Inflation and Quantum Gravity

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University of Wisconsin-Madison

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Chaos and complementarity in de Sitter space

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Abstract

We consider small perturbations to a static three-dimensional de Sitter geometry. For early enough perturbations that satisfy the null energy condition, the result is a shock-wave geometry that leads to a time advance in the trajectory of geodesics crossing it. This brings the opposite poles of de Sitter space into causal contact with each other, much like a traversable wormhole in Anti-de Sitter space. In this background, we compute out-of-time-order correlators (OTOCs) to asses the chaotic nature of the de Sitter horizon and find that it is maximally chaotic: one of the OTOCs we study decays exponentially with a Lyapunov exponent that saturates the chaos bound. We discuss the consequences of our results for de Sitter complementarity and inflation.

The String Swampland and Quantum Gravity Constraints on Effective Theories

Coordinators: Hiroshi Ooguri, Gary Shiu, Cumrun Vafa, and Irene Valenzuela

The idea that the string landscape is too large to lead to concrete predictions has been countered by the idea that most of the naively consistent effective theories of gravity coupled to matter are actually inconsistent and belong to the swampland. The identification of criteria distinguishing the true string landscape from the swampland, which has been studied for more than a decade now, is beginning to reach a more mature stage with the developments of the last few years. In particular a conjectured consistency condition for quantum gravity known as the Weak Gravity Conjecture (WGC), which postulates that gravity is always the weakest force among all the forces, has found an unexpectedly broad range of applications.

The WGC on the one hand has been used to constrain cosmological models of inflation including scenarios being tested by the present generation of CMB experiments and on the other hand has been connected to the cosmic censorship conjecture of general relativity. Furthermore, ideas from holography have been found to be nicely consistent with the WGC. Moreover a sharpened version of the WGC has been used to put constraints on particle phenomenology and in particular has been used to place bounds on the neutrino masses. This program will bring together the diverse communities of string theorists, cosmologists, general relativists, particle phenomenologists and researchers working on holography and the conformal bootstrap to further develop consistency criteria for quantum theories of gravity and possibly extract concrete predictions from these ideas for the observable universe as well as deepen our understanding of the structure of string vacua.

DATES

Feb 18, 2020 - Mar 13, 2020

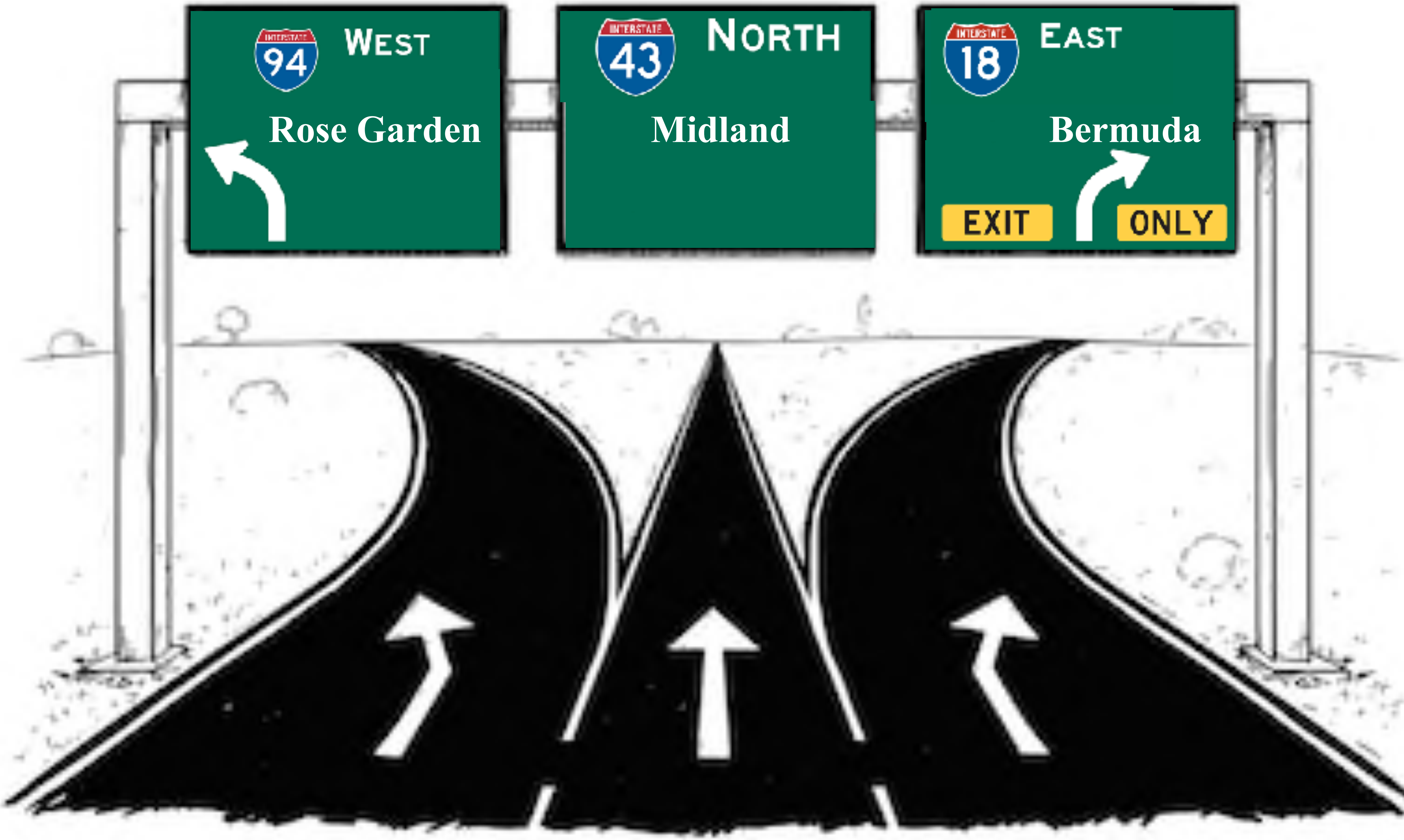
INFORMATION

Apply

Application deadline is:
Nov 18, 2018.
Applications will be considered and invitations will be issued after the above deadline.

QUICK LINKS

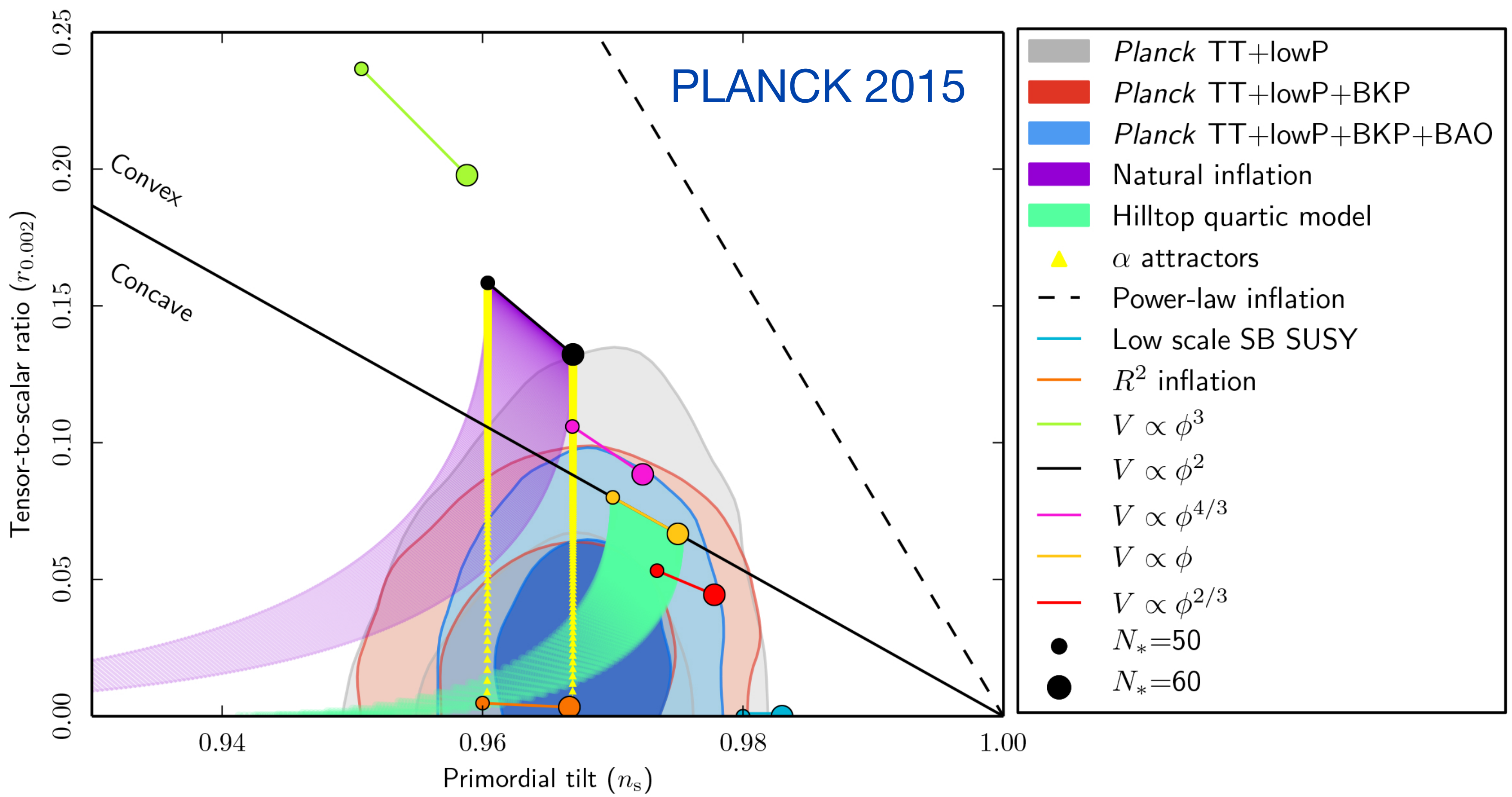
Three Roads to the Swampland

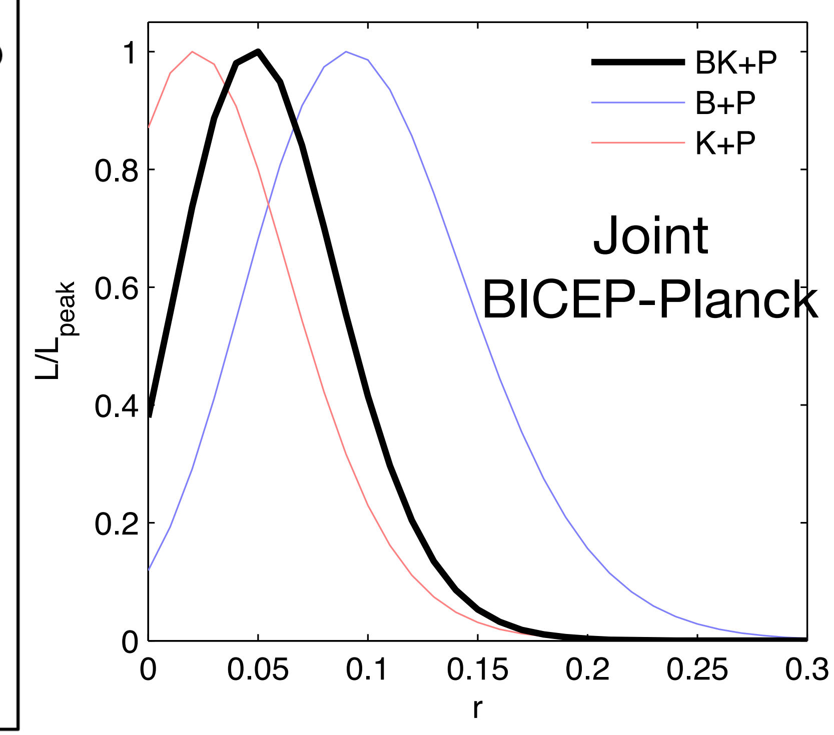



Landscape
Swampland

Inflation and Gravity Waves

Primordial Gravitational Waves





Ongoing experiments can potentially detect primordial B-mode with a tensor-to-scalar ratio r as small as $\sim 10^{-2}$.

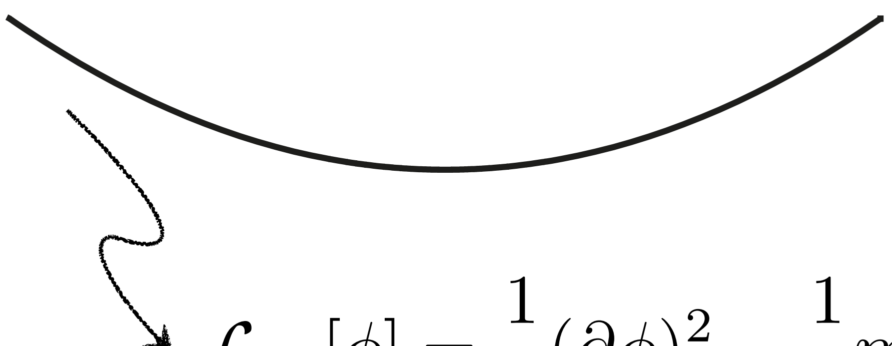
Further experiments, such as CMB-S4 and LiteBIRD, .. may improve further the sensitivity to r as small as $\sim 10^{-3}$.

