Simons Collaboration on Special Holonomy in Geometry
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M-Branes, T-banes in G_2 Holonomy Backgrounds

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Background/Outline

I. Motivation: M-theory in space-time dimensions 4D [3D] with
N=1 supersymmetry → on G_2 [Spin(7)] holonomy spaces

II. M-theory as a classical gravity background -11D supergravity
→ 11D on G_2 [Spin(7)] holonomy spaces
& $F_{(4)}$-flux - four-form field strength → typically smooth

→ M2, M5-branes (brief); highlight new insights - M3-branes

III. M-theory describing gauge degrees (QFT- Standard Model?)
→ 11D on G_2 [Spin(7)] holonomy spaces with
co-dimension four singularities →
gauge degrees governed by a Hitchin-type system

Add Hitchin flux (T-brane type configurations) →
Localized ``matter'' modes

IV. Summary/Outlook

Background:

II. M-theory – in classical gravity backgrounds with $F_{(4)}$ flux (M-branes)

M.C., Gary W. Gibbons, Hong Lü, Christopher N. Pope ‘01-’04
(ALC non-compact special holonomy spaces & M – branes)

Review (Les Houches ’01 lectures, M.C.) hep-th 0206154

M.C., Jonathan J. Heckman – work in progress

III. M-theory – gauge degrees Hitchin-type system with fluxes (T-brane type configurations)

Rodrigo Barbosa, M.C., Jonathan J. Heckman, Craig Lawrie, Ethan Torres, Gianluca Zoccarato – to appear 1904…

II. 11D supergravity in 3D and 4D with N=1 supersymmetry

11D metric on special holonomy space and $F_{(4)}$-flux:

Prototype in 3D: Fractional M2-brane

$$d\hat{s}_{11}^2 = H^{-2/3}\, dx^\mu\, dx^\nu\, \eta_{\mu\nu} + H^{1/3}\, ds_8^2$$

$R^{(1,2)}$ Spin(7)*

$$F_{(4)} = d^3x \wedge dH^{-1} \;\; + m\, L_{(4)}$$

$$\Box_8 H = \;\; -\tfrac{1}{48} m^2\, L_{(4)}^2$$

H - Harmonic function in Spin(7)

$L_{(4)}$ - harmonic self-dual 4-form in transverse Spin(7) space

M.C., Pope, Lü
hep-th/0011023…

* - explicit non-compact co-homogeneity-one Spin(7) metrics

AC – Bryant, Salamon

ALC – M.C., Gibbons, Lü, Pope 0103.155…Foscolo, Haskins, Nordström'17…

Regular solutions w/ N=1/2 supersymmetry → AdS_4/CFT_3 correspondence

Example in 4D: M3-brane

M.C., Pope, Lü, 0105.096
0106.026

Metric: $ds^2_{11} = H^2\, dx^\mu\, dx_\mu + 2H^{-7}\, U^{-1}\, dr^2 + \frac{1}{2} r^2\, H^{-1}\, U\, D\mu^i\, D\mu^i + r^2\, H^{-1}\, d\Omega_4^2$

$R^{(1,3)}$

7D - a deformation of G_2 holonomy space w/ topology of the $\mathbf{R}^3$ bundle over S^4

$\mu^i\, \mu^i = 1 \quad D\mu^i = d\mu^i + \epsilon_{ijk}\, A^j_{(1)}\, \mu^k$

$A^i_{(1)}$ - SU(2) Yang-Mills instanton on S^4
$F^i_{(2)}$ - field strength

Flux: $F_{(4)} = f\,(2\Omega_{(4)} + X_{(2)} \wedge Y_{(2)}) + \frac{1}{2} f'\, d\rho \wedge Y_{(3)}$

$$X_{(2)} \equiv \tfrac{1}{2}\epsilon_{ijk}\, \mu^i\, D\mu^j \wedge D\mu^k\,, \qquad Y_{(2)} \equiv \mu^i\, F^i_{(2)} \qquad Y_{(3)} \equiv \epsilon_{ijk}\, \mu^i\, D\mu^j \wedge F^k_{(2)}$$

$$U = 1 - \frac{\ell^4}{r^4}\,, \qquad H = \left(1 + \frac{c^2}{2r^{12}\, U^2}\right)^{1/6} \qquad f = \frac{c}{r^3\, H^{3/2}\, U^{1/2}}$$

