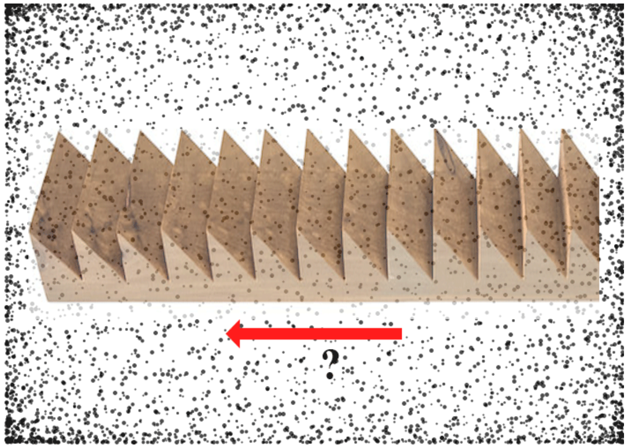
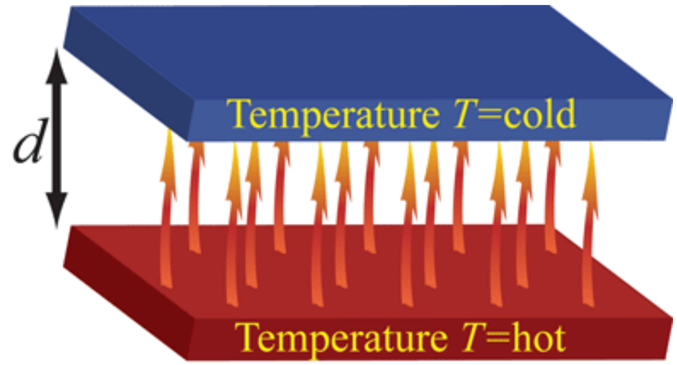
Casimir force heat engine





David Gelbwaser, Noah Graham, Matthias Krüger

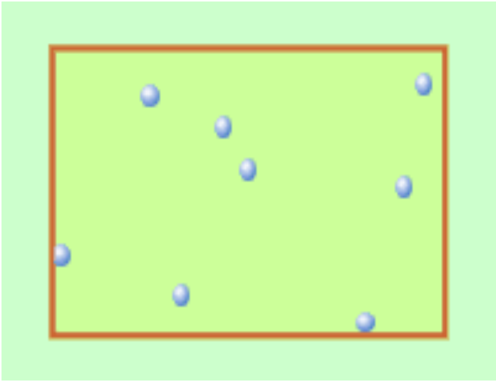
Outline

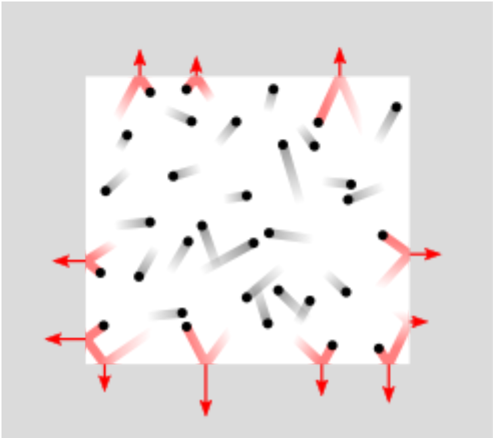
Outline

Force from Equilibrium Fluctuation

(from classical to quantum and back!)

- **Pressure** is a force from restriction of particle fluctuations





$$P = k_B T \frac{\partial \ln Z}{\partial V} \quad , \quad Z = \int \prod_i d^3q_i \, d^3p_i \, e^{-\beta \mathcal{H}}$$

- Force per unit area is a bulk property (function of equilibrium state)
  - independent of the shape of the container and potentials at the wall
  - independent of the dynamics of the particles (momentum integrations)
- For a dilute gas: $P = n \, k_B T + \cdots$ (non-universal corrections)

- **Radiation:** Photon density is temperature dependent:

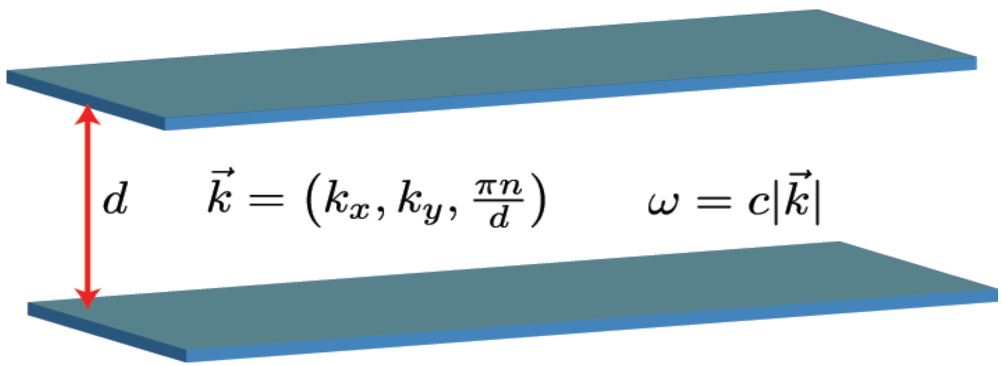
$$n(T) \propto \frac{1}{\lambda_T^3}\,,\ \text{with} \quad \lambda_T = \frac{\hbar c}{k_B T}$$

Stefan-Boltzmann law of black-body pressure:

$$P = \sigma' T^4$$

Casimir Force: Virtual photons (QED fluctuations) at low temperatures: $\lambda_T > d$

- "On the attraction between two perfectly conducting plates," H.B.G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948)



- Quantum fluctuations of electromagnetic field in the vacuum between leads to an **attractive force** between perfect mirrors

