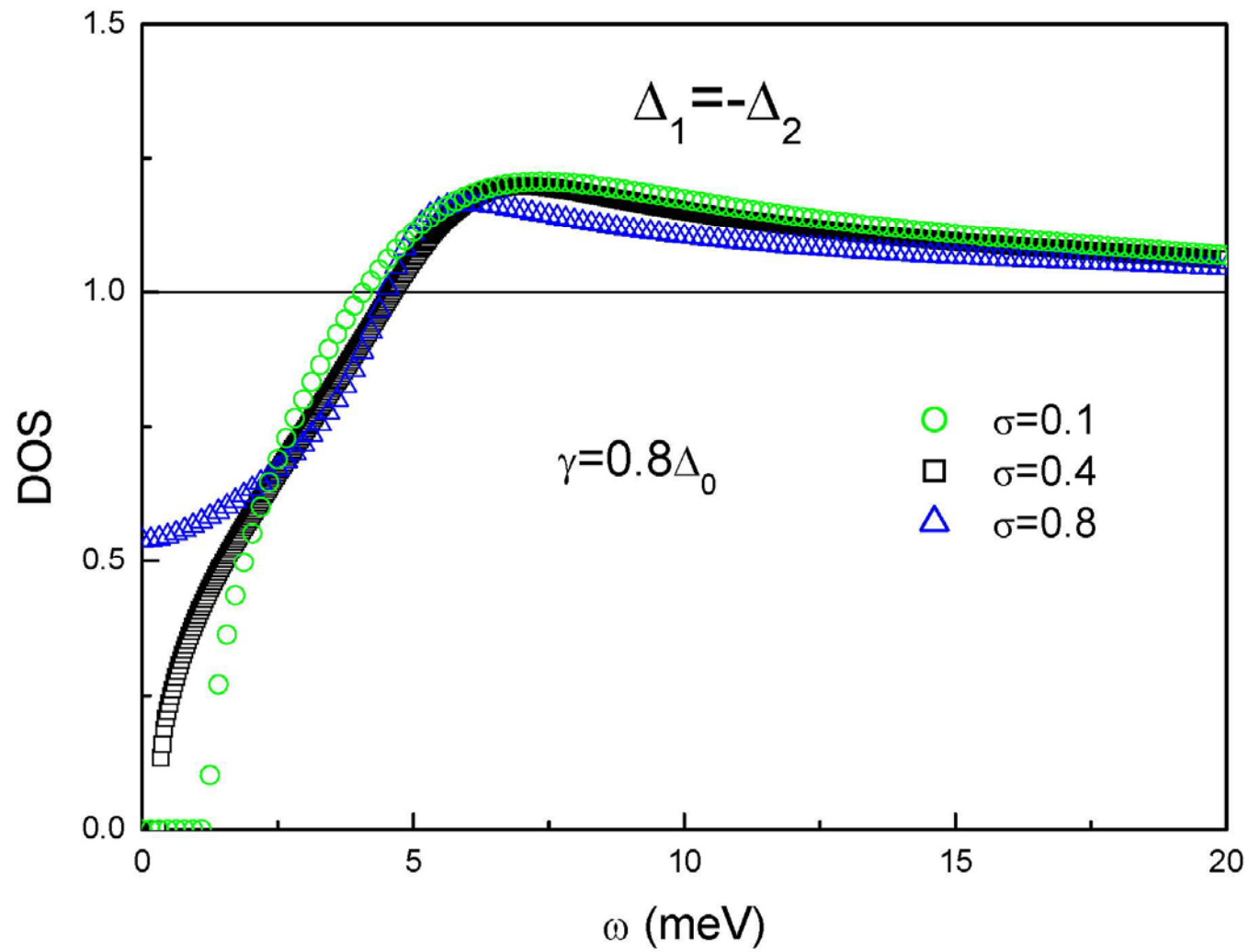
# Magnetic field penetration depth: calculations for various scattering rates