M3-brane configurations do not carry any conserved charge or mass (H, f –fall-off too fast) → ``3-branes without 3-branes”

but in the interior: r → ℓ…

M.C., Heckman - work in progress

In the interior r → ℓ; ρ ~ (r – ℓ) → 0

Metric singular: $ds^2_{11} = \rho^{-2}\, dx^\mu \wedge dx_\mu + \frac{1}{2}\rho^{2}\, D\mu^i\, D\mu^i + \rho^{4}\, d\Omega_4^2 + d\rho^2$

co-dim 4-singul.

Transverse internal space locally: **R^3**

Flux: $F_{(4)} = f\,(2\Omega_{(4)} + X_{(2)} \wedge Y_{(2)})$ f-constant

$$X_{(2)} \equiv \frac{1}{2}\epsilon_{ijk}\, \mu^i\, D\mu^j \wedge D\mu^k\,, \qquad Y_{(2)} \equiv \mu^i\, F^i_{(2)}$$

$\mu^i\,\mu^i = 1 \qquad D\mu^i = d\mu^i + \epsilon_{ijk}\, A^j_{(1)}\, \mu^k,$

$A^i_{(1)}$ - SU(2) Yang-Mills instanton on S^4
$F^i_{(2)}$ - field strength

How it is related to appearance of gauge (QFT) degrees.

Further exploration including also deformation of other G_2 holonomy spaces

Motivation for the second part of the talk: study of Hitchin-type system

III. Hitchin-type system in G_2 background

Non-Abelian gauge degrees of M-theory in G_2 background realized on three-manifold M_3, associated with co-dimension 4 ADE singularities, described as Pantev, Wijnholt 0905.1968

c.f., S. Schäfer-Nameki's talk; R. Barbosa's talk

- a partial topological twist of a six-brane wrapped on three-manifold M_3, dictated in the six-brane supersymmetric gauge theory by an adjoint-valued one-form ϕ (parameterizes normal deformations in the local geometry TM_3) and one-form gauge field A.
  - Chiral matter studied by allowing ϕ to vanish at various locations (co-dimension 7 singularities).

Braun, Cizel, Hübner, Schäfer-Nameki 1812.06072

- Extensive analysis further developed and extended to co-dimension 6 singularities (non-chiral matter). Appealing feature: they could possibly connect to building compact G_2 manifolds via twisted connected sums (TCS). Kovalev; Corti, Haskins, Nordström, Pacini
- Most prior analyses of localized matter have assumed one-form ϕ is diagonal and no A in M_3 →

Barbosa, M.C., Heckman, Lawrie, Torres, Zoccarato 1904…

Hitchin-type system generalized to include non-diagonal one-form ϕ & non-zero flux A

Summary

$$\Phi \sim \begin{bmatrix} \lambda & 1 & 0 \\ z & \lambda & 0 \\ 0 & 0 & -2\lambda \end{bmatrix}$$

- ϕ components will not commute & A turned on: refer to this as a ``T-brane type configuration”
  (Naturally fit in the broader scheme of T-brane like phenomena: ``invisible" to the bulk G_2 geometry & characterized by limiting behavior M-theory flux $F_{(4)}$.)
- Local model: three-manifold M_3 as a Riemann surface Σ fibered over an interval I: The gauge theory on Σ is a mild deformation of a Hitchin system on a complex curve Σ.
- As a Hitchin system on Σ describes a local Calabi-Yau threefold geometry → obtain a local deformation of a TCS-like construction → Interpreted as building up a local G_2 background.
- Study resulting localized matter obtained from such T-brane configurations → solving a second order differential equation. [Their existence reduces to a linear algebraic problem: à la localized zero modes of T-branes in F-theory.]