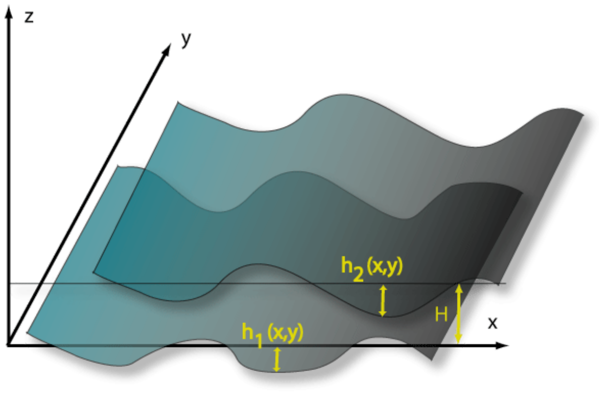
$$\begin{aligned} F &= -\frac{\partial E_0}{\partial d} = -\text{Area} \times \frac{\pi^2}{240} \times \frac{\hbar c}{d^4} \\ P &= \frac{F}{\text{Area}} \approx -1.3 \times 10^{-5}\text{atm.} \left(\frac{1\mu\text{m}}{d}\right)^4 \end{aligned}$$

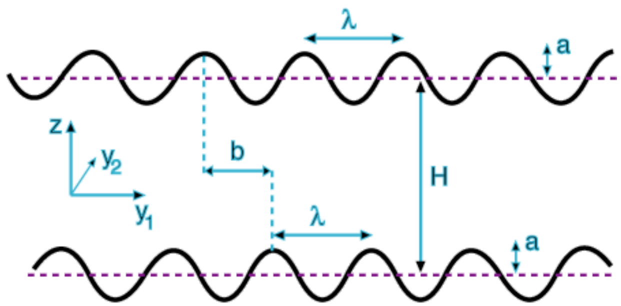
- "The Theory of Molecular Attractive Forces Between Solids," E.M. Lifshitz, Soviet Physics **2**, 73 (1956),

  Generalizes above by considering fluctuating current sources in the bodies, and at finite temperatures.

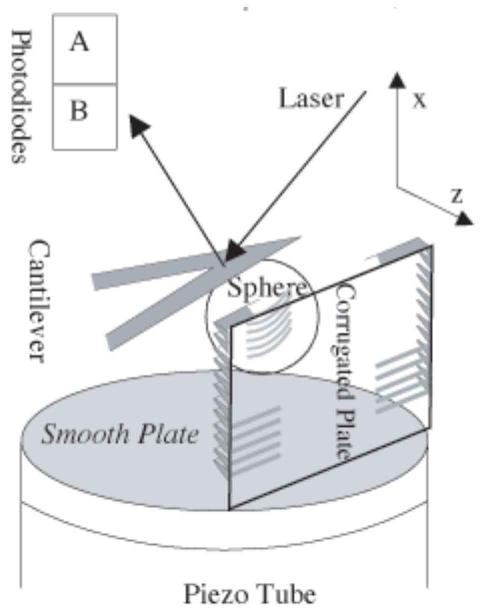
- Since 1995 confirmed in myriad experiments.

Lateral Casimir Force: A sideways force to "align" corrugated plates into position of minimum energy:

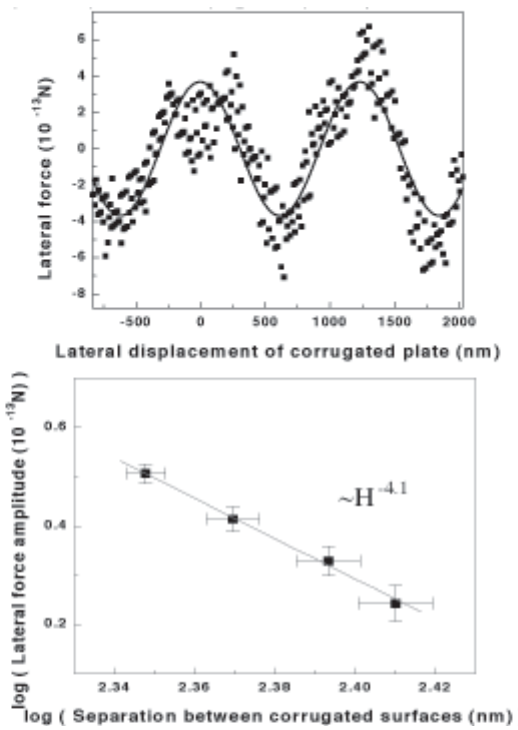




- "Mechanical Response of Vacuum," Golestanian, & Kardar, [Phys. Rev. Lett. **78**, 3421 (1997)]
- "Demonstration of the Lateral Casimir Force," F. Chen, U. Mohideen, *et. al* , [Phys. Rev. Lett. **88**, 101801 (2002)]

