On the Arithmetics of D-brane Superpotentials

Program

study PF eq analytic invariants ($\mathcal{W}$)
of algebraic cycles (D-brane) (co-dim 2
on families of CY 3-folds. Interpret via
MS, expansion in 2CSL
arithmetic

D-brane superpotentials

$E \to Y$ hol. vb

$$\mathcal{W}(u,z) = \int_{\Gamma} \Omega \wedge \mathrm{tr}\left(\frac{1}{2} A\, \bar\partial A + \frac{1}{3} A \wedge A \wedge A\right)$$

simpler, invariant

Example

$$Y = \{ W_\psi = \sum x_i^5 - 5\psi x_i - x_5 = 0 \} \in \mathbb{P}^4$$

$$C_I = \{ x_1 + x_2 = 0, \quad x_3 + x_4 = 0, \quad x_5^2 = \pm\sqrt{5\psi}\, x_1 x_3$$

$$\Omega = -\operatorname{Res} \frac{\omega}{W}$$

$$T_\rho(z) = \int_{C^-}^{C_+} \Omega \qquad\qquad z = (5\psi)^{-5}, \quad \theta = z\frac{\partial}{\partial z}$$

if $\mathcal{D} = \theta^4 - 5z(5\theta$

$$\mathcal{P} T_B(z) = \frac{15}{16\pi^2}\sqrt{z}$$

\# real lines on a quintic

$$\varpi(z, H) = \sum \frac{\Gamma(5(n+H)+1)}{\Gamma(n+H+1)^5} = \sum_{i=0}^{3} \omega_i H^i$$

$$\mathcal{P}\varpi = 0 \mod H^4$$

$$T_B = \frac{\omega_1}{2} + \frac{\omega_0}{4} \pm \frac{1}{4}\omega\left(z, \frac{1}{2}\right) \qquad \text{mirror map}$$

$$\mathcal{J}_A(q) = \frac{T_B(z(q))}{\omega(z(q))} = \sum_{d \text{ odd}} \hat{n}_d\, q^{d/2} + \frac{\pm}{2} + \frac{1}{4}$$

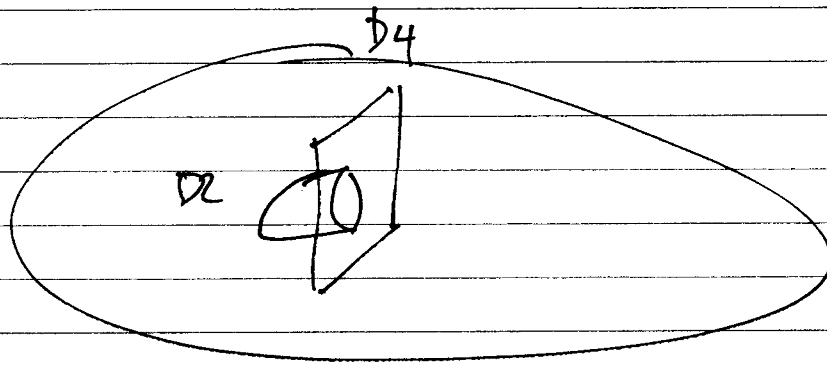
$\hat{n}_d \in \mathbb{Q}$

$$= \sum_{\substack{d,t\\ \text{odd}}} \frac{n_d}{t^2}\, \varepsilon^{dt/2}$$

$n_d \in \mathbb{Z}$

Enumerative interpretation

$\mathbb{RP}^3 = L \subset X$ real lines in Fermat quintic



$\hat{n} = \#$

$d \in H_2(X, L)$

$n_d =$ BPS invariant