Where is the quantum critical point in the cuprate superconductors ?

Subir Sachdev

arXiv:0907.0008

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Destruction of Neel order in the cuprates by electron doping,
R. K. Kaul, M. Metlitksi, S. Sachdev, and C. Xu,
Physical Review B **78**, 045110 (2008).

Paired electron pockets in the underdoped cuprates,
V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Competition between spin density wave order and superconductivity in the underdoped cuprates,
Eun Gook Moon and S. Sachdev,
arXiv:0905.2608

Fractionalization of the spin density wave transition in metals
M. Metlitski, Y. Qi, S. Sachdev, and C. Xu
to appear....

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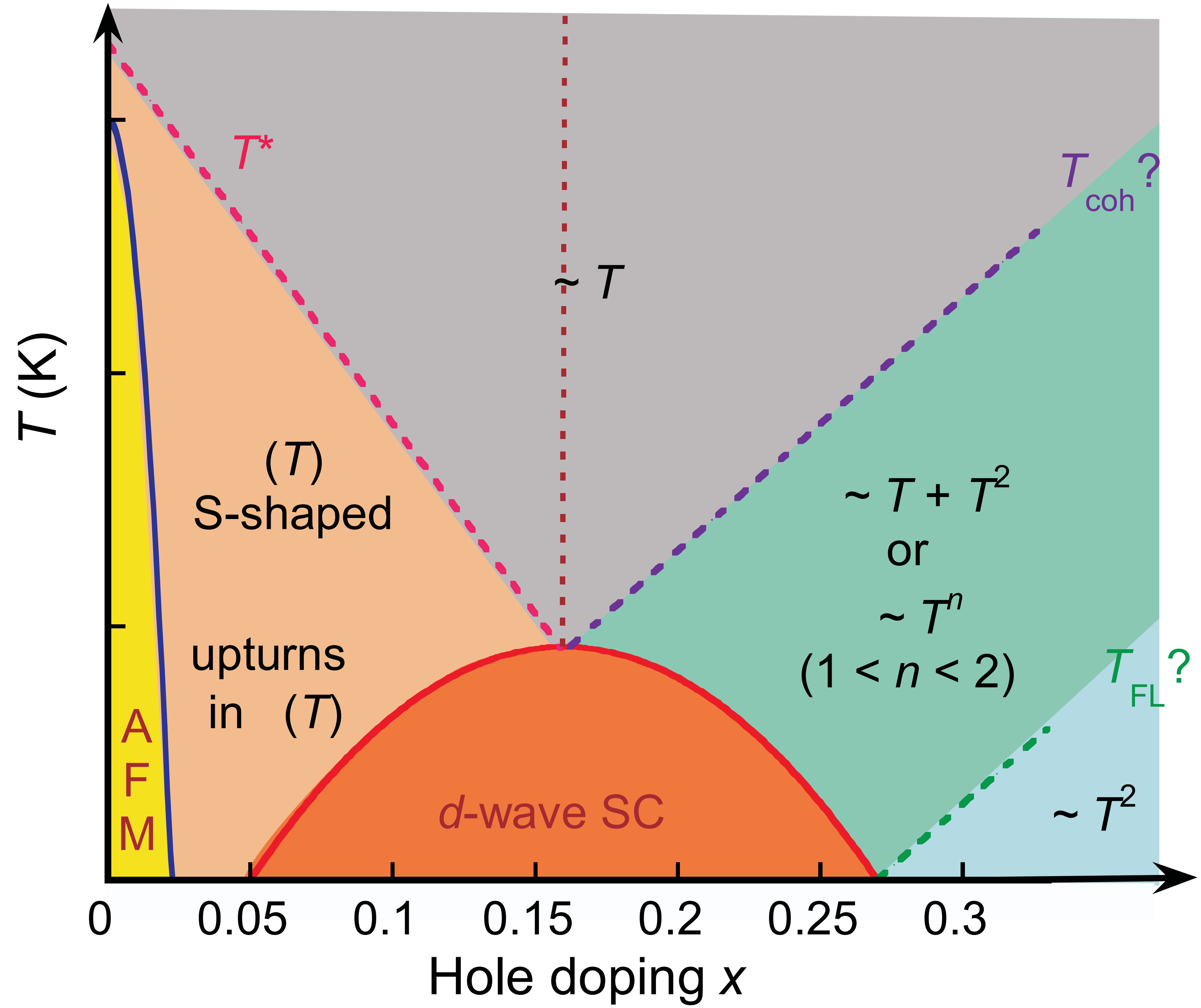
Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

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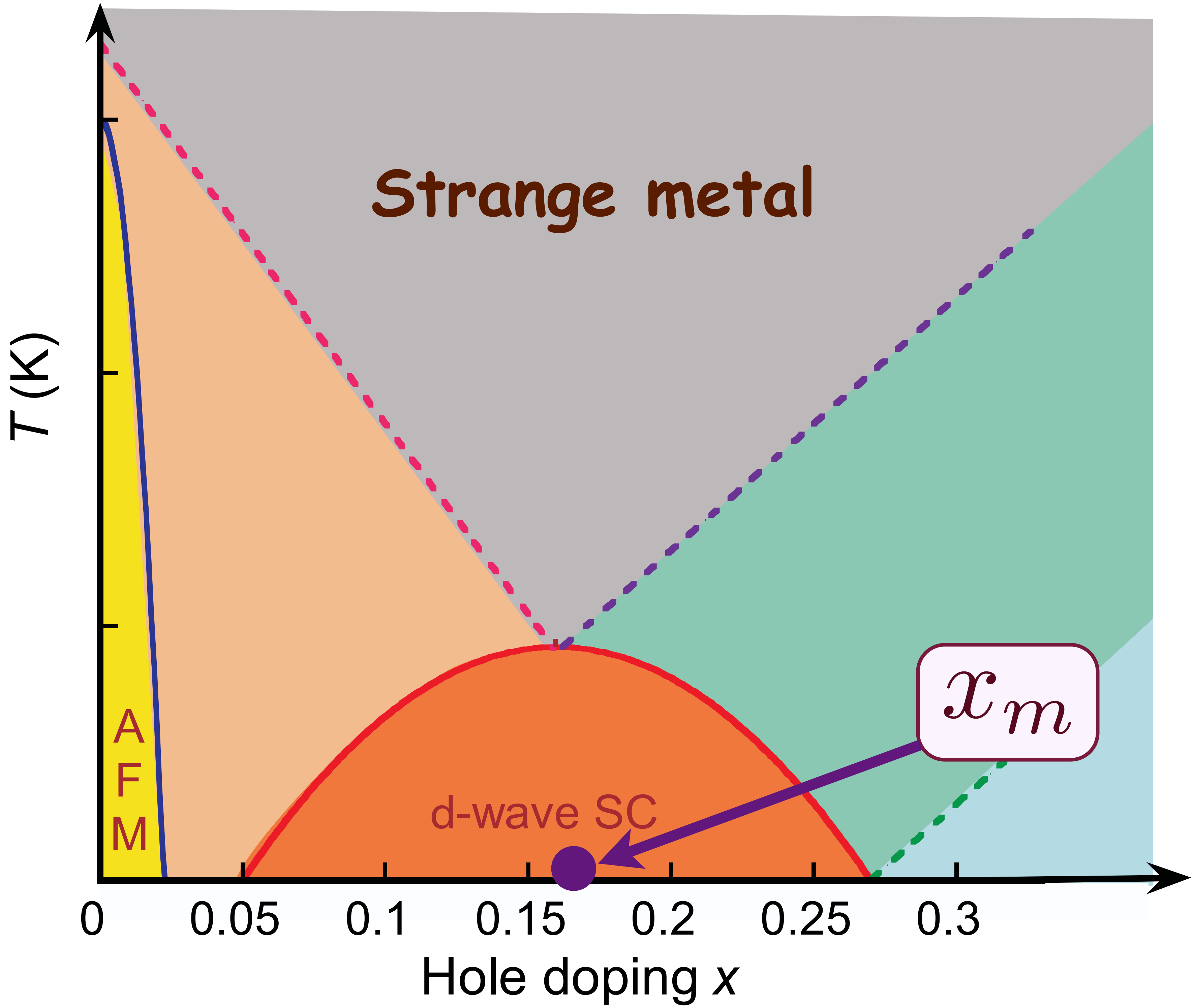
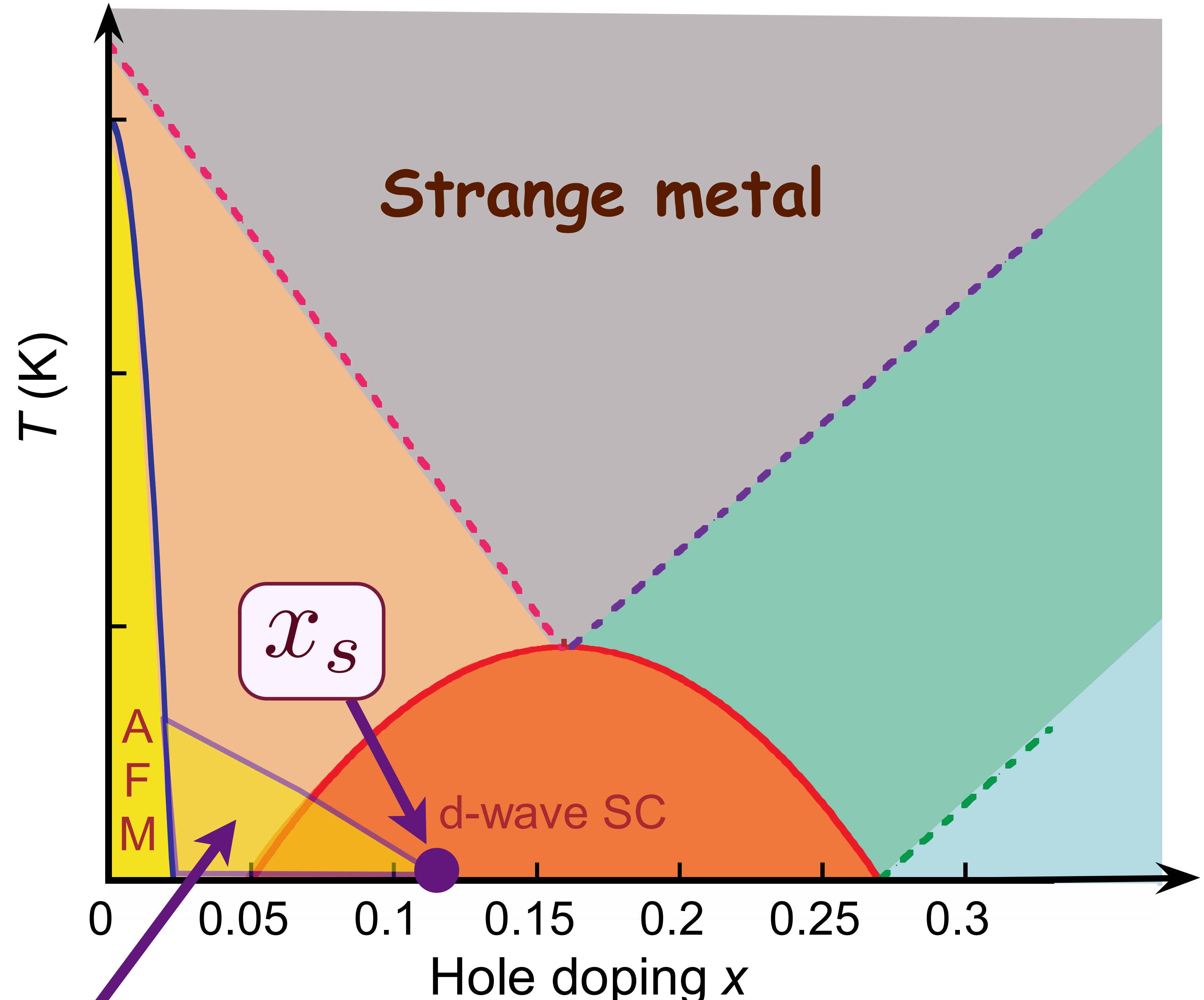


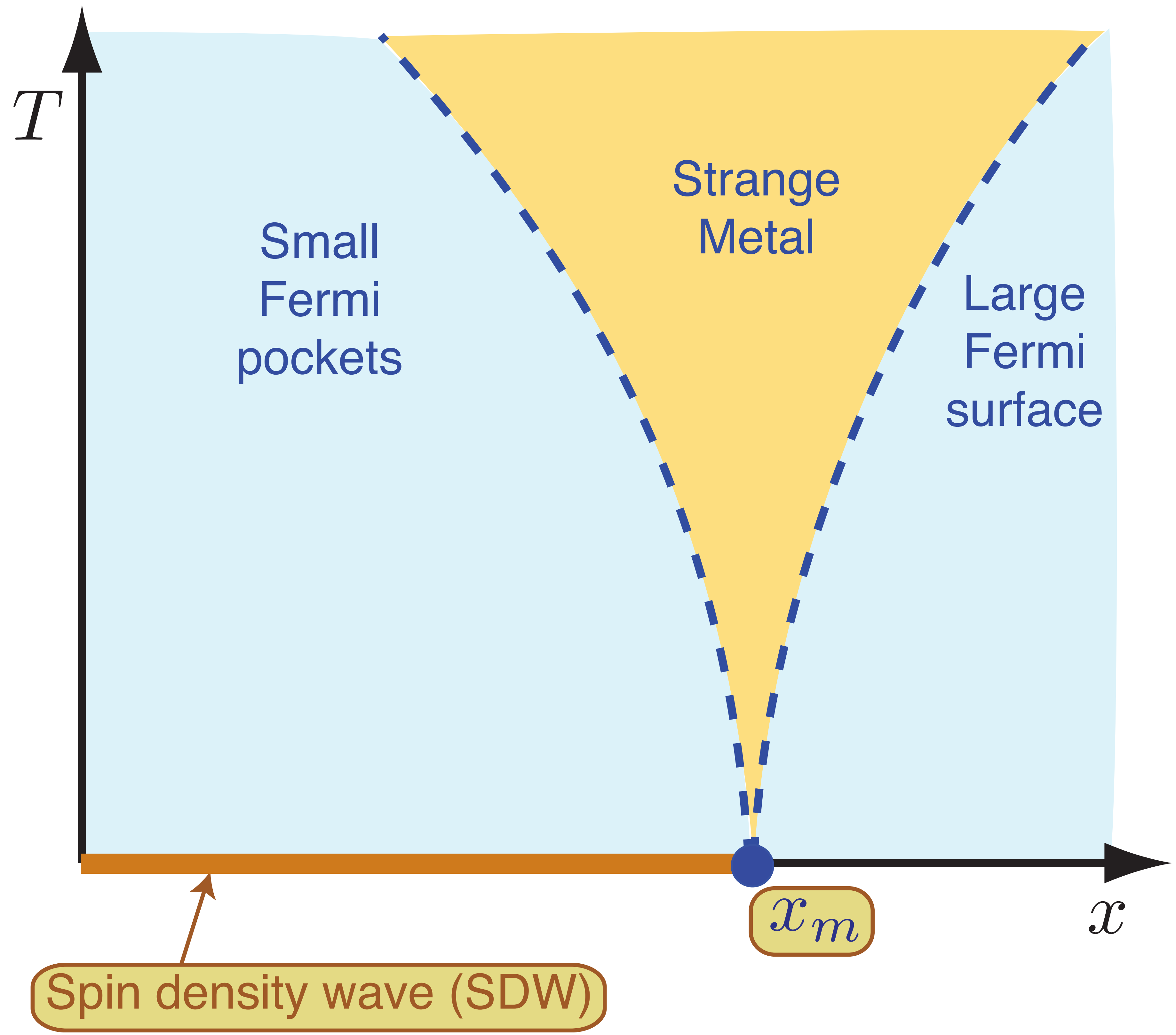
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



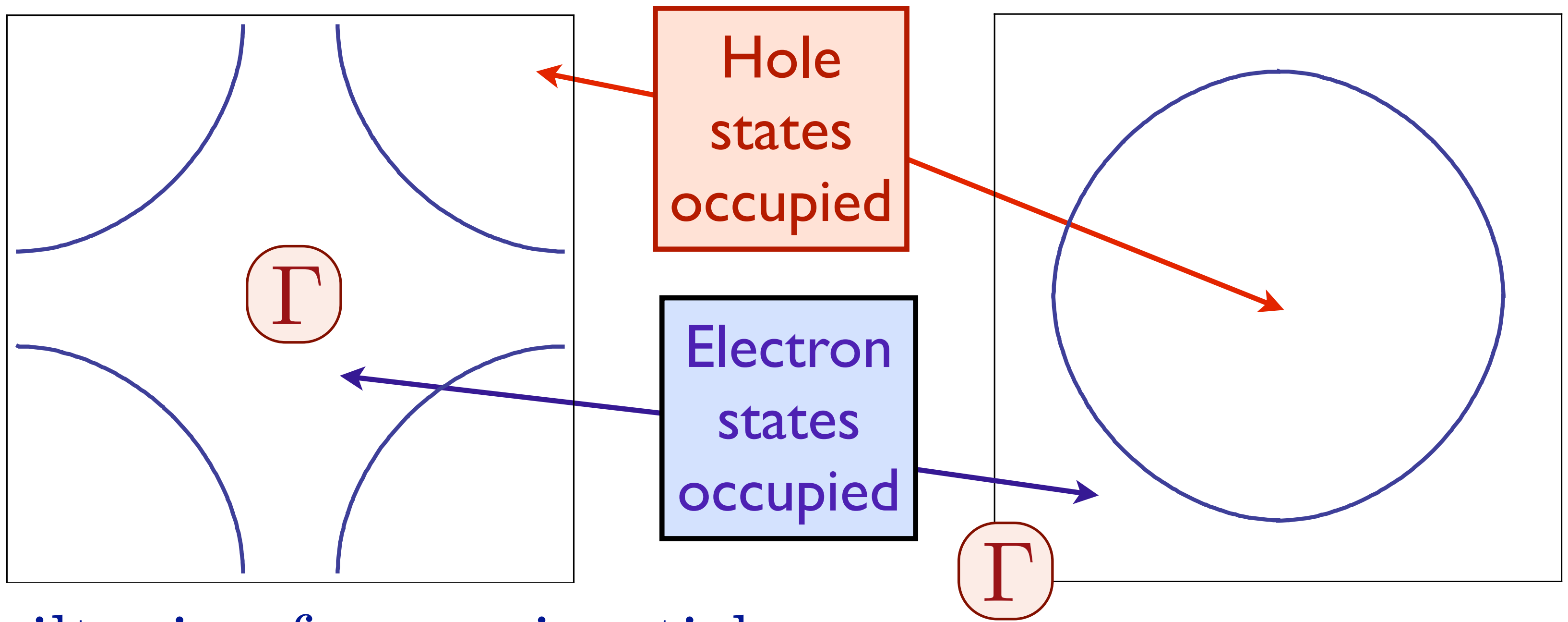
Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

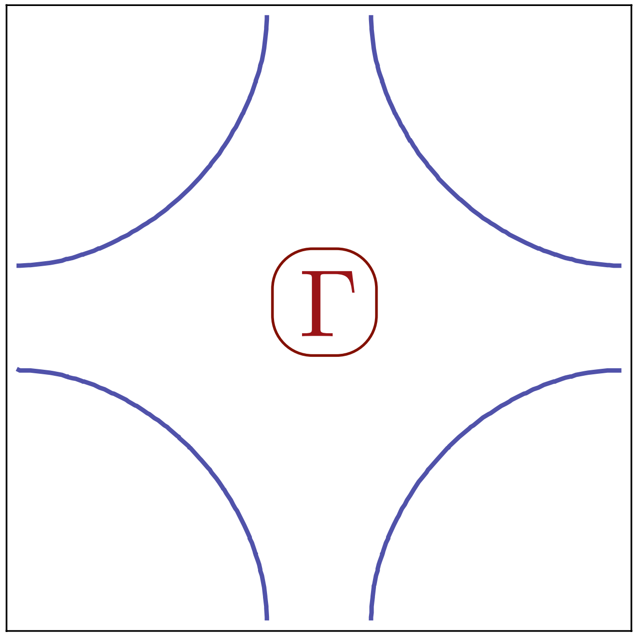
$$H_{\text{sdw}} = -\vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} = (0, 0, \varphi)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

This leads to the Fermi surfaces shown in the following slides for hole doping.

Spin density wave theory in hole-doped cuprates



S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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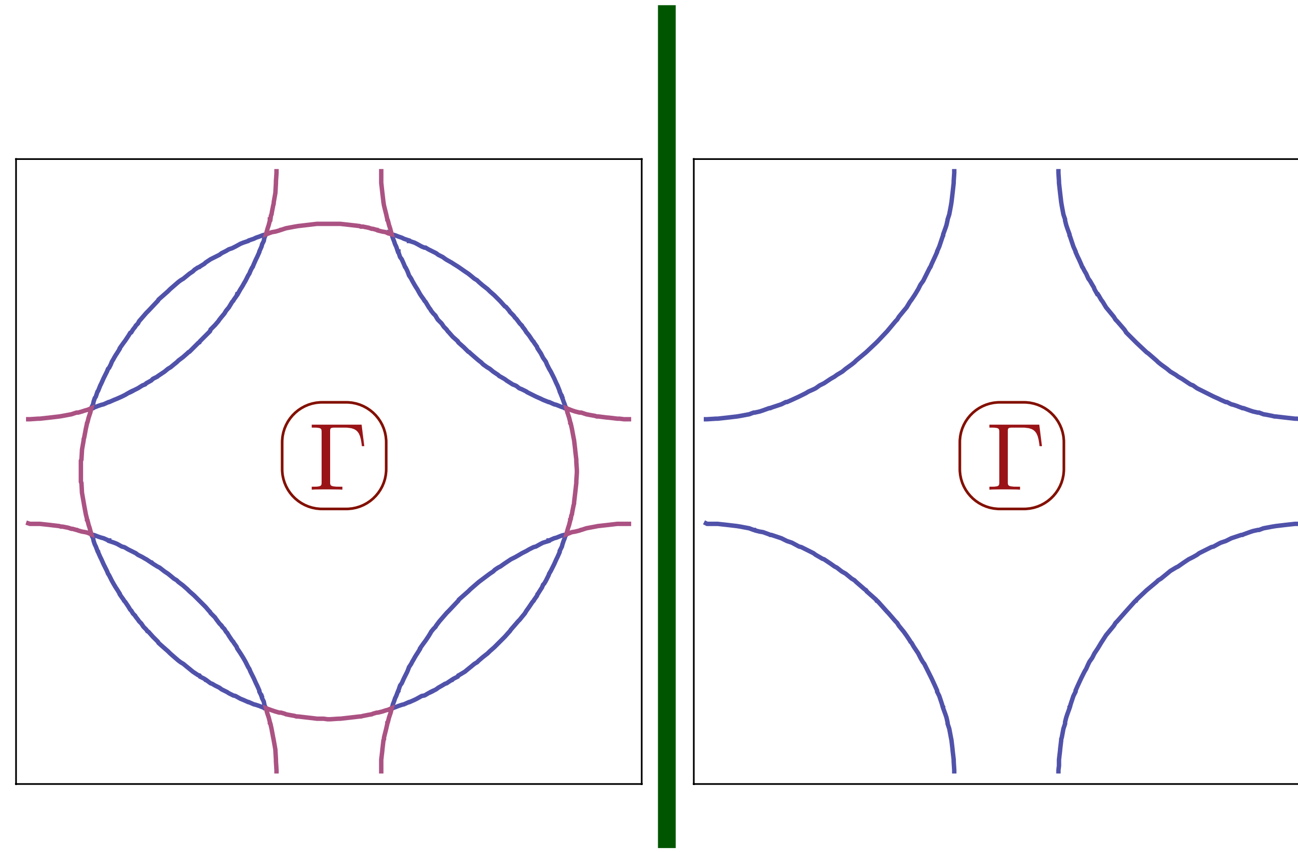

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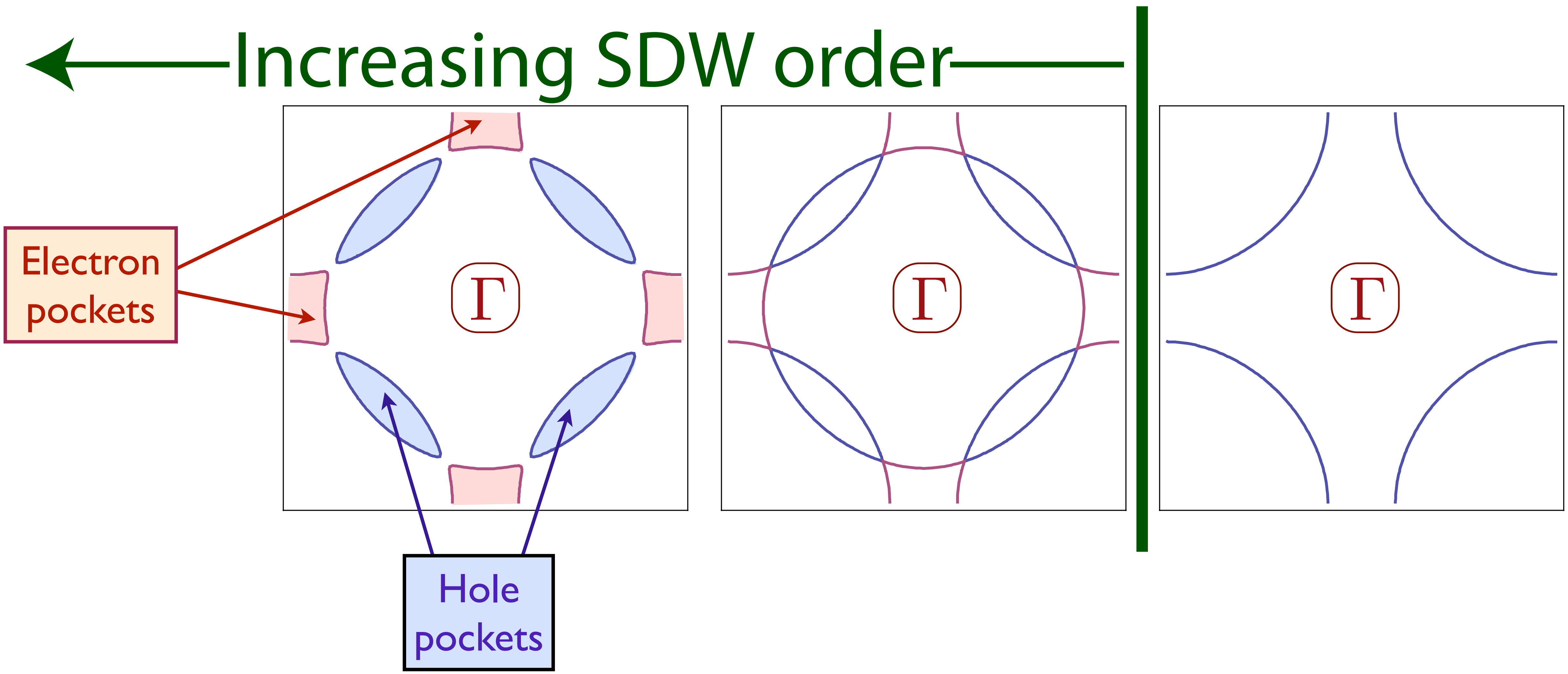

SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

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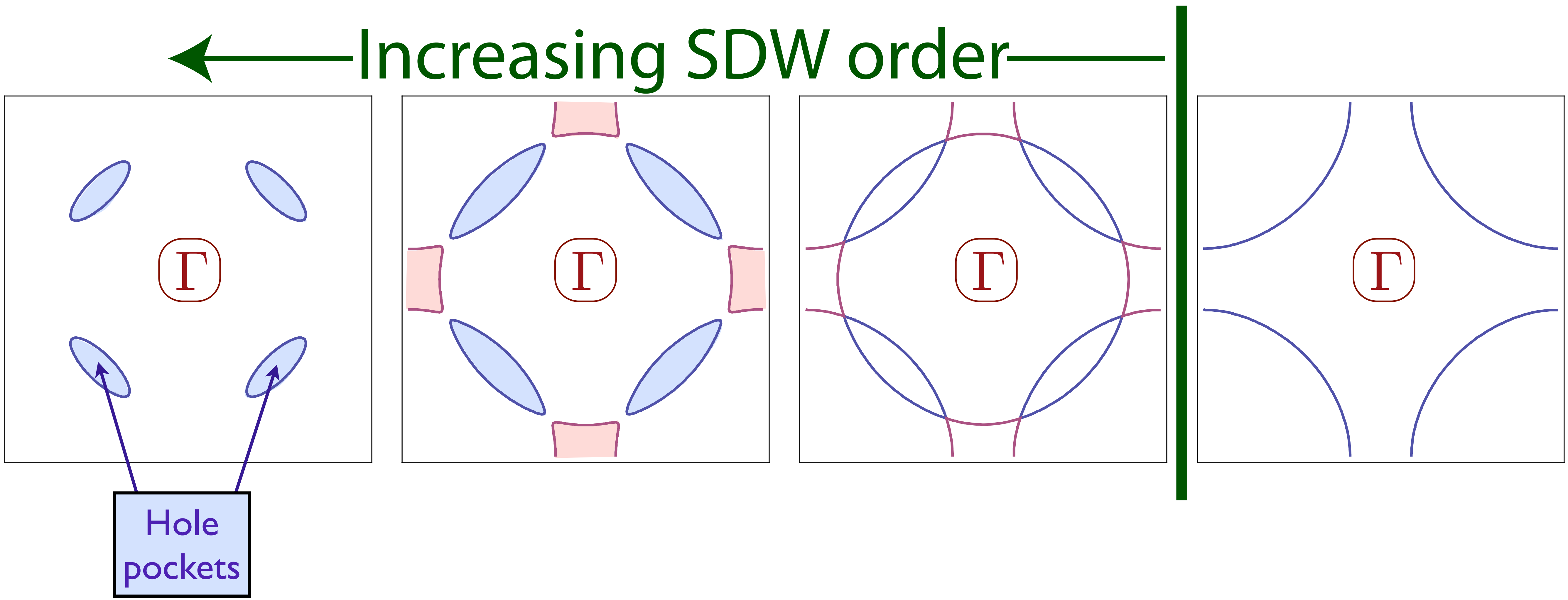

