An Upper Bound on Transport

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Order, fluctuations and strong correlations @ KITP
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Unconventional transport

- Unconventional transport regimes are ubiquitous and represent a long-standing challenge to theory.

- Some of these systems may be beyond well-established methods (Boltzmann equations, large N, etc.) which are typically framed around a quasiparticle lifetime, so that:

$$\rho = \frac{m}{ne^2}\frac{1}{\tau} \qquad (*)$$

- (Although $\tau \to \tau_{\rm tr}$ in general).

- Objective: find results on transport that hold with or without quasiparticles, where (*) cannot be assumed.

Diffusion bound — ’15 version

- One confusing aspect of T-linear resistivity in cuprates is that it seems oblivious to the change in scattering mechanisms from low to high temperatures.
- Also potentially surprising similarities between T-linear resistivity in different materials.
- A logical perspective from which these facts make sense is that these materials are saturating a fundamental bound on transport. How to formulate this bound?

Diffusion bound — '15 version

- Some history to the idea that the timescale $\tau \sim \frac{\hbar}{(k_B T)}$ is the "fastest possible". [Sachdev '99, Zaanen '04, Bruin et al. '13]

- Was not clear (to me) exactly how this timescale would feed into transport in the absence of Drude-like formulae.

- In [Nat. Phys. '15] I proposed that it would instead be a transport observable that is directly subject to a fundamental bound.

diffusivity → $$D \gtrsim v_F^2 \frac{\hbar}{k_B T}$$

- This bound was inspired by the [Kovtun-Son-Starinets '05] bound on η/s, which also determines a diffusivity.

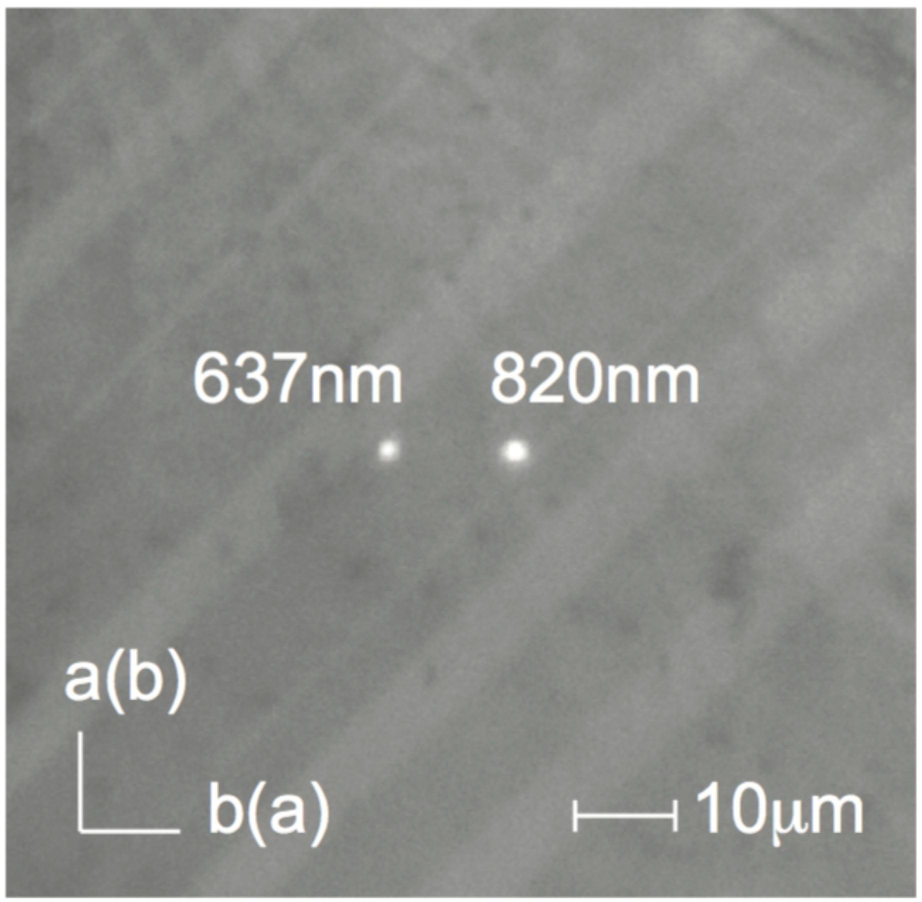
Diffusion bound — ’15 version

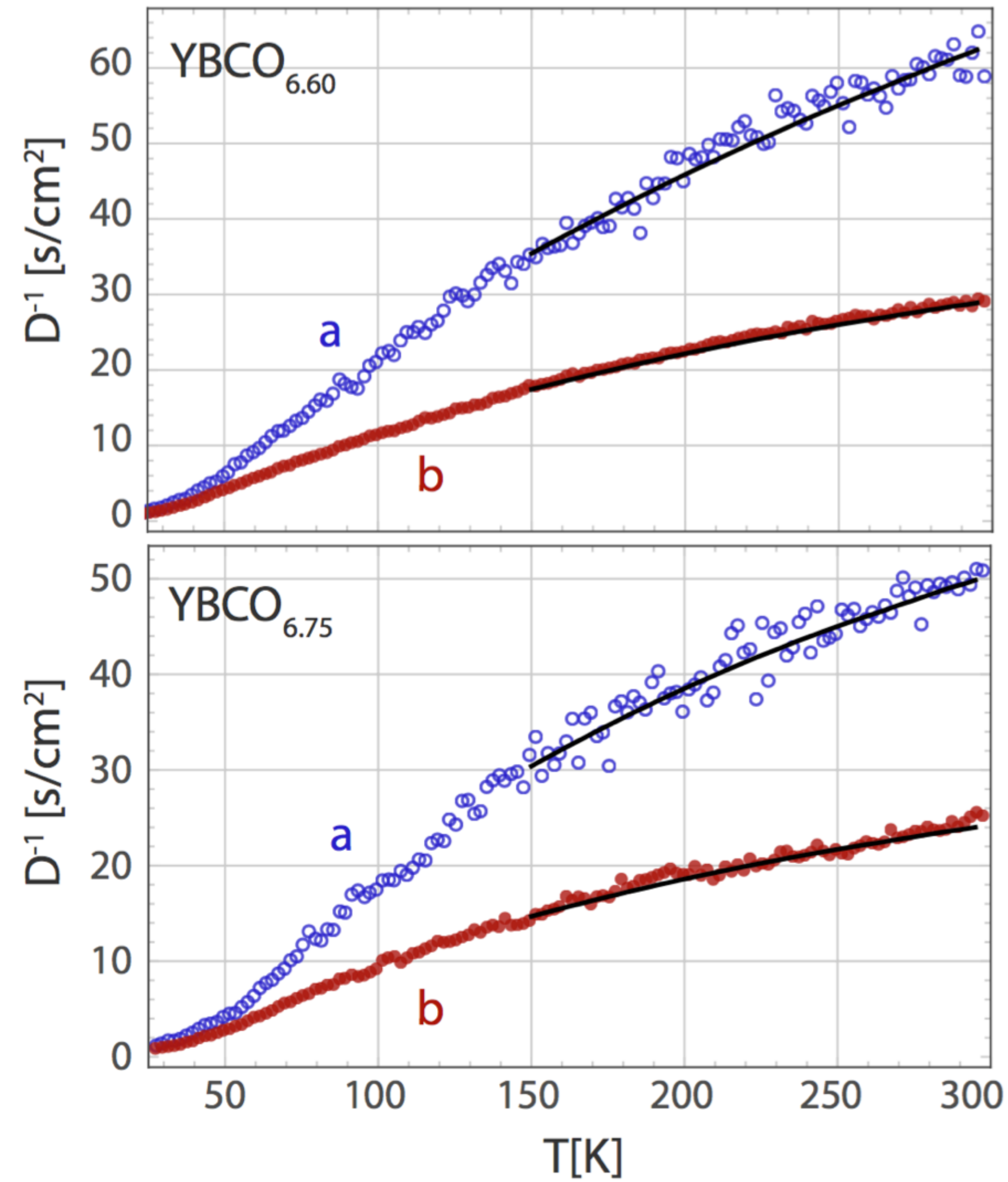
- The bound has some problems.
  (i) It is incompatible with residual resistivities.
  (ii) Not clear what v_F is.

- We will return to these points.

- Nonetheless, this bound has motivated new, different types of measurements on unconventional materials, and seems to be a useful way to think about the results of these measurements ….

Thermal diffusivity in YBCO





[J.-C. Zhang, E.M. Levenson-Falk, B.J. Ramshaw, D.A. Bonn, R. Liang, W.N. Hardy, S.A. Hartnoll, A. Kapitulnik. PNAS ’17]

Thermal diffusivity in YBCO

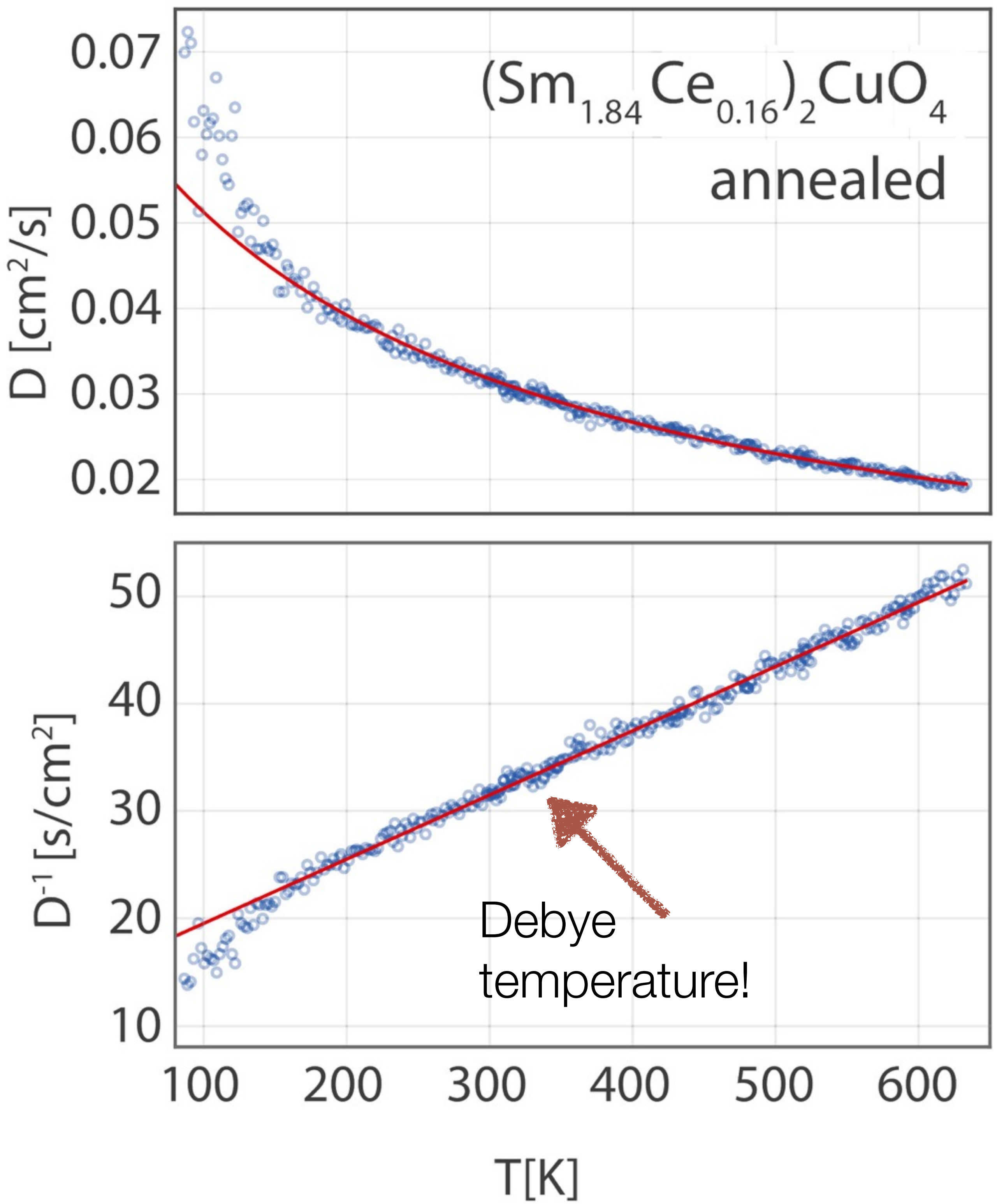
- There are many more phonons than electrons: $c_{ph} >> c_{el}$.
- But the electrons are much faster: $v_F >> v_S$.
- The crossover between these two effects occurs on the temperature scales of the experiment.
- Excellent fit of diffusivity to