B-mode and UV Sensitivity

A detection at the targeted level implies that the inflaton potential is nearly flat over a **super-Planckian** field range:

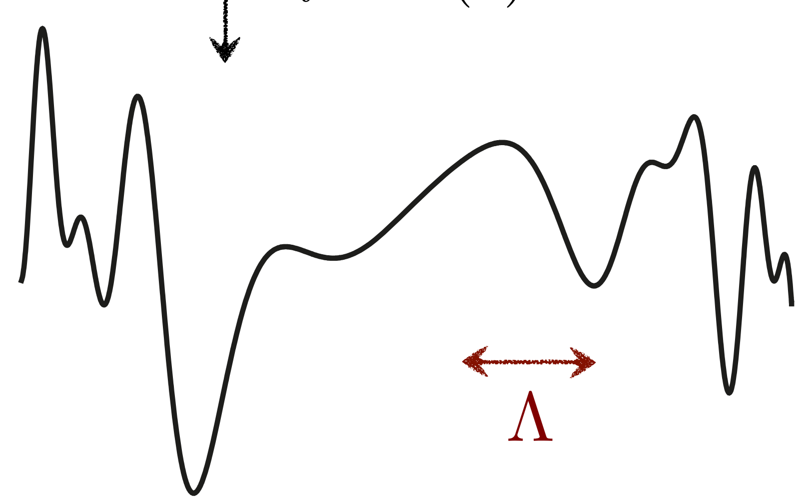
$$\Delta\phi \gtrsim \left(\frac{r}{0.01}\right)^{1/2} M_{\mathrm{Pl}}$$

Lyth '96



$$\mathcal{L}_{\mathrm{eff}}[\phi] = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\left(1 + \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}} + \cdots\right)$$

$c_i \sim \mathcal{O}(1)$

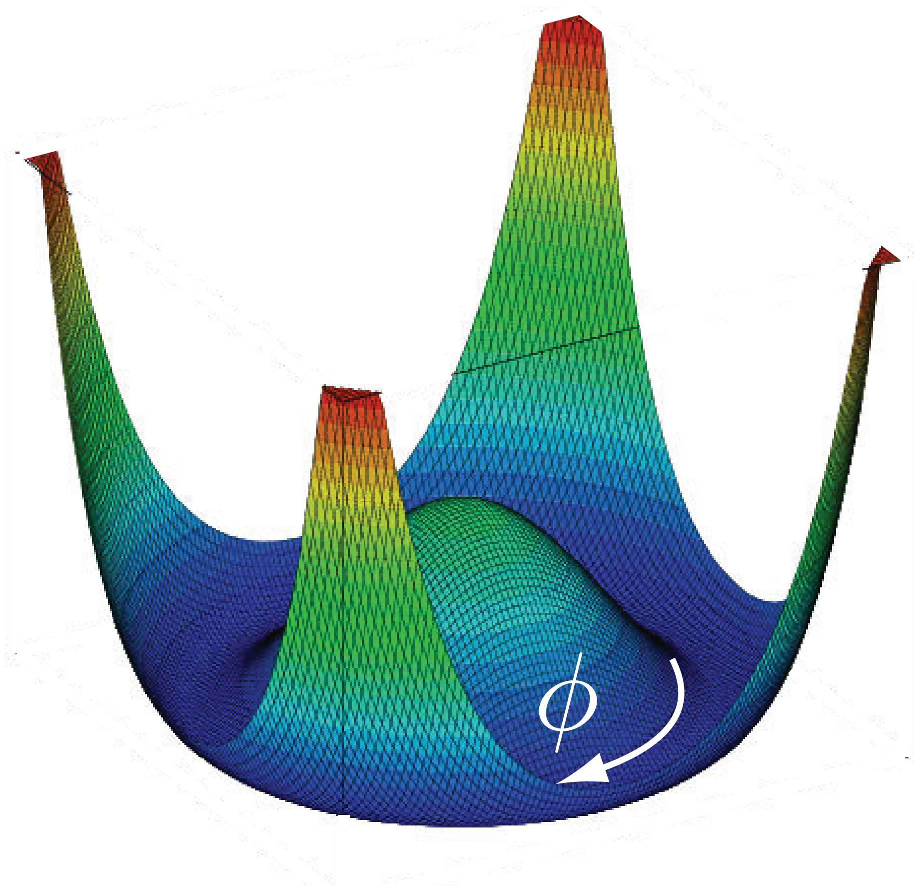


"Large field inflation" are highly **sensitive to UV physics**

Axions & Large Field Inflation

Natural Inflation [Freese, Frieman, Olinto]

Pseudo-Nambu-Goldstone bosons are natural inflaton candidates.



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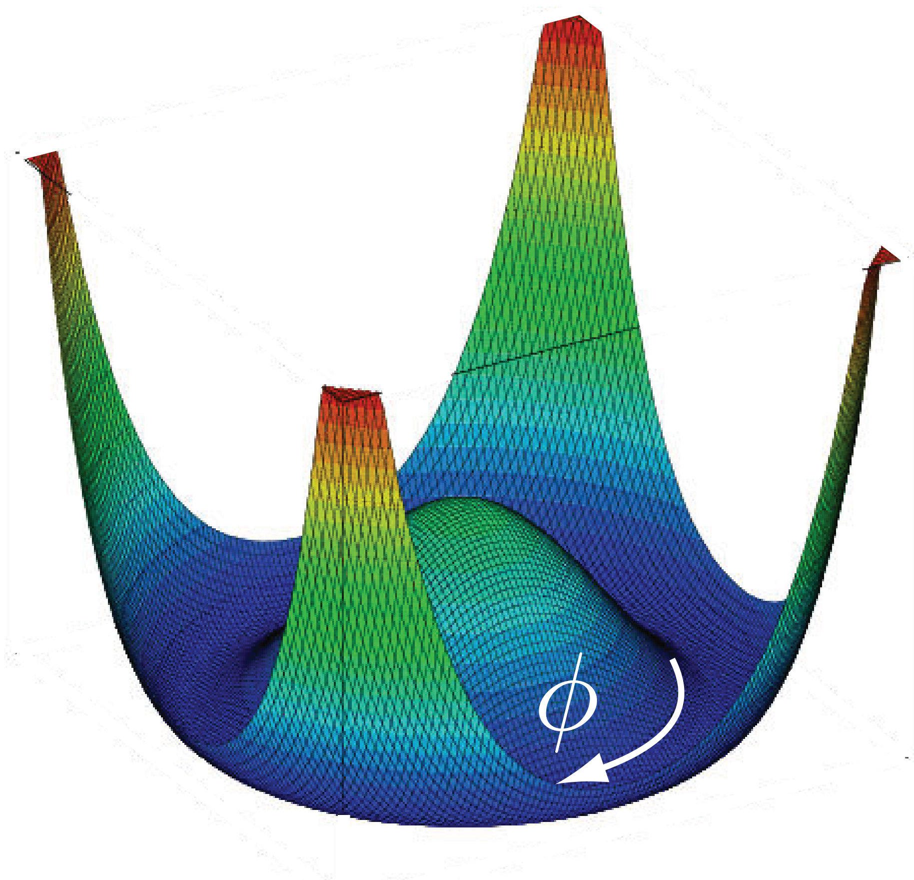
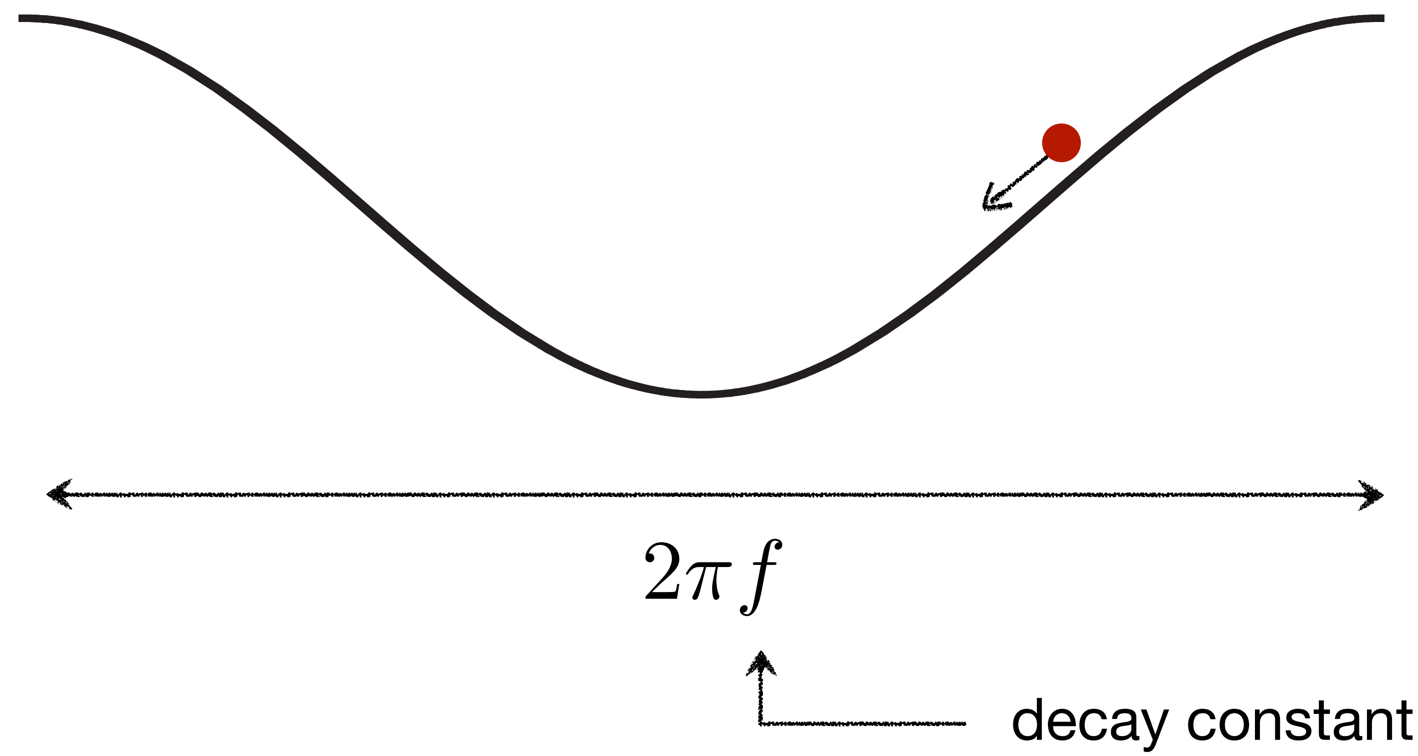


They satisfy a shift symmetry that is only broken by non-perturbative effects:



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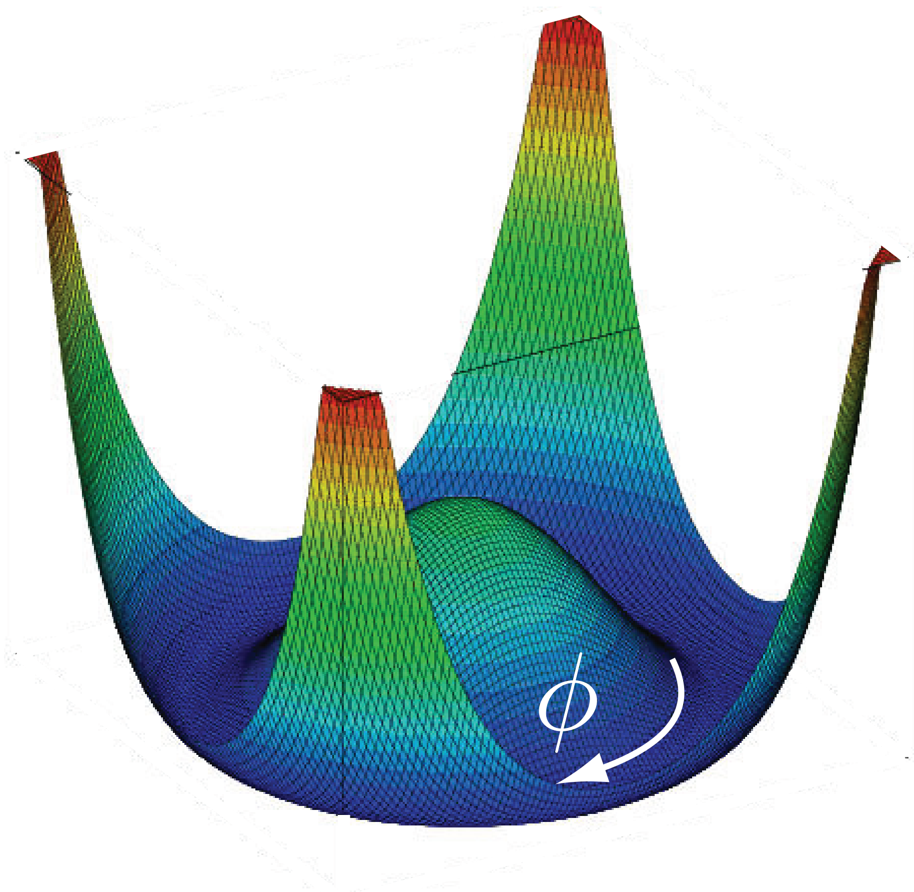

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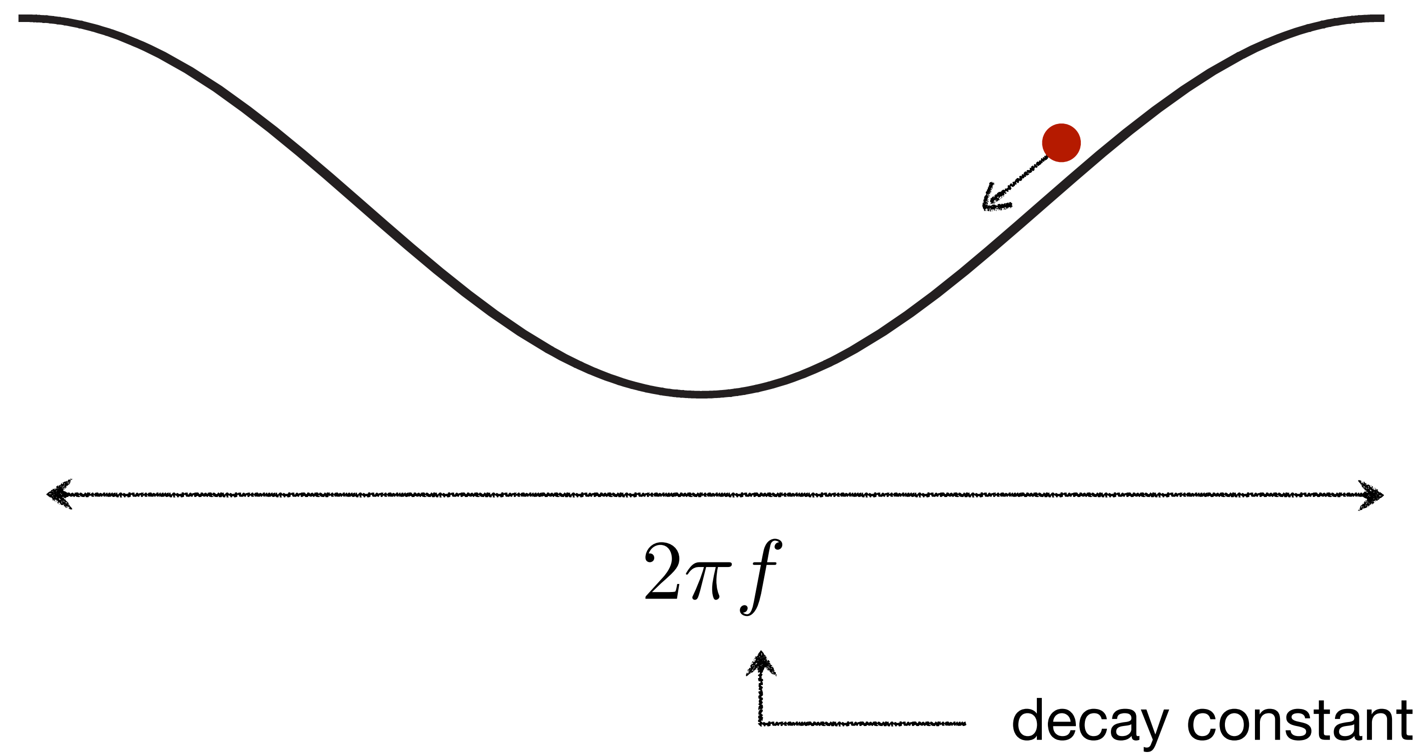

Slow roll: $f > M_P$

$$V(\phi) = 1 - \Lambda^{(1)} \cos\left(\frac{\phi}{f}\right) + \sum_{k>1} \Lambda^{(k)} \left[1 - \cos\left(\frac{k\phi}{f}\right)\right] \quad \text{if} \quad \frac{\Lambda^{(n+1)}}{\Lambda^{(n)}} \sim e^{-m} \ll 1$$

Axions in String Theory

String theory has many **higher-dimensional form-fields**:

e.g. $$F = \mathrm{d}A$$

3-form flux → F ; A ← 2-form gauge potential:

gauge symmetry: $A \to A + \mathrm{d}\Lambda$

Integrating the 2-form over a 2-cycle gives an **axion**:

$$a(x) \equiv \int_{\Sigma_2} A$$

The gauge symmetry becomes a **shift symmetry**.

