Enlarging the Space of Viable Inflation Models: A Slingshot Mechanism

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Based on work done in collaboration with:

- **Keith Dienes and Jeff Kost [arXiv:1907.10074].**

KITP From Inflation to the Hot Big Bang Workshop, Feb. 12th, 2020.

Initial Conditions for Inflation

- In order to give rise to a prolonged period of accelerated expansion, the inflation potential $V(\phi)$, must satisfy the slow-roll criteria:

$$\epsilon = \frac{M_P^2}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 < 1$$

Slow Roll

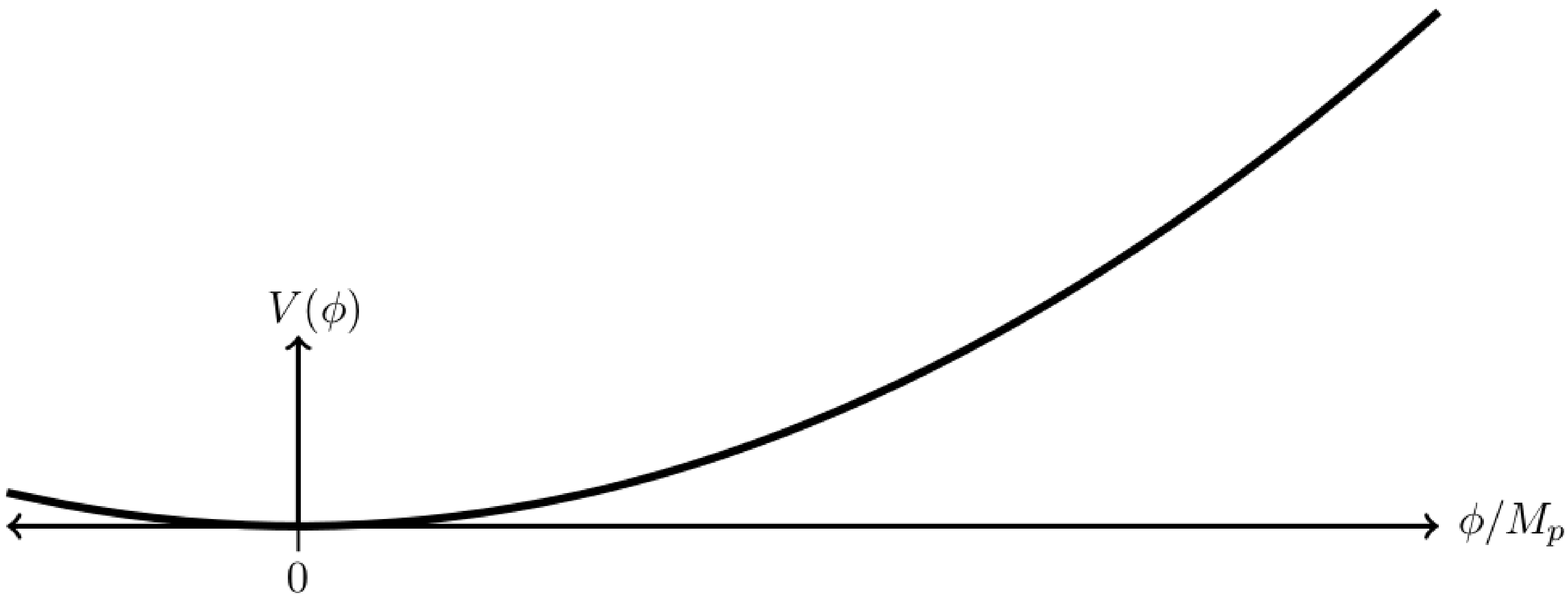
$$\eta = M_P^2 \left|\frac{V''(\phi)}{V(\phi)}\right| < 1$$

Prolonged Duration

- Successful inflation is not contingent on the shape of $V(\phi)$ alone, however. Appropriate ***initial conditions*** for ϕ and $\dot{\phi}$ must also be established.

Example:

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2$$



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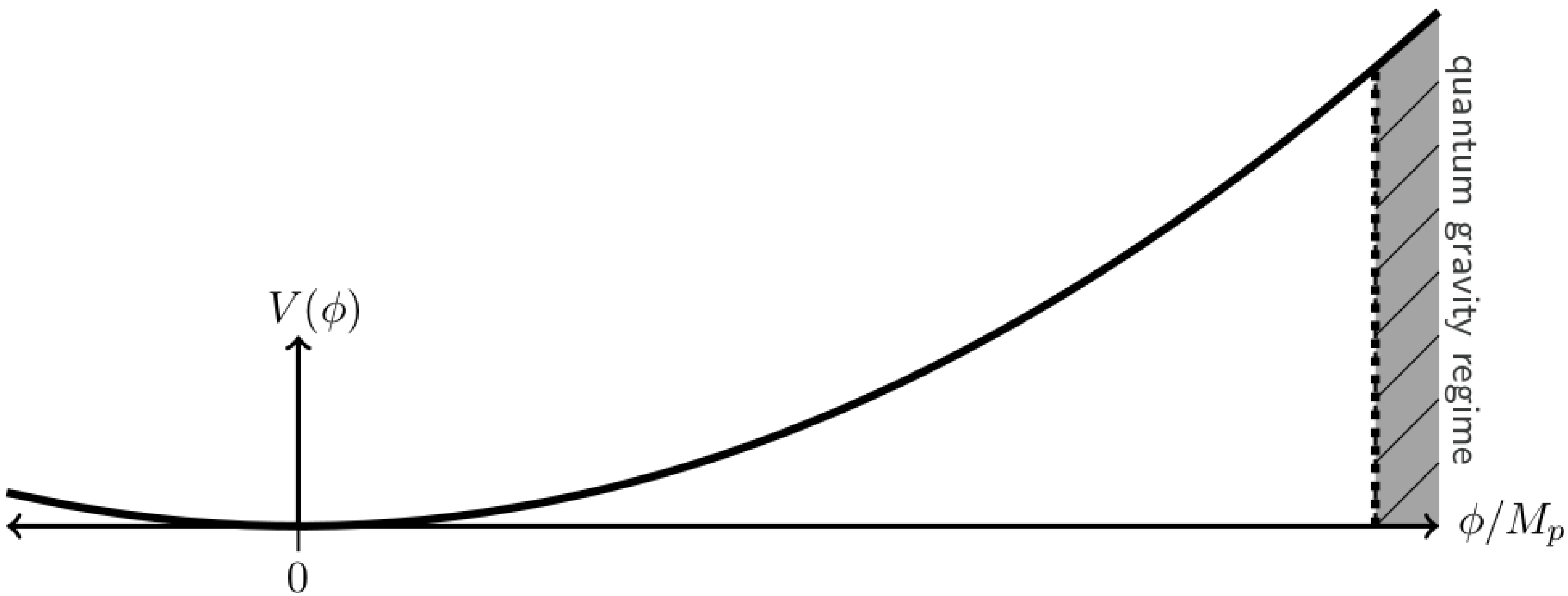

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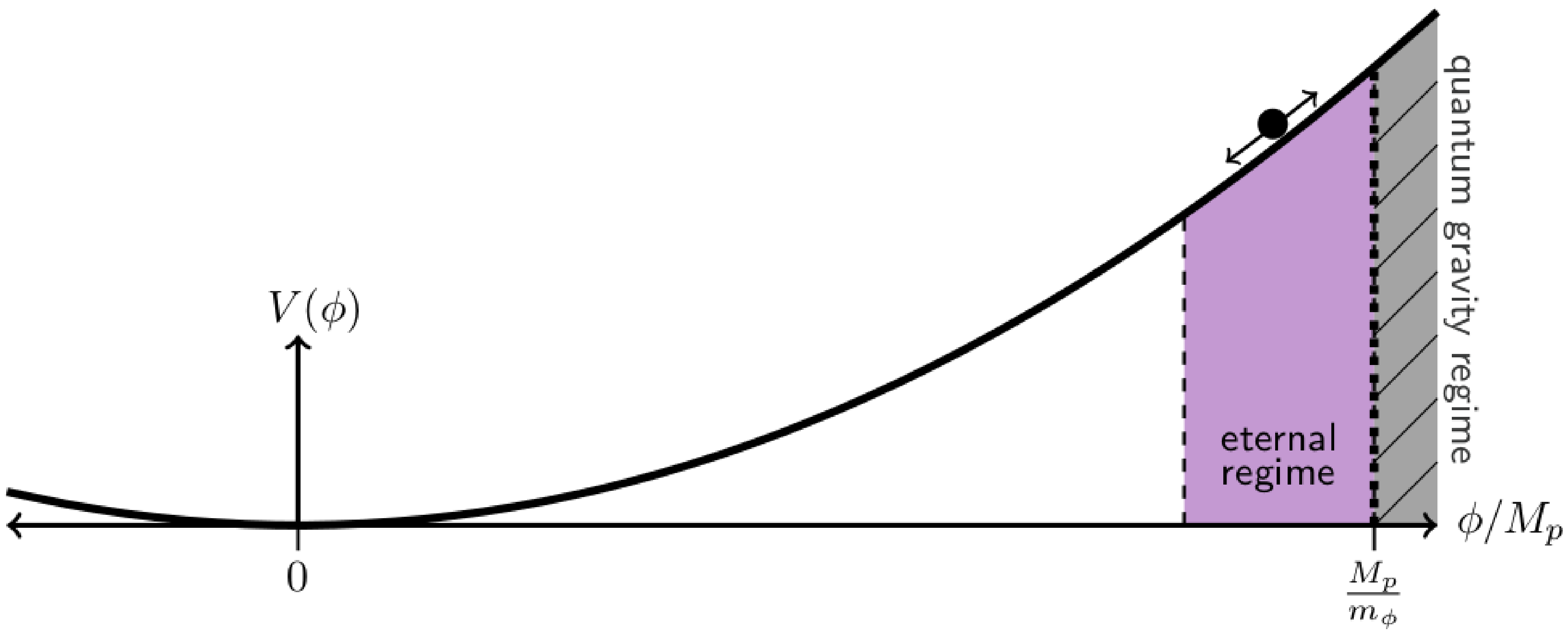

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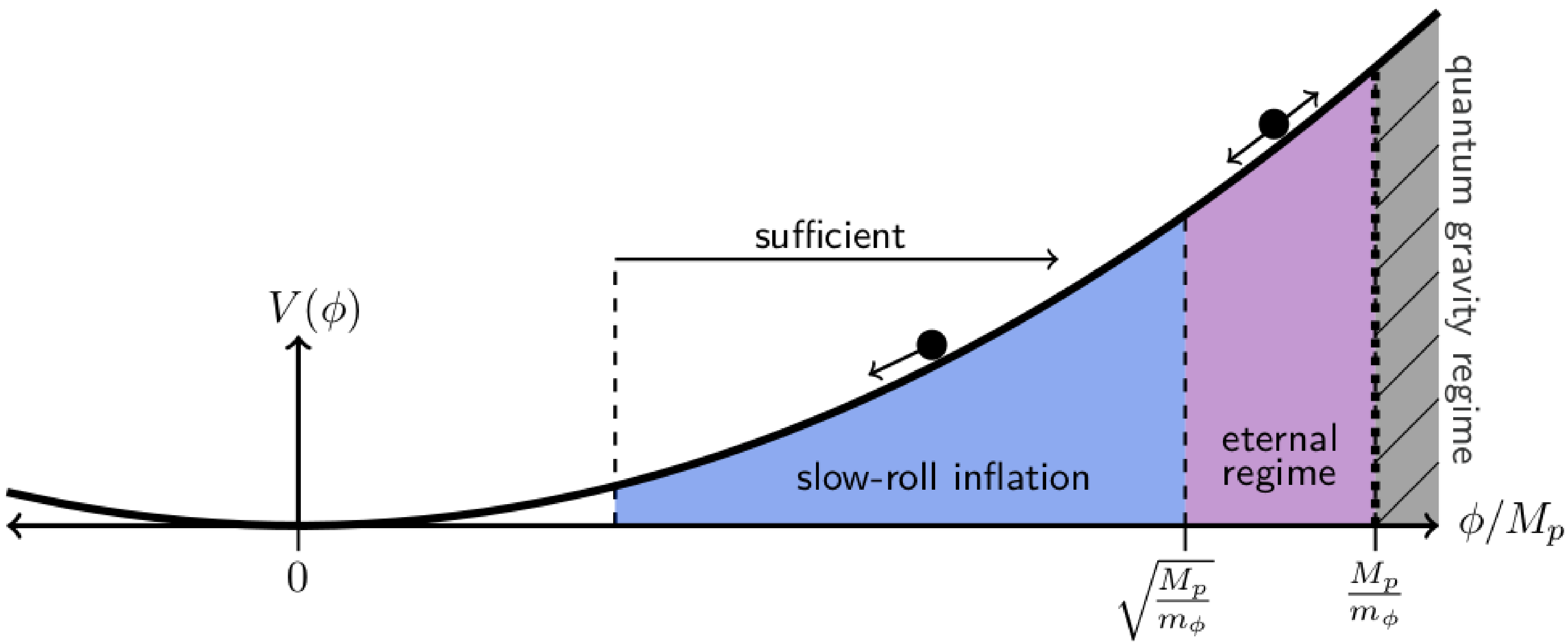