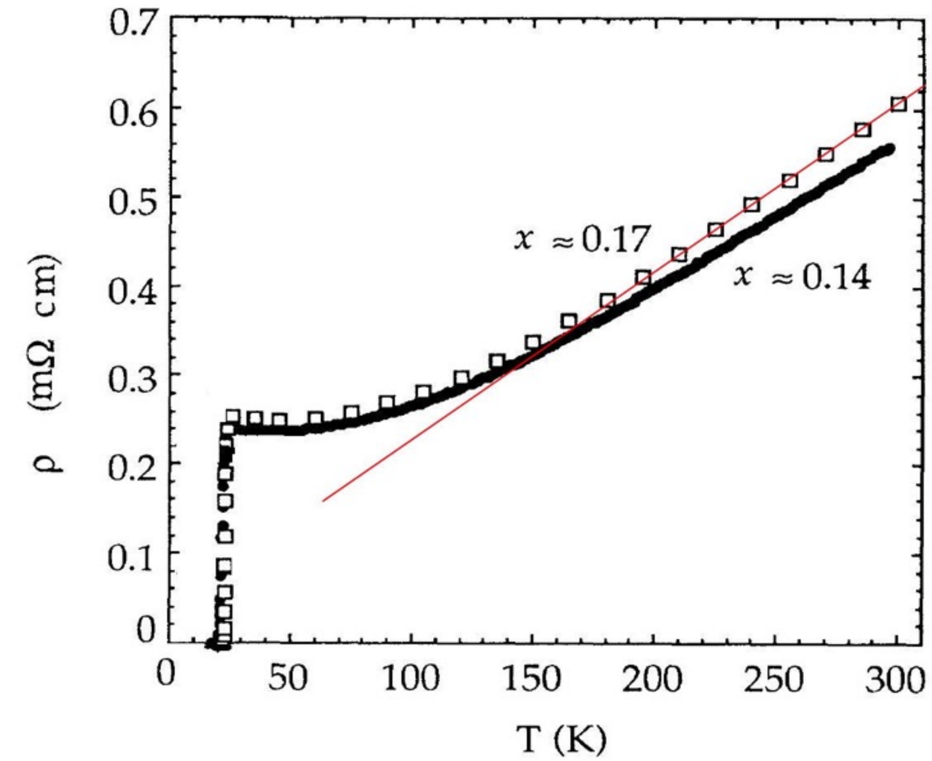
$$D_{\text{heat}} \sim v_B^2 \frac{\hbar}{k_B T} \, ,$$

- Where $v_s < v_B < v_F$:

$$v_B^2 = \alpha \, \frac{c_{\text{el}}}{c} v_F^2 + \beta \, \frac{c_{\text{ph}}}{c} v_s^2$$

Thermal diffusivity in SmCeCuO





$$D^{-1} = \left(\frac{v_F^2}{3}\frac{\hbar}{k_B T}\right)^{-1} + \left(\frac{\hbar}{3m^*}\right)^{-1}$$

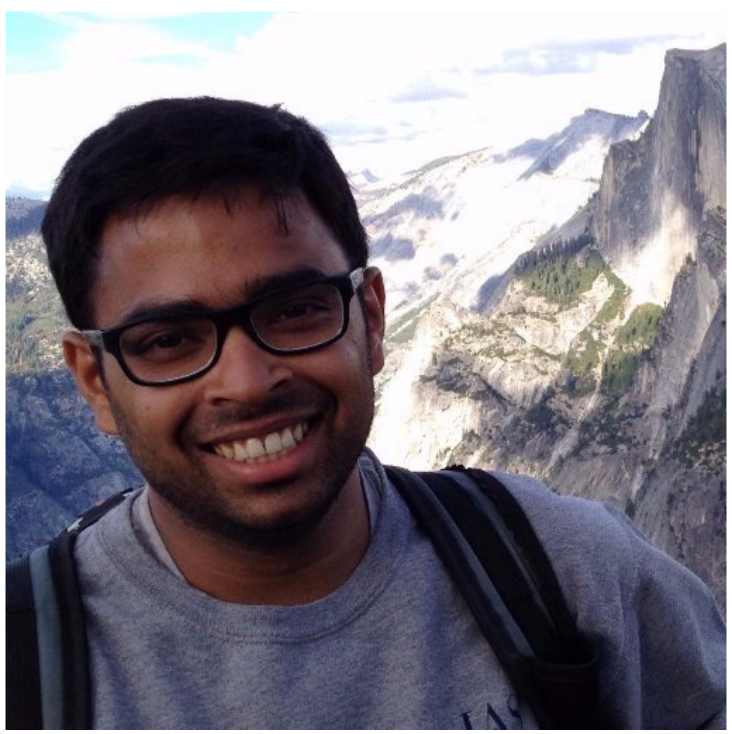
[J.-C. Zhang, A. Kapitulnik. unpublished]

Diffusion bound — ’17 version

- I have spent some time trying to prove a diffusion bound.
- These efforts recently landed on a result that has a different character to the bound discussed so far, but addresses some of the same puzzles.
- The new bound will go the other direction to previously conjectured diffusivity bounds.

An Upper Bound on Transport

- Based on 1706.00019 [hep-th]
- With Raghu Mahajan and Thomas Hartman



Implications of locality I

- Even non-relativistic systems have a 'lightcone': bounded propagation of signals from locality. [Lieb-Robinson '72]

$$||[A(t,x),B(0,0)]|| \lesssim ||A||||B||e^{-\mu(|x|-vt)}$$

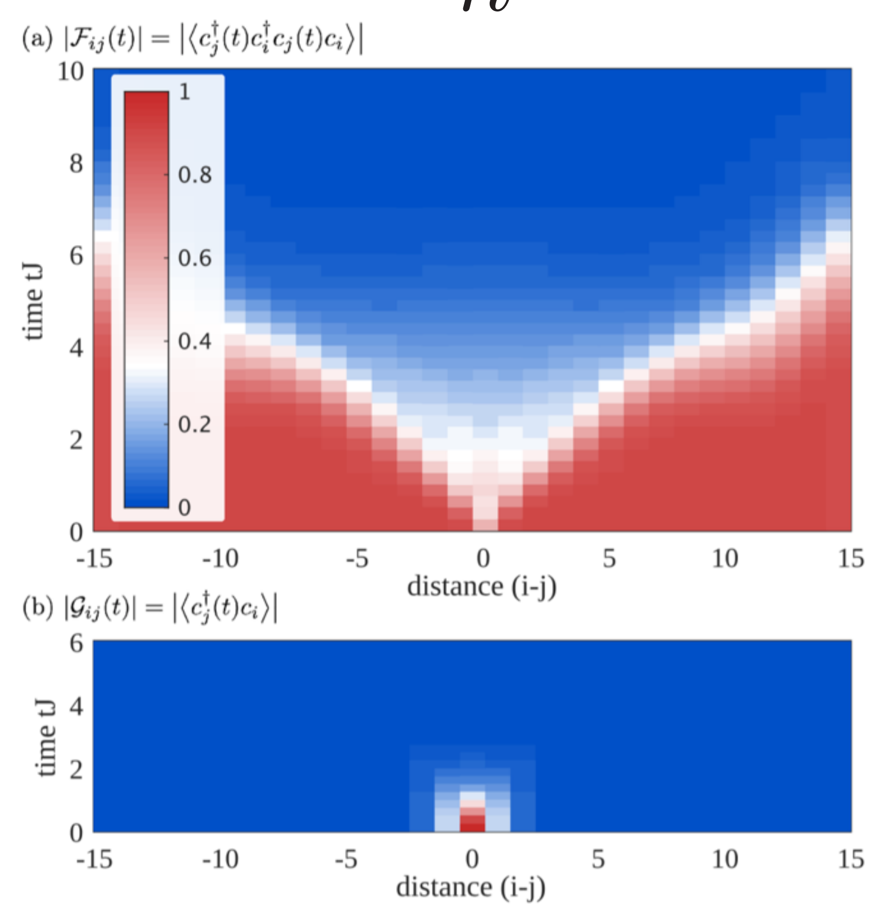
- The "Lieb-Robinson" velocity: $v \sim \frac{J\,a}{\hbar}$

- In some systems a less microscopic "butterfly velocity" defined by

$$\langle [A(t,x),B(0,0)]^2 \rangle \sim e^{\lambda_L(t-|x|/v_B)}$$

also bounds signals.

[Roberts et al '14]



[Bohrdt et al '17 (Bose-Hubbard)]

Implications of locality II

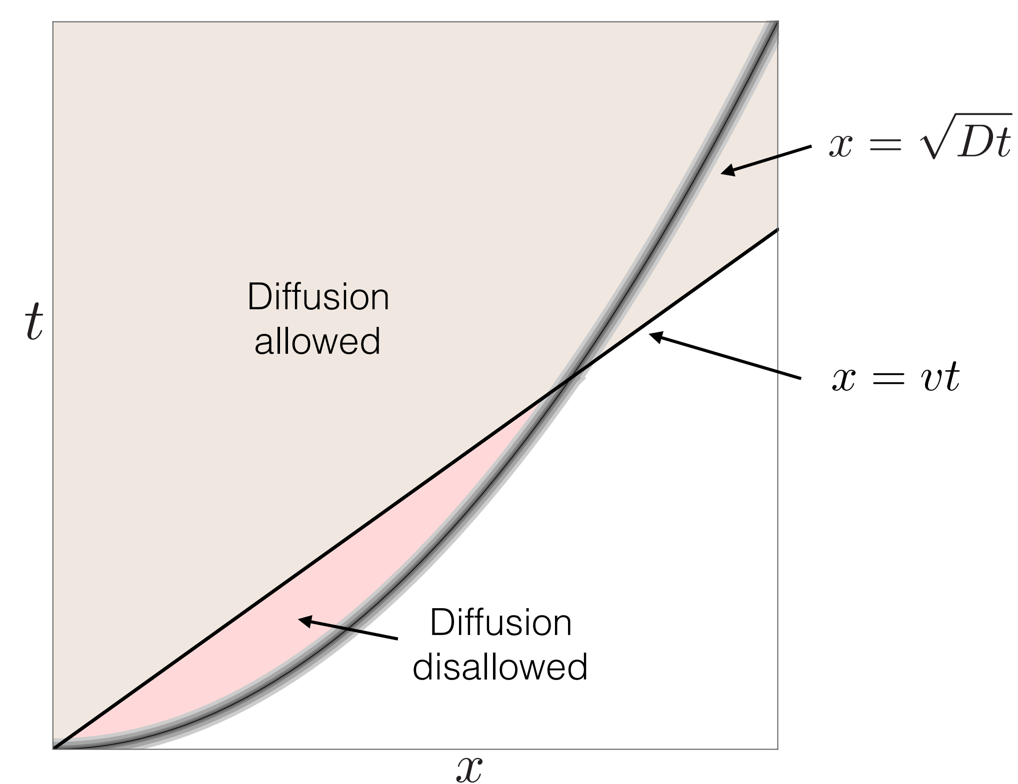
- Conserved densities diffuse (assume no sound modes):

$$\langle [n(t,x), n(0,0)] \rangle \propto \nabla^2 \frac{e^{-x^2/(4Dt)}}{t^{d/2}} \qquad (t \gtrsim \tau_{\text{eq}}, \quad |x| \gtrsim \ell_{\text{eq}}) \, .$$

- The diffusivity controls transport, e.g.:

$$\sigma = \chi \, D_{\text{charge}} \, , \qquad \kappa = c \, D_{\text{heat}} \, , \qquad \eta = \chi_{\pi\pi} D_{\text{momentum}} \, .$$

- At short times, diffusion is too fast!



Transport bound

- To avoid contradiction with the lightcone, disallowed region must not be diffusive — i.e. must occur before the local equilibration time, so that:

$$D \lesssim v^2 \tau_{\mathrm{eq}}$$

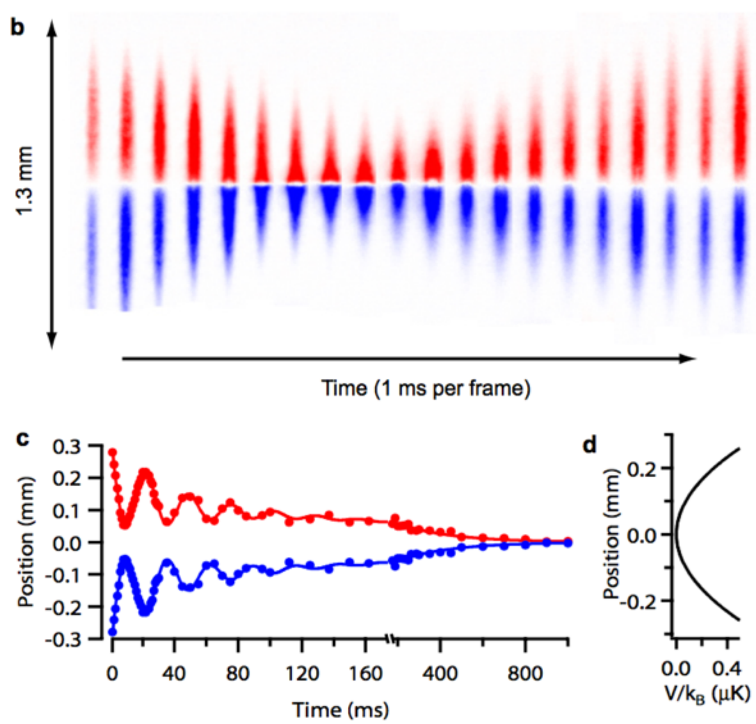
- In a quasiparticle system, $\tau_{\mathrm{eq}} \sim \tau \text{ or } \tau_{\mathrm{tr}}$. The inequality is saturated in quasiparticle regimes, where $D \sim v_{\mathrm{qp}}^2 \tau$.

- More generally, the inequality relates transport to a relaxation timescale, without assuming the existence of quasiparticles. D, v, τ_{eq} independently defined with no reference to quasiparticles.

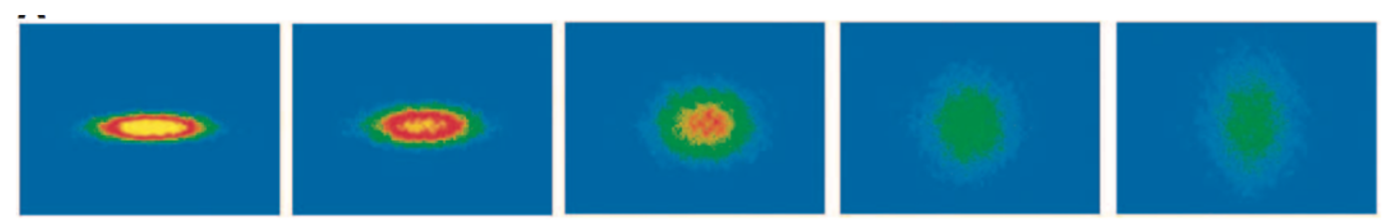
Diffusion in ultracold atomic (non)-Fermi liquids

- Unitary cold Fermions: spin and momentum diffusion

[Sommer et al. '11]



[Cao et al. '11]

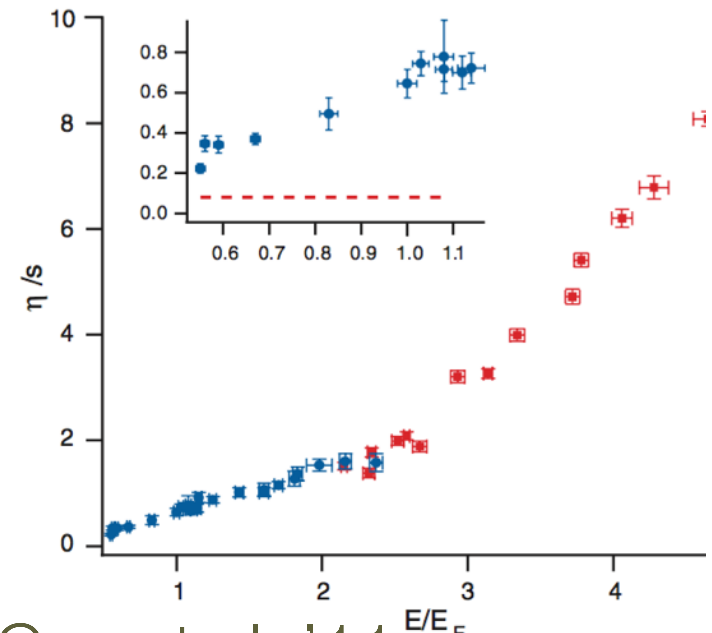


$$\frac{\eta}{n} \lesssim E_F\,\tau_{\rm eq}$$

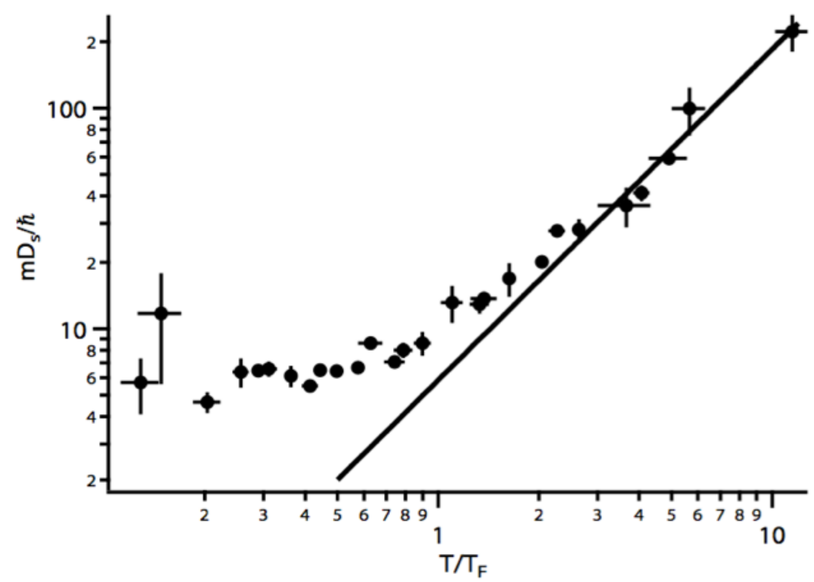
$$mD \lesssim E_F\,\tau_{\rm eq}$$

Diffusivities and relaxation rates

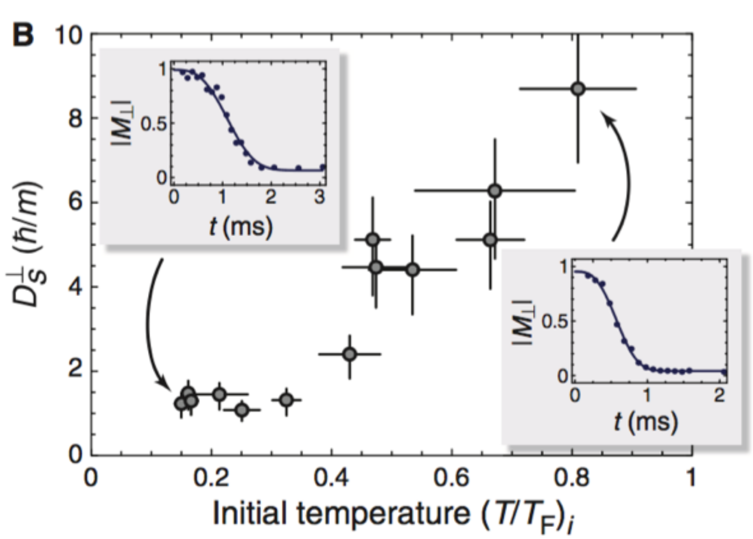
Diffusivities



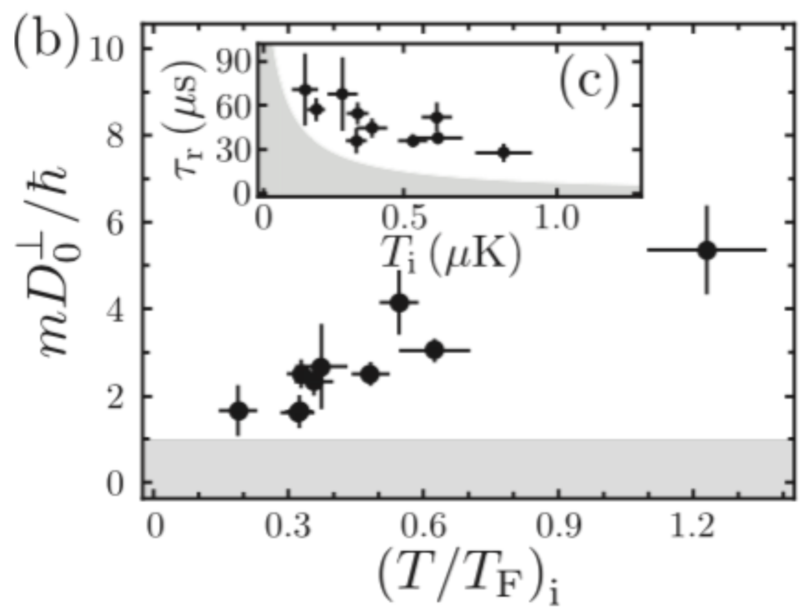
Cao et al. ’11



Sommer et al. ’11

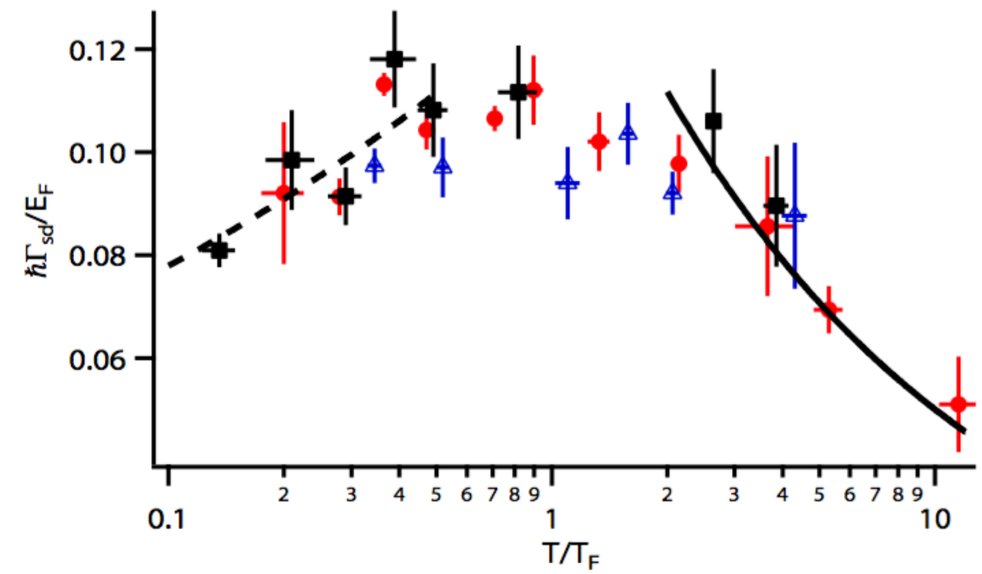


Bardon et al. ’14

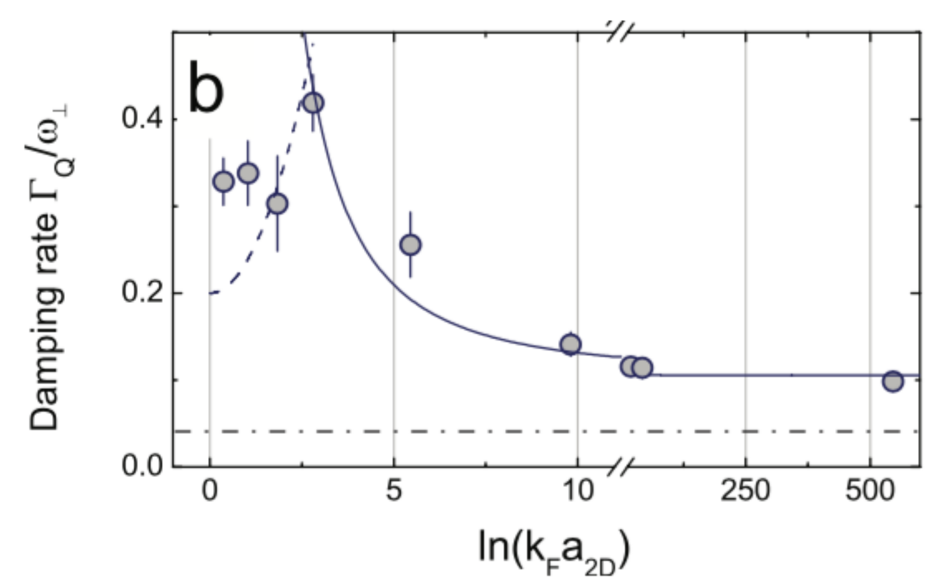


Luciuk et al. ’17

Relaxation rates (?)



Sommer et al. ’11



Vogt et al. ’12

Transport in metals

- In a metal τ_{eq} can be measured from both transport and single-particle probes, e.g. $\sigma(\omega)$ or $\Sigma(\omega)$.