Axions with super-Planckian decay constants don't seem to exist in controlled limits of string theory. [Banks, Dine, Fox, Gorbatov, '03]

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

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- The conjecture:

"Gravity is the Weakest Force"

- This is a scale-dependent statement, but as we'll see, the WGC comes with a UV cutoff Λ (magnetic WGC).
- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{“}1\text{”}$$

- This implies an extremal BH can decay.
- Applying the WGC to magnetically charged states imply:

$$q_{mag} \sim 1/g, \quad m_{mag} \sim \Lambda/g^2 \quad \Rightarrow \quad \Lambda \lesssim g(\Lambda) M_P$$

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$$\frac{q}{m} M_P \geq \text{“}1\text{”} \equiv \frac{Q_{Ext}}{M_{Ext}} M_P$$

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- Applying the WGC to magnetically charged states imply:

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WGC for p-forms

Arkani-Hamed, Motl, Nicolis, Vafa ‘06

- One can generalize the WGC for 1-forms to the WGC for p-forms which couple to (p-1)-branes:

$$\frac{Q_{p-1}}{T_{p-1}} \geq \left(\frac{Q_{p-1}}{T_{p-1}} \right)_{\mathrm{Ext}}$$

- However, the 0-form gauge field (axion) case is more subtle as the “branes” that couple to it are **instantons**.

WGC and Axions

- Consider a U(1) gauge theory in 5d, and compactify on S to 4d. Shift symmetry originates from gauge symmetry: $A_M(x, x_4) \to (A_\mu(x), \phi(x))$

$$S = \int d^5x \, \frac{-1}{4g_5^2} F_{MN}F^{MN} \quad \longrightarrow \quad \int d^4x \left(\frac{-1}{4g_4^2} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \right)$$

- Topologically non-trivial Euclidean configurations (instantons) with charged fields wrapping the 5d circle generate a potential

$$V(\phi) = e^{-S_{inst}} \cos\left(\frac{\phi}{f}\right)$$

$$S_{inst} = 2\pi R m_5$$
$$f^{-1} = q_5\sqrt{2\pi R}$$

- The 5d WGC for charged particles $m_5 < q_5 M_{p,5d}^3$ translates into:

$$f \cdot S_{inst} \leq M_p$$

- Duality [Brown, Cottrell, GS, Soler] or dimensional reduction [Heidenreich, Reece, Rudelius] maps the WGC for 1-forms to that for axions. Attempt for a direct argument [Andriolo, Huang, Noumi, Ooguri, GS, in progress].

Loopholes

[Brown, Cottrell, GS, Soler];[Rudelius]

- Naively, the WGC rules out "natural inflation".

- However the WGC requires f · m < 1 for ONE instanton, but not ALL

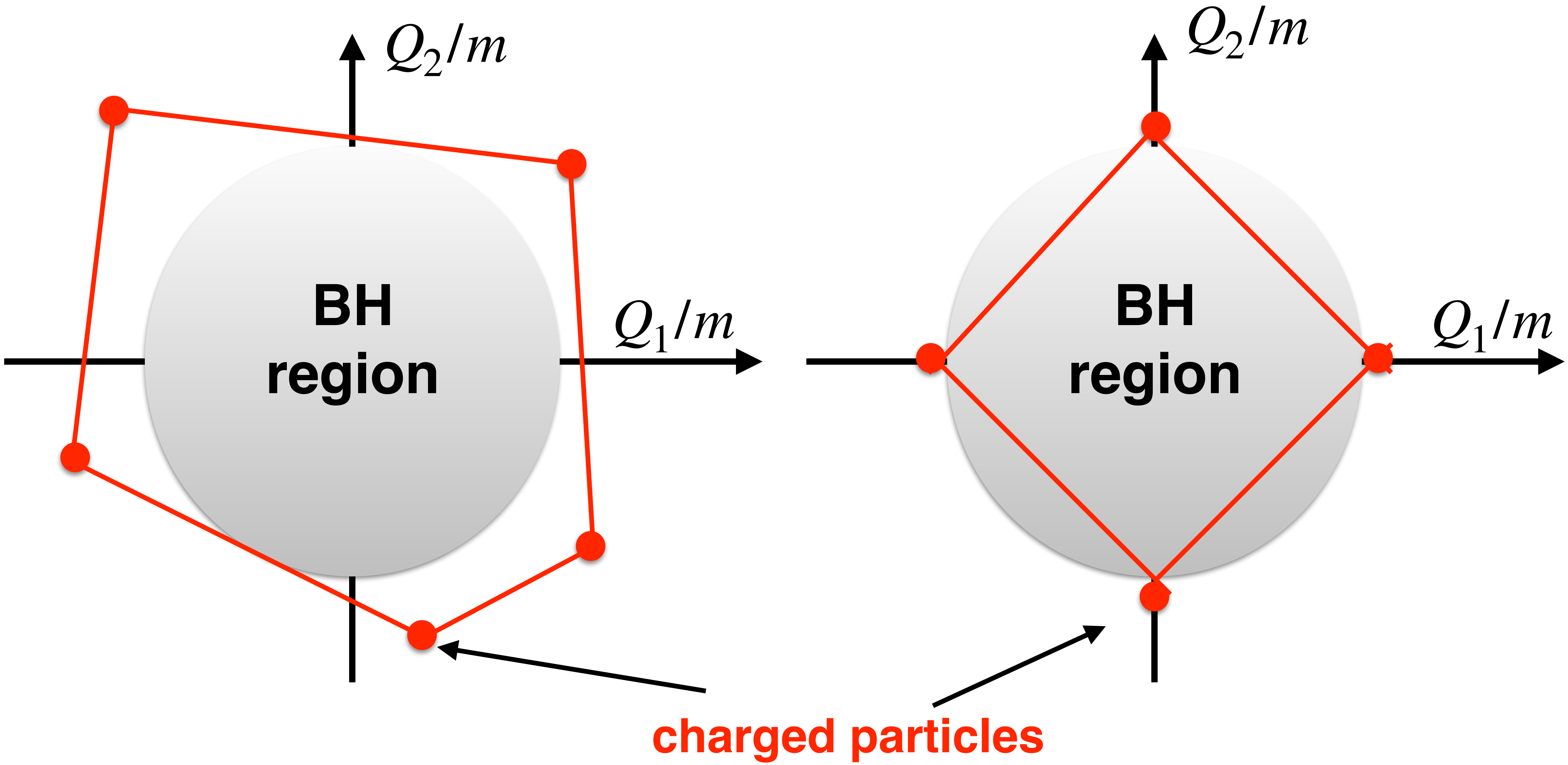
$$V = e^{-m}\left[1-\cos\left(\frac{\Phi}{F}\right)\right] + e^{-M}\left[1-\cos\left(\frac{\Phi}{f}\right)\right]$$

- with $1 < m \ll M, \qquad F \gg M_P > f, \qquad M \times f \ll 1$

- The second instanton fulfills the WGC, but is negligible, an "spectator". Inflation is governed by the first term.

- Another loophole is non-periodic axions (aka axion monodromy) as they are not mapped to long-range gauge fields.

Convex Hull Condition

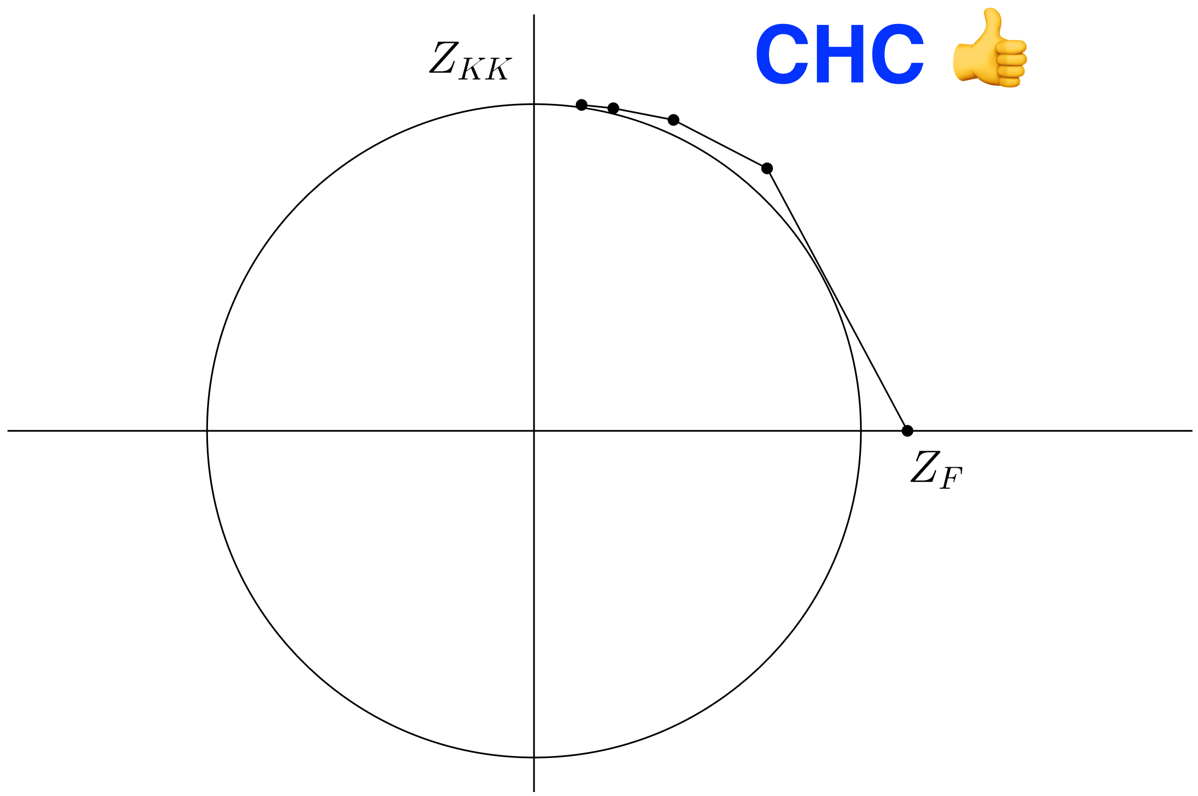


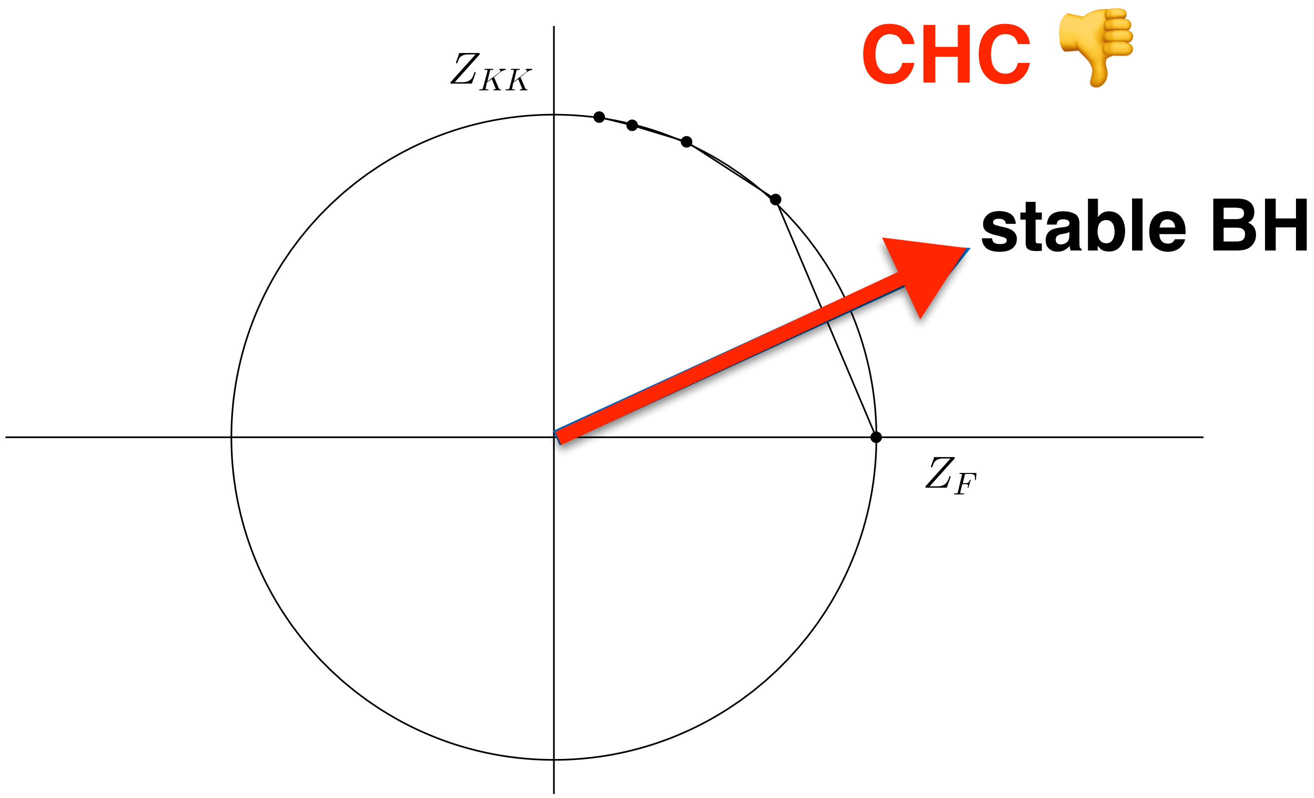
[Cheung, Remmen, '14]

Tower/Sub-Lattice WGC

- Compactifying a theory on a circle gives rise to an additional $U(1)_{KK}$. Apply the WGC to black holes with general charges.