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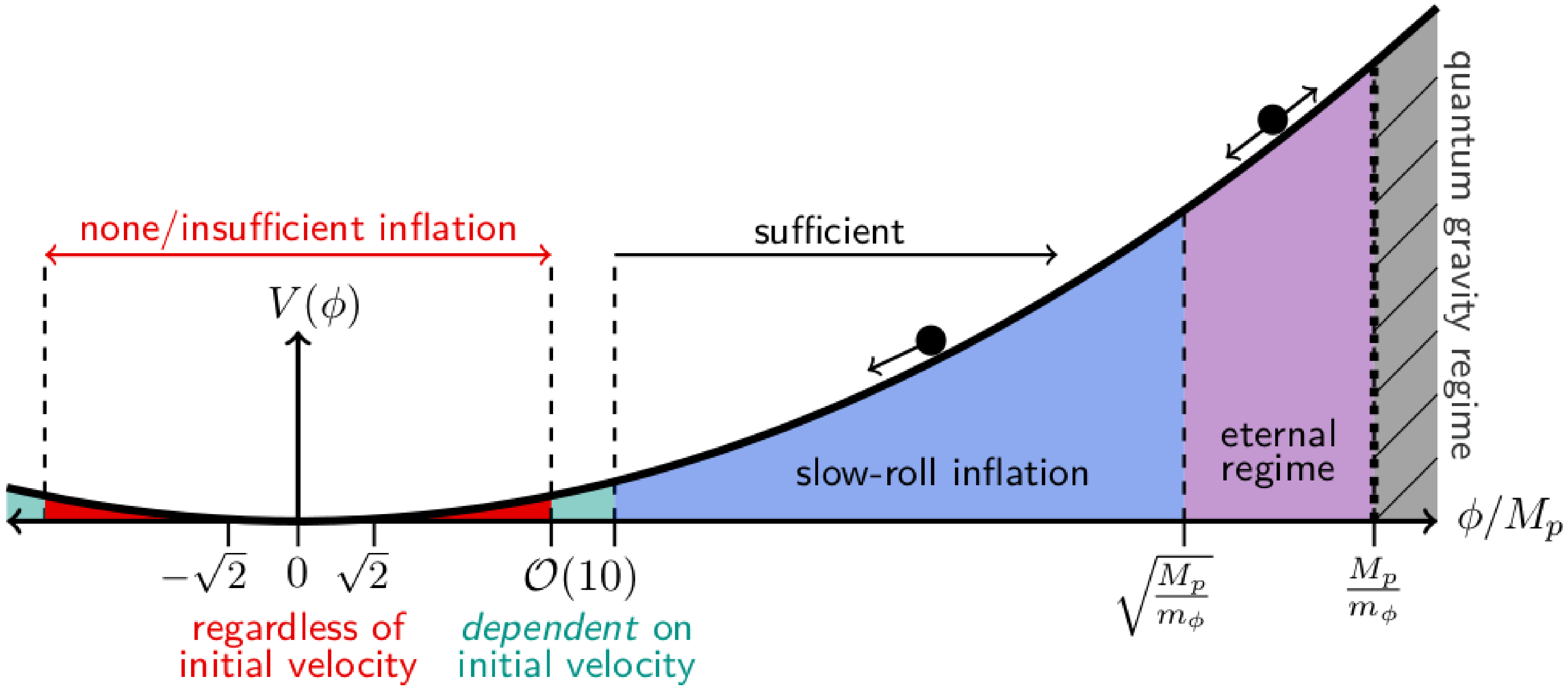


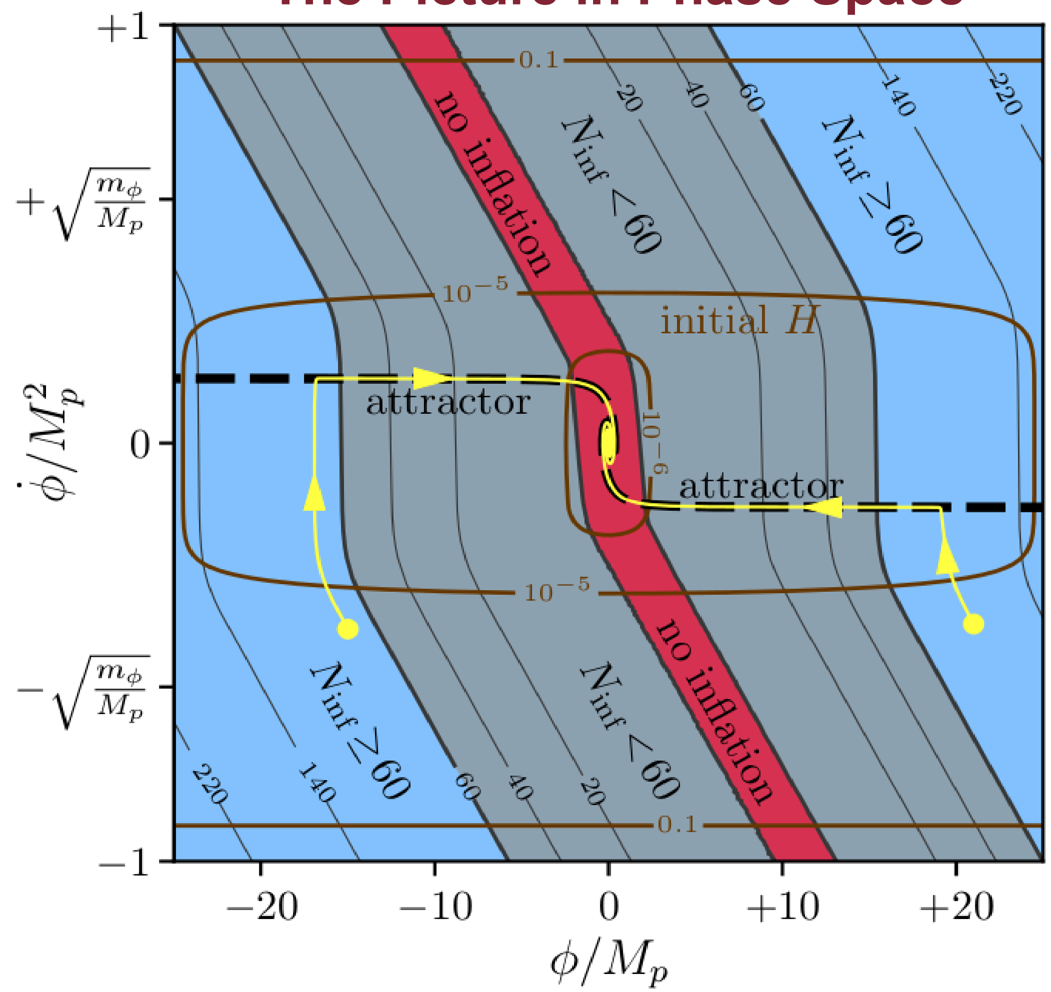
Initial Conditions for Inflation

- Appropriate initial conditions for inflation that include large or even trans-Planckian field VEVs arise naturally many scenarios, e.g.,
  - Chaotic inflation [Linde '83]
  - Natural inflation [Freese, Frieman, Olinto, '90; Freese, Kinney '04]

- ***The question***: might we be able to "salvage" regions of phase space that might not give rise to successful inflation?

- In other words, might there be a ***dynamical mechanism*** which could drive a scalar field into a region of phase space from which inflation of sufficient duration can occur?

The Picture in Phase Space



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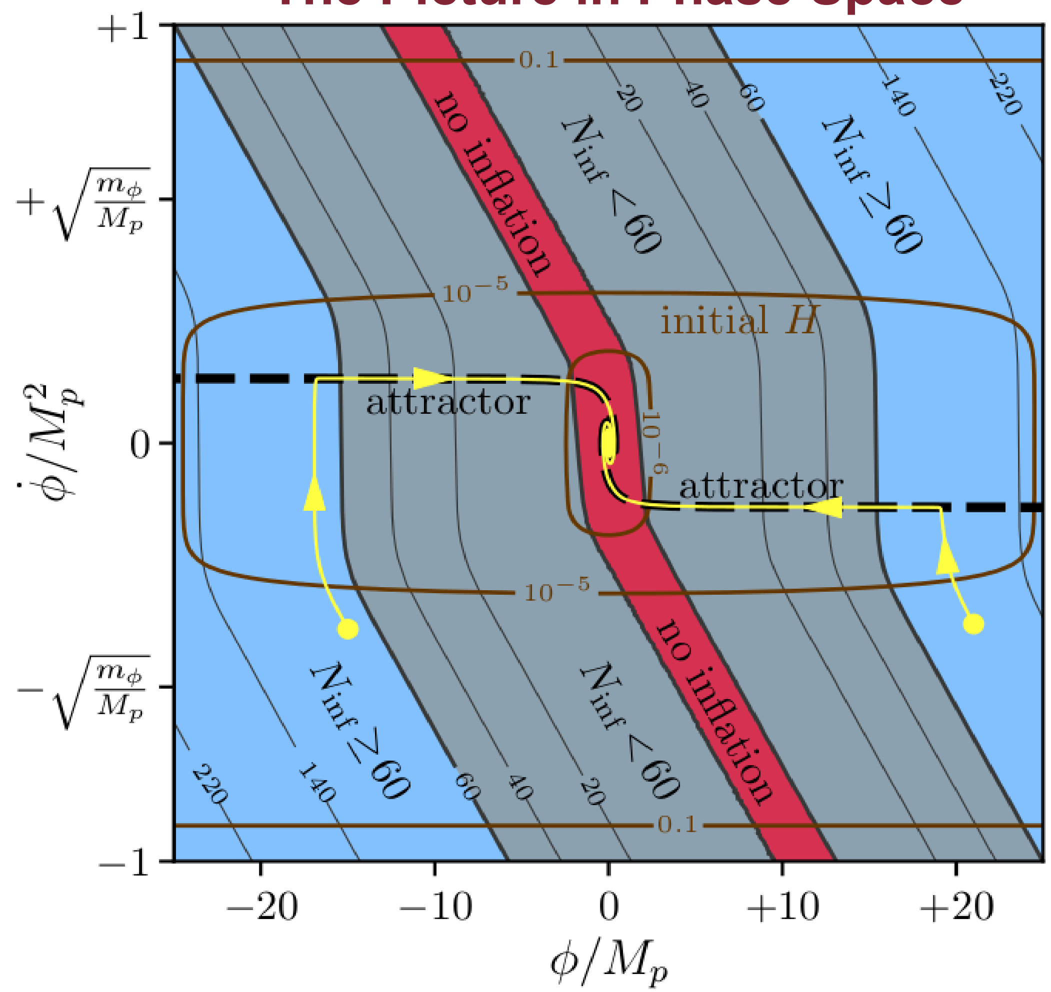
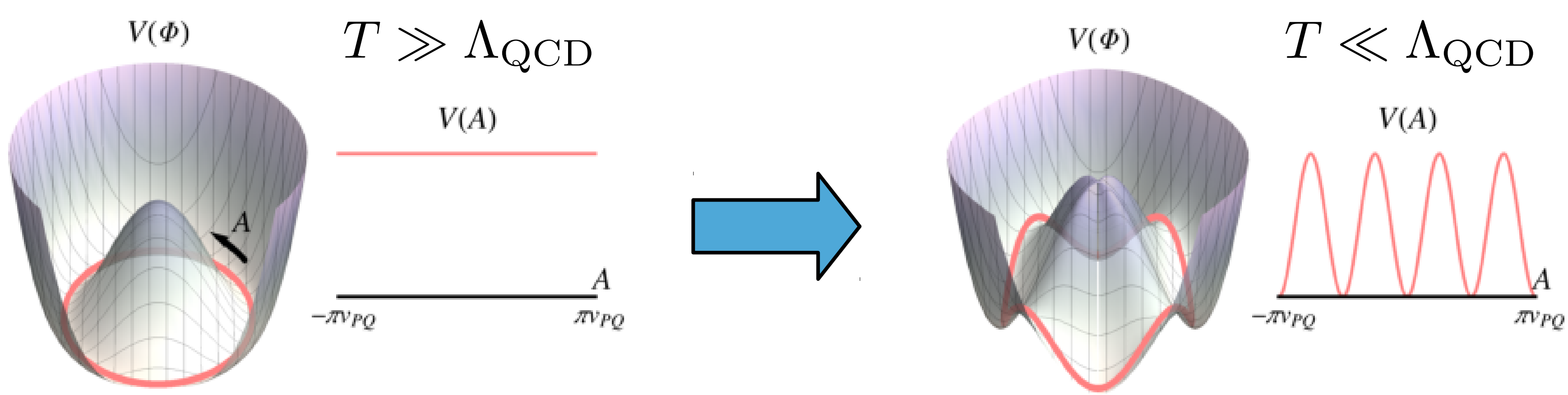


The answer: yes, there is, and the ingredients for this mechanism are standard tools in the cosmologist's toolbox.

