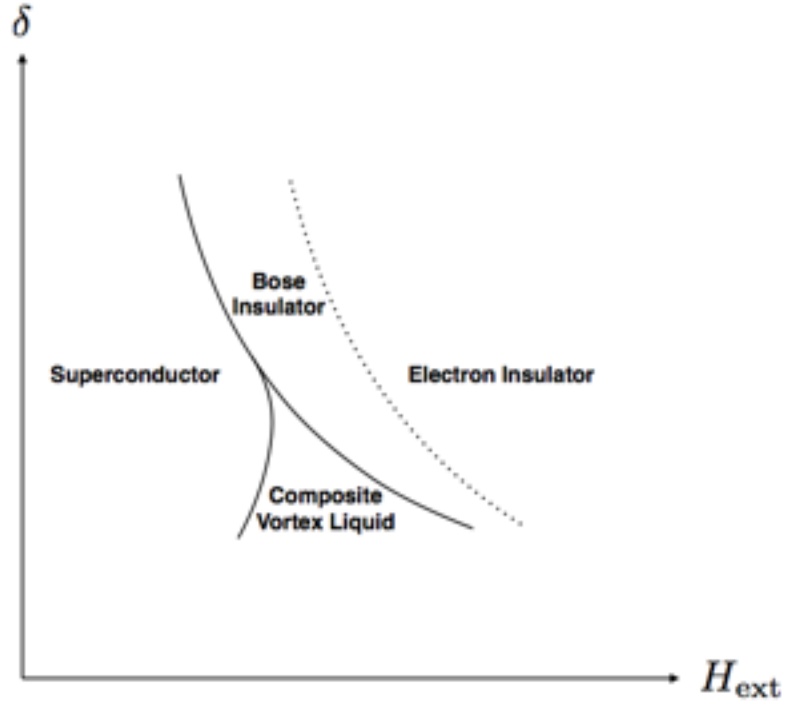
Self-duality and metallic behavior near 2d superconductor-insulator transitions

Srinivas Raghu (Stanford)



Acknowledgments

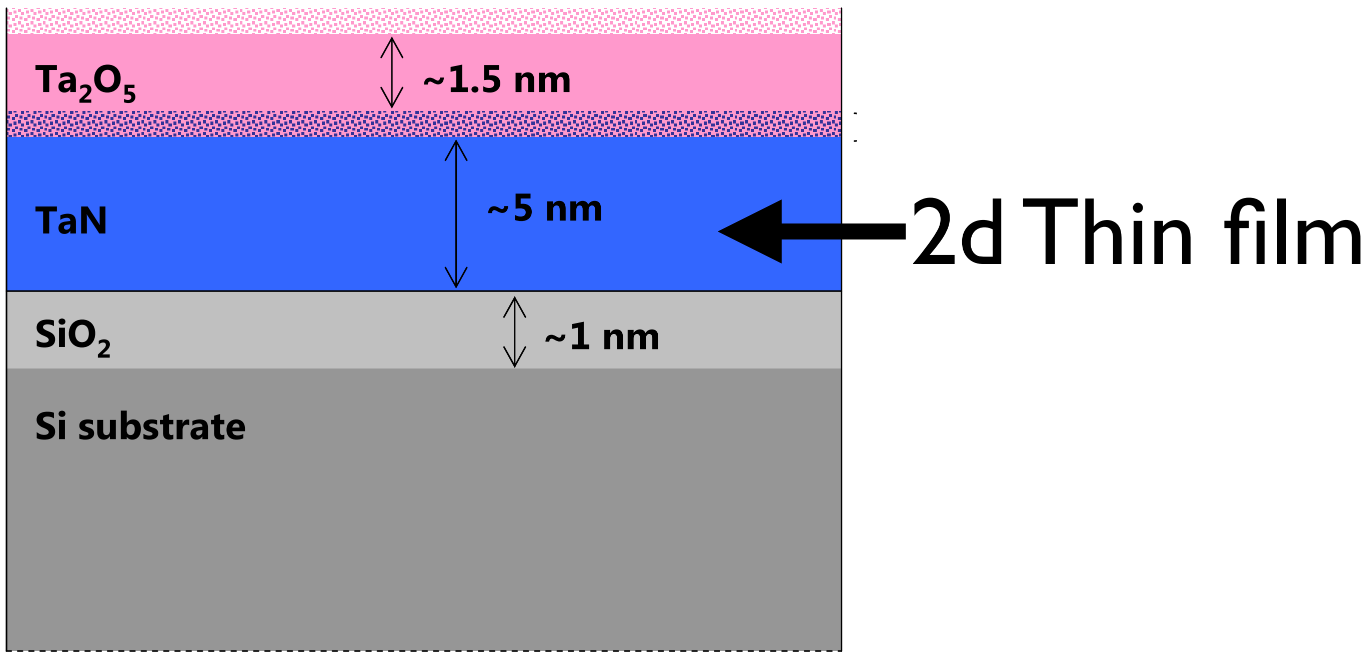
Michael Mulligan (UC Riverside)

(Collaborators on related issues):

Jing-Yuan Chen, Alfred Cheung, Hart Goldman, Pallab Goswami, Prashant Kumar, Jun Ho Son, Gonzalo Torroba, Max Zimet

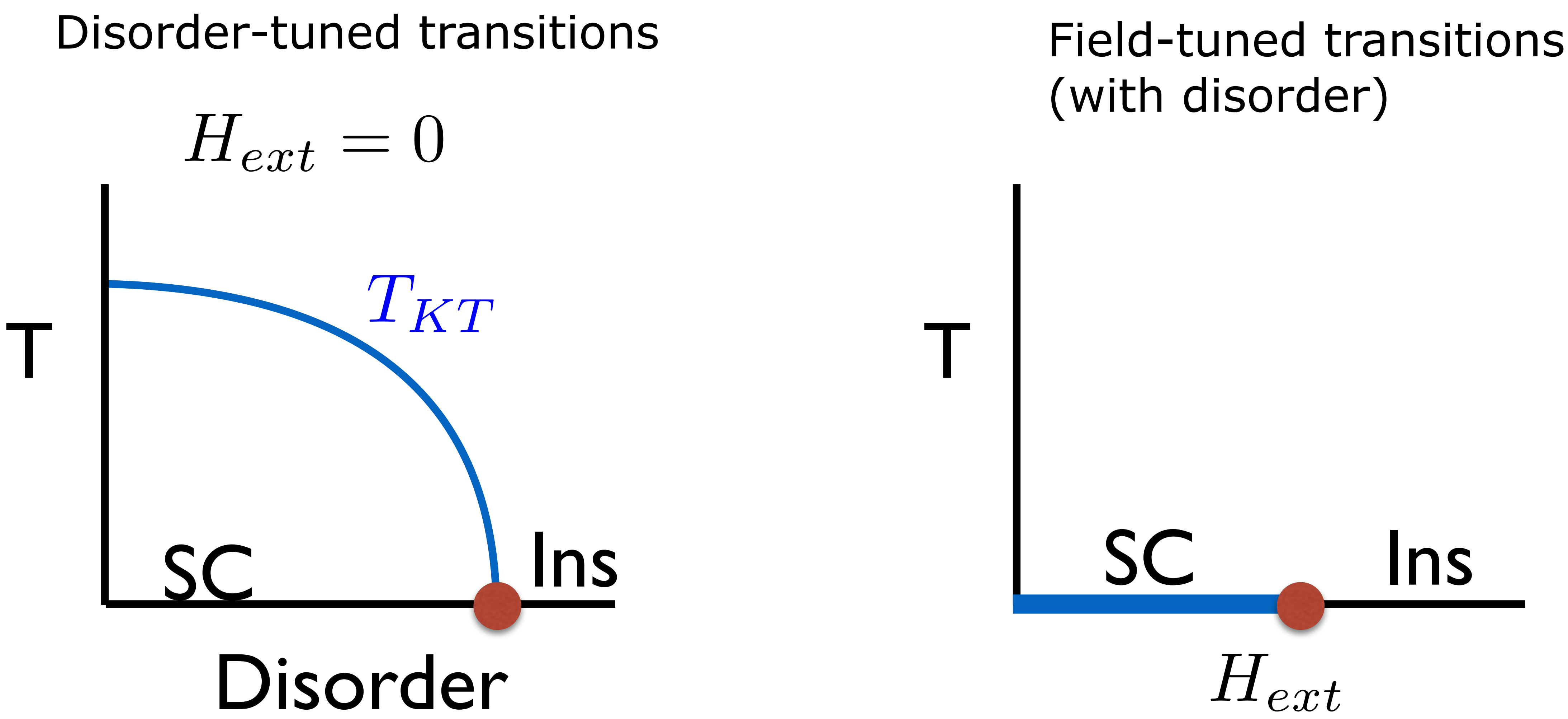
Basic setting: 2d disordered, superconducting thin films

N. Breznay thesis (Stanford)



Destroy superconductivity by tuning disorder or perpendicular magnetic field

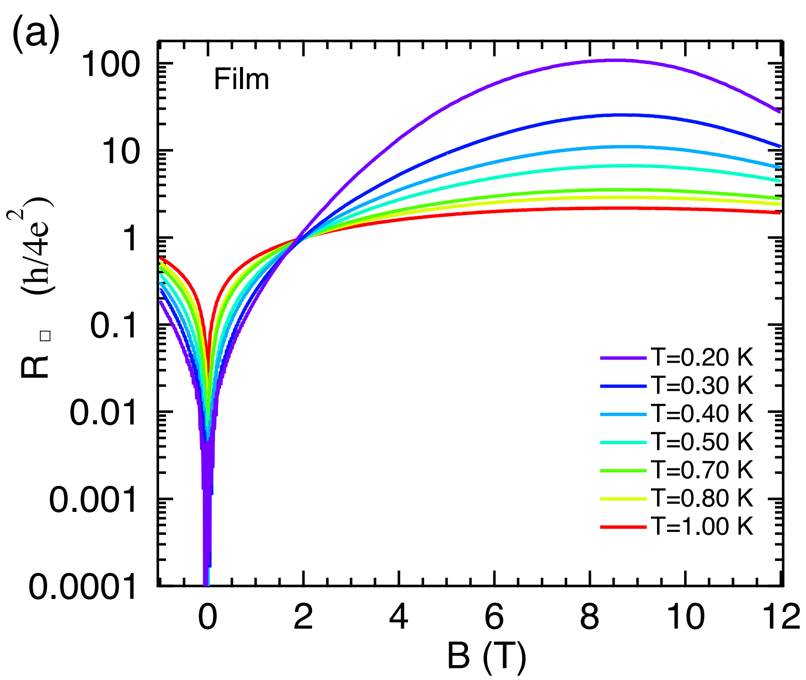
Magnetic field vs. disorder tuning in 2d