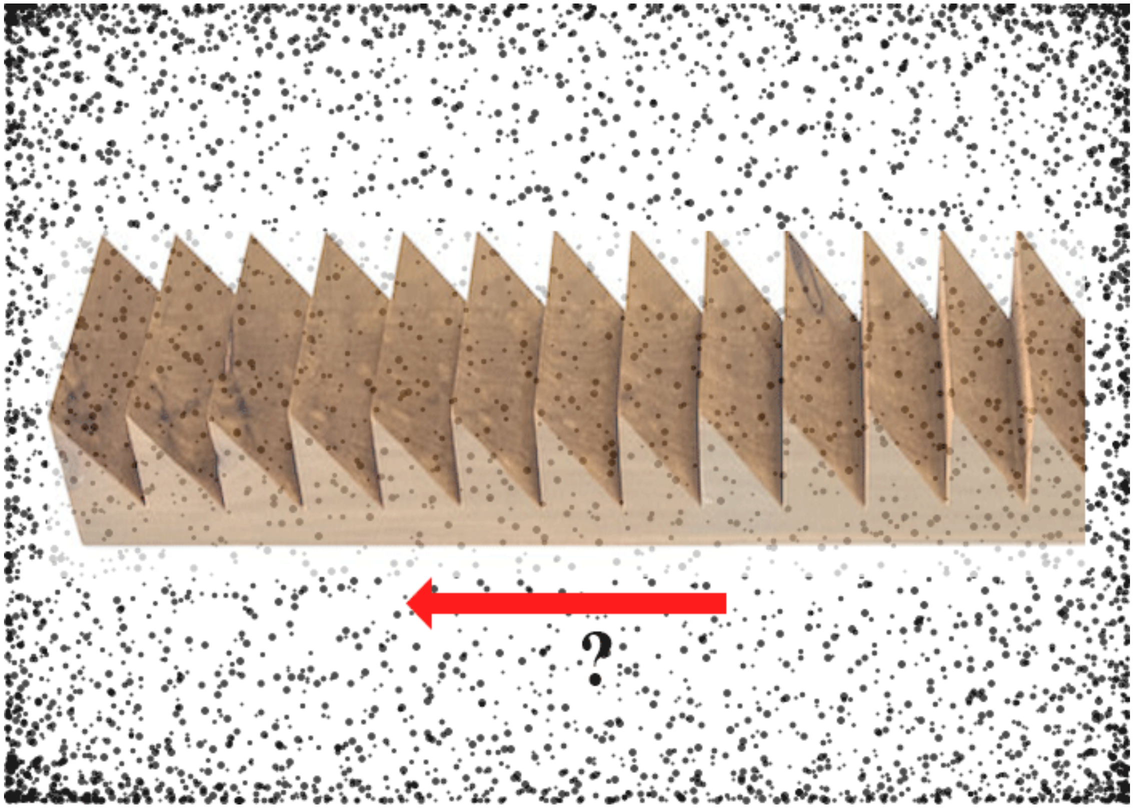


separation H=221nm, wave length λ=1200nm



Motion of Ratchet

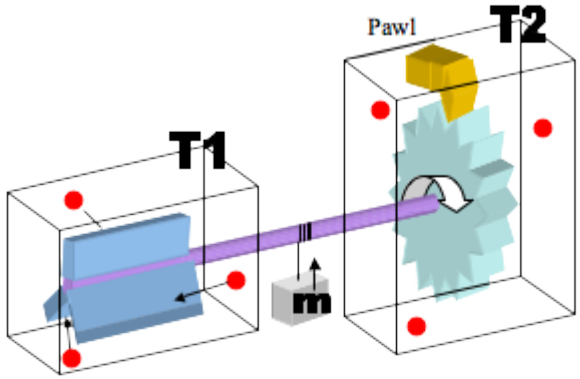
Asymmetry: Can asymmetric boundaries rectify fluctuations to force and motion?



This is clearly not possible in thermal equilibrium (time reversal symmetry).

Non-equilibrium fluctuations can indeed generate force and motion

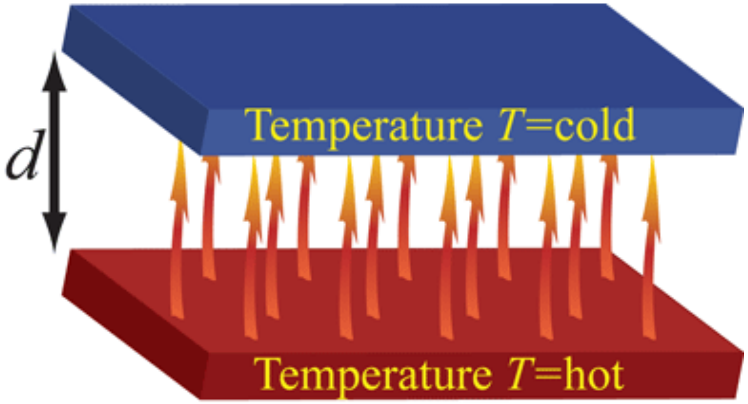
- [Feynman's ratchet and pawl]: ([youtube])

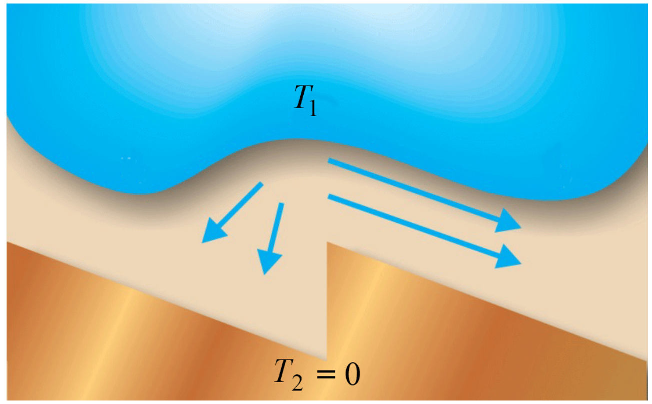


- Random motion of bacteria:

Nonequilibrium fluctuations + asymmetry can potentially lead to motive force

Radiative heat transfer as source of nonequilibrium fluctuations



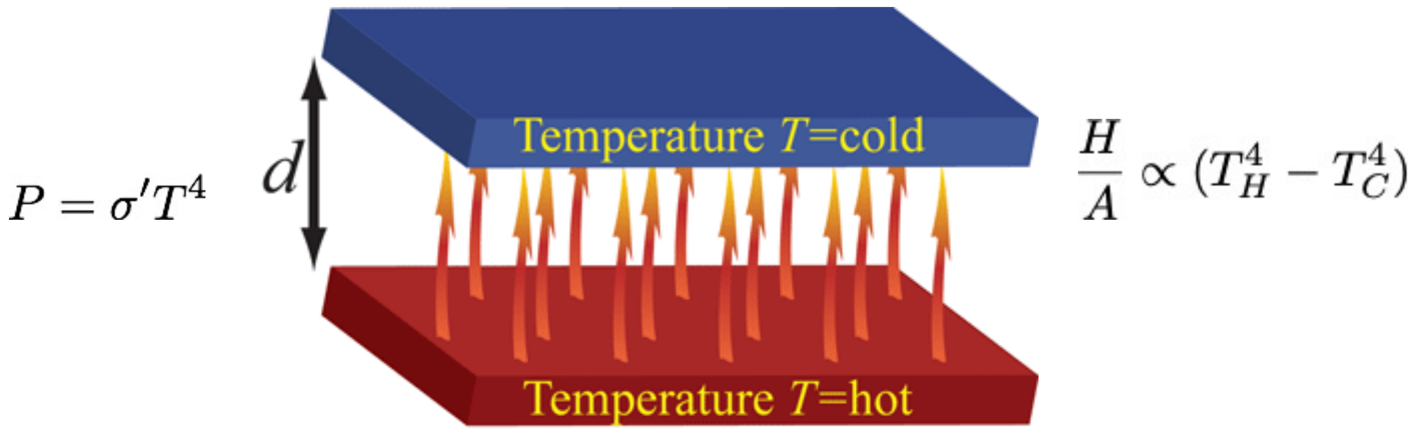


Motion

Heat

(... transport in non-equilibrium QED)