Scalars and Phase Transitions

- Additional ***light scalars*** are a common feature of many scenarios for physics beyond the Standard Model.
- Such fields are often light because they are protected from acquiring masses by classical symmetries which are broken at the quantum level by non-perturbative effects.
- Such non-perturbative effects can arise dynamically, as a consequence of a ***cosmological phase transition***.
- A canonical example is the QCD axion, which is protected from acquiring a mass at high temperatures $T \gg \Lambda_{\rm QCD}$ by a shift symmetry, but acquires a small mass at temperatures $T \lesssim \Lambda_{\rm QCD}$ due to instanton effects associated with the QCD phase transition.



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Important:

Scalar masses generated in this way are ***time-dependent*** and evolve dynamically over the course of the phase transition. As we shall see, this time-dependence will play a crucial role in establishing the initial conditions for the inflaton field at the beginning of inflation.

Dynamics of the Phase Transition

[Dienes, Kost, BT '15]

- We consider a mass-squared matrix of the form

$$\mathcal{M}^2 = \begin{bmatrix} 0 & 0 \\ 0 & M^2 \end{bmatrix} + \begin{bmatrix} m_{00}^2(t) & m_{01}^2(t) \\ m_{01}^2(t) & m_{11}^2(t) \end{bmatrix}$$

constant

uniform time dependence

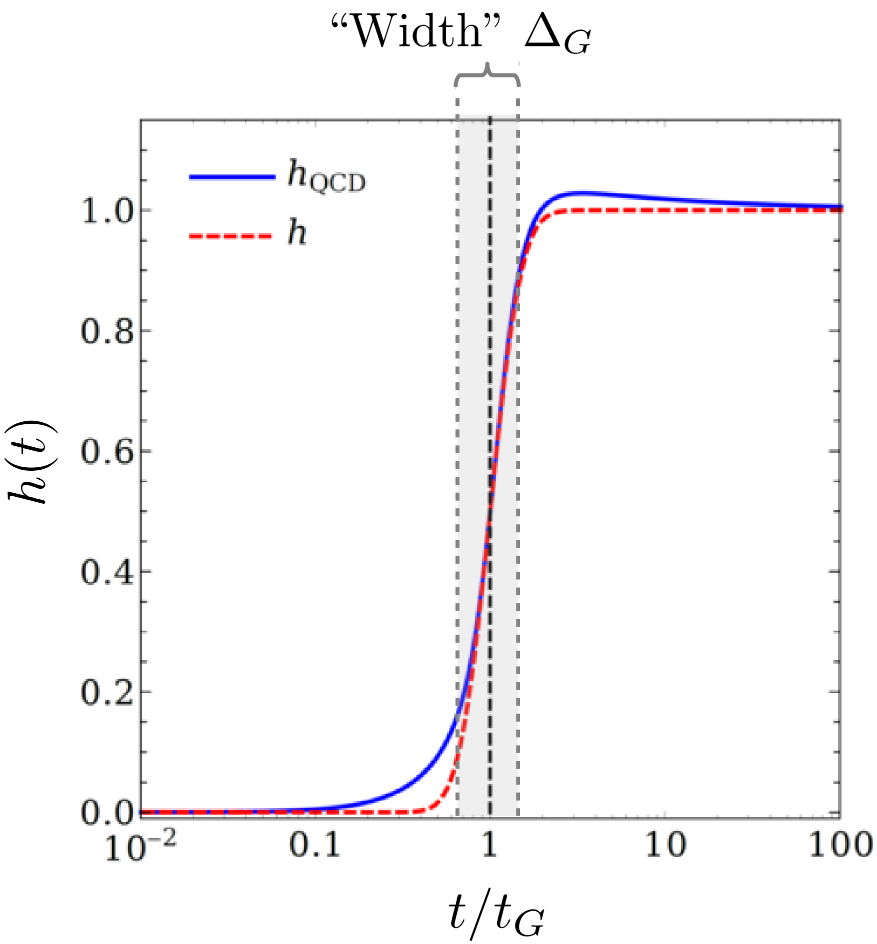
$$m_{ij}^2 = \overline{m}_{ij}^2 \; h(t)$$

- It is useful to parametrize the resulting mass matrix

$$m_{\text{sum}}^2 \equiv M^2 + m_{00}^2 + m_{11}^2$$

$$\alpha \equiv \frac{M^2 - m_{00}^2 + m_{11}^2}{M^2 + m_{00}^2 + m_{11}^2}$$

$$\beta \equiv 1 - \frac{m_{01}^2}{\sqrt{m_{00}^2(M^2 + m_{11}^2)}}$$



- In order that $\mathcal{M}^2$ always be positive semi-definite, we require that

$$m_{\text{sum}}^2 \geq 0 \qquad |\alpha|^2 \leq 1 \qquad 0 \leq \beta \leq 1$$

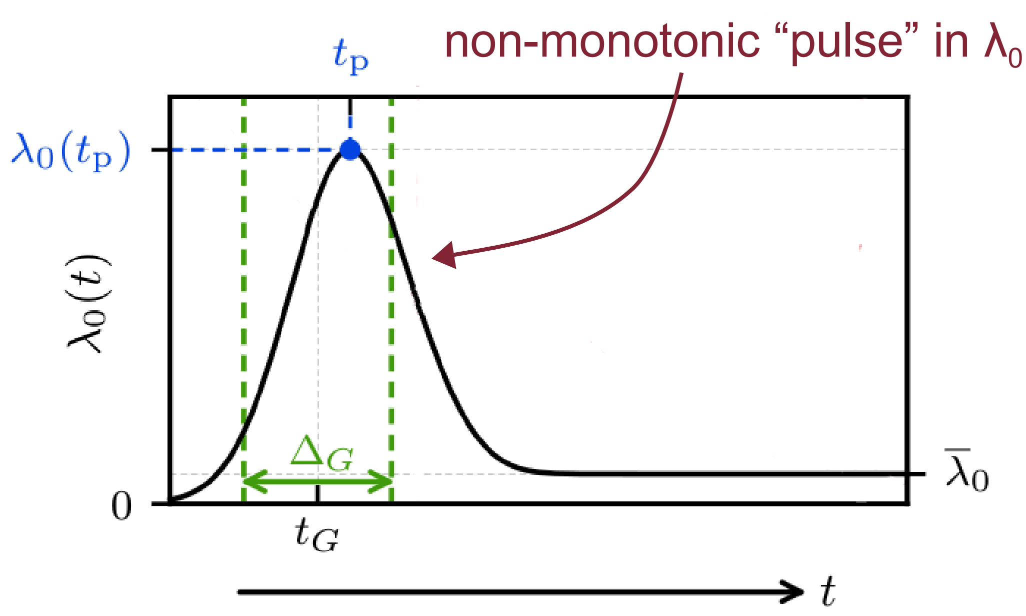
Evolution of the Masses

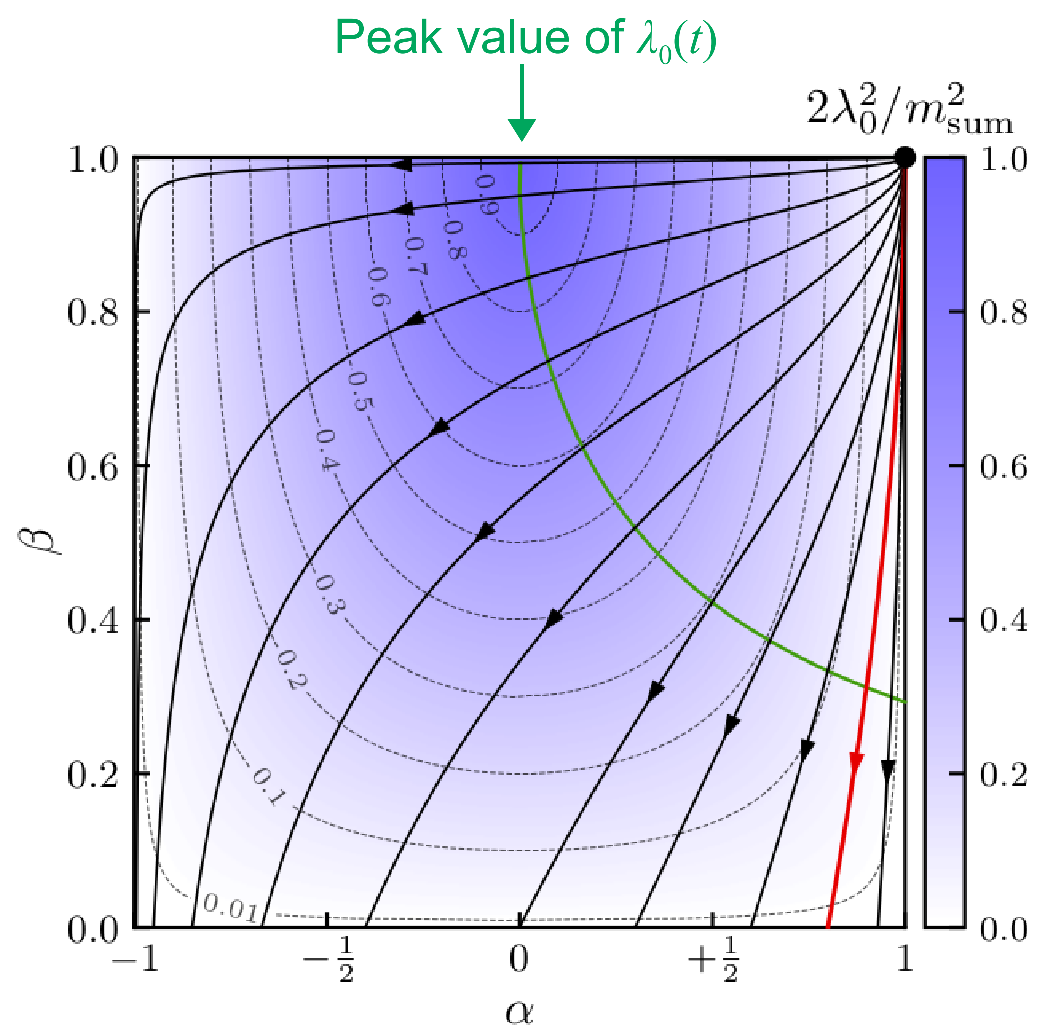
- Mass-squared eigenvalues: $$\lambda_{0,1}^2 = \frac{1}{2} m_{\text{sum}}^2 \left[1 \mp \sqrt{\alpha^2 + (1-\alpha^2)(1-\beta)^2} \right]$$

- The system evolves in time along particular flow lines within $(m_{\text{sum}}^2, \alpha, \beta)$-space.

- The mass eigenvalues evolve as well, and can even be ***non-monotonic*** functions of t.