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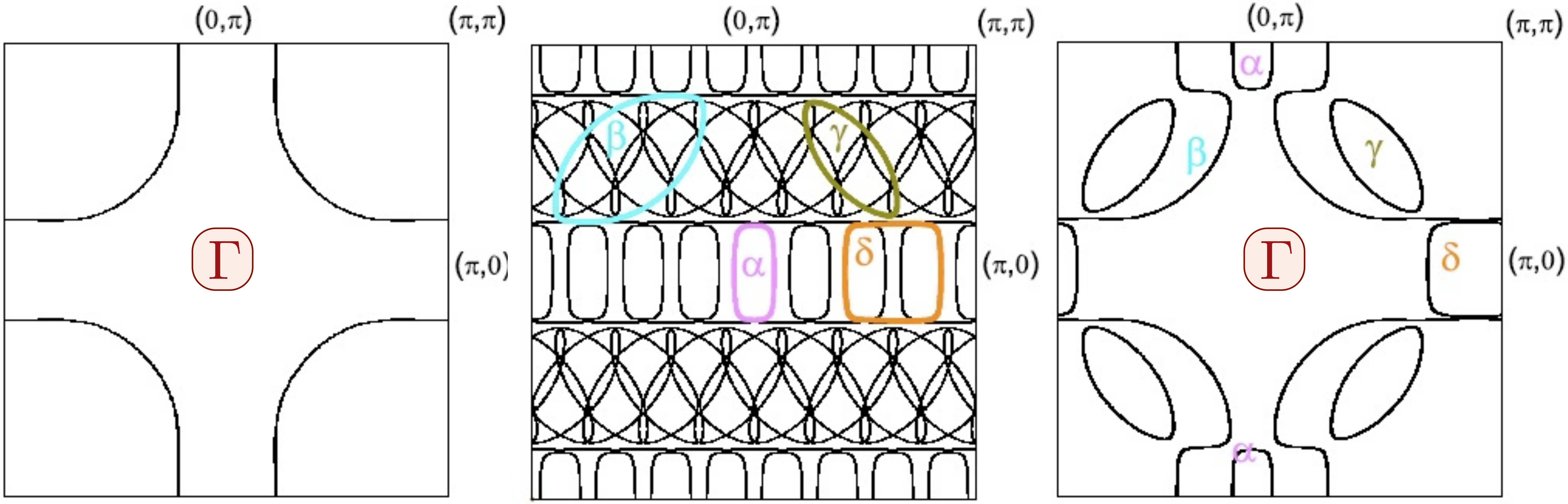
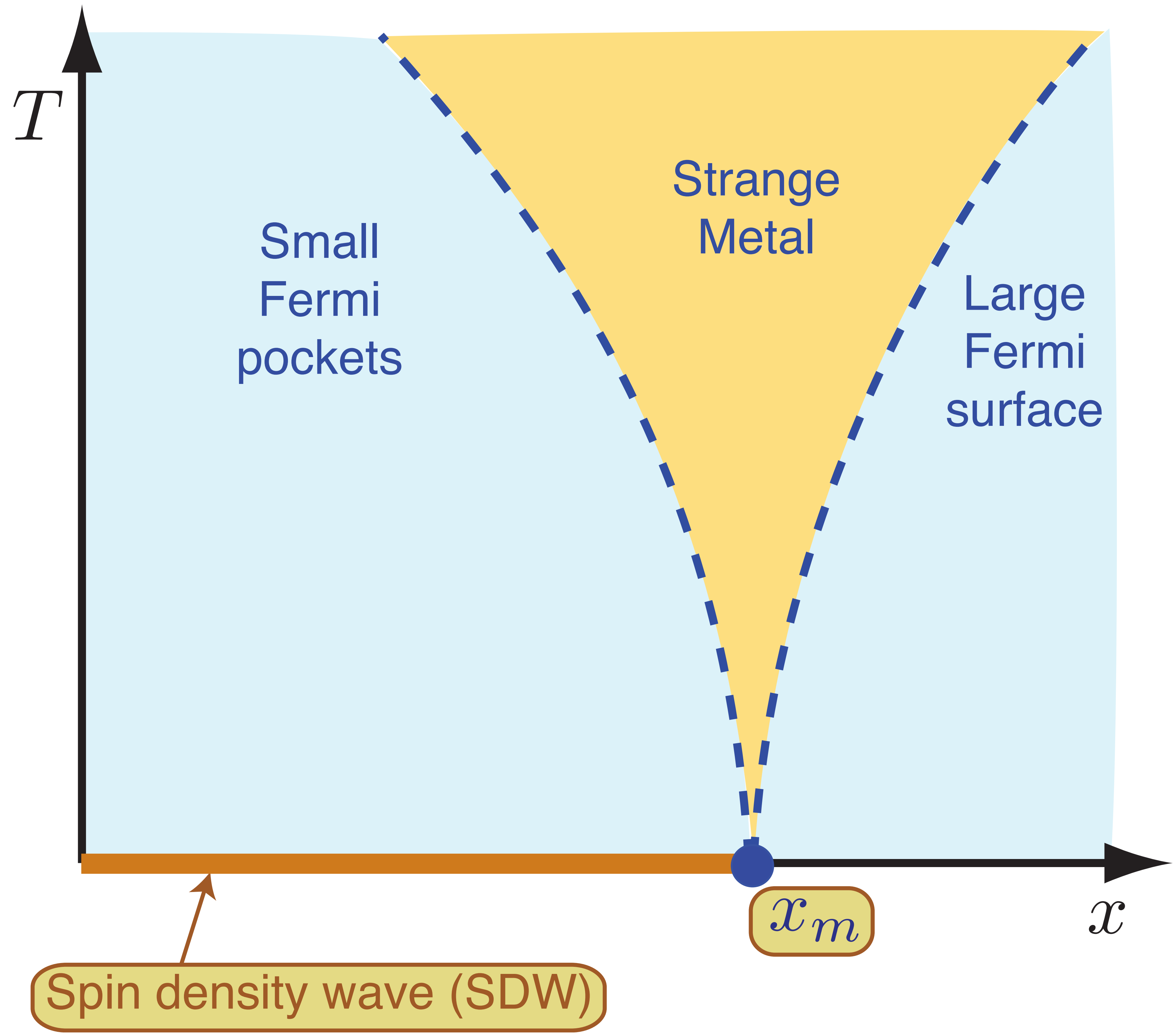


Incommensurate order in $YBa_2Cu_3O_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point
in **metal** at $x = x_m$

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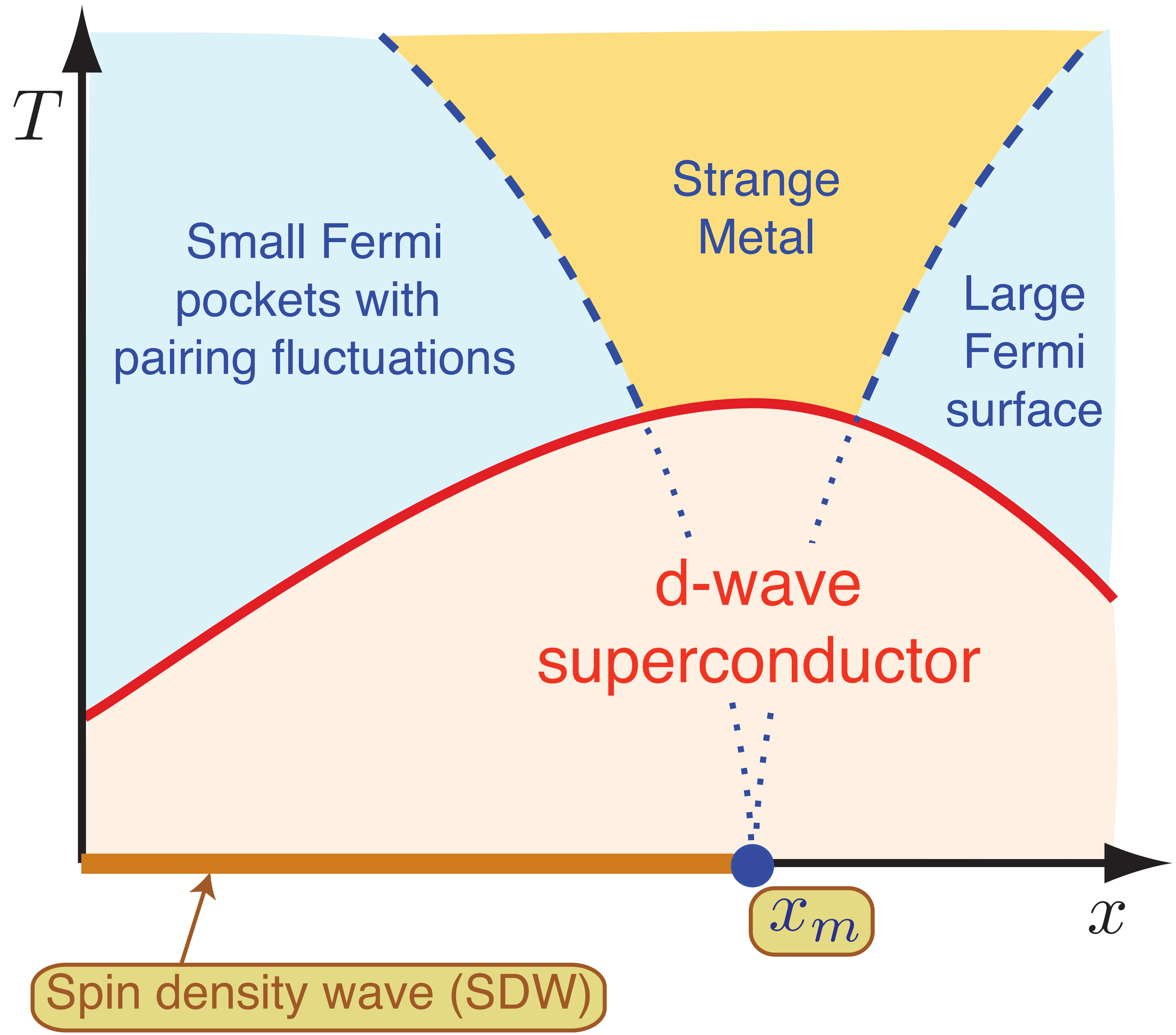
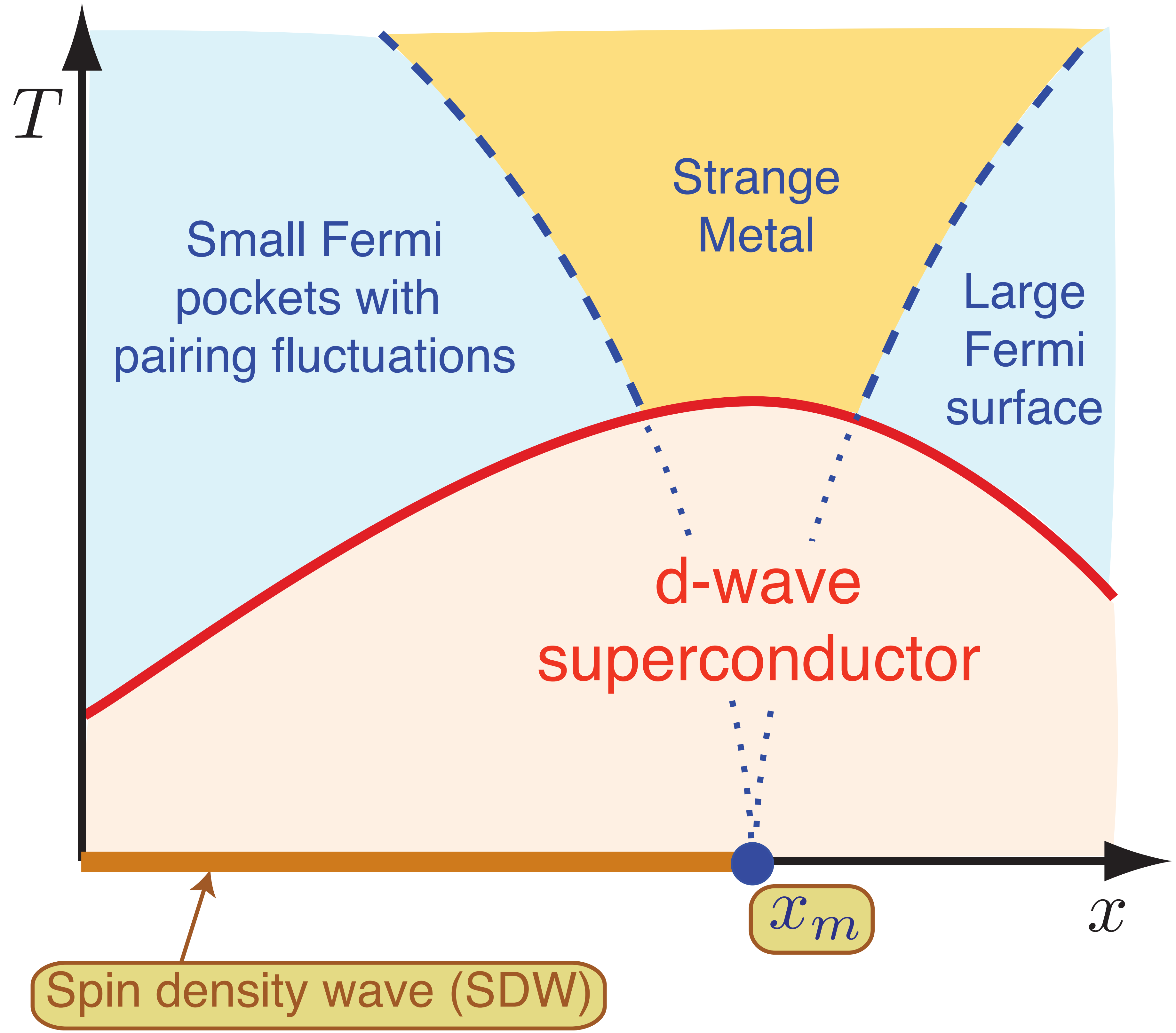


Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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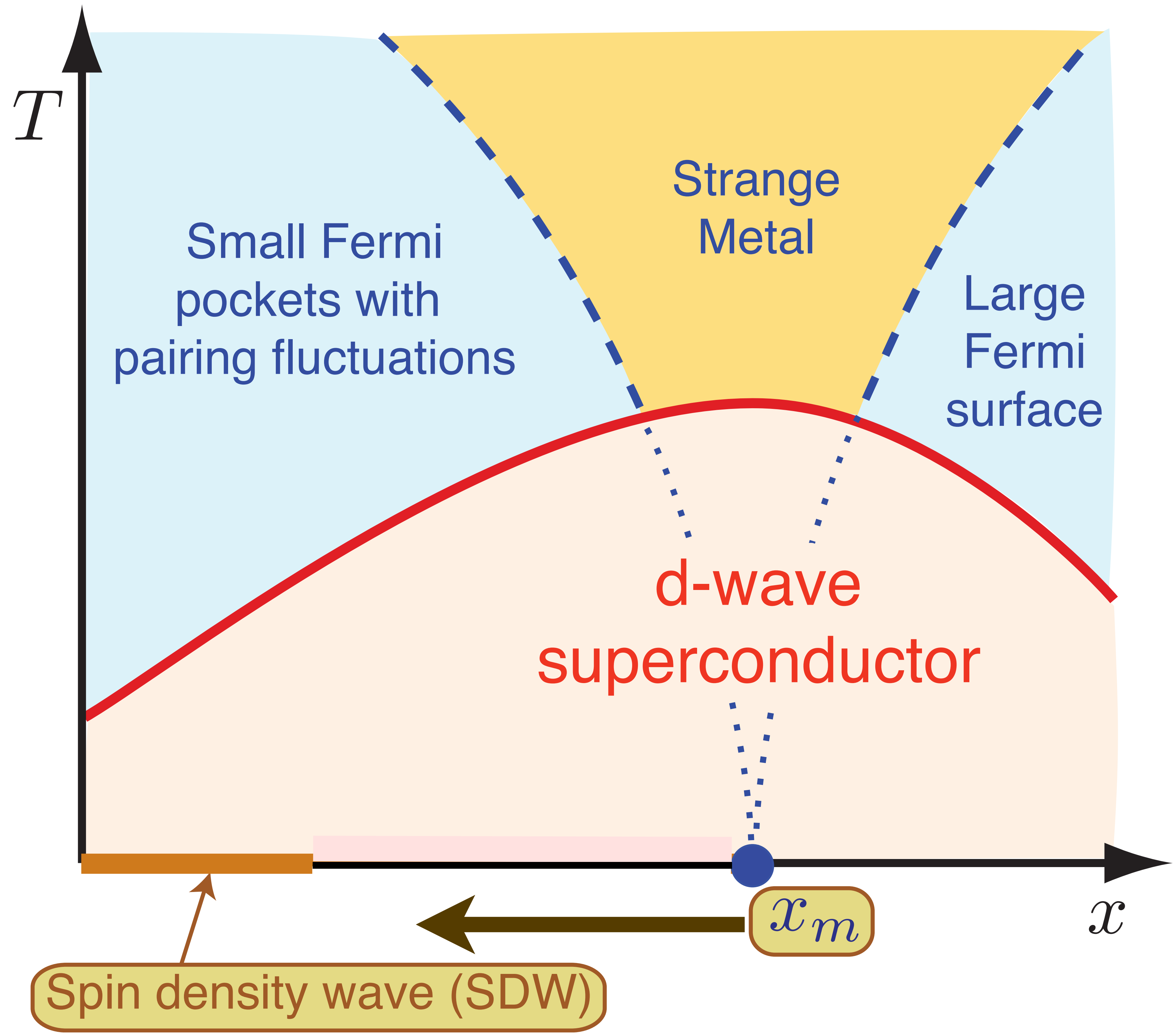

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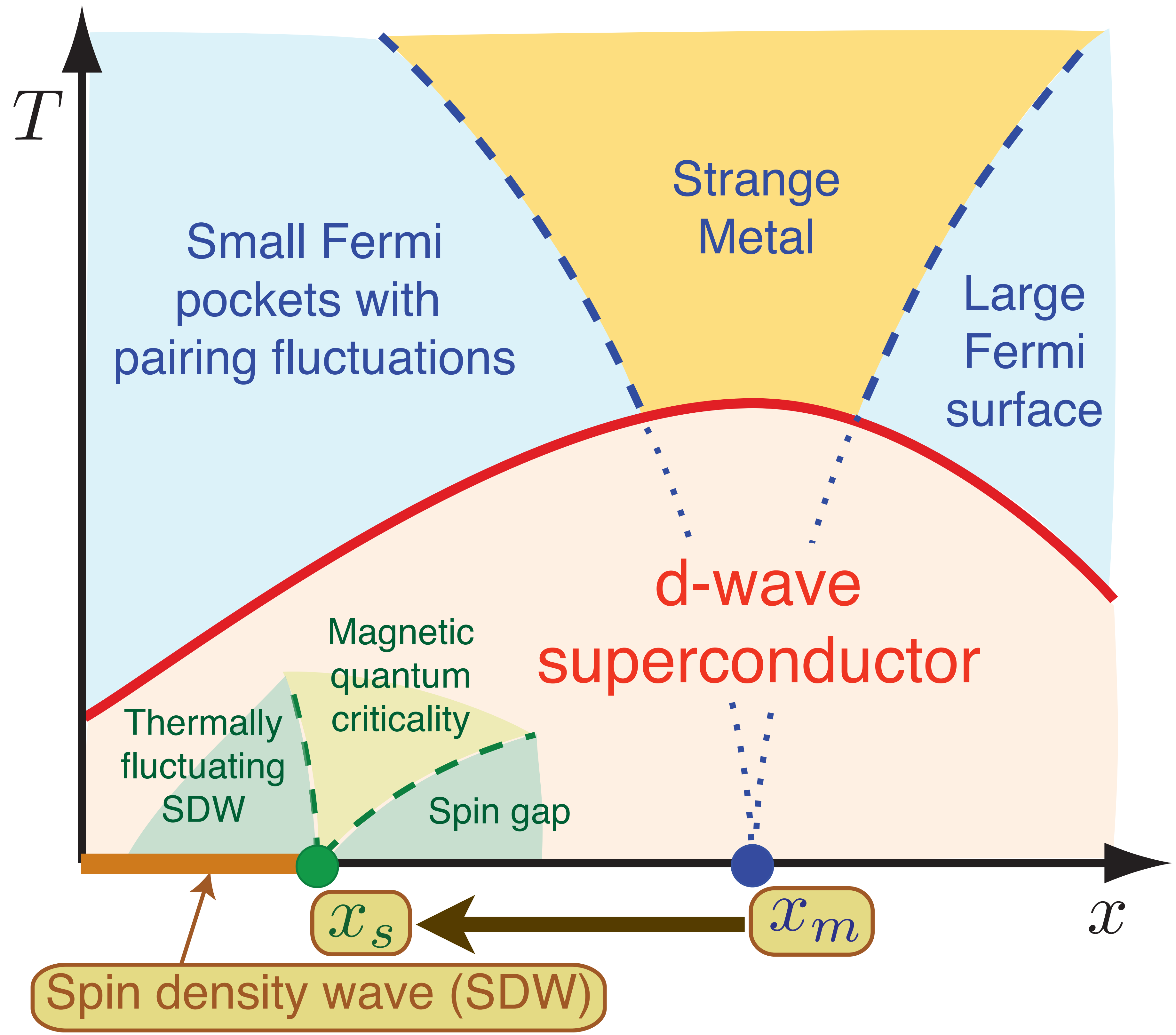

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Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order

   Survey of recent experiments

2. Superconductivity in the overdoped regime

   BCS pairing by spin fluctuation exchange

3. Superconductivity in the underdoped regime

   U(1) gauge theory of fluctuating SDW order

4. A unified theory

   SU(2) gauge theory of transition from Fermi pockets to a large Fermi surface

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Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2 r d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \frac{u}{4} \left(\vec{\varphi}^2 \right)^2 + \kappa \, \vec{\varphi}^2 \, |\psi|^2 \right] + \int d^2 r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004),
S. A. Kivelson, D.-H. Lee, E. Fradkin, and V. Oganesyan, *Phys. Rev. B* **66**, 144516

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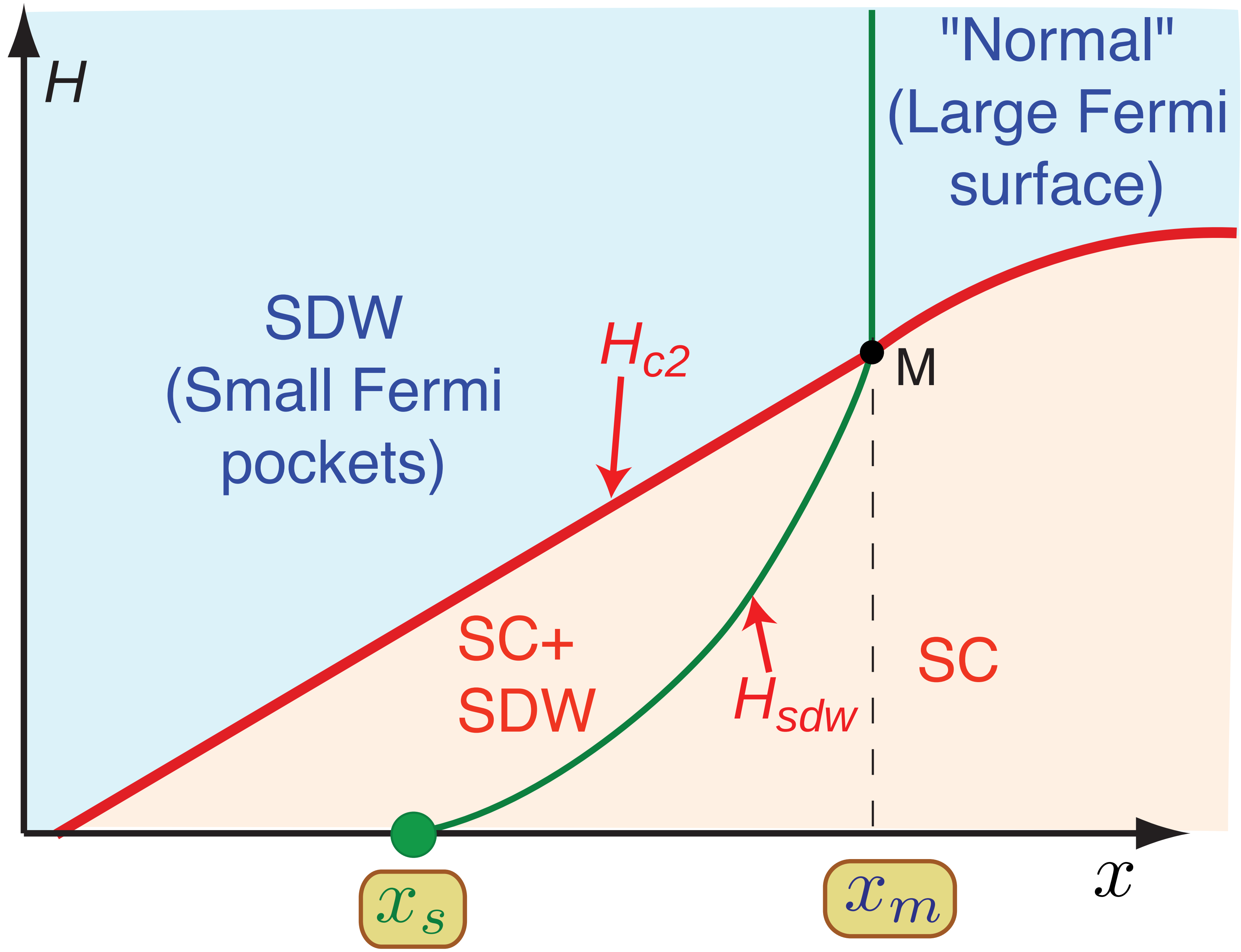

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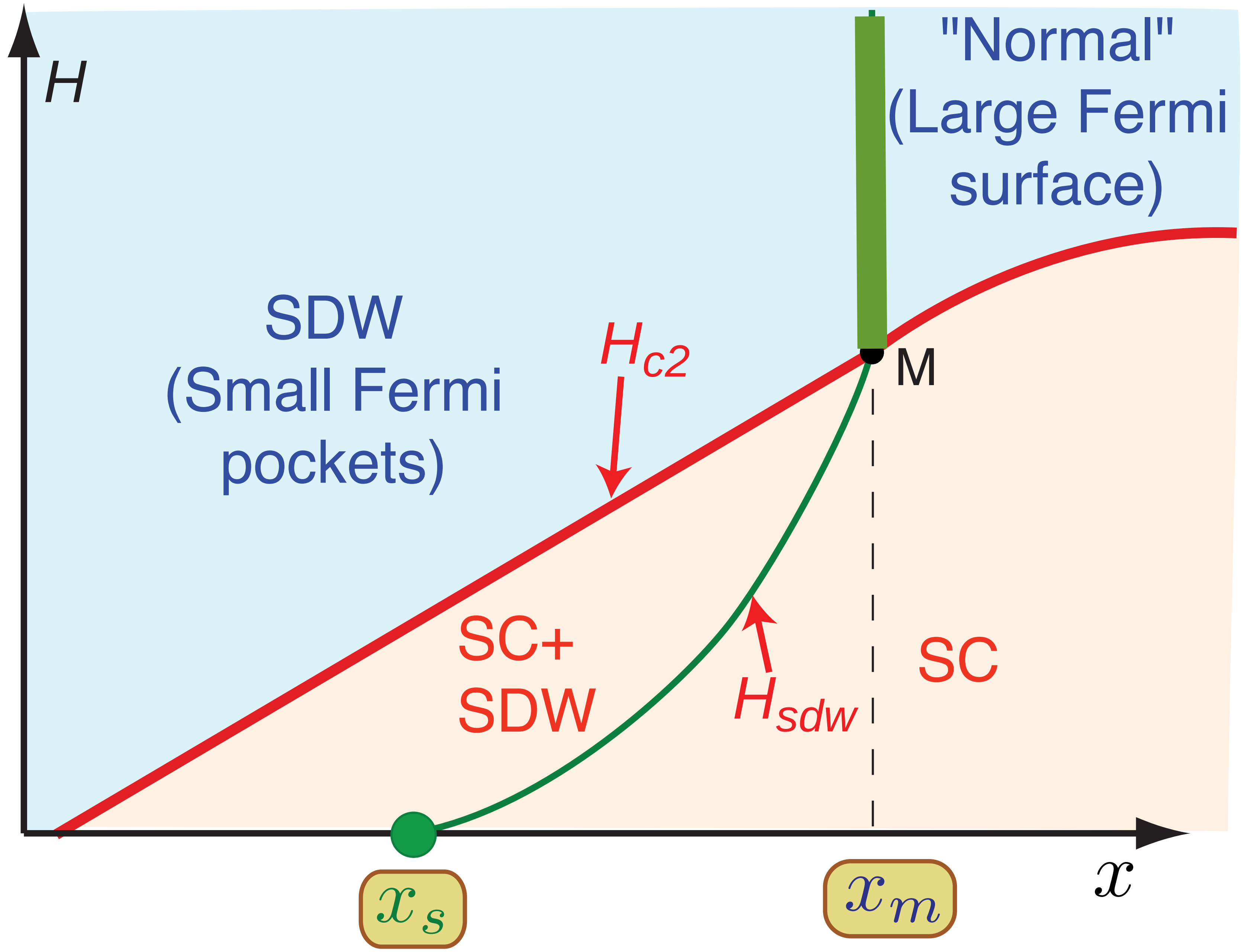

- SDW order is more stable in the metal than in the superconductor: $x_m > x_s$.

