13 July 2011

S Gukov

S.G. '03 ← Solve $sl(2)$ Chern-Simons via algebraic curves

S.G., T. Dimofte, L. Hollands '10
S.G. + T. Dimofte '11 } Same algebraic curves in 3d/2d $\mathcal{N}=2$ SUSY gauge theories

S.G., T. Dimofte, D. Gaiotto ← cutting and gluing

SL(2) Chern-Simons on 3-manifold M ⟷ 3d $\mathcal{N}=2$ SUSY gauge theory $T(M)$

$$Z_{SS}(M;\hbar) = Z^{3d}_{\varepsilon_1,\varepsilon_2}(T(M))$$ = partition function on $S^3_b$

$$\boxed{\hbar = b^2 = \varepsilon_1/\varepsilon_2}$$

reduce on $S^3$

reduce on M

"geometric engineering"

6d five-brane theory on $M \times S^3$

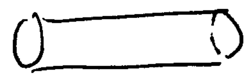
SUSY: put theory in $\Omega$-background

SUSY: top. twist along M

## Semi-classical limit

$$\hbar \sim b^2 \sim \frac{\varepsilon_1}{\varepsilon_2} \to 0$$

$$S^3_b \rightsquigarrow \mathbb{R}^2 \times S^1$$



In $\hbar \to 0$ limit, replace 6d five-brane theory by 5d max. SUSY Yang-Mills theory (D4-brane)

symmetry group of this theory (in Euclidean signature)

$$SO(5)_E \times \underbrace{SO(5)_R}_{\text{transverse rotations}}$$

bosonic fields: $\underset{\text{gauge fields}}{(5,1)} \oplus \underset{\text{Higgs field}}{(1,5)}$

fermionic fields: $(4,4)$

$$SO(5)_E = \left( \begin{array}{c|c} SO(3)_E & \\ \hline & SO(2)_E \end{array} \right)$$ ; $SO(5)_R$ similarly.

twist

$SO(3)'_E$ = diagonal subgroup of $SO(3)_E \times SO(3)_R$

In top. twisted theory:

Symmetry $SO(3)'_E \times SO(2)_E \times SO(2)_R$

bosons: $3 \oplus 3 \oplus \cdots$ (w.r.t $SO(3)'_E$)

$\quad A \quad \phi \quad$ vectors

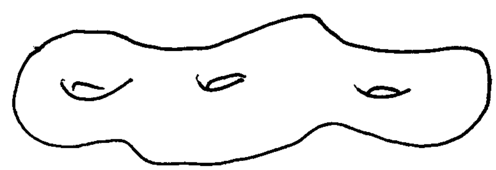
$$\mathcal{A} = A + i\phi \quad \text{complex-valued gauge connection}$$

$Q_{BRST}$ - fixed pts: $\mathcal{F} = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = 0.$

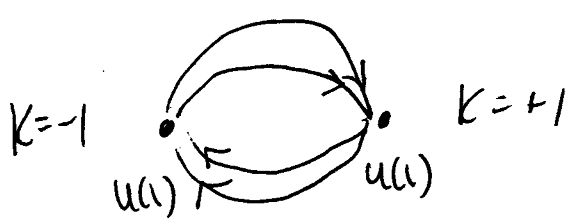
$\Longleftrightarrow$ SUSY vacua of $T(M)$ $\boxed{\text{on } S^1 \times \mathbb{R}^2}$

Example

$$A(x,y) = y + y^{-1} - (x^{-4} - x^{-2} - 2 - x^2 + x^4)$$



In this example, $T(M)$ = quiver 3d $\mathcal{N}=2$ gauge theory



<u>SUSY moduli</u>:

$S^1 \times \mathbb{R}^2_\hbar$

$\hbar \to 0$

$$Z_{3d}(T(M)) = \exp\left(\frac{1}{\hbar} W + \cdots\right)$$

twisted superpotential

<u>Example</u> 3d $\mathcal{N}=2$ SQED

$$W = \sum_{\substack{\text{charged}\\ \text{chirals}}} (\sigma - \tilde{m}_i)\log(\sigma - \tilde{m}_i)$$

$$W_{S^1 \times \mathbb{R}^2} = \sum \mathrm{Li}_2(\cdots)$$

(Kaluza-Klein modes)

- minimize $W$ wrt all dynamical fields

$\rightsquigarrow W_{eff}(v_i)$ ← parameters (= vevs of non-dynamical fields)

- $u_i = \dfrac{\partial W_{eff}(v_i)}{v_i}$

$x_i = e^{u_i}$, $y_i = e^{v_i}$

get nice algebraic variety in $x_i, y_i$

in the example, get the A-polynomial $A(x,y)$ written above.

---

3-manifolds $M$ $\longleftrightarrow$ 3d $\mathcal{N}=2$ theory $T(M)$

flat $SL(2)$ connections $\longleftrightarrow$ SUSY moduli on $S^1 \times \mathbb{R}^2$

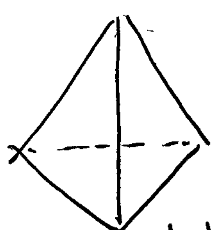
? $\updownarrow$ $\qquad\qquad$ $\swarrow\!\!\nearrow$ ?

Quantum Integrable Systems

---

$M$ $\longleftrightarrow$ $T(M)$

flat conn. $\longleftrightarrow$ SUSY moduli



tetrahedron

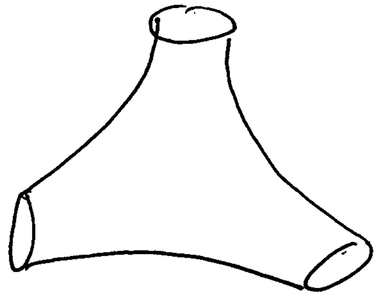
$\longleftrightarrow$ $T_1 = T($  $)$

$T_1$ = a charged chiral field
coupled to a background, non-dynamical $U(1)$

$k = -\frac{1}{2}$ $\leftarrow$

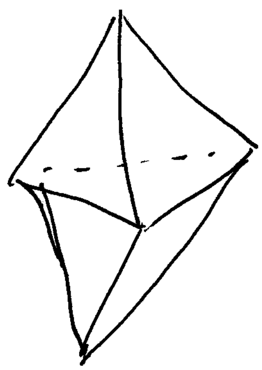
(combine with $k=\frac{1}{2}$ version ...)

Space of SUSY moduli:

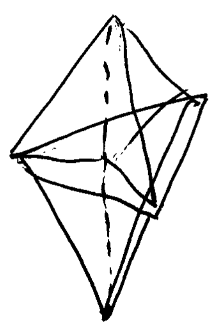


$$A(x,y) = 1 + x + y^{-1}$$

glue two tetrahedra:

$M =$ 

$T(M)$ = U(1) gauge theory w/ two charged chirals of charge $\pm 1$

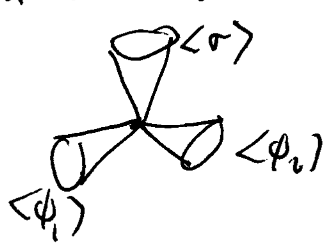
or $M =$ 

"(2,3) Pachner move"

triangulates the same manifold by 3 tetrahedra

This is 3D mirror symmetry!

U(1) with two charged chirals of charge $\pm 1$



$W_{IR} = XYZ$ IR thy = 3 chirals.

---

also pentagon identity on $Li_2$.