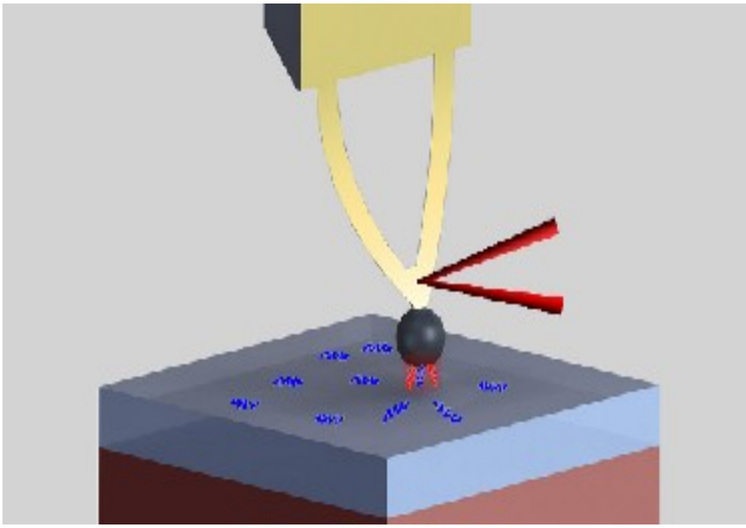
Radiation Pressure is a feature of non-equilibrium ***steady states*** with different temperatures

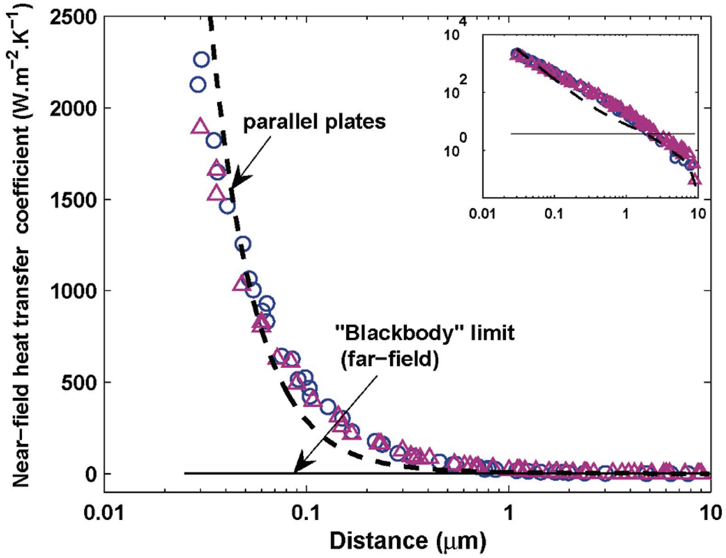


At short scales **"near-field effects"** due to **evanescent waves** modify classical "Stefan-Boltzmann" law:

"Surface Phonon Polaritons Mediated Energy Transfer between Nanoscale Gaps," S.Shen, A. Narayanaswamy, & G. Chen,

Nano Lett. **9**, 2909 (2009) Breaking the law, at the nanoscale (MIT news, July 29, 2009)

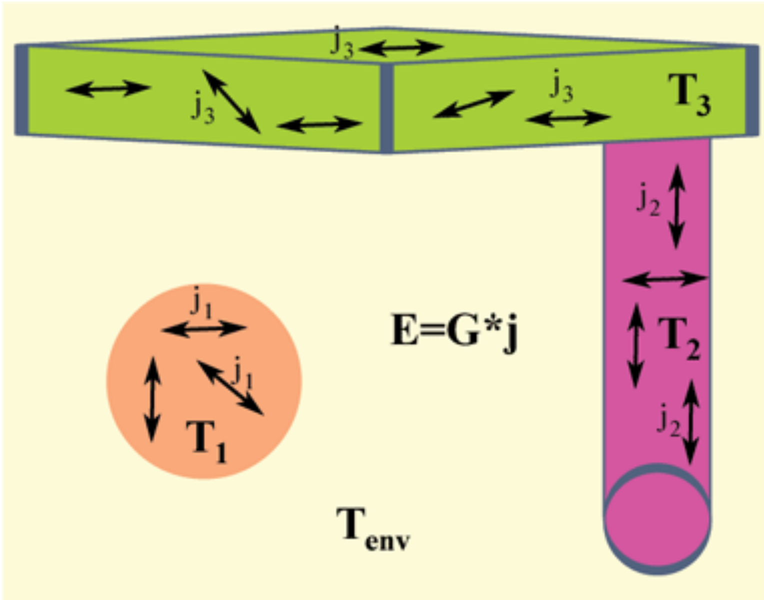




- A generalized approach for computation of Casimir forces, as well as radiation and heat transfer.
  - "Nonequilibrium Fluctuational QED: Heat Radiation, Heat Transfer and Force,"

    G. Bimonte, T. Emig, M. Kardar, and M. Krüger, [Annual Review of Condensed Matter Physics **8**, 119 (2017)]

Rytov (1959):



"Fluctuational QED"

- Fluctuating currents in each object are related to its temperature by a *fluctuation-dissipation* condition:

$$\langle j_\alpha(\mathbf{r},\omega) j_\beta(\mathbf{r}',\omega')\rangle = \hbar\omega^2 \left(\frac{1}{2} + \frac{1}{e^{\hbar\omega/k_B T} - 1}\right) \Im\varepsilon(\mathbf{r},\omega)\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega')$$

- The EM field due to *thermal fluctuations* of *one object* is related to overall Green's function by:

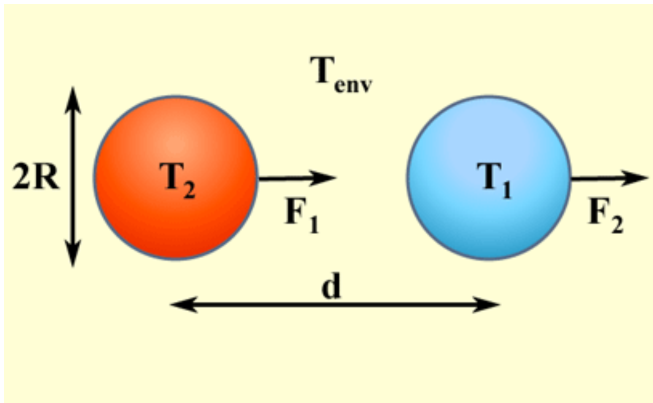
$$C_\alpha^{sc}(T) \equiv \langle E_i(\mathbf{r},\omega) E_j^*(\mathbf{r}',\omega')\rangle = \frac{8\pi\hbar\omega^4}{c^4} \coth\left(\frac{\hbar\omega}{2k_B T}\right)\delta(\omega-\omega')$$
$$\times \int d^3\mathbf{r}'' \mathcal{G}_{ik}(\omega;\mathbf{r},\mathbf{r}'')\Im\varepsilon(\omega,\mathbf{r}'')\mathcal{G}^*_{kj}(\omega;\mathbf{r}'',\mathbf{r}')$$

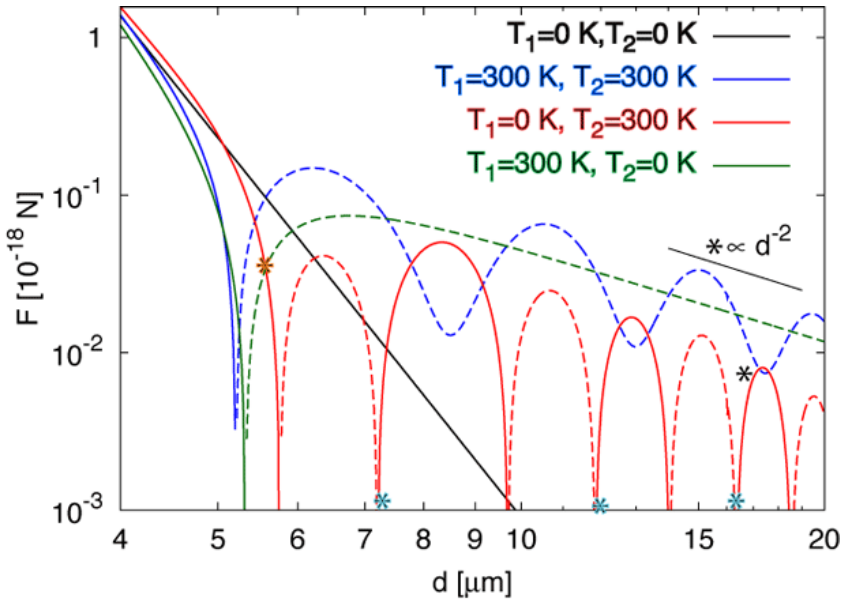
- The overall fluctuations with many objects at different temperatures is then given by:

$$C^{neq}(T_{env},\{T_\alpha\}) = C^{eq}(T_{env}) + \sum_\alpha \left[C_\alpha^{sc}(T_\alpha) - C_\alpha^{sc}(T_{env})\right]$$

- From EM correlations follow the stress tensor and the Poynting vector, hence forces and radiation.

- As an example with asymmetry, consider forces between two micro-spheres at different temperatures:





- "Non-equilibrium Casimir forces: Spheres and sphere-plate,"

  M. Krüger, T. Emig,G. Bimonte and M. Kardar, [Europhys. Lett. **95**, 21002 (2011)] (1 micron Si O2 spheres, force on #2)

- Whereas the *equilibrium* force (attractive) falls off as $1/d^6$, the *non-equilibrium* force decays as $1/d^2$. (*)

- The non-equilibrium force can be attractive and repulsive. (*)

- Unlike in thermal equilibrium, there are points of stable levitation. (*)

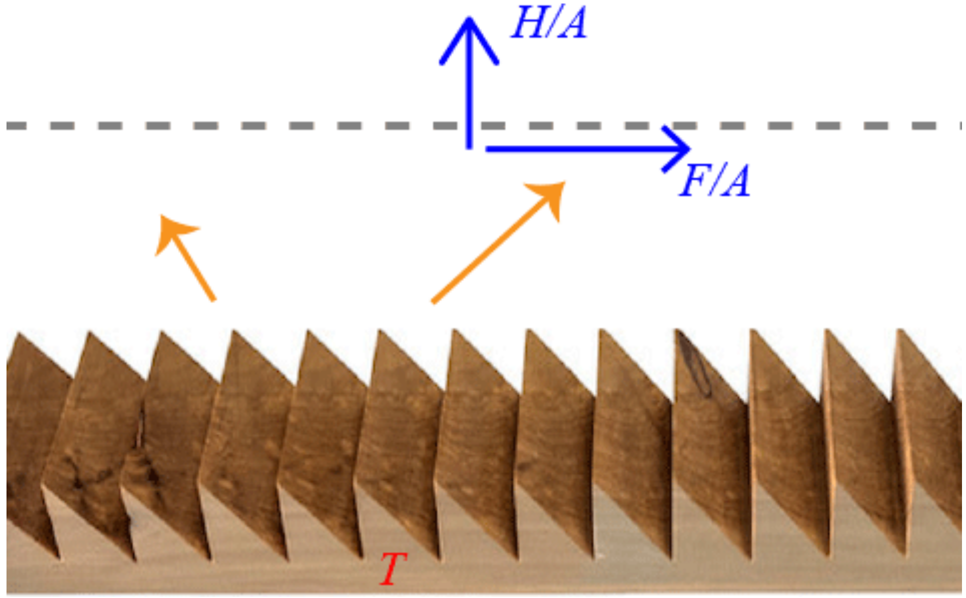
- Forces are not equal and opposite, with points of equal force in the same direction! (*)

Near field effects strongly modify force and heat transfer.