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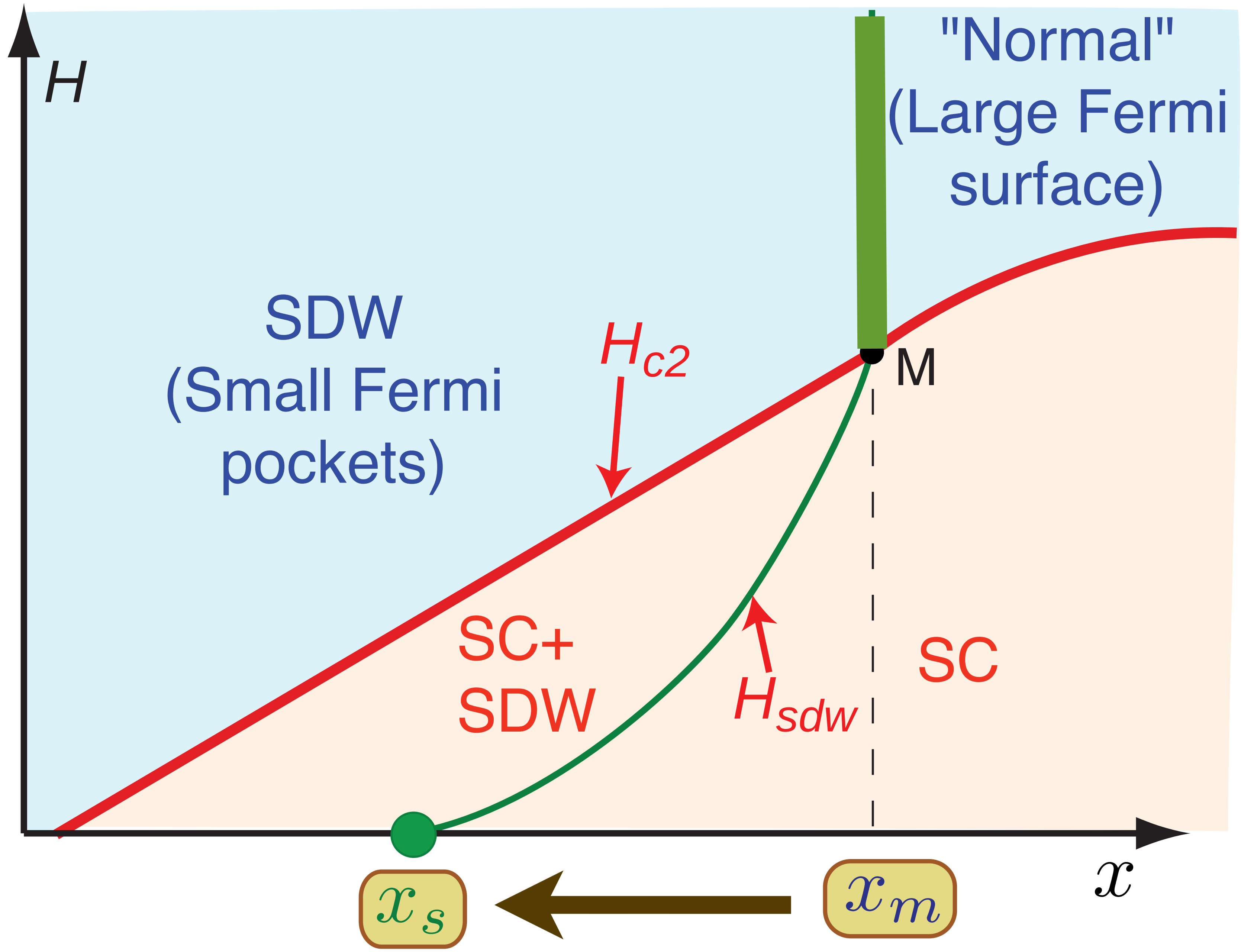

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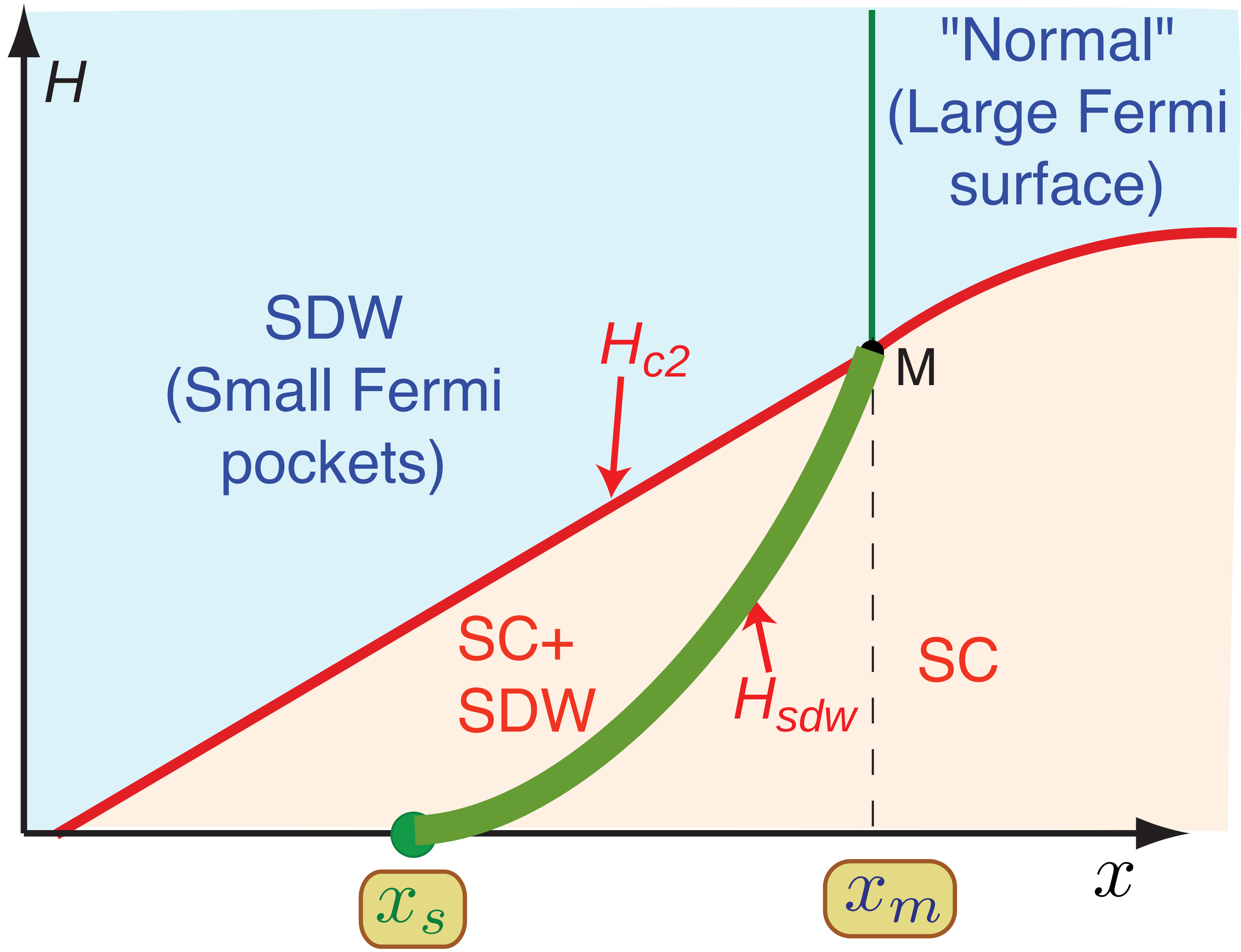

- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

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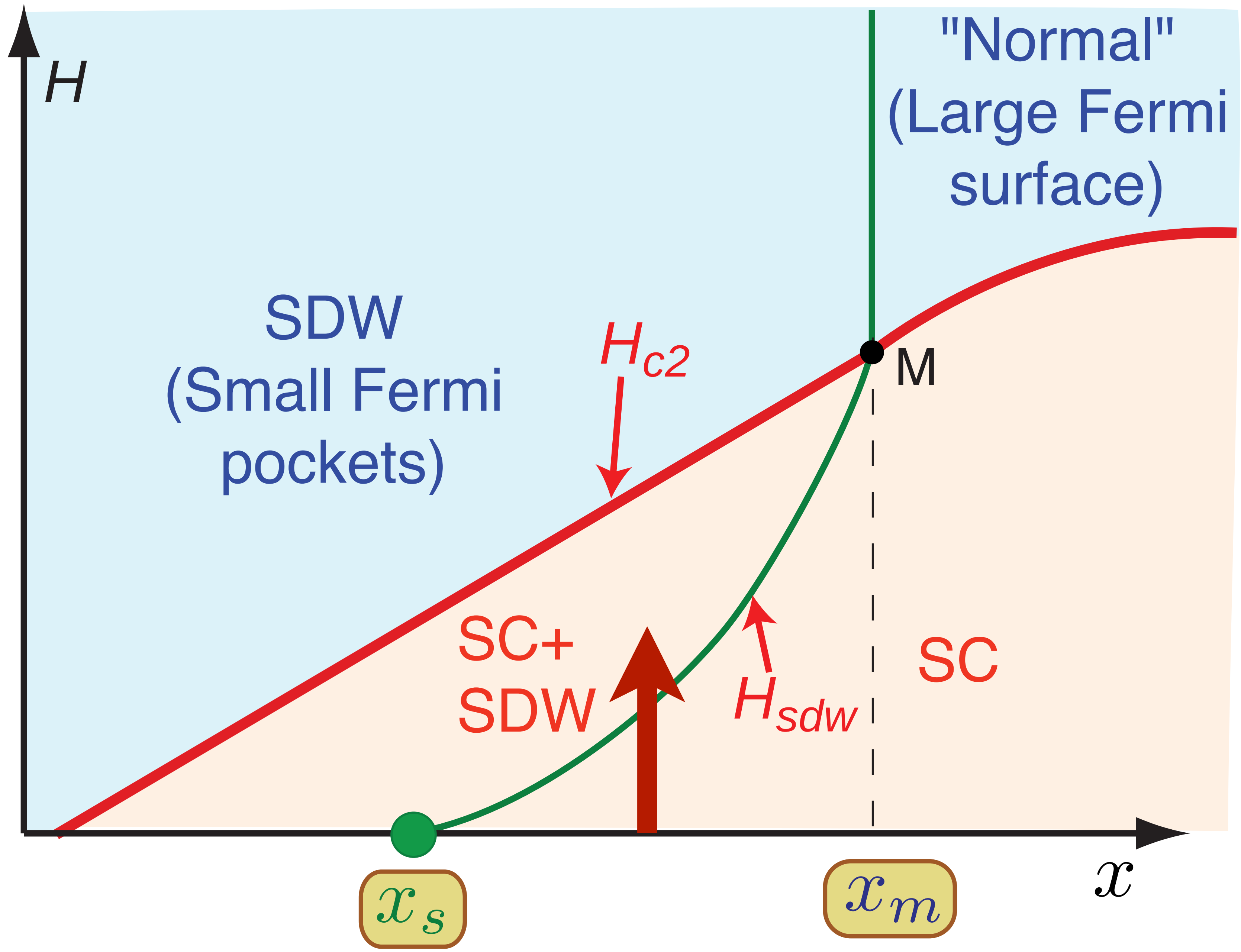

Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, *Phys. Rev. Lett.* 102, 177006 (2009).

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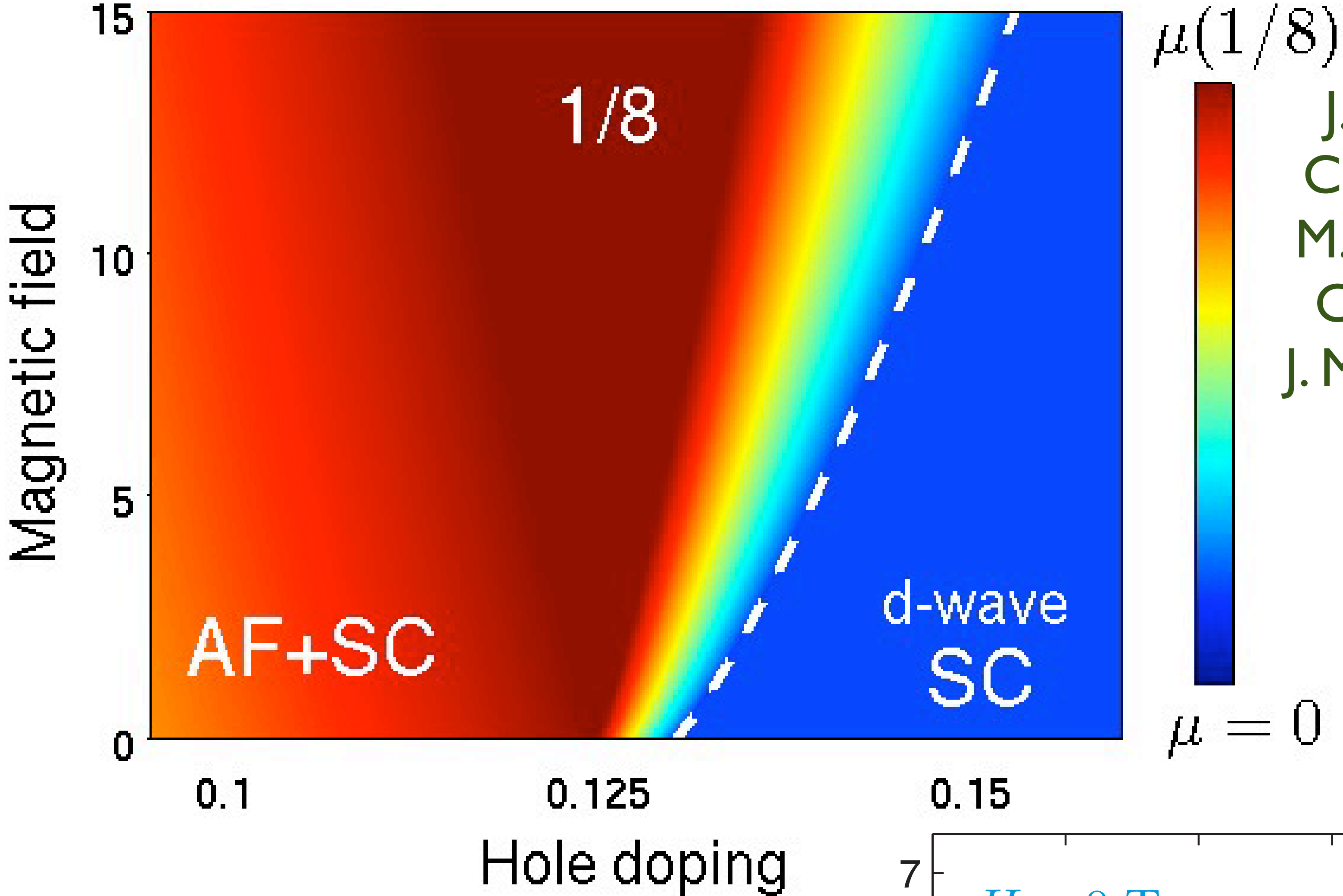
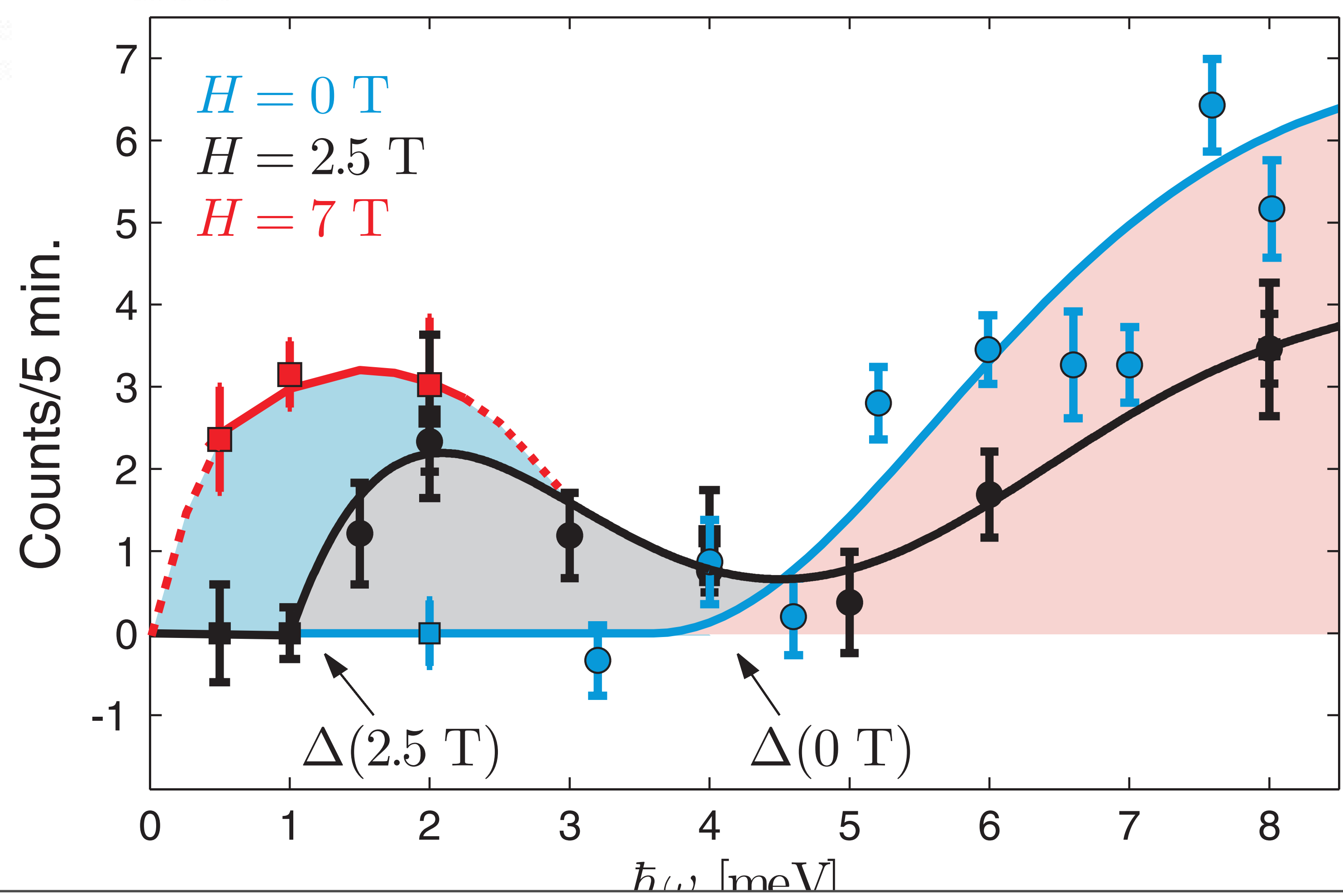


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

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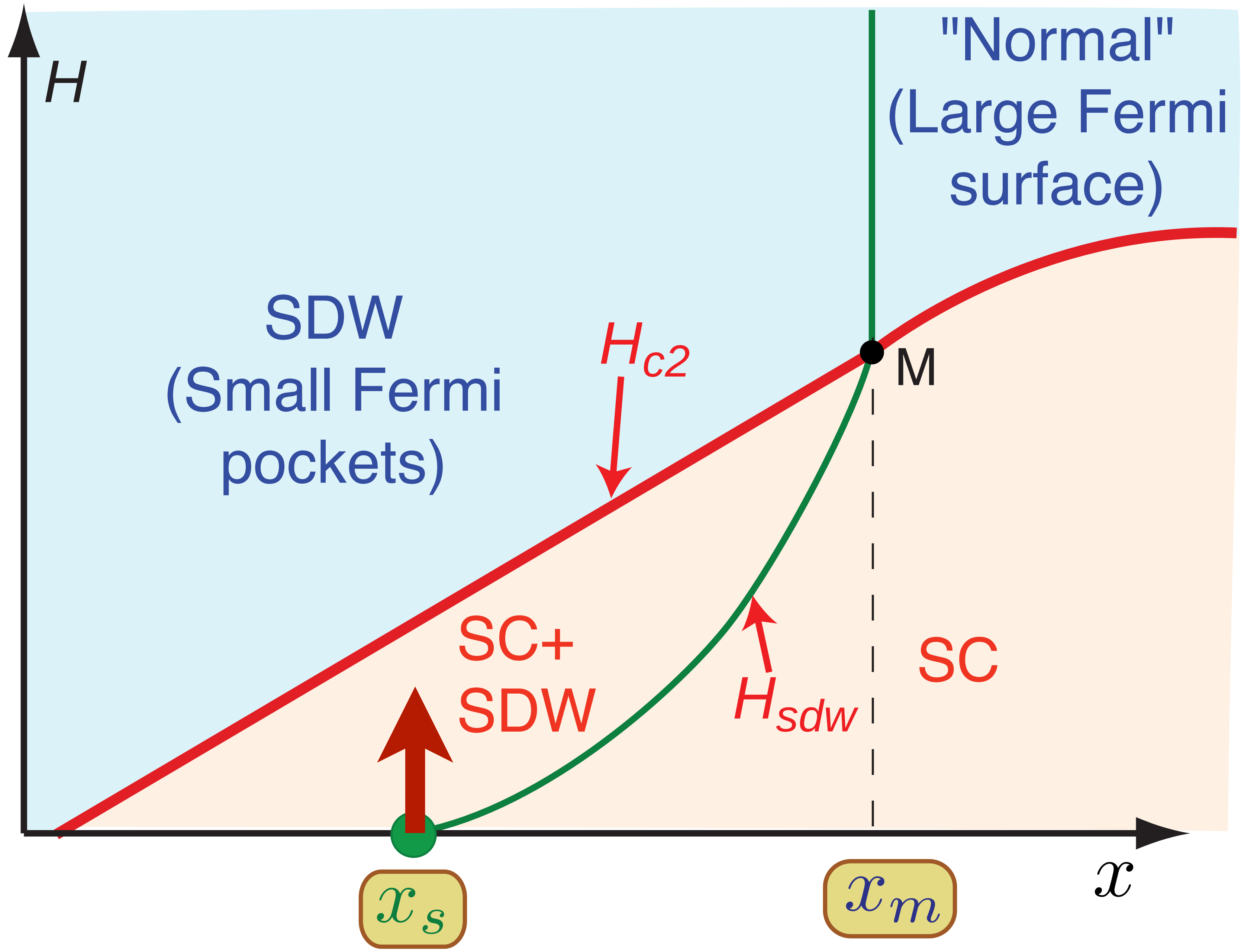

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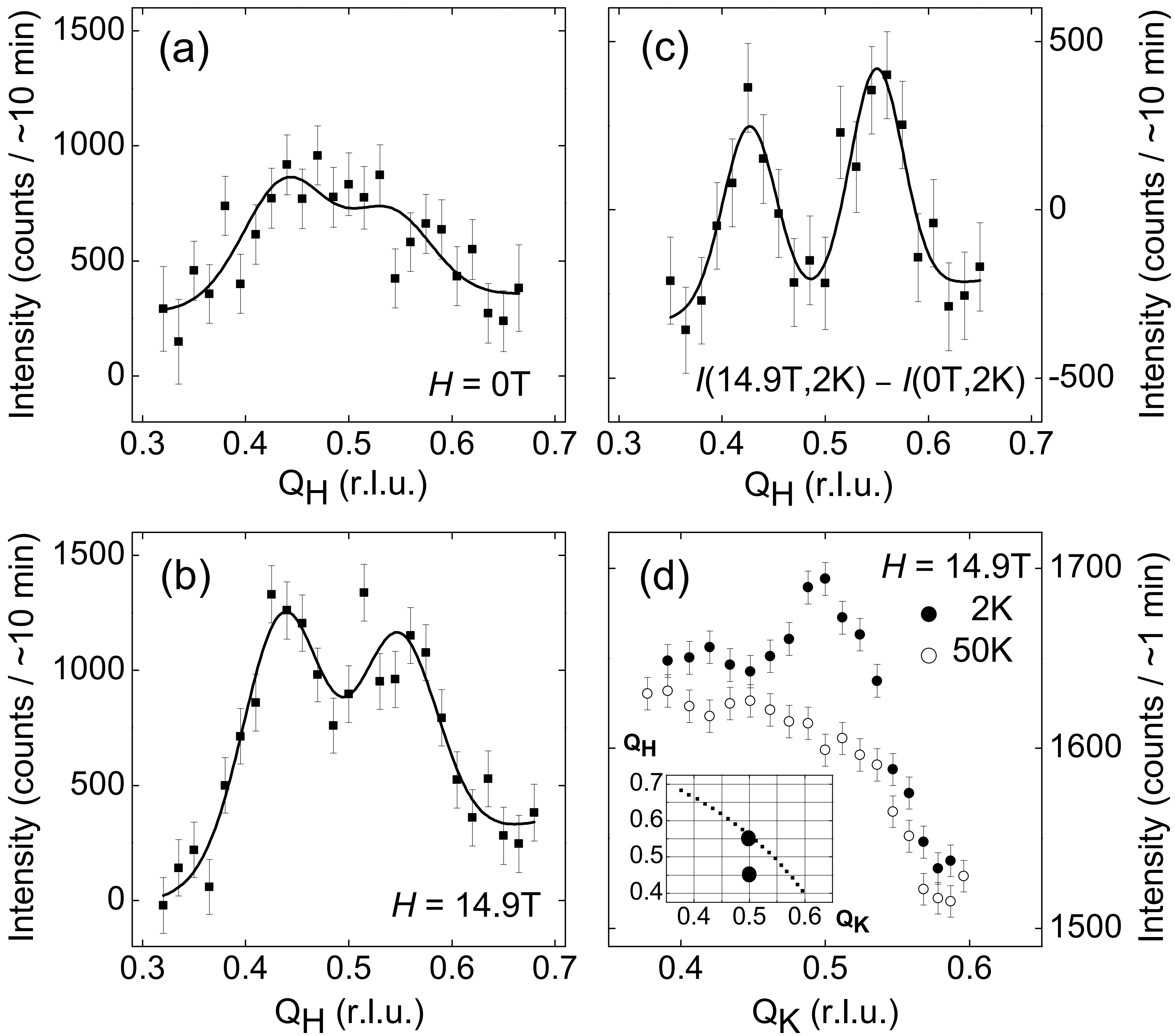