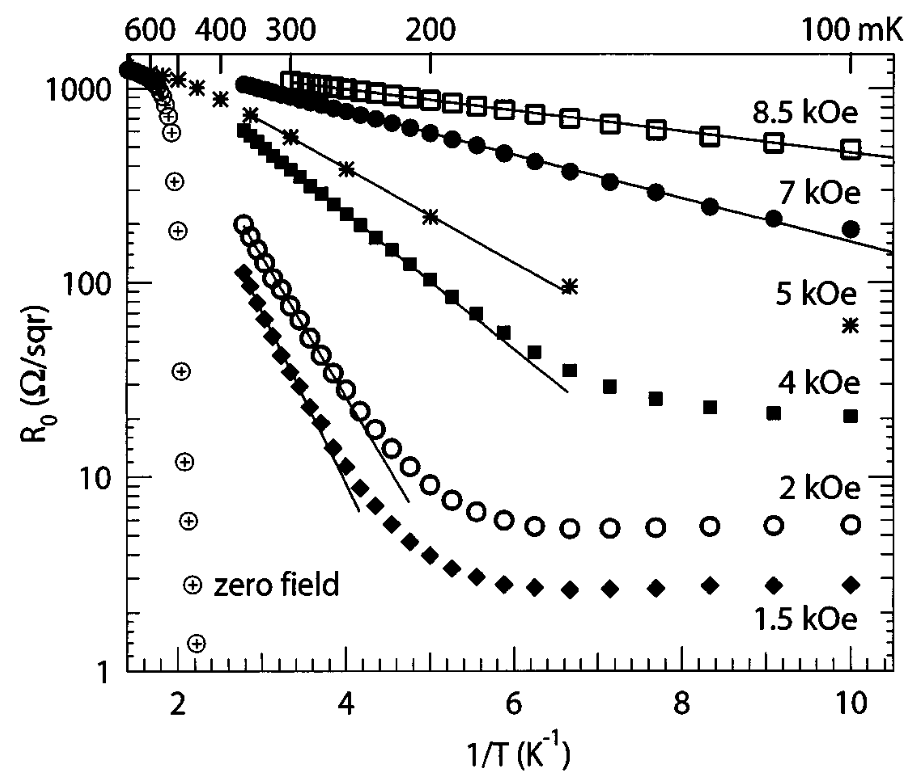
This talk: field-tuned transitions

Motivation: 2 sets of experimental observations:

1) Self-duality near the SIT

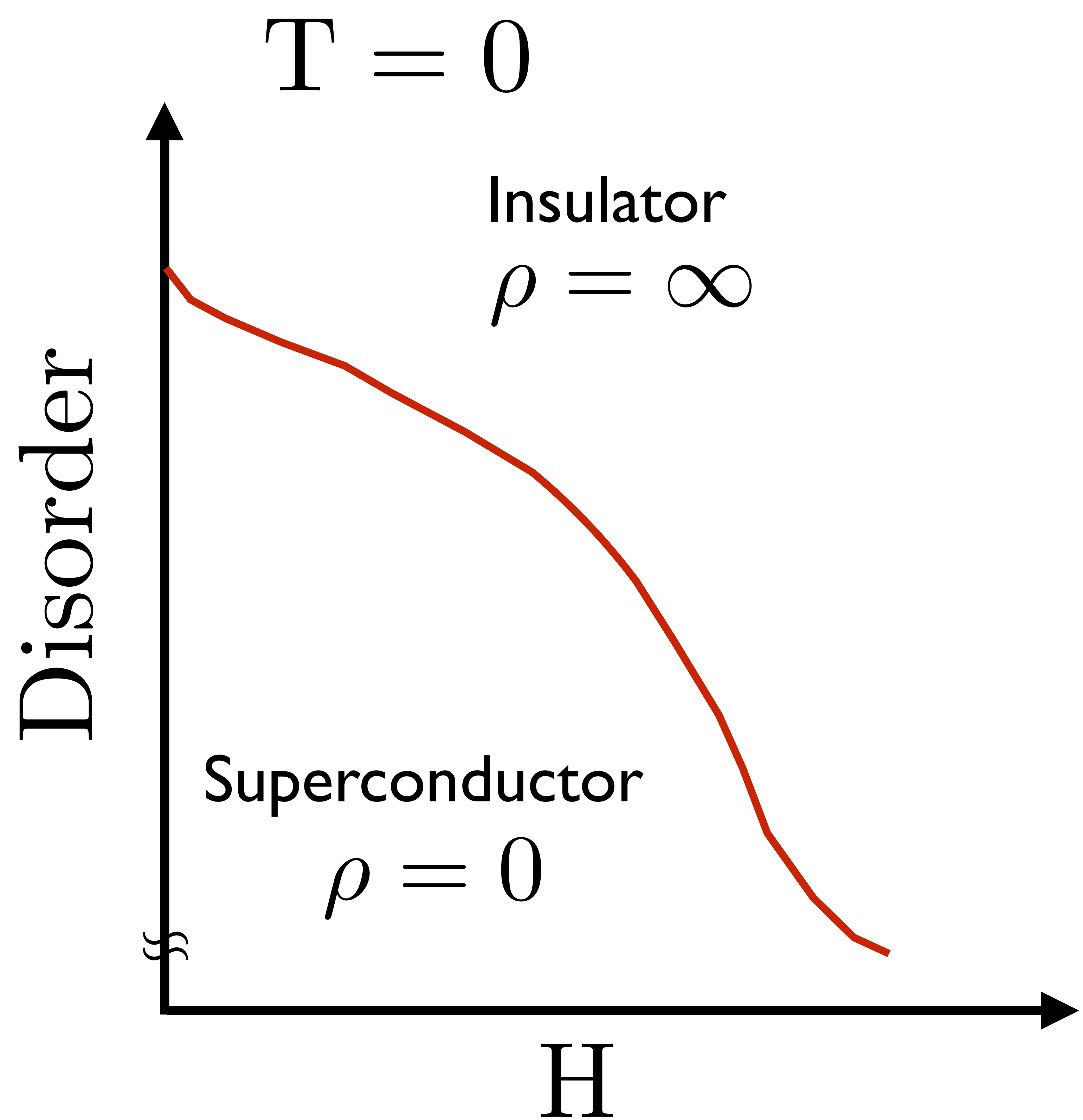


2) metallic phases near the SIT



Are these phenomena related?

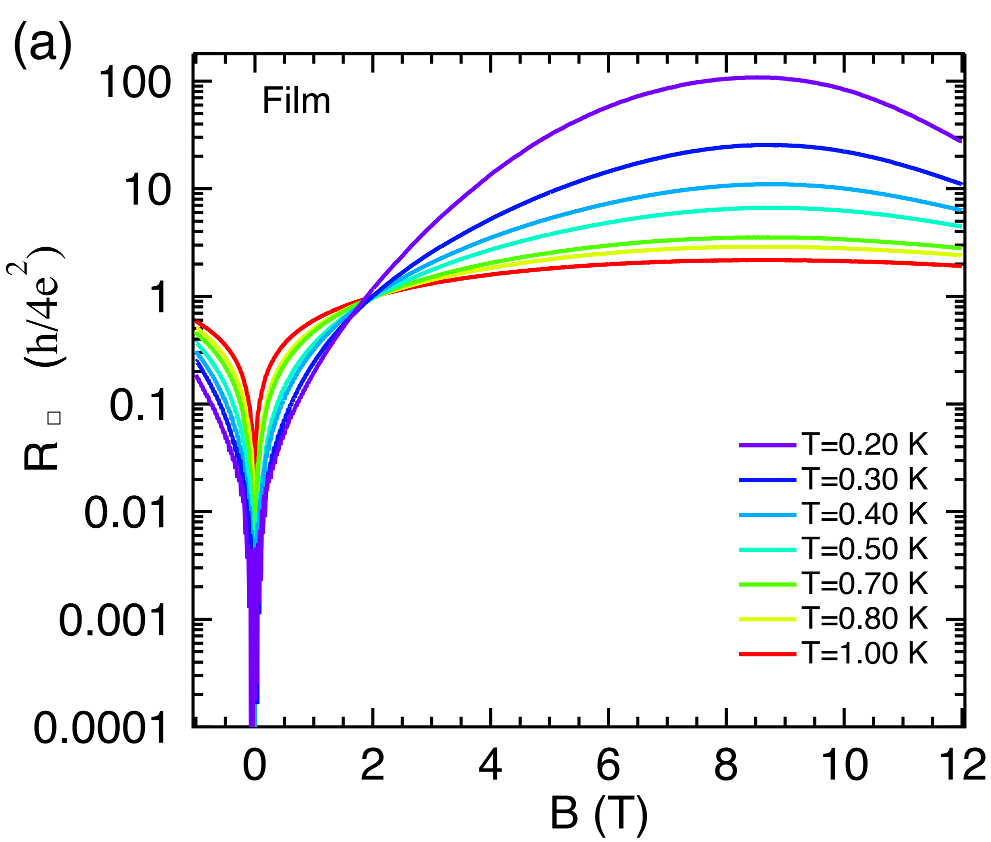
In 2d, we expect a T=0 phase diagram like this:



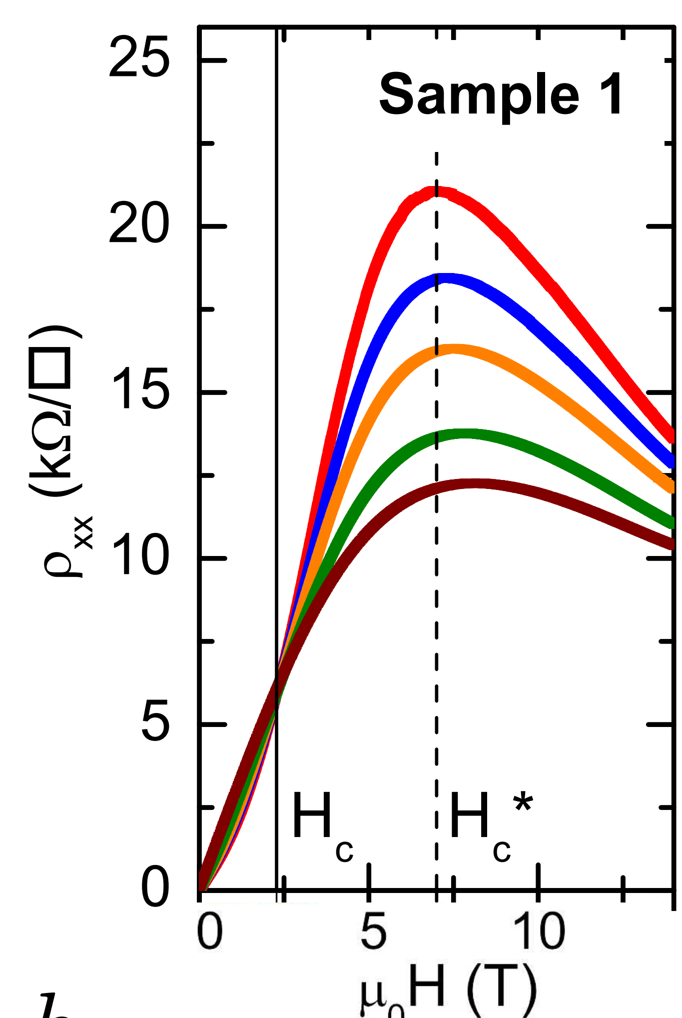
Self-duality: experiment

Paalanen, Hebard, Ruel, PRL (1992)