Heat

Reciprocity in EM

Radiation from a heated ratchet:



$\beta = (k_B T)^{-1}$

$$\frac{H}{A} = \int_0^\infty d\omega \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1} \int d^2 k_{\|} \; \mathcal{S}_\omega(\vec{k}_{\|})$$

$$\frac{\vec{F}}{A} = \int_0^\infty d\omega \frac{1}{e^{\beta\hbar\omega} - 1} \int d^2 k_{\|} \; \hbar\vec{k}_{\|} \; \mathcal{S}_\omega(\vec{k}_{\|})$$

"Near Field Propulsion Forces from Nonreciprocal Media," D. Gelbwaser-Klimovsky, N. Graham, M. Kardar, and Matthias Krüger,

Phys. Rev. Lett. **126**, 170401 (2021).

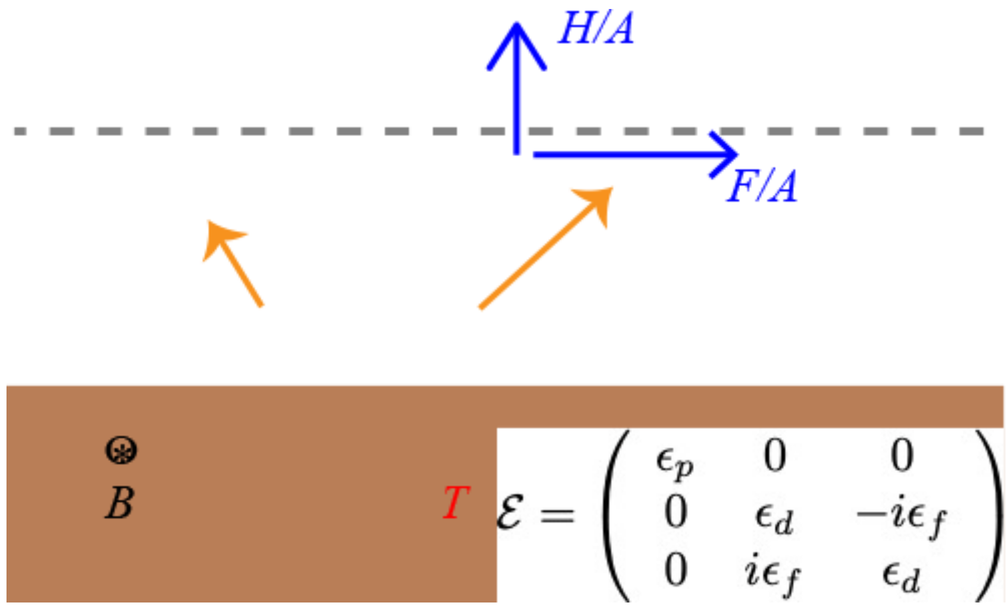
- Dispersion relation of photons (relevant at far fields):

$$\omega = c\sqrt{k_\perp^2 + k_\parallel^2} \quad \Longrightarrow \quad |\vec{F}| < H/c$$

- Reciprocity of electromagnetic field equations: $\mathcal{S}_\omega(\vec{k}_\parallel) = \mathcal{S}_\omega(-\vec{k}_\parallel) \quad \Longrightarrow \quad \vec{F} = 0$

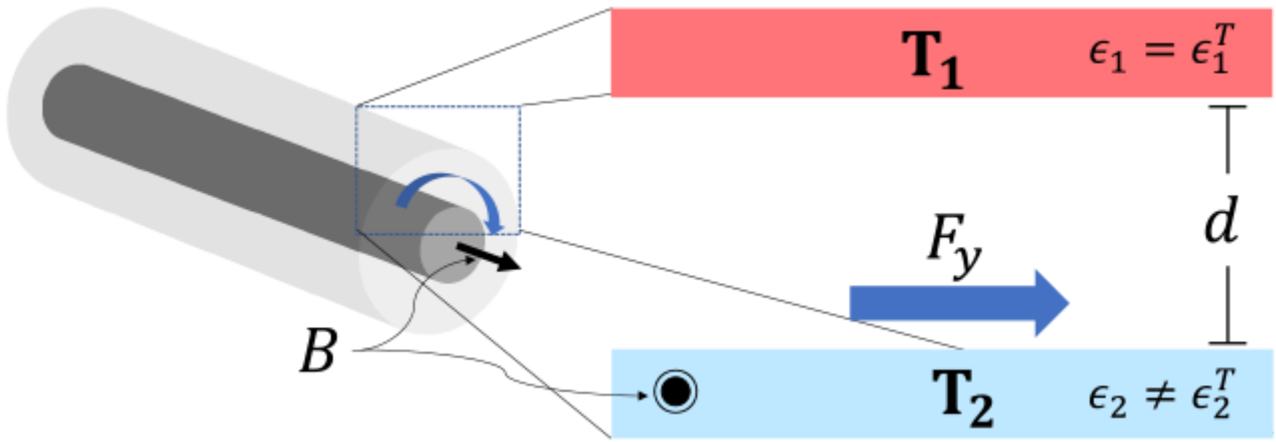
"Nonreciprocal radiative heat transfer between two planar bodies," L. Fan, ..., C. Fan, Phys. Rev. B **101**, 085407 (2020).