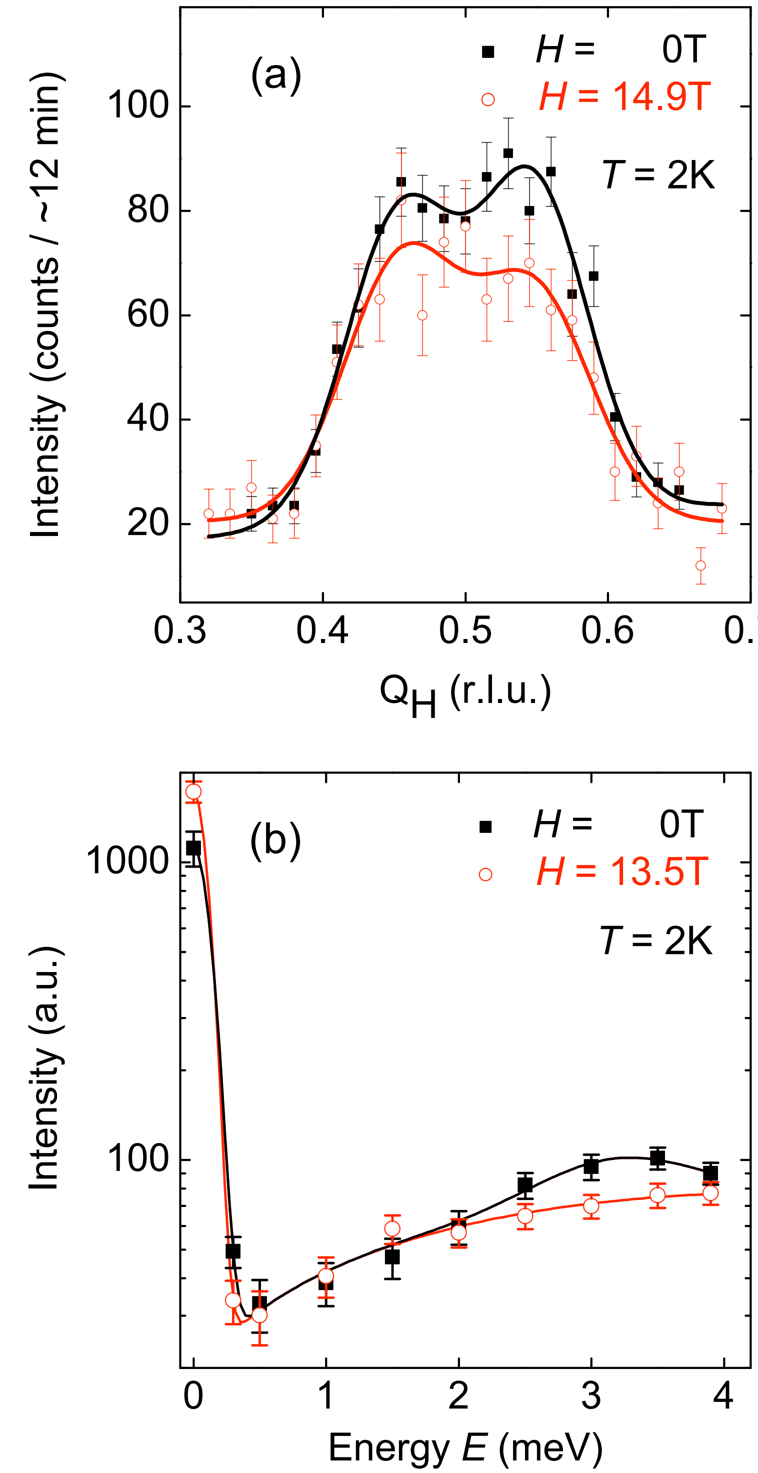

D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2001)

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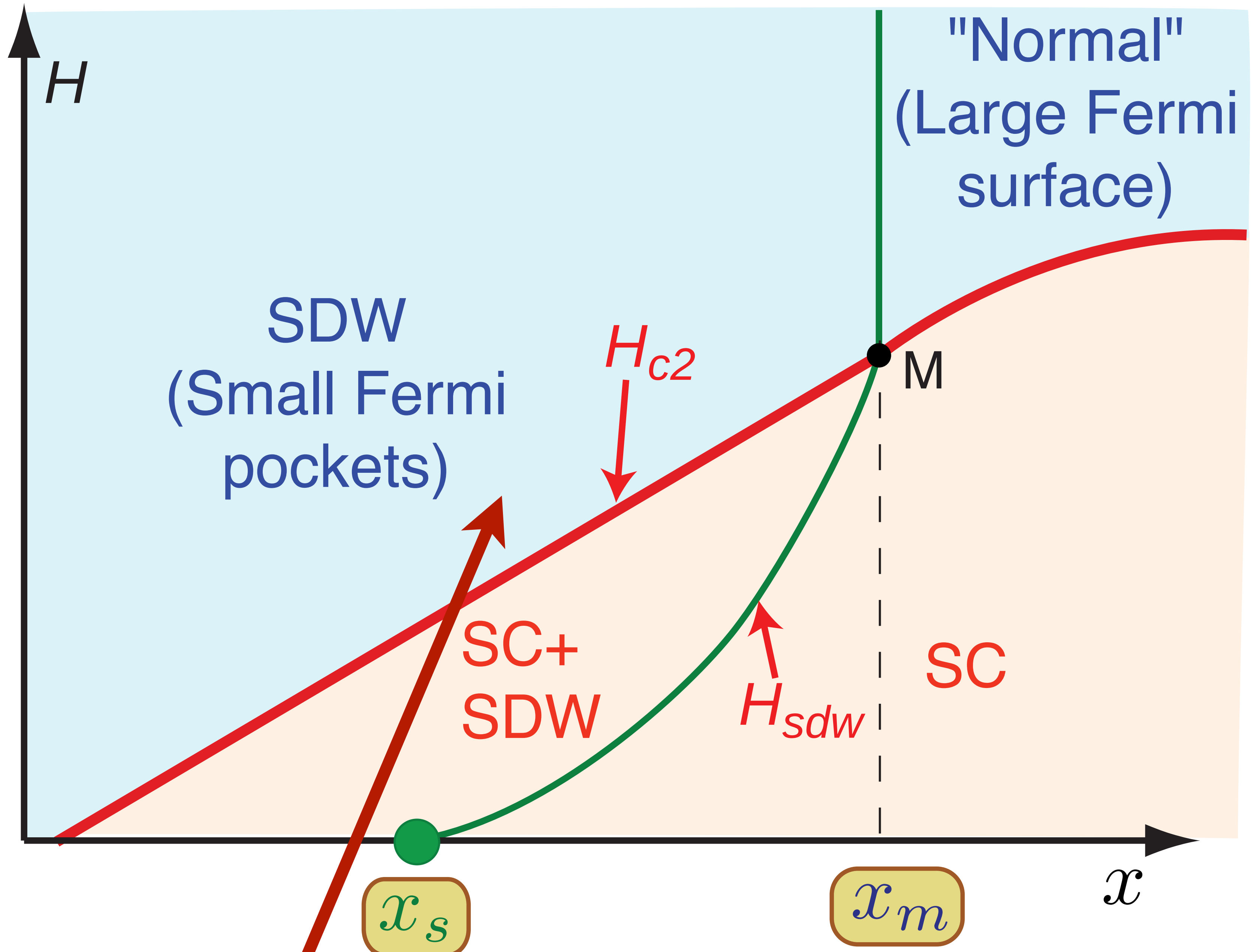


Quantum oscillations without Zeeman splitting

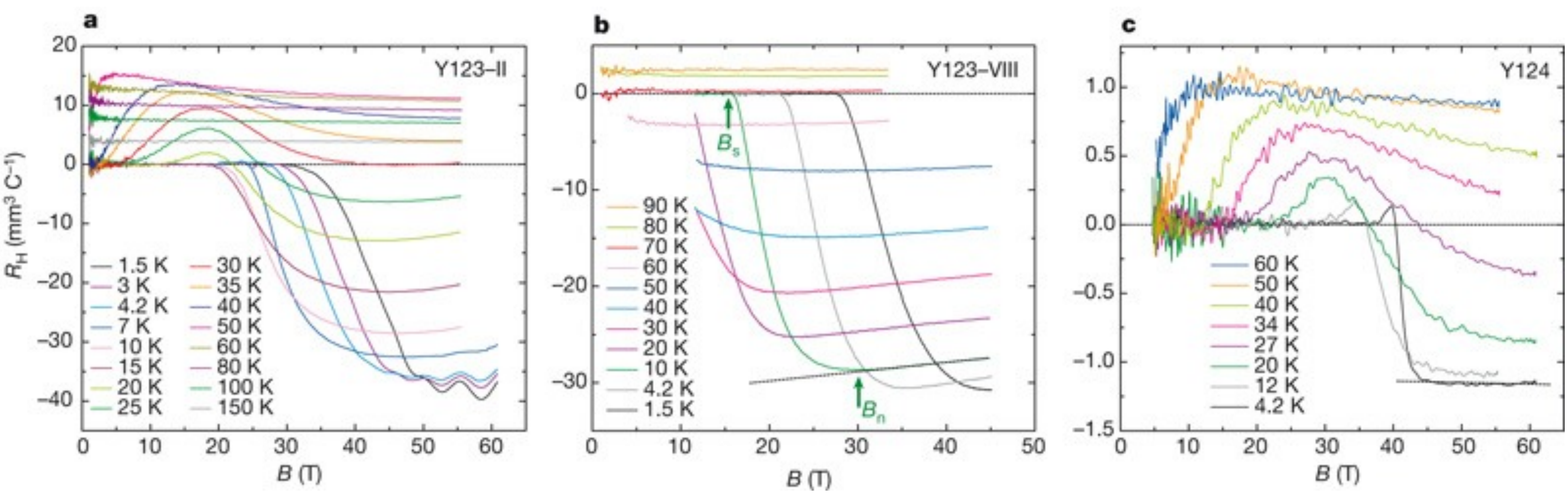
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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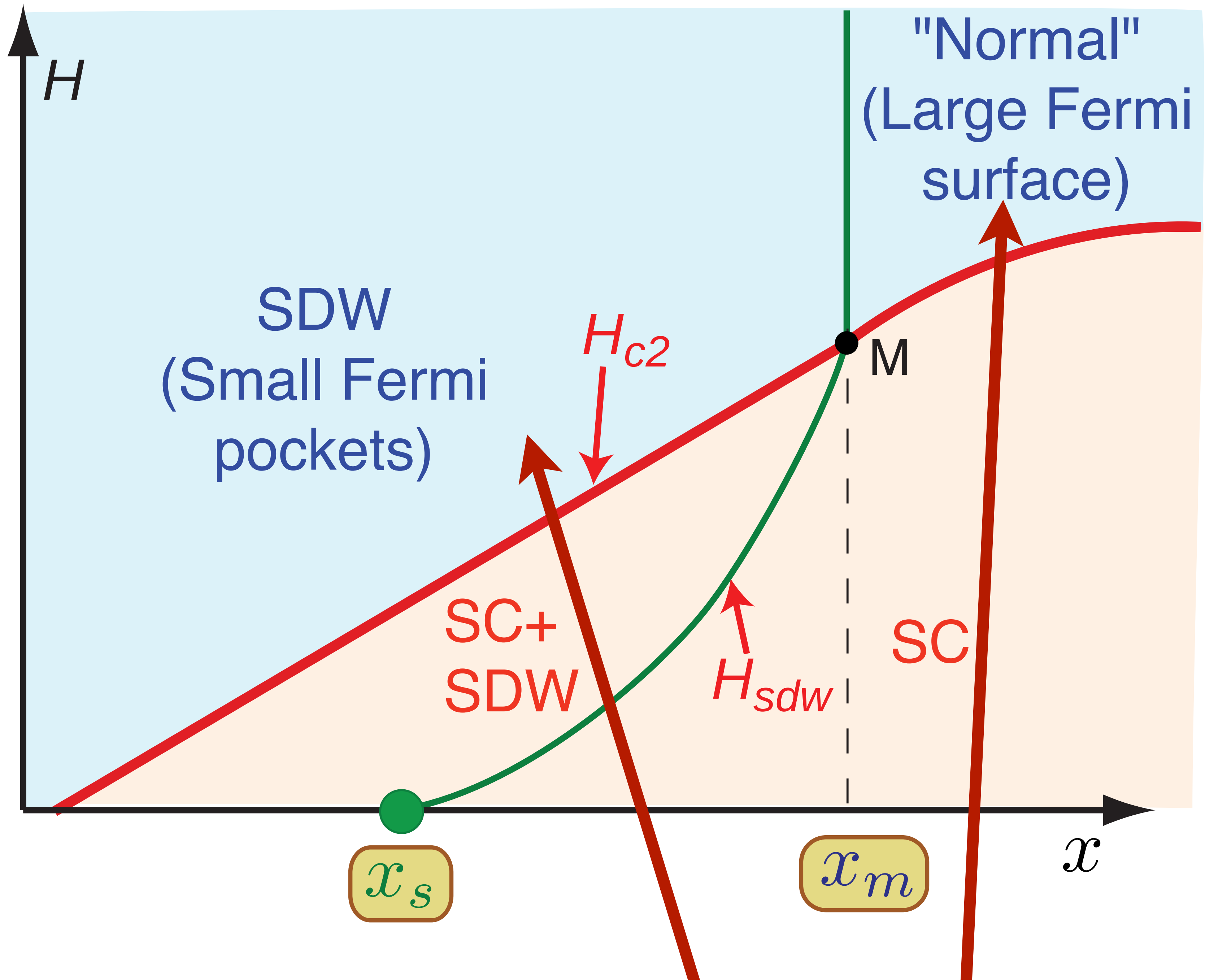
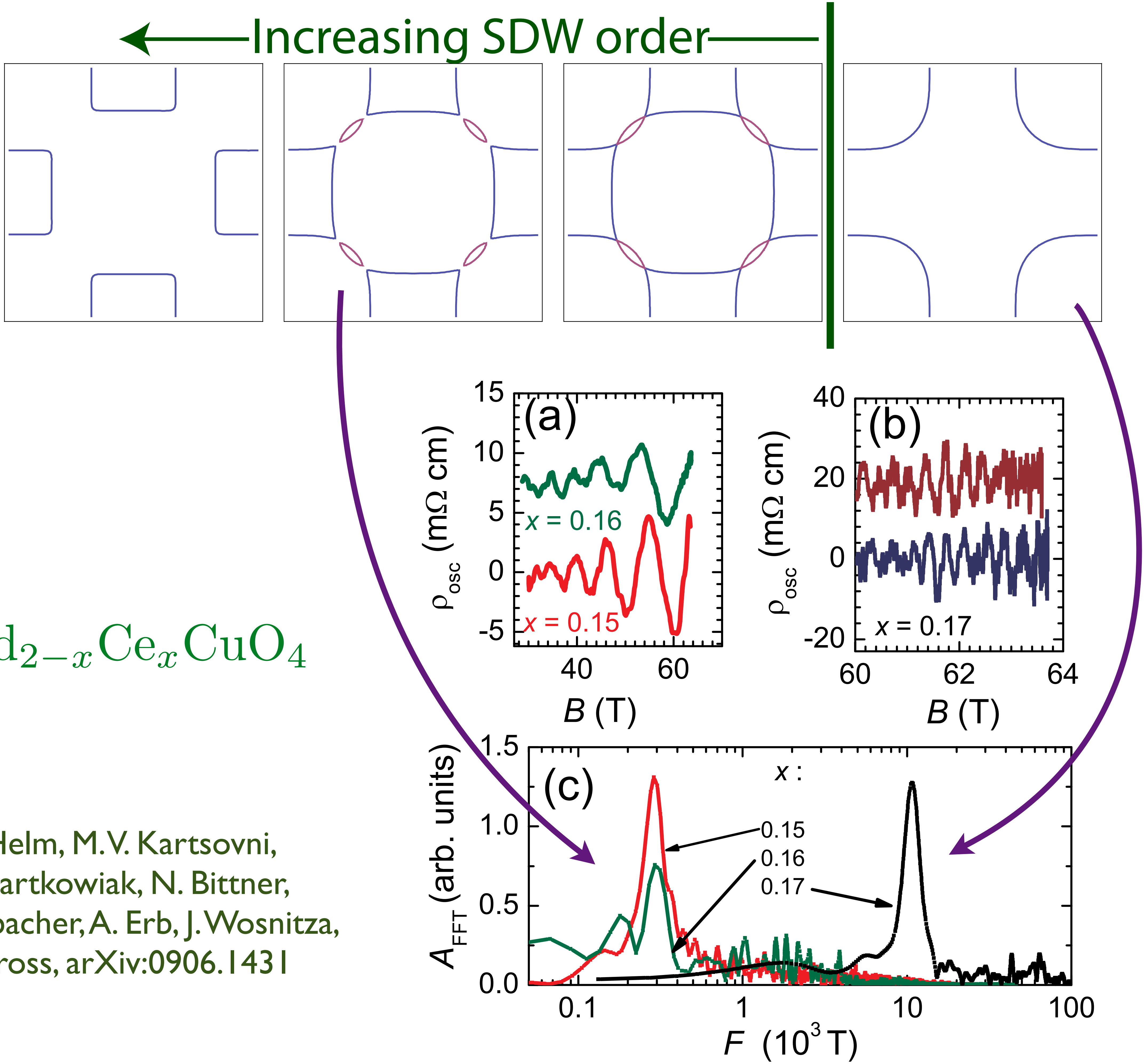


Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$



$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovni, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, R. Gross, arXiv:0906.1431

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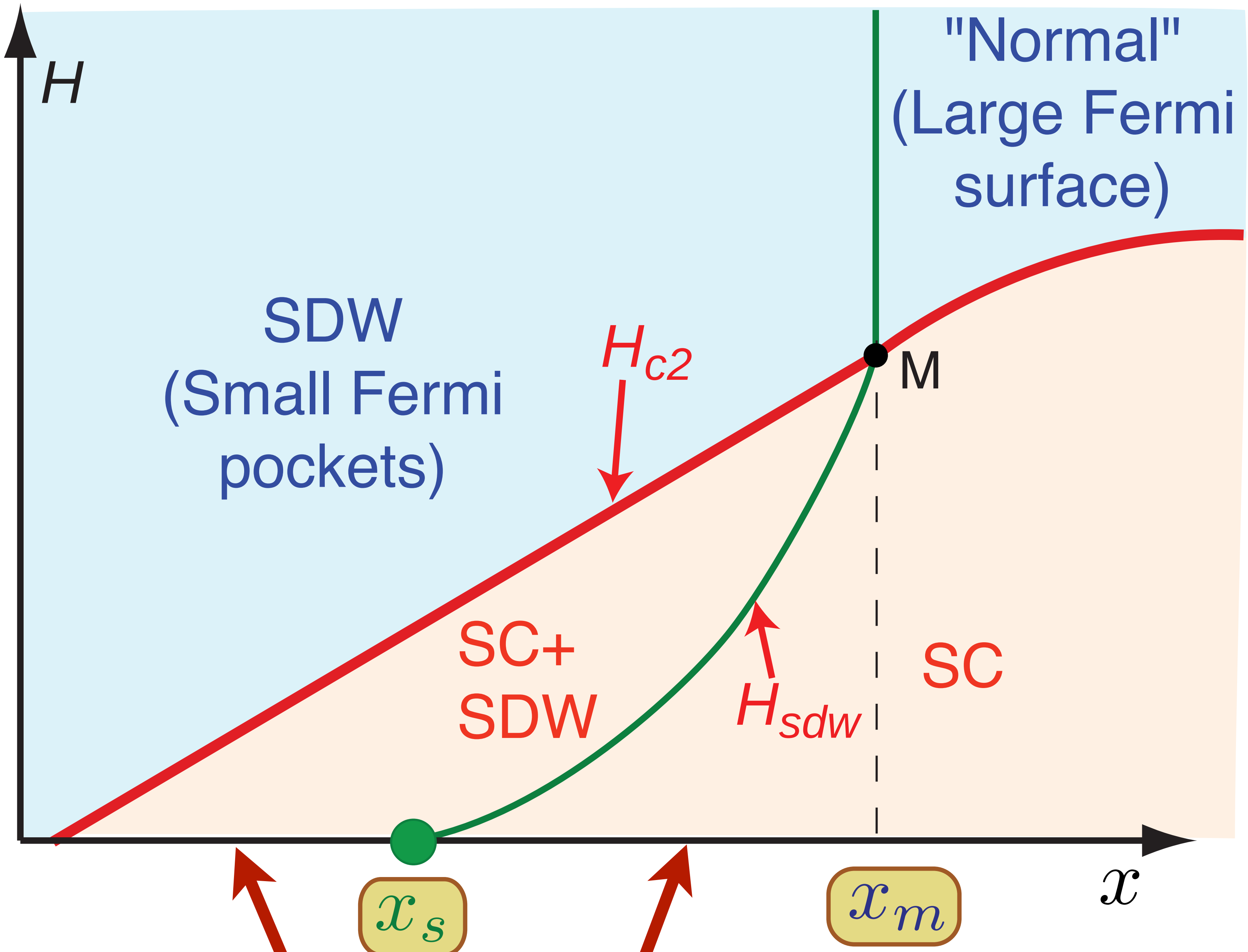
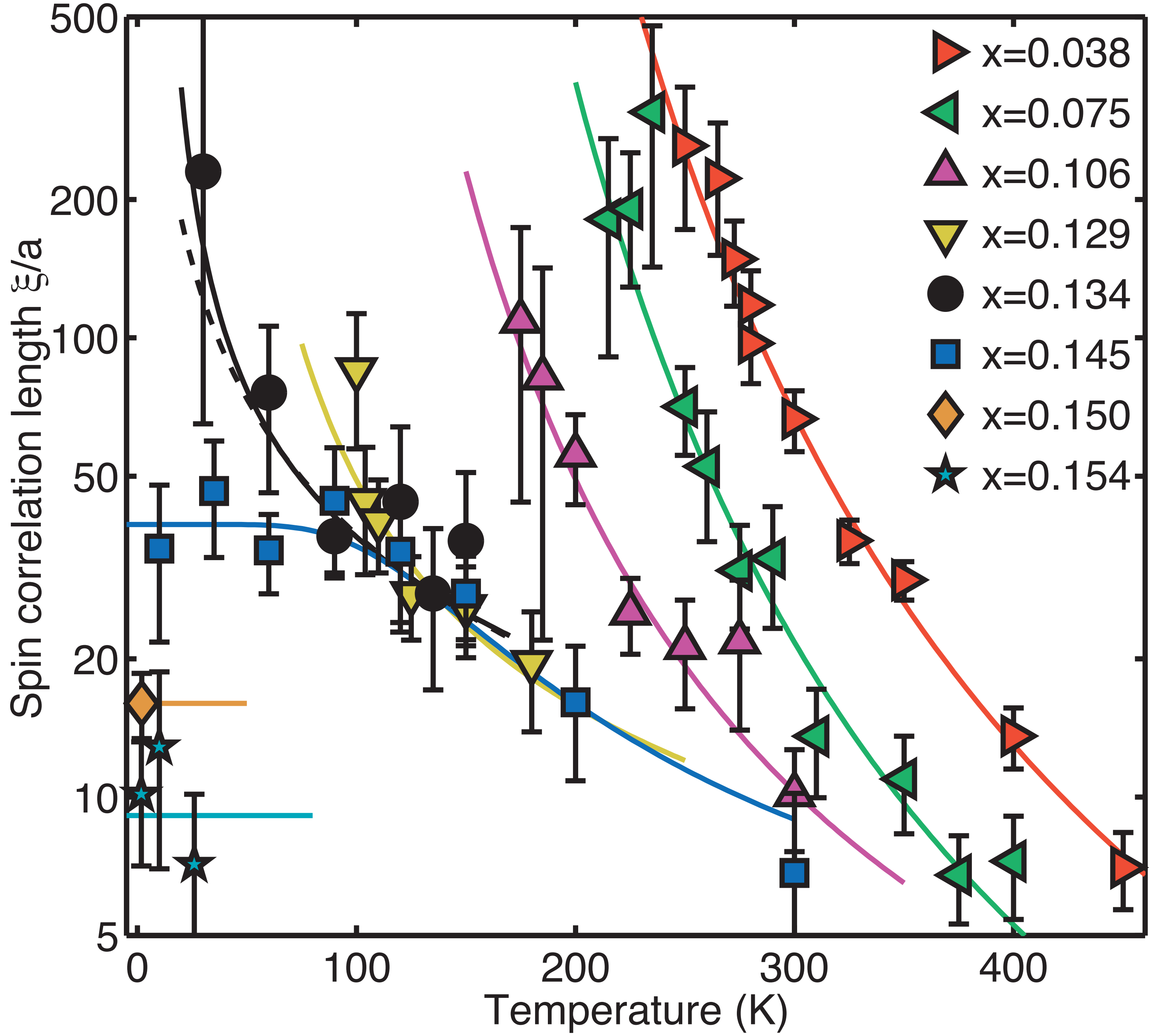
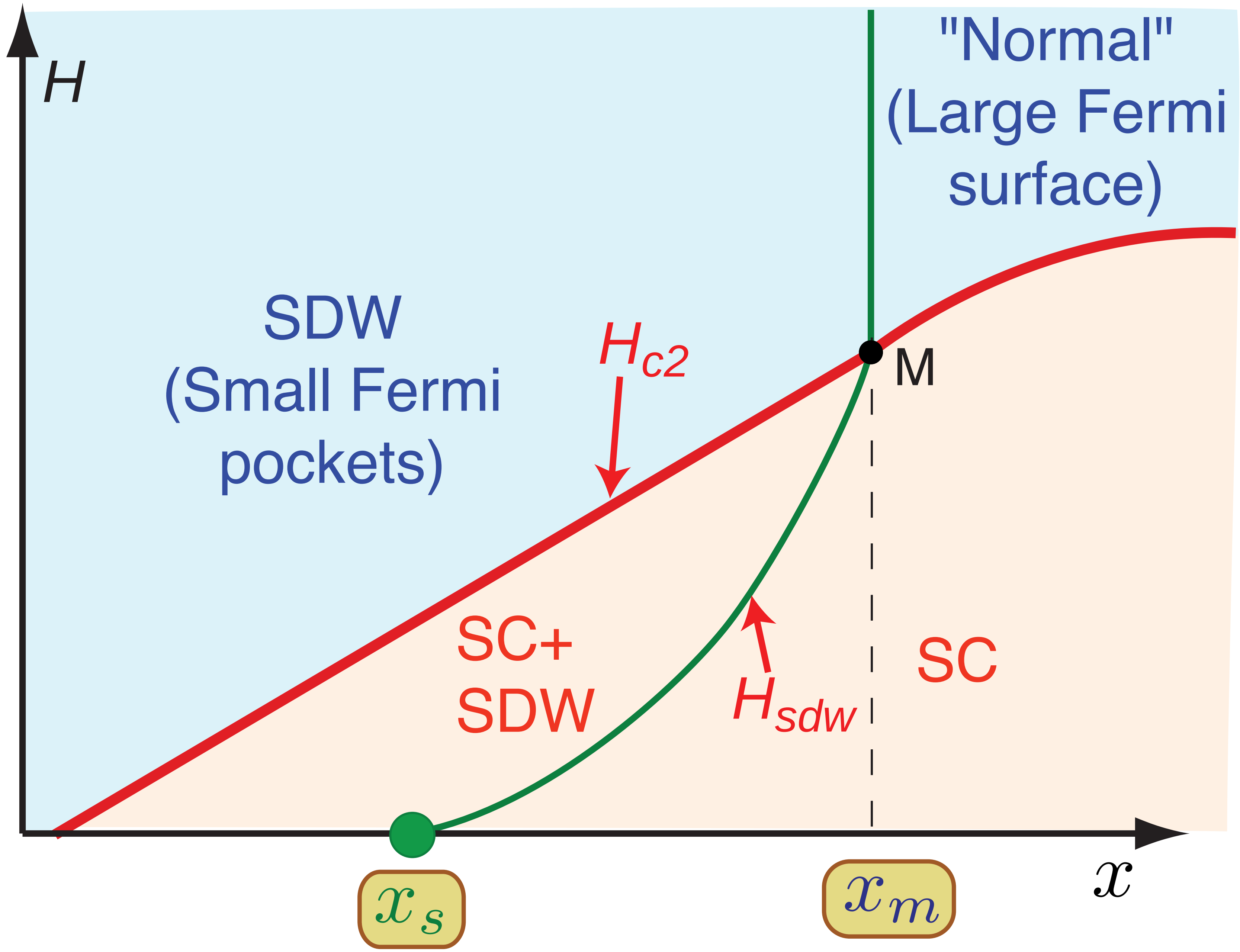


Neutron scattering at $H=0$ in **same** material identifies $x_s = 0.14 < x_m$

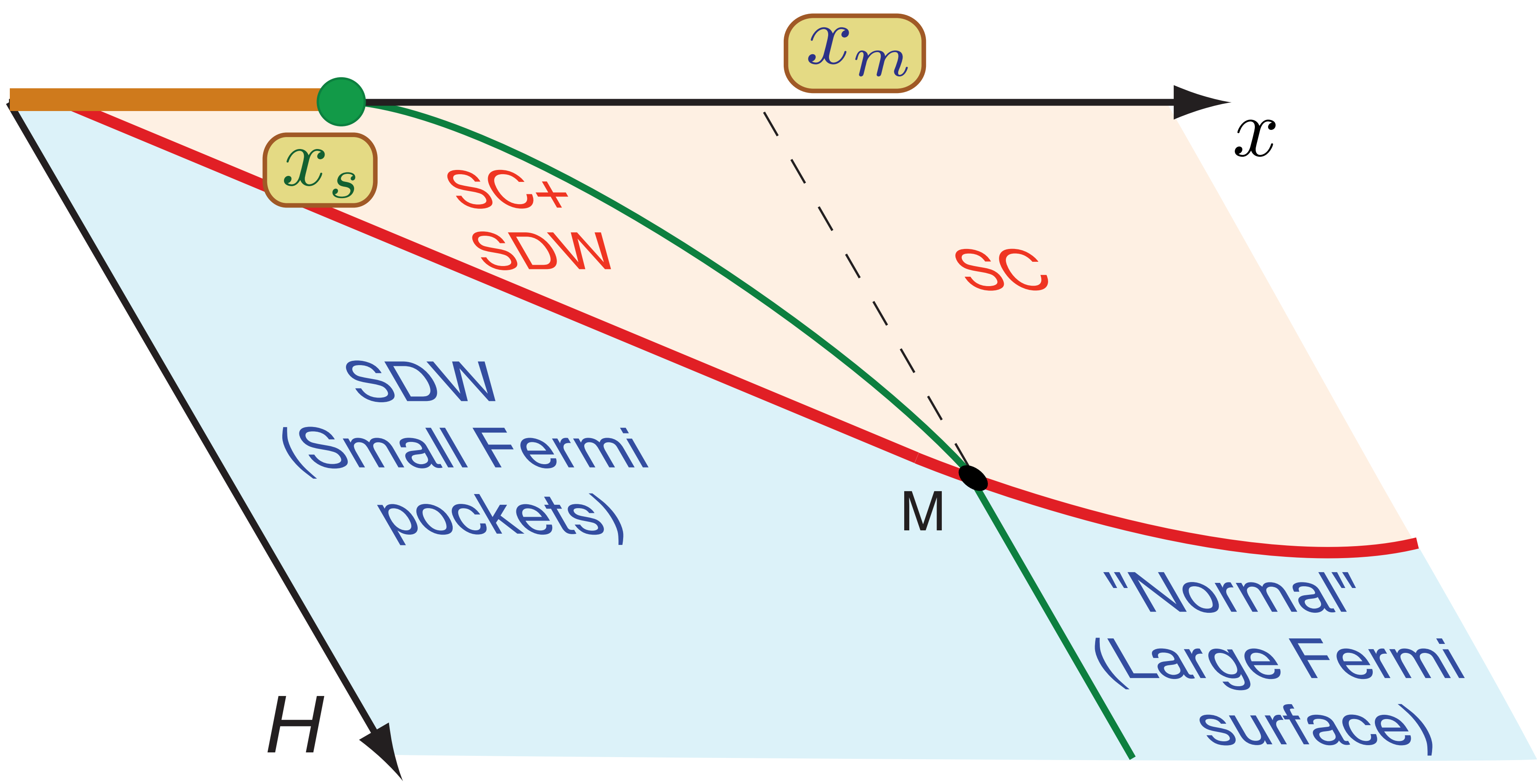
$Nd_{2-x}Ce_xCuO_4$



E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).