[Dienes, Kost, BT '15]





Consequences of Non-Monotonicity

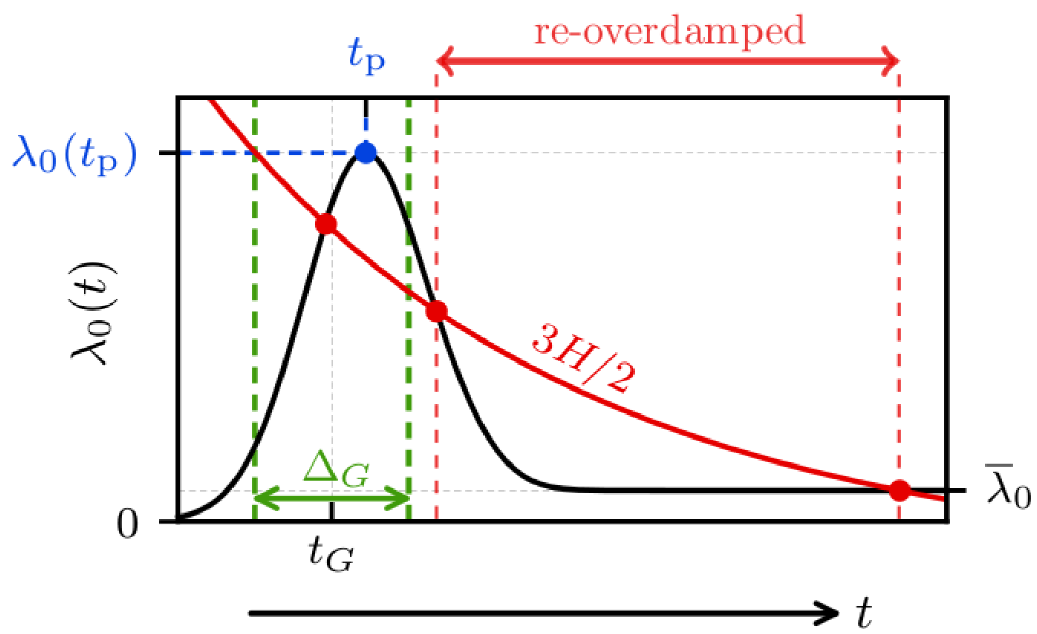
1. Re-Overdamping

- The way in which the energy density $\rho_{\lambda 0}$ associated with the lighter mass eigenstate $\phi_{\lambda 0}$ evolves depends on its mass.

$3H \gtrsim 2\lambda_0$ ⇨ **Overdamped**: behaves like vacuum energy $(\rho_{\lambda_0} \sim [\text{const.}])$

$3H \lesssim 2\lambda_0$ ⇨ **Underdamped**: behaves like massive matter $(\rho_{\lambda_0} \propto a^{-3})$

- Thus, a typical field will behave like vacuum energy at early times and like matter at late times.
- However, when $\lambda_0(t)$ is non-monotonic, a field can transition from the overdamped to the underdamped regime and then ***back*** to the overdamped regime.



Consequences of Non-Monotonicity

2. Parametric Resonance

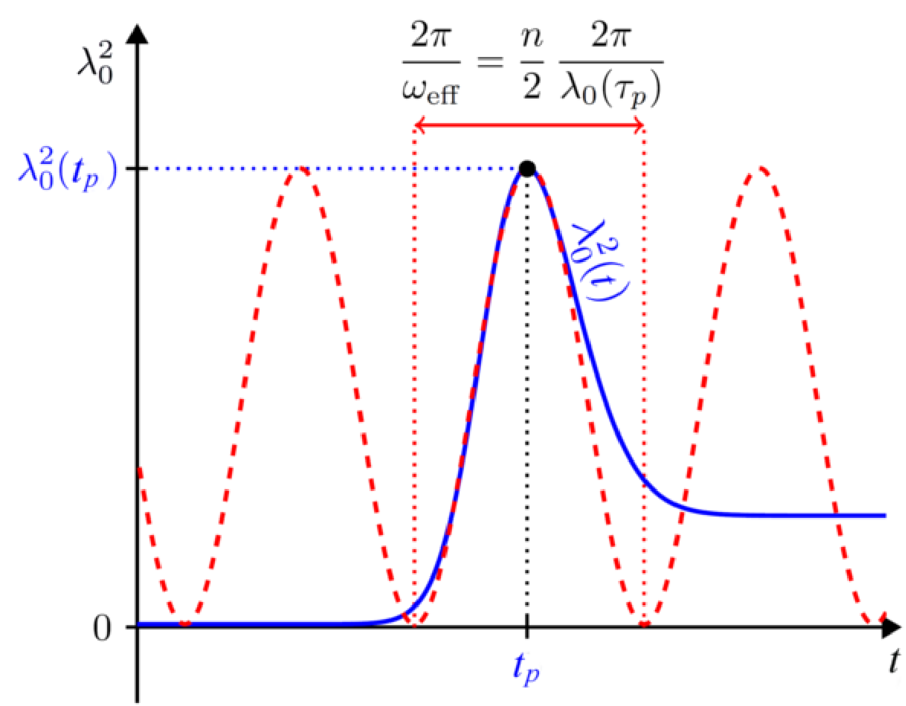
- The "pulse" behavior in which λ_0 rises and falls during the phase transition occurs over a particular timescale.
- Indeed, one can view this pulse as having an effective "frequency":

$$\lambda_0^2(t) \sim \frac{1}{2}\lambda_0^2(t_p)\left\{1+\cos[\omega_{\rm eff}(t-t_p)]\right\} \Rightarrow \omega_{\rm eff}^2 = -4\left(\frac{\ddot{\lambda}_0}{\lambda_0}\right)\Bigg|_{t=t_p}$$

- It therefore follows that when a certain relationship exists between λ_0 and λ_0, ***parametric resonances*** arise.

Resonance Condition

$$\left(\frac{\ddot{\lambda}_0}{\lambda_0^3}\right) = \frac{1}{n^2} \qquad n\in\mathbb{Z}^+$$

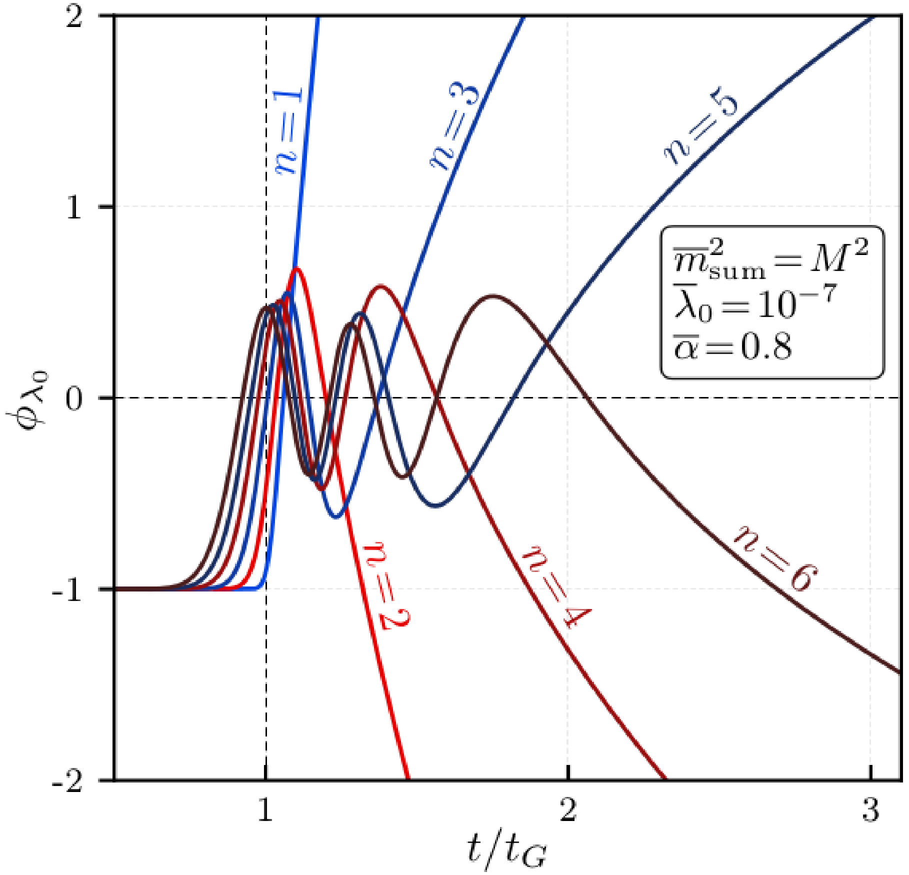


- Even though this resonance exists for only half an "oscillation," it can have a dramatic effect on $\phi_{\lambda 0}$ and $\rho_{\lambda 0}$.

“Propulsion” of the Field

- The interplay between these two effects can serve to propel the field to a large value of $|\phi_{\lambda 0}|$.
- The phase transition carries the field through $n/2$ oscillations and then releases it at maximum velocity $\dot{\phi}_{\lambda 0}$ into the re-overdamped phase.

Field trajectories (where Δ_G has been chosen appropriately for each value of n)



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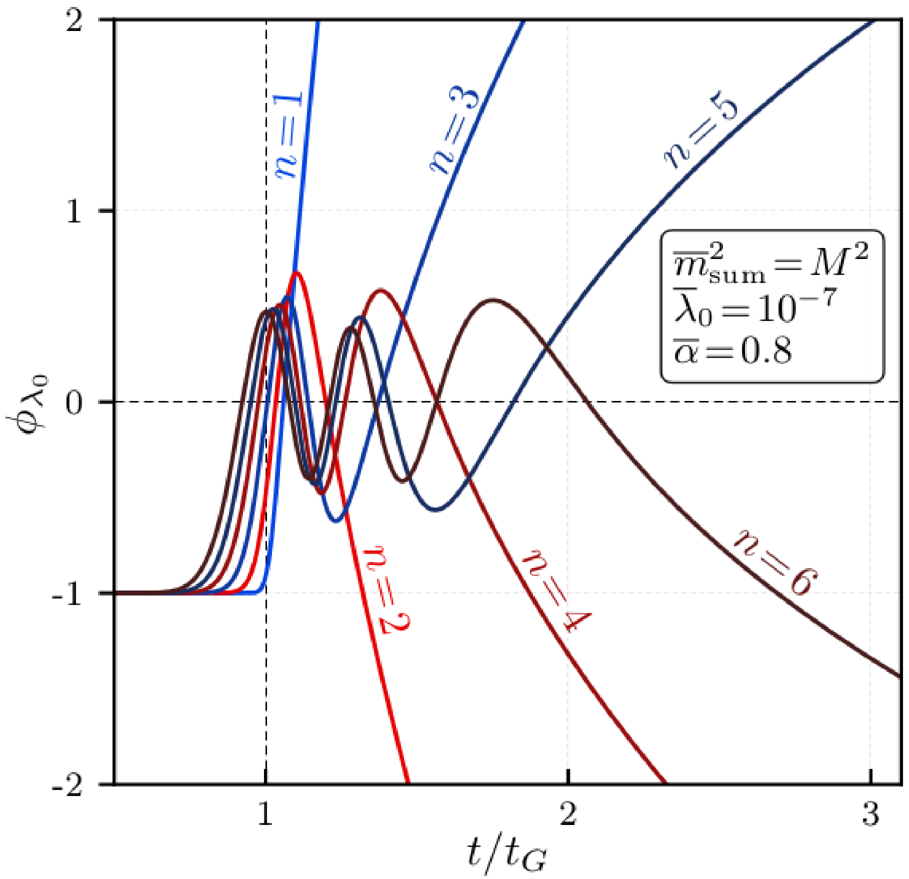

- **So…**

in other words...