Cecotti, Cheng, Heckman, Vafa 0910.0477
Cecotti, Cordova, Heckman, Vafa 1010.5780

c.f., S. Schäfer-Nameki's talk; R. Barbosa's talk

Building blocks of Hitchin-type system in G_2 background

M-theory in G_2 background: associative three-form ρ naturally pairs with the three-form flux C potential [$dC=F_{(4)}$] of M-theory, along the singular (co-dimension 4) ADE fibers decompose:

$$C = A_\alpha \wedge \omega^\alpha, \quad \rho = \phi_\alpha \wedge \omega^\alpha$$

ω_α - harmonic (1,1) forms on the local ADE singularity

ϕ_α and A_α are one-forms on three-manifold M_3
in the adjoint rep. of 3D gauge theory

Complexified connection: $\mathcal{A} = A + \phi$ w/

$$\mathcal{F}_{ij} = \partial_i \mathcal{A}_j - \partial_j \mathcal{A}_i + [\mathcal{A}_i, \mathcal{A}_j]$$

$$\mathcal{D}_{ij} = \partial_i \overline{\mathcal{A}}_j - \partial_j \mathcal{A}_i + [\mathcal{A}_i, \overline{\mathcal{A}}_j]$$

Eq. preserving supersymmetry
[F- and D-flatness conditions]:

$$\{\mathcal{F} = 0, \quad g^{ij}\mathcal{D}_{ij} = 0\}/G_U^{\text{gauge}}$$

↑ Flat complexified connection

↑ unitary gauge transformations

Three-manifold M_3 chosen as Riemann surface Σ over interval I (t)

$$ds^2_M = g_{tt}dt^2 + g_{ab}dx^a dx^b$$

I Σ

Metric

$$F_\Sigma + [\phi_\Sigma, \phi_\Sigma] = 0$$

$$d_\Sigma \phi_\Sigma = 0$$

Deformed Hitchin system

$$d_\Sigma *_\Sigma \phi_\Sigma = -g^{tt}\nabla_t\phi_t$$

Hitchin system (on CY_3)

Donagi, Diaconescu, Pantev

$$\mathcal{F}_{ta} = \partial_t \mathcal{A}_a - \partial_a \mathcal{A}_t + [\mathcal{A}_t, \mathcal{A}_a] = 0$$

Flow of A_Σ: interpret the flow equations as a gluing construction for local Calabi-Yau threefolds (à la TCS).

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Background solutions in a local patch

Introduce g : M_3 → G_C takes values in complexified gauge group G_C

* $$\mathcal{A} = g^{-1} dg$$ *g* – preserve asymptotics of $\mathcal{A}$

F- flatness D-flatness → fixes g(x)

A special case of infinitesimal h : M_3 → $\mathbf{g_C}$ Lie algebra valued function

$$\mathcal{A} = dh + \ldots.$$ F-flatness

$$g^{ij}\mathcal{D}_{ij} = g^{ij}\partial_i\partial_j(h - h^\dagger) = 0$$ D-flatness

(Im h - Lie algebra valued harmonic map)

Local patch (Σ x R):

z – holomorphic coordinates on Σ

3D background Ansatz:. $\mathcal{A} = \mathcal{A}_z dz + \mathcal{A}_{\bar{z}} d\bar{z} + \mathcal{A}_t dt,$

with a suitable metric on M_3 consistently solve the D-term constraints.

→ an example with an analytic result.

[Obtained also explicitly as a power series in t which is resummed.]

Localized zero modes

Background: $\mathcal{A}^{(o)}$ takes values in subalgebra $k_C \subset g_C$ with commutant subalgebra $h_C \subset g_C$.

$$\mathfrak{g}_{\mathbb{C}} \supset h_{\mathbb{C}} \times k_{\mathbb{C}}$$

$$\mathrm{adj}\,(\mathfrak{g}_{\mathbb{C}}) = \bigoplus_i (\mathcal{T}_i, \mathcal{R}_i)$$

Zero modes ψ fluctuations around $\mathcal{A}^{(o)}$ background

$$\mathcal{A} = \mathcal{A}^{(0)} + \Psi$$

$$\Psi \rightarrow \Psi + d_{\mathcal{A}}\chi$$

For R of k_C - action of $\mathcal{A}_R$ on zero mode ψ

Take a direct approach: expanding around a given background and seek out zero modes in a linearized Hitchin-type system:

$$\partial_i \Psi_j - \partial_j \Psi_i + [\Psi_i, \mathcal{A}_j] + [\mathcal{A}_i, \Psi_j] = 0$$

$$g^{ij}\left(\partial_i \overline{\Psi}_j - \partial_j \Psi_i + \left[\Psi_i, \overline{\mathcal{A}}_j\right] + \left[\mathcal{A}_i, \overline{\Psi}_j\right]\right) = 0$$

Second order differential equations → focus on localization → examples

[Also, work on developing differential and algebraic approach.]