Thursday, July 16, 2009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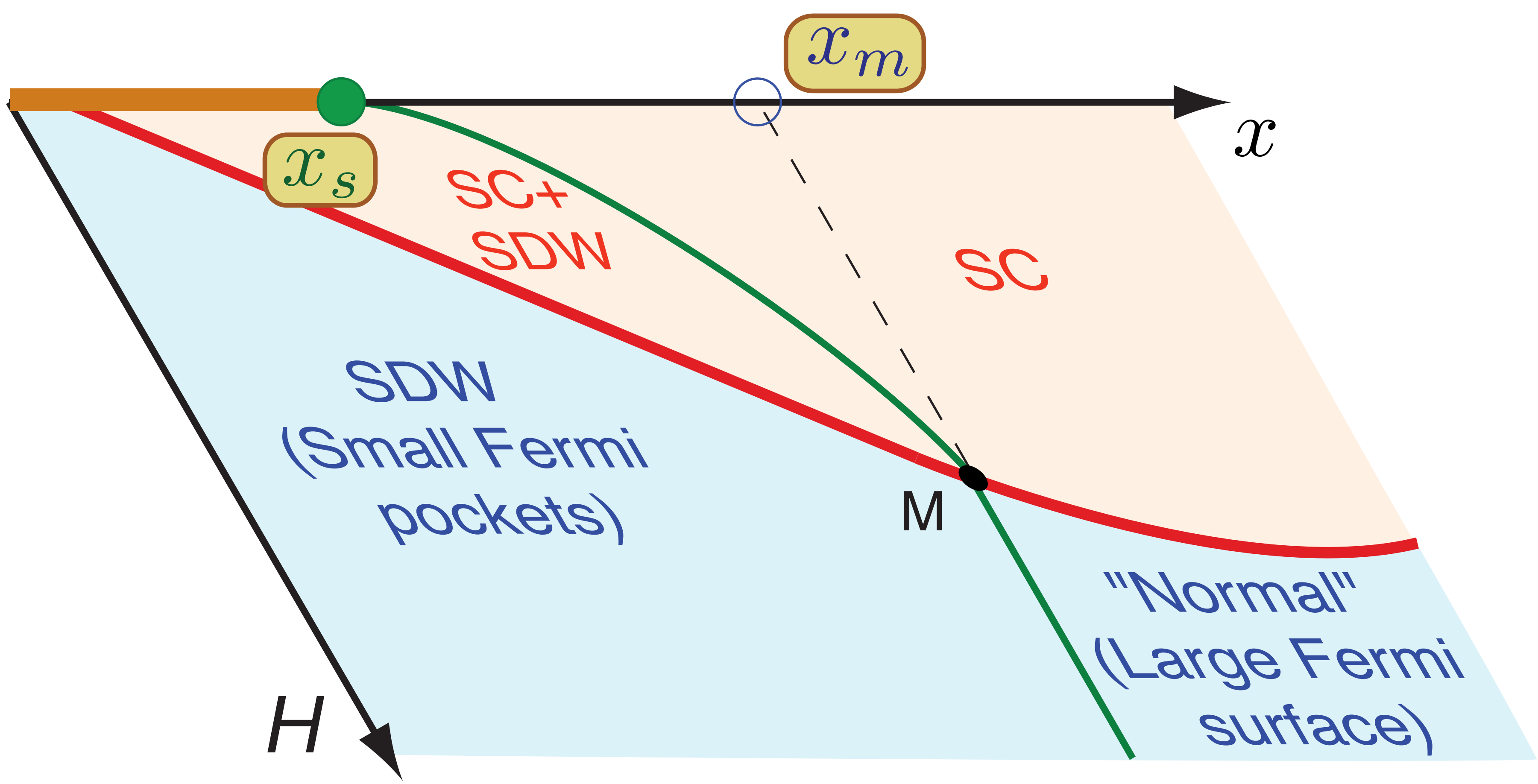
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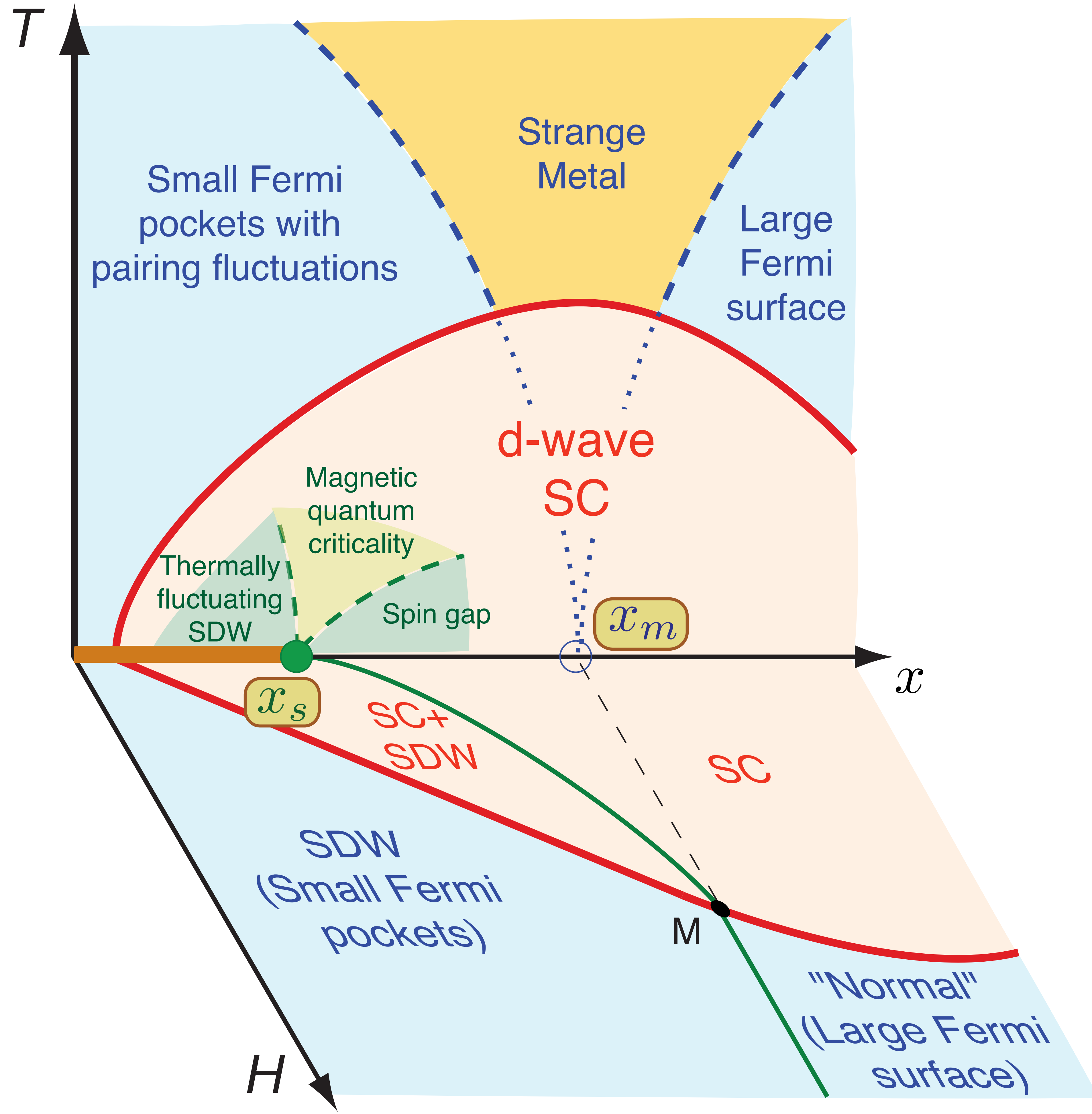
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Strange
Metal
Small Fermi
pockets with
pairing fluctuations
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x
x_s
SC+
SDW
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M
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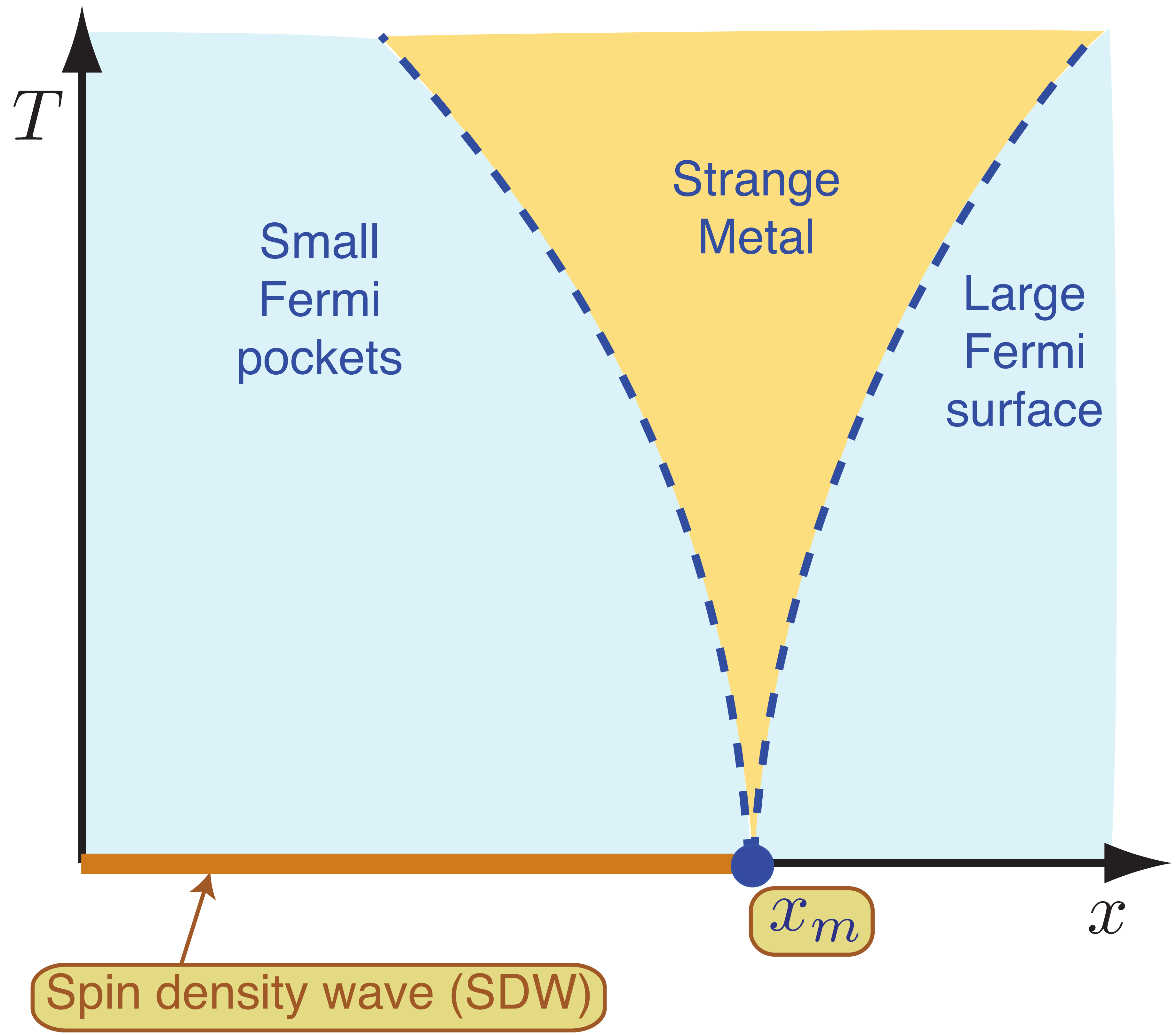
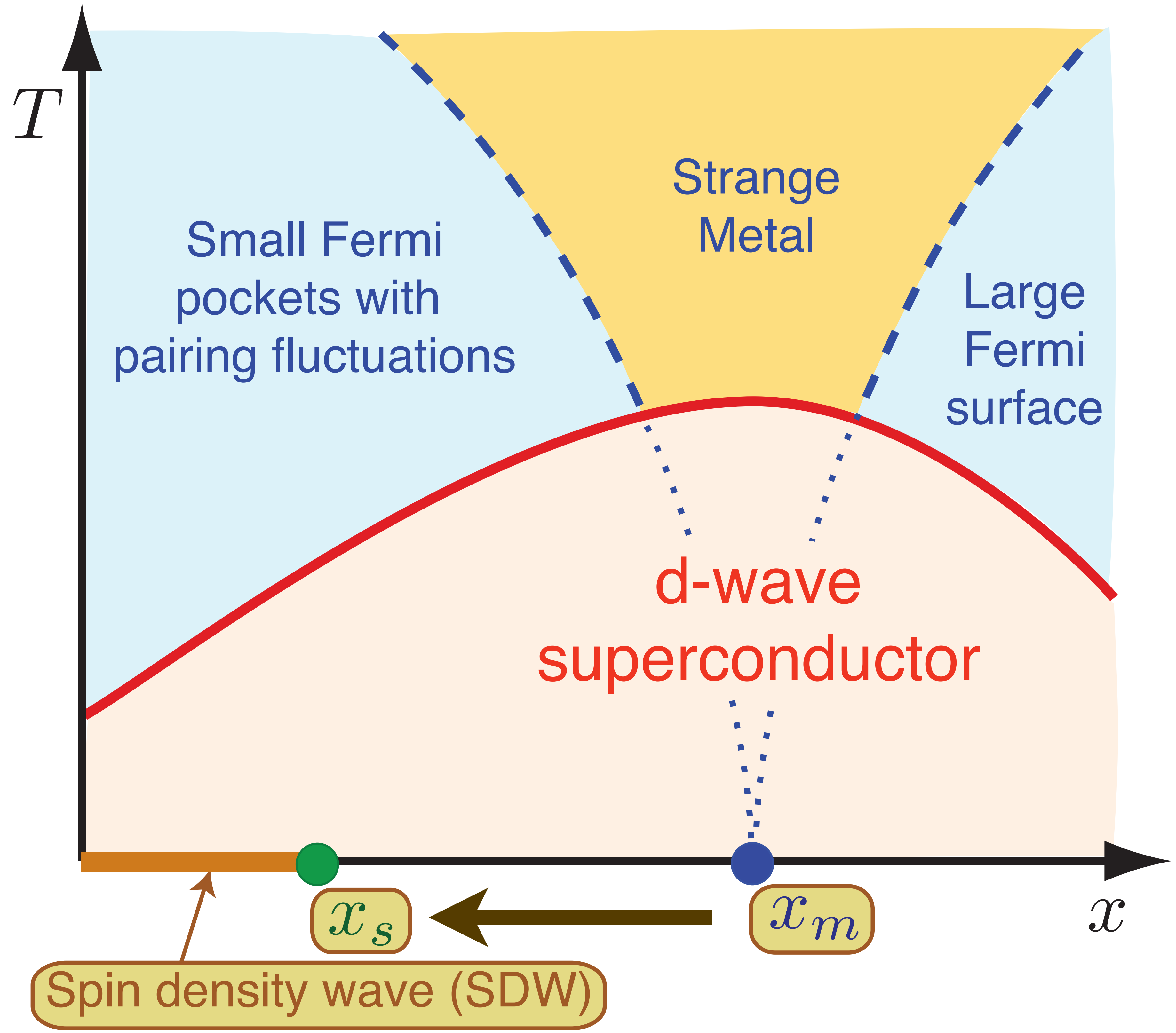
Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in metal at $x = x_m$

Theory of quantum criticality in the cuprates



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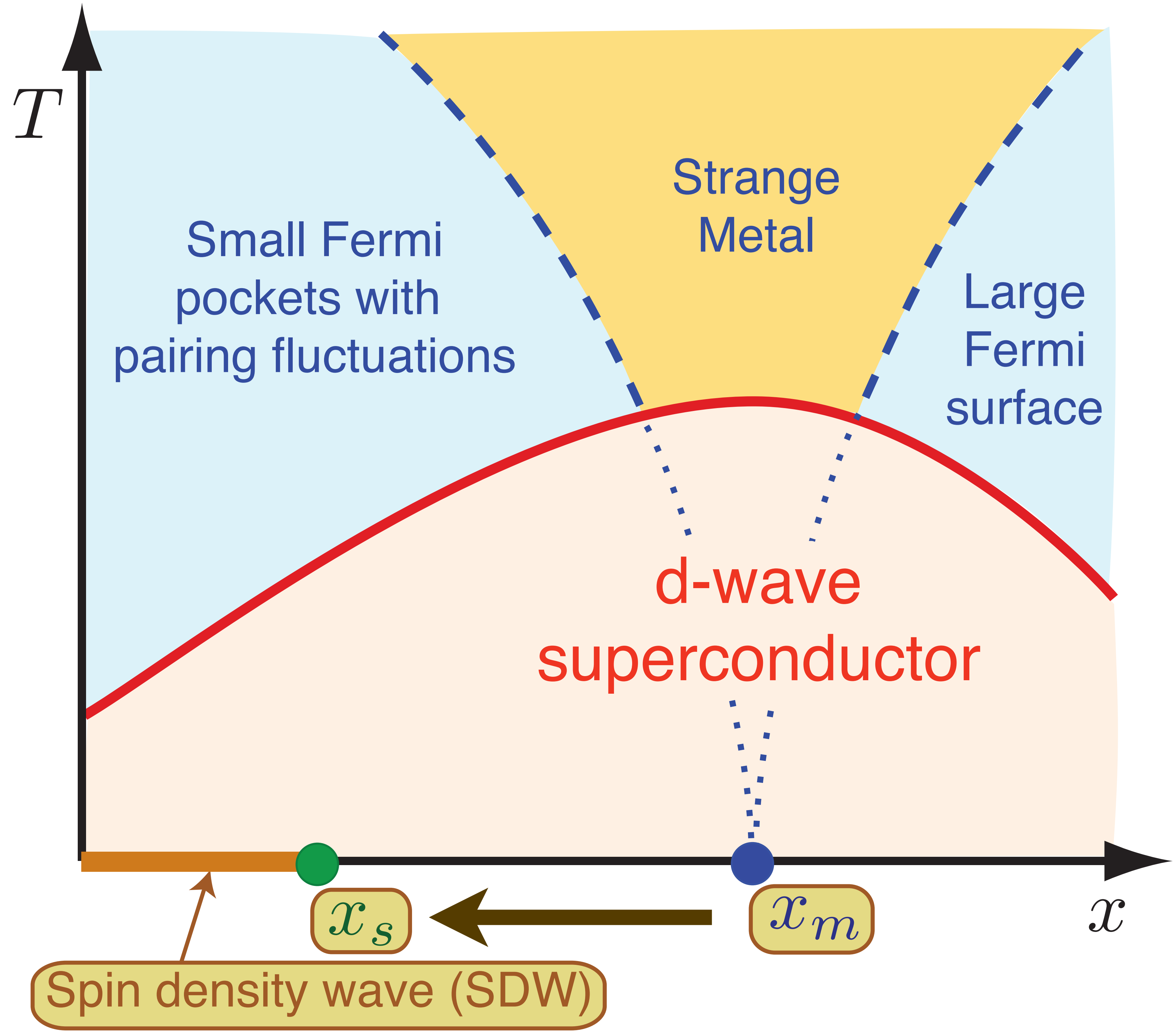
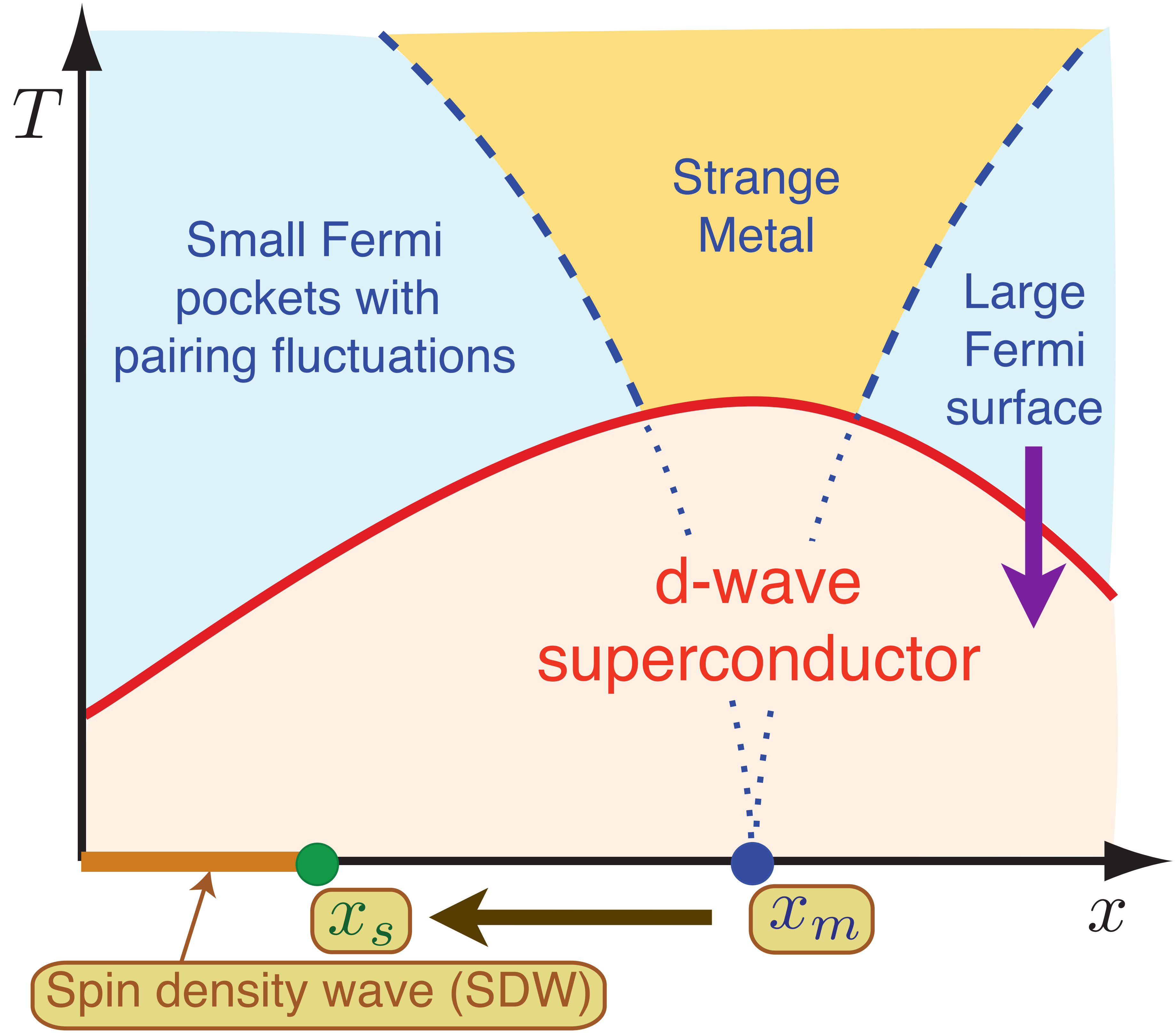


Theory of the onset of d-wave superconductivity from a large Fermi surface

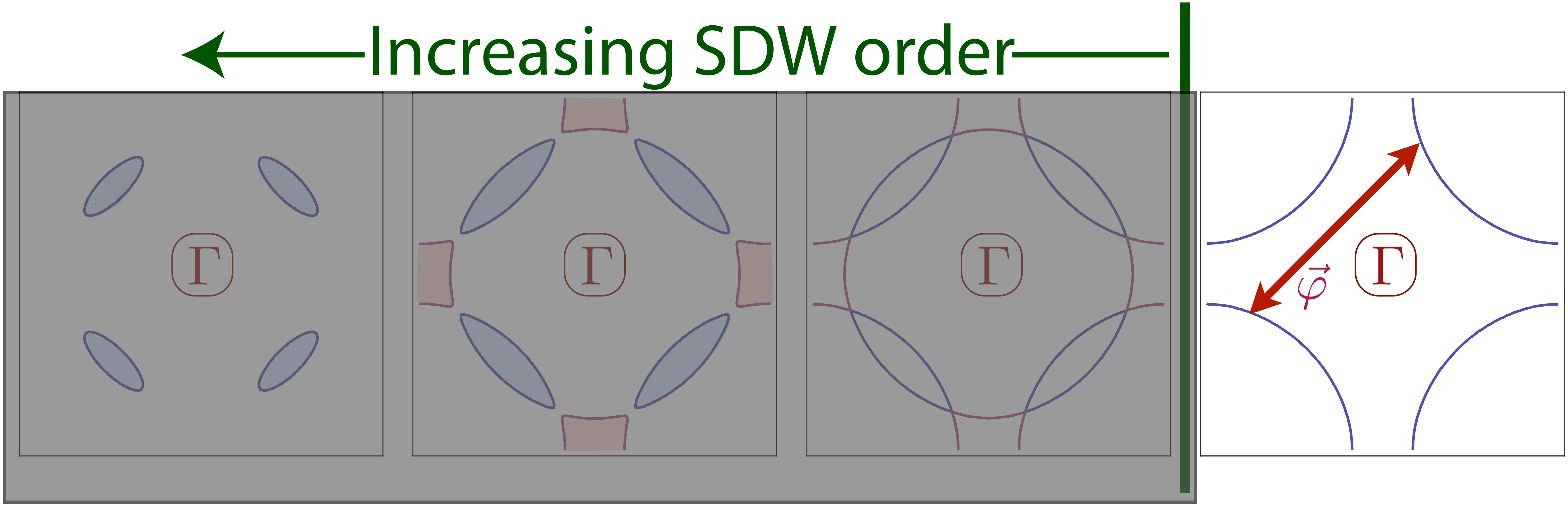
Thursday, July 16, 2009

Theory of quantum criticality in the cuprates



Theory of the onset of d-wave superconductivity from a large Fermi surface

Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2}\sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^{\dagger}_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