J. Seidemann *et al*, arXiv1609.07105

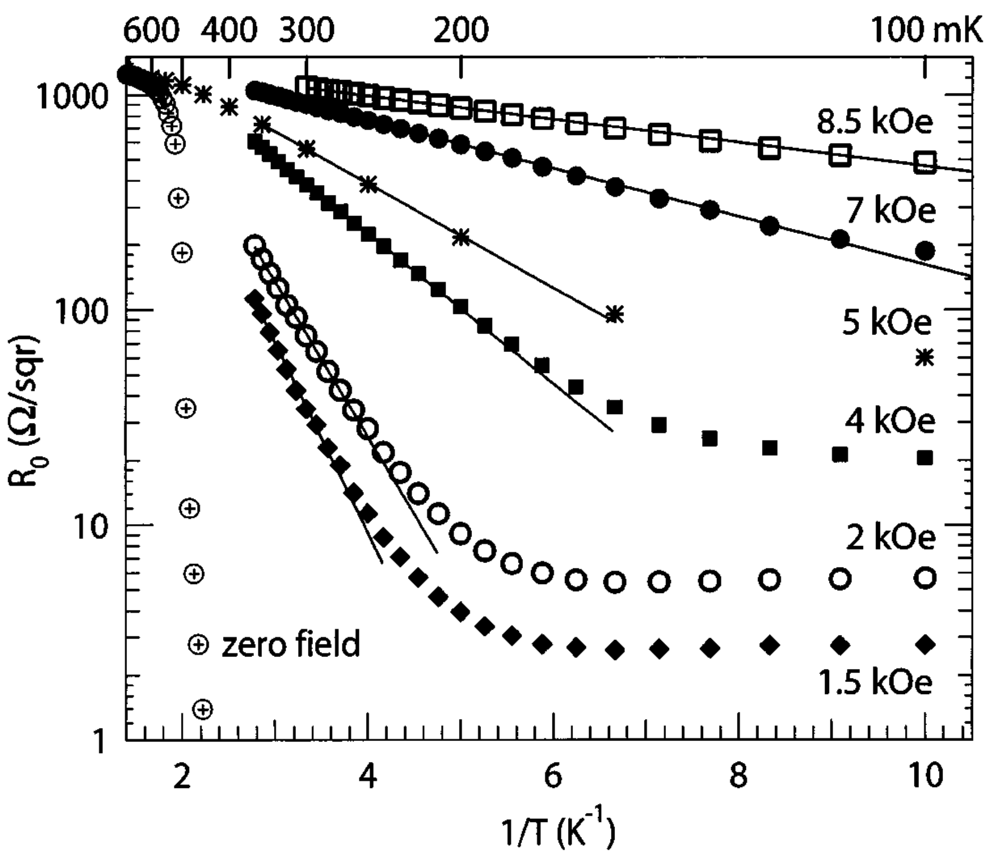


Breznay *et al.*, PNAS (2016)