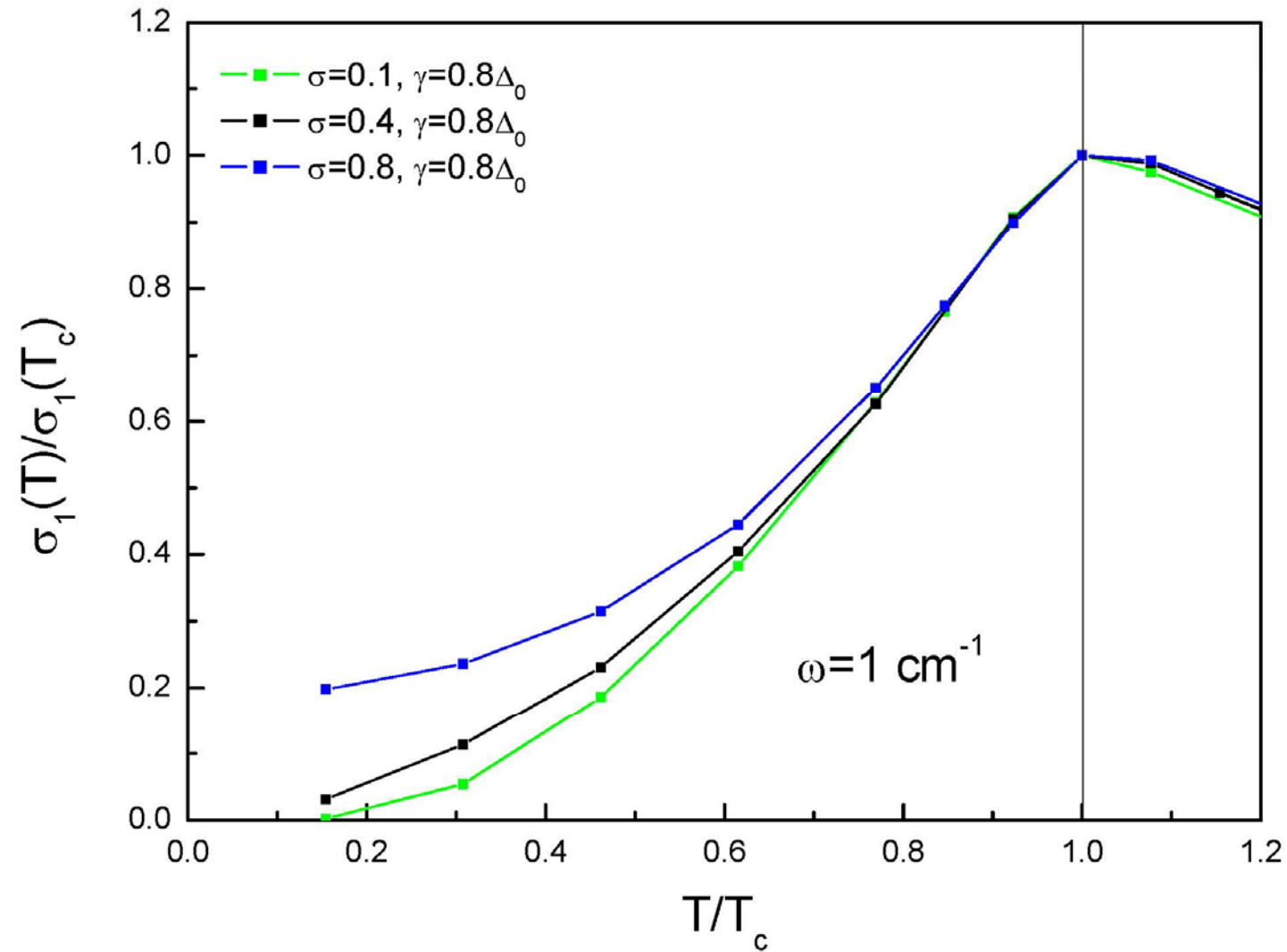
# Magnetic field penetration depth: low T



# Low energy density of states



# The microwave conductivity $\sigma_1(T)$



# Tunneling in N/S junction: extending Andreev reflection formalism (the BTK model) to two bands

A.A. Golubov et al, PRL 103, 077003 (2009)