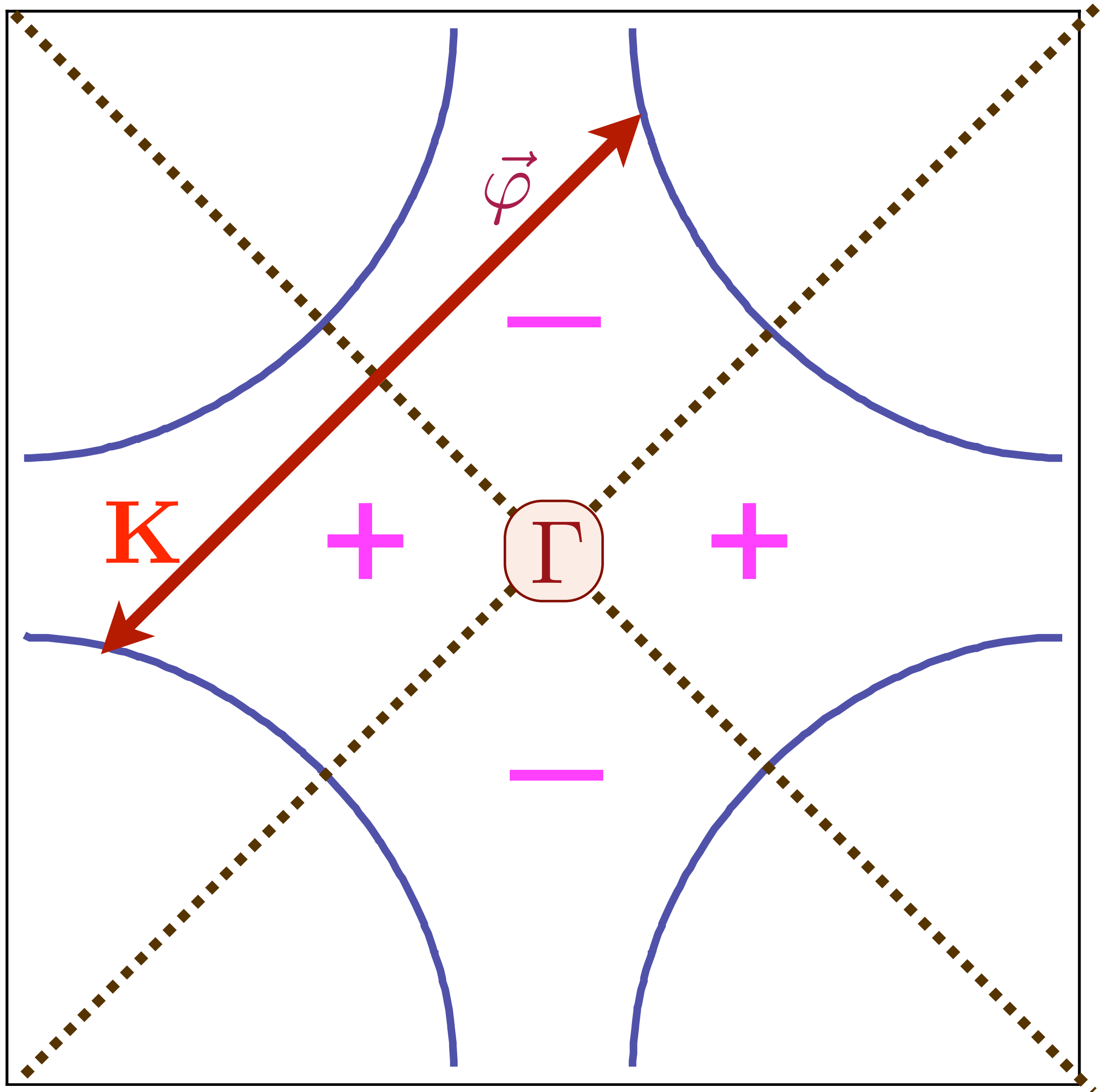
where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q}-\mathbf{K})^2},$$

with $\chi_0\xi^2$ the SDW susceptibility and ξ the SDW correlation length.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

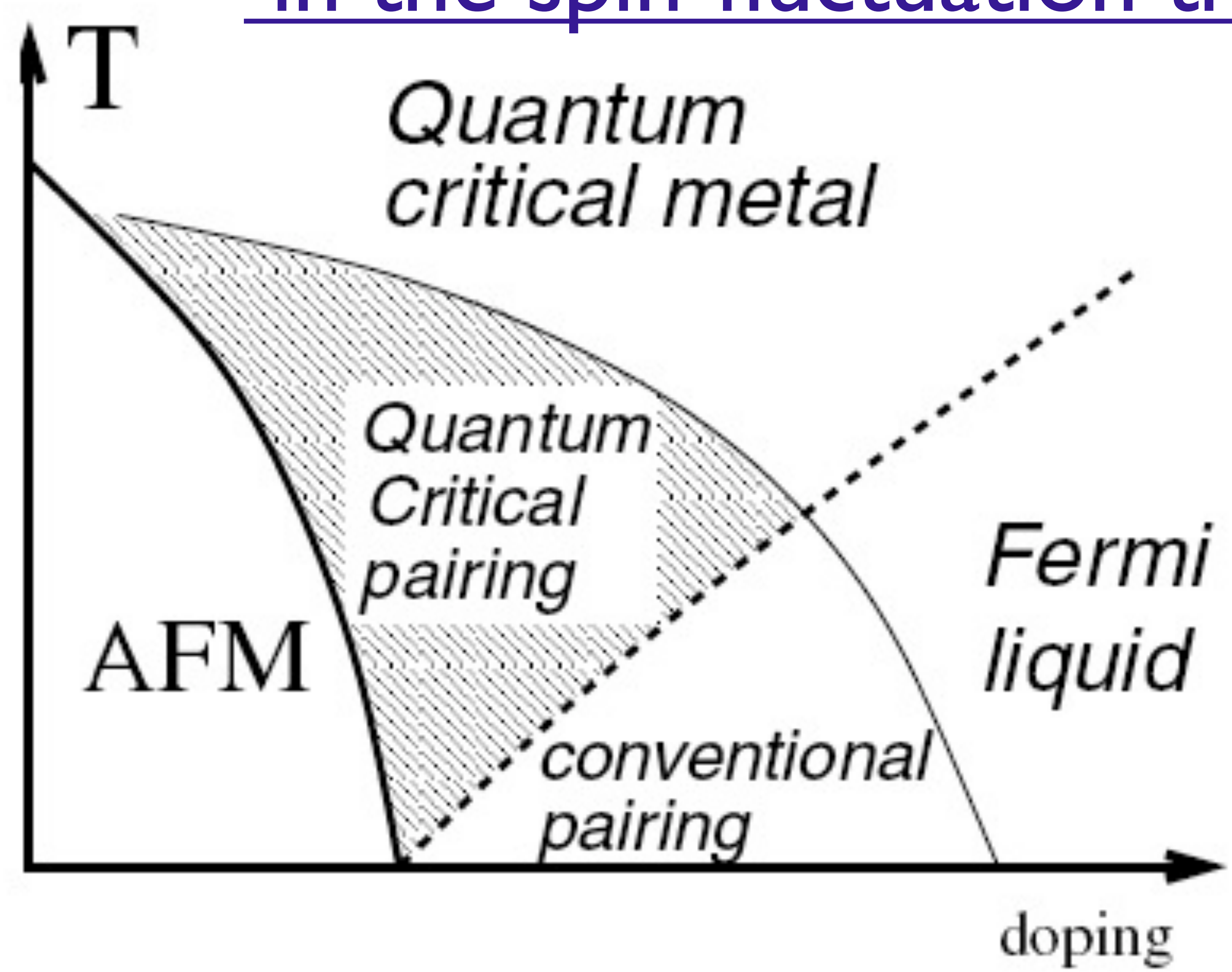
d-wave pairing of the large Fermi surface



$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

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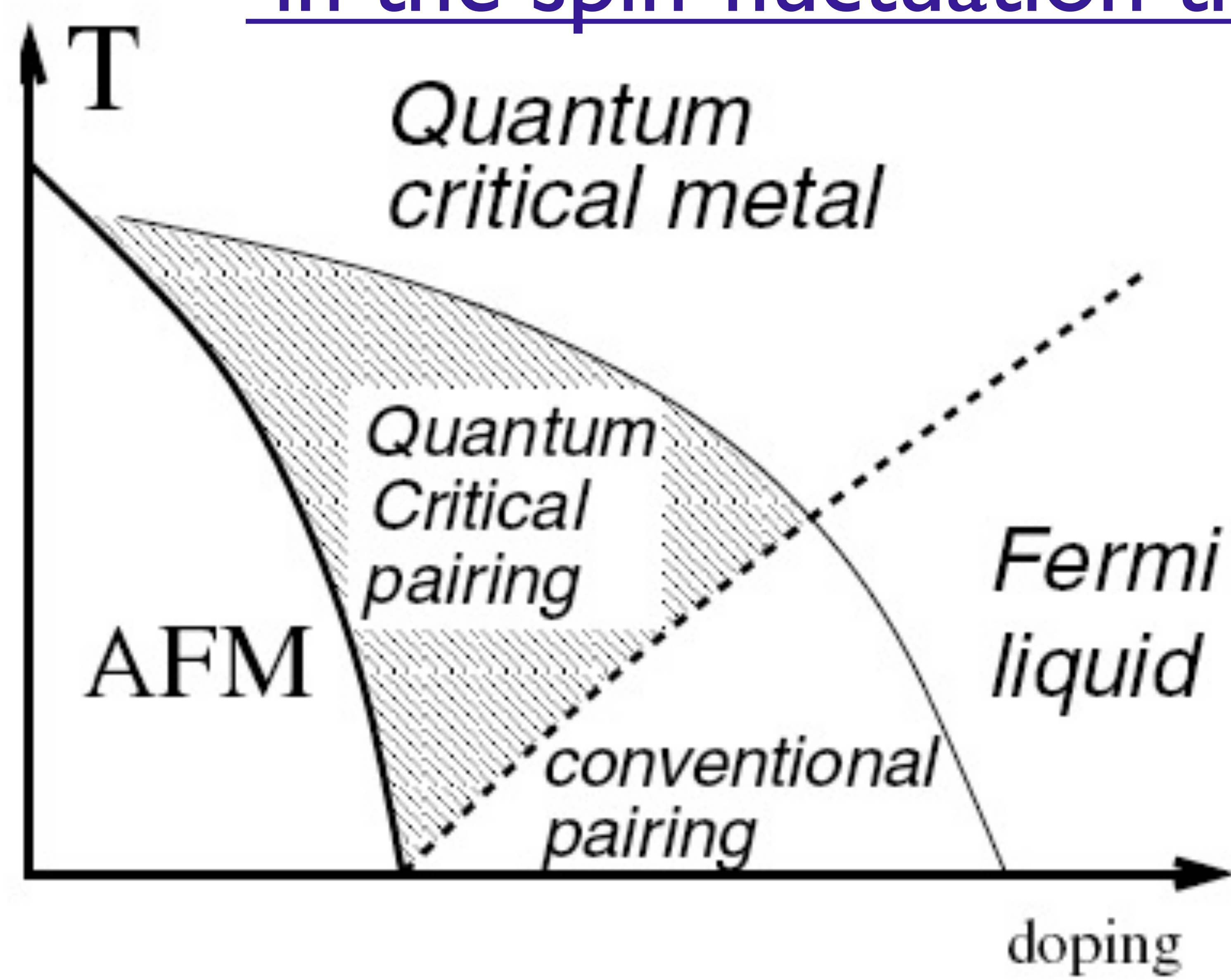

- T_c *increases* upon approaching the SDW transition.
  SDW and SC orders do not compete, but attract each other.

- No simple mechanism for nodal-anti-nodal dichotomy.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

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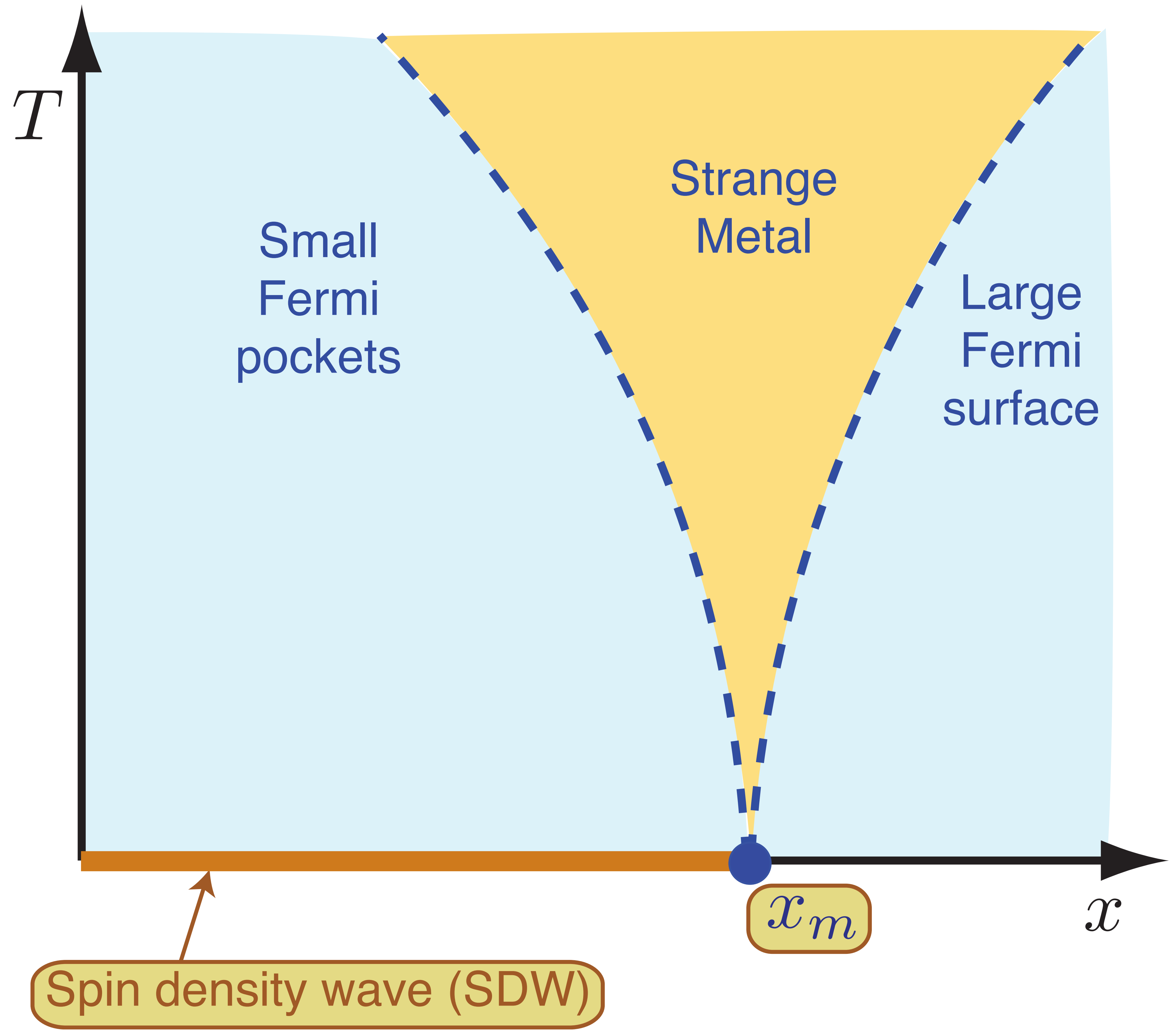

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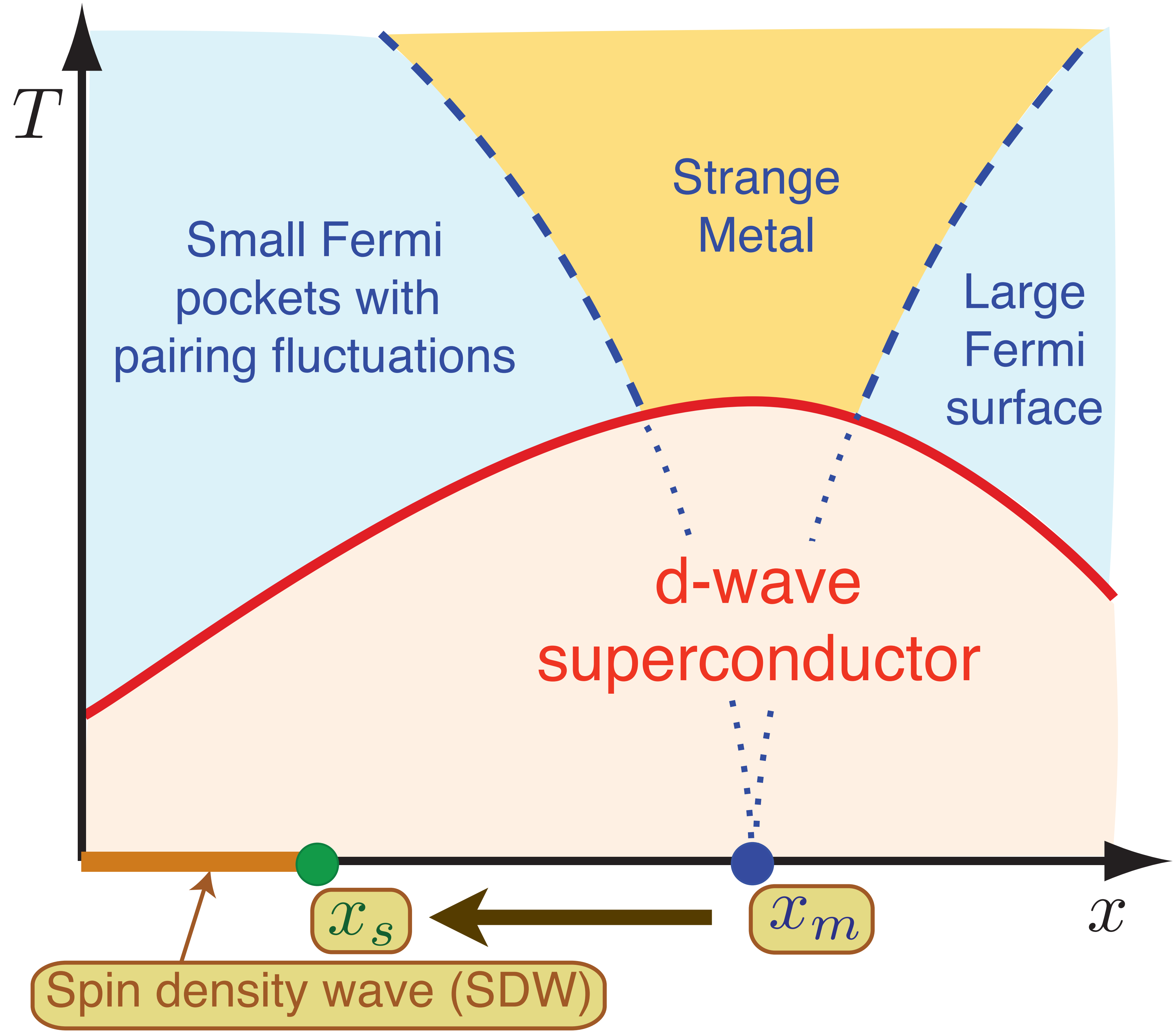

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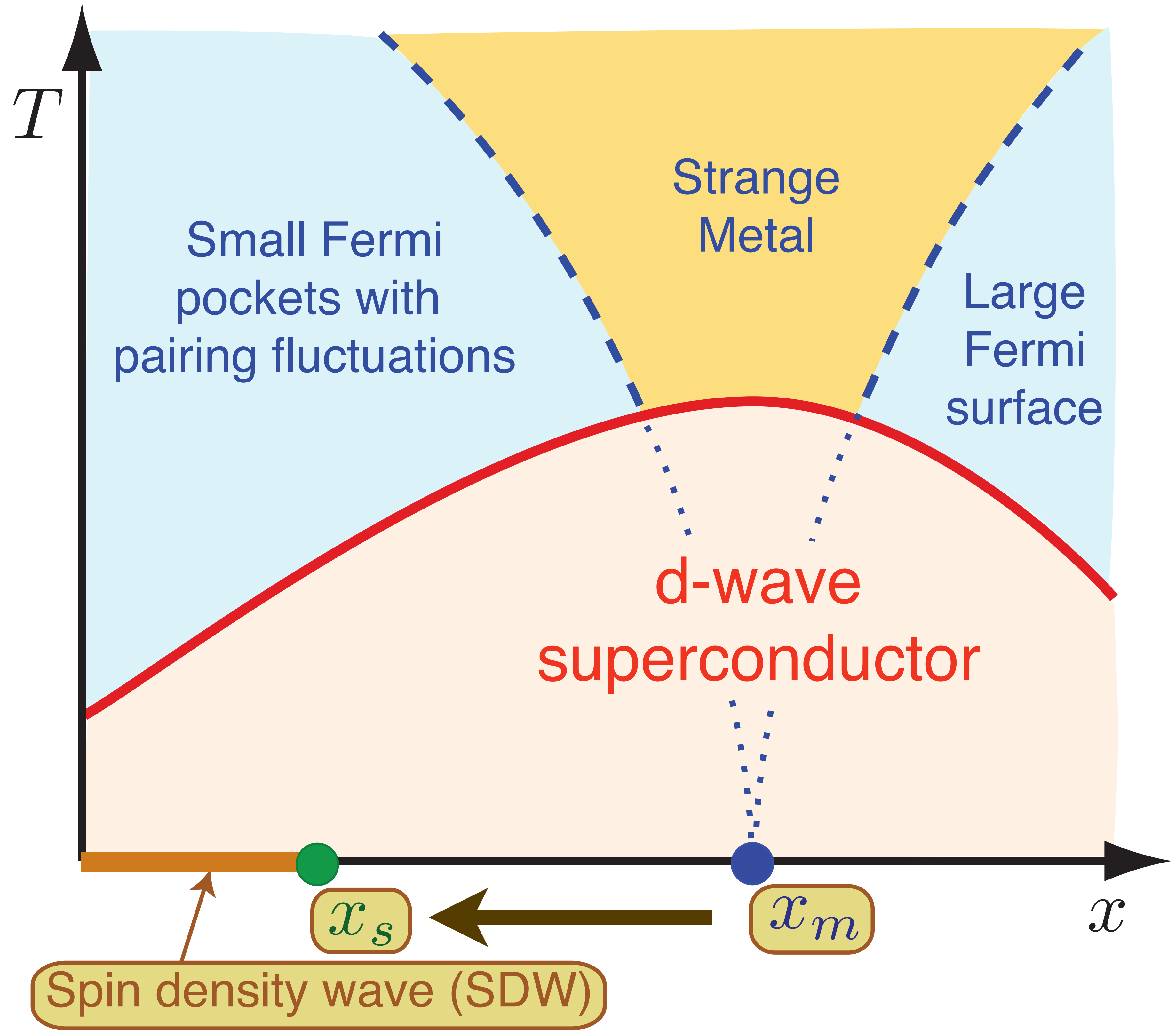
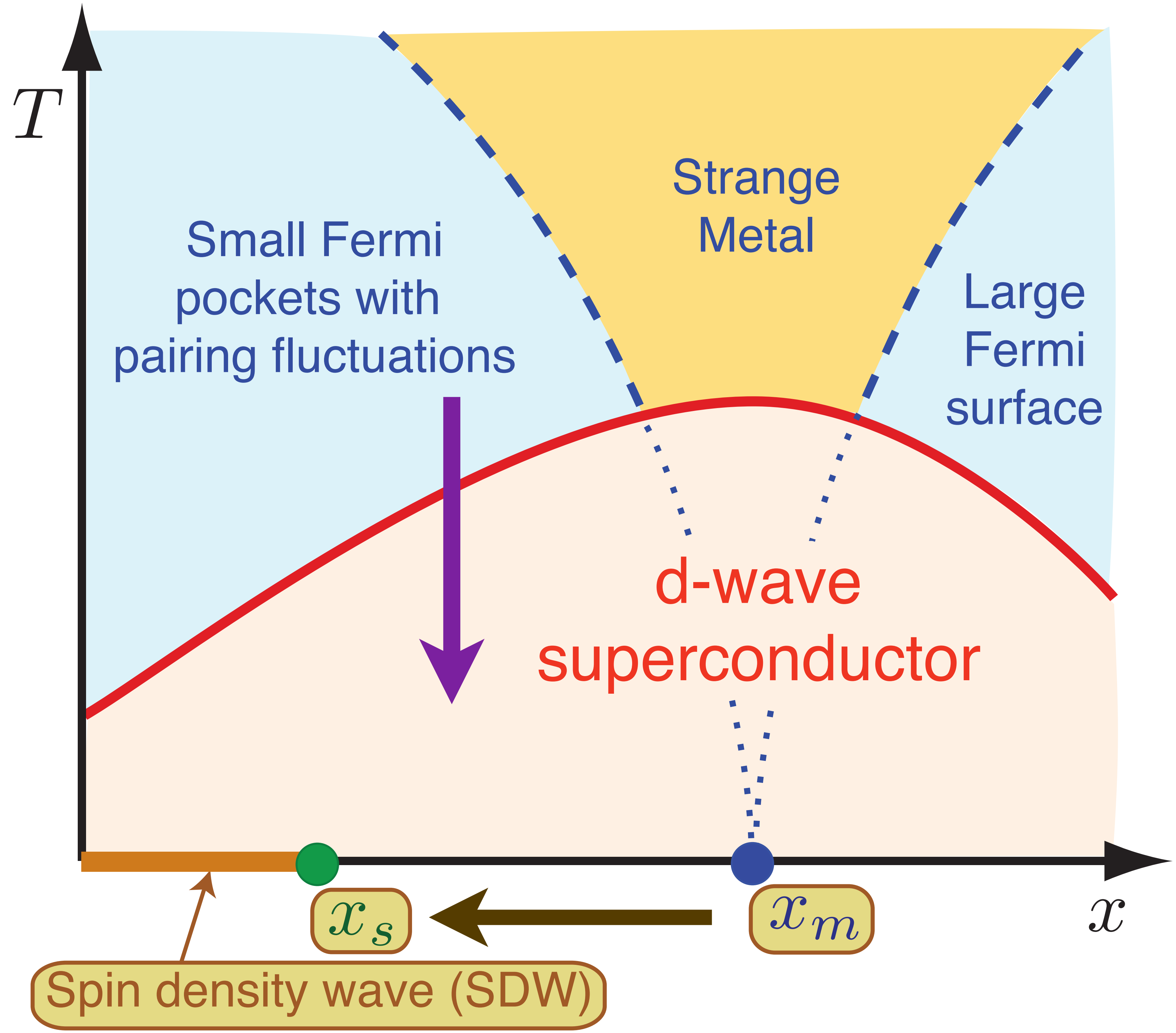


Theory of the onset of d-wave superconductivity from small Fermi pockets

Theory of quantum criticality in the cuprates



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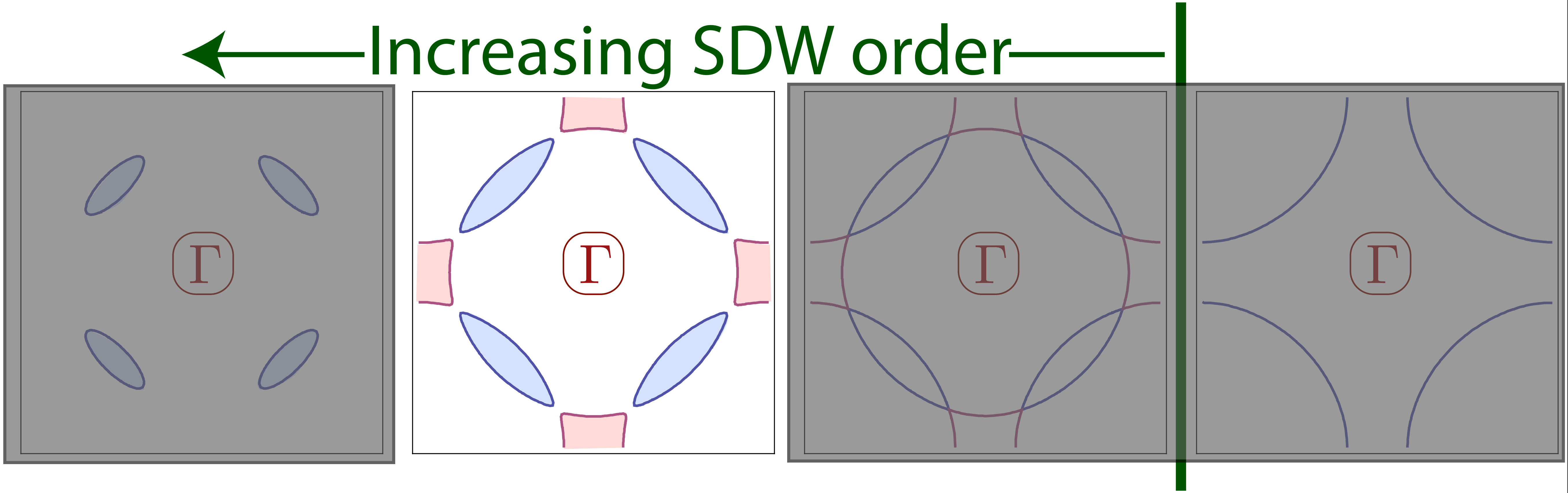
Theory of underdoped cuprates