- Expect $v \sim v_F$, in which case the bound implies:

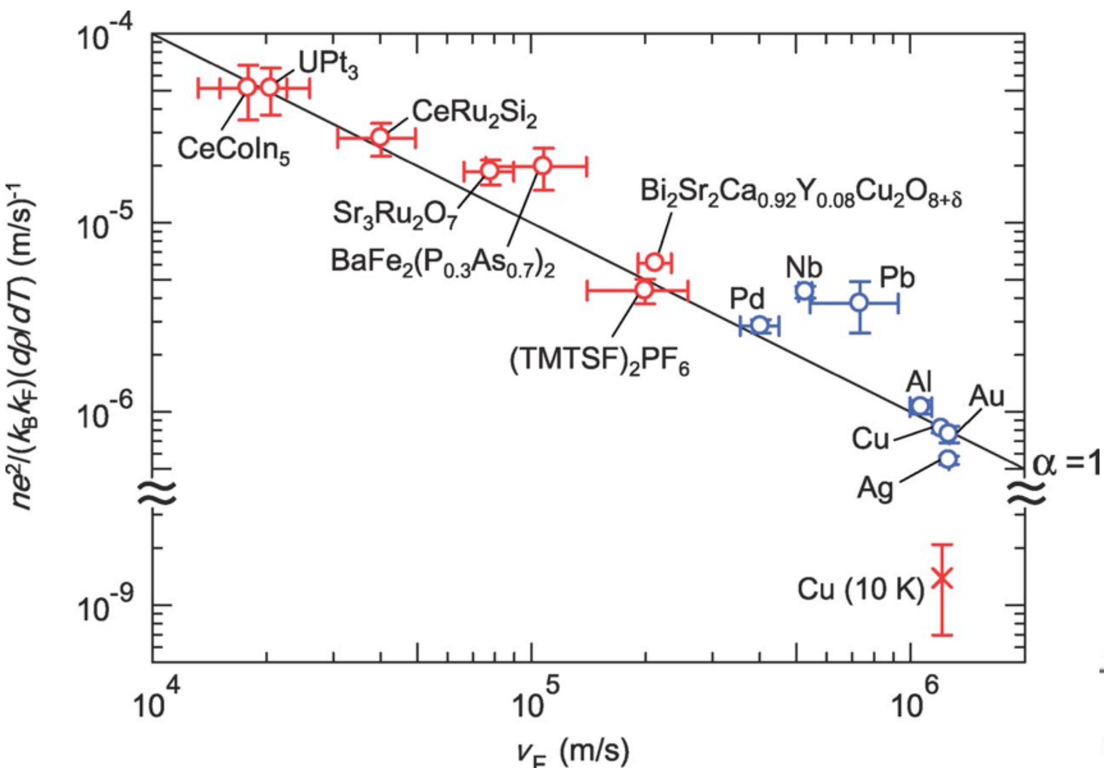
$$\rho \gtrsim \frac{m}{e^2 n} \frac{1}{\tau_{\mathrm{eq}}}$$

- Consistent with transport data in unconventional metals.

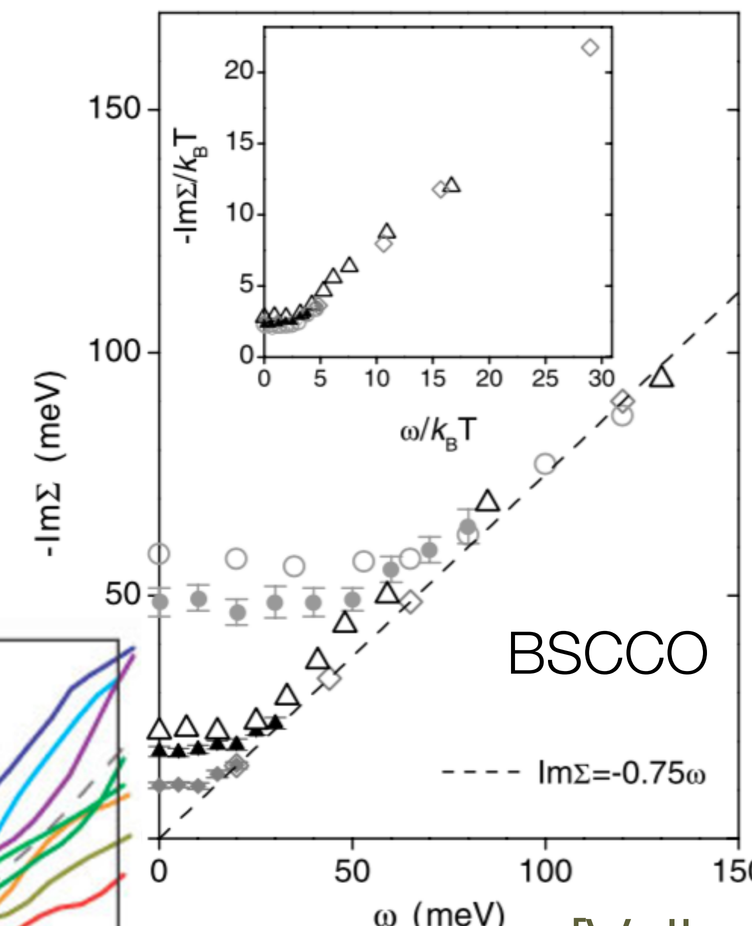
- The existence of the $\tau \sim \hbar/(k_B T)$ timescale may not be the most mysterious aspect of these materials. What is lacking is a non-quasiparticle way to translate this timescale into a resistivity. Drude formula not allowed!