- The reciprocity condition can be broken in a number of ways, e.g. by magneto-optical couplings



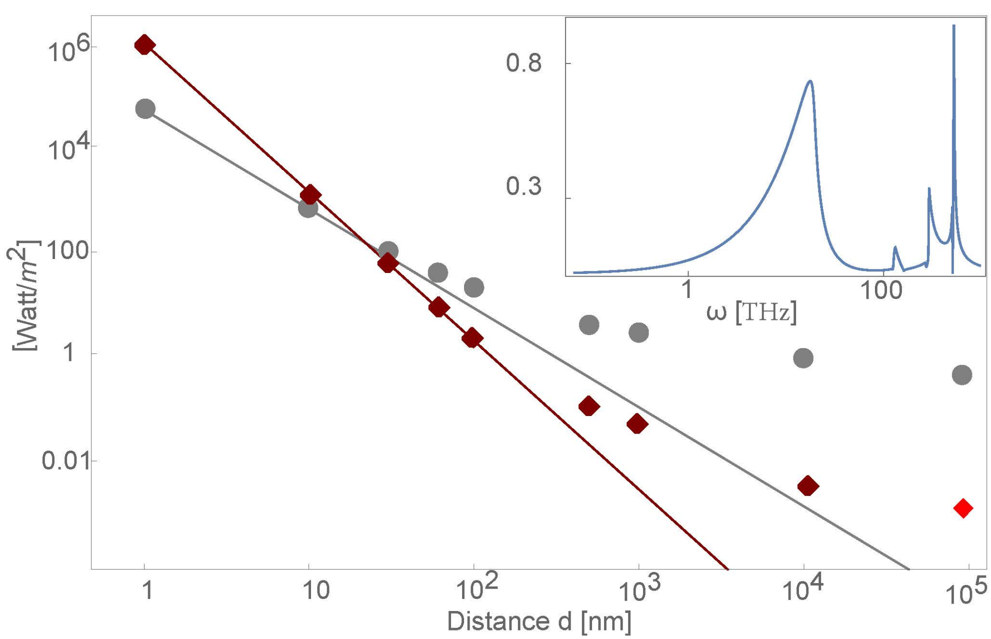
$$\mathcal{E} = \begin{pmatrix} \epsilon_p & 0 & 0 \\ 0 & \epsilon_d & -i\epsilon_f \\ 0 & i\epsilon_f & \epsilon_d \end{pmatrix}$$

- There is a motive force with non-reciprocal media, which can be used to construct a heat engine



As with heat transfer, the motive force is much larger in the near-field regime $d < \frac{\bar{\omega}}{c}$, where $\bar{\omega}$ is a characteristic material frequency.

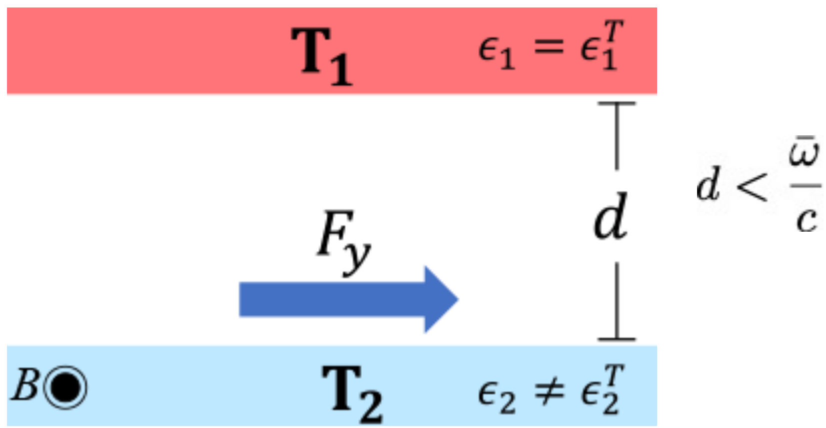
$$\frac{F}{H}c < 1 \qquad \xrightarrow{d \ll c/\omega} \qquad \frac{f(\omega)}{h(\omega)}(d\omega) < 1$$



Scaled motive force F (red diamonds), and heat transfer H (gray circles) for a SiC plate and one of n-InSb subject to a magnetic field along the x axis. The dots cor-respond to numeric calculations and the continuous lines to the small d asymptotes. Note that the force changes sign from +y to -y (dark red to light red diamond) at large separation. Inset: $[f(\omega)/h(\omega)](\omega d)$ for d= 1nm. Figures parameters: $B= 10T, T1= 300K, T2= 270K$.

Reciprocity

Power and efficiency in the near field



- For simplicity, the results will be expressed in terms of the **Carnot efficiency**:

$$\eta_c = \frac{T_H - T_C}{T_H} \approx \frac{\Delta T}{T}, \quad \text{for} \quad T_H - T_C \ll T_H = T$$

  - To extract power, the plate must move with some velocity *v*, leading to efficiency

$$\eta = \frac{F_y}{H} v + \mathcal{O}(v^2)$$

  - A naive interpretation of the above formula is that the Carnot efficiency is reached at a velocity scale

$$v_c = \frac{H}{F_y} \eta_c$$

- **Onsager's relations**, however, imply that if heat exchange drives motion, motion must modify heat exchange according to

$$\mathcal{H}(v) = H + \frac{T}{\Delta T} F_y v + \mathcal{O}(v^2) = H \left[1 + \frac{v}{v_c} + \mathcal{O}(v^2) \right]$$

while there are also frictional forces (even with a vacuum gap) reducing the propulsive force to

$$\mathcal{F}_y(v) = F_y - \Gamma v + \mathcal{O}(v^2)$$

Focusing on a single material frequency $\bar{\omega}$, and expressing the degree of non-reciprocity in terms of a single dimensionless parameter α :

$$\frac{F_y}{A} = C\frac{\alpha\eta_c\hbar\bar{\omega}}{d^3}\,, \qquad \frac{H}{A} = 2C\frac{\eta_c\hbar\bar{\omega}^2}{d^2}\,, \qquad \frac{\Gamma}{A} = \frac{3}{2}C\frac{\hbar}{d^4}$$

(C is a dimensionless function of $h\omega/kT$ and reflectivities of the two bodies)

This leads to a final expression for the efficiency of

$$\eta = \eta_c \frac{v}{v_c}\frac{1 - 3v/(\alpha^2 v_c) + \cdots}{1 + v/v_c + \cdots}$$