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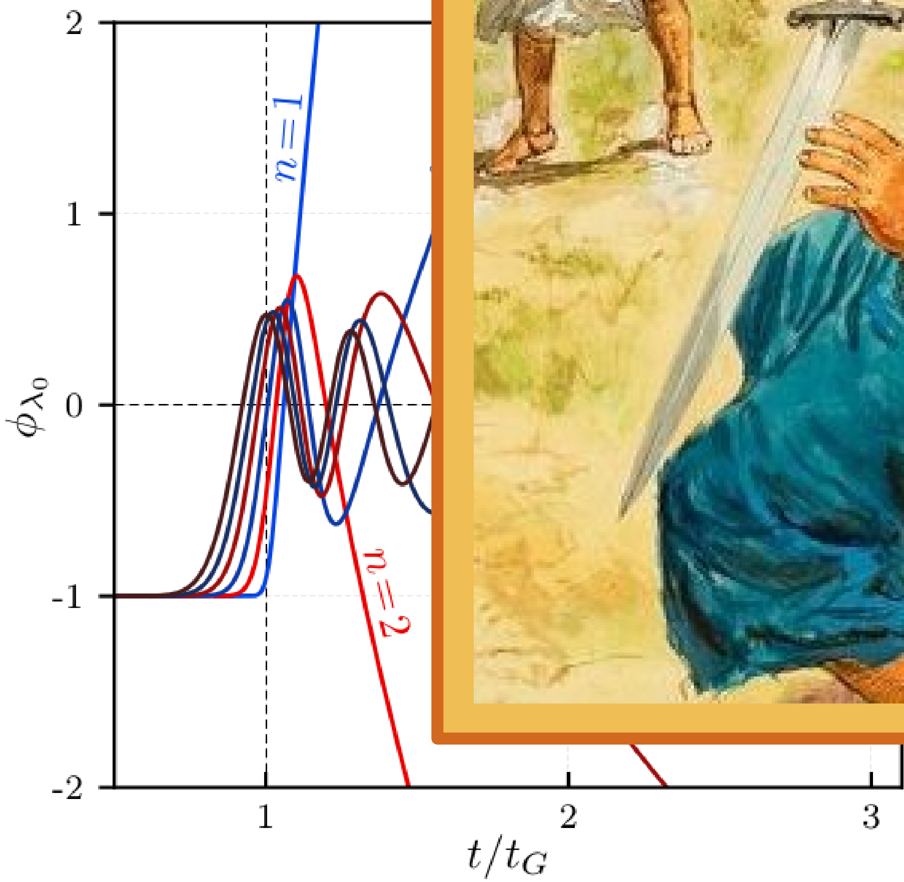
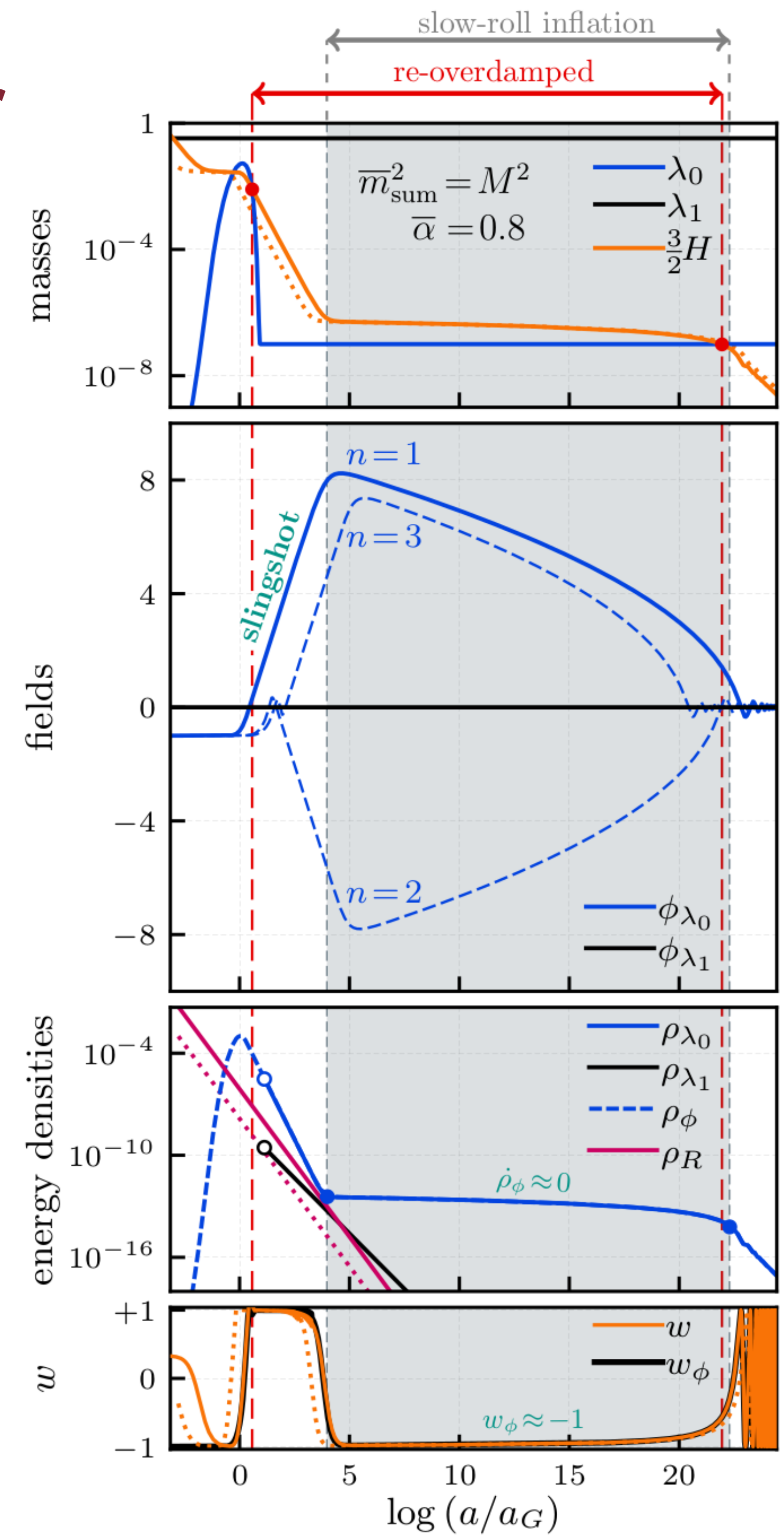


(Yeah, I know… it's really technically a sling rather than a slingshot, but "slingshot" has a nicer ring to it.)

Evolution of the Scalar Sector

- Assume that the universe is initially radiation-dominated, with no energy density stored in the scalar sector.
- For concreteness, start from an initial field value $\mathcal{A}_\phi \equiv \phi_{\lambda_0}(t \ll t_G) \sim -M_P$.



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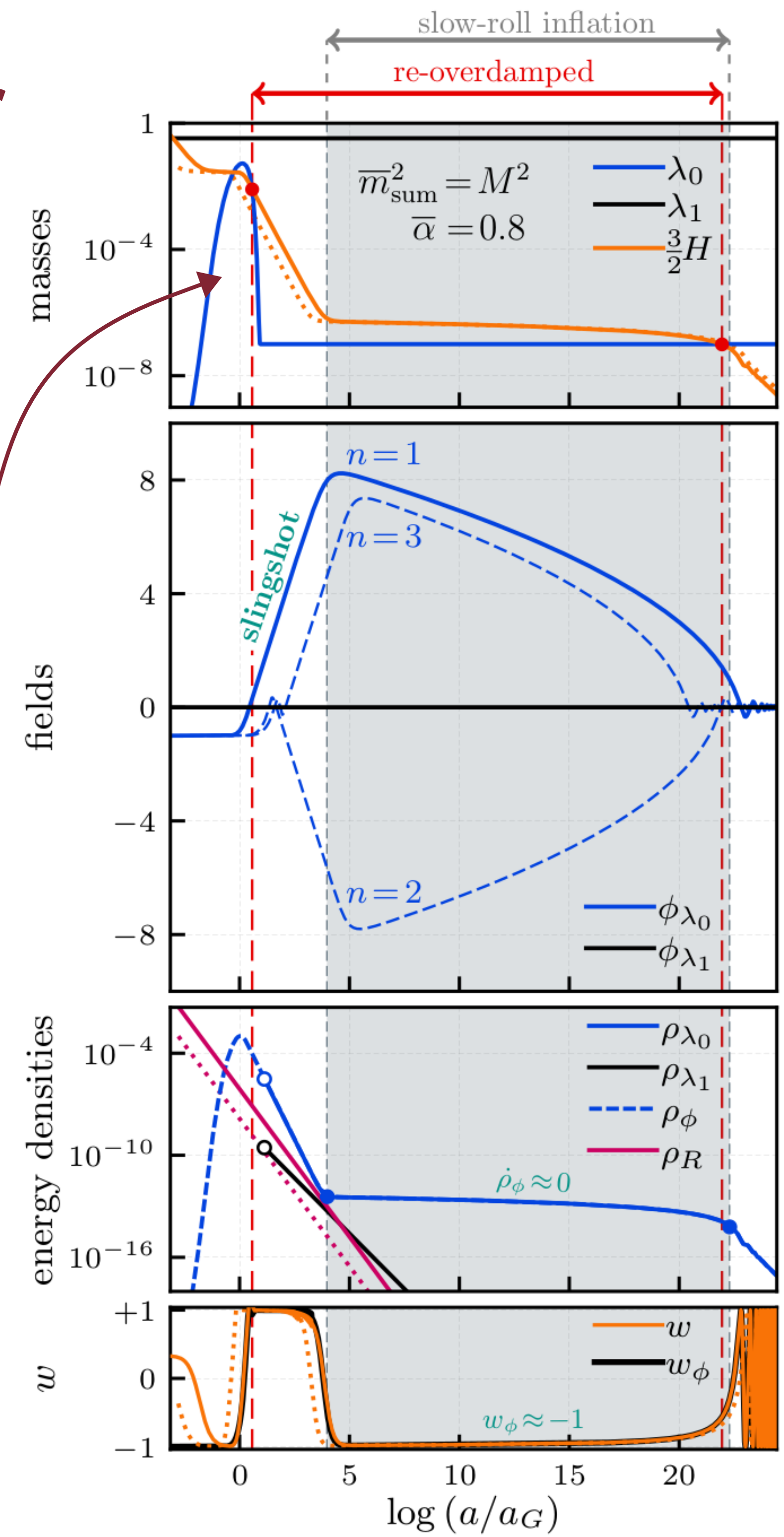

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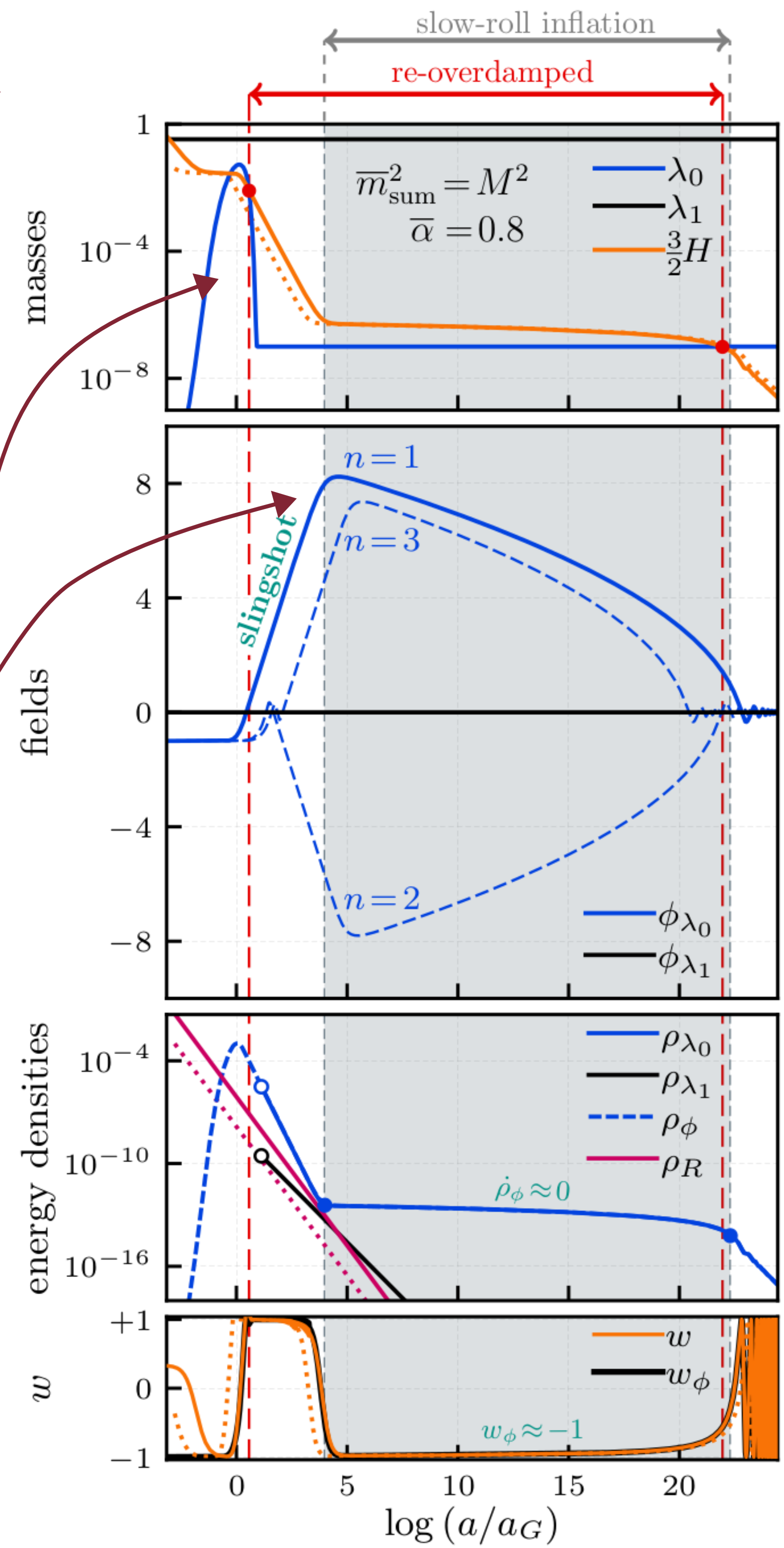

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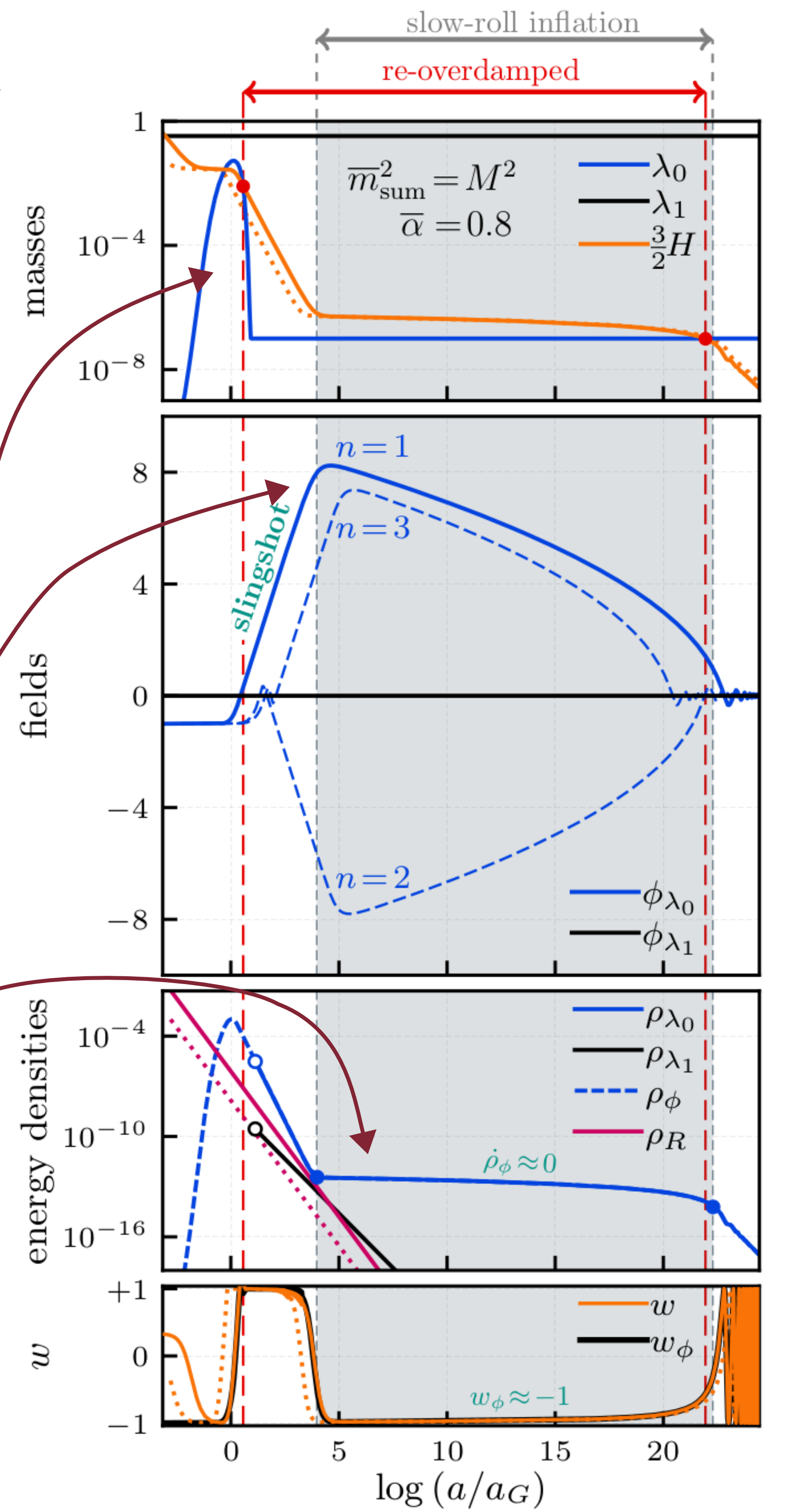

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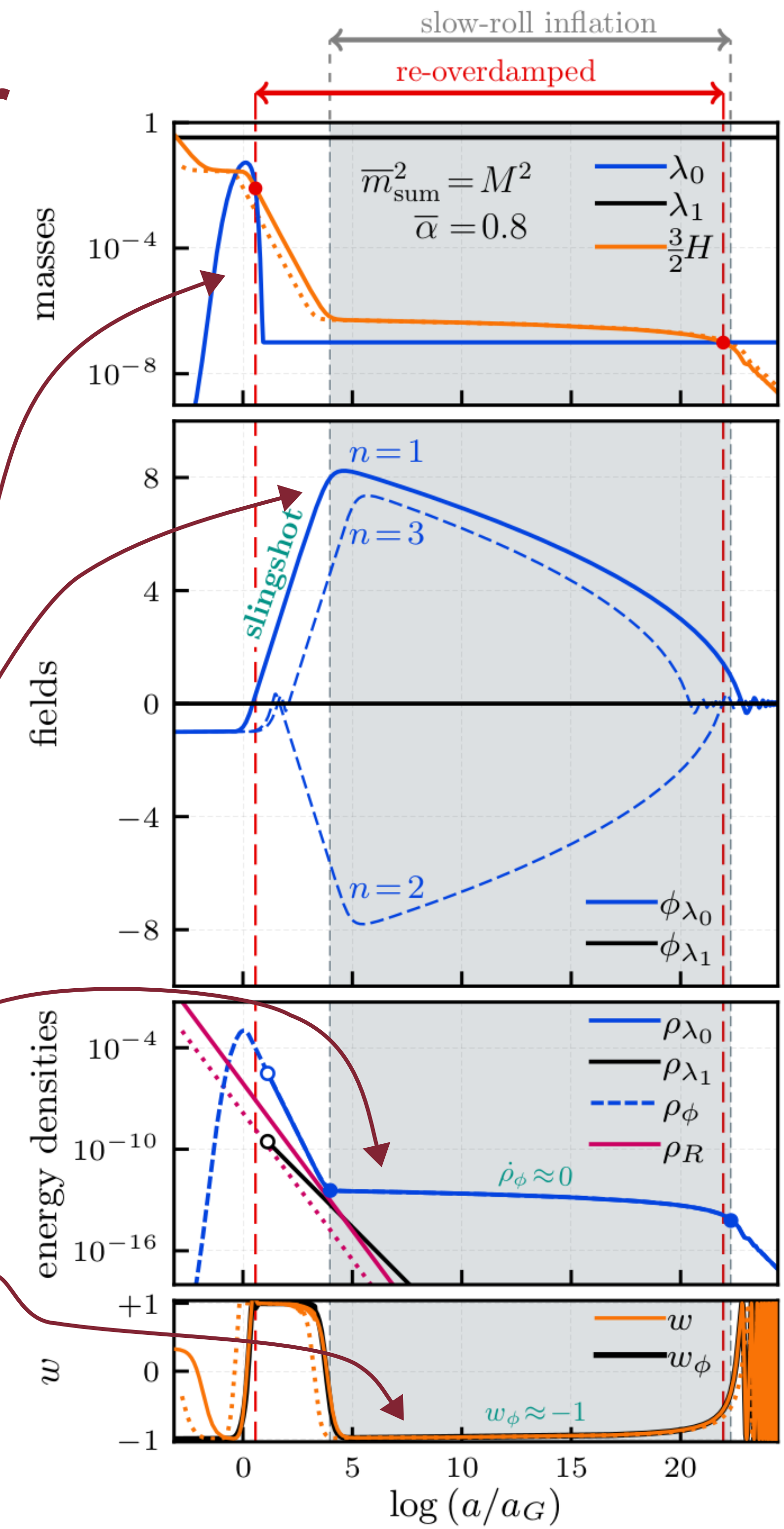

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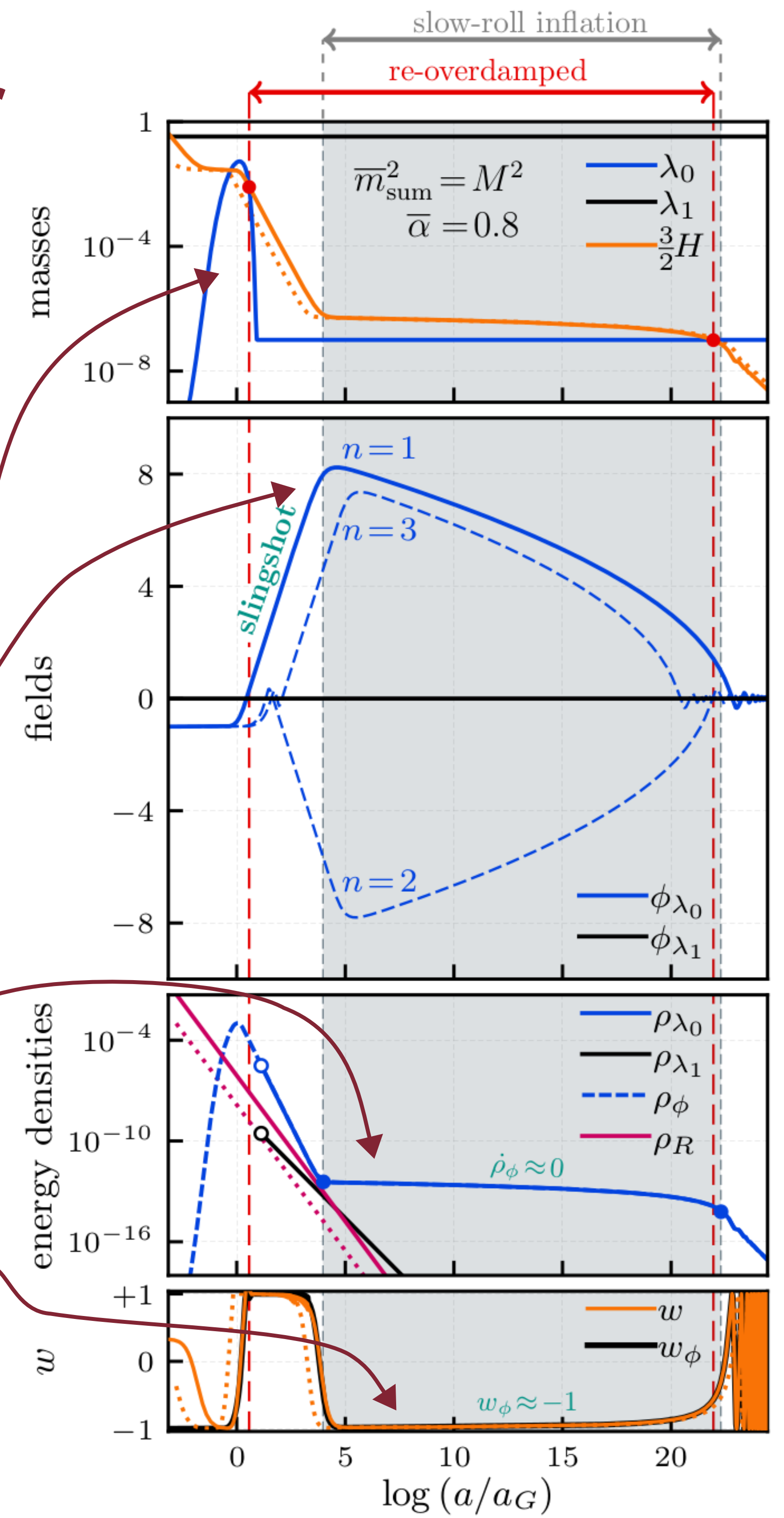
In this way, an inflationary epoch emerges!



