Cosmological Questions for G2 Compactifications at Large N

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Simons G2 Meeting @ KITP

influenced heavily: by collaborations with C. Long, D. Morrison, B. Nelson, F. Ruehle, G. Salinas, B. Sung, and W. Taylor.

Today: Lessons at Large N

- N ~ topological complexity (cycles, branes, fluxes, etc)

- **Lessons thus far:**

  - at large N in CY, many axions, large gauge sectors
  - most of known landscape lives at large N
    (e.g. flux vacua, F-theory bases)

- **Selection:** if cosmology picks generic bases or flux vacua, will find large gauge sectors and many axions.

- **Big caveat:** could be sharply peaked away from large N, would mean that these considerations don't apply.

  e.g. complexity measure, a la proteins and spin glasses.
  [Denef, Douglas, Greene, Zukowski]

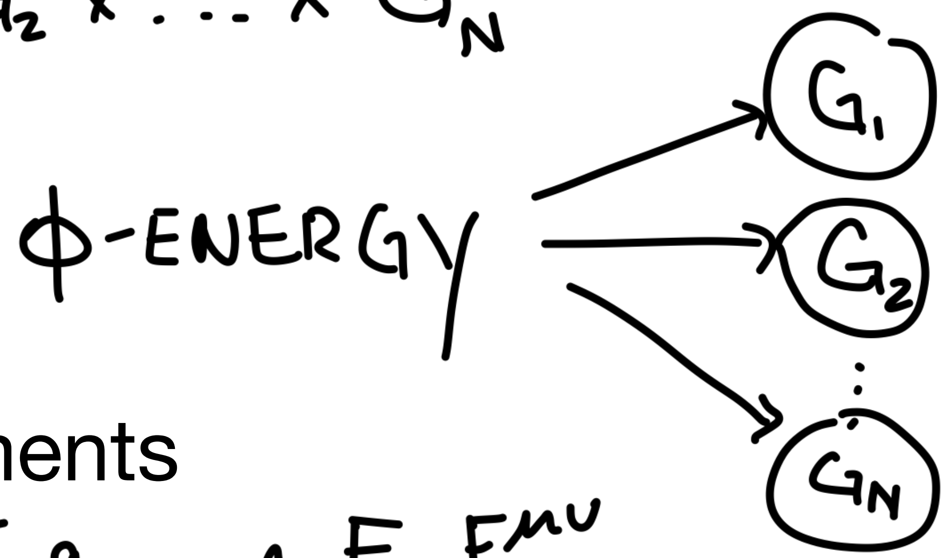
- Do lessons apply in G2 at large N = large b3?

Outline

- Large Gauge Sectors

$$G = G_1 \times G_2 \times \ldots \times G_N$$

- Axions and Asymmetric Reheating



- Axion - Photon Couplings and Experiments

$$\mathcal{L} = \sum_i g_{a_i \gamma\gamma}\, a_i F_{\mu\nu} F^{\mu\nu}$$

Some goals for each:

1) explain origin and relevance of large N
2) give G2 context and questions
3) convey why physically relevant

Large Gauge Sectors

section outline:

1. **F-theory**
2. **Non-Higgsable Clusters**
3. **NHC in ensemble**
4. **Tree Ensemble**
5. **Combinatoric Picture**
6. **NHC in TCS G2**
7. **Questions for NHC in G2**
8. **Questions for Cosmology**

F-theory

- Strongly coupled regime generalizing IIB
- Geometry $\mathbb{R}^{3,1} \times Y_4$ $\quad Y_4 \xrightarrow{\pi} B_3$ ell. fib.

WEIERSTRASS FORM

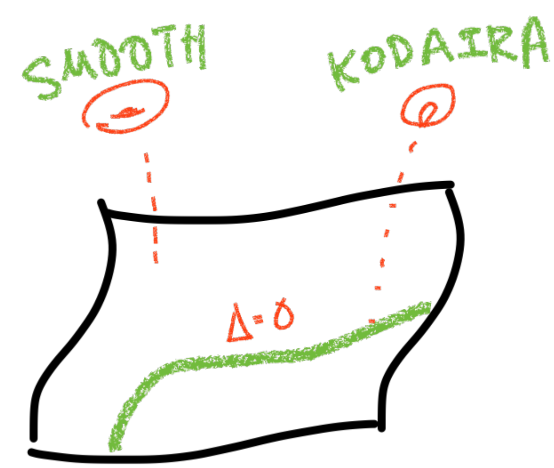
$$y^2 = x^3 + f x + g$$

$$f \in \Gamma(-4K_B) \quad g \in \Gamma(-6K_B)$$

- Physics from geometry

DISCRIMINANT LOC

$$\Delta = 4f^3 + 27g^2 = 0 \implies$$



- G4-flux for chirality and moduli stabilization

$$G_4 + \tfrac{1}{2} c_2 \in H_4(Y_4, \mathbb{Z}) \qquad n_{D3} = \frac{\chi(Y_4)}{24} + \frac{1}{2}\int_X G_4 \wedge G_4 \in \mathbb{Z}_{\geq 0}$$

Non-Higgsable Clusters

Some selective progress: Anderson, Braun, del Zotto, Halverson, Heckman, Grassi, Morrison, Schafer-Nameki, Shaneson, Taylor, Vafa, Wang
esp. [Morrison, Taylor]

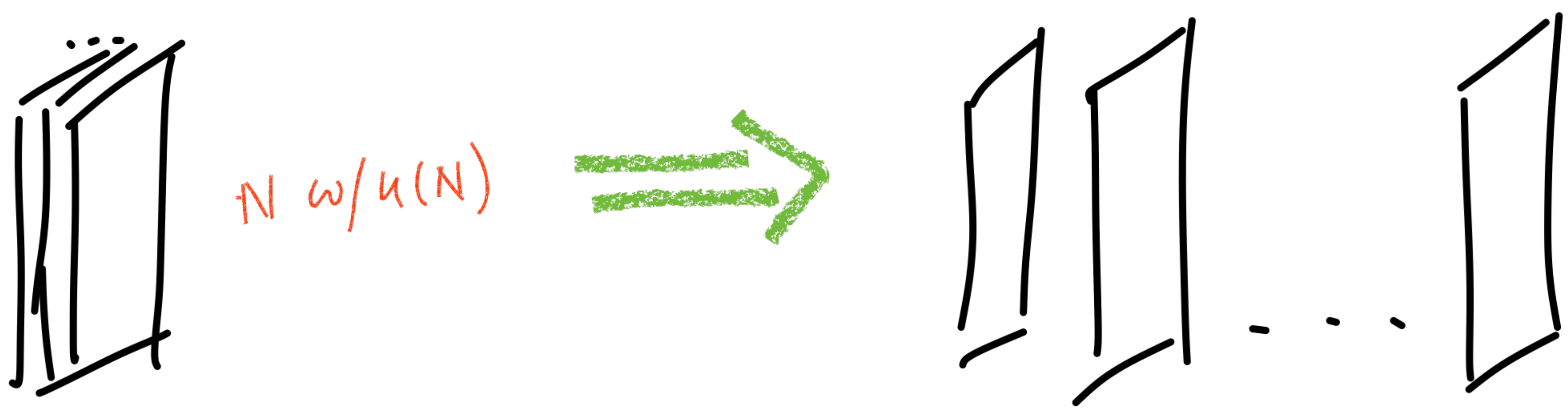
- Math Story: f and g factorize for generic CS.