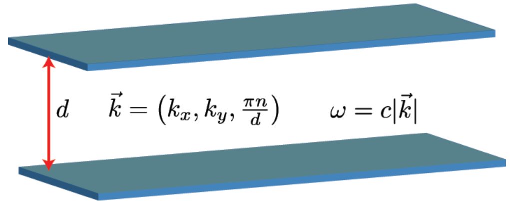
For maximum power

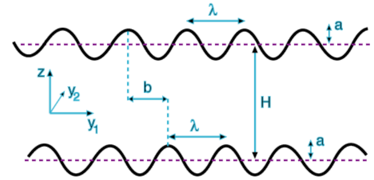
$$v_{MP} = \frac{(\bar{\omega}d)}{3}\,\alpha\eta_c \quad \text{and} \quad \eta_{MP} = \eta_c\,\frac{\alpha^2}{2(6+\alpha^2)}$$

Efficiency

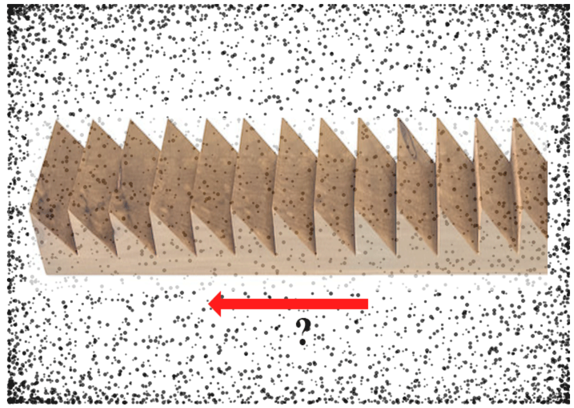
Summary

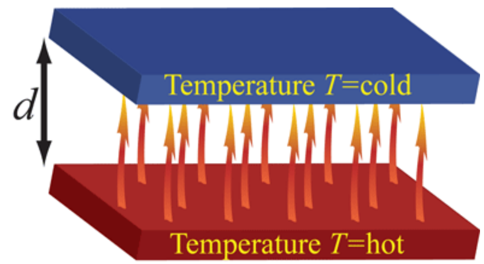
I. Force: Casimir force, normal and lateral



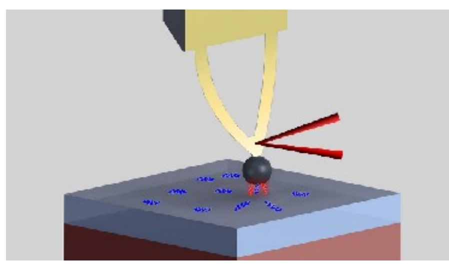


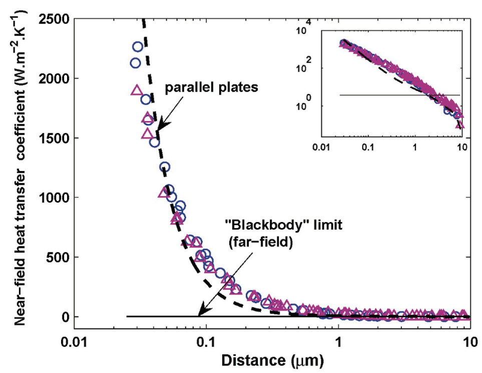
II. Motion: Ratchets, and propulsion

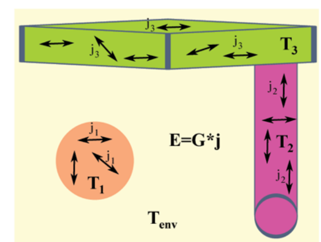




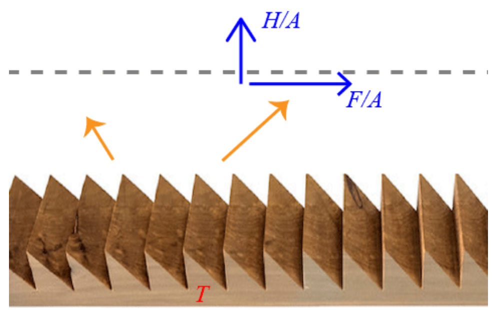
III. Heat/force in near field

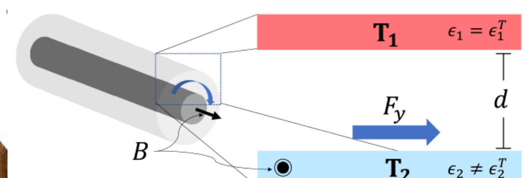




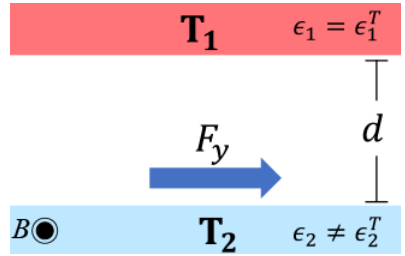


IV. Reciprocity: Constraints on propulsive force



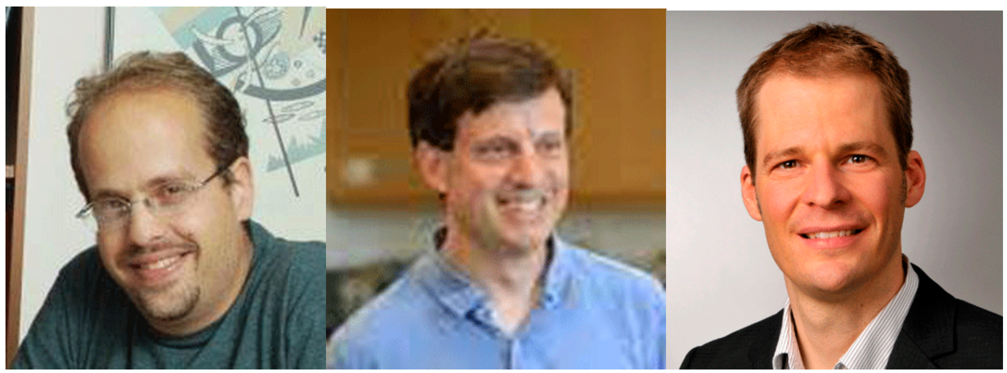


V. [Power](): Work, motion, Onsager, friction, and efficiency



VI. [Summary]()

Acknowledgments



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G. Bimonte, T. Emig, R.L. Jaffe

KITP program [infoversality21]() 25 August 2021

Summary