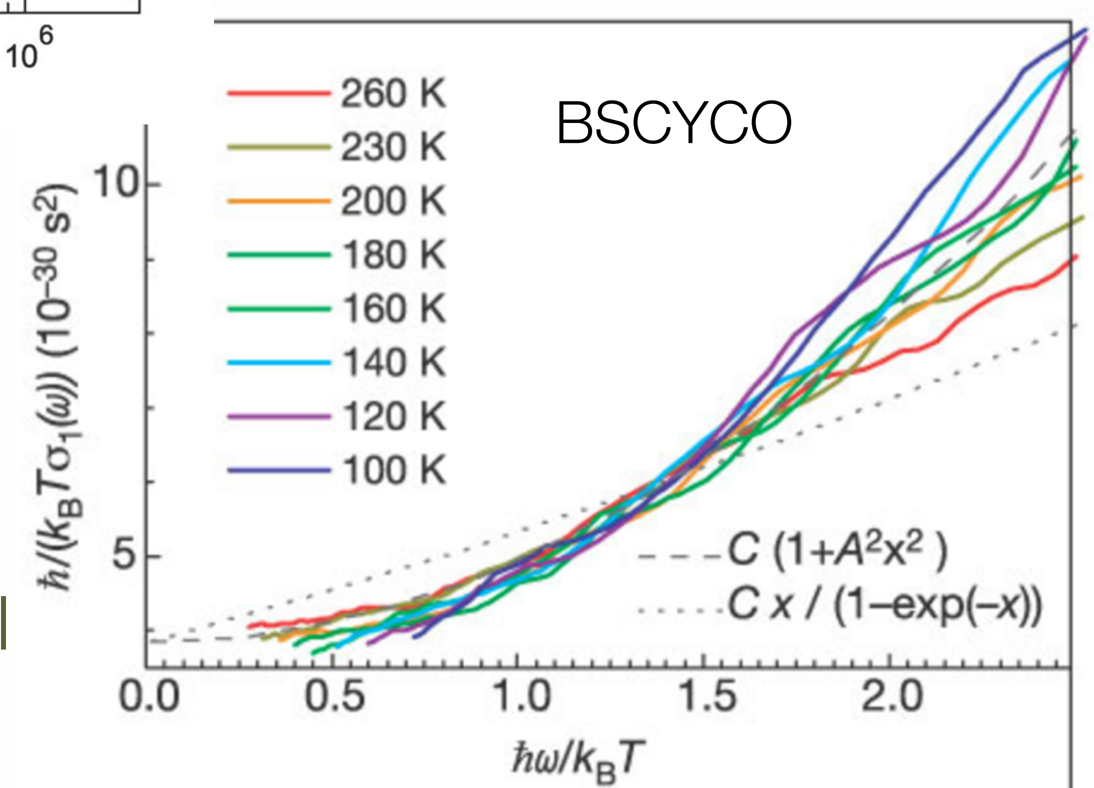
Relaxation rates in T-linear metals

- The T-linear resistivity is indeed due to $\tau \sim \hbar/(k_B T)$



[Bruin et al '13]



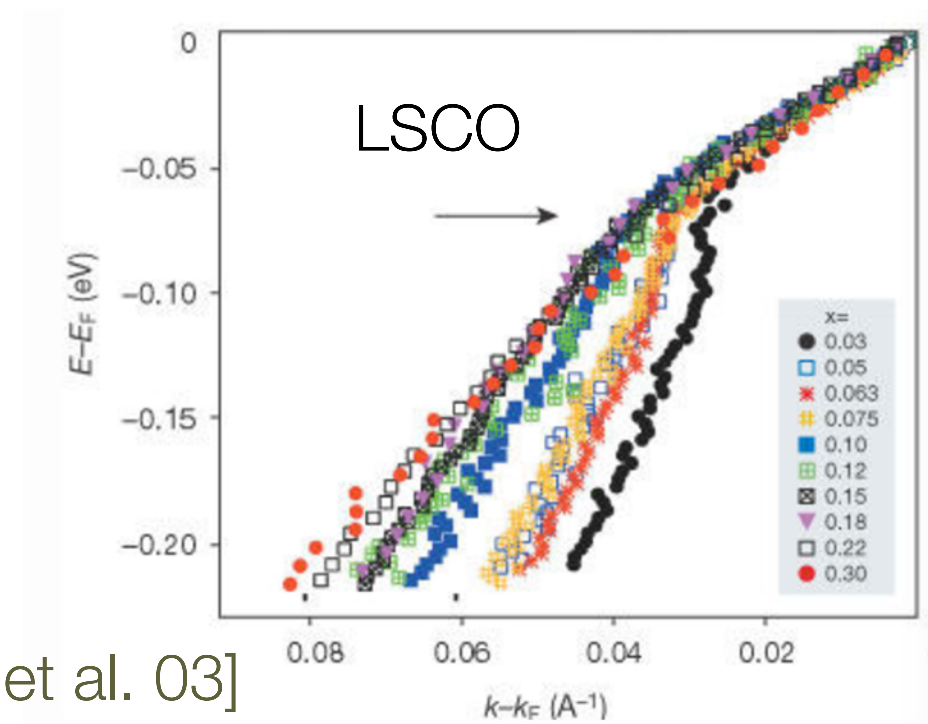


[Valla et al '99]

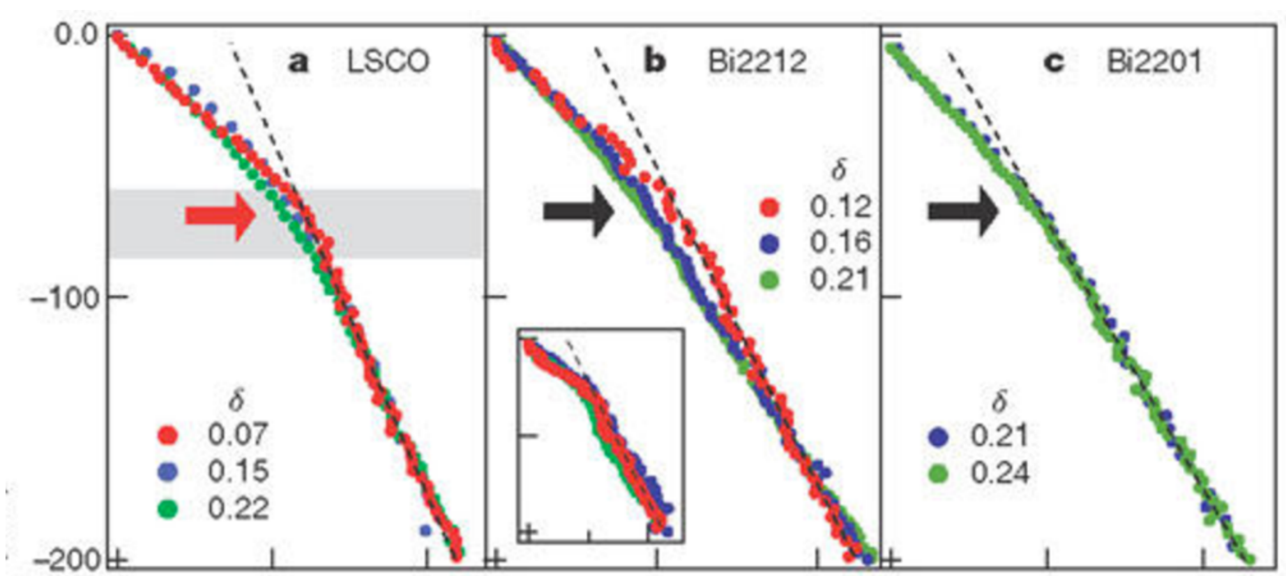
[van der Marel et al '03]

Looking forward

- Lieb-Robinson velocity best understood in spin systems. Experiments and numerics in e.g. 1d Bose-Hubbard.
- Theoretical and experimental opportunity to understand the role of a non-quasiparticle velocity in metals.
- Some interesting structure in measured velocities. E.g.



[Zhou et al. 03]



[Lanzara et al. 01]

Summary

- Bounds may help to organize our thinking about non-quasiparticle transport.
- A conjectured (and currently imperfect) lower bound on diffusion has motivated, and is consistent with the results of, new experiments in cuprates.
- We obtained an upper bound on diffusion in terms of the lightcone velocity and local equilibration time.
- A better understanding of characteristic velocities in non-quasiparticle metals may lead to further insights.