$$f = z^a F \qquad F \in \Gamma(-4K_B \otimes \mathcal{O}(-aZ))$$

$$g = z^b G \qquad G \in \Gamma(-6K_B \otimes \mathcal{O}(-bZ))$$

$$\Delta = z^c \delta \qquad \delta \in \Gamma(-12K_B \otimes \mathcal{O}(-cZ)) \qquad c = \min(3a, 2b)$$

- Physics story: no CS deformation that splits branes (Higgses)



- 6d story: no Higgs b/c SYM or not enough matter. 4d: W

- Facts: $G = \prod_i G_i \qquad G_i \in \{E_8, E_7, E_6, F_4, D_4, B_3, G_2, A_2, A_1\}$

NHC in Ensemble

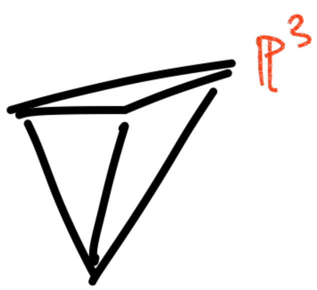
- Rule of thumb: NHC when B not weak Fano

WEAK FANO: $-K_B \cdot C \geq 0 \quad \forall C$

- Toric weak Fanos:

2D: 

.... & 12 MORE

3D: 

... & 4318 MORE REFL POLY

FRST ⟹ $O(10^{15})$ B_3

- Toric not weak Fanos: [2D torics, see Morrison, Taylor]

3D:

"SKELETON" ENSEMBLE: $O(10^{3000})$ ESTIMATE FROM $\mathbb{P}^3$ BLW-UPS

[Taylor, Wang]

"TREE" ENSEMBLE: $\frac{4}{3} \times 2.96 \times 10^{755}$ BLW-UPS

[J.H., Long, Sung]

PHYSICS $N_{ax} = 1883 \pm 29$ $\quad rk(G) = 1609 \pm 17$ $\quad$ #FAC $= 762 \pm 11$

Tree Ensemble

[J.H., Long, Sung]

- **4D F-theory:** 3-fld B, gauge structure det'd by B topology, called "non-Higgsable cluster."
- **Starting point:** B a weak Fano toric threefold, encoded in a certain triangulation (fine regular star) of a 3d reflexive polytope.
- **Topological transitions:** systematically perform sequences of toric blowups along toric curves and points.
- **Sequences are bounded:** if all singularities are canonical, geom. that transition is done from is at finite distance in CS moduli space.

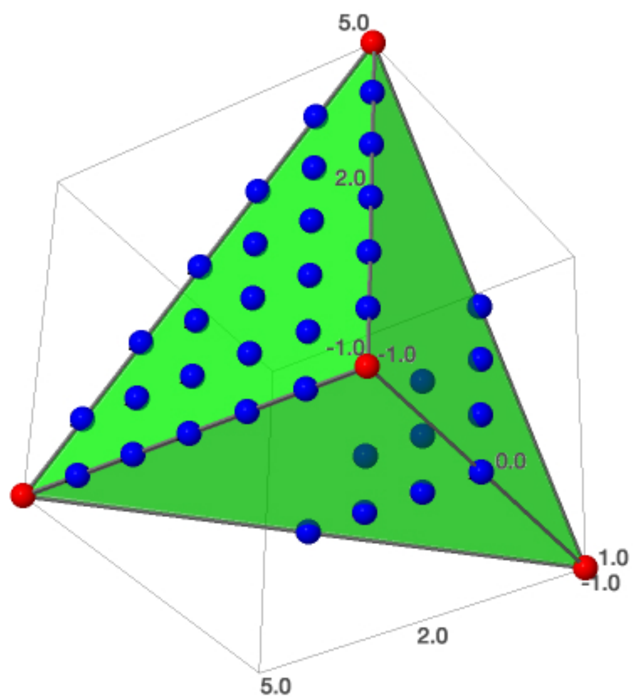
  Alg. Geom: [Hayakawa] [Wang] **in F-theory: [Morrison]**

- **Classification:** there are 82 (41,873,645) sequences over curves (points) that satisfy a sufficient condition for canonical singularities.
- **Ensemble:** all ways of performing these sequences of blowups. from an initial, fixed, triangulated polytope.

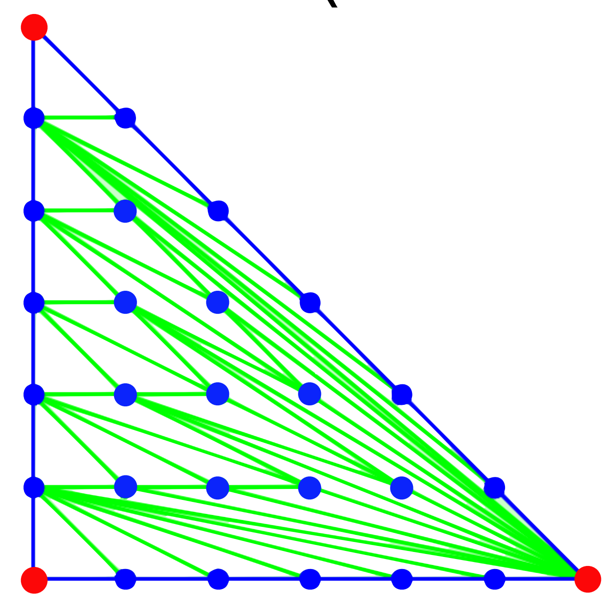
The Combinatoric Picture

[J.H., Long, Sung]

- **Polytope:**

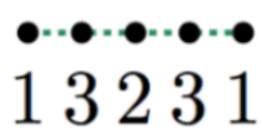


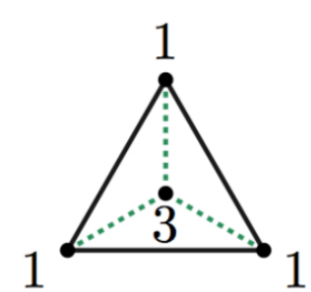
Triangulation: (codim 1 faces)

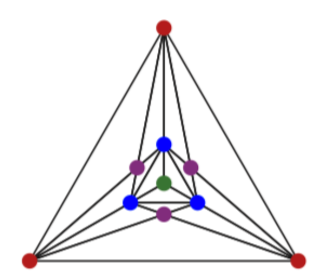


Fact: any FRS triangulation of this has 108 edges, 72 faces.