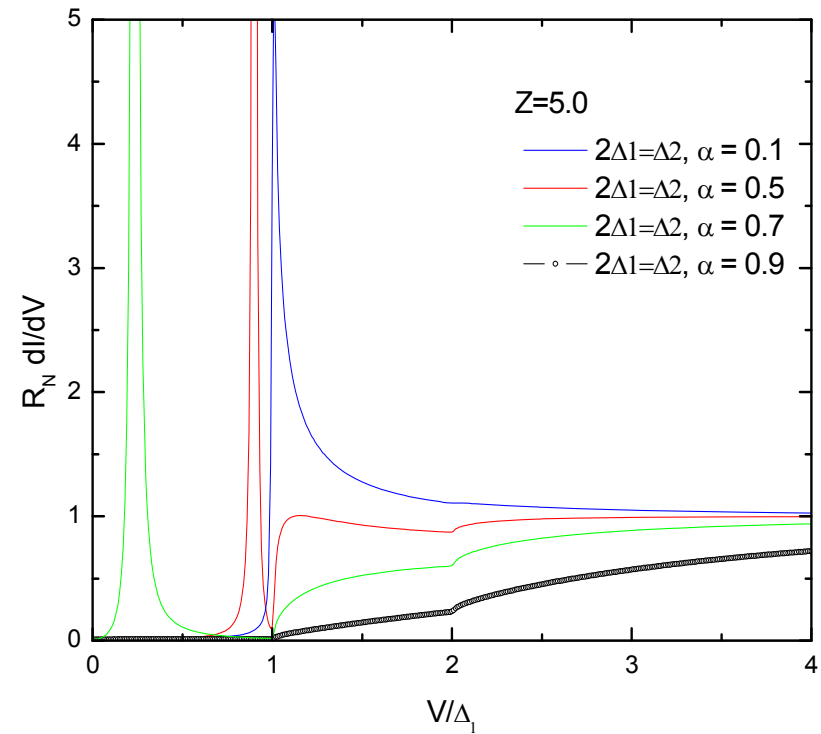
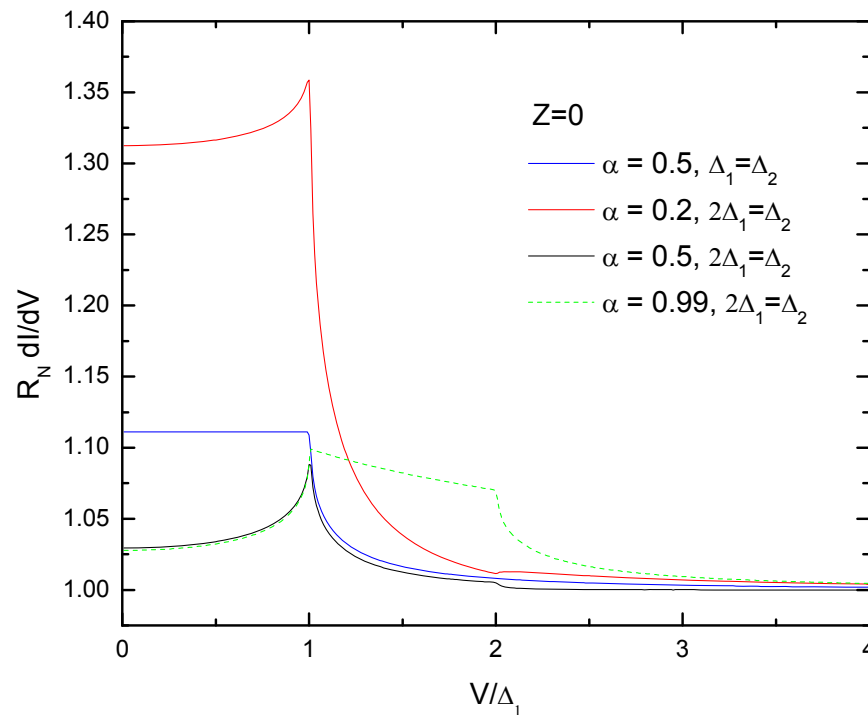
$$\begin{aligned}\Psi &= \Psi_N \theta(-x) + \Psi_S \theta(x), \\ \Psi_N &= \psi_k \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a \psi_k \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b \psi_{-k} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \\ \Psi_S &= c \left[ \phi_p \begin{pmatrix} u_1 \\ v_1 e^{-i\varphi_1} \end{pmatrix} + \alpha \phi_q \begin{pmatrix} u_2 \\ v_2 e^{-i\varphi_2} \end{pmatrix} \right] \\ &+ d \left[ \phi_{-p} \begin{pmatrix} v_1 \\ u_1 e^{-i\varphi_1} \end{pmatrix} + \alpha \phi_{-q} \begin{pmatrix} v_2 \\ u_2 e^{-i\varphi_2} \end{pmatrix} \right]\end{aligned}$$

$$S_{++} \text{ model: } \varphi_1 = \varphi_2 \quad S_{+-} \text{ model: } \varphi_1 = \varphi_2 + \pi$$