Example:

Local patch Σ x R

Take: $\mathfrak{g}_{\mathbb{C}} \supset h_{\mathbb{C}} \times k_{\mathbb{C}}$

z - local holomorphic coordinate on Σ

SU(3) → SU(2) x U(1)

Ansatz:

$$\phi = \begin{pmatrix} \frac{1}{3}dh & -\bar{z}e^{-f(z,\bar{z})}d\bar{z} & 0 \\ ze^{-f(z,\bar{z})}dz & \frac{1}{3}dh & 0 \\ 0 & 0 & -\frac{2}{3}dh \end{pmatrix},$$

$$A = \frac{i}{2}\left[\partial_{\bar{z}} f(z,\bar{z})d\bar{z} - \partial_z f(z,\bar{z})dz\right] \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Ansatz builds on the Hitchin system with which we would localize a zero mode at a point z=0 of the Riemann surface Σ, and then check it is also localized in t.

Hitchin-type equations:

$$4\partial_z\partial_{\bar z} h + \partial_t^2 h = 0$$
$$\partial_z\partial_{\bar z} f = -|z|^2 e^{-2f}$$

Solution:

$$h = \kappa/8(z+\bar z)^2 - \kappa/2t^2$$
$$f(z,\bar z) = -\log\left[\frac{2i}{1+|z|^4}\right]$$

Zero mode solution:

SU(3) → SU(2) x U(1)

Ψ ~ (2 , 1)

$$\psi = e^{-\kappa/2t^2}\left[\begin{pmatrix} \tau_1(z,\bar z) \\ i\alpha\beta(z,\bar z)\end{pmatrix} dz + \begin{pmatrix} \alpha\beta(z,\bar z) \\ \tau_2(z,\bar z)\end{pmatrix} d\bar z\right]$$

Solved in expansion in κ:

$$\beta(z,\bar z) = e^{-z\bar z} + \mathcal{O}(\kappa^2)\,,$$
$$\tau_1(z,\bar z) = i\alpha\kappa\left[e^{-z\bar z}\frac{1+z^2+z\bar z}{4z^2} - \frac{1}{4z^2}\right] + \mathcal{O}(\kappa^2)$$
$$\tau_2(z,\bar z) = \alpha\kappa\left[e^{-z\bar z}\frac{1+\bar z^2+z\bar z}{4\bar z^2} - \frac{1}{4\bar z^2}\right] + \mathcal{O}(\kappa^2)\,.$$

Square normalizable mode, localized at $z=\bar z=t=0$!

Summary

M theory on G_2 holonomy backgrounds & fluxes

- 11D supergravity: M3-branes with co-dimension four singularity and constant $F_{(4)}$ flux there
  → relevance for studying gauge degrees

- Gauge degrees: Hitchin-type system with flux: constructed T-brane type configurations

The local gauge degree description in G_2 backgrounds can be understood as a deformation of Calabi-Yau threefolds fibered over an interval, captured by a gradient flow equation in a deformation of a Hitchin-like system on a Riemann surface.

An explicit constructions of localized zero modes.

Outlook

- Further exploration of gauge degrees from 11D supergravity perspective, possibly relating it to Hitchin-type systems.

- Gauge degrees: fibering a 2D gauge theory over an interval produce a 3D gauge theory with moduli space matching onto that of G_2 background.

  Proposed extension: take these 3D gauge theories fibered over another interval, thereby producing solutions to 4D gauge theories, which we expect to build up local Spin(7) backgrounds given by a four-manifold of ADE singularities.

  Physical applications of the results: Interpretation of these 3D N=1 backgrounds as N=1 domain walls in one dimension higher.

- Ultimate goal: to embed these local geometries into a globally defined G_2 backgrounds with chiral matter.

Thank you!