- **Rep seq. of blowups:** (topological transitions, project into board)







- **Ensemble Size:** (put the widgets on the triangulation)

$$82^{108} \times 41873645^{72} = 2.96 \times 10^{755}$$

NHC in TCS G2

[Braun, Schafer-Nameki]

TCS Construction:

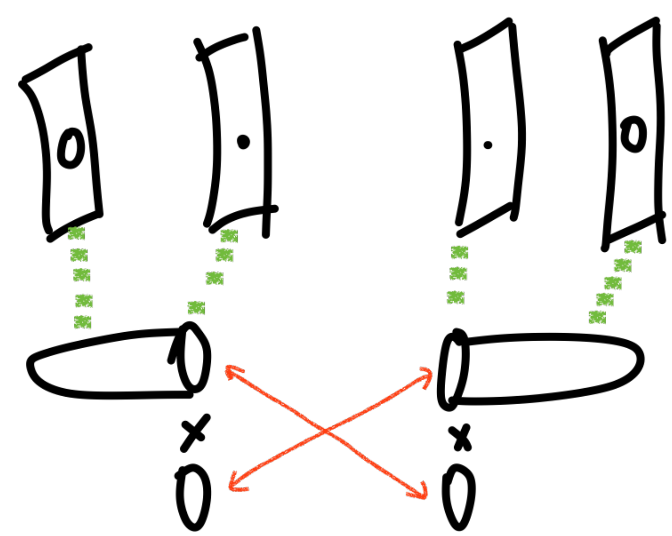
- K3-fibered building blocks
- Asymptotically cylindrical CY3
- Glued together at infinity by particular Hyper Kahler rotation, matching condition.

$$N_+ \cap N_- = G .$$

TCS NHC:

- build block K3 fiber with lattice of resolved ADE.
- those cycles forced to vanish by matching.

(i.e. building block Kahler form forced to integrate to 0 on curves of ADE resolution)