$\alpha$  is the mixing coefficient between electron and hole bands

# Tunneling conductance in the $s_{\pm}$ case

$$Z = H / \hbar v_{FN} \quad H - \text{the barrier height}$$

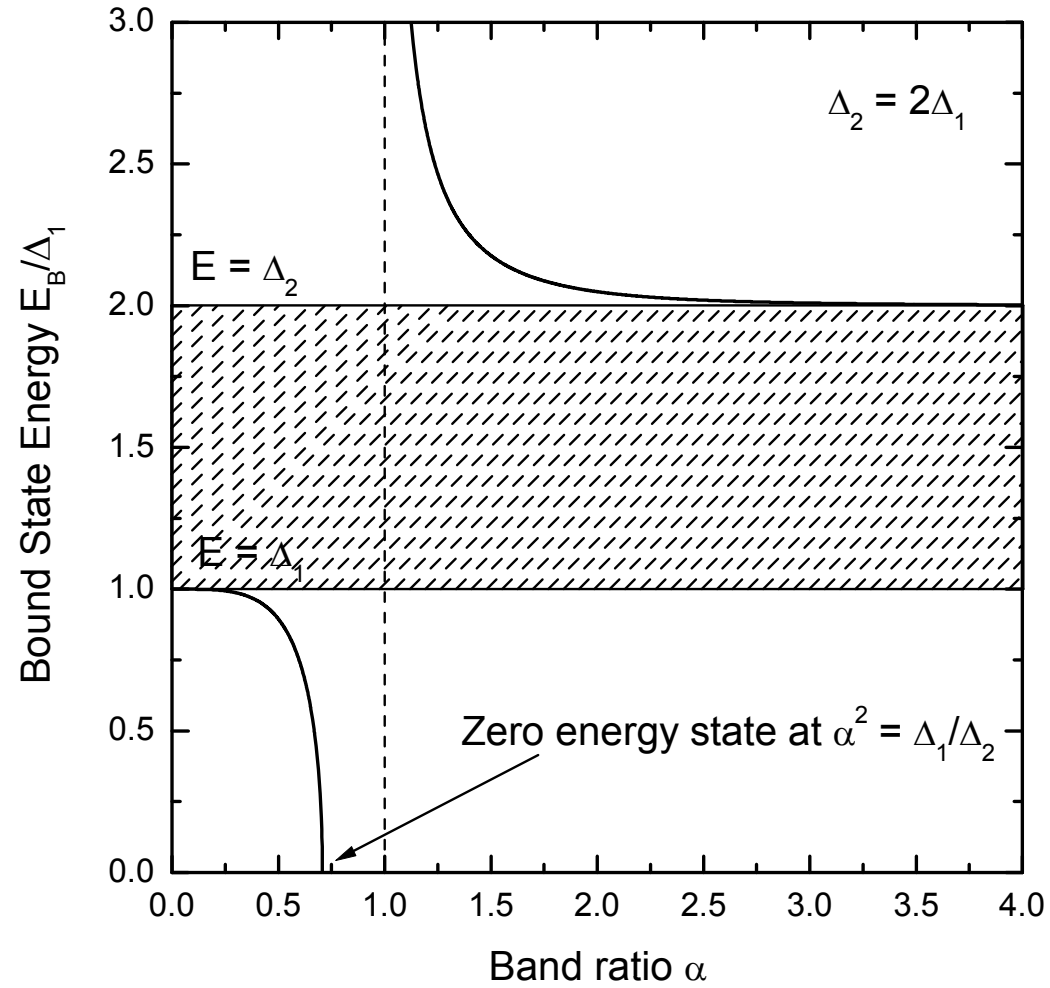


**Andreev** conductance is suppressed due to destructive **interband interference**

**Bound states** appear at **finite energy** for large  $Z$



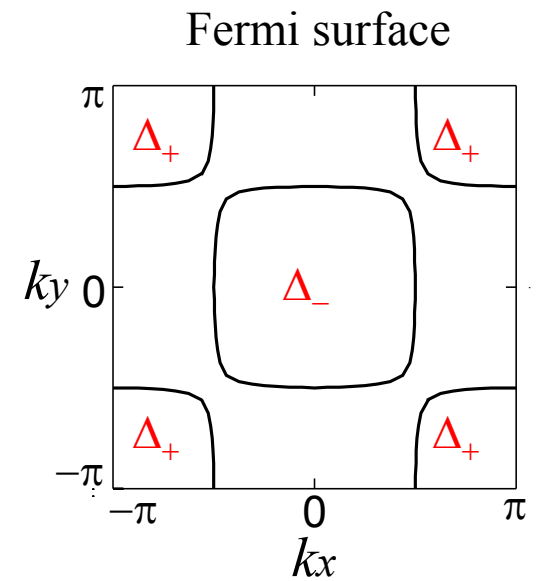
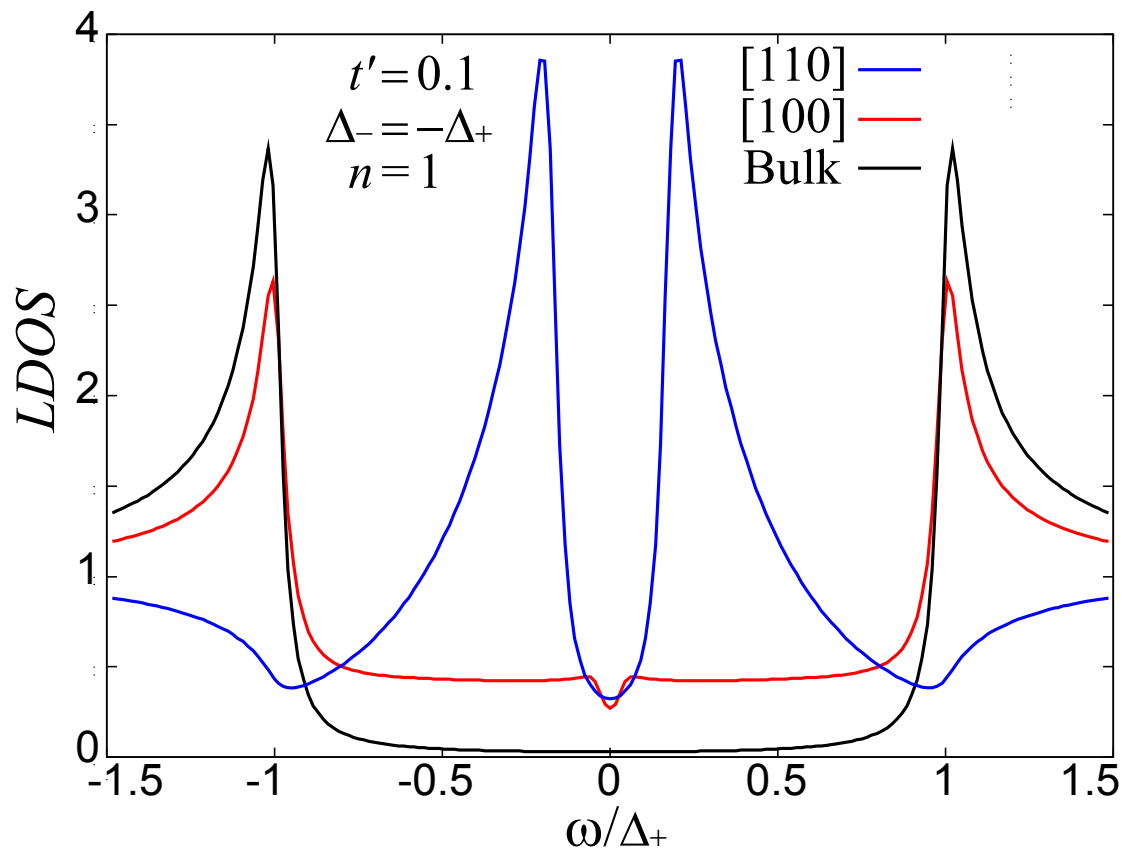
# Tunneling regime: Surface bound states



# LDOS at surface

$s+ -$ wave

$$\Delta_{+(-)} = +(-)0.1$$



Andreev bound state with non zero energy (Onari and Tanaka)

# Conclusions

- $s_{\pm}$  pairing can explain some properties of superconducting *Fe-pnictides*.
- $T_c$  is robust against *unitary* interband scattering.
- The lack of an NMR Hebel-Slichter peak is consistent with the nodeless  $s_{\pm}$  wave symmetry of the order parameter (whether in the clean or dirty limit).
- The low-temperature power-law behavior of  $1/T_1$  can be also explained in the framework of the  $s_{\pm}$  model but requires the impurity scattering beyond the Born limit.
- *Conductance Peak* can appear in *Andreev* and tunneling experiments, but, unlike nodal superconductors, at finite energy.