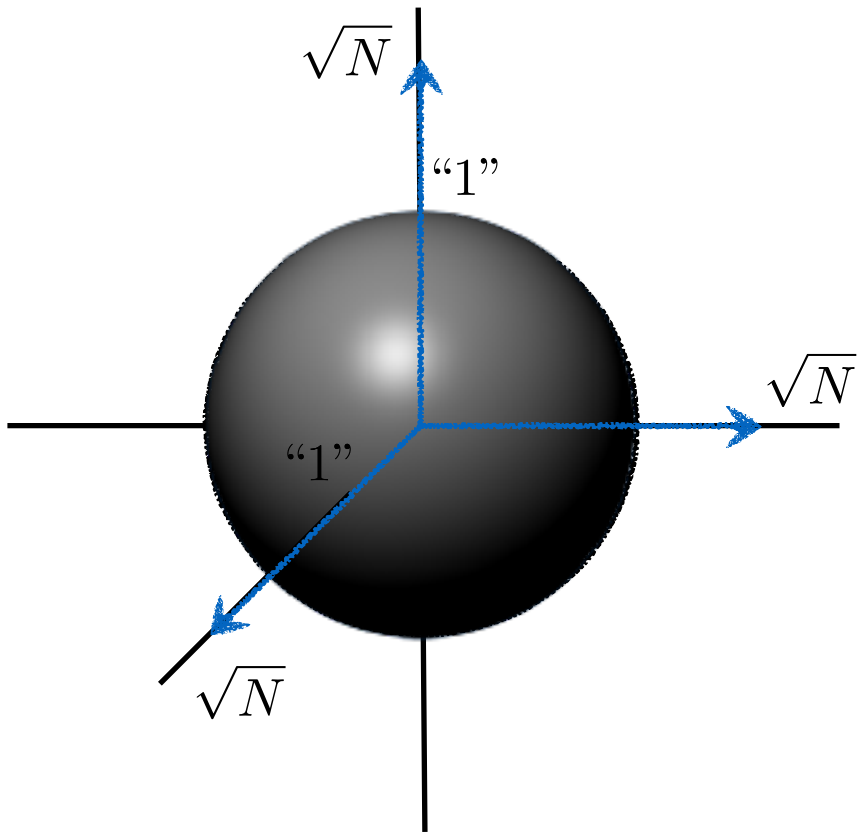
- Infinite tower of (super)extremal KK states. Charge-to-mass ratio depends on the radius:





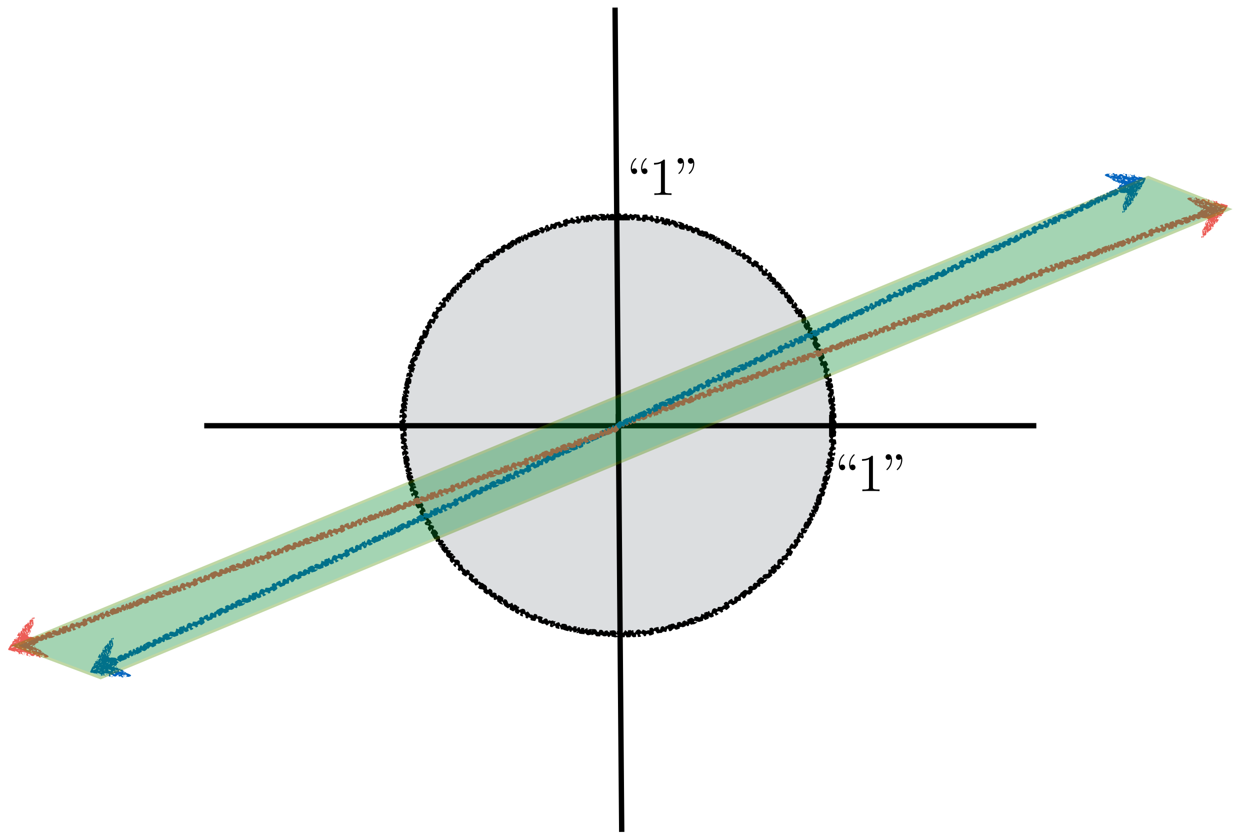
- Convex hull not guaranteed to contain the BH region. This motivates a stronger version of the WGC known as Tower/Sub-Lattice WGC
[Andriolo,Junghans, Noumi, GS];[Heidenreich, Reece, Rudelius];[Montero, GS, Soler]

Multi-Axion Inflation



N-flation

[Dimopoulos, Kachru, McGreevy, Wacker, '05]

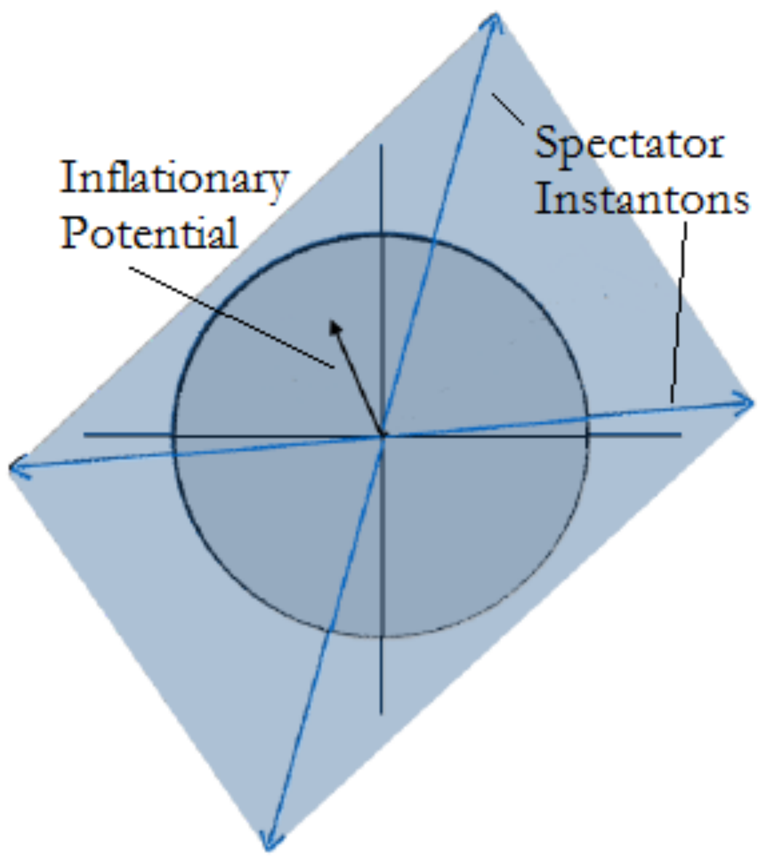


Alignment/Clockwork

[Kim, Nilles, Peloso, ’04];[Choi, Kim, Yun, '14];[Choi, Im, '15];[Kaplan, Rattazzi, ’15]

Naively they violate the WGC, but one can come up with loopholes…

Loopholes



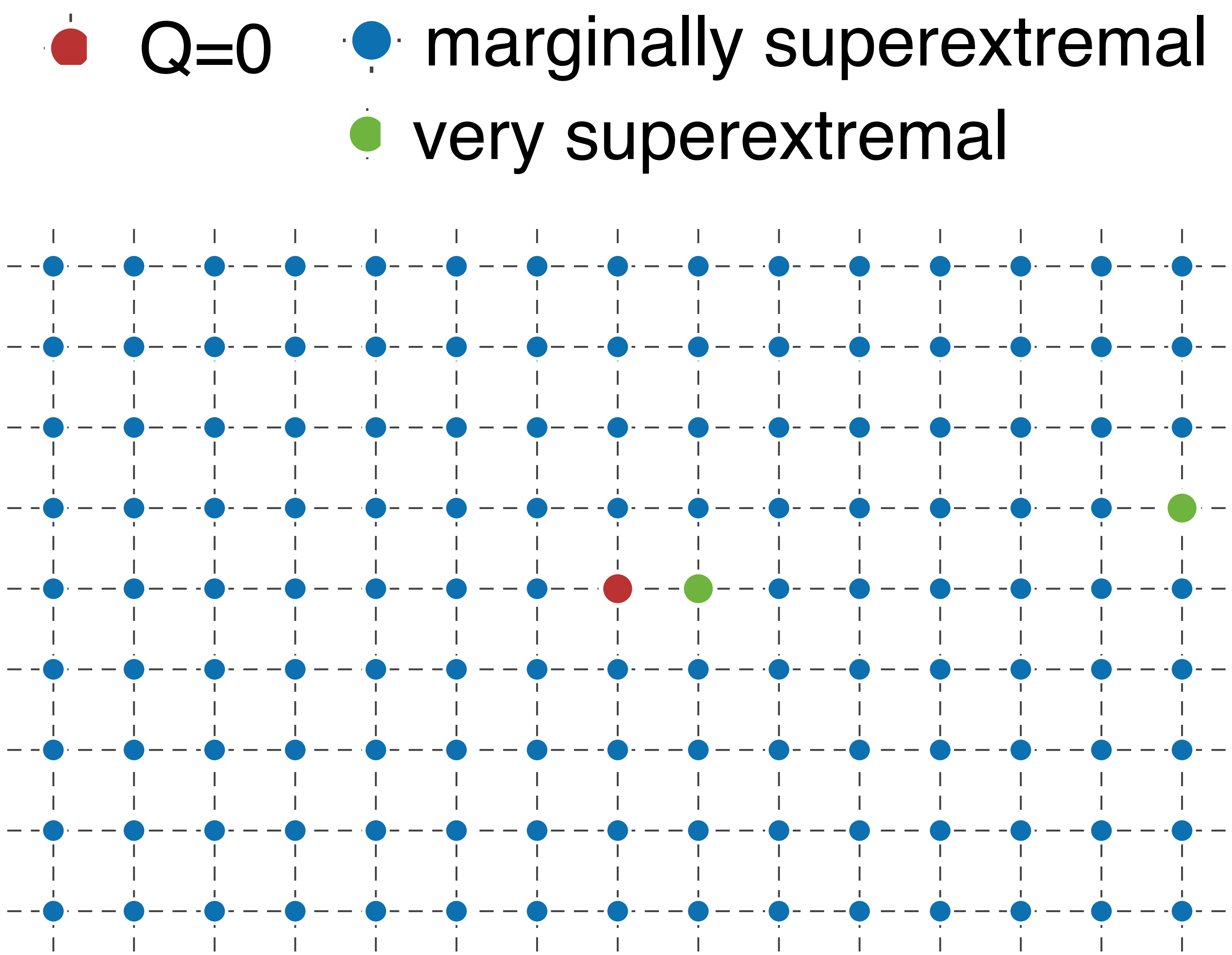
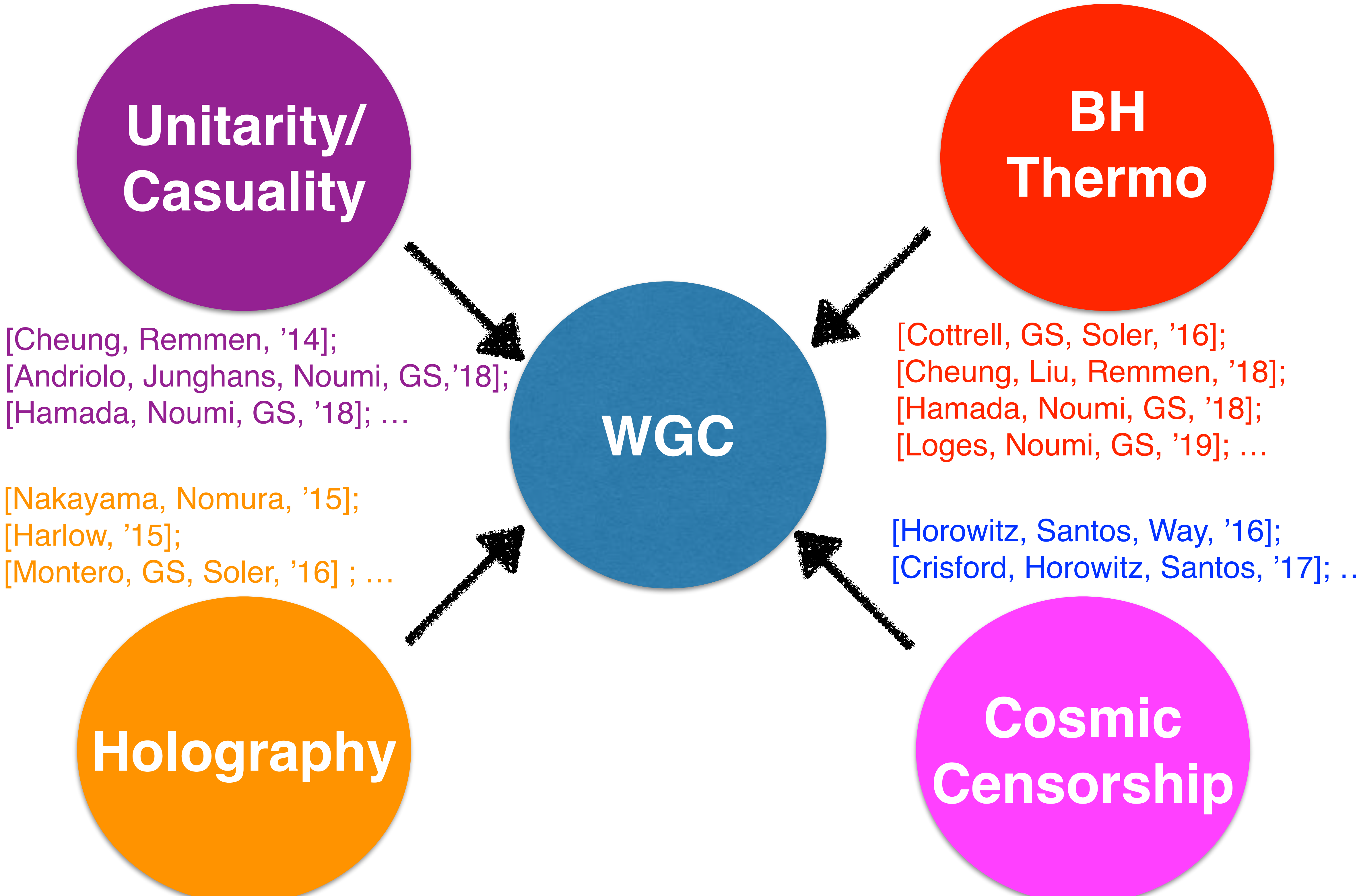