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$$

the theory is invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha \quad ; \quad \psi_+ \rightarrow e^{-i\theta} \psi_+ \quad ; \quad \psi_- \rightarrow e^{i\theta} \psi_-$$

and the SDW order is given by

$$\hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

Theory of underdoped cuprates

Starting from the "SDW-fermion" model with Lagrangian

$$\begin{aligned}\mathcal{L} \quad = \quad & \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\ & - E_{sdw} \sum_{i} c_{i\alpha}^{\dagger} \hat{\vec{\varphi}}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\ & + \frac{1}{2t} \left(\partial_\mu \hat{\vec{\varphi}} \right)^2\end{aligned}$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

$$\mathcal{L} = \sum_{\mathbf{k}, p=\pm} \Bigg[\psi^\dagger_{\mathbf{k}p} \left(\frac{\partial}{\partial \tau} - i A_\tau + \varepsilon_{\mathbf{k} - p\mathbf{A}} \right) \psi_{\mathbf{k}p} - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \Bigg]$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order

$$\mathcal{L} = \sum_{\mathbf{k}, p=\pm} \left[\psi^\dagger_{\mathbf{k}p} \left(\frac{\partial}{\partial \tau} - i A_\tau + \varepsilon_{\mathbf{k} - p\mathbf{A}} \right) \psi_{\mathbf{k}p} \right.$$

$$\left. - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \right]$$

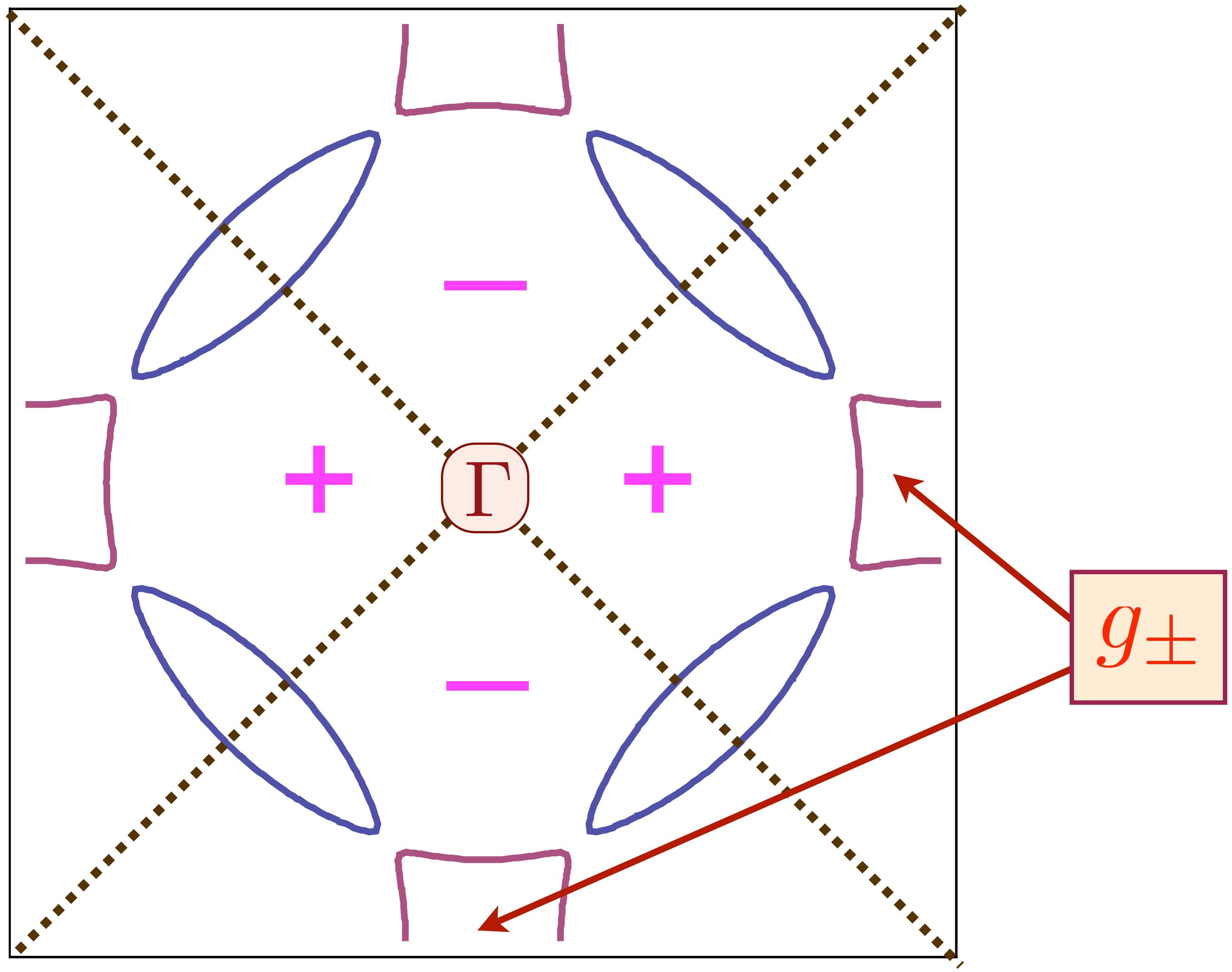
$$+ \frac{1}{t} \left[|(\partial_\tau - i A_\tau) z_\alpha|^2 + v^2 |(\nabla - i\mathbf{A}) z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1) \right]$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

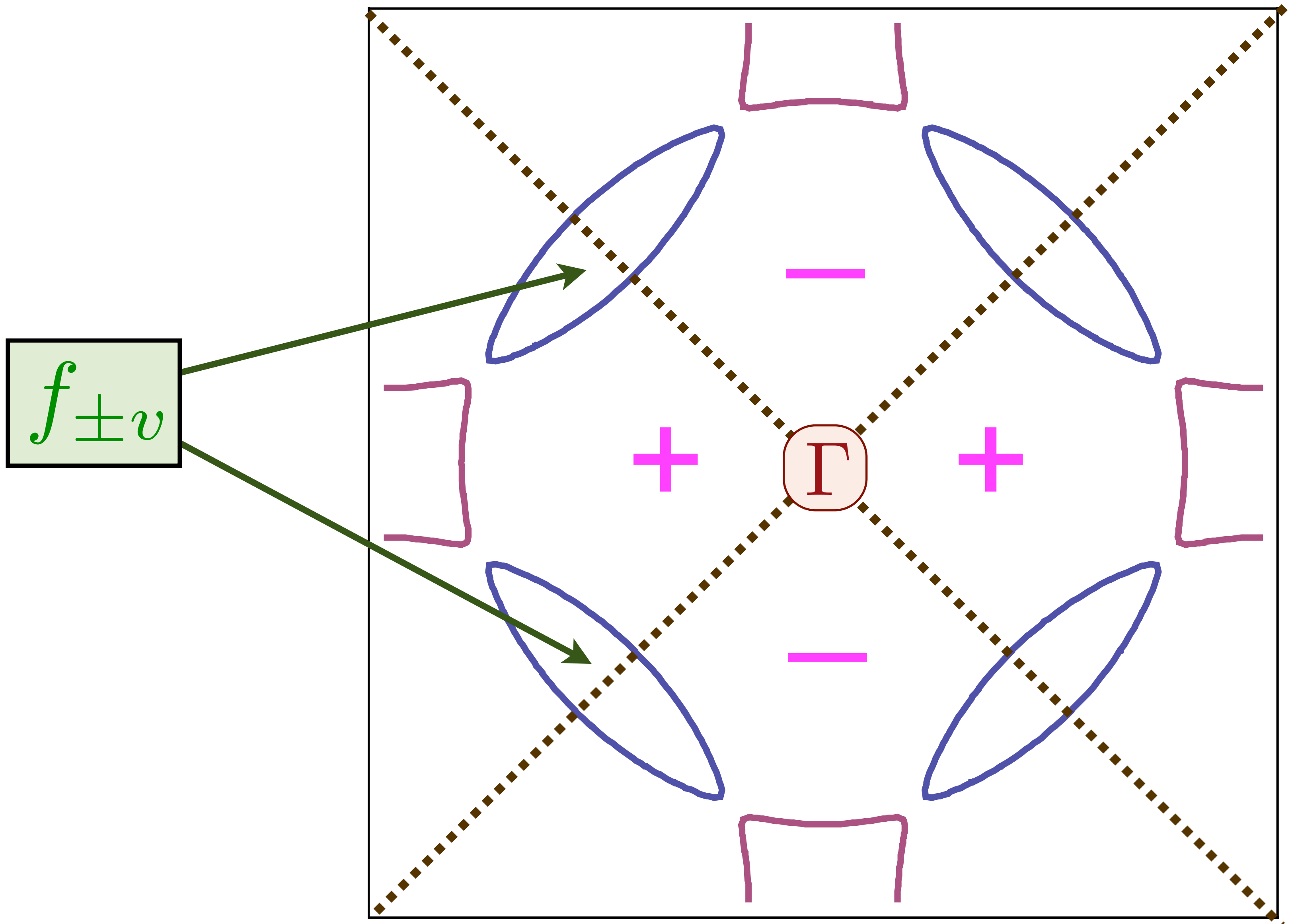
- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order
- Monopoles carrying Berry phases; onset of superconductivity leads to confinement via condensation of monopoles, which induces charge order.

Strong pairing of the $g_\pm$ electron pockets

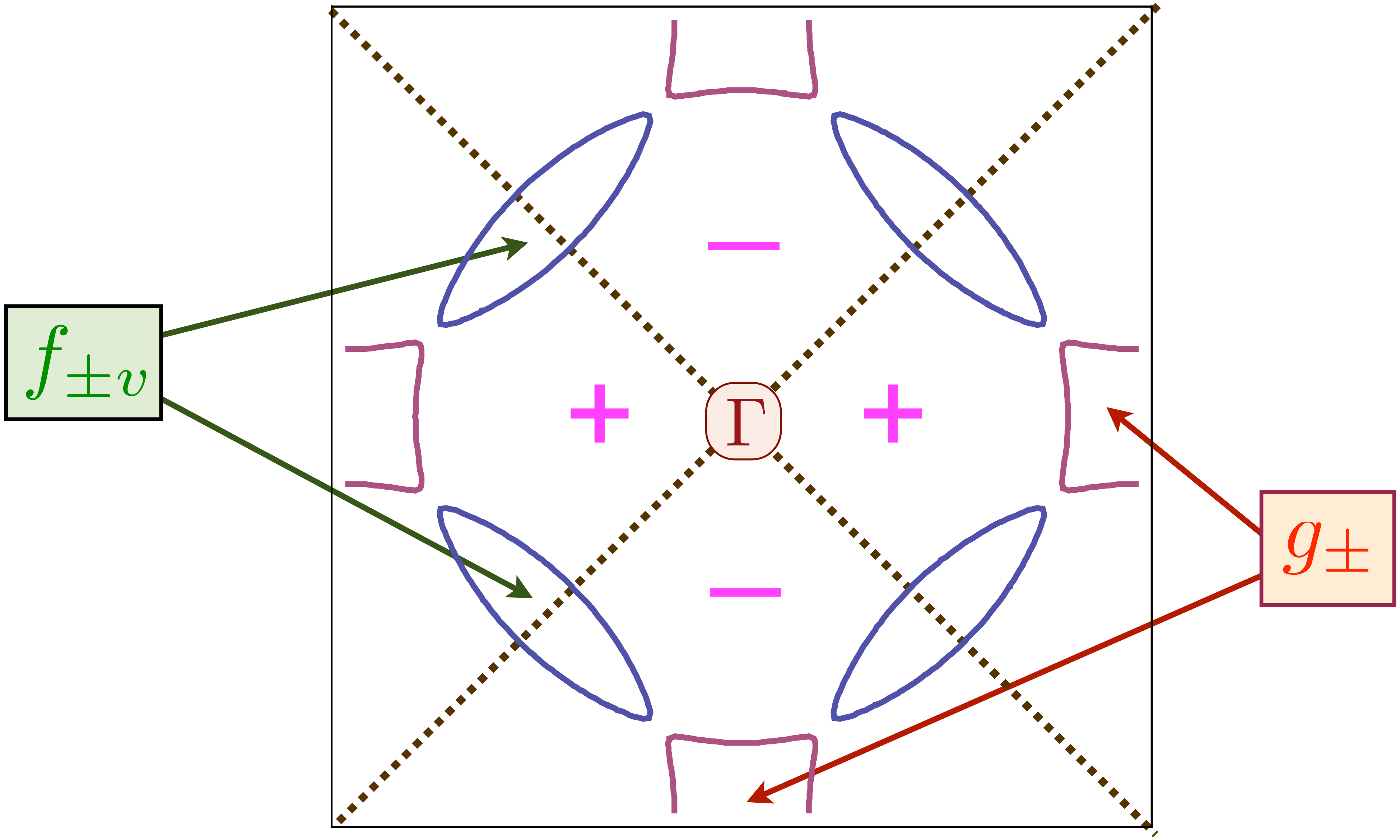


$$\langle g_+ g_- \rangle = \Delta$$

Weak pairing of the $f_\pm$ hole pockets



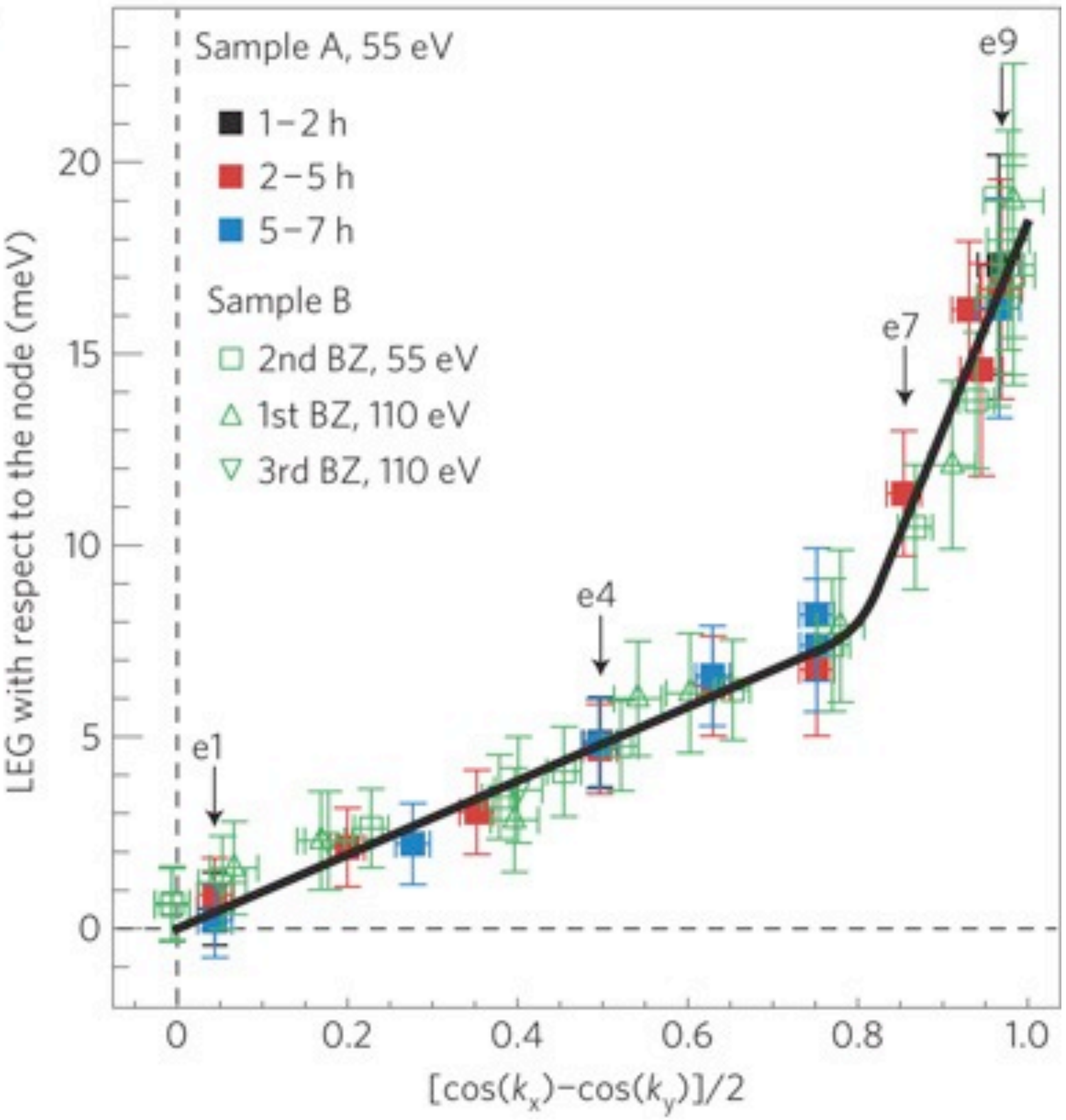
$$\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle \sim (k_x - k_y) J \langle g_+ g_-\rangle;$$
$$\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle \sim (k_x + k_y) J \langle g_+ g_-\rangle;$$
$$\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle = 0,$$



d-wave pairing of the electrons is associated with

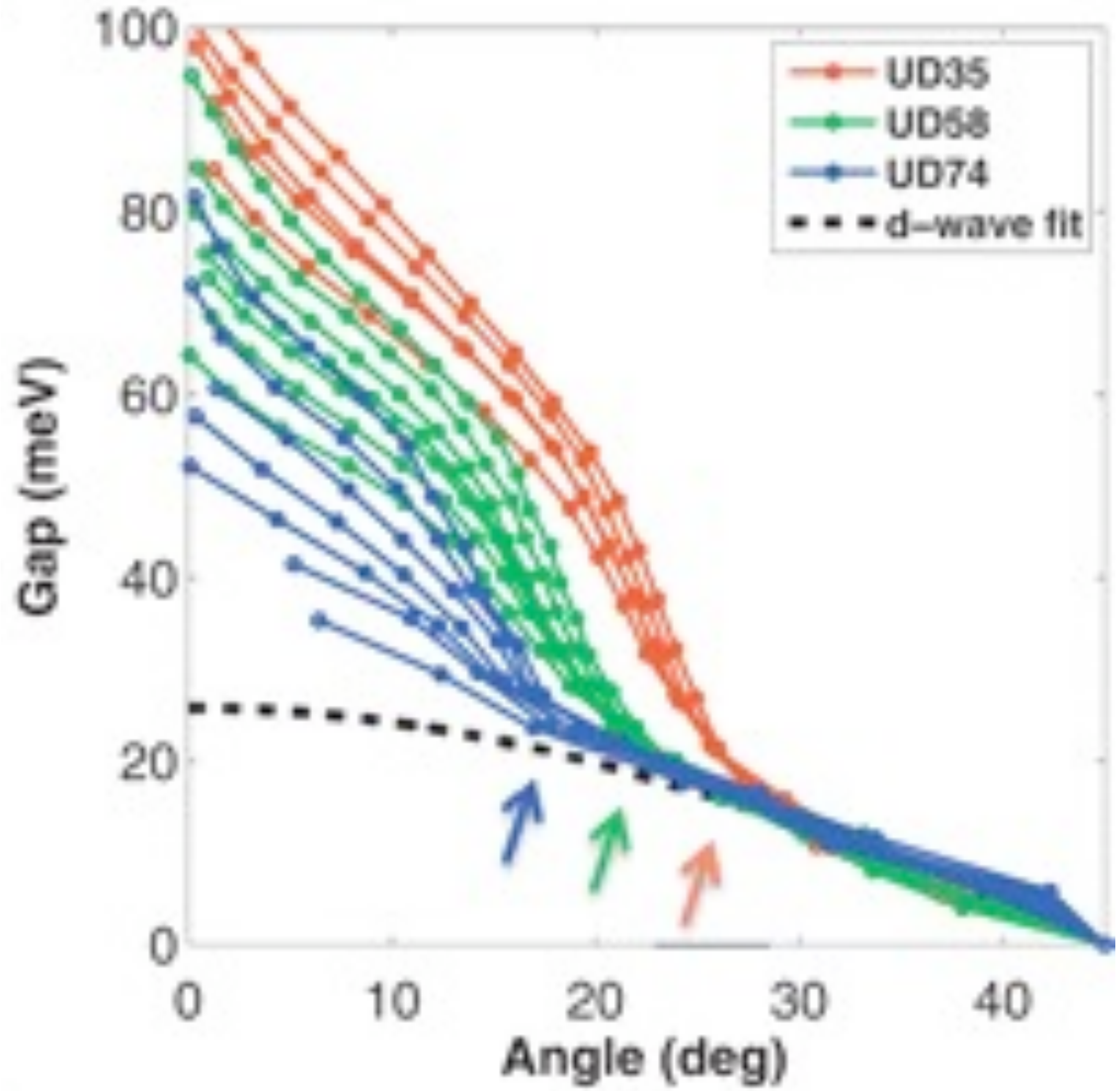
- Strong s-wave pairing of $g_{\pm}$
- Weak p-wave pairing of $f_{\pm v}$.

Photoemission in LBCO

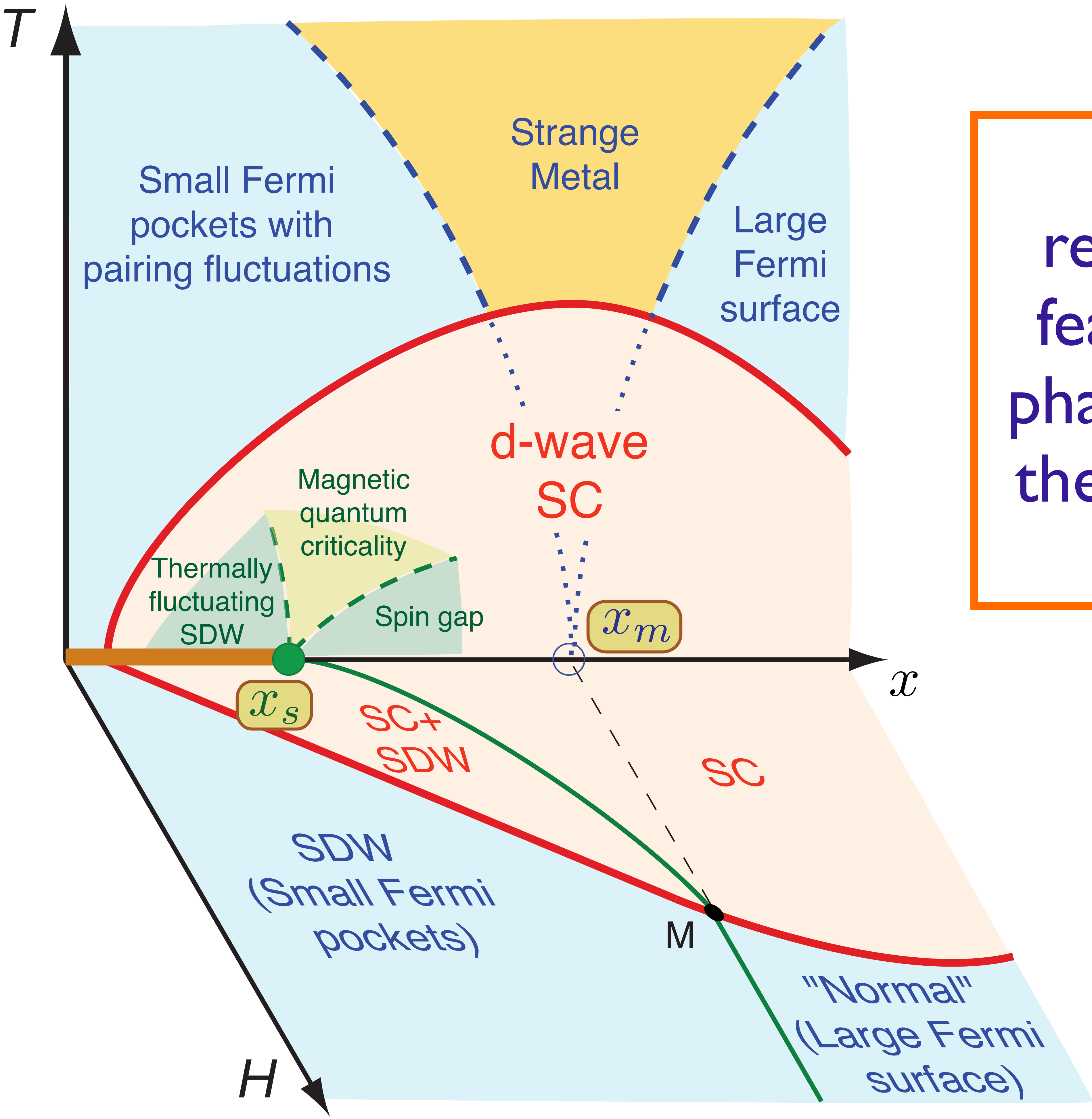


R.-H. He, K. Tanaka, S.-K. Mo, T. Sasagawa, M. Fujita, T. Adachi, N. Mannella, K. Yamada, Y. Koike, Z. Hussain and Z.-X. Shen, *Nature Physics* **5**, 119 (2008)

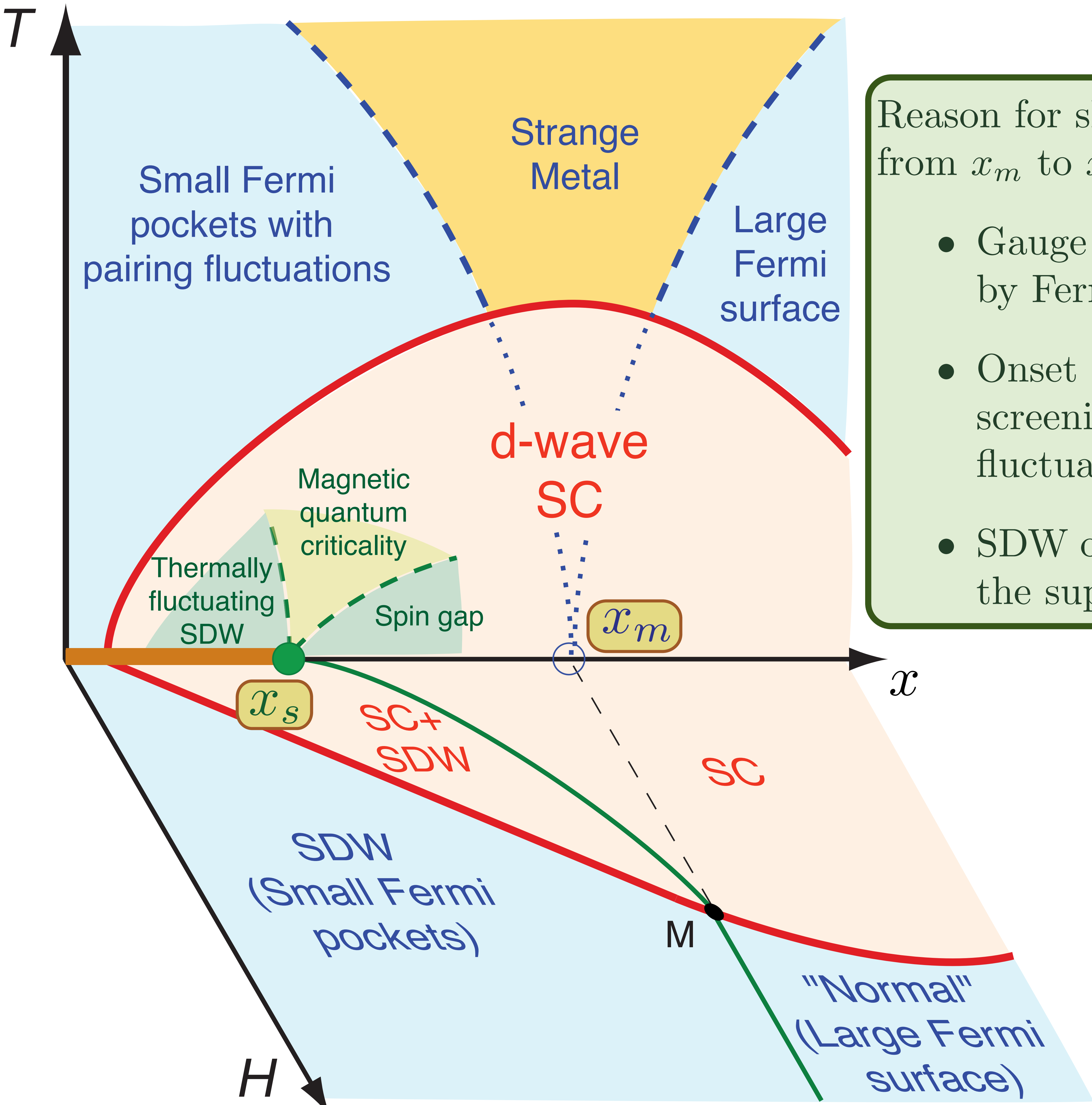
STM in BSCCO



A. Pushp, C. V. Parker, A. N. Pasupathy, K. K. Gomes, S. Ono, J. Wen, Z. Xu, G. Gu, and A. Yazdani, *Science* **324**, 1689 (2009)



Theory reproduces all features of the phase diagram in the underdoped regime



Reason for shift in onset of SDW from x_m to x_s:

- Gauge fluctuations are screened by Fermi surface in metal
- Onset of pairing suppresses screening, and enhances gauge fluctuations
- SDW order is suppressed in the superconductor