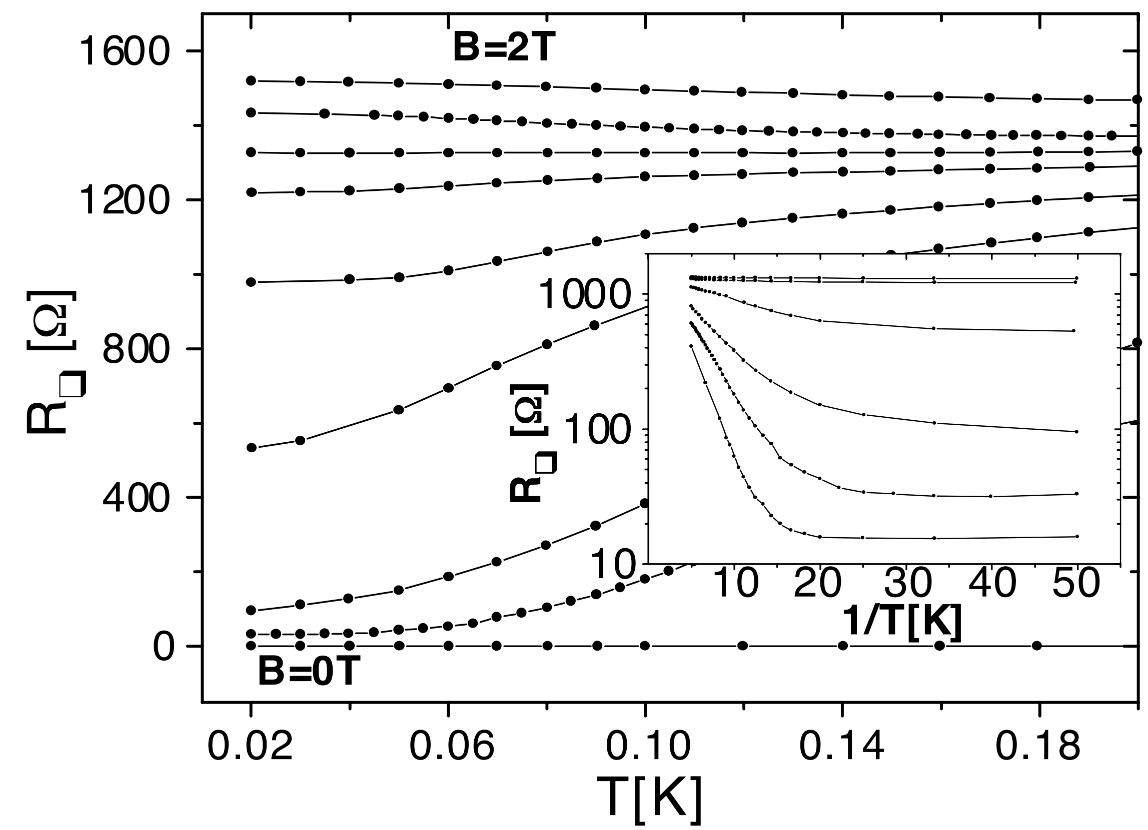


Critical resistance: $R_c = R_Q = \frac{h}{(2e)^2} \approx 6.45k\Omega$

Metallic behavior: experiment

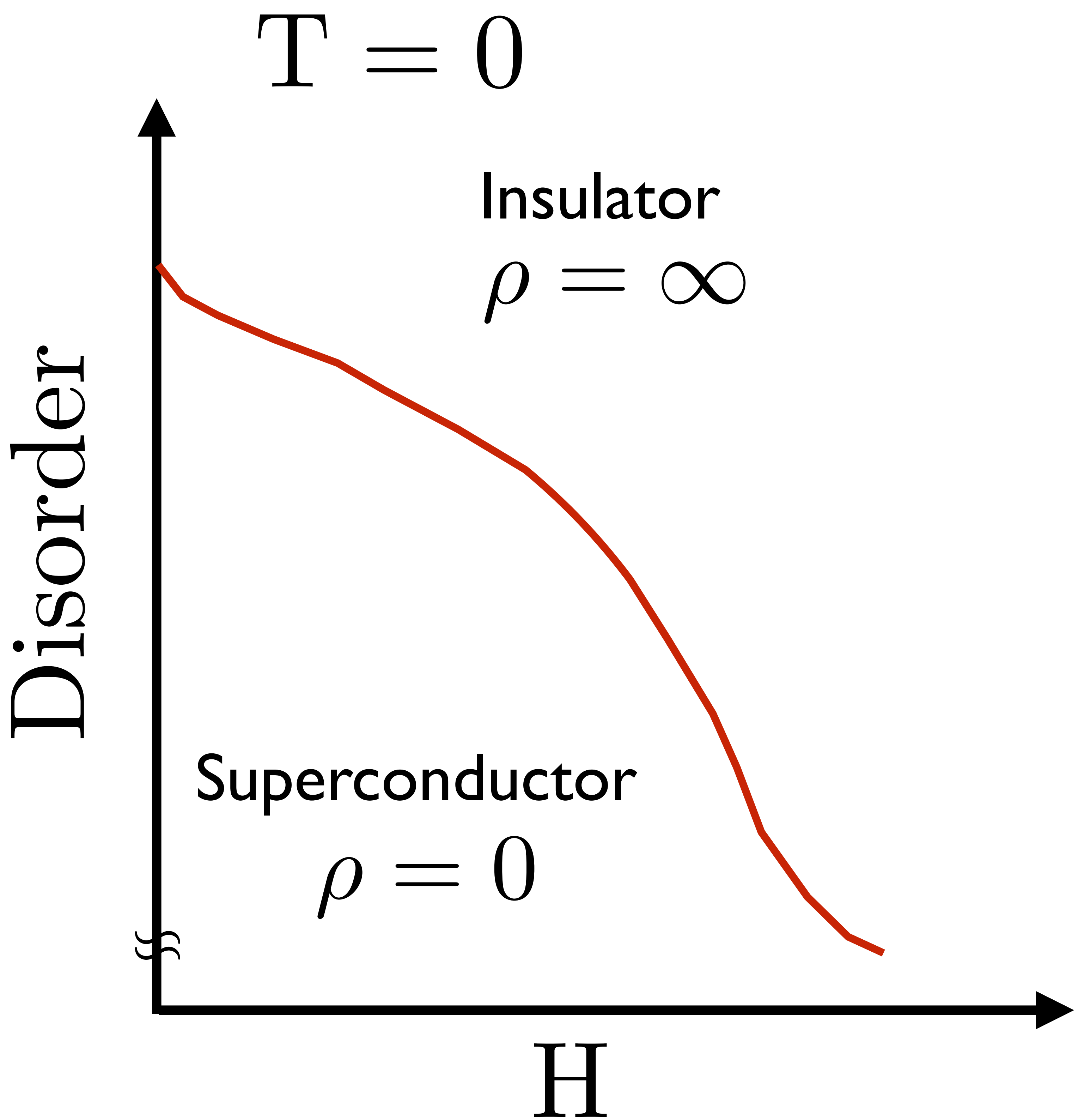


D. Ephron et al, PRL 76, 1529 (1996)

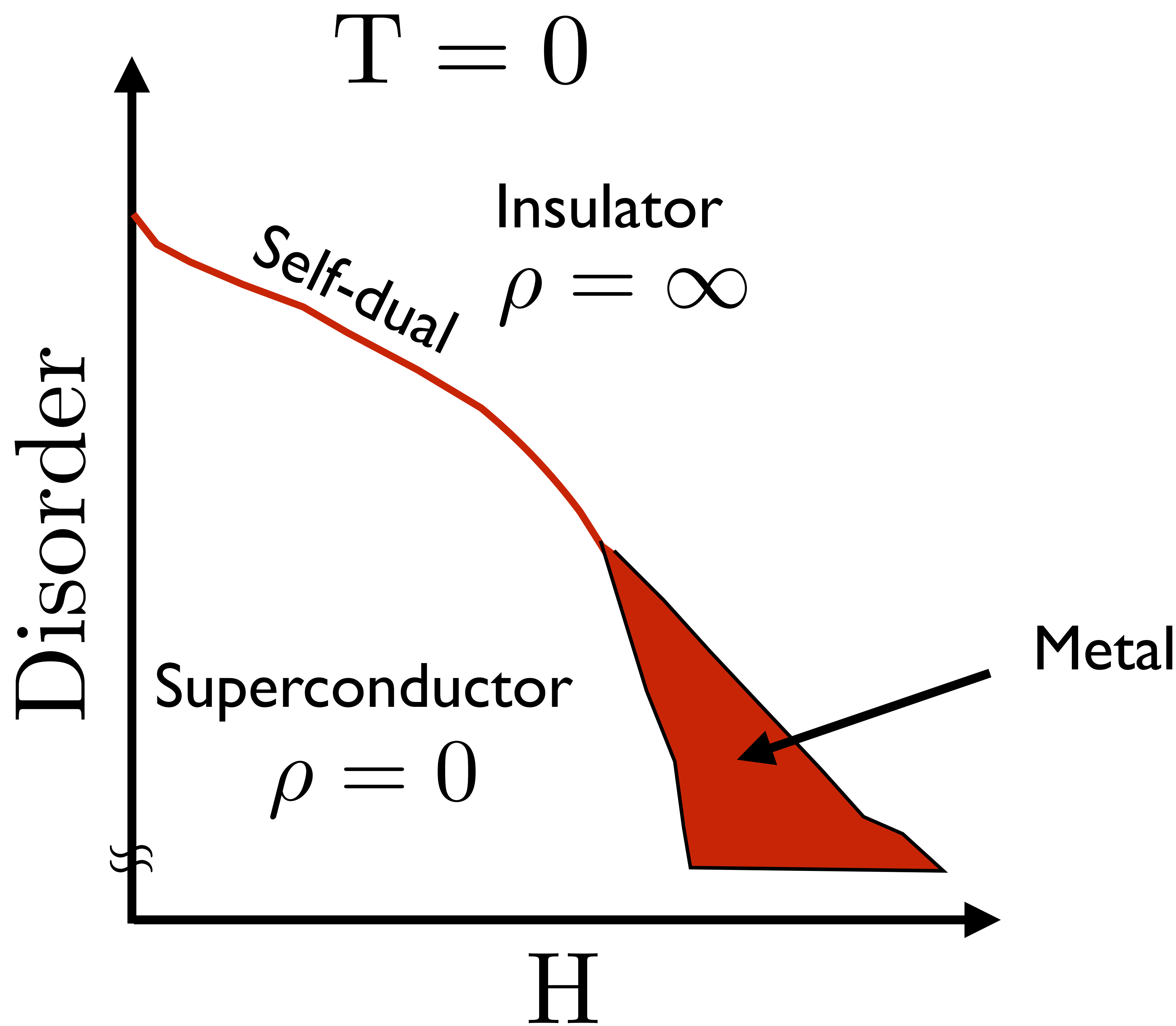


N. Mason et al, PRL 82, 5341 (1998)

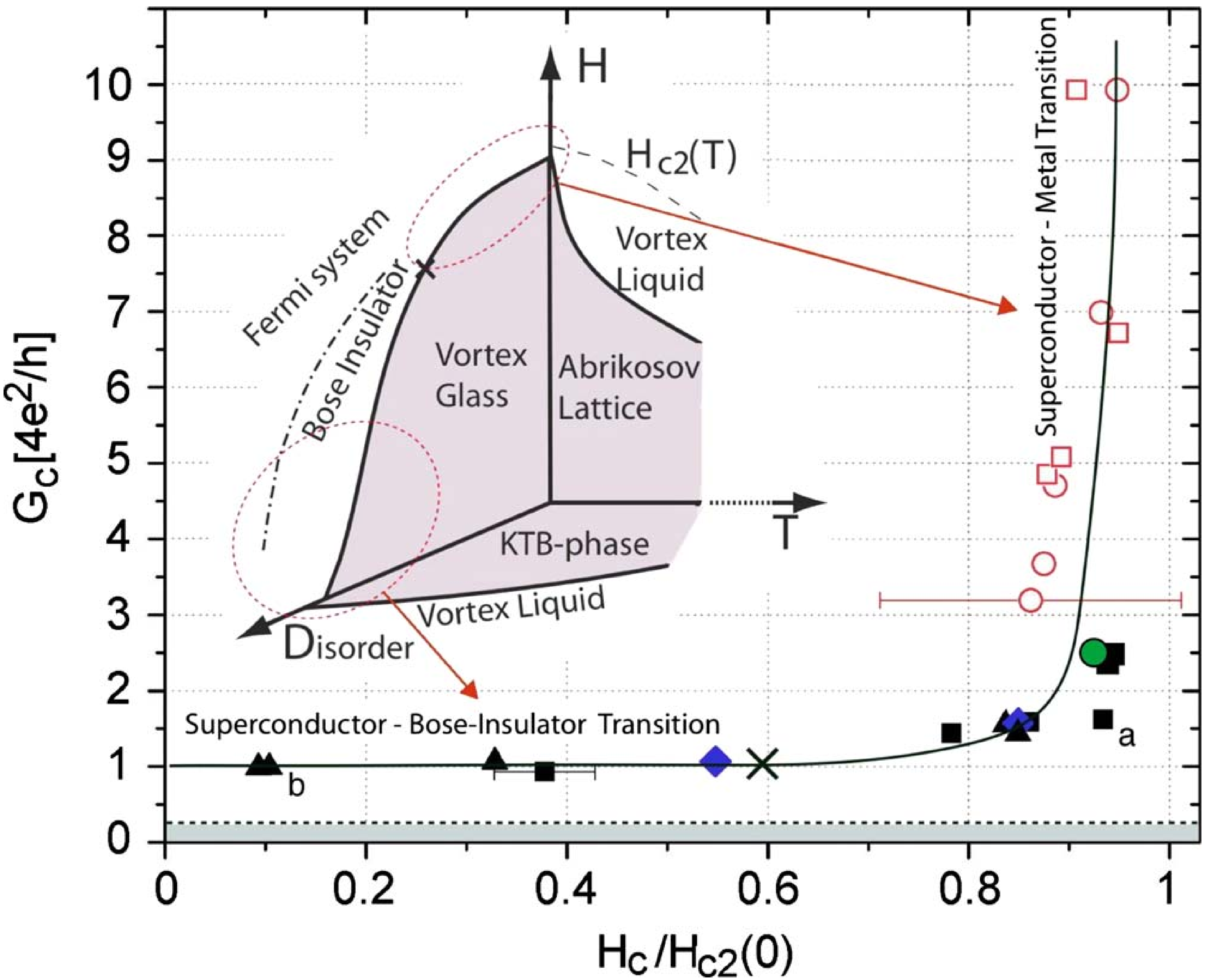
In 2d, we expect a T=0 phase diagram like this:



Instead, experiments suggest a phase diagram like this:



Steiner, Breznay, Kapitulnik, PRB (2008).



Basic ideas

Superconducting order parameter:

$$\Psi = \underbrace{|\Psi|}\underbrace{e^{i\theta}}$$

Maekawa, Fukuyama (1981)
Finkelstein (1987)
Belitz (1989)

MPA Fisher (1989, 1990,…)
Wen, Zee (1990)

Superconducting order parameter:

$$\Psi = \underbrace{|\Psi|}\underbrace{e^{i\theta}}$$

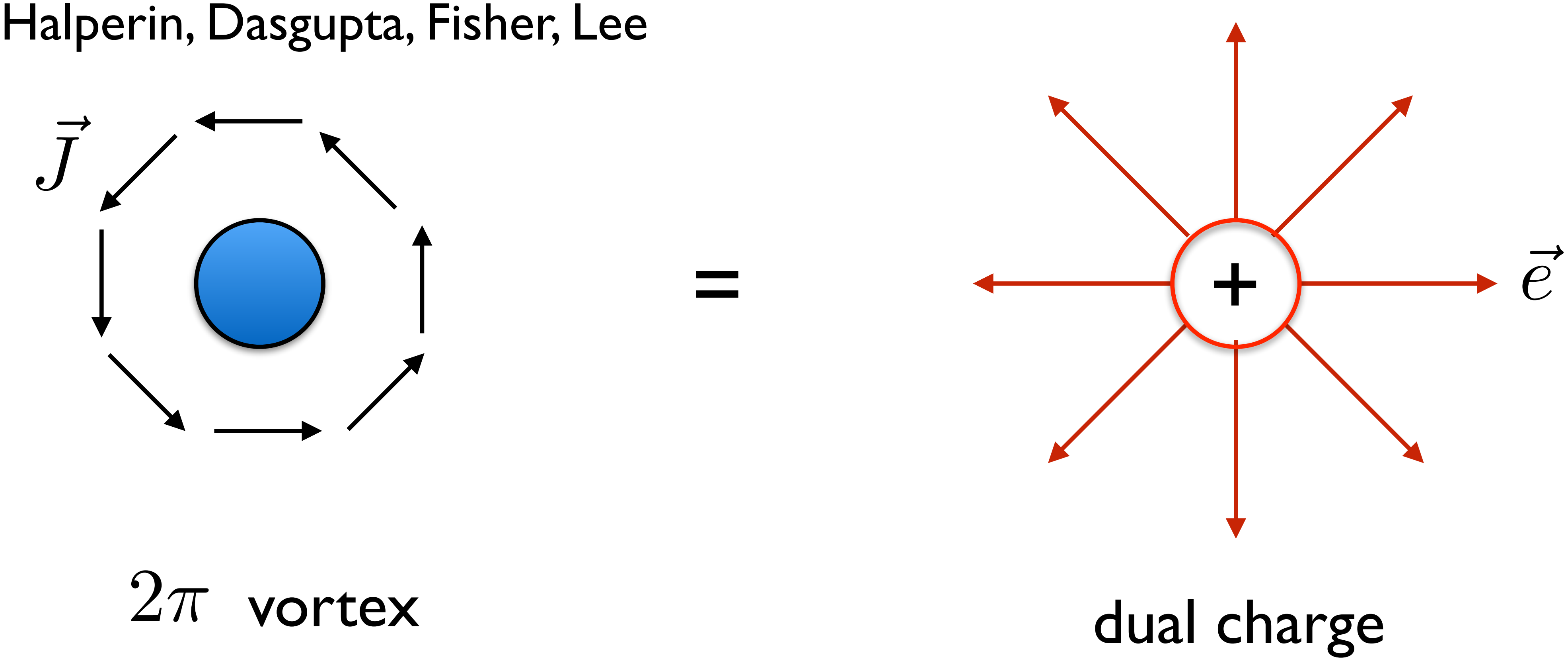
Maekawa, Fukuyama (1981)
Finkelstein (1987)
Belitz (1989)

MPA Fisher (1989, 1990,…)
Wen, Zee (1990)

Basic framework:
particle-vortex duality + dirt

Particle-vortex duality

Peskin, Halperin, Dasgupta, Fisher, Lee



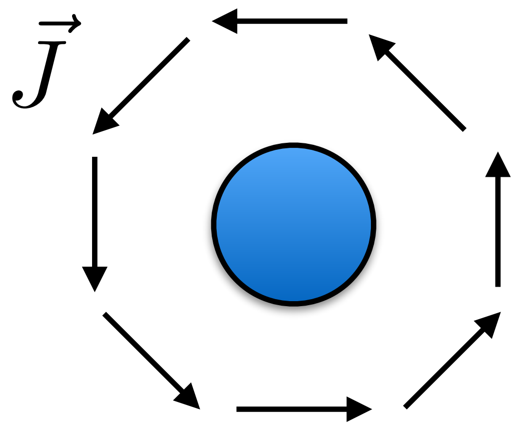
$$J_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda \qquad j_\mu = -\frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$$

A_μ : External gauge field (electromagnetism)

a_μ : Emergent gauge field (2d photon)

Particle-vortex duality

Peskin, Halperin, Dasgupta, Fisher, Lee



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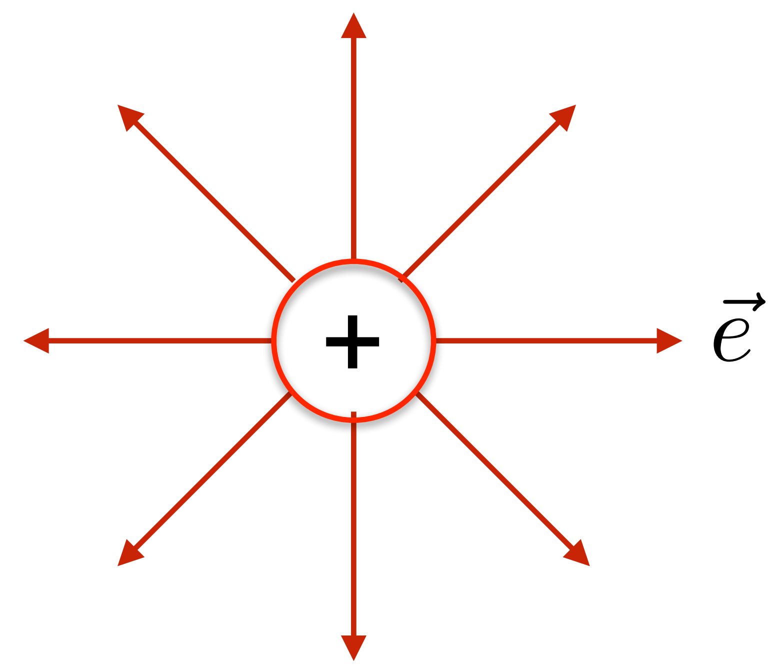