Figure from
[Brown, Cottrell, GS, Soler, ’15]

Figure from
[Heidenreich, Long, McAllister, Rudelius, Stout, ‘19]

Evidence for the WGC

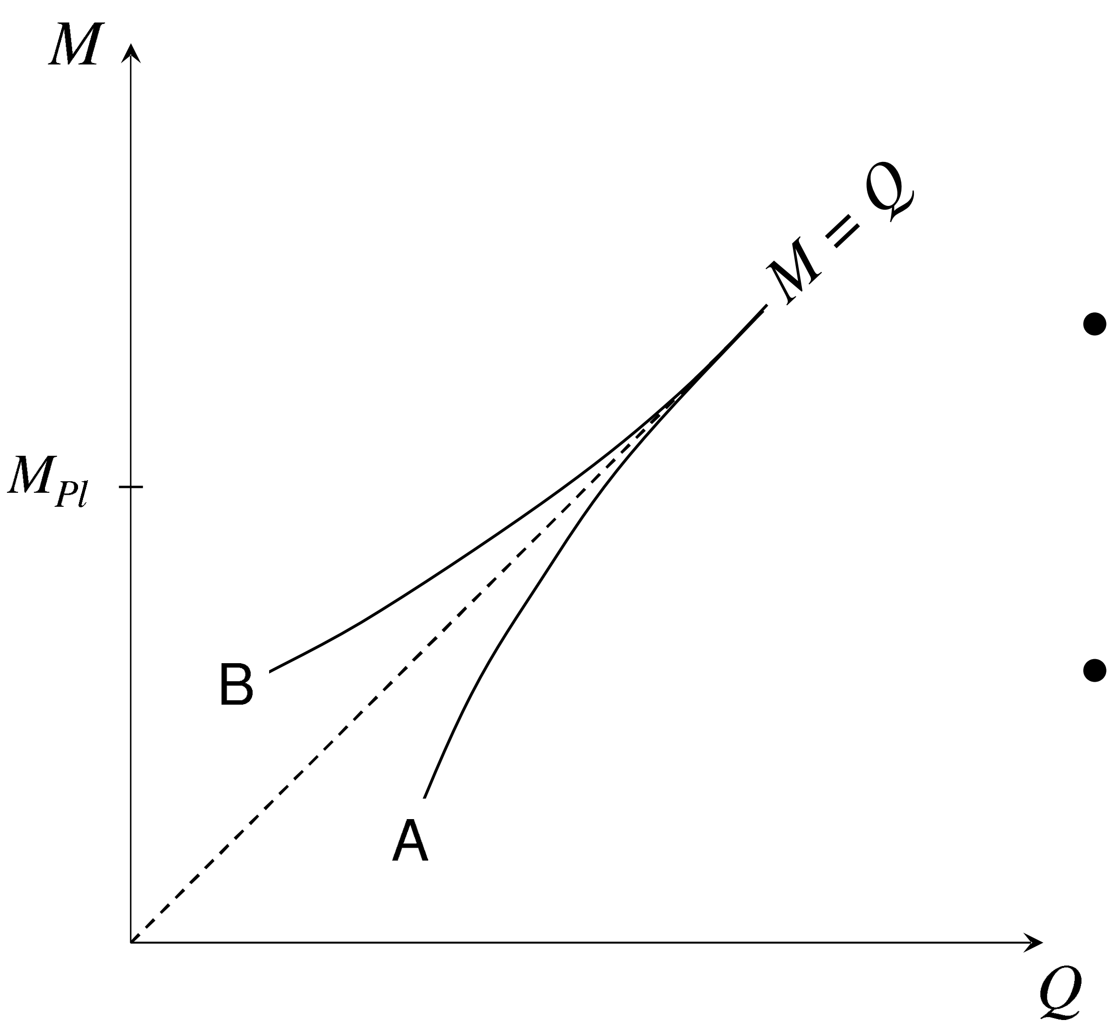
Evidence for the WGC



WGC and Black Holes

Extremality of Black Holes

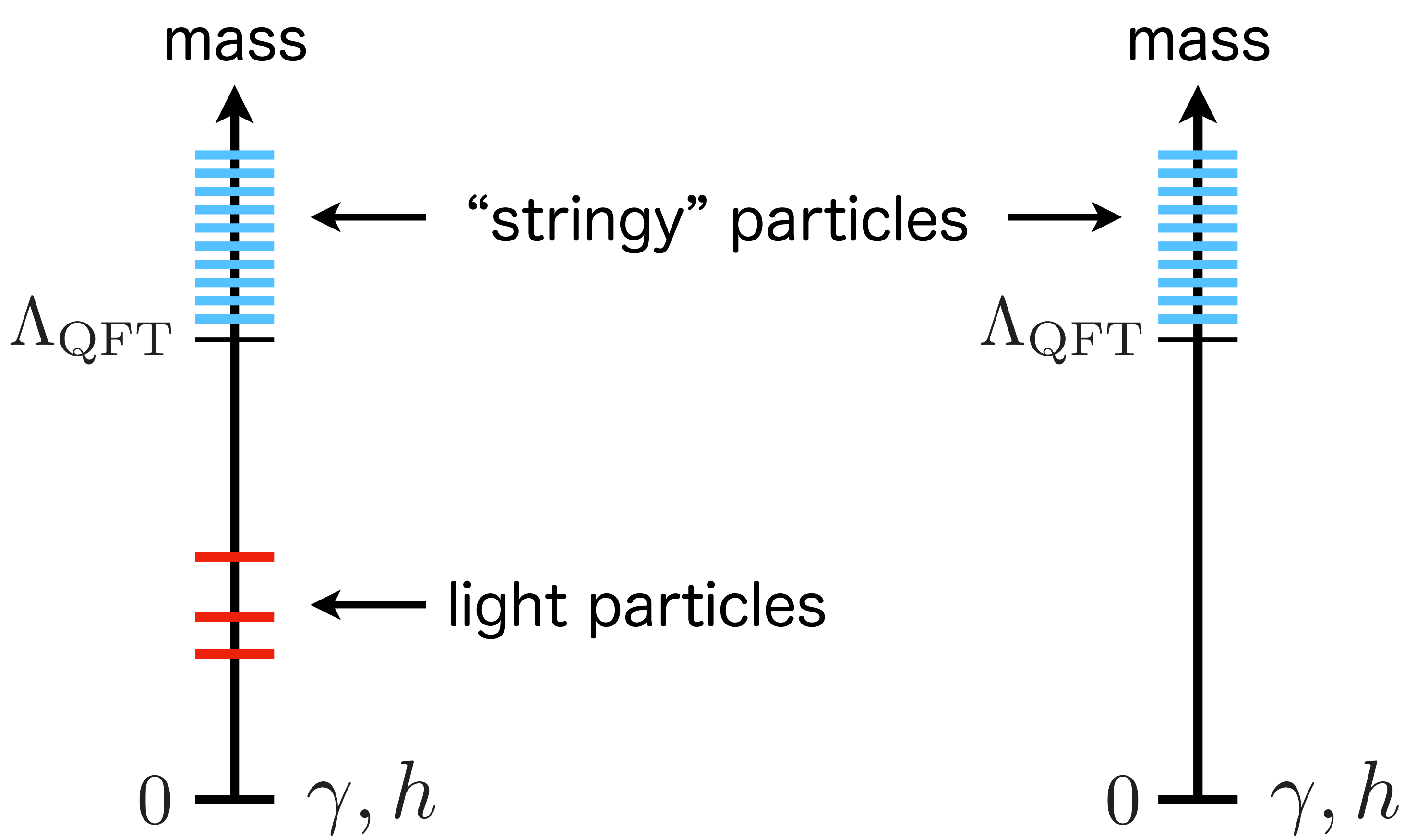
- The mild form of the WGC requires only ***some*** state for an extremal BH to decay to.
- **Can an extremal BH satisfy the WGC?**



- Higher derivative corrections can make extremal BHs lighter than the **classical bound** Q=M
- Demonstrated to be the case for 4D heterotic extremal BHs. **[Kats, Motl, Padi, '06]**
- We showed that this behavior (A) follows from unitarity (at least for some classes of theories). **[Hamada, Noumi, GS]**

WGC from Unitarity and Causality

- We assume a ***weakly coupled UV completion*** at scale Λ_{QFT}. Our proof for the strict WGC bound applies to at least two classes of theories:



  - Theories with ***light*** (compared with Λ_{QFT})***, neutral i) parity-even scalars*** (e.g., dilaton, moduli), or **ii) spin ≥ 2 particles**
  - ***UV completion*** where the photon & the graviton are accompanied by different sets of Regge states (as in open string theory).

Higher Derivative Corrections

- In the IR, the BH dynamics is described by an EFT of photon & graviton.
- In D=4, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$S = \int d^4x \sqrt{-g} \left[\frac{2M_{\rm Pl}^2}{4} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Delta \mathcal{L} \right]$$

where

$$\begin{aligned} \Delta\mathcal{L} = \quad & c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & + c_4 R F_{\mu\nu} F^{\mu\nu} + c_5 R_{\mu\nu} F^{\mu\rho} F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \\ & + c_7 F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + c_8 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}. \end{aligned}$$

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- In D=4, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$S = \int d^4x\sqrt{-g}\left[\frac{2M_{\rm Pl}^2}{4}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_1}{4M_{\rm Pl}^4}(F_{\mu\nu}F^{\mu\nu})^2 \right.$$
$$\left. + \frac{\alpha_2}{4M_{\rm Pl}^4}(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \frac{\alpha_3}{2M_{\rm Pl}^2}F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}\right]$$

by field redefinition. Here, $W^{\mu\nu\rho\sigma}$ is the **Weyl tensor**:

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + \frac{1}{2}\left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) - \frac{1}{3}Rg_{\mu[\rho}g_{\sigma]\nu}$$

Extremality Condition

- The higher derivative operators modify the BH solutions, so the charge-to-mass ratio of an extremal BH is corrected:

$$z = \frac{\sqrt{2} M_{\rm Pl} |Q|}{M} = 1 + \frac{2}{5} \frac{(4\pi)^2}{Q^2} (2\alpha_1 - \alpha_3)$$

[Kats, Motl, Padi, '06]

applicable when the BH is sufficiently heavy: $M^2 \sim Q^2 M_{\rm Pl}^2 \gg \alpha_i M_{\rm Pl}^2$

because extremal BHs in Einstein-Maxwell theory satisfy:

$$R \sim M_{\rm Pl}^4 / M^2 \text{ and } F^2 \sim M_{\rm Pl}^6 / M^2$$

- Proving the WGC (mild form) amounts to showing:

$$2\alpha_1 - \alpha_3 \geq 0\,.$$

so large extremal BHs can decay into smaller extremal BHs.

Sketch of the Proof: Step 1

[Hamada, Noumi, GS]

- We first show that for the aforementioned theories, **causality** implies

$$|\alpha_1| \gg |\alpha_3|$$

because α_3 leads to causality violation and an infinite tower of massive higher spin states is required to UV complete the EFT at tree-level **[Camanho, Edelstein, Maldacena, Zhiboedov].**

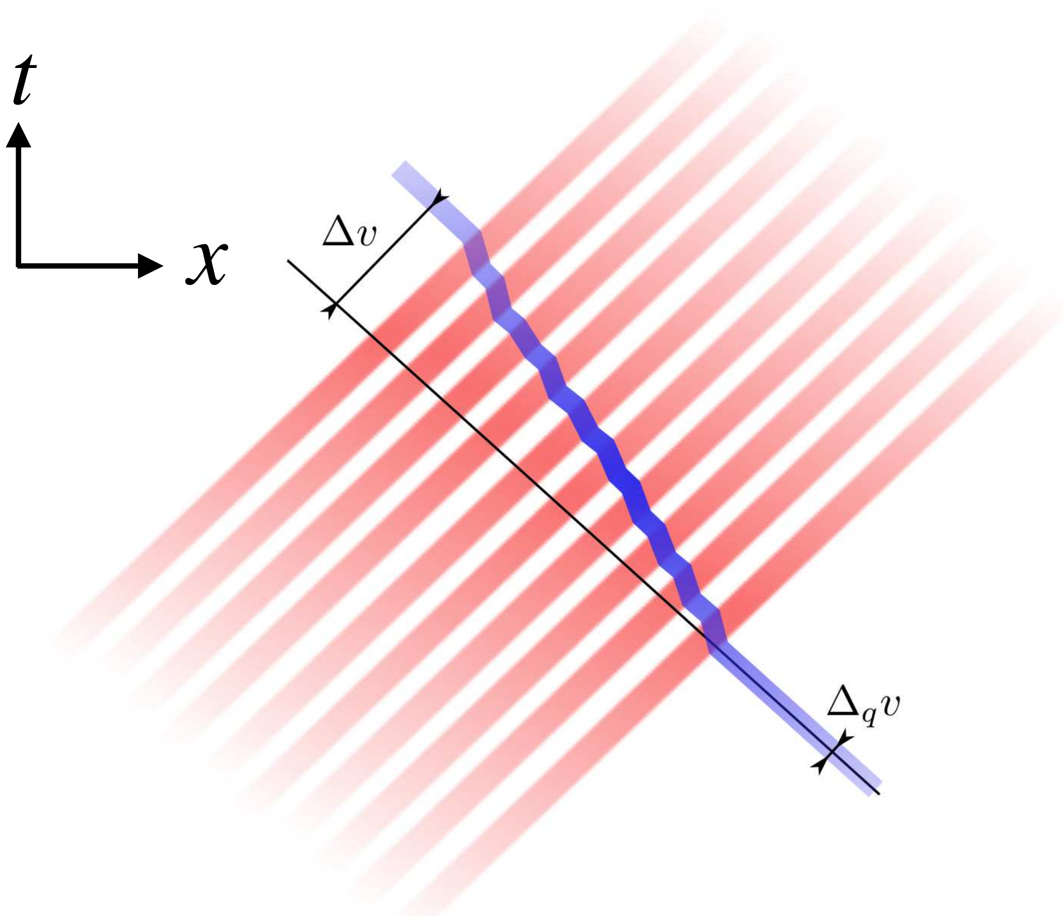


fig: Camanho et al '14

phase shift of photon propagation:

$$\delta \sim s\left(\ln(L_{\rm IR}/b) \pm \frac{|\alpha_3|}{b^2} + \dots\right)$$

time delay in GR

helicity dependent phase shit

b : impact parameter $L_{\rm IR}$: IR cutoff

Sketch of the Proof: Step 1

[Hamada, Noumi, GS]

