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Heterotic Duals of M-Theory on Joyce Orbifolds

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Overview

- Want to understand M-theory and its compactifications on G2 spaces
- Tool: If the G2 space admits a coassociative K3 fibration, expect a dual heterotic gauge bundle over SYZ fibered CY3
- Goal: An algorithm to produce the geometry and gauge bundles of these heterotic duals
    - Braun and Schafer-Nameki did this for TCS G2s with elliptic K3 fibers
    - What about for Joyce orbifolds without elliptic data?

Plan

1. Review of M-theory and the E8 heterotic string
2. M-Theory / Heterotic Duality
   - Relevant limits in moduli space
   - Duality in 7D
   - Duality in 4D
3. Heterotic duals of Joyce orbifolds
   - Orbifold with an M-theory background
   - Dual heterotic geometry
   - Constraints on dual heterotic bundle

M-Theory

- At low energies, M-theory is effectively described by 11D supergravity + effects of M2-branes & M5-branes
- 11D supergravity has three fields:

  Bosons: 3-form C , metric g

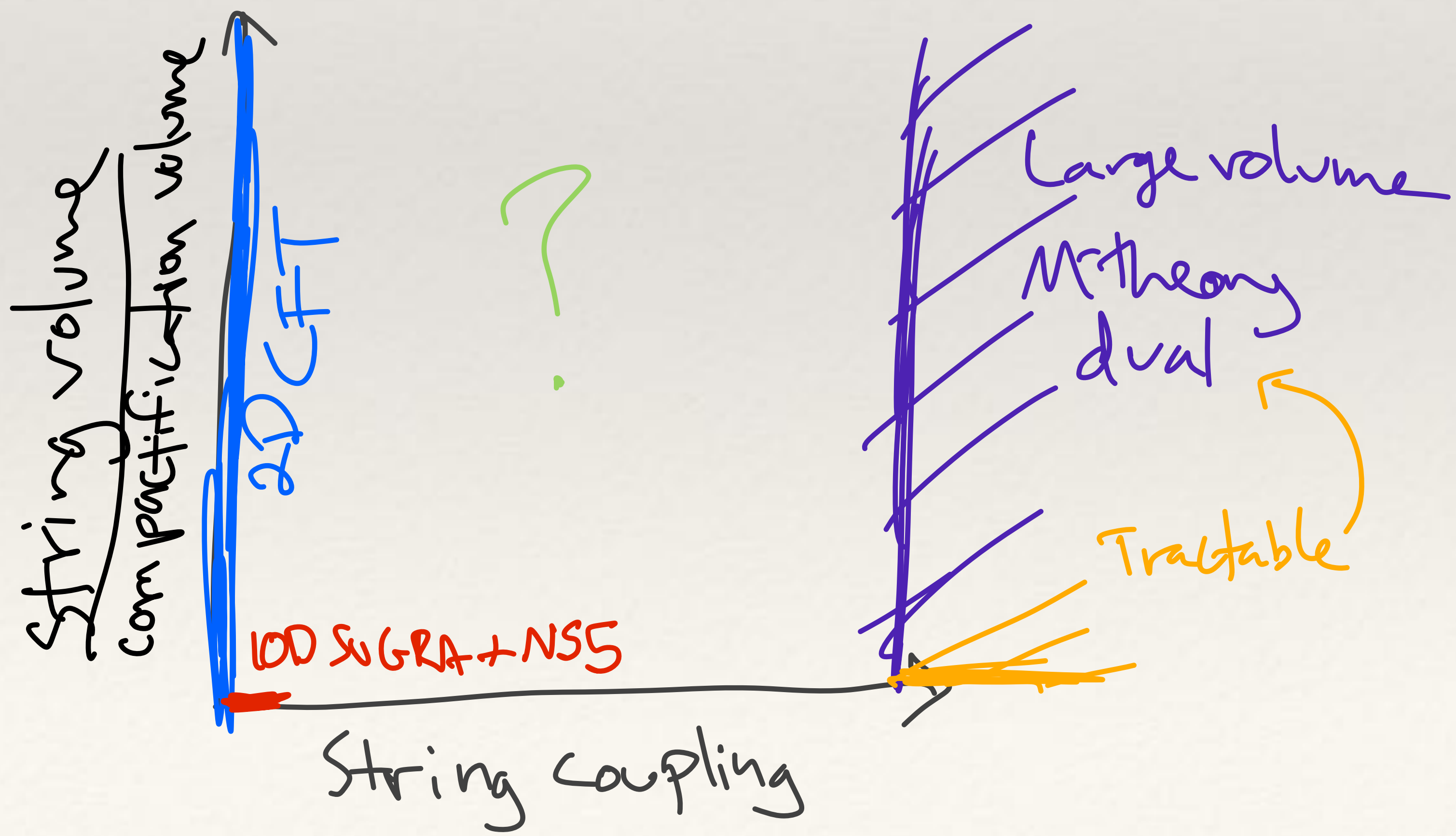
  Fermions: gravitino ψ
- To specify a low energy M-theory background, we need to select a configuration for each of these fields that solves the equations of motion and specify an M-brane background
- More specifically, we restrict to solutions that are
  - Bosonic: Fermion backgrounds vanish
  - Supersymmetric: SUSY variations of configurations vanishes

4D Effective Theory

- If we take our background geometry to be a metric product $Y^7 \times \mathbb{R}^{3,1}$ where Y^7 has small volume, then we get an "effective" 4D theory on $\mathbb{R}^{3,1}$
- We decouple gravity and study the gauge sector only
- Abelian gauge symmetry comes from C-field, and this is enhanced to non-abelian by M2-branes wrapped on orbifold loci

The E8 Heterotic String

- Perturbatively in the string coupling, we can understand the theory as a 2D CFT
- At strong string coupling, our best description for the E8 string is via a dual M-theory description



Heterotic Effective Theory

- For large compactification volumes, we may regard the heterotic string as 10D heterotic SUGRA + NS5-branes
- The bosonic fields are
  - Dilaton (scalar)
  - Metric
  - B-field (locally a 2-form field, globally connection on gerbe?)
  - Gauge field (connection on heterotic bundle)
- Again, compactification on a metric product $X^6 \times \mathbb{R}^{3,1}$ where X^6 is at small volume lets us approximate with a 4D gauge theory

Heterotic-M Duality

Limits in the 7D Moduli Space

- In regions of the 7D string/M moduli space with maximal unbroken SUSY, we expect dual descriptions by M-theory and the heterotic string

$$[SO(3,19;\mathbb{Z})\backslash SO(3,19;\mathbb{R})/SO(3)\times SO(19)]\times\mathbb{R}^{+}$$

- There are three limits that we impose:

	M perspective	Het perspective
1.	Orbifold limit	Non-generic flat connection
2.	Small K3 volume	Weak string coupling
3.	Half-K3 limit	Large T^3 volume

Limit 1: Orbifold

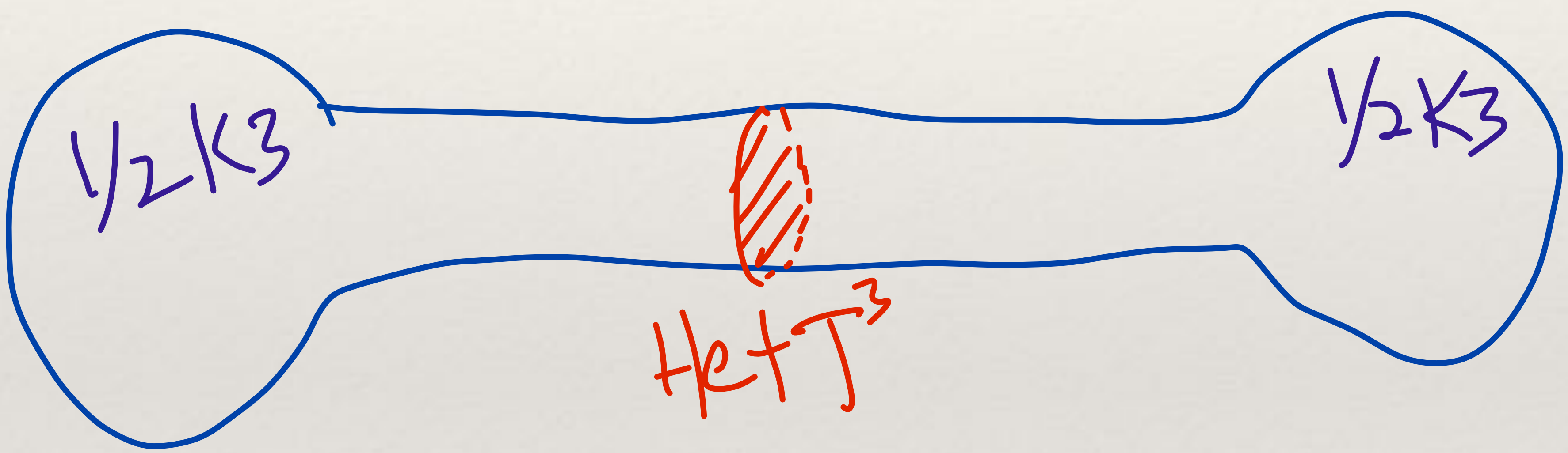
- This is the limit that is required so that we have non-abelian gauge symmetry in the effective 7D theory
- M theory perspective is geometric: K3 orbifold
- Heterotic perspective is gauge theoretic: non-generic holonomies of a flat connection

Limit 2: Small string coupling

- We want this limit so that we may treat the heterotic string semiclassically in the string coupling
- In the effective theory, this translates to working semiclassically in the Yang-Mills coupling
- M-theory perspective: small K3 volume

Limit 3: Large Heterotic Volume

- Want to treat the heterotic string as 10D SUGRA + NS5-branes
- M-theory perspective: half-K3 limit



- Heterotic T3 is the space transverse to the throat
- Analogous to stable degeneration limit in het/F
- Geometry of half-K3 determines an E_8 bundle on T^3

7D Duality

- In the limit we have described, we expect dual descriptions by M and heterotic compactifications

- Example: M-Theory on $T^4/\mathbb{Z}_2$ with flat C-field

- Dual: Heterotic on T^3 with flat $E_8 \times E_8$ connection with holonomies generating $H < E_8 \times E_8$ such that $Z(H) = \mathrm{SU}(2)^{16}$

4D Duality

- SYZ conjecture: CY3 with mirror manifolds admit special Lagrangian T^3 fibrations
- Apply 7D duality fiberwise to a coassociative K3 fibration of a G_2 space. Supersymmetry suggests we will obtain an SYZ-fibered CY3 with a heterotic gauge bundle.
- Requires adiabatic limit: fiber geometry varies slowly compared to base
- This is violated at singular fibers, which are necessarily present

4D Limits

- 1: Orbifold limit —> Codimension 4 singular locus
- 2: Small string coupling —> $\frac{\text{Vol(fiber)}}{\text{Vol(base)}} \to 0$
- 3: Large het volume —> half-K3 limit on each fiber
- 4: Adiabatic limit

Duality for a Joyce Orbifold

Our Example: A Joyce Orbifold

- We want to understand this duality for the particular example of a Joyce orbifold $Y = T^7/\mathbb{Z}_2^3$, where the group is generated by:

$$\alpha : (x_1, x_2, x_3, x_4, x_5, x_6, x_7) \mapsto (-x_1, -x_2, -x_3, -x_4, x_5, x_6, x_7)$$
$$\beta : (x_1, x_2, x_3, x_4, x_5, x_6, x_7) \mapsto (-x_1, \frac{1}{2} - x_2, x_3, x_4, -x_5, -x_6, x_7)$$
$$\gamma : (x_1, x_2, x_3, x_4, x_5, x_6, x_7) \mapsto (\frac{1}{2} - x_1, x_2, \frac{1}{2} - x_3, x_4, -x_5, x_6, -x_7)$$

- This example has 12 disjoint T^3 loci of A_1 orbifold singularities

- Invariant harmonic forms:

$$b^2_G(Y) = 0 \qquad\qquad b^3_G(Y) = 7$$

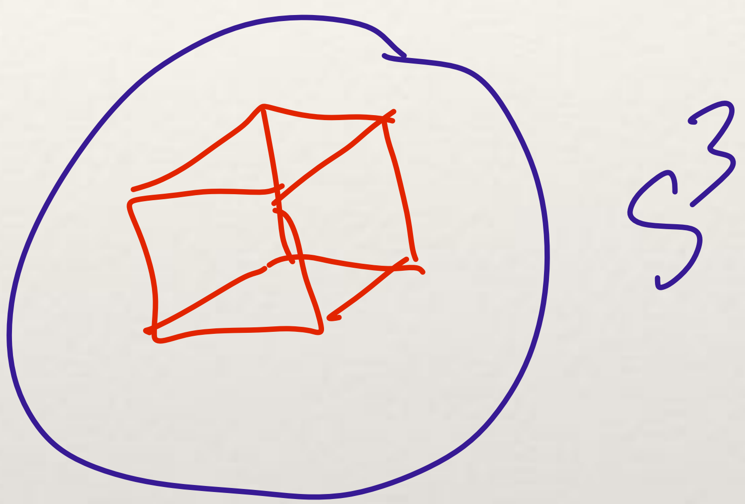
M-Theory Background on Y

- To consider a heterotic dual, we need to choose an M-theory background on Y
- g: Flat orbifold metric inherited from that on $\mathbb{R}^7$
- C: We choose the flat C field with no holonomies
- ψ: Vanishes
- The effective 4D theory then has $\mathrm{SU}(2)^{12}$ gauge symmetry with adjoint matter

Geometry of a K3 Fibration

- The orbifold has three immediate orbifold K3 fibrations [Liu '98]

$$\pi_{567} : Y \to T^3_{567}/\langle \beta, \gamma \rangle$$
$$\pi_{246} : Y \to T^3_{246}/\langle \alpha, \beta \rangle$$
$$\pi_{347} : Y \to T^3_{347}/\langle \alpha, \gamma \rangle$$



- We must choose one for duality: take π_{567}
- Then the generic fiber is $T^4_{1234}/\langle \alpha \rangle$, which has 16 A_1 singularities
- The (extra) singular fibers lie above the 1-skeleton of a cube in the base

The Dual Heterotic Geometry

- In the half-K3 limit, it is straightforward to identify CY3:

$$T^6_{123567}/\langle \beta, \gamma \rangle \to T^3_{567}/\langle \beta, \gamma \rangle$$

- Orbifold loci: 16 T^2 of A_1 singularities
- Complex structure dictated by SYZ and G-action holomorphy:

$$z_1 = x_5 + ix_1$$
$$z_2 = x_6 + ix_2$$
$$z_3 = x_7 + ix_3$$

- (Note that different choices of K3 fibration give non-biholomorphic complex structures on *X*)

The Heterotic Gauge Bundle

- To complete our heterotic description, we need to specify the gauge bundle with connection over the geometry and also the B-field
- Ideal: a rigorous algorithm to determine a gauge bundle from the G_2 geometry
- F-theory analogue: Line bundle over spectral cover to determine the total bundle
- Dualizing K3 fiber data gives flat connections on T^3 fibers
- Horizontal data in K3 holonomies must give HYM

Perturbative vs. Non-Perturbative Gauge Symmetry

- On the M-theory side, all of the gauge symmetry is on the same footing: comes from C-field + loci of orbifold singularities in the space
- On the heterotic side, the choice of K3 fibration introduces a new quality: whether or not a particular enhancement may be seen perturbatively
- (This means whether or not the gauge symmetry comes from the 2D CFT perspective of the string theory)
- Expectation: The gauge symmetry corresponding to an orbifold locus in G2 may be seen perturbatively iff the locus is transverse to the fibers (c.f. F-theory)

Point-Like Instantons

- This criterion suggests $SU(2)^8$ non-perturbative gauge symmetry
- The simplest way to achieve gauge symmetry that is not visible perturbatively is to have bundle singularities
- The simplest type of bundle singularity that gives extra gauge symmetry is an instanton whose curvature is localized on an orbifold singularity
- “Small instanton” or “point-like instanton” or “idealized instanton"

Anomaly Cancellation

- In fact, point-like instantons are forced upon us by anomaly cancellation
- The condition for heterotic anomaly cancellation is that

$$c_2(X) = c_2(V) + [\mathrm{NS5}]$$

- The second Chern class measures the number of instantons localized on each curve class
- Point-like instantons on orbifold singularities may be thought of as fractional NS5-branes

The Tangent Bundle

- The tangent bundle of $T^4/\mathbb{Z}_2$ has second Chern number 3/2 on each of the 16 orbifold singularities
- So the tangent bundle has point-like instantons built in!
- The simplest way to cancel anomalies is to take the gauge bundle to be the tangent bundle ("standard embedding"), but this will be tentatively ruled out later

Spectrum

- A necessary condition on a candidate dual pair is to produce the same massless matter spectrum

M-Theory	Heterotic
$b_1(M)$ adjoint chiral multiplets for each factor in gauge group	Perturbative matter spectrum + point-like instanton matter spectrum

- Each point-like instanton on an orbifold singularity comes with gauge bosons and fundamental multiplets
- This rules out standard embedding!

Bundle Constraints

We require a heterotic bundle with connection that satisfies

1. An $E_8 \times E_8$ HYM connection such that the centralizer of the reduced structure group is $\mathrm{SU}(2)^4$
2. Curvature is localized to the 16 orbifold loci
3. Second Chern class $\frac{3}{2}\sum(\text{orbifold loci})$
4. The enhanced gauge symmetry from these point-like instantons is $\mathrm{SU}(2)^8$
5. All matter is in the adjoint representation

Future Directions

1. Classification of bundle singularities on orbifolds and their associated heterotic spectra
2. Detailed understanding of bundle and B-field reconstruction from M-theory data
3. Generalize M-theory backgrounds on G_2
4. Is this duality useful to mathematicians?