Refined Topological Strings, Quantum Geometry, and Integrable Systems

Daniel Krefl

1105.0630 w/ AGDV
many related works

Outline

(I) Def^n of refined strings
$\epsilon_2 \to 0$ limit

(II) "quantum geometry"

(III) Connection to quantum integrable systems

(I) Refined topological strings

Def^n M-thy based def^n

$$(TN \times S_1) \times X, \quad X \text{ local CY 3-fold}$$

$$Z_{BPS}(\epsilon_1, \epsilon_2) = \mathrm{Tr}\, (-q_L)^{2m_L} (-q_R)^{2m_R} e^{-\vec{\lambda}\cdot\vec{t}}$$

spinning M2 with $SU(4) = \cancel{SO}\, SU(2)_R \times SU(2)_L$

$$\Lambda \in H_2(X, \mathbb{Z})$$

$$\mathcal{F}(\varepsilon_1, \varepsilon_2) = \log Z_{BPS} = \sum_{\pi} \sum \sum N^{\vec{d}}_{j_1 j_2} \frac{[m/2]\ [\ \ ]e^{m \vec{d} \cdot}}{m[\ \ ][\ \ ]}$$

$$[m]_{\Lambda} = \left(e^{im\Lambda} - e^{-im\Lambda}\right)$$

<u>Def^n</u> $Z_{BPS}(\varepsilon_1, \varepsilon_2; X)$ = refined top string partition functions

$$\varepsilon_1 = \sqrt{\beta} \cdot g_s$$
$$\varepsilon_2 = \frac{-\Lambda}{\sqrt{\beta}} g_s$$

$$\mathcal{F}(\beta, g_s) = \sum \mathcal{F}_{\beta}^{(g)} g_s^{2g-2}$$

$$\mathcal{F}(\varepsilon_1, \varepsilon_2) = \sum_{m_1, m_2} \mathcal{F}^{(m_1, m_2)} (\varepsilon_1 \cdot \varepsilon_2)^{\#} (\varepsilon_1 + \varepsilon_2)^{2m_2}$$

<u>resum</u>

Holloway et al, Iqbal

$$Z_{BPS}(\varepsilon_1, \varepsilon_2) \xrightarrow[EFT]{4D} Z_{Nek}(\varepsilon_1, \varepsilon_2)$$

NS $\varepsilon_2 \to 0$

$$W(\hbar) = \lim_{\varepsilon_2 \to 0} \varepsilon_2 \mathcal{F}(\hbar, \varepsilon_2) = \sum_{\Lambda} \sum \sum \dot{N}$$

$\boxed{\text{II}}$ Quantum Geometry

Classical B-model geometry $\qquad w + H(x,p) = 0$

Σ: $H(x,p) = 0$, $\quad$ 1-form $\lambda = p\,dx$

$\omega = \frac{du}{u} \wedge dp \wedge dx$

via special geometry with

$$\Pi_G = \oint \lambda$$

Consider brane probe localized at a point on Σ

$$\psi_{cl} = \int_{\Gamma} e^{\frac{1}{\hbar}\int^x \lambda} \qquad \lambda = p\,dx$$

WKB

Hamilton-Jacobi equation

$$H(x, \;) = 0 \qquad \text{2 complex dim phase space}$$

$$\langle \psi \rangle = \left[-\varepsilon_x \partial_x^2 + V(x) + \underbrace{g_s^2 D}_{} \right] \langle \psi(x) \rangle = 0$$

Disappears in $g_s \to 0$ limit

$$\psi \sim e^{\frac{1}{\hbar}\int^x \partial S}$$

Solve $\partial S = 0$

$$\Pi_G = \oint_G \partial S \qquad \Pi_A(\hbar) \quad \to \quad W(\hbar)$$

$$\Pi_B = \partial \mathcal{F}_\hbar^{(0)} \qquad \Pi_B(\hbar)$$