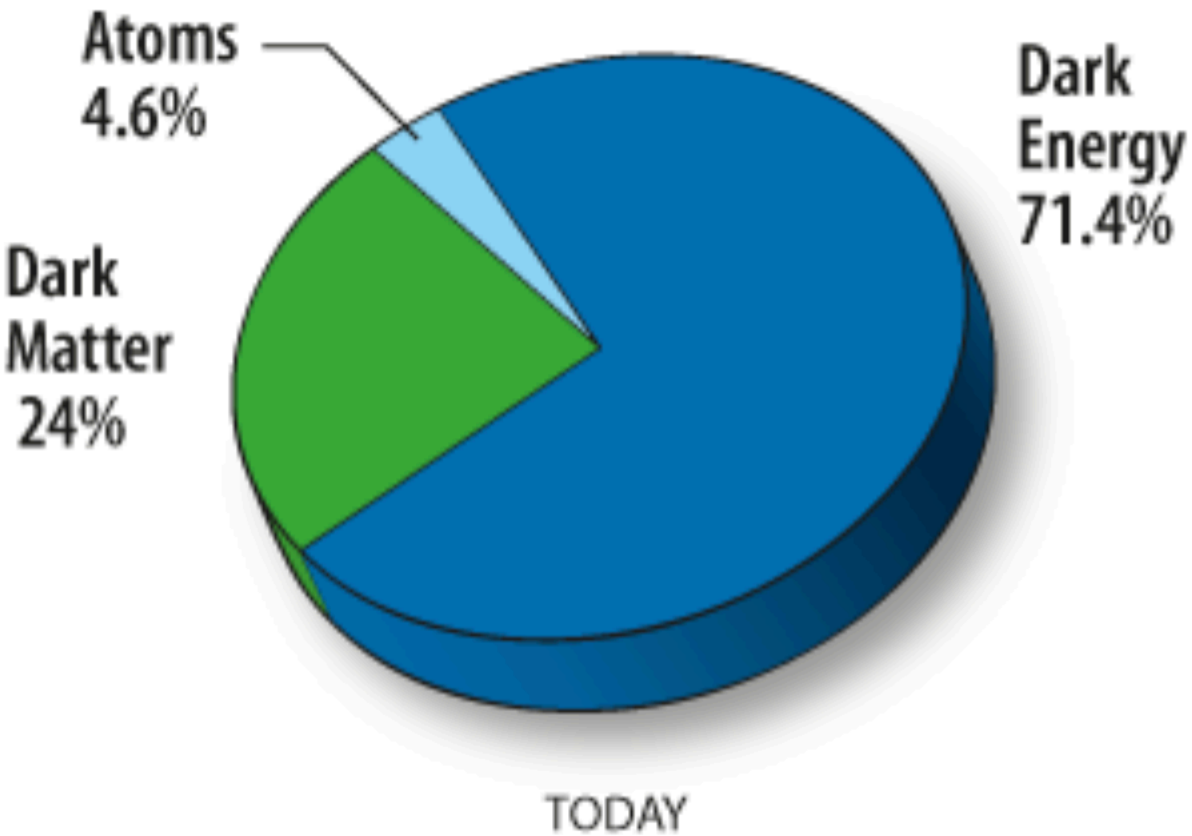


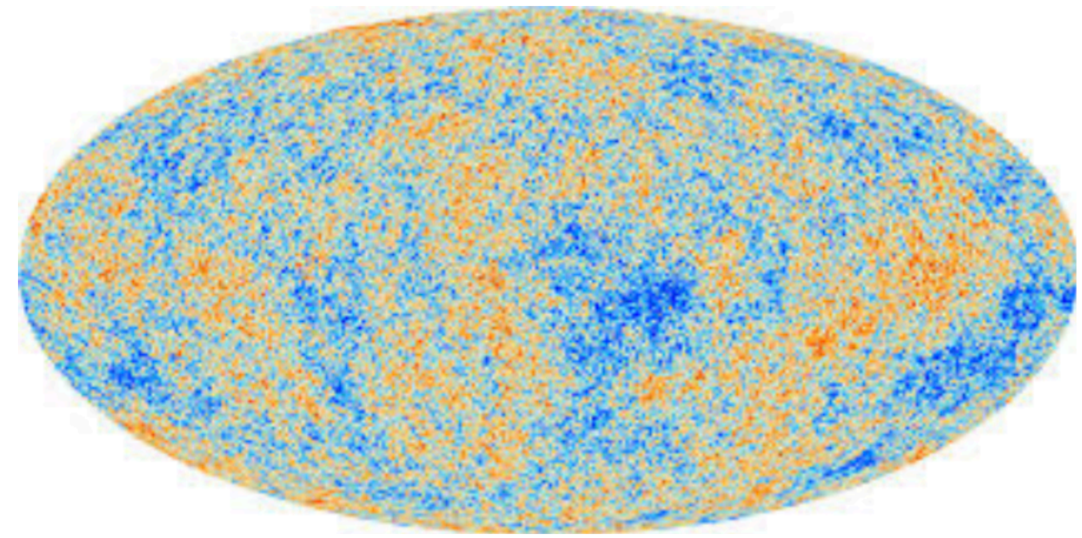
Questions for NHC in G2

- NHC in G2:   Singular with no smoothing G2 deformation.
- What can be said about these? Anything beyond TCS?
- Any reason to expect they are common at large N = large b3? (analog to CY)
- Do they emerge as we do more and more topological transitions to larger b3? (analog to CY)
- If yes, what is the physics of those NHC-yielding transitions?

6d analogs as in e.g. [Heckman, Morrison, Vafa]

Questions for Cosmology





Planck

- Cosmology: precise science about evolution of the universe that has important open questions, and also gives constraints.

- Many gauge sectors —> dark matter!?

- Can motivate EFTs that work, but can also lead to problem: too much DM, that becomes worse at large N = large number of gauge sectors.

[JH, Nelson, Ruehle] [Acharya, Fairbairn, Hardy]

Axions and Reheating

section outline:

1. **Origins of axions**
2. **in CY and G2**
3. **Axion Cosmology**
4. **Reheating Issues**
5. **Axion Reheating in F**
6. **Reheating Q’s for G2**
7. **Ultralight @ large N**

Origins of Axions

- Theories with extra dimensions and higher form gauge fields
- Examples: supergravity, and therefore string / M

CONTEXT $\mathbb{R}^{3,1} \times X$ $\quad C_p = \theta_I \wedge \omega_I \quad \omega_I \in H^p(X)$

ORIGIN KK REDUCTION $\Rightarrow$ 4D FIELDS θ_I

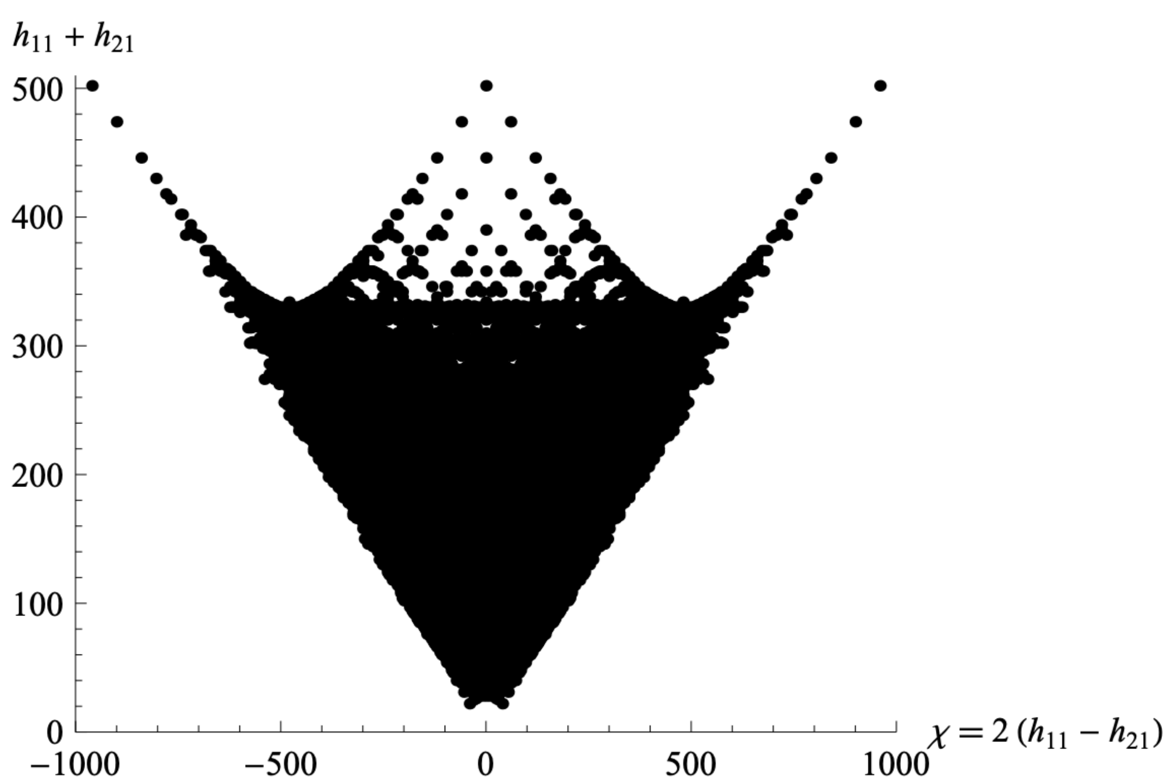
SYMMETRY $C_p \mapsto C_p + d\Lambda_{p-1} \Rightarrow \theta$ shift symm

- One for each p-cycle in theory with p-form
- Topological complexity ~ num axions