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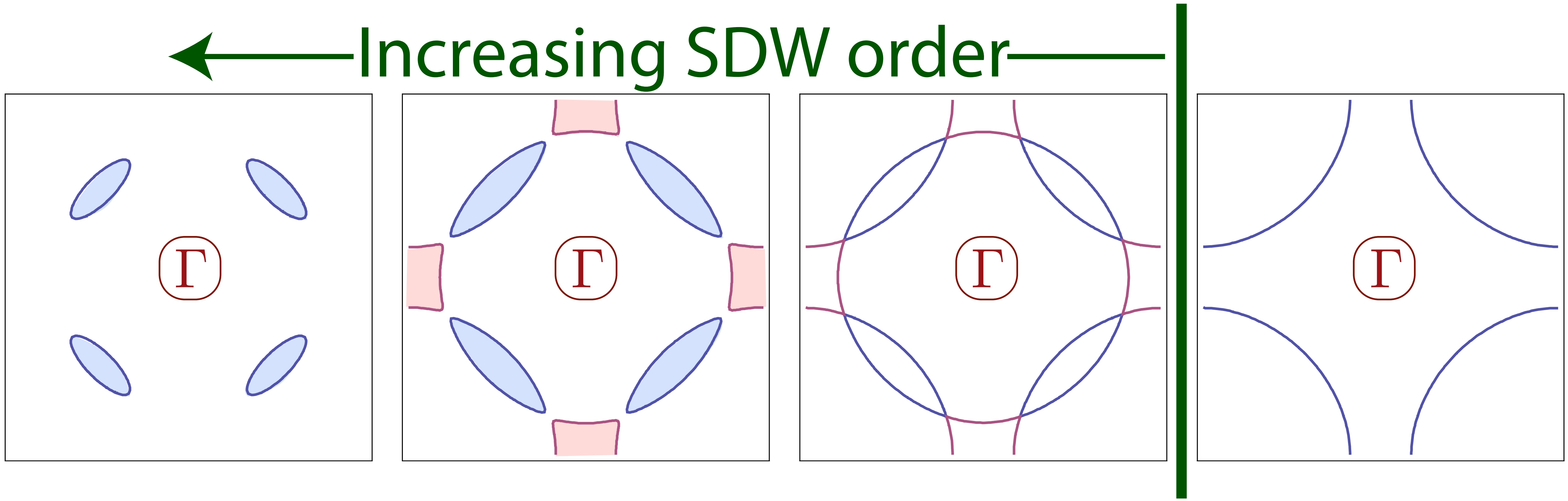
Unified theory



The parameterization $$\left(\begin{array}{c} c_\uparrow \\ c_\downarrow \end{array} \right) = R \left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right)$$

is actually invariant under a SU(2) gauge transformation

$$\left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) \rightarrow U \left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) \quad ; \quad R \rightarrow R U^\dagger$$

Unified theory

The theory has $\mathrm{SU}(2)_{\mathrm{gauge}} \otimes \mathrm{SU}(2)_{\mathrm{spin}} \otimes \mathrm{U}(1)_{\mathrm{em\ charge}}$ $\otimes$(lattice space group) invariance,

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and matter content

- fermion ψ transforming as $(\mathbf{2}, \mathbf{1}, 1)$, and with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure,

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- relativistic real scalar N transforming as $(\mathbf{3}, \mathbf{1}, 0)$, measuring the local SDW amplitude,

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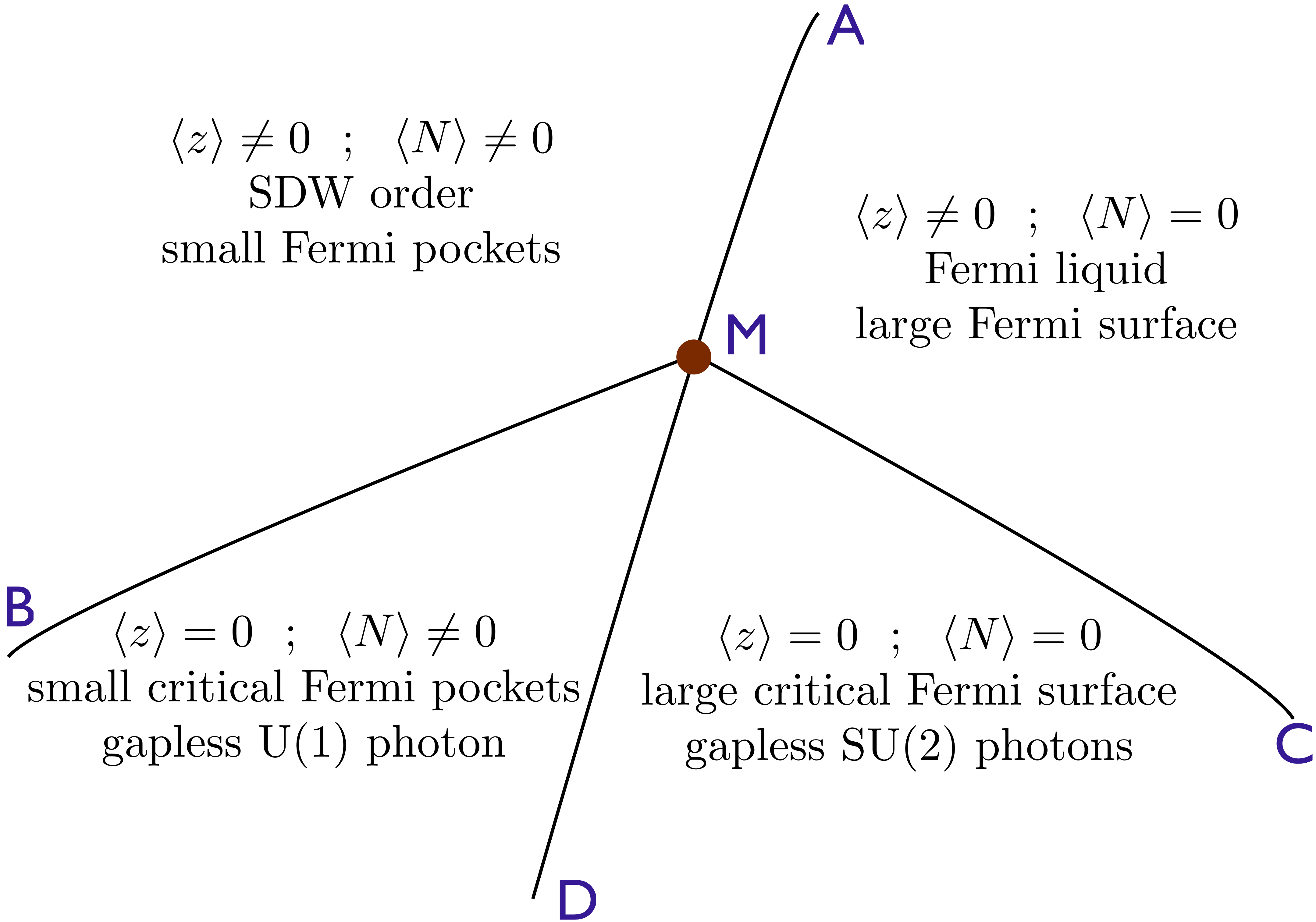
- relativistic real scalar N transforming as $(\mathbf{3}, \mathbf{1}, 0)$, measuring the local SDW amplitude,

- a Yukawa coupling between N and ψ, which $\sim e^{i\mathbf{K}\cdot\mathbf{r}}$ because of space group transformations.

Unified theory

Conjectured phase diagram (assuming a phase with gapless SU (2) photons is possible)



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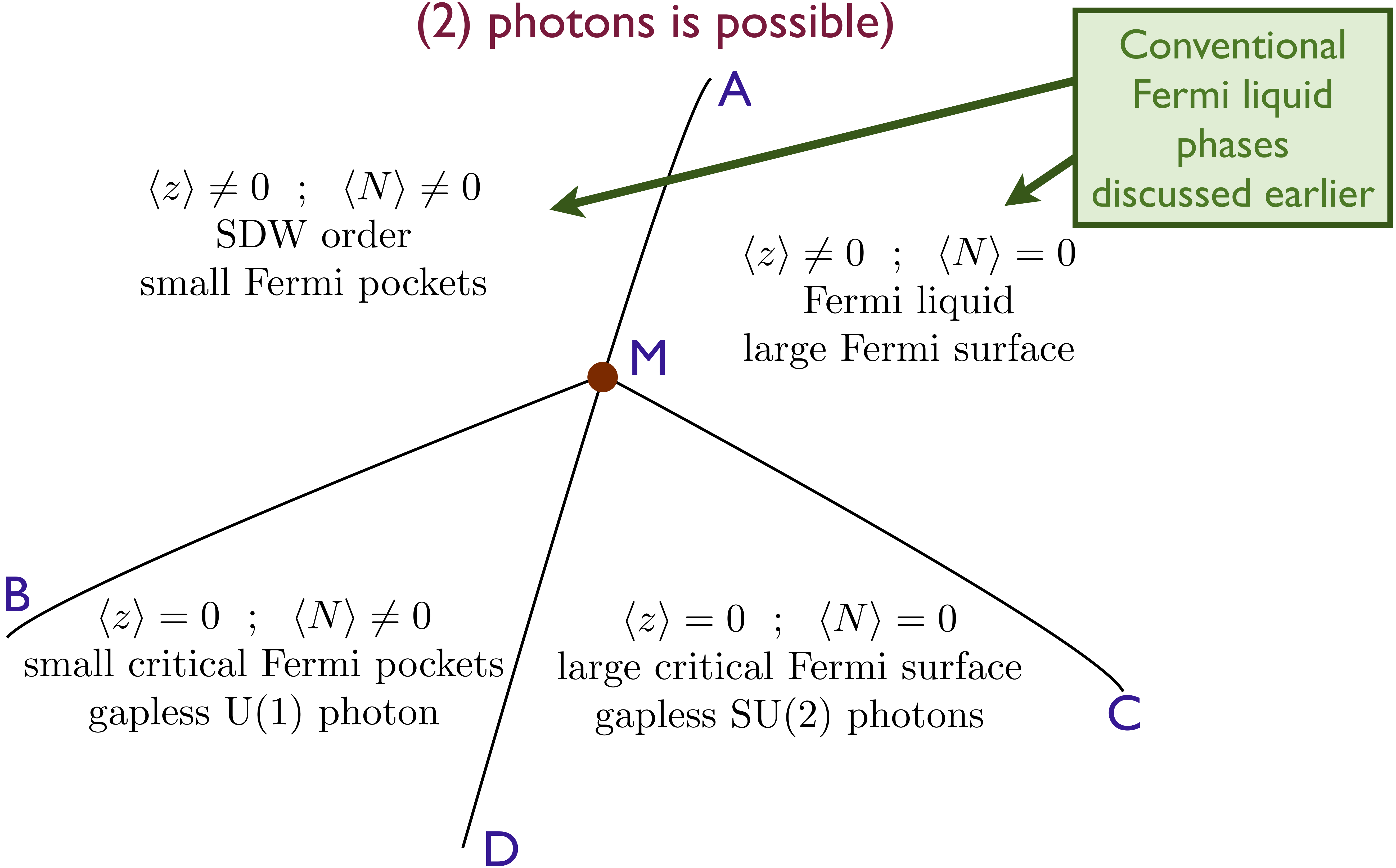

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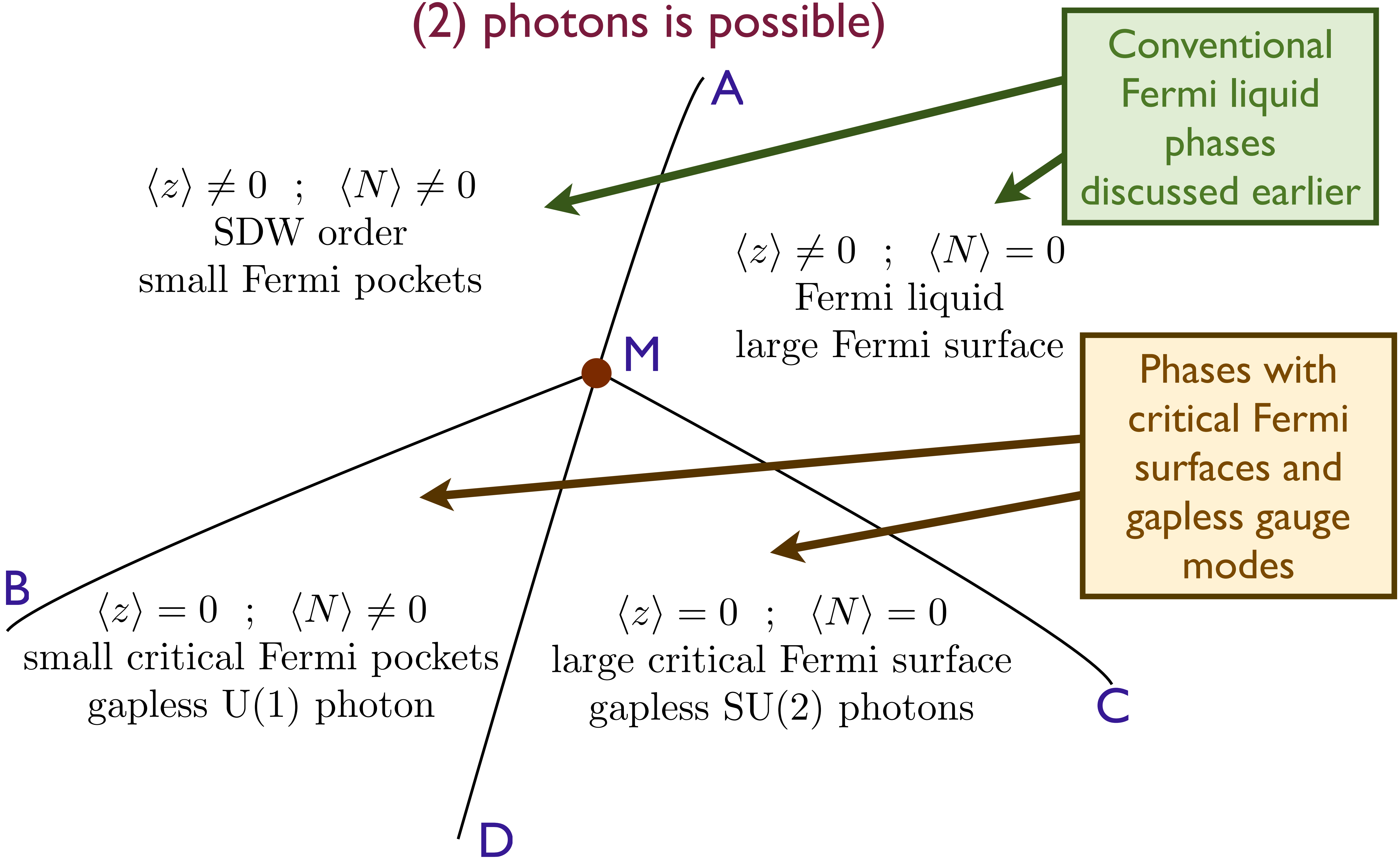

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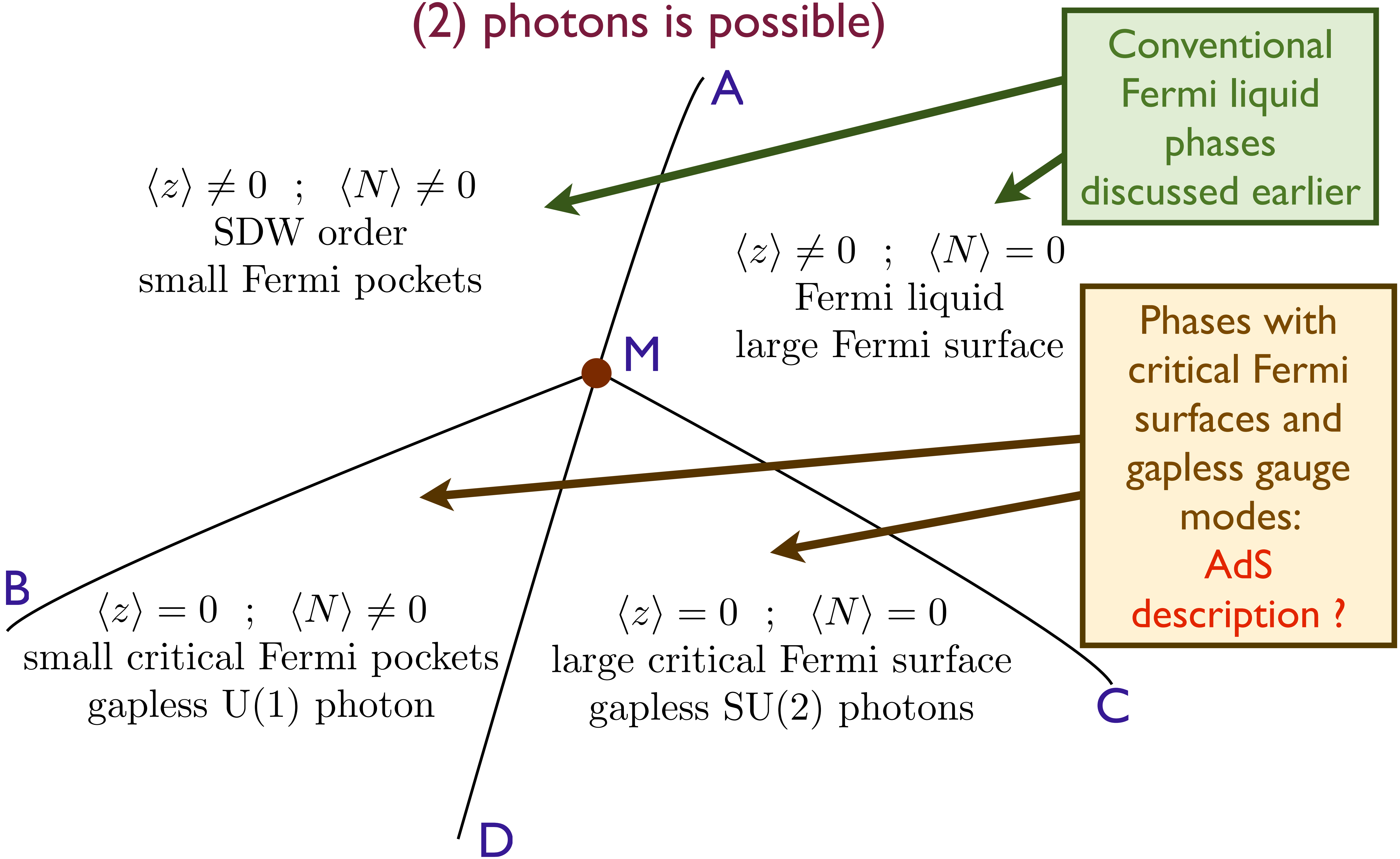
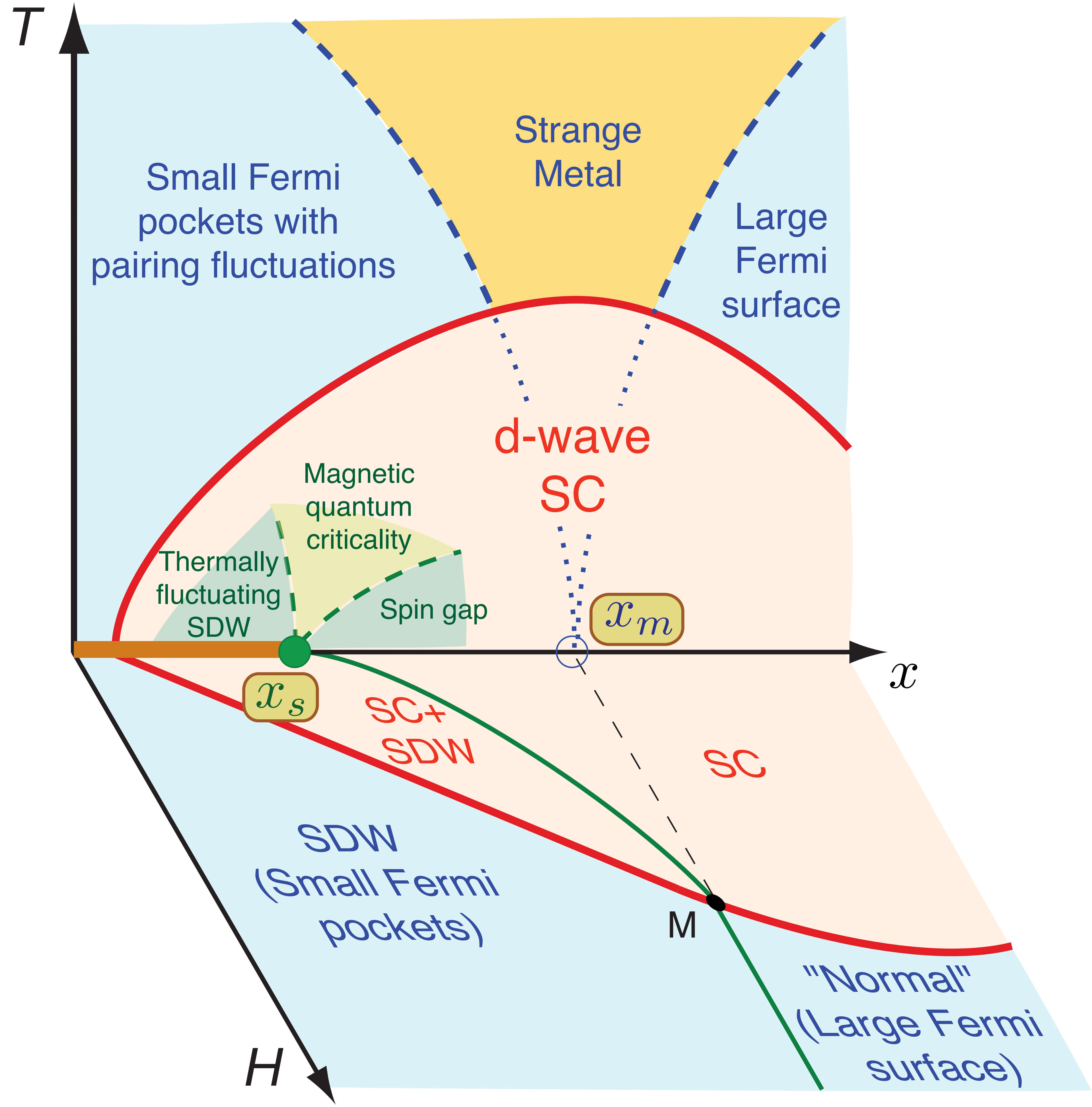
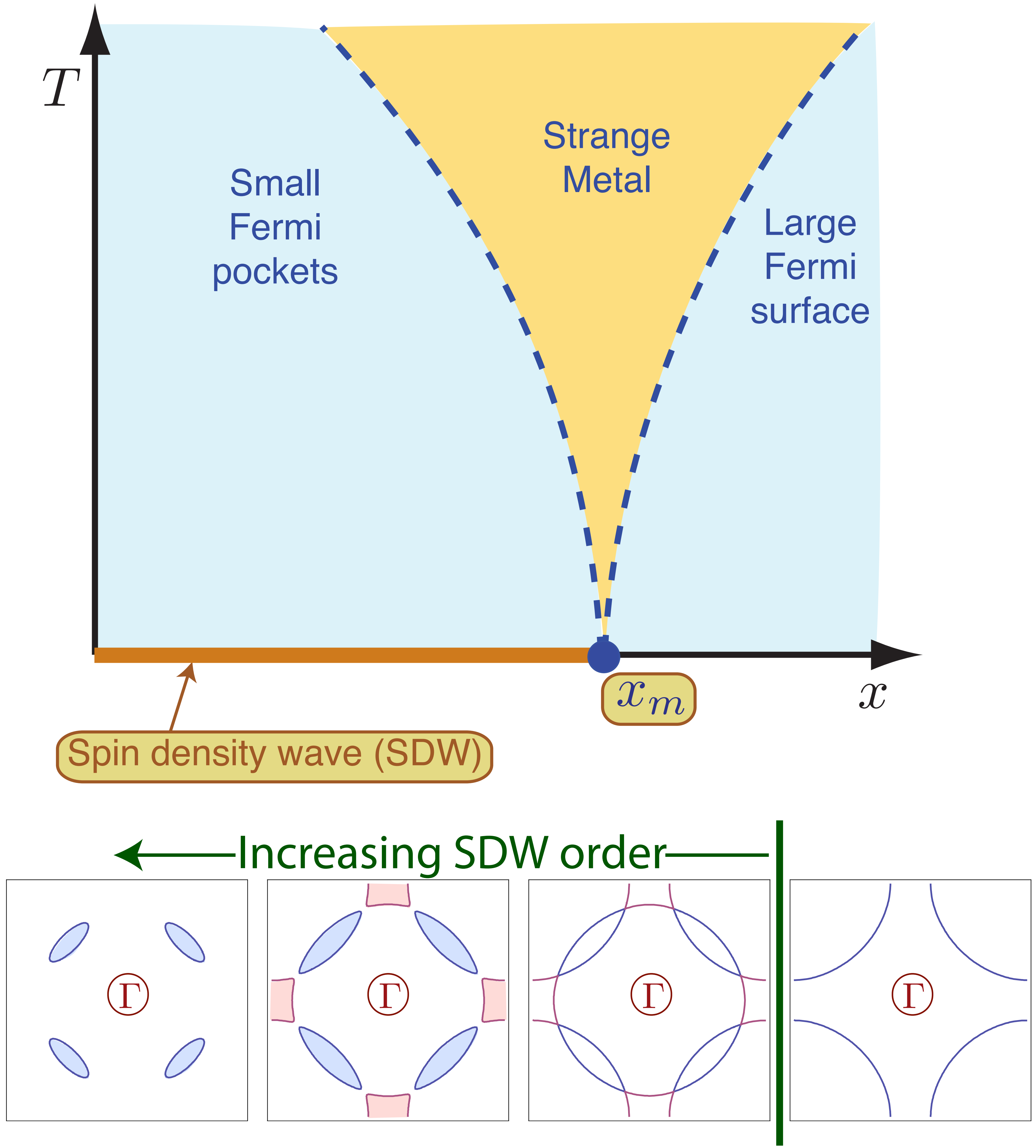


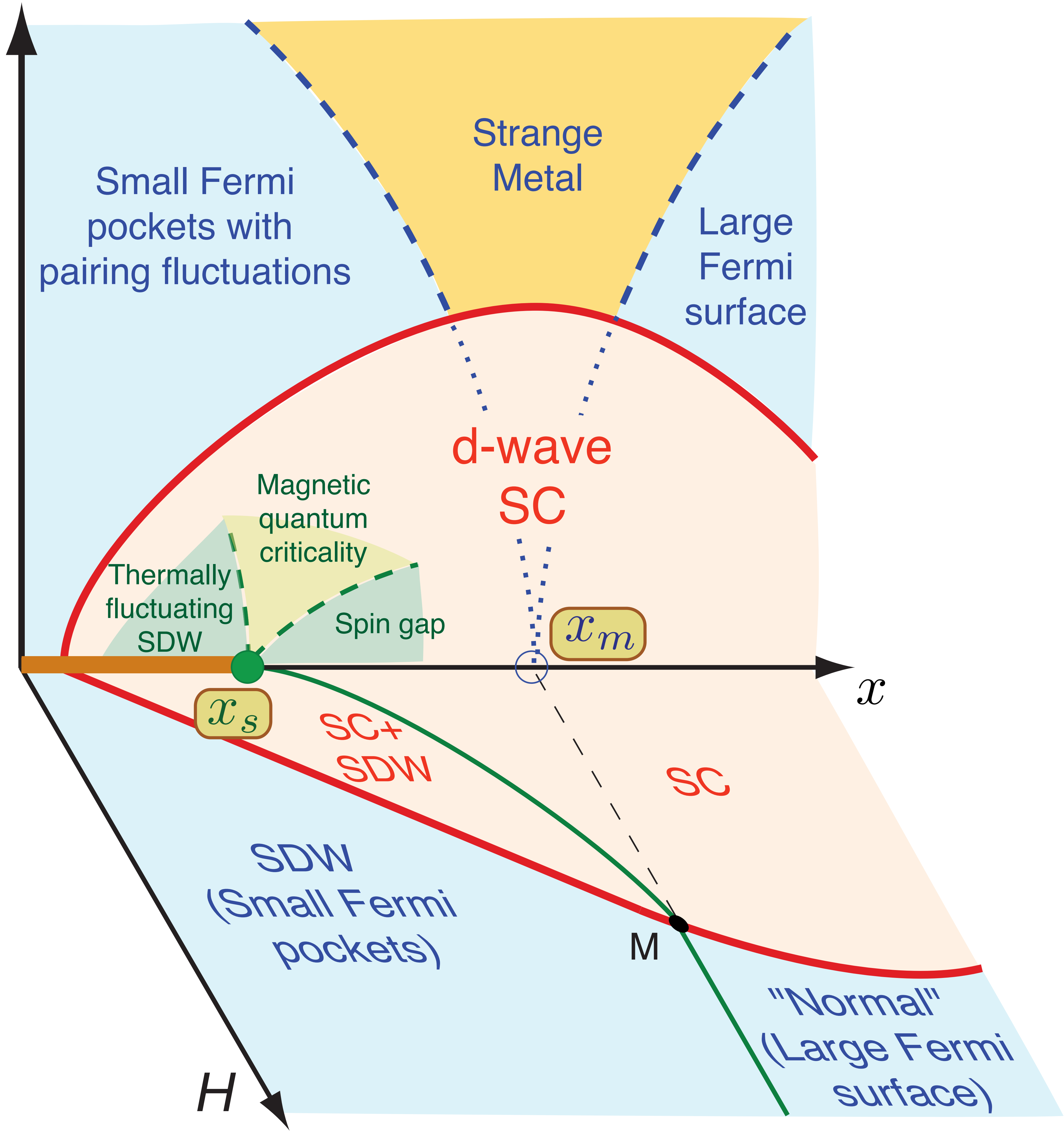


T
Small Fermi pockets with pairing fluctuations
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x
x_s
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H



T
Small Fermi pockets
Strange Metal
Large Fermi surface
x_m
x
Spin density wave (SDW)
Increasing SDW order
Γ
Γ
Γ
Γ

U(1) theory reproduces all features of the phase diagram in the underdoped regime



Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity