Theories of Neutrino Mass

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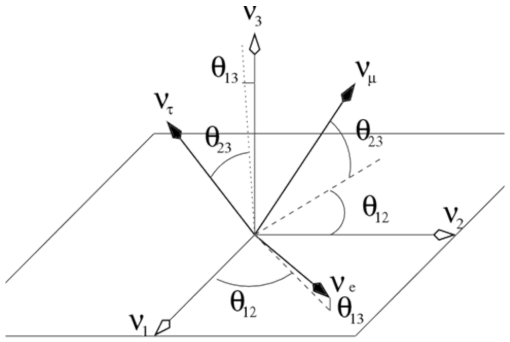
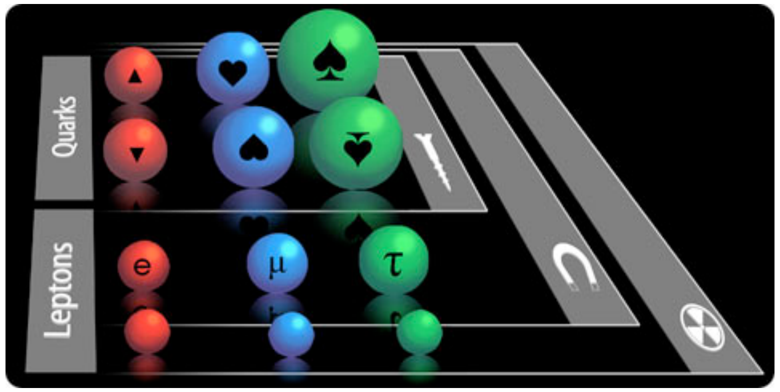
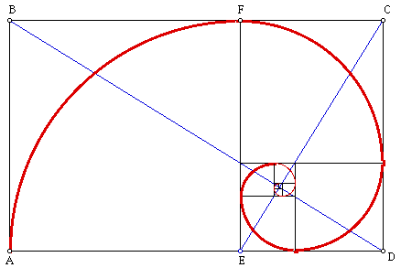
INFLATION 20, KITP, January 2020



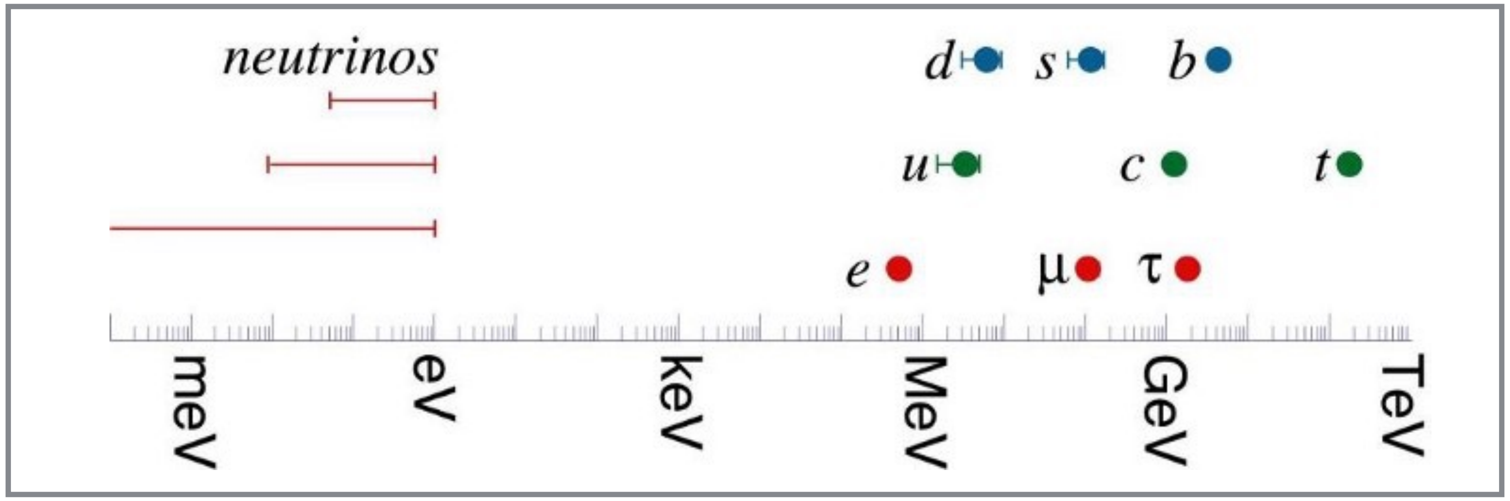




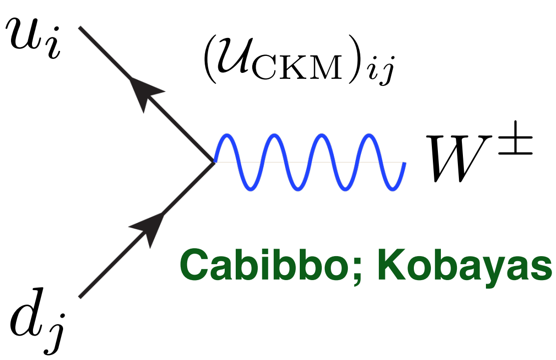
(image credits: Wikipedia, E. Palti)

The Standard Model Flavor Puzzle:

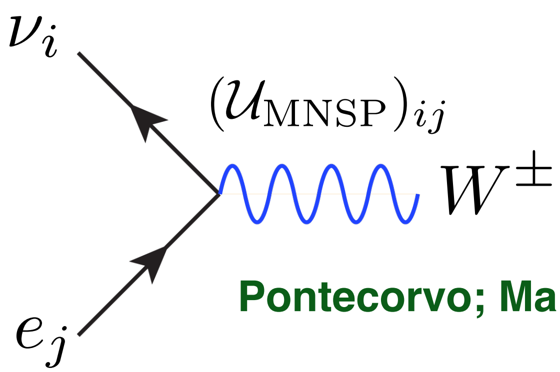
the origin of the fermion masses and mixing angles in the SM



(image credit:
H. Murayama)



Cabibbo; Kobayashi, Maskawa



Pontecorvo; Maki, Nakagawa, Sakata

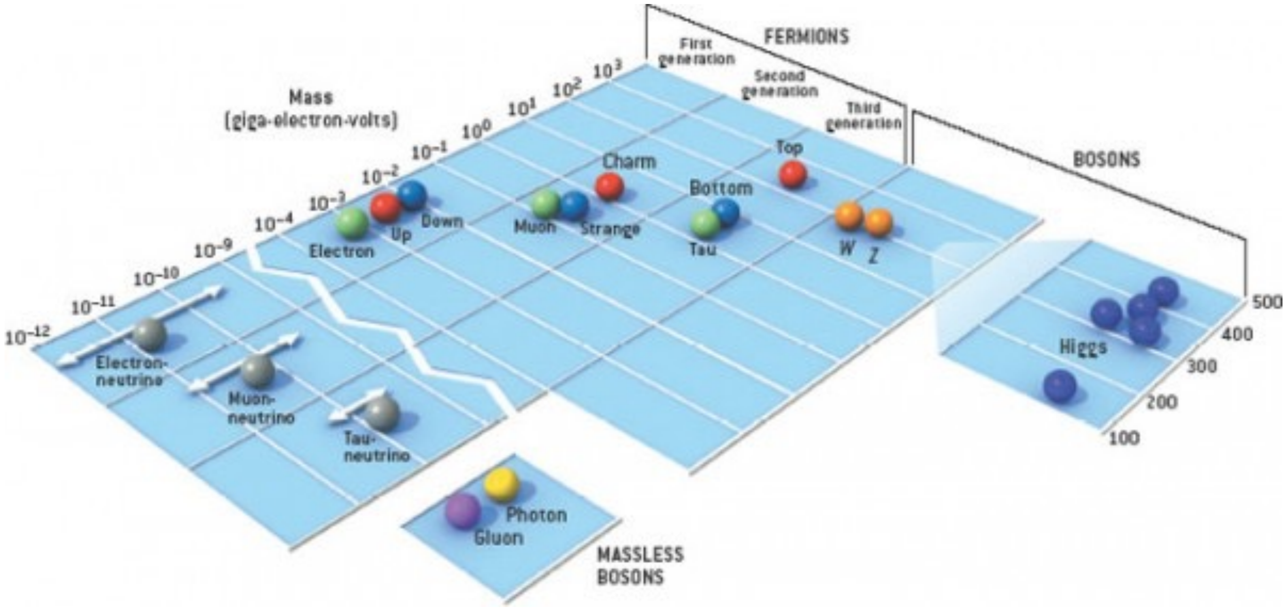
traditional SM definition: 3 generations, massless neutrinos →

13 out of 19 SM parameters

w/neutrino masses: at least 7 more (9 more if Majorana)

SM → ν **SM**

A long-standing question that predates and runs in parallel with the theoretical development of the SM itself (and its possible extensions). It has resulted in its own rich history, and multitude of ideas.



(image credit: Scientific American/G. Kane)

Key elements:

✓	V-A interactions	Marshak, Sudarshan (1957), Feynman, Gell-Mann (1958),…
✓	Cabibbo angle, weak interactions universality	Cabibbo (1963),…
✓	GIM mechanism and FCNC suppression	Glashow, Iliopoulos, Maiani (1970),…
✓	quark sector CP violation	Kobayashi, Maskawa (1973),…

Much experimental input! In recent decades:

	✓	discovery of top quark	CDF, D0 (1995)
	✓	precise determination of quark mixing matrix entries	many exps/analyses
→	✓	neutrino oscillations discovery and measurements	SuperK (1998) + many others
	✓	Higgs discovery and coupling measurements to fermions	ATLAS, CMS (2012),…

Discovery of neutrino oscillations (nonzero neutrino masses)

+ subsequent detailed measurements of lepton sector →

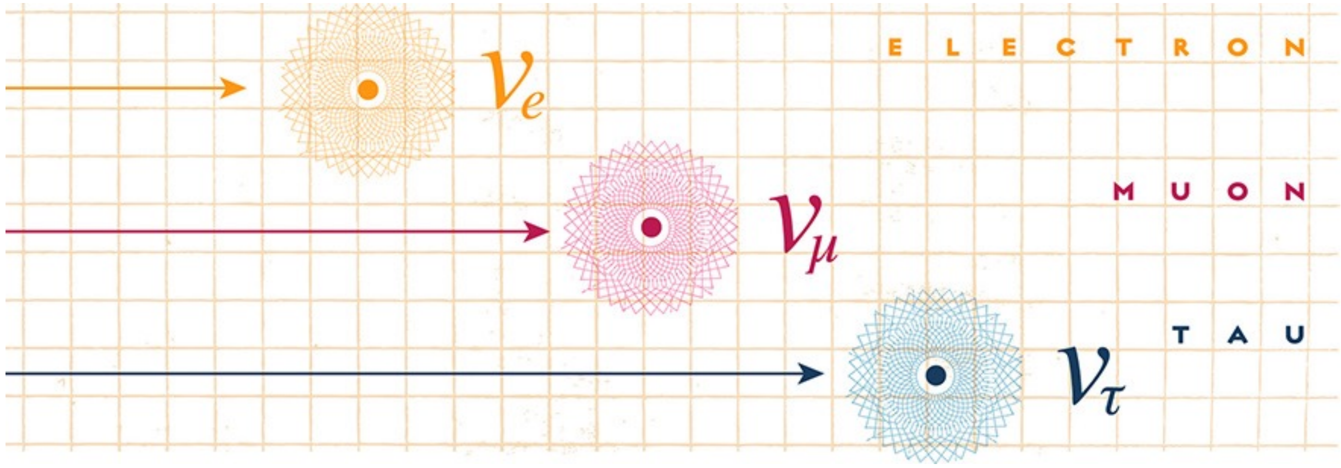
a paradigm shift

(image credit: Sandbox Studio via symmetry magazine)

Goal here: **overview of current status (not fully comprehensive)**
challenges and future outlook

But first, a (brief) summary of the data

- **Neutrino masses** **SM** → ν **SM**



(image credit: C. Wiens)

neutrino oscillations discovery!

SuperK ’98

2 decades+ of results…

Some highlights:

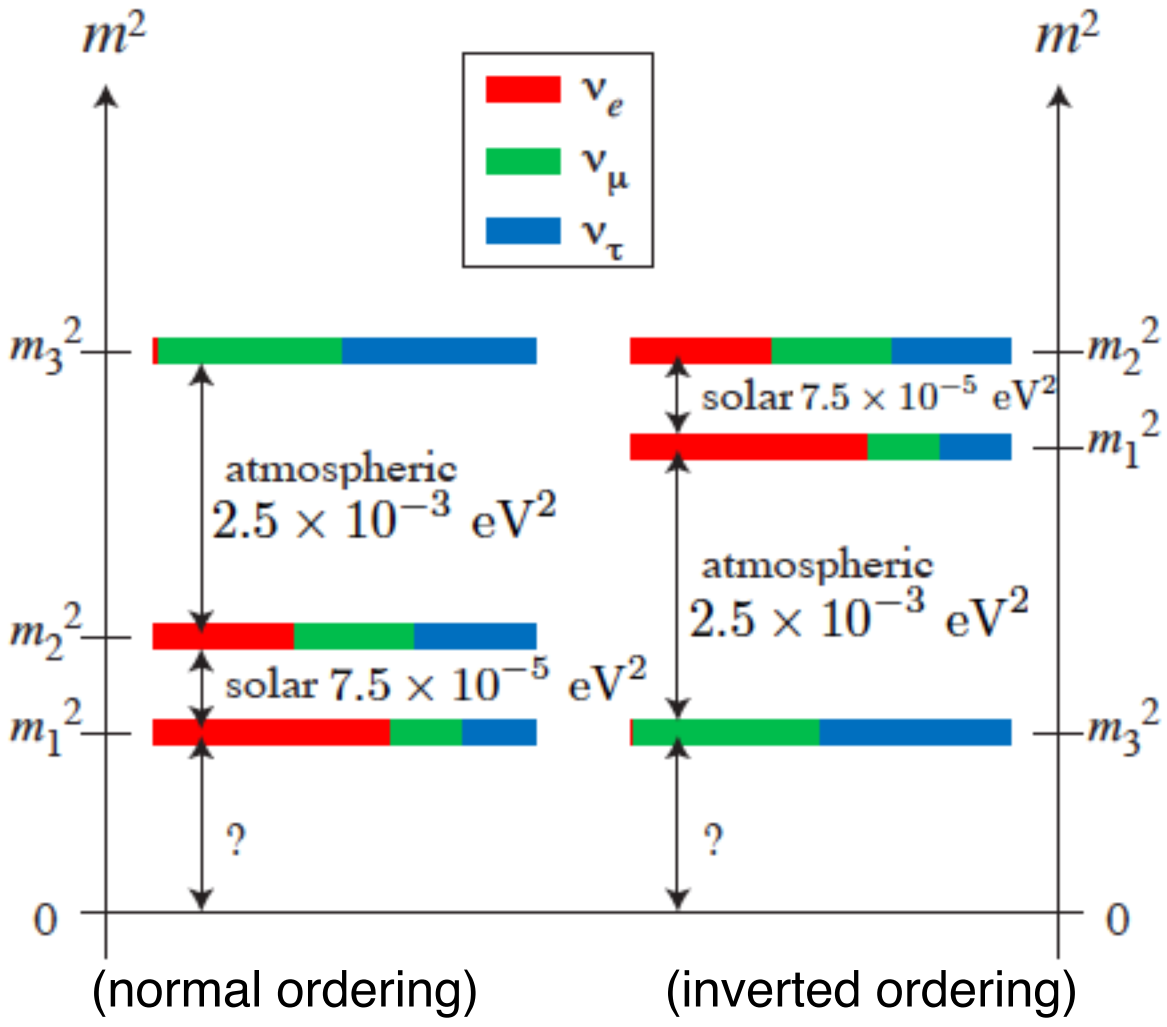
1998: atmospheric ν_μ disappearance **(SK)**
2002: solar ν_e disappearance **(SK)**
2002: solar ν_e appear as ν_μ, ν_τ **(SNO)**
2004: reactor $\overline{\nu}_e$ oscillations **(KamLAND)**
2004: accelerator ν_μ disappearance **(K2K)**
2006: accelerator ν_μ disappearance **(MINOS)**

2011: accel. ν_μ appear as ν_e **(T2K,MINOS)**
2012: reactor $\overline{\nu}_e$ disappear
reactor angle measured! **(Daya Bay, RENO,…)**
2014: CP violation hint? **(T2K)**
2015: normal hierarchy hint? **(SK, T2K, NOvA)**
2016: non-maximal atm hint? **(NOvA)**
2018: CP cons disfavored at 2σ **(T2K)**
2019: improved direct mass limit **(KATRIN)**

The emergent picture…

a (seemingly) robust 3-neutrino mixing scheme

✓ **mass-squared differences** $\Delta m^2_{ij} \equiv m^2_i - m^2_j$



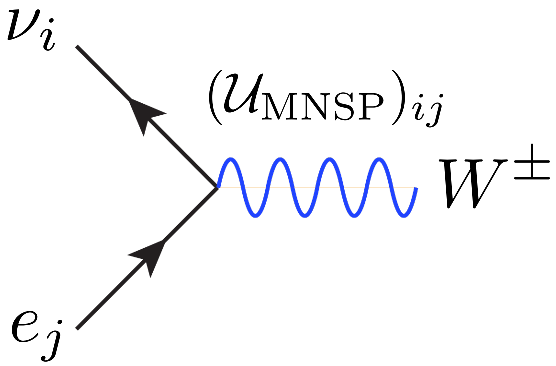
(normal ordering) (inverted ordering)

(image credits: King, Luhn)

✓ **overall mass scale: limits from direct searches, cosmology** $\sum m_\nu < 0.7$ eV

$m_{\bar{\nu}_e} < 1.1$ eV **KATRIN (2019)**

Lepton mixing:



"standard" PDG parametrization

$$\mathcal{U}_{\rm MNSP} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13},\delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{-i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{P}$$

Global Fits:

Forero et al., '17

Capozzi et al.,'18

Gonzalez-Garcia et al., (www.nu-fit.org)

NuFIT 4.1 (2019)

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.244 \to 0.496 & 0.467 \to 0.678 & 0.646 \to 0.772 \\ 0.287 \to 0.525 & 0.488 \to 0.693 & 0.618 \to 0.749 \end{pmatrix}$$

$$|U|_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.243 \to 0.490 & 0.473 \to 0.674 & 0.651 \to 0.772 \\ 0.295 \to 0.525 & 0.493 \to 0.688 & 0.618 \to 0.744 \end{pmatrix}$$

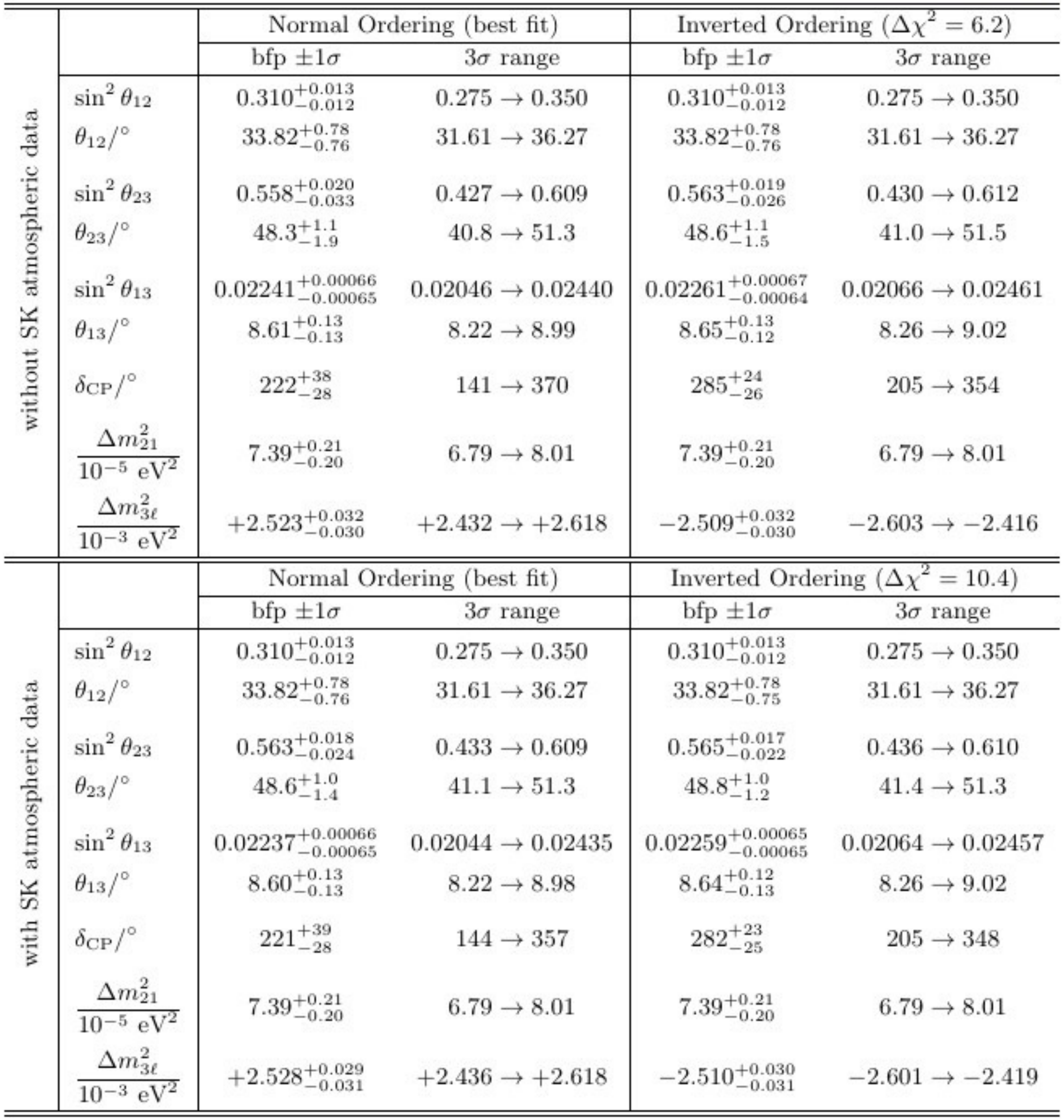
(assumptions: 3 active neutrinos only, unitarity)

Note

here: "MNSP"
more often: "PMNS"

More details (NuFit 2019)

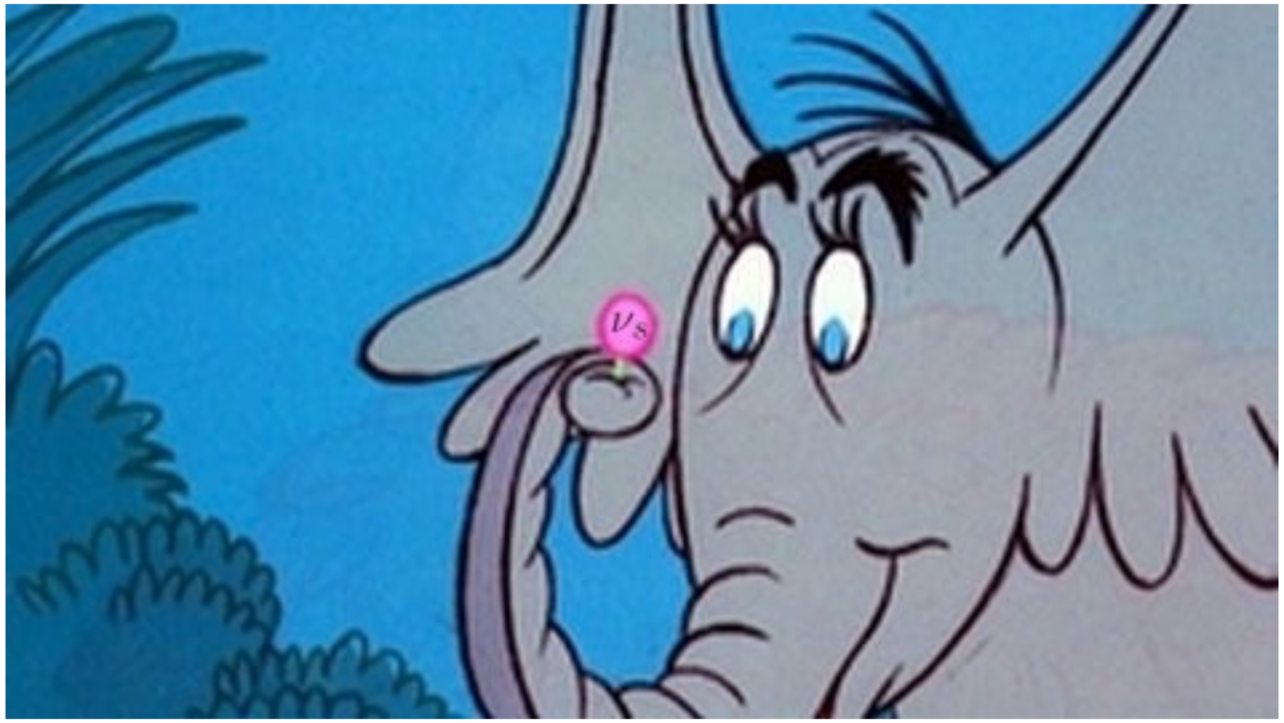
NuFIT 4.1 (2019)



		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.2$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$	$0.563^{+0.019}_{-0.026}$	$0.430 \to 0.612$
	$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$
	$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$	$0.02261^{+0.00067}_{-0.00064}$	$0.02066 \to 0.02461$
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$
	$\delta_{\rm CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	285^{+24}_{-26}	$205 \to 354$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.523^{+0.032}_{-0.030}$	$+2.432 \to +2.618$	$-2.509^{+0.032}_{-0.030}$	$-2.603 \to -2.416$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 10.4$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.563^{+0.018}_{-0.024}$	$0.433 \to 0.609$	$0.565^{+0.017}_{-0.022}$	$0.436 \to 0.610$
	$\theta_{23}/^\circ$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$
	$\sin^2\theta_{13}$	$0.02237^{+0.00066}_{-0.00065}$	$0.02044 \to 0.02435$	$0.02259^{+0.00065}_{-0.00065}$	$0.02064 \to 0.02457$
	$\theta_{13}/^\circ$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$
	$\delta_{\rm CP}/^\circ$	221^{+39}_{-28}	$144 \to 357$	282^{+23}_{-25}	$205 \to 348$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.528^{+0.029}_{-0.031}$	$+2.436 \to +2.618$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$

Caveat: sterile neutrino(s)?

(image credit: ParticleBites)



Anomalies:

1995: $\overline{\nu}_e$ appearance **(LSND)**

2007: $\overline{\nu}_e$ appearance **(MiniBooNE)**

2012: ν_e appearance **(MiniBooNE)**

1995: ν_e disappearance **(Gallium)**

2011: ν_e disappearance **(Reactor)**

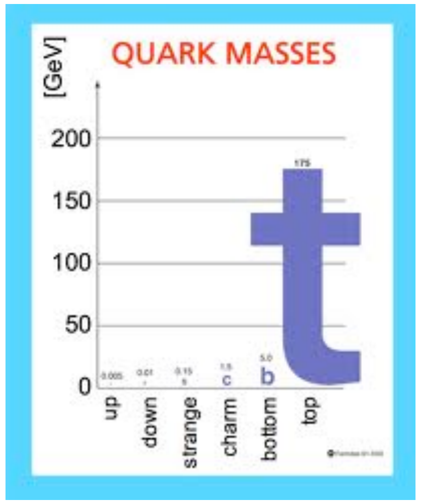
[well-documented tension between appearance and disappearance data]

See: Huber's IPA 2017 talk for "scorecard"
Maltoni's talk at Neutrino 2018

Restrict focus here to 3 active light families only

Compare to the quark sector:

(image credit: D0 Single Top group)



- **Quark and charged lepton masses**

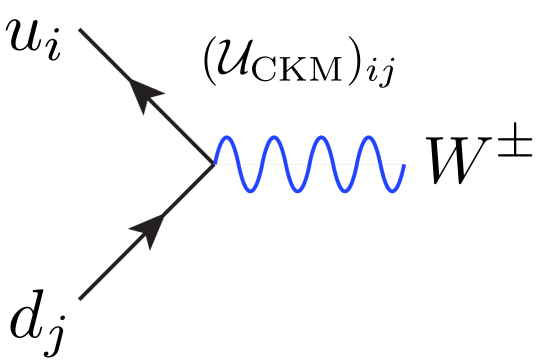
$m_u \simeq 2-3\ \text{MeV}$ $m_c \simeq 1.3\ \text{GeV}$ $m_t \simeq 173\ \text{GeV}$

$m_d \simeq 4-6\ \text{MeV}$ $m_s \simeq 90-100\ \text{MeV}$ $m_b \simeq 4\ \text{GeV}$

$m_e = 0.511\ \text{MeV}$ $m_\mu \simeq 106\ \text{MeV}$ $m_\tau \simeq 1.8\ \text{GeV}$

PDG (2019)

Quark mixing:



$$\mathcal{U}_{\text{CKM}} = \mathcal{R}_1(\theta_{23}^{\text{CKM}})\mathcal{R}_2(\theta_{13}^{\text{CKM}}, \delta_{\text{CKM}})\mathcal{R}_3(\theta_{12}^{\text{CKM}})$$

“standard” PDG parametrization

$$s_{13} \ll s_{23} \ll s_{12}$$

Wolfenstein parametrization: $\lambda = \sin\theta_c$ **(Cabibbo angle)**

$$s_{12} = \lambda \quad s_{23} = A\lambda^2 \quad s_{13} = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}(1-A^2\lambda^4(\bar\rho + i\bar\eta))}$$

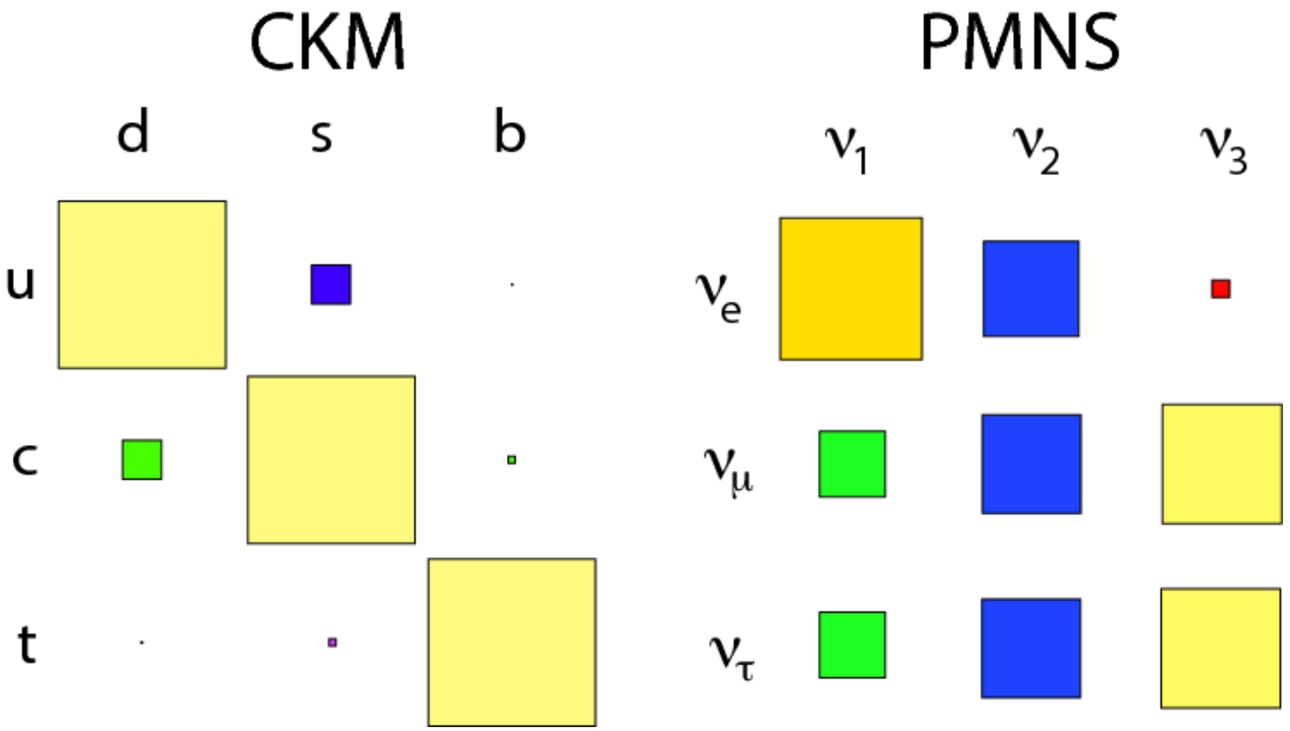
(Cabibbo expansion)

$$\lambda \simeq 0.225 \qquad A \simeq 0.83 \qquad \bar\rho \simeq 0.1 \qquad \bar\eta \simeq 0.35$$

PDG (2018)

A plethora of interesting results to (try to) explain!

- **Quark and lepton sectors look very different!**
  - ✓ **Quarks: hierarchical masses, small mixings, O(1) CP violation**
  - ✓ **Leptons: hierarchical charged lepton masses**
    suppression of overall neutrino mass scale
    hierarchy apparently "milder" for neutrino masses
    two large mixing angles (or more)**



(image credit: S.Stone)

→ **implications for quark-lepton unification and other BSM physics**

Mass Generation

- **Quarks and charged leptons:**

Mass generation must proceed via Yukawa interactions w/Higgs:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix},\ L_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} \qquad u_{Ri},\ d_{Ri},\ e_{Ri}$$

$$\boxed{Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj}}$$

Dirac masses upon electroweak symmetry breaking (EWSB)

$$\boxed{\mathcal{M}_{ij}^{\text{Dirac}} = Y_{ij} \langle H \rangle}$$

Yukawas arbitrary in SM: diagonalize

$$\boxed{\mathcal{U}_{fL}^{\dagger} \mathcal{M}_f^{\text{Dirac}} \mathcal{U}_{fR} = \mathcal{M}_f^{\text{diag}}} \qquad \boxed{\mathcal{U}_{\text{CKM}} = \mathcal{U}_{uL}^{\dagger} \mathcal{U}_{dL}}$$

Dirac

FB "The Same Oddly Asymmetric Picture of Paul Dirac Each Day"

Masses tied to EW scale: suppressions required (all but top)

(image credits: FB, Wikipedia)

- **Neutrino Masses**

main question: are neutrinos Dirac or Majorana?

Dirac

$\Delta L = 0$

Majorana

$\Delta L = 2$

Critically important question, to be settled by experiment!

Many options for neutrino mass suppression in each case.

Consider each in turn (Majorana first)

Majorana neutrinos

lepton number violating

$$\Delta L = 2$$

- **SM at NR level: Weinberg operator**

$$\frac{\lambda_{ij}}{\Lambda} L_i H L_j H$$

Majorana neutrino masses upon EWSB

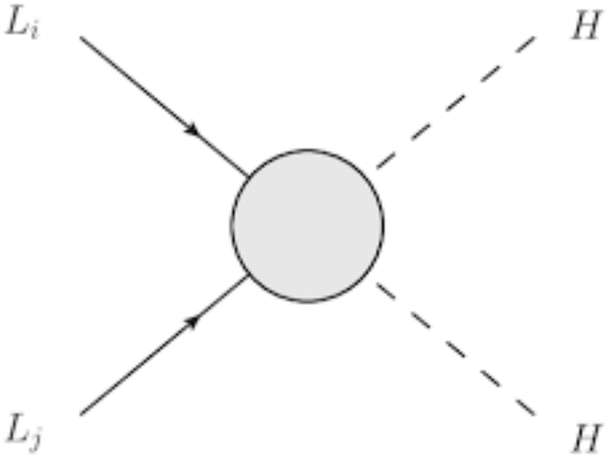
$$\boxed{\mathcal{M}_\nu^{\rm Maj} \simeq \frac{\lambda_{ij} \langle H \rangle^2}{\Lambda}}$$

if $\lambda \sim O(1)$

$$\Lambda \gg \langle H \rangle \sim O(100 \text{ GeV})$$

mass suppression via ratio of EW to heavy scale

"seesaw"



Tree level possibilities

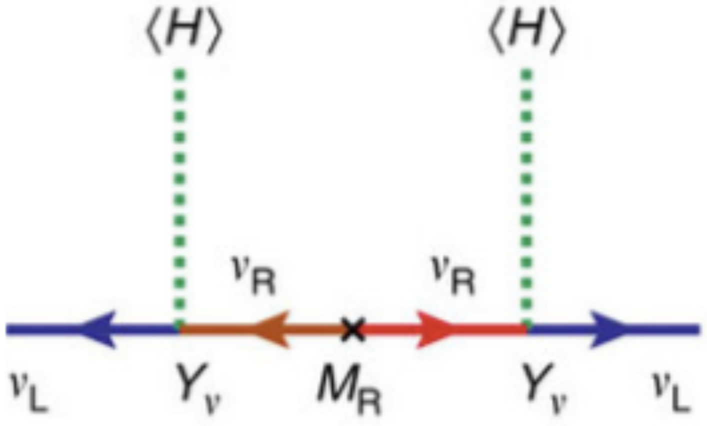
see e.g. Ma, '98

- ✓ **Type I seesaw** ν_R **(fermion singlet)**
- ✓ **Type II seesaw** Δ **(EW triplet scalar)**
- ✓ **Type III seesaw** $\sum$ **(EW triplet fermion)**

Type I seesaw

Minkowski ’77, Gell-Mann, Ramond, Slansky ’79, Yanagida ’79
Mohapatra, Senjanovic ’80, Schecter, Valle ’80…

✓ introduce right-handed neutrinos



(image credit: T. Ohlsson et al., Nat. Comm.)

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$m_1 \sim \frac{m^2}{M} \qquad m_2 \sim M \gg m_1$$

$$\nu_{1,2} \sim \nu_{L,R} + \frac{m}{M}\nu_{R,L}$$

$$Y_{ij} L_i \nu_{Rj} H + M_{R\,ij} \nu_{Ri} \nu^c_{Rj}$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y M_R^{-1} Y^T}$$

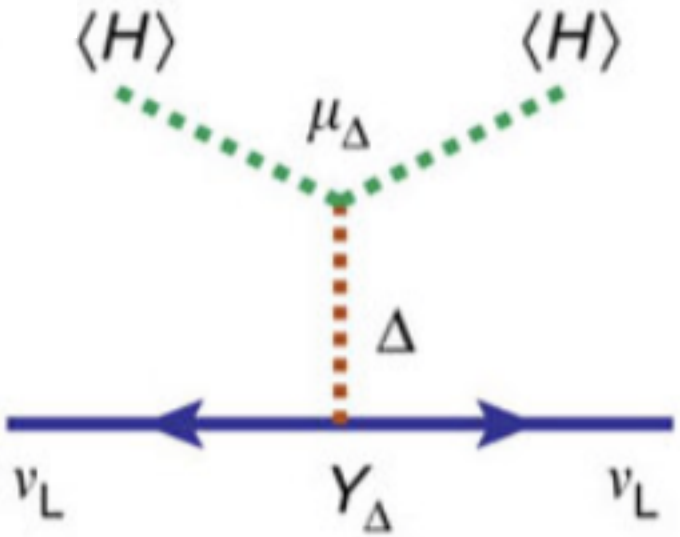
Advantages: economical, connection to grand unification, leptogenesis**

Disadvantages: testability without model assumptions

Type II seesaw

Magg, Wetterich ’80, Lazarides, Shafi, Wetterich ’81, Mohapatra, Senjanovic ’81, Cheng, Li ’80,…

✓ **introduce triplet Higgs scalar**



(image credit: T. Ohlsson et al., Nat. Comm.)

$$\Delta \sim (\mathbf{3}, 2)$$
$$(SU(2)_L, U(1)_Y)$$

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

$$(Y_\Delta)_{ij} L_i L_j \Delta + \mu_\Delta H H \Delta$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2}$$

can have clean LHC signatures of lepton # violation via decays of H^+, H^{++}

if $M_\Delta \leq O(\mathrm{TeV})$ Fileviez Perez et al. ’08, Gavela et al. ’09,…

(also LFV)

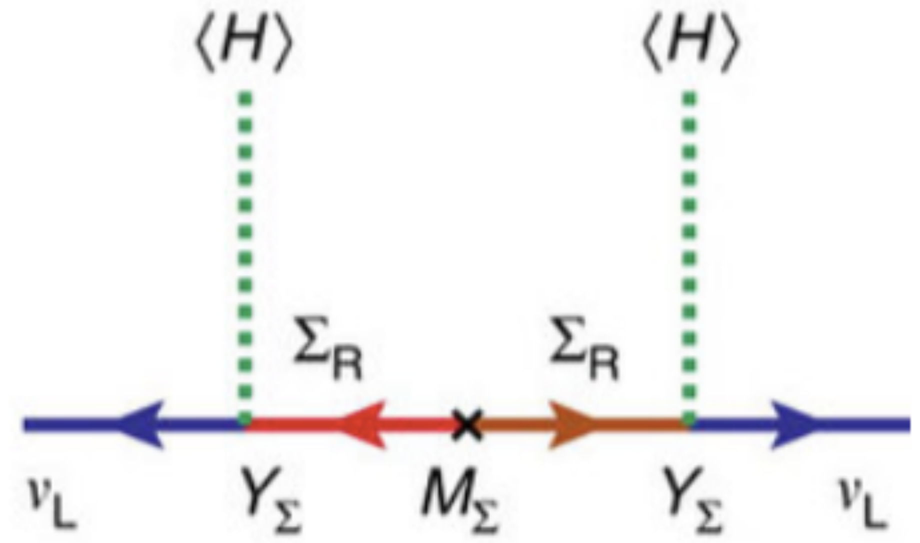
Advantages: testability (charged Higgs states probed at LHC)

Disadvantages: not as economical/minimal as Type I

Type III seesaw

Foot, He, Joshi ’89, Ma ’98,…

✓ **introduce electroweak triplet fermions**



(image credit: T. Ohlsson et al., Nat. Comm.)

$$\Sigma \sim (\mathbf{3}, 0)$$
$$(SU(2)_L, U(1)_Y)$$

$$\Sigma_i = \left(\Sigma_i^0, \Sigma_i^\pm\right)$$

$$(Y_\Sigma)_{ij} L_i \Sigma_j H + (M_\Sigma)_{ij} \Sigma_i \Sigma_j$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T}$$

can have clean LHC signatures via mixing w/charged leptons

$M_\Sigma \sim O(\mathrm{TeV})$

Franchesini, Hambye, Strumia ’08,…

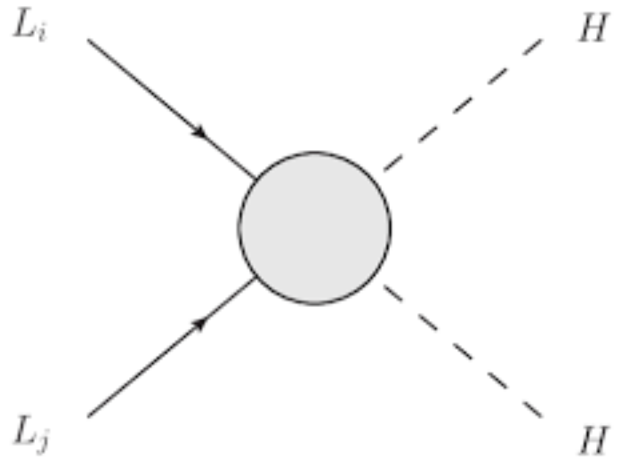
also highly predictive pattern of LFV signals

Abada et al. ’07, Gavela et al. ’09,…

Advantages: testability (new charged states probed at LHC, LFV)
Disadvantages: not as economical/minimal as Type I

Zee ’86, Babu ’88, Ma ’98,…

Radiative possibilities:

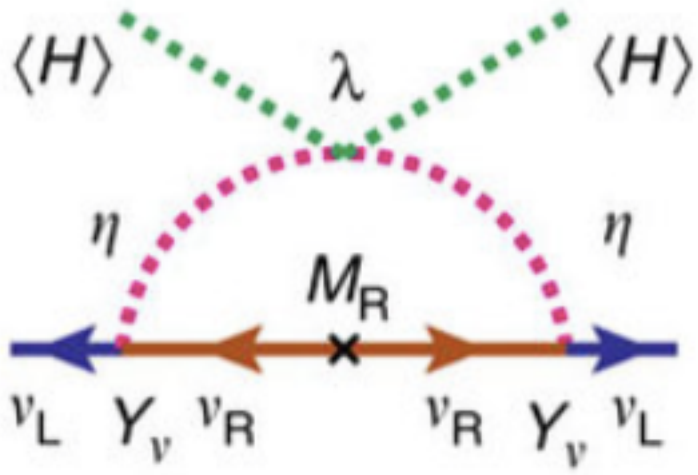


complete Weinberg operator via loops

Canonical example: “scotogenic” model

$$\boxed{\mathcal{M}_\nu \sim \lambda \frac{\langle H\rangle^2}{16\pi^2} Y M_R^{-1} Y^T}$$

“radiative Type I seesaw” Ma ’06



(image credit: T. Ohlsson et al., Nat. Comm.)

analogous construction with fermion triplet:

“radiative Type III seesaw” Ma, Suematsu ’09

can also construct a “radiative Type II seesaw” Fraser, Kownacki, Ma, Popov ’15

Required: new symmetry (usually discrete) to forbid tree level term, new states

Zee ’86, Babu ’88, Ma ’98,…

Can also consider alternatives to Weinberg operator

Many other $\Delta L = 2$ NR operators **(odd mass dimension d>5)**

Classification

Babu and Leung ’01

de Gouvea and Jenkins ’07

Bonnet et al. ’12

d=7	**d=9**
$LLLe^cH$	$LLLe^cLe^c$ **(Zee-Babu)**
$LLQd^cH$	$LLQd^cQd^c$
$LL\overline{Q}\overline{u}^cH$	+ many others…
$L\overline{e}^c\overline{u}^cd^cH$	

…

Generic advantage of radiative models:

New physics scale can be accessible at LHC (subject to LFV bounds)

Many explicit realizations!

See e.g. excellent review:

Cai, Herrero-Garcia, Schmidt, Vicente, Volkas ’17

Also potential connections to other, possibly accessible, new physics:

✓ **possible connections to** $\boxed{\text{dark matter}}$

symmetry for dark matter stability → **radiative ν mass**

potential DM candidates in loops

many authors!

✓ **possible connections to** $\boxed{\text{flavor physics anomalies}}$

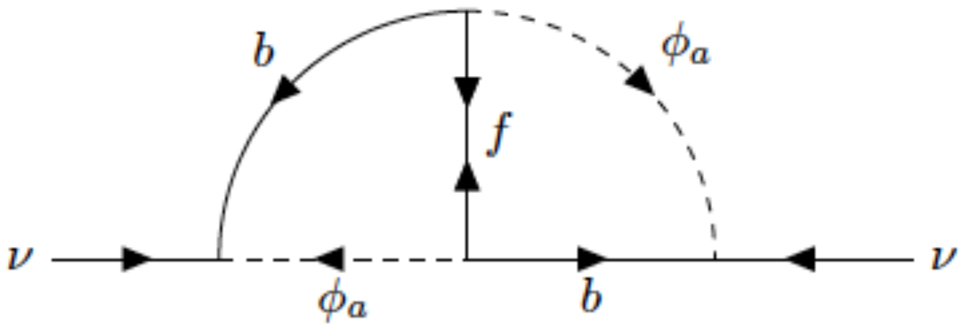
Päs, Schumacher ’15,
Deppisch et al. ’16,…

One way leptoquarks can manifest themselves:

Two-loop representative example:

scalar leptoquark $\phi \sim (\mathbf{3}, \mathbf{1}, -1/3)$

+ octet fermion $f \sim (\mathbf{8}, \mathbf{1}, 0)$



Angel, Cai, Rodd, Schmidt, Volkas ’13
Cai, Gargalones, Schmidt, Volkas ’17

Many other ideas for Majorana masses

✓ **more complicated seesaws** → **lower scales**

e.g. double/inverse seesaw: **3** ν_R **and 3 new singlet fermions**

Mohapatra, Valle '86

...

$$Y_{ij} L_i (\nu_R)_j H + M_{ij} S_i (\nu_R)_j + \mu_{ij} S_i S_j \qquad \mu, m \ll M$$

$$M_{\text{eff}} = M^T \mu^{-1} M$$

$$\mathcal{M}_\nu \sim \begin{pmatrix} 0 & m & 0 \\ m & 0 & M \\ 0 & M & \mu \end{pmatrix}$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 (Y^T M_{\text{eff}}^{-1} Y)}$$

✓ **SUSY with R-parity violation**

Aulakh, Mohapatra '82, Hall, Suzuki '84,...

Borzumati et al. '96, Grossman et al. '03

✓ **Warped extra dimensions**

see e.g. Csaki, et al. '08,...

...

lepton number violation → **Majorana** ν **masses**

Dirac neutrinos

$\Delta L = 0$

- Analogous to charged fermions,
  but much stronger suppression $\boxed{Y_\nu \sim 10^{-14}}$

→ Must forbid both types of “bare” mass terms
Less intuitive, but mechanisms exist.

Example: radiative mass generation

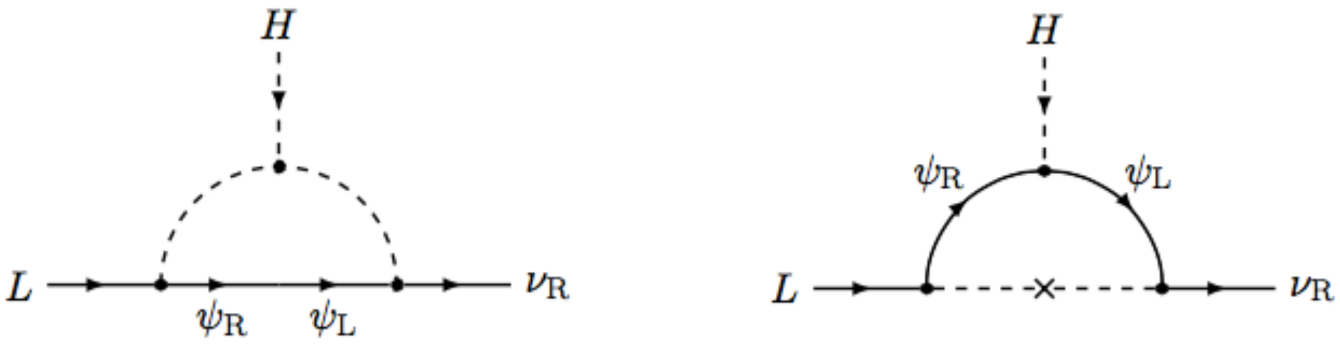
Cheng, Li ’78, Mohapatra ’87, ’88

Balakrishna, Mohaptra ’89, Rajpoot ’01

…

e.g. discrete symmetry: ν_R nontrivial
one-loop: 2 topologies

Ma, Popov ’16
Wang et al. ’16, ’17
Review: Cai et al. ’17



variety of possibilities
for new states

(many studies in
GUT contexts)

other options: higher loops, loop-induced vev,…

✓ **extended gauge sectors**

non-singlet ν_R → **forbids “bare” terms, simplest seesaws**

higher-dimensional operators: e.g. $U(1)'$ **symmetry**

see e.g. Langacker ’11 for review

✓ **SUSY breaking**

symm+holomorphy forbids superpotential

neutrino masses via Kahler potential terms

Arkani-Hamed et al. ’00
Borzumati et al. ’00,’01
Demir, LE, Langacker ’08
...

✓ **string constructions**

exponentially suppressed interactions from stringy instanton effects…

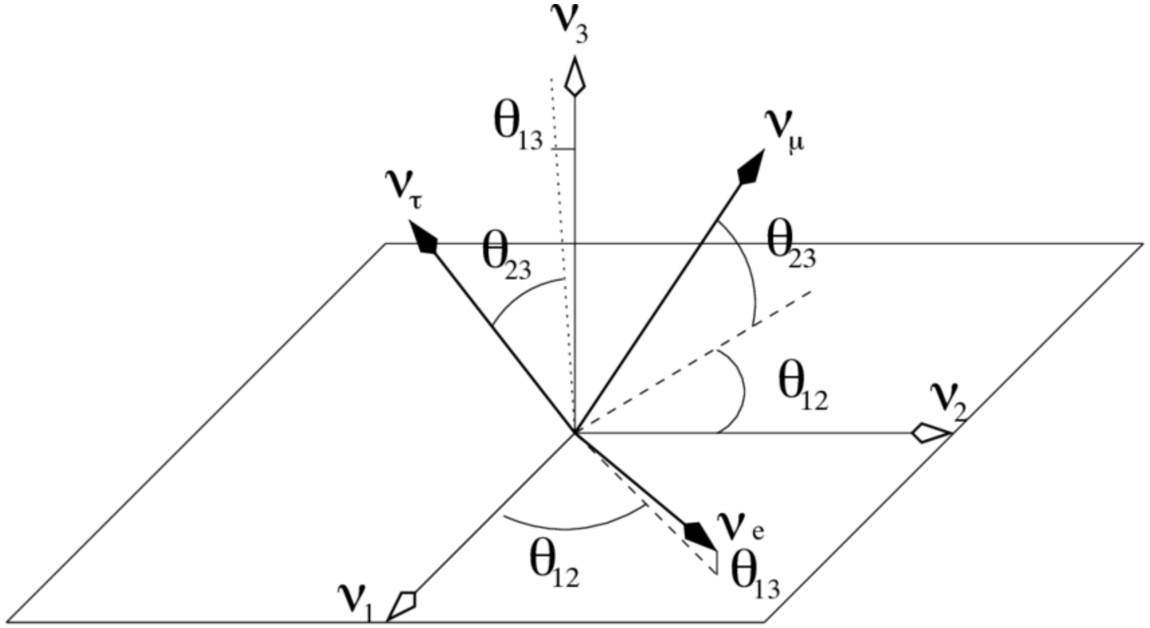
...

Cvetic et al. ’08,…
Langacker review, ’11
...

Neutrino sector much richer than charged fermions

→ **but trade-off between simplicity and testability**

Mass hierarchies and lepton mixing



(image credit: S. King)

(Dirac or Majorana neutrinos)

$$\boxed{\mathcal{U}_\nu^T \mathcal{M}_\nu^{\text{Maj}} \mathcal{U}_\nu = \mathcal{M}_\nu^{\text{diag}}}$$

$$\boxed{\mathcal{U}_{fL}^\dagger \mathcal{M}_f^{\text{Dirac}} \mathcal{U}_{fR} = \mathcal{M}_f^{\text{diag}}}$$

$$\boxed{\mathcal{U}_{\text{MNSP}} = \mathcal{U}_{eL}^\dagger \mathcal{U}_{\nu L}}$$

$$\mathcal{U}_{\text{MNSP}} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13},\delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$$

Lepton sector properties:

charged lepton masses: strongly hierarchical

hierarchy apparently “milder” for neutrino masses

two large lepton mixing angles (or more)**

(can work within any framework for mass suppression. vast majority: Type I seesaw)

Lepton mixing structure:

Two **large mixing angles**: $\theta_{23},\ \theta_{12}$

CP violation

Dirac phase: important goal of DUNE, HyperK

Majorana phases: challenging

A basic question: is θ_{13} “large” or “small”?

large reactor angle ↓ the case for anarchy

vs.

small reactor angle ↓ the case for structure

Anarchy

Hall, Murayama, Weiner ’99

(character: Watterson)

$\mathcal{U}_\nu$ **from a random draw of unbiased distribution of 3x3 unitary matrices**

statistical tests: lower bound on $|\mathcal{U}_{e3}|^2$

basis independence:

distribution invariant upon unitary transformations

→ **flat in Haar measure**

Haba, Murayama ’00

Post-reactor angle measurement: renewed focus

de Gouvea, Murayama ’12
Altarelli et al. ’12
Bai and Torroba ’12
...

Some recent highlights: RG analysis

model-building + quark sector

Brdar, Konig, Kopp ’15
Babu et al. ’16,...
Fortin et al. ’17

Note: anarchy hypothesis alone does not provide information on Δm^2

Structure

Standard assumption for SM flavor puzzle: structure from **symmetry**

Usual paradigm for quark sector:

Mixing angles as functions of mass ratios

Gatto-Sartori-Tonin relation $\boxed{\sin\theta_c = \sqrt{m_d/m_s}}$ Gatto, Sartori, Tonin '68

Special Yukawa structures: **“texture zeros”**

canonical three-family predictive examples: **(ruled out: top quark mass + CKM)**

- ✓ **Fritzsch texture** Fritzsch '78
- ✓ **Georgi-Jarlskog texture** Georgi, Jarlskog '79
- ✓ **“Yukawa quilt”** Ramond, Roberts, Ross '93

What persists: more intricate structures, lessons for high-scale embedding

GJ mass relations
(high scale)

$$\boxed{m_b = m_\tau \qquad m_\mu = 3m_s \qquad m_d = 3m_e}$$

Chanowitz et al. '77 Buras et al. '78

(factors of 3: # of colors (RG effects))

Achieve special structures via symmetries:

- **Postulate family symmetry** G_f

spontaneously broken by “flavon” fields $\{\varphi_a\}$

$$\boxed{Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \left(\frac{\langle\varphi\rangle}{\Lambda}\right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj}}$$

$\epsilon = \left(\frac{\langle\varphi\rangle}{\Lambda}\right)$ $\Lambda > \langle\varphi\rangle > \langle H\rangle$ $\epsilon \ll 1$ **Froggatt, Nielsen ’79**

Heavy sector >> TeV (avoid too-large FCNC)

natural identification: $\boxed{\epsilon \sim O(\lambda)}$ $\lambda = \sin\theta_c$

Cabibbo angle (or some power) as a flavor expansion parameter

Unique theoretical starting point: $\mathcal{U}_{\rm CKM} \sim 1 + O(\lambda)$

- **Possibilities for G_f:**

canonical example: $\boxed{U(1) \text{ family symmetries}}$

Froggatt, Nielsen ’79

Have guidance from enhanced symmetry for vanishing Yukawas:

✓ **SM (no neutrino masses):** $U(3)^5$

$$U(3)_Q \otimes U(3)_{u^c} \otimes U(3)_{d^c} \otimes U(3)_L \otimes U(3)_{e^c}$$

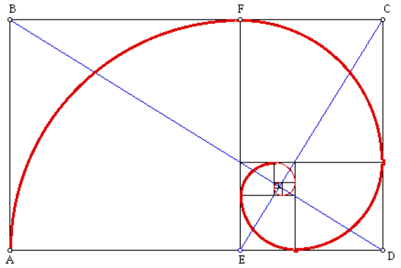
→ G_f **subgroup of** $U(3)^5$

Key input: ***O(1)*** **top quark Yukawa coupling**

$$U(3)_Q \otimes U(3)_{u^c} \xrightarrow{m_t} U(2)_Q \otimes U(2)_{u^c}$$

Many examples! continuous Abelian, non-Abelian, discrete non-Abelian

Structure



Now, lepton sector:

Can still introduce family symmetry

G_f **spontaneously broken at scale M** $\epsilon \sim \langle \varphi \rangle / M$

But quite different from the quark sector!

diagonalize $\mathcal{M}_\nu$**: 1 small, 2 large mixing angles**
(diagonal $\mathcal{M}_e$ **basis)**

Arguably the most challenging* leading order pattern: **(* for 3 families)**

3 small angles ⟶ $\sim$ **recall quarks**
2 small, 1 large ⟶ $\sim \ \mathrm{Rank}\mathcal{M}_\nu < 3$
3 large angles ⟶ **anarchical** $\mathcal{M}_\nu$
} **relatively easy**

2 large, 1 small ⟶ **fine-tuning, non-Abelian**

Family symmetry approach

✓ *spontaneously broken G_f

typical choice: →

discrete non-Abelian group

*But see recent interesting work in *symmetric* limit

Reyiumaji and Romanino, '18

✓ No unique theoretical starting point for “Cabbibo-like” expansion

$$\mathcal{U}_{\rm MNSP} \sim \mathcal{W} + O(\lambda') \qquad \lambda' \ll 1$$

↑

“bare” mixing angles (diagonal charged lepton basis)

$$(\theta^\nu_{12}, \theta^\nu_{23}, \theta^\nu_{13})$$

First stage: symmetry breaking to generate nontrivial $\mathcal{W}$

different unbroken subgroups for neutrinos, charged leptons

→ large mixing angles

Next stage: corrections as expansion in λ'

“Bare” mixing angles generically shift due to $O(\lambda')$ corrections

✓ ***A priori,*** **expansions in quark and lepton sectors unrelated**

unification paradigm (broad sense): $\lambda' = \lambda$

Ideas of quark-lepton complementarity and “Cabibbo haze”

↑

Raidal ’04, Minakata+Smirnov ’04,...

$$\theta_{23} = \theta_{12} + \theta_c$$

(empirical)

(“haze” terminology from Datta, L.E., Ramond ’05)

not an unreasonable approach given the data

$$\boxed{\theta_{13} \sim O(\lambda)}$$

pre-measurement, idea that θ_{13} **might be a Cabibbo effect:**

$$\theta^{\nu}_{13} = 0 \qquad \theta_{13} = \lambda/\sqrt{2}$$

Vissiani ’98, ’01

Ramond ’04

Possible theoretical starting points:

✓ **Most studied: maximal atm, zero reactor** $\boxed{\sin^2\theta^\nu_{23} = 1/2 \quad \sin^2\theta^\nu_{13} = 0}$

→ **classify scenarios by** **bare solar angle**

tri-bimaximal mixing: $\sin^2\theta^\nu_{12} = 1/3$ **Harrison, Perkins, Scott ’02; Xing ’02, He, Zee ’02, Ma ’03…**

bimaximal mixing: $\sin^2\theta^\nu_{12} = 1/2$ **Petcov ’82, Vissiani ’97, Barger et al. ’98, Baltz et al. ’98**

golden ratio (A) mixing: $\sin^2\theta^\nu_{12} = 1/(2+r) \sim 0.276$ **Datta, Ling, Ramond ’03; Kajiyama, Raidal, Strumia ’08;…**

$r = (1+\sqrt{5})/2$

golden ratio (B) mixing: $\sin^2\theta^\nu_{12} = (3-r)/4 \sim 0.345$ **Rodejohann ’09,…**

hexagonal mixing: $\sin^2\theta^\nu_{12} = 1/4$ **Albright, Duecht, Rodejohann ’10, Kimand, Seo ’11,…**

✓ **can also study nonzero reactor:** $\boxed{\sin^2\theta^\nu_{13} \neq 0}$ **Lam ’13, Holthausen et al. ’12,…**

…

All can be obtained via SSB of discrete non-Abelian family symmetries

Family symmetry models

✓ **usual choices:** $SU(3),\ SO(3)$ **subgroups:**

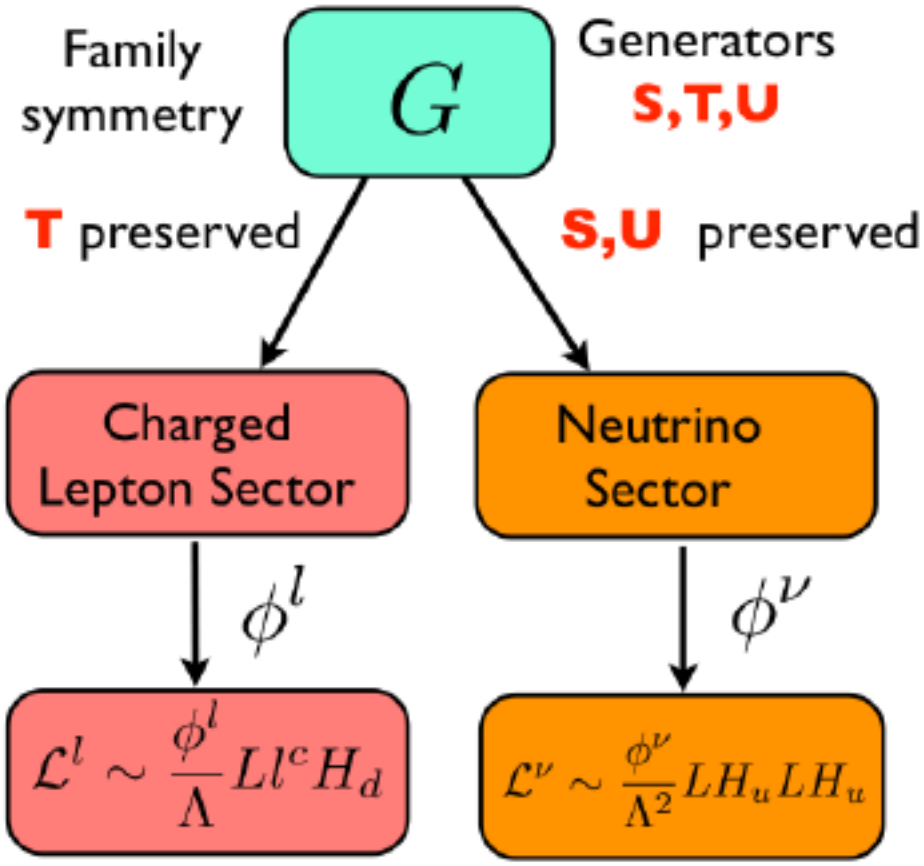
$$\mathcal{A}_4 \quad \mathcal{S}_4 \quad \mathcal{A}_5 \quad \Delta(3n^2) \quad \Delta(6n^2) \quad \mathcal{D}_n \quad \mathcal{T}' \quad \mathcal{I}' \quad \ldots$$

"Platonic solid" groups
+ double covers

example (Majorana):

(image credit: King, Luhn)

see reviews by King, Luhn '13, King '17



Flavons:

$$\phi^l, \phi^\nu$$

Residual symmetries:

$$T\langle\phi^l\rangle \approx \langle\phi^l\rangle$$
$$S, U\langle\phi^\nu\rangle \approx \langle\phi^\nu\rangle$$

(or broken further, e.g. only *S* or *U* unbroken)

also needed:
"driving fields" (singlets)

Many papers and authors! Some authors (not comprehensive):

Babu, Chen, Ding, L.E., Feruglio, Grimus, Hagedorn, King, Lam, Luhn, Ma, Merle, Ohlsson, Rodejohann, Stuart,…

Residual Symmetries

Lam ’08, ’09, Grimus et al. ’09,
Ge et al. ’11, Toroop et al. ’11, He et al. ’12, Hernandez et al. ’12,’13
Holthausen et al. ’12, King et al. ’13,
Hagedorn et al. ’14, Lavoura, Ludl ’14,
Fonseca, Grimus ’14
...

- **model-independent approach:**

determine rows and columns in $\mathcal{U}_{\mathrm{MNSP}}$

as pure numbers, independent of masses,

depending on preserved subgroups of finite group G_f

$$T^\dagger \mathcal{M}_e \mathcal{M}_e^\dagger T = \mathcal{M}_e \mathcal{M}_e^\dagger \qquad S^\dagger \mathcal{M}_\nu S = \mathcal{M}_\nu$$

$$G_f \to G_e,\ T \in G_e \qquad G_f \to G_\nu,\ S \in G_\nu$$

→ $$\mathcal{U}_{eL}^\dagger T \, \mathcal{U}_{eL} = T^{\mathrm{diag}} \qquad \mathcal{U}_\nu^\dagger S \, \mathcal{U}_\nu = S^{\mathrm{diag}}$$

Majorana: $G_\nu \supseteq Z_2 \times Z_2$ **(Klein group)**

systematic classification of possible mixing matrices Fonseca, Grimus ’14

Very different from texture zeros (mixing angles as ratios of masses)!

Corrections

✓ **charged lepton corrections**

source the reactor angle: $\mathcal{U}_{eL} \sim 1 + O(\lambda)$

correlations among observables → **“sum rules”**

Ge, Dicus, Repko ’11, Hanlon et al. ’12,

Marzocca et al. ’13, Petcov ’14,
Girardi et al. ’14-16,

Ballet et al. ’14

example: $\mathcal{U}_{eL} \sim \mathcal{R}_1(\theta^e_{23}, \delta^e_{23})\mathcal{R}_3(\theta^e_{12}, \delta^e_{12})$

Prediction for Dirac phase δ !

$$\boxed{\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right]}$$

1 model parameter!

guideline for “distinguishing power” needed from data

Marzocca et al. ’13, Petcov ’14, Girardi et al. ’14-16

LE, Ramos, Rock, Stuart ’19

✓ **canonical normalization** **(Kahler potential corrections)**

Many authors.
King ’17 (review)

✓ **RG effects**

more significant for IO,
heavy neutrino masses
(can be significant for sum rule analysis)

Antusch et al. ‘03
…
Gehrlein et al. ’16

Example: tri-bimaximal mixing (TBM/HPS)

(Type I seesaw)

$$\mathcal{U}_{\mathrm{MNSP}}^{(\mathrm{HPS})} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(~Clebsch-Gordan coeffs!)

Meshkov, Zee…

✓ **Many models predated reactor angle measurements**

see e.g. Albright et al. ’10

→ **data now requires Cabibbo-sized corrections**

$L_i \sim \mathbf{3}$ $\mathcal{A}_4$ $\mathcal{S}_4$ $\mathcal{T}'$ **(typically SUSY/SUSY-GUT)**

Many authors!!

Aranda, Carone, Lebed ’00, Ma et al. ’03,
Altarelli, Feruglio ’05, Chen et al. ’07 …

↑

“minimal” flavor group (contains *S*,*T*,*U* generators)

Lam ’11

✓ **Residual symmetries:** $\mathcal{Z}_3$ ~*T* $\mathcal{Z}_2 \times \mathcal{Z}_2$ ~*S*,*U*,*SU*

Can further break down Klein symmetry:

1 column only of HPS matrix preserved

see e.g. King ’17 for review

Many ways to correct for reactor angle.

Chen et al. ’13,
many authors…

Example: asymmetric charged lepton corrections to TBM/HPS
(with a dash of grand unification)

Rahat, Ramond, Xu ’18

***SU(5), SO(10)* GUT-inspired relations:**

<u>symmetric</u> Yukawas → **insufficient corrections to** θ_{13}

Kile, Perez, Ramond, Zhang ’14

<u>asymmetric</u> Yukawas → **possible for specific corrections**
Y_e **(via** $Y_{\bar{5}}$**)**

discrete non-Abelian family symmetry embedding

Perez et al. ’19

$$\mathcal{T}_{13} = Z_{13} \rtimes Z_3$$

Notable (often-found) feature: phase input needed

phase required in $\mathcal{U}_\nu \sim \mathcal{U}^{(\mathrm{HPS})}$ **for consistency with mixing angle data**

numerical example: $\delta \simeq \pm 1.3\pi, \quad J \simeq \mp 0.03$

CP Violation

- **spontaneous CP violation**

Generalized CP

$$X^T \mathcal{M}_\nu X = \mathcal{M}_\nu^* \qquad Y^\dagger \mathcal{M}_e \mathcal{M}_e^\dagger Y = (\mathcal{M}_e \mathcal{M}_e^\dagger)^*$$

“ordinary” CP has $X = Y = 1$ **Branco, Lavoura, Rebelo ’86…**

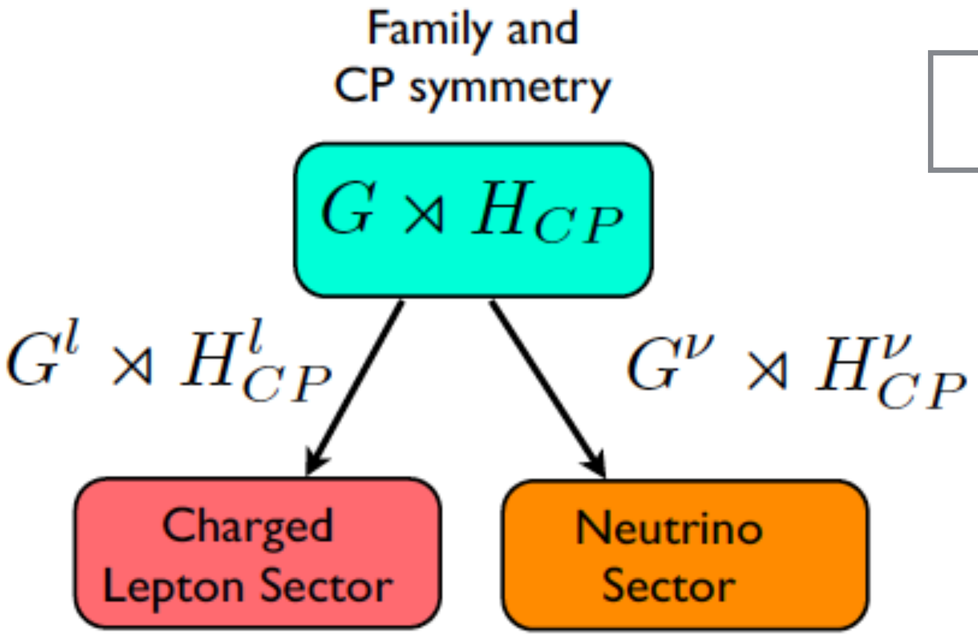
automorphisms of discrete family symmetry

Grimus, Rebelo ’95
Holthausen et al. ’12, Feruglio et al. ’12, Chen et al. ’14, Ding et al. ’14, Branco et al. ’15…

$$X \rho(g)^* X^{-1} = \rho(g')$$

consistency condition

family symmetry



many recent papers! see King ’17 for review

Residual/generalized CP symmetries

existence of “CP basis” **Holthausen, et al. ’12**

group classification **Chen et al. ’14**

bottom-up approach (Klein symm preserved)

Feruglio et al. ’12
L.E., Garon, Stuart ’15
L.E., Stuart ’16

…

Connection (or not) to quark sector

- not particularly straightforward (subjective!)

many examples, authors

recent notable case:
Hagedorn et al. ’18**

✓ discrete non-Abelian symmetry models:

quarks can require alternate embeddings

e.g. often $L_i \sim \mathbf{3}$ but $Q_i \sim \mathbf{2} \oplus \mathbf{1}$

want groups with both doublets and triplets

(larger groups, double covers $\mathcal{T}'$, $\mathcal{I}'$)

✓ work explicitly in SUSY GUT framework (Type I seesaw)

Dermisek et al. ’05, ’06

Poh, Raby ’15

$SO(10)$ with family symmetry:
$\mathcal{D}_3 \times U(1) \times Z_3 \times Z_3$ $\quad \mathbf{2} \oplus \mathbf{1}$

14 fermion sector inputs → 6 fermion sector predictions

Pati-Salam version (lighter superpartners)

Poh, Raby, Wang ’17

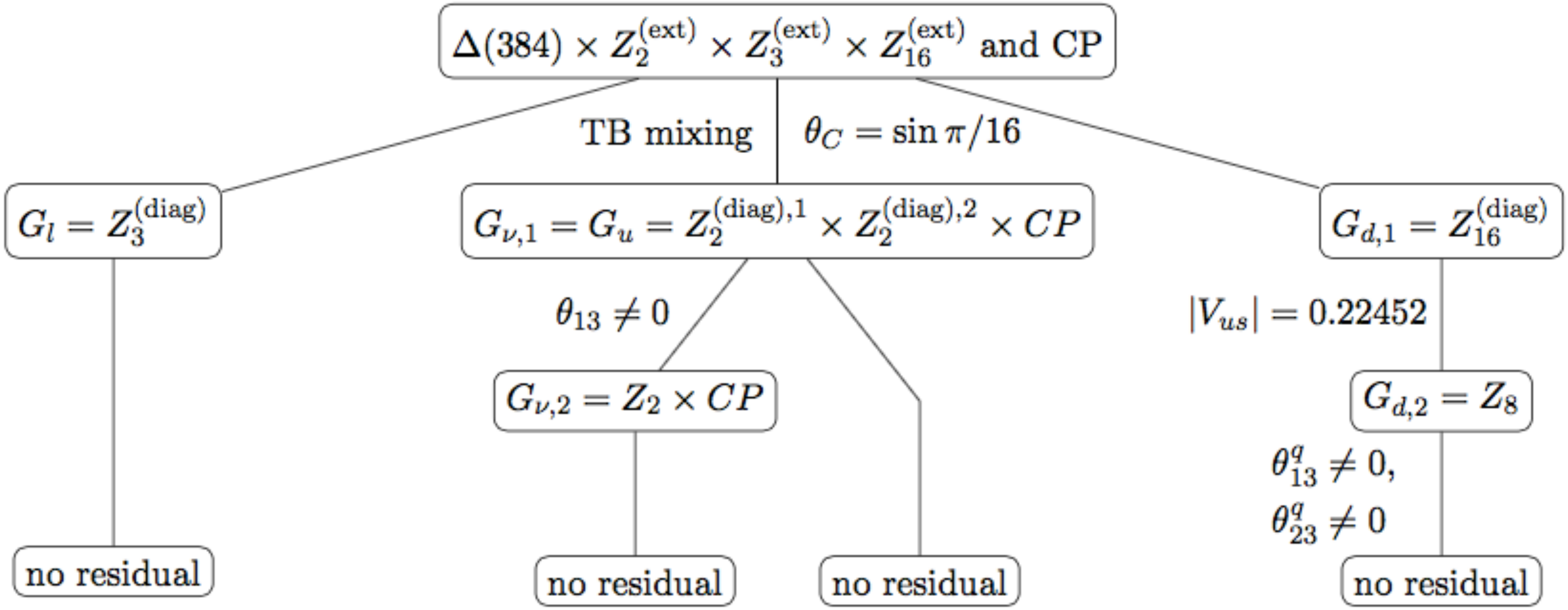
(17 fermion sector inputs)

Example: full quark, lepton flavor model with $\Delta(384)$

Hagedorn and Konig, ’18

MSSM framework, Type I seesaw

$Q_i, L_i \sim \mathbf{3}$


$\Delta(384) \times Z_2^{(\mathrm{ext})} \times Z_3^{(\mathrm{ext})} \times Z_{16}^{(\mathrm{ext})}$ and CP

TB mixing

$\theta_C = \sin \pi/16$

$G_l = Z_3^{(\mathrm{diag})}$

$G_{\nu,1} = G_u = Z_2^{(\mathrm{diag}),1} \times Z_2^{(\mathrm{diag}),2} \times CP$

$G_{d,1} = Z_{16}^{(\mathrm{diag})}$

$\theta_{13} \neq 0$

$|V_{us}| = 0.22452$

$G_{\nu,2} = Z_2 \times CP$

$G_{d,2} = Z_8$

$\theta_{13}^q \neq 0,$

$\theta_{23}^q \neq 0$

no residual

no residual

no residual

no residual

leptons: first break to TBM → reactor angle generated

quarks: first step breaking, $\lambda \sim \pi/16$ → Cabibbo angle corrected

requires intricate sector of flavons+ driving fields (characteristic)

Top-down

- **String constructions:**

Many authors!
see e.g. Langacker'11 for review

✓ variety of possibilities for mass suppression

higher-dimensional operators (field theoretic)

geometric suppression (braneworlds) $Y \sim e^{-A}$

worldsheet instantons (nonperturbative)

✓ Yukawa unification often not retained even in GUT scenarios

✓ not necessarily just minimal Type I seesaw

ν_R candidates often not pure gauge singlets

explorations in heterotic orbifolds Giedt et al. '05, Buchmuller et al.'07…

"Mixed" scenarios (e.g. seesaw + R-parity violation)

e.g. G2 models Acharya et al. '08…
F theory GUTs Beasley, Heckman, Vafa '08,…

Concluding remarks

- Neutrino masses and lepton mixing angles: paradigm shift

many ways to suppress overall neutrino mass scale

lepton mixings: anarchical or structural

masses/mass ordering “secondary”

not easy to unify/wed with quark sector

needed physics often at high scales

A great amount of theoretical effort, but still seeking

compelling, complete, testable theories

More observational handles needed!

New ideas welcomed!