2π vortex

dual charge

$$J_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda$$

$$J_0 = -\frac{1}{2\pi}\nabla \times \vec{a} = b/2\pi$$

$$\vec{J} = \frac{1}{2\pi}\hat{z} \times \vec{e}$$

$$j_\mu = -\frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$$

$$j_0 = -\frac{1}{2\pi}\nabla \times \vec{A} = -B/2\pi$$

$$\vec{j} = -\frac{1}{2\pi}\hat{z} \times \vec{E}$$

Particle-vortex duality

Linear response:

$$\hat{\sigma}_v = \left(\frac{1}{2\pi}\right)^2 \hat{\sigma}^{-1}$$

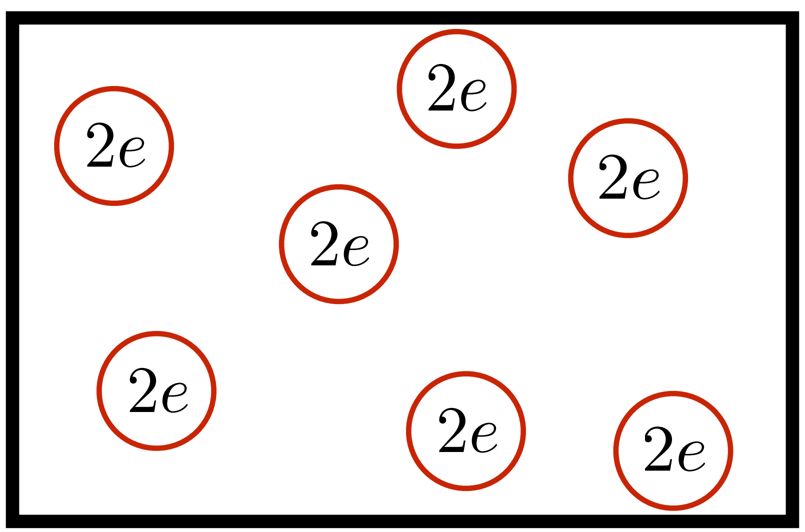
Vortex conductivity ($\hat{\sigma}_v$) — Electrical resistivity ($\hat{\sigma}^{-1}$)

Phases

1) Superconductor: Cooper pair condensate, gapped vortices

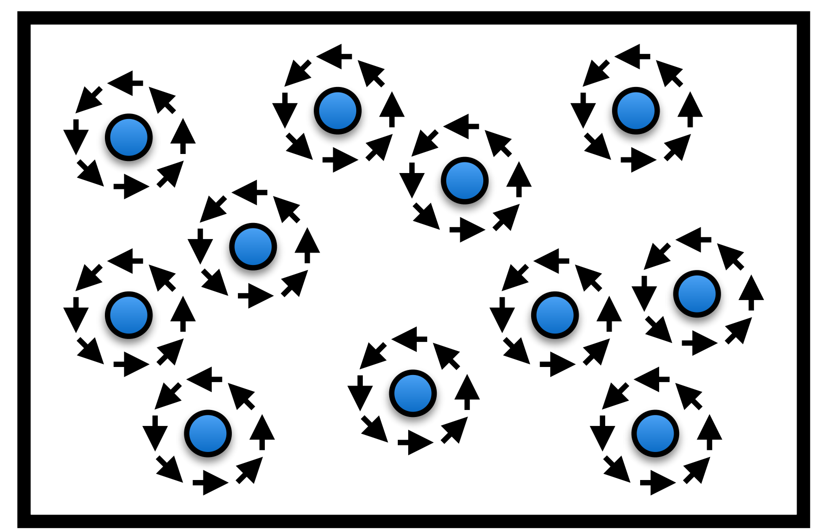
2) Insulator: vortex condensate (Higgs), gapped Cooper pairs

Superconductor ($H \ll H_c$)



Condensate of mobile Cooper pairs

Insulator ($H \gg H_c$)

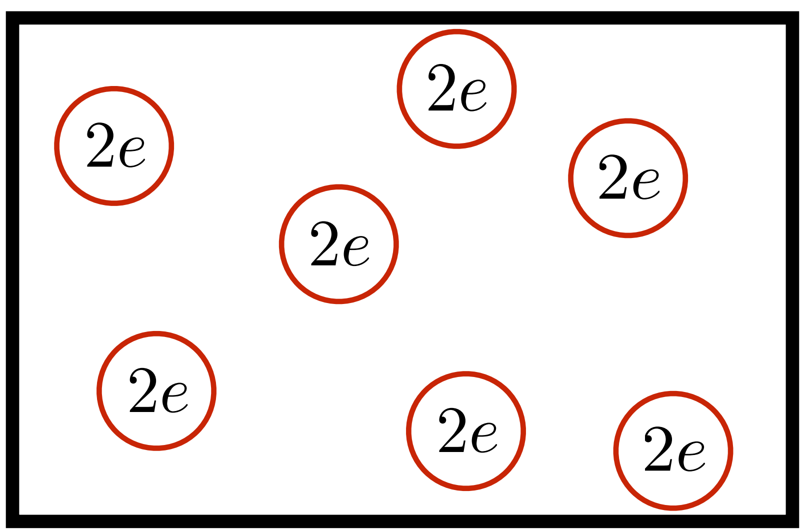


Condensate of mobile vortices

What about the critical point (SIT)?

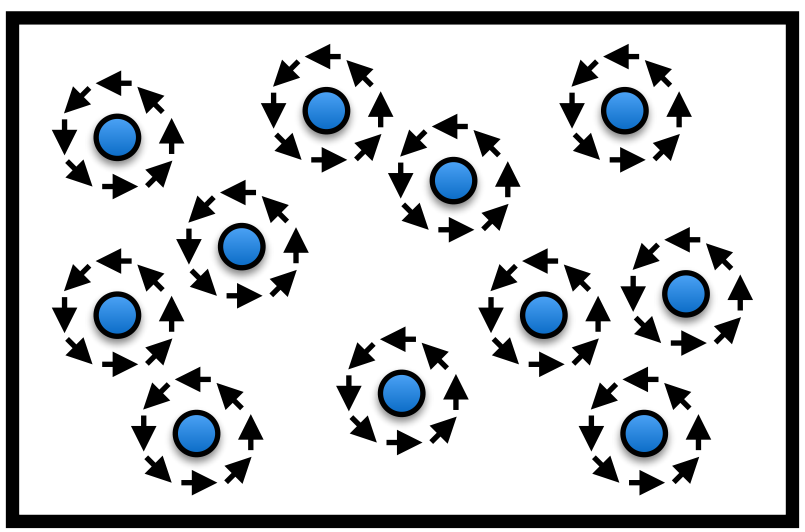
Disorder: most of the Cooper pairs are localized (frozen).

Superconductor ($H \ll H_c$)



Condensate of mobile Cooper pairs

Insulator ($H \gg H_c$)



Condensate of mobile vortices

What about the critical point (SIT)?

Self-duality at the SIT

Duality:

$$\hat{\sigma}_v = \left(\frac{1}{2\pi}\right)^2 \hat{\sigma}^{-1} \qquad [\hbar = c = 2e = 1]$$

Self-duality: $\hat{\sigma}_v = \hat{\sigma}^T$

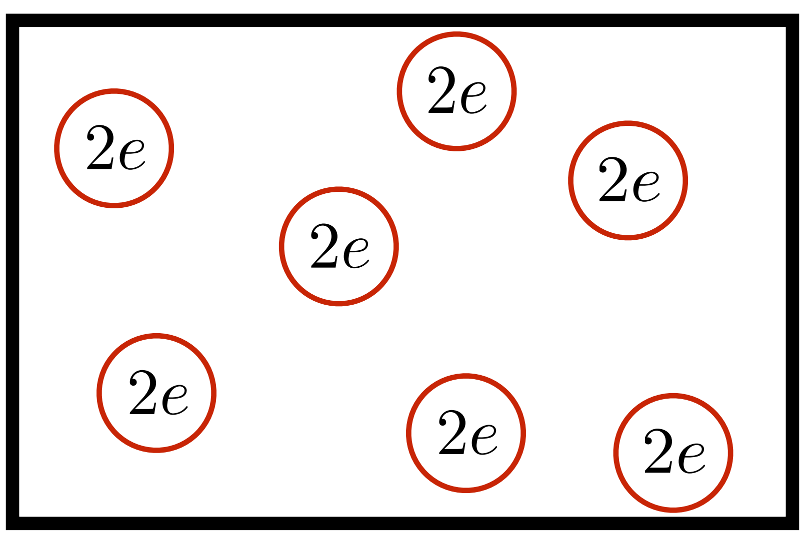
Consequence:

$$R_{xx}^2 + R_{xy}^2 = R_Q^2 \qquad R_Q = \frac{h}{(2e)^2} \approx 6.45 k\Omega$$

Requirement for self-duality: Cooper pairs and vortices both have the same response Lagrangian

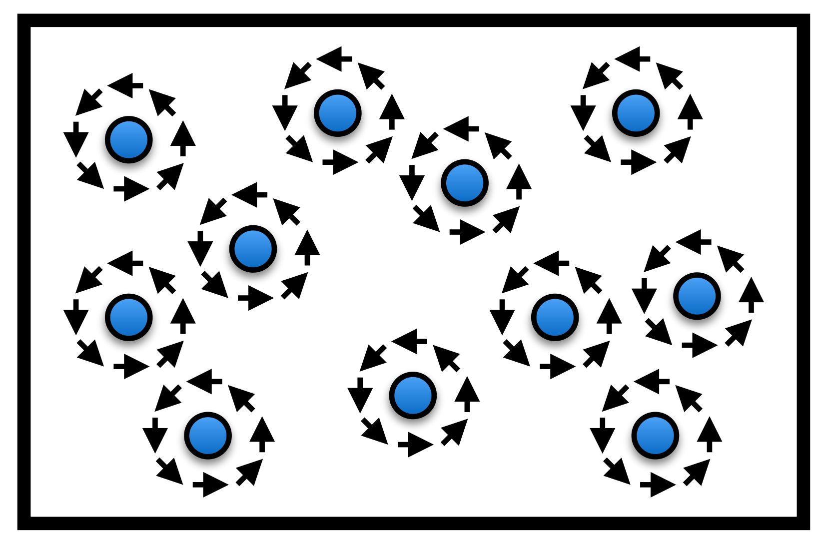
Disorder: most of the Cooper pairs are localized (frozen).

Superconductor ($H \ll H_c$)



Condensate of mobile Cooper pairs

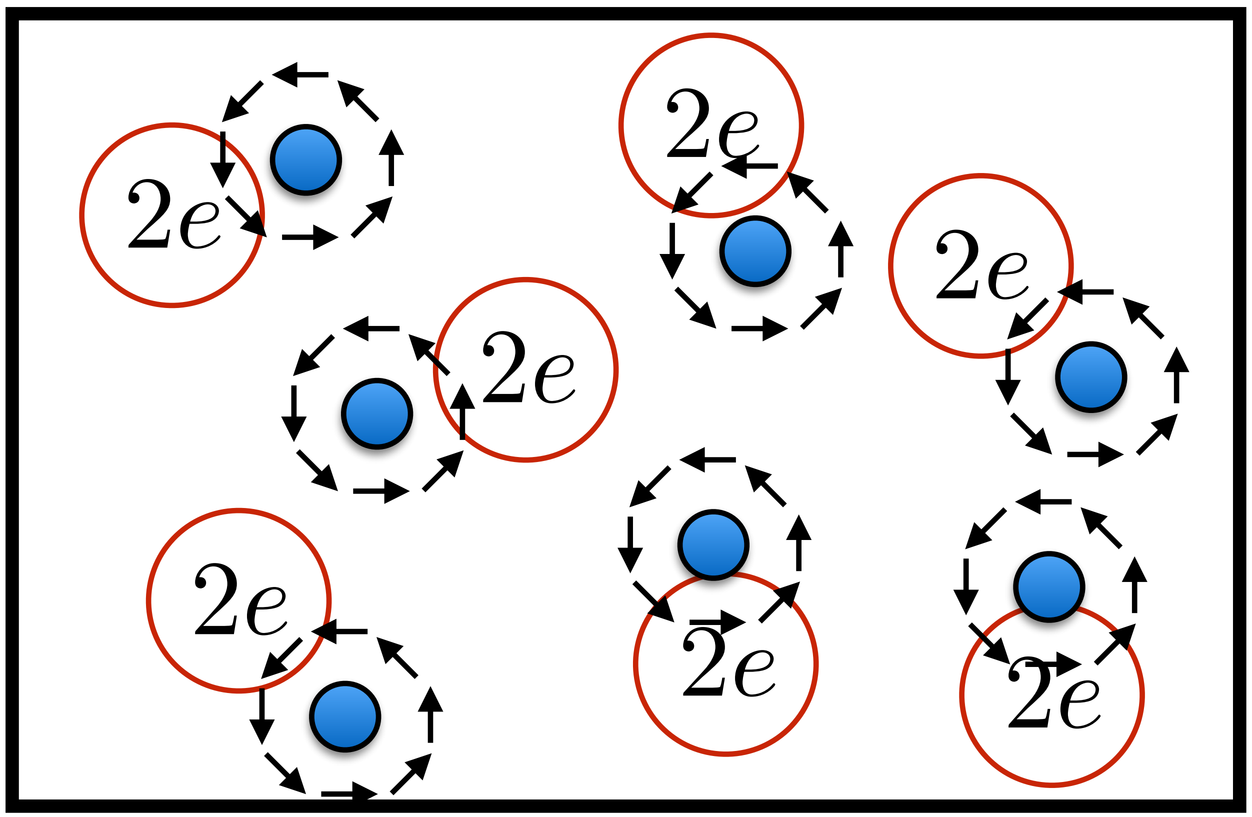
Insulator ($H \gg H_c$)

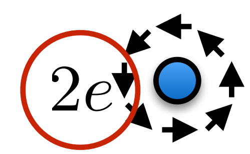


Condensate of mobile vortices

Our proposal for the SIT with self-duality

Self-dual SIT ($H = H_c$)



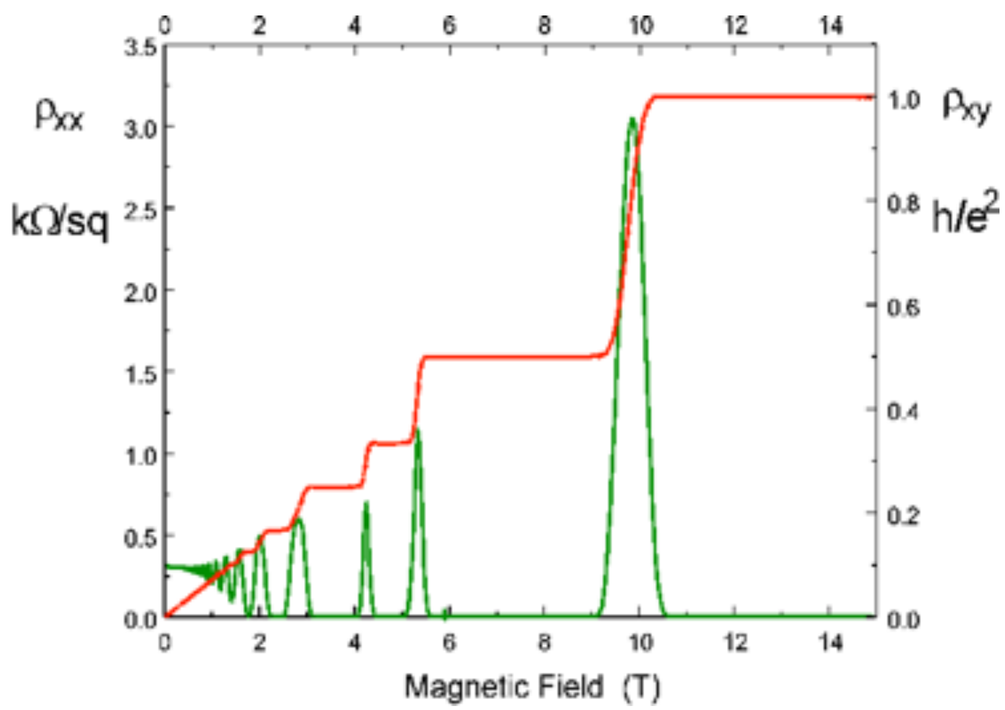


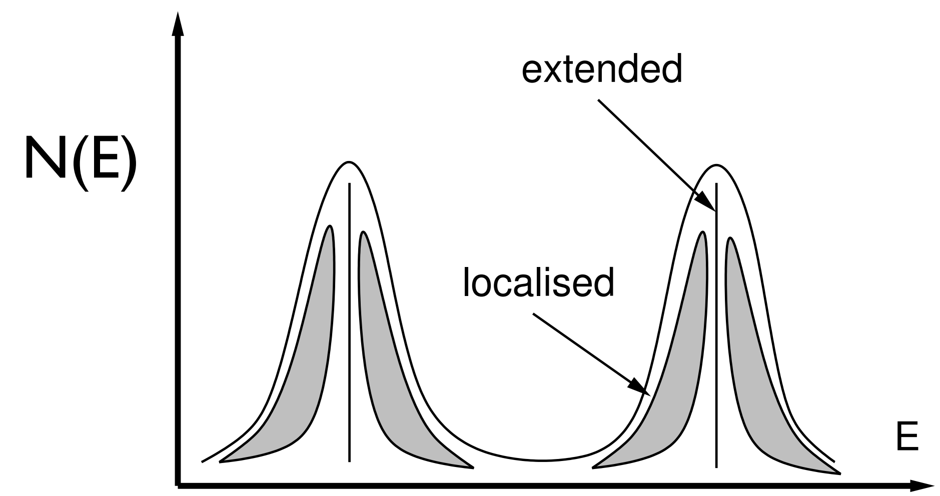
= A fermion

(composite vortex)

Finite resistance at the SIT due to emergent fermions

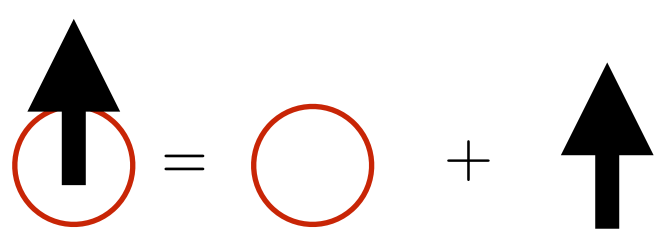
Analogous story: integer quantum Hall-insulator transition (QHIT)



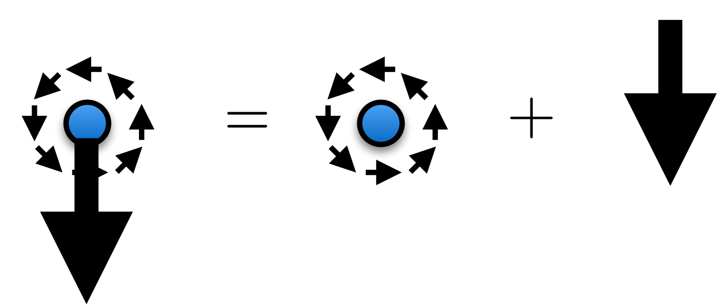


J. Chalker Lecture notes

Bosonic description:



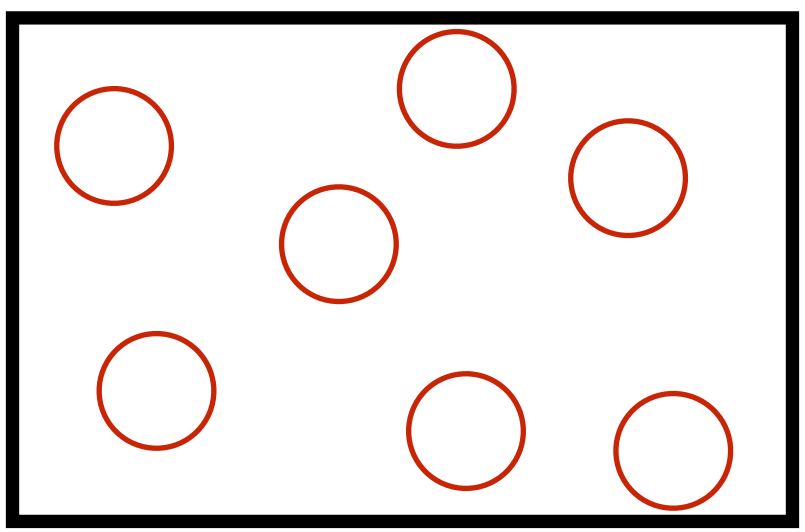
electron = boson + flux quantum



hole = vortex + flux quantum

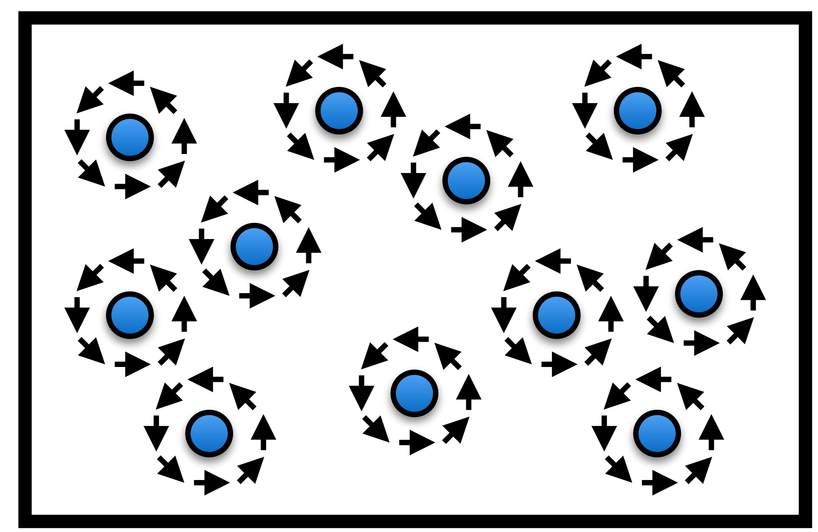
S.-C. Zhang, T. Hansson, S. Kivelson

Integer QH $(\nu = 1)$



Condensate of mobile
Composite bosons

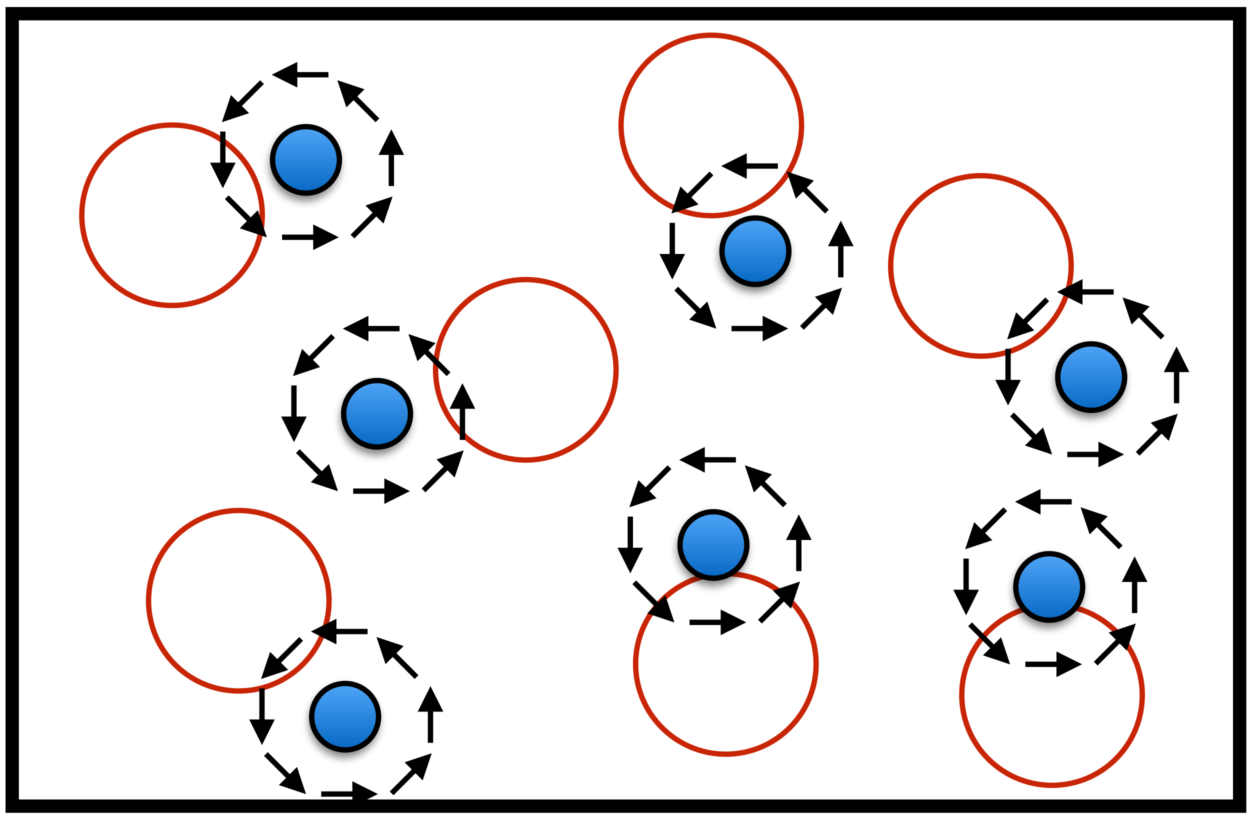
Insulator $(\nu = 0)$

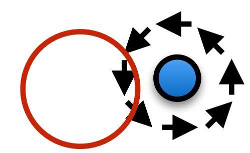


Condensate of mobile
vortices of comp. bosons

Integer quantum Hall - Insulator transition (QHIT)

QHIT with particle-hole symmetry





= A fermion

(composite fermion)

Finite resistance at the QHIT due to emergent fermions

Dictionary

QHIT	SIT
Integer QH $(\nu = 1)$	Superconductor
Insulator $(\nu = 0)$	Insulator
Particle-hole symmetric QHIT $(\nu = 1/2)$	Self-dual SIT

In both cases the critical point involves composite fermions

Slightly more quantitative discussion

Cooper pairs:

$$L_A = |D_A \Phi|^2 - V(|\Phi|) + \text{dirt}$$

Φ : Cooper pair

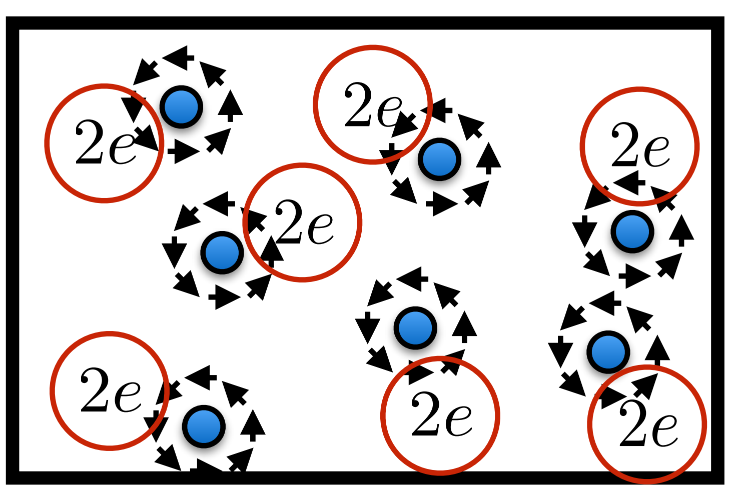
ϕ : Vortex

Vortices:

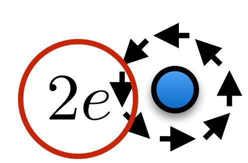
$$L_B = |D_a \phi|^2 - \tilde{V}(|\phi|) + \frac{1}{2\pi} \underbrace{adA} + \text{dirt}'$$

$\rightarrow \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda$

SIT: vortices at unit filling $(\nu = 1)$.



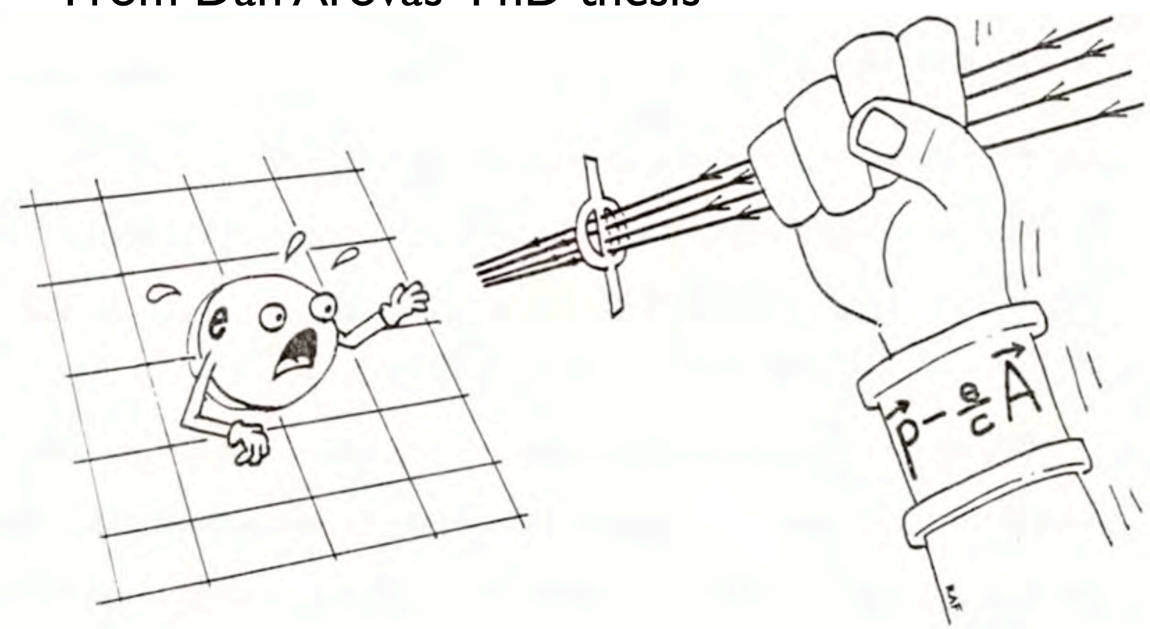
$\Longrightarrow$ Composite vortex metal

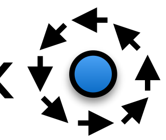


= A fermion

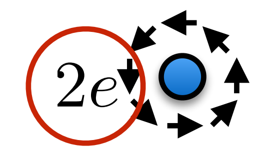
This can be implemented with flux attachment

From Dan Arovas' PhD thesis



Vortex  (boson)

$\Downarrow$ flux attachment

Composite vortex 

(fermion)

M. Mulligan and SR, PRB 2016

The SIT then maps onto the problem of a half-filled Landau level

2 choices for theoretical description of composite vortex metal:

1) Halperin-Lee-Read (HLR) composite Fermi liquid

M. Mulligan and SR, PRB 2016

2) D. Son: Dirac composite Fermi liquid

M. Mulligan, PRB 2017

QHIT: HLR theory seems to lack particle-hole symmetry

SIT: HLR theory for composite vortices seems to lack self-duality

QHIT: Son theory manifestly preserves particle-hole symmetry

SIT: Son theory for composite vortices is manifestly self-dual

M. Mulligan, PRB 2017

“Modern” discussion of self-dual theory

M. Mulligan, PRB 2017

Cooper pairs:

$$L_A = |D_A \Phi|^2 - V(|\Phi|) + \mathrm{dirt}$$

$$\updownarrow$$

Vortices:

$$L_B = |D_a \phi|^2 - \tilde{V}(|\phi|) + \frac{1}{2\pi} a dA + \mathrm{dirt}'$$

$$\updownarrow$$

composite vortices:

$$L_C = \bar{\chi} \not{D}_c \chi - \frac{1}{8\pi} cdc - \frac{1}{2\pi} cdA - \frac{1}{4\pi} AdA + \mathrm{dirt}'$$

2^{nd}-3^{rd} Lines: make use of a conjectured (and recently well-established) boson-fermion duality

Boson-fermion “duality” (t < 2015)

Polyakov (1988),
Zhang, Hansson, Kivelson (1989),
Chen, Fisher, Wu (1993),
Fradkin, Kivelson (1996),
Barkeshli, Mcgreevy (2014)

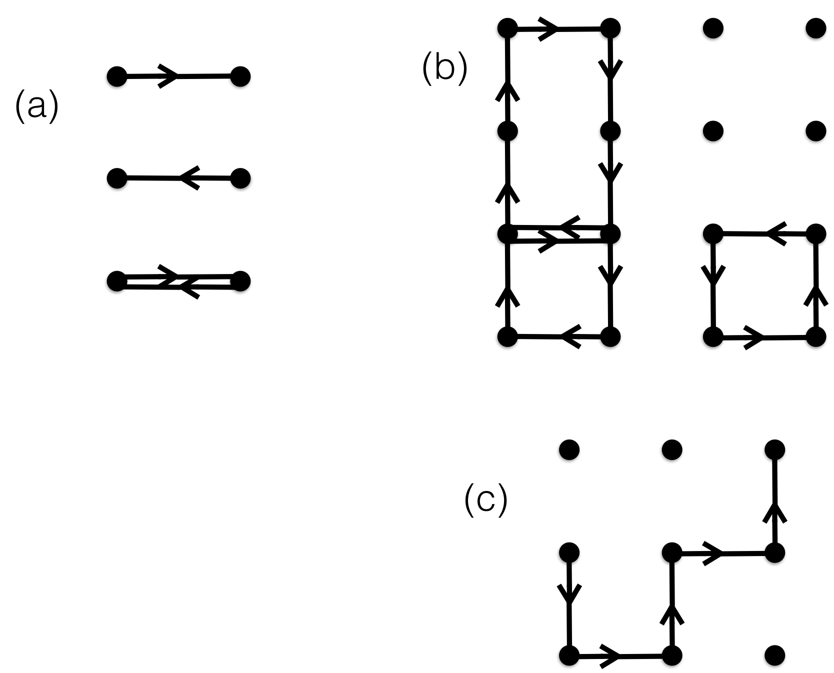
Boson-fermion duality (t > 2015)

Karch, Tong (2016),
Seiberg, Senthil, Wang, Witten (2016),
Kachru, Mulligan, Torroba, Wang (2016),
Metlitski, Vishwanath, Xu (2016),
Mross, Alicea, Motrunich (2017),
Chen, Son, Wang, SR (2017),
…

arXiv:1705.05841

Exact Boson-Fermion Duality on a 3D Euclidean Lattice

Jing-Yuan Chen[1], Jun Ho Son[1], Chao Wang[1] and S. Raghu[1,2]