Mapping to Inflationary Quantities

- In general, we can construct a map between the parameters of our two-field model and initial conditions for the inflaton field.

$$\{M^2, \overline{m}^2_{ij}, t_G, \Delta_G\} \longrightarrow \phi^{(I)}_{\lambda_0}$$

- Certain regions of our parameter space give rise to suitable initial field VEVs, while others do not. The most auspicious region of parameter space is that within which...
  - ...there is ***substantial mixing***.

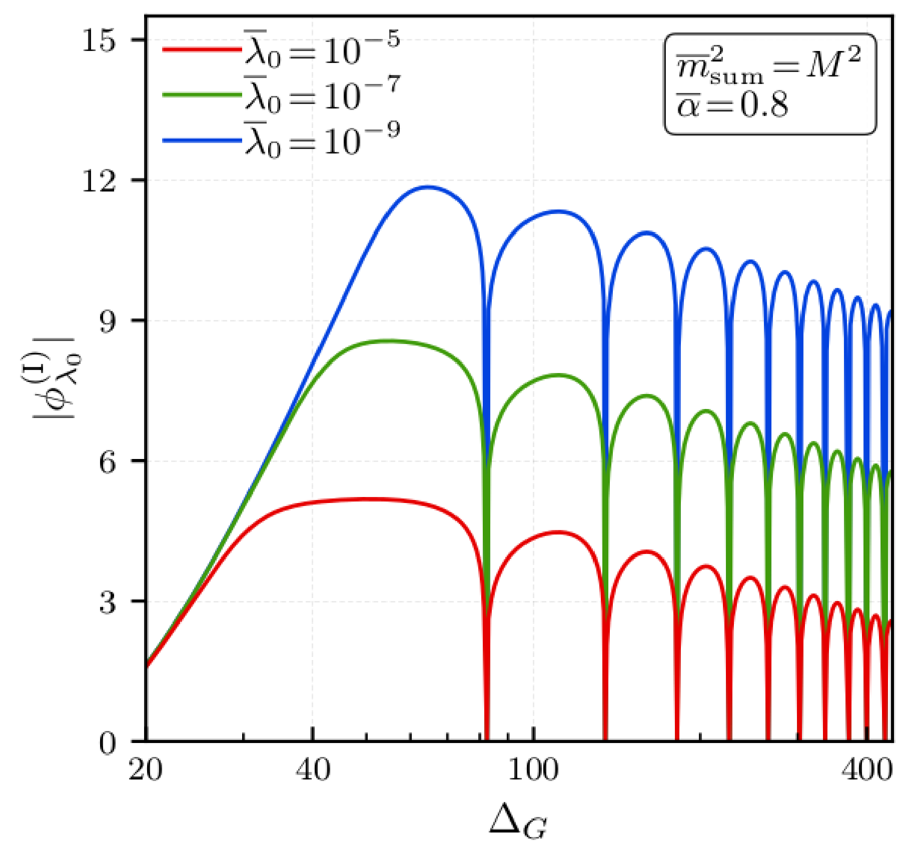
$$\overline{\beta} \ll 1$$

  - ...the late-time inflaton mass is ***small***.

$$\overline{\lambda}_0^2 \approx \frac{1}{2}\overline{m}^2_{\text{sum}}(1-\overline{\alpha}^2)\overline{\beta} \ll M^2$$

  - ...the width of the phase transition is at least ***approximately on resonance***, though fine-tuning is not required.

$$\Delta_G \approx \Delta_G^{(n)}$$



Number of e-folds

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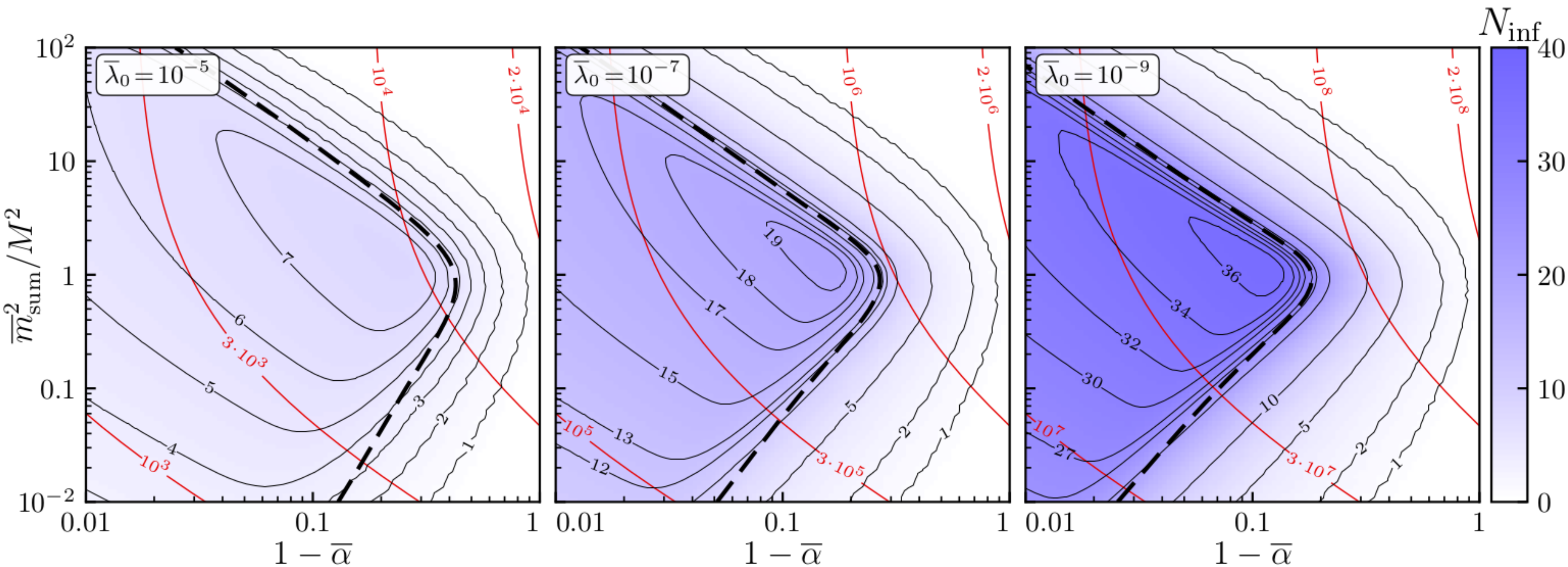

Contour where $\rho_{\lambda 0} = \rho_{\lambda 1}$ at the start of inflation.

Contours of the ratio $\mathcal{Q}$, where...

$$\mathcal{Q} \equiv \frac{\lambda_0(t_p)}{\bar{\lambda}_0}$$

Mass at the peak of the pulse

Late-time asymptotic mass

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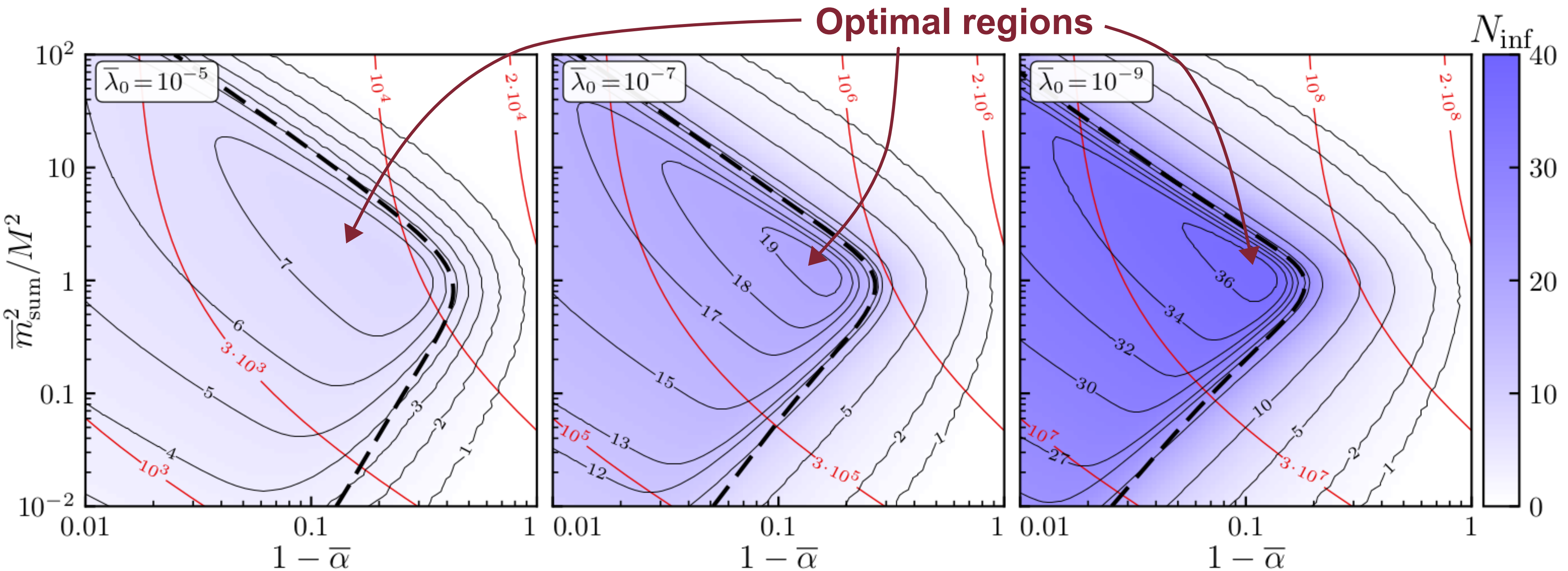

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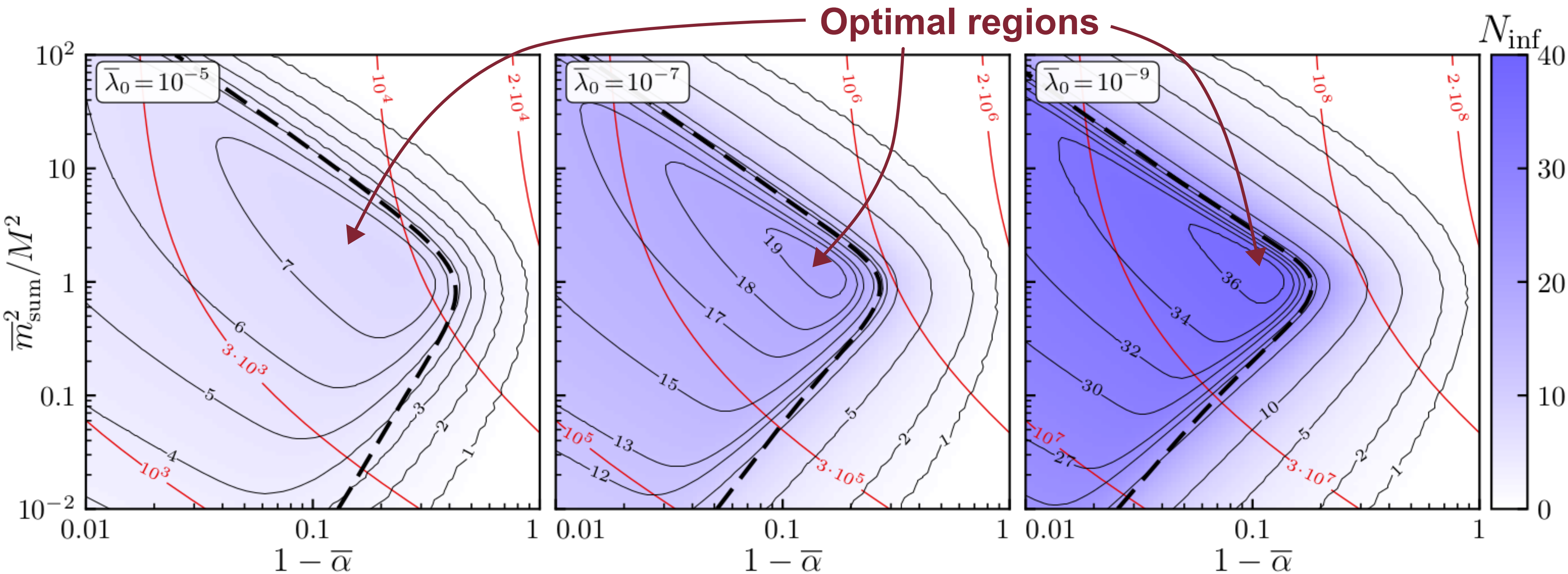

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Mass at the peak of the pulse

Late-time asymptotic mass

Clearly, a significant number of e-folds of inflation can be induced by this slingshot mechanism!