- **Time advancement** if $b^2 \ln(L/b) \ll |\alpha_3|$
- Phase shift generated by spin J is $\delta \sim s^{J-1}$. A finite # of higher spin particles does not help → infinite tower of higher spin states.
- **Causality violation** unless the scale $M_P/\alpha_3^{1/2}$ is above Λ_{QFT}.
- Integrating out light neutral scalars does not give significant contributions to α_3 and so $|\alpha_1| \gg |\alpha_3|$
- If there are different Regge towers as in theories with open strings:

$$\alpha_{1,2,3}^{\text{closed}} \sim \frac{M_{\text{Pl}}^2}{M_s^2} \quad \ll \quad \alpha_{1,2}^{\text{open}} \sim \frac{M_{\text{Pl}}^2}{g_s M_s^2}, \qquad g_{\text{open}} \sim \sqrt{g_s} \gg g_s$$

- If there are light fields or different Regge towers, α_3 is **subdominant** compared with the causality preserving terms α_1 and α_2.

Sketch of the Proof: Step 2

[Hamada, Noumi, GS]

- The forward limit t→0 of γγ scattering for the aforementioned theories:

$$\mathcal{M}^{1234}(s) = \sum_n \left[\frac{g_{h_1 h_2 n} g_{\bar{h}_3 \bar{h}_4 n}}{m_n^2 - s} P^{1234}_{\mathbf{s}_n}(1) + \frac{g_{h_1 h_4 n} g_{\bar{h}_3 \bar{h}_2 n}}{m_n^2 + s} P^{1432}_{\mathbf{s}_n}(1) \right] + \text{analytic}$$

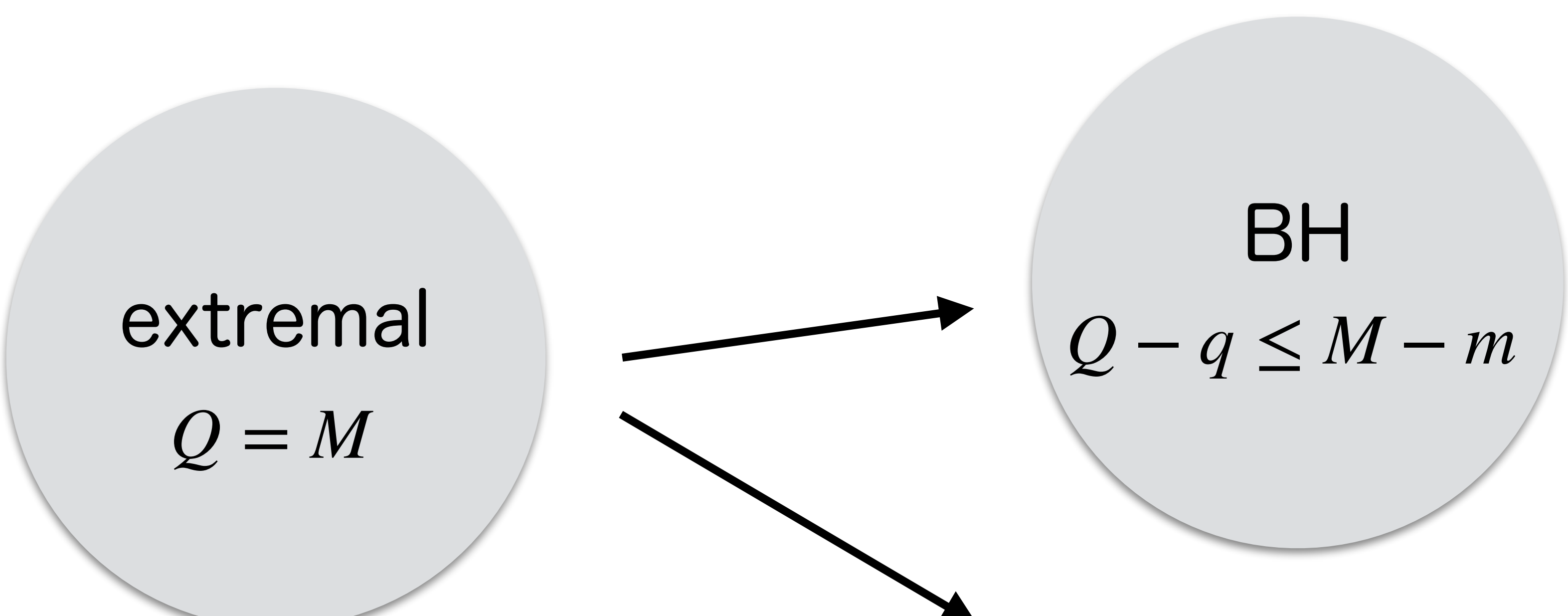
Spinning polynomials
[Arkani-Hamed, Huang, Huang, '17]

Froissart bound $a_n + b_n s$

- The higher derivative operator parametrized by α_1 leads to:

$$\alpha_1 (F_{\mu\nu} F^{\mu\mu})^2 \quad \Rightarrow \quad \mathcal{M} \sim \alpha_1 s^2$$

Unitarity ⇒ $\alpha_1 > 0$



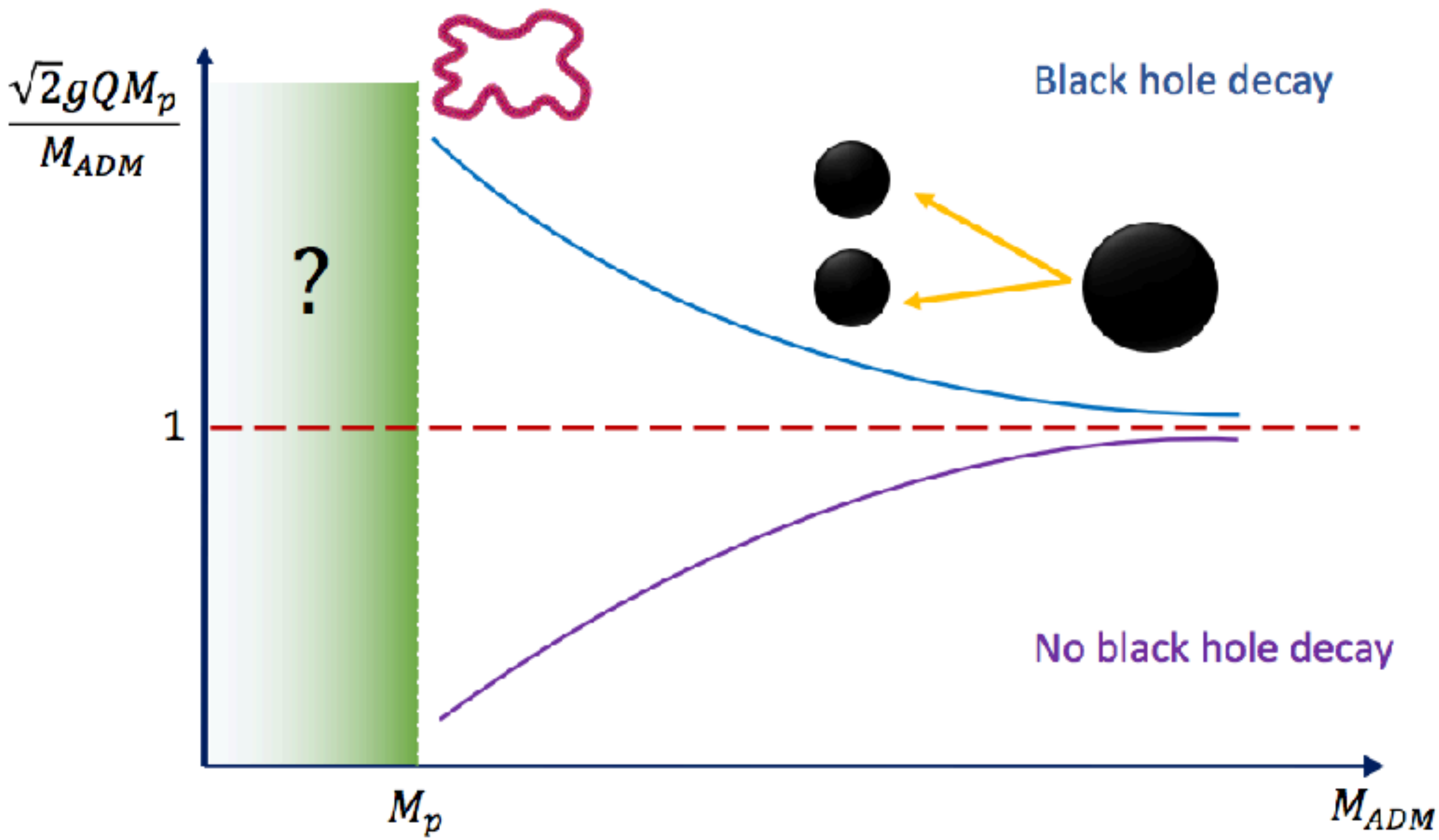
- a state $q \geq m$ **can be an extremal BH!**

WGC: Mild vs Strong

- Moreover, higher derivative corrections in WGC-satisfying theories increase the BH entropy as $z_{ext} > 1 \Leftrightarrow \Delta S > 0$ **[Hamada, Noumi, GS].**
- Similar results hold for **dilatonic, dyonic BHs** **[Loges, Noumi, GS].**
- **1-loop effects** of these higher derivative terms (dominated for asymptotically large BHs) increase z_{ext} **[Arkani-Hamed, Huang, Liu, unpublished];[Charles]** while for N=2 BPS BHs, z_{ext} remains 1.
- While the WGC can be satisfied by a very massive state, this mild form of the WGC seems rather toothless.
- For example, the 0-form version of the WGC can be satisfied by a large instanton action with negligible contribution to the potential (spectator instanton) **[Rudelius];[Brown, Cottrell, GS, Soler].**
- Could the underlying structure of string theory allows us to infer from the existence of a super-extremal BH the *stronger forms* mentioned?

WGC and Modular Invariance

- In **[Aalsma, Cole, GS],** we argued that for extremal BHs with a near horizon AdS_3 geometry, we can use modular invariance and anomalies to infer that there is a tower of superextremal states interpolating between perturbative string states and BHs.



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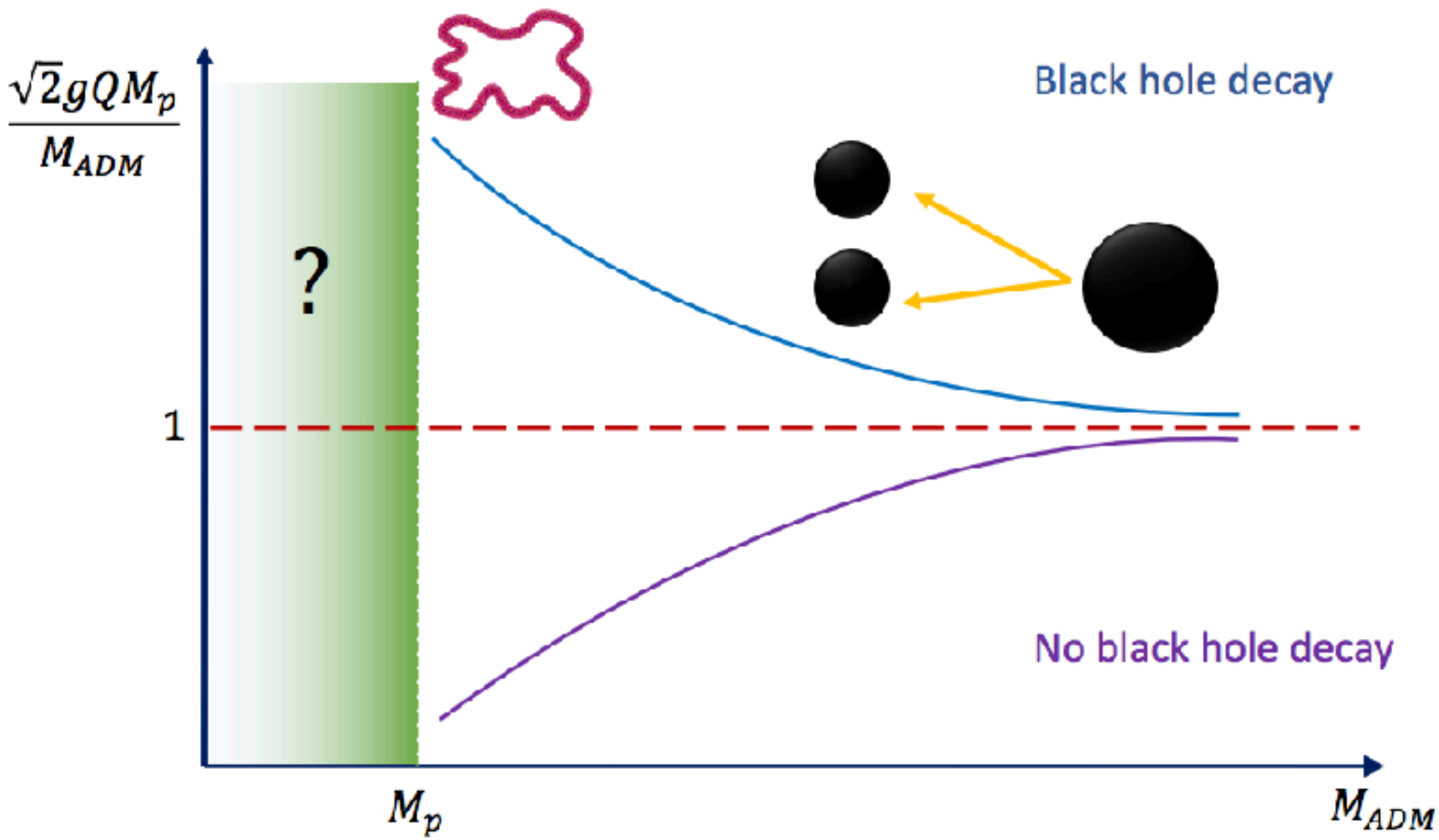