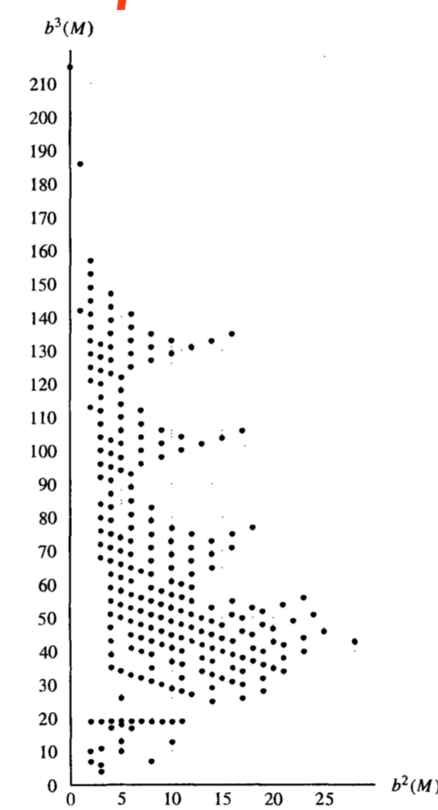
in CY and G2

- E.g. IIB on CY3 $\int_{\Sigma_4} C_4, \int_{\Sigma_2} C_2, \int_{\Sigma_2} B_2 \Longrightarrow$ 3 TYPES
- M on G2 $\int_{\Sigma_3} C_3 \Longrightarrow \int_{\Sigma_3} \Phi + i C_3$ (complexified G_2 modulus)
- Ensemble examples:

KREUZER-SKARKE



JOYCE MANIFOLDS



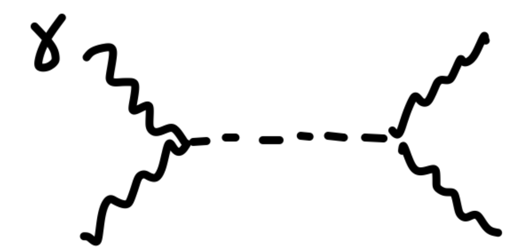
- Tree ensemble $\int_{\Sigma_4} C_4 \Longrightarrow$ # axions ~ 1800

Axion Cosmology

laundry list of ways they can be relevant

[see review by Marsh]

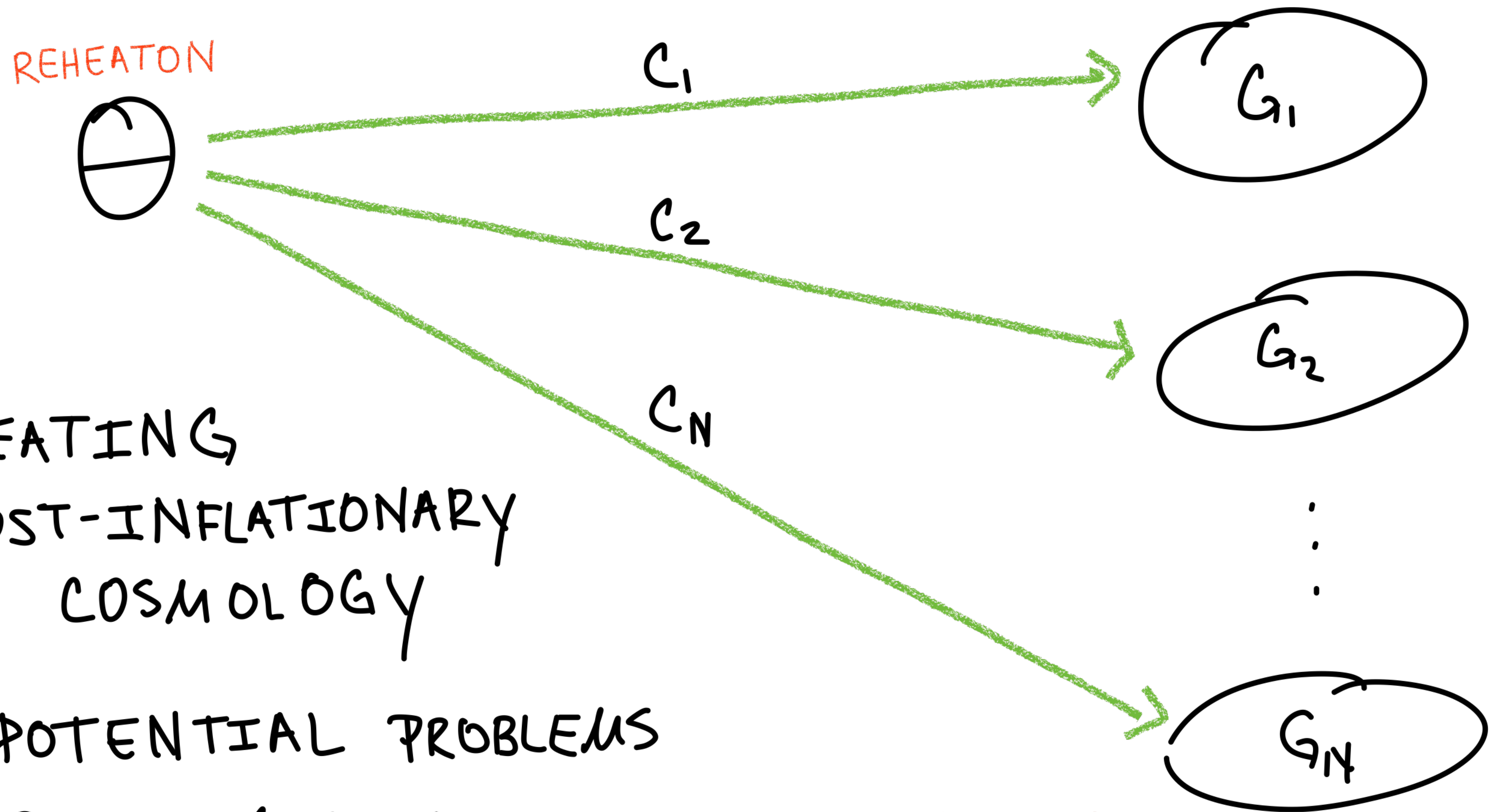
- **Inflation:** e.g. axion inflation of axion monodromy.
- **Reheating:** field that reheats not necessarily inflaton.
- **Dark matter:** QCD axion or general ALP (e.g. FDM)
- **γ couplings:** light-by-light scattering



- **Dark radiation:** relativistic DOF in universe

a single string compactification could realize many.
idea: “axiverse” with many axions realizing many possibilities.
[Arvanitaki et al.]

Reheating Issues



1) REHEATING ⟹ POST-INFLATIONARY COSMOLOGY

2) ∃ POTENTIAL PROBLEMS @ LARGE N

3) CAN BE SOLVED BY <u>ASYMMETRIC</u> <u>REHEATING</u>

coupling distributions?