How High Can We Get?

- Within the optimal parameter-space region identified above, the inflaton VEV at the onset of inflation turns out to be well approximated (for $n = 1$) by

$$|\phi^{(I)}_{\lambda_0}| \approx \sqrt{\frac{2}{3}} M_P W\left(\frac{2\sqrt{3}Q}{e^{\sqrt{\frac{3}{2}}\frac{|\mathcal{A}_\phi|}{M_P}}} \frac{|\mathcal{A}_\phi|}{M_P} \right)$$

Lambert W-function

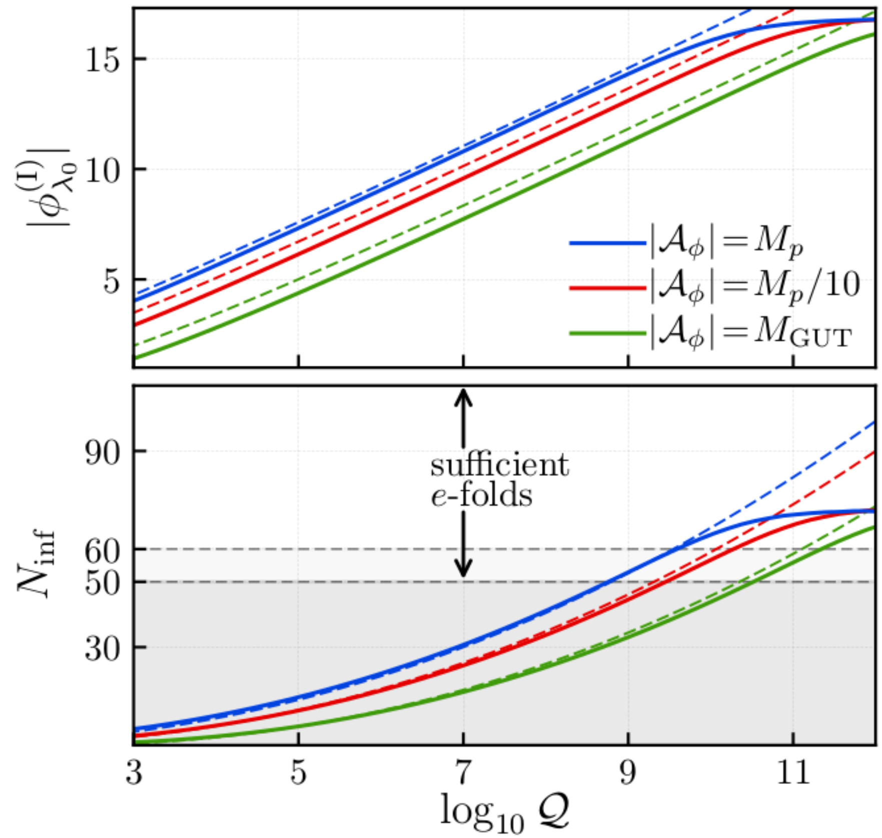
- Taking $|\mathcal{A}_\phi| = M_P$, we find that the corresponding number of e-folds of inflation is approximately

$$N_{\rm inf} \approx \frac{1}{6}\log^2\left(\frac{2Q}{1+\sqrt{3}\log Q}\right)$$

- A full numerical calculation reveals that indeed this mechanism can generate $N_{\rm inf} \gtrsim (50-60)$ e-folds of inflation.
- Interestingly, the value of $N_{\rm inf}$ plateaus only slightly above this threshold.

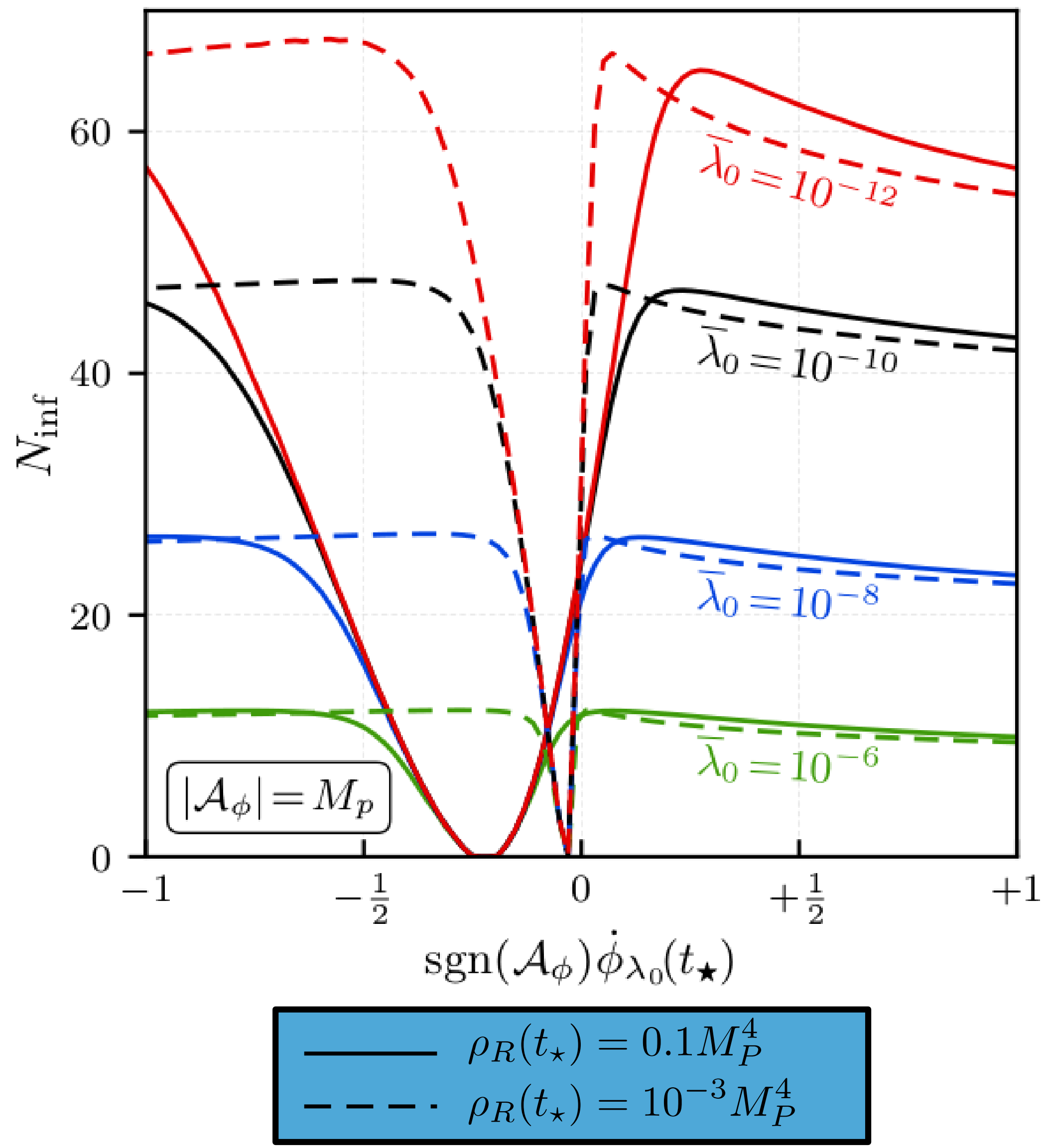
Solid lines: full numerical results

Dashed lines: approximate values



Initial Field Velocities

- It is also interesting to consider how are results are modified in the case in which the fields in our scalar sector have nonzero initial velocities – and in particular, when $\dot{\phi}_{\lambda_0}(t_\star) \neq 0$, where $t_\star \ll t_G$.
- In cases in which $\mathcal{A}_\phi$ and $\dot{\phi}_{\lambda_0}(t_\star)$ have opposite sign and the field is propelled to smaller field values by its initial velocity, N_{inf} is significantly suppressed for certain values of $\dot{\phi}_{\lambda_0}(t_\star)$.
- However, other than this local suppression, N_{inf} remains large, even in the presence of an initial field velocity.
- Thus, our slingshot mechanism is robust against modifications of the initial field velocities.



Inflationary Observables

Important: Once the slingshot has done its job and inflation begins, we just have ***regular old quadratic inflation***.

- The power spectra for scalar and tensor perturbations are typically parameterized as

$$\mathcal{P}_s = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \qquad \mathcal{P}_T = A_T \left(\frac{k}{k_*}\right)^{n_T}$$

pivot scale

- Quantities like $r = A_T/A_s$, the spectral index n_s, etc., are ***essentially the same*** as in other models of quadratic inflation.
- By contrast, A_s also depends of the scale of inflation – or, equivalently in our scenario, on the late-time inflaton mass $\overline{\lambda}_0$:

$$A_s = \frac{1}{12\pi^2 M_P^6}\left[\frac{V^{3/2}(\phi_{\lambda_0})}{V'(\phi_{\lambda_0})}\right]^2\Bigg|_{\phi_{\lambda_0}=\phi^*_{\lambda_0}} = \frac{1}{6}\left[\frac{(\phi^*_{\lambda_0})^2\overline{\lambda}_0}{M_P^3}\right]^2$$

- Planck data with $k_* = 0.05\ \mathrm{Mpc}^{-1}$ yield $A_s = 2 \times 10^{-10}$. [Akrami et al. '18] This implies that $\overline{\lambda}_0 \sim \mathcal{O}(10^{-6} M_P)$.
- While some tension exists between this bound and the bound on N_{inf}, it can be reconciled in many ways – a curvaton, modifications to $V(\phi)$, etc.

Particle Production

- The masses of our scalars change quite abruptly during the phase transition. This non-adiabatic evolution can lead to the production of ***inflaton field quanta***.
- The total energy density associated with such quanta can be determined by evaluating the ***Bogoliubov coefficients*** between the asymptotic states at $t \ll t_G$ and the asymptotic states at $t \gg t_G$.
- These turn out to be mathematically equivalent to the transmission coefficients for scattering off a potential $V(x) \propto -\text{sech}^2(x)$ in non-relativistic quantum mechanics.

$$\frac{\rho^{(\mathrm{p})}_{\lambda_0}}{\rho_\lambda} \approx \frac{12\zeta(3)}{\pi^6 \lambda_0^2(t_p)\mathcal{A}_\phi^2}\left(\frac{1}{\Delta_G^4}\right)$$

Total energy density of inflaton quanta

Total energy density of background fields

- This implies a constraint on $\mathcal{Q}$ of the form

$$\mathcal{Q} \ll \mathcal{Q}_{\max} \approx \frac{\pi^9 \lambda_0^3(t_p)\Delta_G^6|\phi^{(I)}_{\lambda_0}|^2\mathcal{A}_\phi^2}{24\sqrt{3}\zeta^{3/2}(3)}$$

Plugging in some numbers

$\mathcal{A}_\phi = M_P, \overline{m}_{\text{sum}} = M$

$\overline{\alpha} = 0.9, \Delta_G = \Delta_G^{(1)}$

$$\mathcal{Q}_{\max} \approx 3.2 \times 10^{11}$$

...which is a non-trivial constraint, but one we can easily satisfy.

Summary

- In this talk, we have presented a novel mechanism for realizing the initial conditions for cosmic inflation.
- This mechanism can be realized in theories involving multiple scalar fields in which a cosmological phase transition dynamically gives rise to additional contirbutions to the scalar mass matrix..
- The time-dependent mixing which arises in the mass matrix during the phase transitiongive rise to two crucial effects – a ***parametric resonance*** and a period of ***re-overdamping*** – which work in concert to propel or “slingshot” the inflaton VEV to larger values and precipitate a subsequent epoch of accelerated expansion.
- We have shown that this scenario is capable of yielding an appropriate number of e-folds $N_{\text{inf}} \gtrsim (50-60)$ for successful cosmic inflation.

Future Directions

- While challenging to arrange in practice, it may be possible to driving the inflaton VEV to higher values via multiple successive "slingshots."
- It would be interesting to generalize our results for a quadratic inflaton potential to more general functional forms for $V(\phi)$.
- In scenarios in which $N_{\text{inf}} \sim 60$, the initial conditions which determine the spectrum of large-scale perturbations prior to inflation differ from those associated with the Bunch-Davies vacuum. This has potentially observable consequences. [Ramirez, Schwarz '11; Ramirez '12]
- The presence of additional heavy fields during inflation can affect the pattern of non-Gaussianities in the CMB. This could potentially give rise to "cosmological-collider" signals. [Arkani-Hamed, Maldacena '15]