Spectral Flow

- The partition function of the worldsheet CFT in the NS-NS sector:

$$Z(\tau;\mu) \equiv \mathrm{Tr}\left(q^{\Delta} y^{Q} \overline{q}^{\tilde{\Delta}}\right)$$

where $\quad q = e^{2\pi i\tau}\,, \quad y = e^{2\pi i\mu}\,, \quad \Delta = L_0 - \dfrac{c}{24}$

- Q is the charge associated with the left-moving current J:

$$J_L(z)J_L(0) \sim \frac{k}{z^2} + \ldots$$

- The worldsheet partition function enjoys **modular invariance**:

T: $\quad Z(\tau+1;\mu) = Z(\tau;\mu), \quad$ **S:** $Z(-1/\tau;\mu/\tau) = e^{\pi i k \frac{\mu^2}{\tau}} Z(\tau;\mu)$

- Under a U(1) transformation:

$$Z(\tau;\mu+\rho) = Z(\tau;\mu) \qquad \forall \rho \in \Gamma_Q^* = \{\rho \,|\, \rho Q \in \mathbb{Z}\ \forall Q \in \Gamma_Q\}$$

Spectral Flow

- Performing a U(1) transformation in between two S transformations:

$$Z(\tau;\mu+\tau\rho)=\exp\left(-\pi i k\left[2\mu\rho+\rho^2\tau\right]\right)Z(\tau;\mu)$$

- This implies a spectral flow: $$L_0\to L_0+Q\rho+k\frac{\rho^2}{2}$$ $$Q\to Q+k\rho$$

- Consider a perturbative string state with charge q and mass m:

$$m=\sqrt{\frac{4}{\alpha'}\Delta}=\sqrt{\frac{4}{\alpha'}\tilde{\Delta}}$$

- Define the charge-to-mass ratio: $\mathcal{Z}'^2\equiv\frac{2}{k\alpha'}\frac{q'^2}{m'^2}$

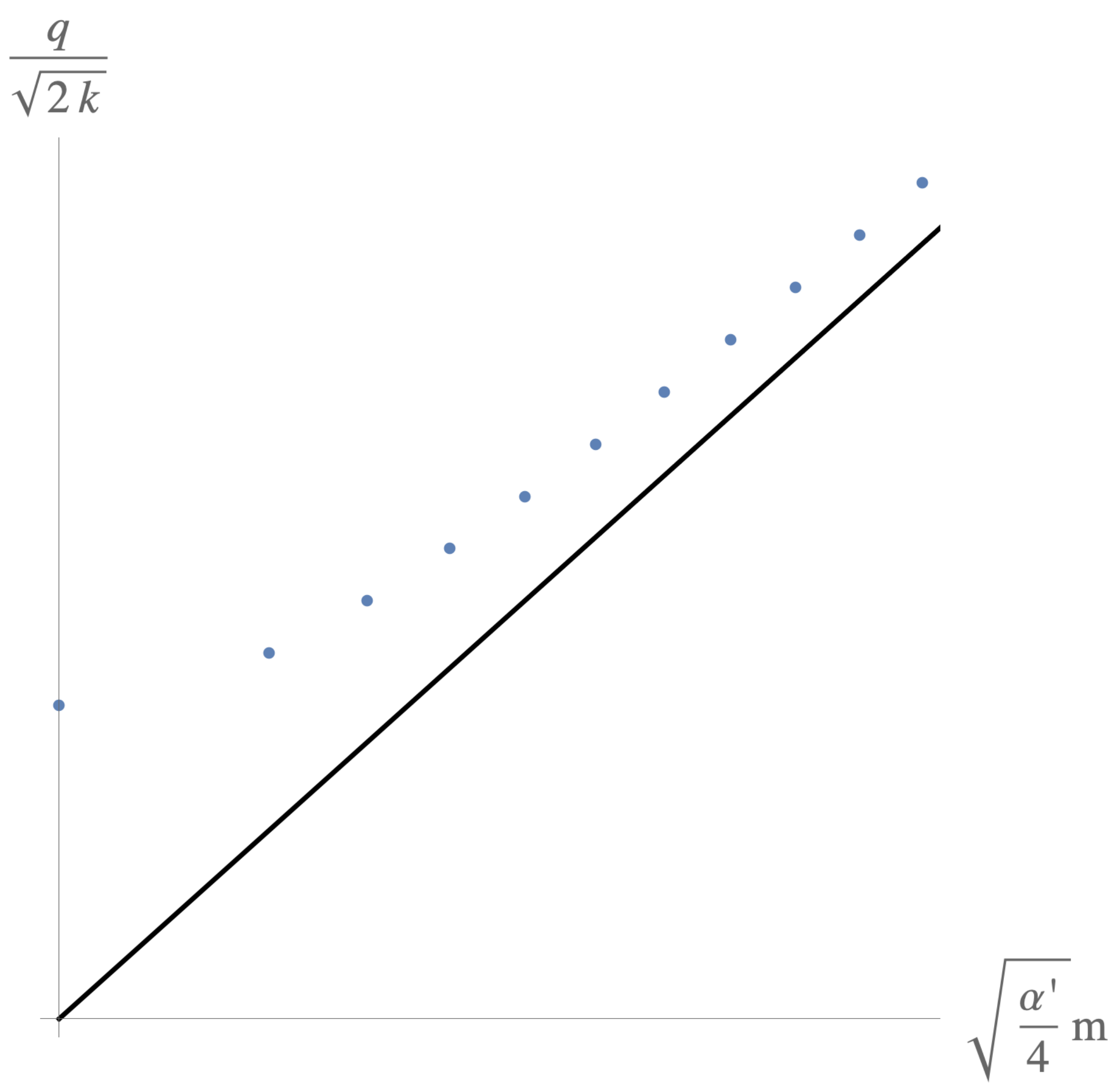
- In the large ρ limit, the spectral flowed states have:

$$\mathcal{Z}'^2=1+\frac{k\alpha'}{2}m^2\frac{\left(\mathcal{Z}^2-1\right)}{k^2\rho^2}+\mathcal{O}(\rho^{-3})\quad\to 1$$

- Moreover, a perturbative string state never crosses the Z=1 line as Δ -$q^2/2k$ = invariant under spectral flow.

Spectral Flow & Black Hole

- Given a Z > 1 state, spectral inflow implies a **tower of states** monotonically approaching Z=1 from above:



Turn on a small string coupling

$$g_c \sim N^{-1/4} \ll 1$$

The excited string state turned into a black hole.

The correspondence principle [Horowitz, Polchinski]:

$$S_{string} = \mathcal{O}(1)\ S_{BH}$$

- This however does not suffice to show that the string states stay Z >1 when we turn on $g_c \sim N^{-1/4}$.

Entropy Matching

- The near horizon geometry of a 4d extremal BH is AdS_2 but if the BH arises from a higher dim theory with an $S^1 \rightarrow AdS_3$ (BTZ).
- The BH entropy is given by the **Cardy's formula**:

$$S_{\rm BH} = 2\pi \left(\sqrt{\frac{c_L}{6} h_L} + \sqrt{\frac{c_R}{6} h_R} \right)$$

- The **left- and right-moving levels** are related to:

$$\ell M_3 = h_L + h_R \ , \quad 8 G_3 J_3 = h_R - h_L \ ,$$

- The AdS radius is related to the Brown-Henneaux central charge:

$$c = \frac{3\ell}{2G_3} = \frac{1}{2}(c_L + c_R)$$

- The extremality bound: $$\frac{8 G_3 |J_3|}{\ell M_3} \leq 1$$

Higher Derivative Corrections

- The entropy for a small BH can receive infinitely many higher derivative corrections, but the AdS_3 geometry allows us to fix them.
- The central charges are fixed by **anomalies** **[Kraus, Larsen, '05]:**
  - $c_L - c_R$: gravitational Chern-Simons term
  - c_R : SU(2) Chern-Simons term
- This fixes the BH entropy:

$$S_{\rm BH} = 2\pi \left(2\sqrt{h_L} + \sqrt{2h_R} \right)$$

- The entropy of heterotic string states at large excitation levels:

$$S_{\rm stat} = 2\pi \left(2\sqrt{N_L} + \sqrt{2N_R} \right)$$

- Exact entropy matching thus identifies at the string/BH transition:

$$h_{L,R} = N_{L,R}$$

also [Dabholkar, '97]

Mass Corrections

- Consider the mass of a non-BPS state with $N_L = 0$, $N_R >> 1$, exact entropy matching implies negligible mass corrections:

$$M_s \simeq \sqrt{\frac{4}{\alpha'} N_R + g_s^2 \Delta M(N_R)}$$

- If the entropy matching is approximate the mass of the string state can be corrected by an O(1) factor. Still, there has been some evidence that the mass is not corrected significantly **[Dabholkar, '97; [Dabholkar, Mandal, Ramadevi, '97].**

- The extremality bound is saturated by $N_L = 0$ or $N_R = 0$:

$$\frac{|J_3|}{M_3} = \frac{c}{12} \frac{|N_R - N_L|}{N_R + N_L} \leq \frac{c}{12}$$

- Thus non-BPS super-extremal states stay super-extremal, while BPS states stay extremal at the string/BH transition:

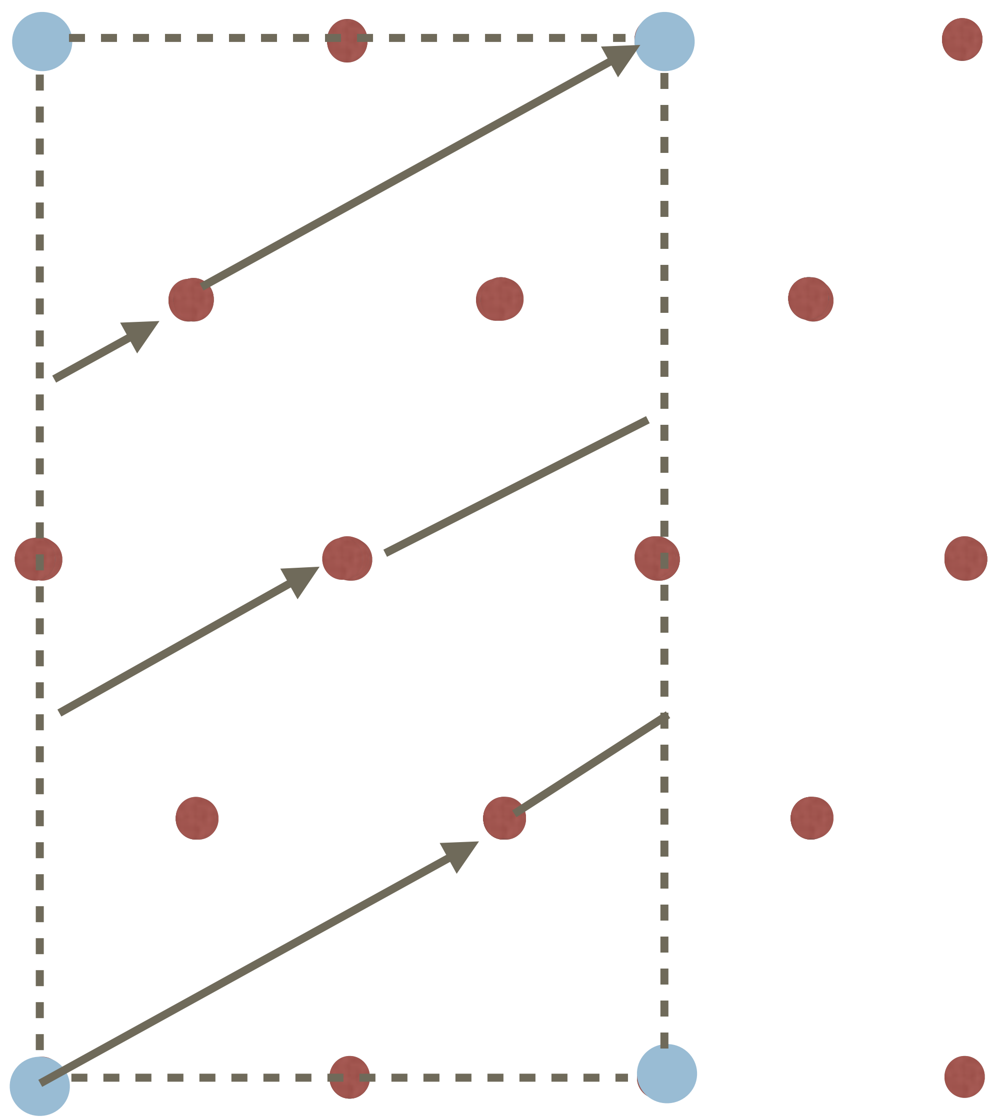
$$\mathcal{Z}^2_{\mathrm{non-BPS}} = \frac{Q_L^2}{M_s^2} = \frac{Q_L^2}{Q_L^2 - 4/\alpha'} > 1 \qquad \mathcal{Z}^2_{\mathrm{BPS}} = \frac{Q_R^2}{M_s^2} = 1$$

WGC from Modular Bootstrap

[Montero, GS, work in progress]

Index/Coarseness of Sublattice

Missing WGC lattice sites form a group: $\Gamma \equiv \Lambda/\Lambda^*$



Index = 8, Coarseness=4

- Two measures of the size of Γ:
  - **Index** of the lattice
  - **Coarseness**

    Smallest c such

    $c\vec{v} \in \Lambda^*$

    for

    $\vec{v} \in \Lambda$

Bounds from the Modular Bootstrap

[Montero, GS, work in progress]

- Consider the **partition function** of the NS sector:

$$Z(\tau) = \mathrm{Tr}_{NS} \left(q^{L_0 - c/24} \bar{q}^{\tilde{L}_0 - \tilde{c}/24} \right)$$

- It is **not** modular invariant but
  - It is **positive**
  - It is **S-invariant** → **Spectral flow/sublattice WGC still works**
- We are putting a bound on the lightest WGC state among the **spacetime bosons**; considering the full partition function and the associated spin structures can only strengthen the bound we find.