G_2 origin?

$$\sum_i c_i \Theta G_{\mu\nu i} G^{\mu\nu i}$$

F-theory and Axion EFT

- F data: roughly, topological intersections, overall volume divisor volumes, curve volumes.

$$\mathcal{V} = \frac{1}{6}\int_B J \wedge J \wedge J = \frac{1}{6}\kappa^{ijk} t_i t_j t_k \qquad K = -2\log(\mathcal{V})$$

$$\mathcal{L} = -\frac{1}{2}K_{ij}(\partial^\mu \theta^i)(\partial_\mu \theta^j) - V(\theta) - \sum_\alpha Q_i^\alpha \left(\tau^i F_\alpha^{\mu\nu} F_{\alpha\mu\nu} + \theta^i \tilde{F}_\alpha^{\mu\nu} F_{\alpha\mu\nu}\right)$$

$$x^{\alpha\beta} \equiv \frac{\mathrm{vol}(D_\alpha \cap D_\beta)}{\mathrm{vol}(D_\alpha) \times \mathrm{vol}(D_\beta)} \qquad c^\alpha = \frac{-\mathcal{V}x^{\check{\alpha}\alpha} + 1}{2\sqrt{-\mathcal{V}x^{\check{\alpha}\check{\alpha}} + 1}}$$

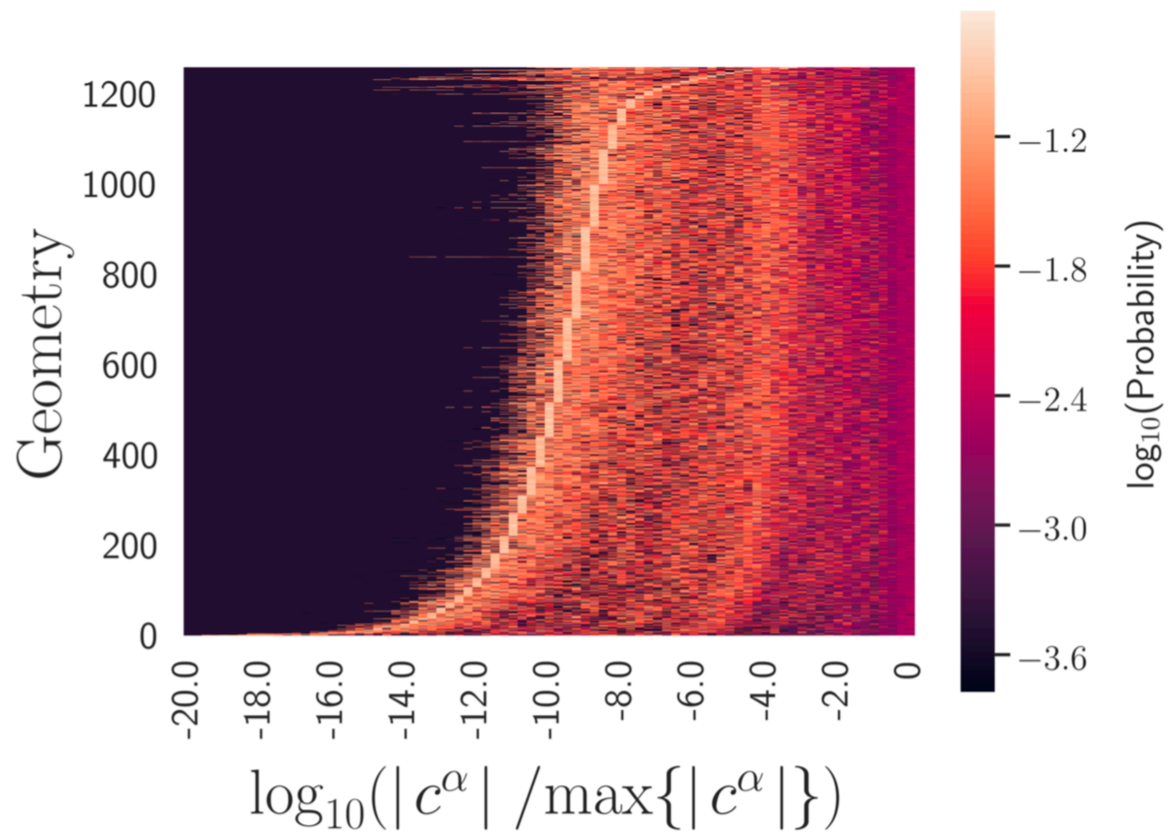
- Effective Lag:

$$\mathcal{L} = -\frac{1}{2}\delta_{ij}(\partial^\mu \phi^i)(\partial_\mu \phi^j) - V(\phi) - \frac{1}{4}\sum_\alpha G_\alpha^{\mu\nu} G_{\alpha\mu\nu} - \sum_\alpha c_i^\alpha \phi^i \tilde{G}_\alpha^{\mu\nu} G_{\alpha\mu\nu}$$