Γ-valued Partition Vector

- The holomorphic & anti-holomorphic U(1) currents define two inner products:

$$(\vec{Q}_1, \vec{Q}_2) = \frac{1}{2}\vec{Q}_L^T \mathbf{N}_L \vec{Q} - \frac{1}{2}\vec{Q}_R^T \mathbf{N}_R \vec{Q}$$

$$\langle \vec{Q}_1, \vec{Q}_2 \rangle = \frac{1}{2}\vec{Q}_L^T \mathbf{N}_L \vec{Q} + \frac{1}{2}\vec{Q}_R^T \mathbf{N}_R \vec{Q}$$

- Current algebra implies $L_0 + \tilde{L}_0 = L_0' + \tilde{L}_0' + \langle \vec{Q}, \vec{Q} \rangle$

$$L_0 - \tilde{L}_0 = L_0' - \tilde{L}_0' + (\vec{Q}, \vec{Q})$$

- We can expand the sum

$$\mathcal{Z}(\tau) = \sum_{\vec{Q}} \mathrm{Tr}_{NS}\left(q^{L_0'} \bar{q}^{\tilde{L}_0'} q^{\frac{1}{2}\left[\langle \vec{Q}, \vec{Q} \rangle + (\vec{Q}, \vec{Q})\right]} \bar{q}^{\frac{1}{2}\left[\langle \vec{Q}, \vec{Q} \rangle - (\vec{Q}, \vec{Q})\right]}\right)$$

- This can be rearranged in terms of a **Γ-valued partition vector**

$$Z = \sum_{\gamma \in \Gamma} \mathcal{Z}_\gamma(\tau) \vartheta(\tau, \gamma), \quad \vartheta(\tau, \gamma) \equiv \sum_{\vec{\lambda} \in \Lambda^*} e^{2\pi i(\tau_1(\vec{\gamma}+\vec{\lambda}, \vec{\gamma}+\vec{\lambda}) - i\tau_2 \langle \vec{\gamma}+\vec{\lambda}, \vec{\gamma}+\vec{\lambda} \rangle}$$

Vector-Valued Modular Form

- The inner product $(\cdot,\cdot)$ on the lattice becomes a **bilinear form on gammas**:

$$(\cdot,\cdot) : \Gamma \times \Gamma \to \mathbb{Z}_{\mathrm{ord}(\Gamma)}$$

$$(\gamma_1, \gamma_2) = (\vec{v}_{\gamma_1}, \vec{v}_{\gamma_2}) \bmod 1$$

- The worldsheet data can be encoded into a vector-valued (non-holomorphic) modular form, with **transformation law**

$$\vec{\mathcal{Z}}\left(-\frac{1}{\tau}\right) = \left(\sqrt{\frac{\tau}{i}}\right)^{n_+} \left(\sqrt{\frac{\bar{\tau}}{-i}}\right)^{n_-} \mathbf{U}_S \vec{\mathcal{Z}}(\tau)$$

where

$$(\mathbf{U}_S)_{\gamma,\gamma'} = \frac{1}{|\Gamma|} \exp\left[2\pi i (\gamma, \gamma')\right]$$

Modular Bootstrap

- The idea of the **modular bootstrap** is to use modular invariance to constrain the partition function **[Hellerman '09, Lin, Shao '19, Collier, Lin, Yin '16, Yin '17, Montero, GS, Soler '16, Qualls '14, '16, Bae, Lee, Song '18…]**

- A good starting point is to consider a **fixed point** of the modular transformation (S here): Parametrize $\boldsymbol{\tau} = i\, e^{s}$, then S maps s→-s

$$\vec{\mathcal{Z}}(-s) = \exp(-\frac{n_d}{2}s)\mathbf{U}_S\vec{\mathcal{Z}}(s)$$

- Derivatives w.r.t $\beta=-2\pi i\, \boldsymbol{\tau}$ are related to s-derivatives

$$\langle\Delta^k\rangle_a\mathcal{Z}_a = (-1)^k\frac{d^k\mathcal{Z}}{d\beta^k} = (-1)^k\sum_{l=0}^{k}\begin{bmatrix}k\\l\end{bmatrix}(2\pi)^{-l}\mathcal{Z}_a^{(l)}$$

Some of these are naturally **positive** or **bounded**!

Warm Up

- Since $U^2 = 1$, we can decompose

$$\vec{\mathcal{Z}}(s) = \vec{\mathcal{Z}}_+(s) + \vec{\mathcal{Z}}_-(s)$$

- In terms of these components:

$$\vec{\mathcal{Z}}_\pm(-s) = \pm \exp(-n_d s/2)\vec{\mathcal{Z}}_\pm(s)$$

- Taking 0, 1 derivatives, we get 4 vectors

$$\vec{\mathcal{Z}}_\pm(0) \equiv \vec{\mathcal{Z}}_\pm, \quad \left.\frac{d\vec{\mathcal{Z}}_\pm(s)}{ds}\right|_{s=0} \equiv \vec{\mathcal{Z}}_\pm^{(1)}$$

with constraints, e.g., for $n_d = 1$:

$$\vec{\mathcal{Z}}_- = \vec{\mathcal{Z}}_+^{(1)} - \frac{1}{4}\vec{\mathcal{Z}}_+ = 0$$

Linear Programming Problem

- What we have is a set of inequalities:

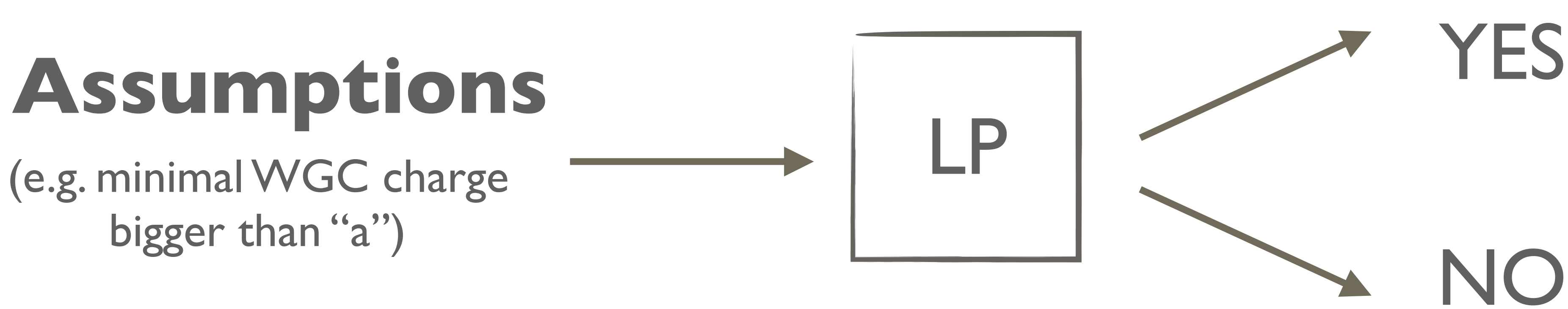
Lowest dim. in each charge sector

$$\mathcal{Z}_a \geq 0,$$

$$\frac{d\mathcal{Z}_a}{d\beta} = -\frac{\mathcal{Z}_a^{(1)}}{2\pi} = \langle \Delta \rangle \mathcal{Z}_a \geq \Delta_{0,a} \mathcal{Z}_a$$

subject to the constraint: $\vec{\mathcal{Z}}_- = \vec{\mathcal{Z}}_+^{(1)} - \frac{1}{4}\vec{\mathcal{Z}}_+ = 0$

- We have a **linear programming** problem!



Universal Constraints

- To get a **universal constraint**, we must do so for **all possible discrete groups Γ** with bilinear forms.
- Mathematicians **[Wall, '63];[Miranda '84]** have proven that this can be reduced to one of **six cases**:

$$\Gamma = \mathbb{Z}_{p_1^{k_1}} \oplus \mathbb{Z}_{p_2^{k_2}} \oplus \ldots$$

$$(\kappa,\kappa)_{A_{p^r}} = \frac{1}{p^r}, \quad (\kappa,\kappa)_{B_{p^r}} = \frac{\xi}{p^r}$$

Quadratic nonresidue (→ ξ)

$$A_{2^r}: \quad (\kappa,\kappa) = \frac{1}{2^r}, \quad B_{2^r}: \quad (\kappa,\kappa) = \frac{-1}{2^r},$$

$$C_{2^r}: \quad (\kappa,\kappa) = \frac{5}{2^r}, \quad D_{2^r}: \quad (\kappa,\kappa) = \frac{-5}{2^r}$$

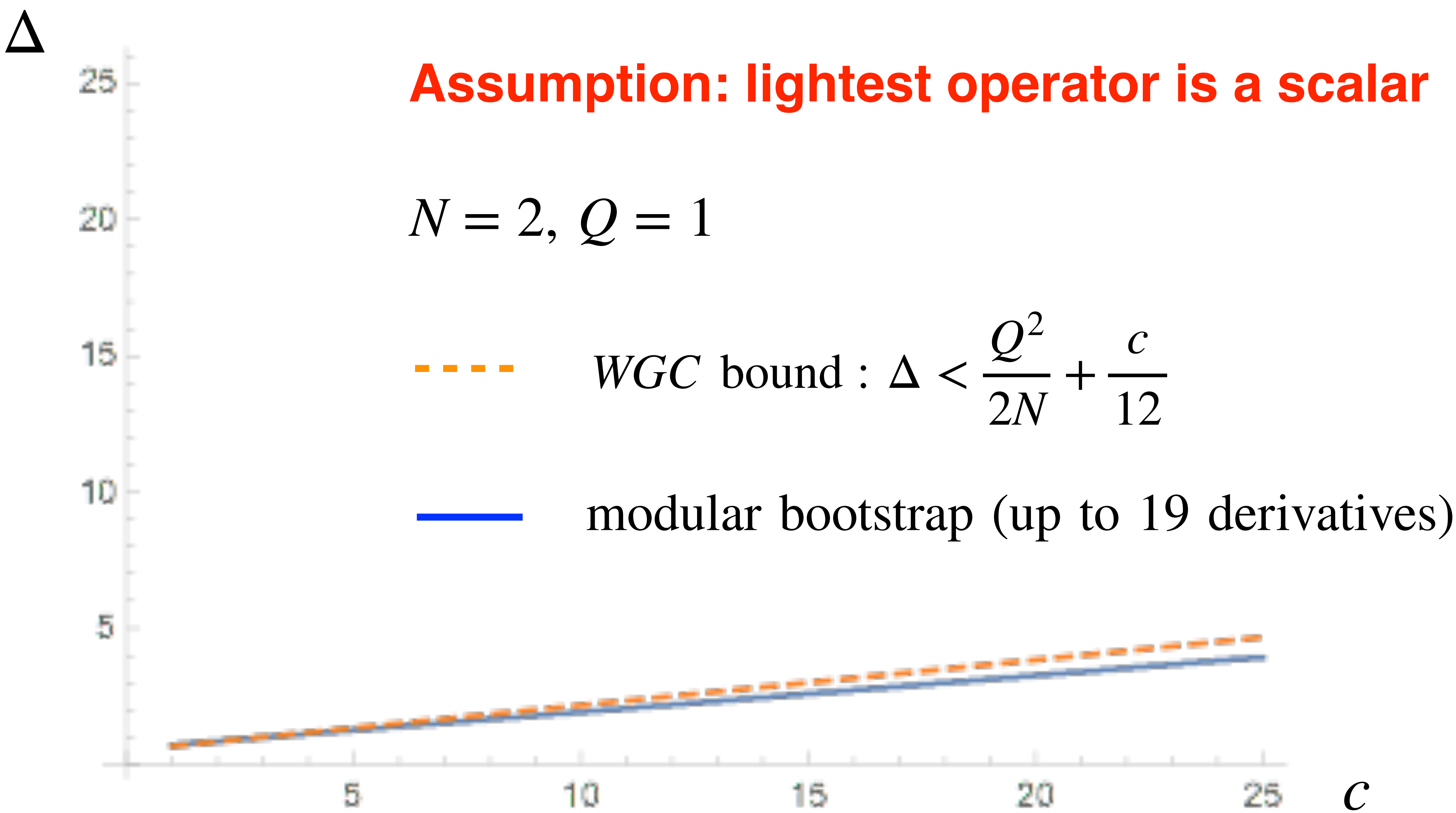
One dimensional

$$E_{2^r}: \quad (\kappa_i,\kappa_j) = \begin{pmatrix} 0 & \frac{1}{2^r} \\ \frac{1}{2^r} & 0 \end{pmatrix},$$

$$F_{2^r}: \quad (\kappa_i,\kappa_j) = \begin{pmatrix} \frac{1}{2^{r-1}} & \frac{1}{2^r} \\ \frac{1}{2^r} & \frac{1}{2^{r-1}} \end{pmatrix}$$

Two dimensional

Preliminary Results



[Montero, GS, in progress]

Summary

Summary

- The Swampland program is an attempt to identify the boundary of possibilities for EFTs that can be consistently coupled to quantum gravity.
- The WGC constrains ***some but not all*** large field inflation models. Loopholes exist; strengthening the WGC can potentially close some of them.
- The mild form of the WGC is essentially established: black holes can be the required superextremal state.
- The WGC can be upgraded to a stronger form (**Tower/Sublattice WGC**) with additional properties of the UV theories (e.g., **modular invariance**).
- The **modular bootstrap** can put a bound on the lightest superextremal state.
- Much more will be discussed in the KITP StringVacua20 Program!

The String Swampland and Quantum Gravity Constraints on Effective Theories

Coordinators: Hiroshi Ooguri, Gary Shiu, Cumrun Vafa, and Irene Valenzuela

The idea that the string landscape is too large to lead to concrete predictions has been countered by the idea that most of the naively consistent effective theories of gravity coupled to matter are actually inconsistent and belong to the swampland. The identification of criteria distinguishing the true string landscape from the swampland, which has been studied for more than a decade now, is beginning to reach a more mature stage with the developments of the last few years. In particular a conjectured consistency condition for quantum gravity known as the Weak Gravity Conjecture (WGC), which postulates that gravity is always the weakest force among all the forces, has found an unexpectedly broad range of applications.

The WGC on the one hand has been used to constrain cosmological models of inflation including scenarios being tested by the present generation of CMB experiments and on the other hand has been connected to the cosmic censorship conjecture of general relativity. Furthermore, ideas from holography have been found to be nicely consistent with the WGC. Moreover a sharpened version of the WGC has been used to put constraints on particle phenomenology and in particular has been used to place bounds on the neutrino masses. This program will bring together the diverse communities of string theorists, cosmologists, general relativists, particle phenomenologists and researchers working on holography and the conformal bootstrap to further develop consistency criteria for quantum theories of gravity and possibly extract concrete predictions from these ideas for the observable universe as well as deepen our understanding of the structure of string vacua.

DATES

Feb 18, 2020 - Mar 13, 2020

INFORMATION

Apply

Application deadline is: *Nov 18, 2018.* Applications will be considered and invitations will be issued after the above deadline.

QUICK LINKS