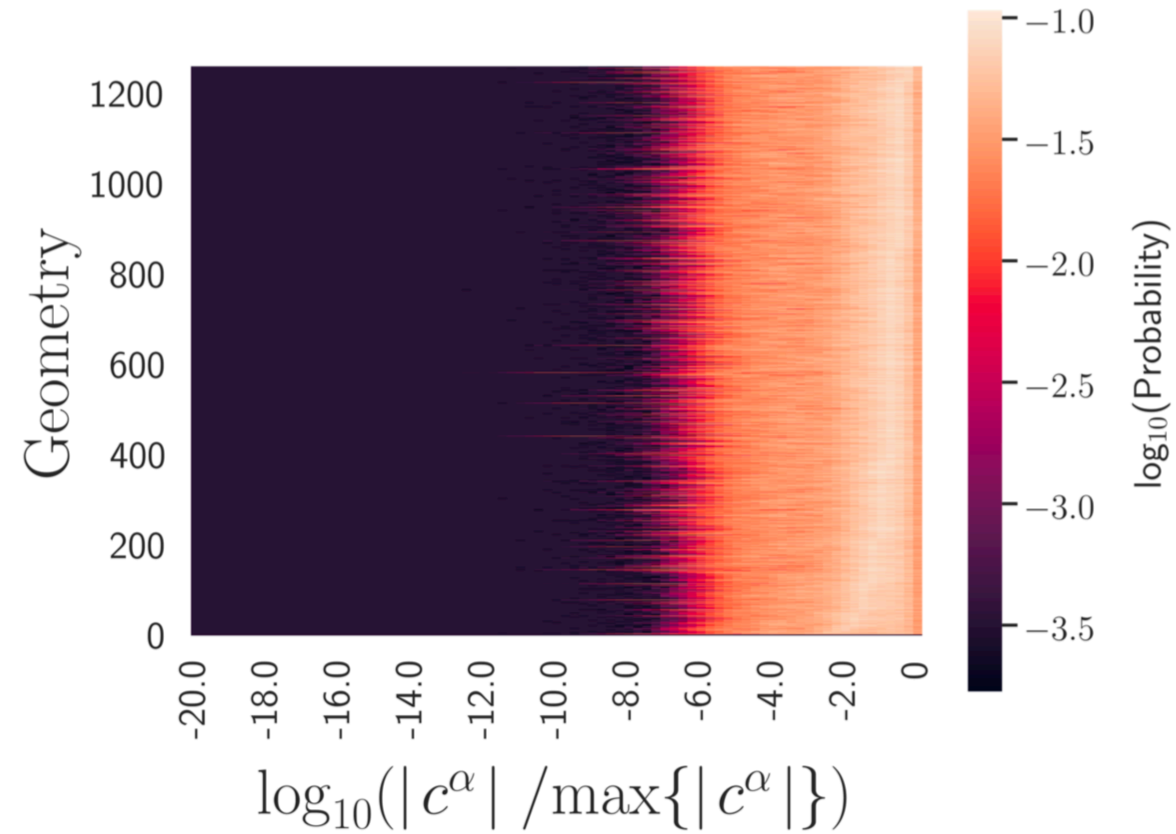
$$-\mathcal{L}_{\mathrm{int}} = c^{\check{\alpha}} \check{\phi} \tilde{G}_{\check{\alpha}}^{\mu\nu} G_{\check{\alpha}\mu\nu} + \sum_{\alpha \neq \check{\alpha}} (c^\alpha \check{\phi} + \ldots) \tilde{G}_\alpha^{\mu\nu} G_{\alpha\mu\nu}$$

Asymmetric Axion Reheating in F

[JH, Long, Nelson, Salinas]



Gauge oriented



Random LC of Gauge-Oriented

- **Setup:** tree ensemble, sample geometries, sample stretched Kahler cone, compute topological axion couplings, canonically normalize.

- **Upshot:** perturbative axion reheating is asymmetric.
  LHS: O(1) coupling to ~ 1% of gauge sectors
  RHS: O(1) coupling to ~ 10% of gauge sectors

also in [Adshead, Heckman, Melchor Watson]?
(though different context)

Q's for G2 Axion Couplings

- G2 data of interest:

  1) Kahler potential: $K = -3 \log\left[\frac{1}{2\pi^2} \frac{1}{7} \int \Phi \wedge * \Phi\right]$

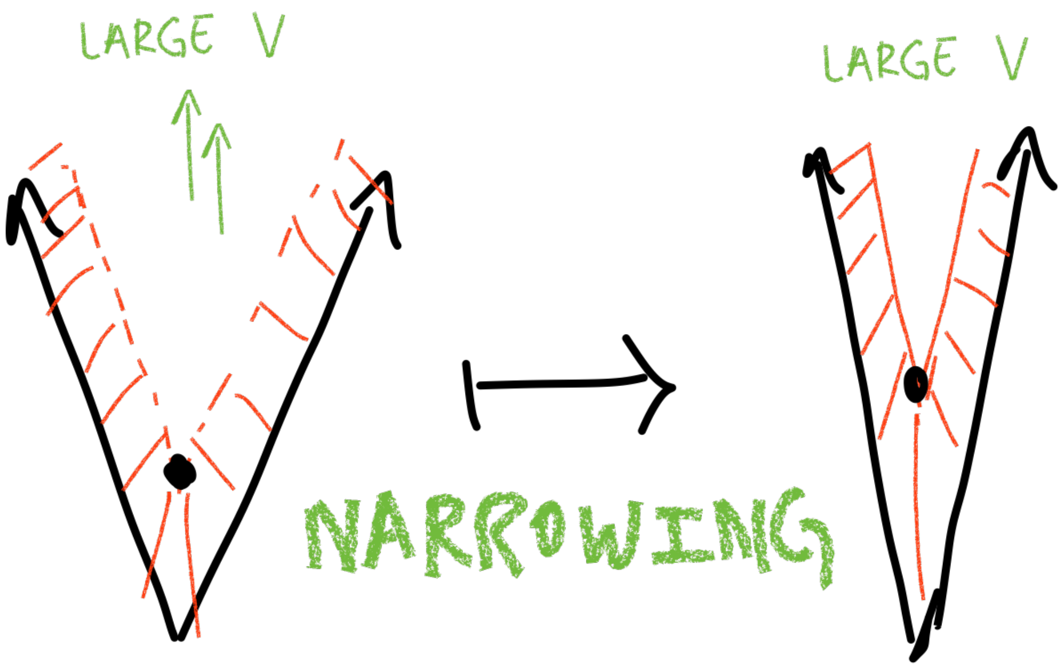
  2) Geometric coupling to gauge sectors: $\int_{M_3} \Phi + i C_3$

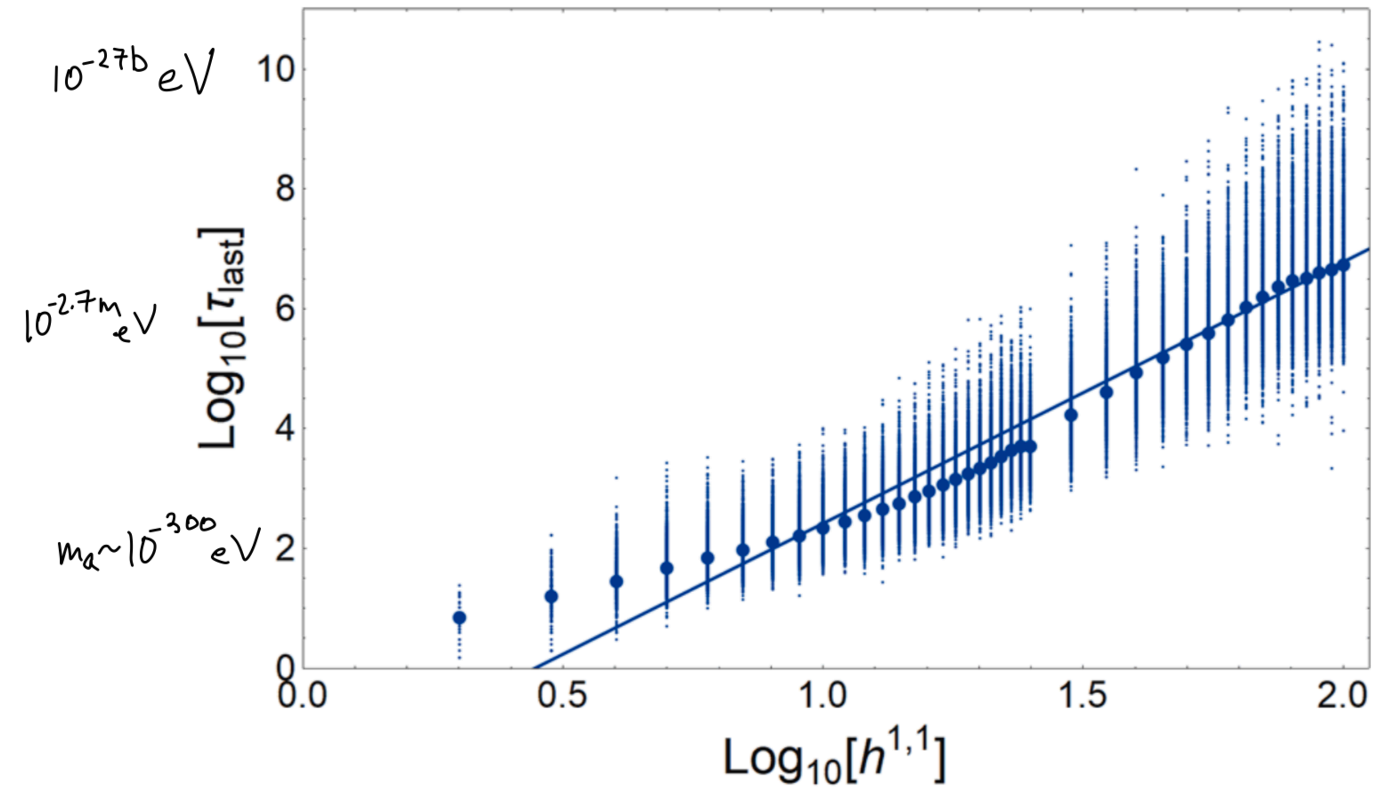
  3) Sampling in G2 moduli space. $M_3 \in H_3(X)$

- Presumably (as usual) it's 2) that's hardest? Seems like major obstruction for now.

Ultralight Axions at Large N

[Demirtas, Long, McAllister, Stillman]





$$m_{\text{last}}^2 \sim e^{-2\pi\tau_{\text{last}}}$$

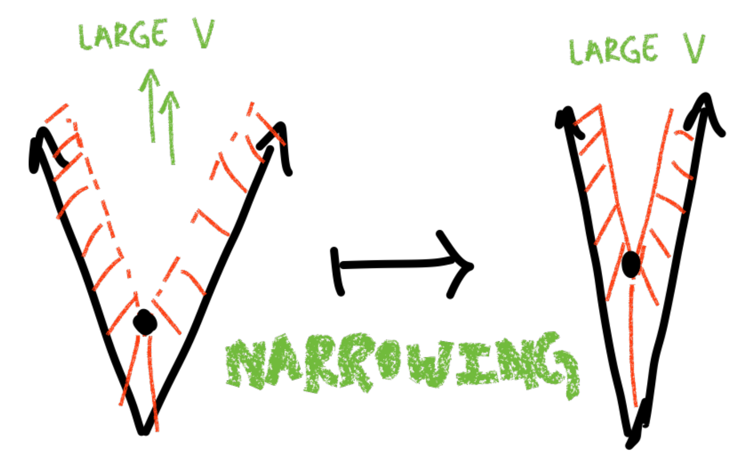
- Stretched Kahler cone:

$$\tilde{K}(X) := \{ J \in H^{1,1}(X) \mid \text{vol}_J(C) \geq 1 \}$$

- Key: ignore WS instantons requires stretched Kahler.
  (needed for KKLT, LVS)

- Physics: ultralight axions. << Hubble today.

G2 Ultralight Axions?



- Stretched Kahler cone: $X = CY$

$$\tilde{K}(X) := \{ J \in H^{1,1}(X) \mid \mathrm{vol}(C) \geq 1 \}$$

- “Stretched G2 Cone”: $X = G_2$

$$\tilde{G}(X) := \{ \Phi \in H^3(X, \mathbb{R}) \mid \mathrm{vol}_{\Phi}(\Sigma) \geq 1 \}$$

- Does this object become very narrow at large N = large b3?
- If so, M2-instanton contributions to axion mass ultralight.

comparison to RMT approach of
[Stott, Marsh, Pongitivanickul, Price, Acharya]?

Conclusions

(wishlists and Q's on next slide)

- Large gauge sectors and # of axions expected in CY.
- Consequences for real-world physics. (constraints + effects).
- In G2 ensembles at large N = large b3, expect similar?

Questions for NHC in G2

- NHC in G2: Singular with no smoothing G2 deformation.
- What can be said about these? Anything beyond TCS?
- Any reason to expect they are common at large N = large b3? (analog to CY)
- Do they emerge as we do more and more topological transitions to larger b3? (analog to CY)
- If yes, what is the physics of those NHC-yielding transitions? (analog of 6d tensor branch transitions)

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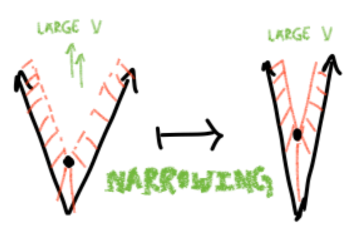

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ALP-photon G2 wishlist

- G2 data similar to before, but extra ingredient crucial:

$$W \simeq \sum_{\Sigma_i \in H_3(X,\mathbb{Z})} A_i\, e^{-\int_{\Sigma_i} \Phi + i C_3}$$

[Harvey, Moore]

G2 dual of infinite # contributions:
[Braun, del Zotto, JH, Larfors, Morrison, Schafer-Nameki]
[Acharya, Braun, Svanes, Valandro]

Requires control over associatives of appropriate type.

- Also: photon can have many origins in G2, e.g. from D5 or A4 ADE singularity and codim 7 singularities.

  Or from A1 singularity + three-cycle of Harmonic two-form.

  or . . . $\sigma \in H^2(X)$ $\quad \Sigma_3 = PD(\sigma \cup \sigma) \in H_3(X)$