Another dictionary

Vortices:

$$L_B = |D_a \phi|^2 - \tilde{V}(|\phi|) + \frac{1}{2\pi} adA + \mathrm{dirt}'$$

$$\updownarrow$$

composite vortices:

$$L_C = \bar{\chi} \not{D}_c \chi - \frac{1}{8\pi} cdc - \frac{1}{2\pi} cdA - \frac{1}{4\pi} AdA + \mathrm{dirt}'$$

	Vortices:	composite vortices:
Superconductor:	insulator	insulator
Insulator:	condensate	Integer QH
Self-dual SIT:	???	composite vortex metal

Self-duality = time-reversal for composite vortices

$$L_C = \bar{\chi} \not{D}_c \chi - \frac{1}{8\pi} cdc - \frac{1}{2\pi} cdA - \frac{1}{4\pi} AdA + \mathrm{dirt}'$$

c_t equation of motion:

$$\langle \chi^\dagger \chi \rangle = \frac{1}{4\pi} \langle \nabla \times \vec{c} \rangle + \frac{B}{2\pi}$$

Self-duality when

$$\langle \nabla \times \vec{c} \rangle = 0$$

Note:

$$\langle J_0 \rangle = -\frac{1}{2\pi} \langle \nabla \times \vec{c} \rangle - \frac{B}{2\pi}$$

$$\langle j_0 \rangle = -\frac{B}{2\pi}$$

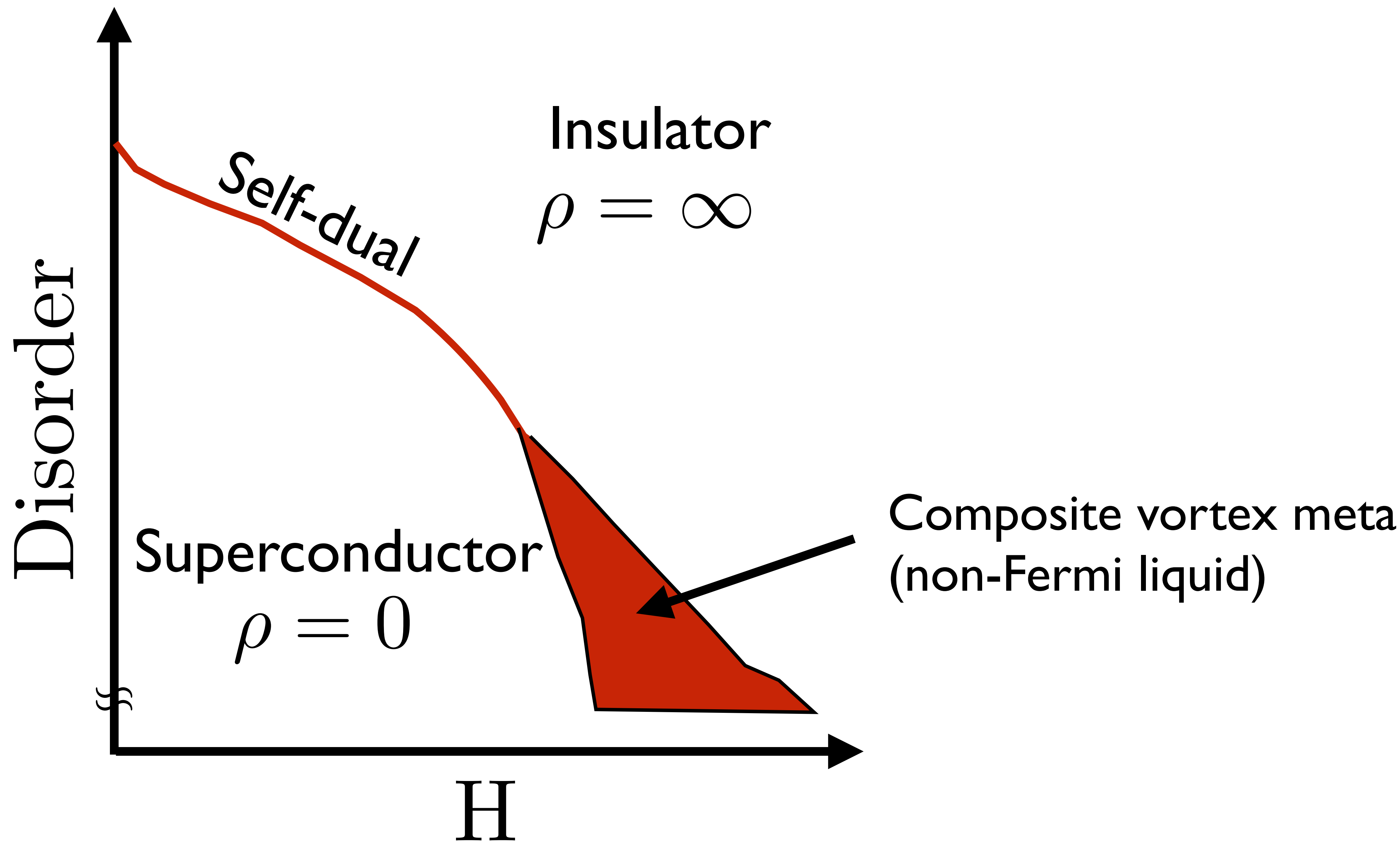
} Mismatch between number of Cooper pairs and vortices = flux for composite vortices

Self-duality = time-reversal for composite vortices

$$\rho_{xx}^2 + \rho_{xy}^2 = (2\pi)^2 \, \frac{\left(\sigma_{xx}^{\chi}\right)^2 + \left(\frac{1}{2} + \sigma_{xy}^{\chi}\right)^2}{\left(\sigma_{xx}^{\chi}\right)^2 + \left(\frac{1}{2} - \sigma_{xy}^{\chi}\right)^2}$$

Metallic phases

Guess: composite vortex metal broadens into a phase



A prediction based on composite vortices

Borrow an old QHE trick:

Composite vortices "see" a $b_{eff} << B$

Huge magnetic length >> mean-free path
Therefore, very large cyclotron radii in the metal.

cleaner systems: quantum oscillations depend on b_{eff} , not B.

What we don't (yet) understand

1) Metallic phases have zero cyclotron resonance.
2) Metallic phases have zero Hall effect (previous talk).
3) Neighboring insulating phases: Hall insulator?
4) Analogous phenomena in quantum Hall context?

Metallic phases

The existence of 2d metallic phases and neighboring transitions: challenging problem of effective field theory

Important consistency checks:

1) Stability of such metals to disorder?

2) Can they exhibit $T=0$ SC-metal transitions?

Metallic phases

The existence of 2d metallic phases and neighboring transitions: challenging problem of effective field theory.

Important consistency checks:

1) Stability of such metals to disorder?

Yes in certain cases

P. Goswami, H. Goldman, SR, PRB (2017)

A. Thomson, S. Sachdev, PRB (2017)

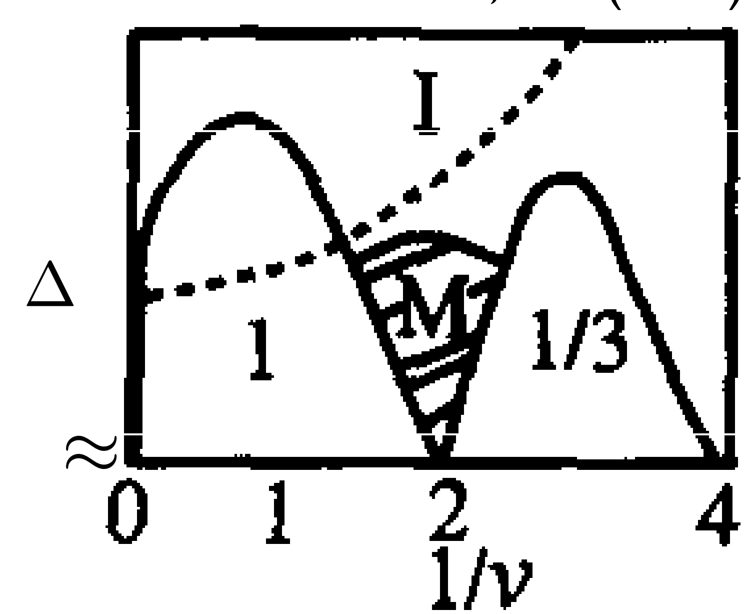
} Perturbative

(see posters)

Non-Perturbative

M. Mulligan, SR, G. Torroba, M. Zimet *(unpublished)*

Sol.St.comm. **102**, 327 (1997)



Metallic phases

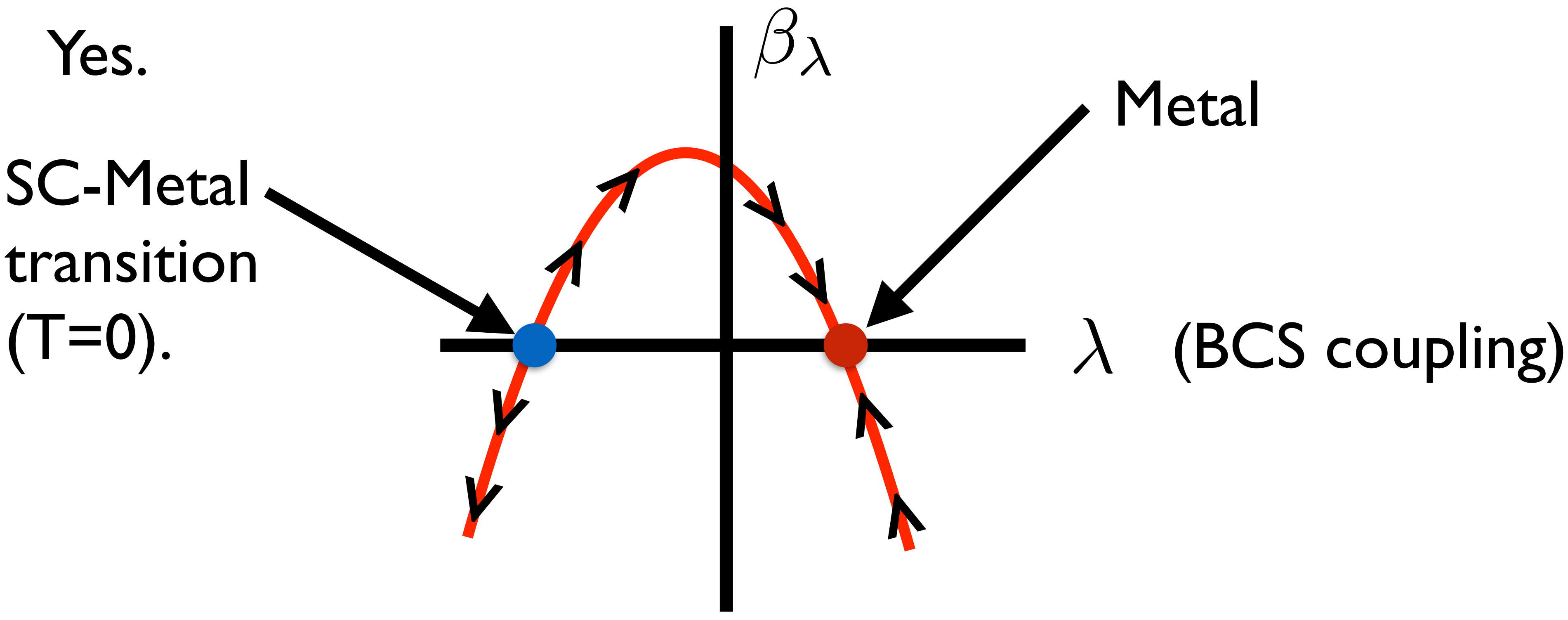
The existence of 2d metallic phases and neighboring transitions: fundamental conceptual problem.

Important consistency checks (effective field theory):

1) Stability of such metals to disorder?

2) Can they exhibit T=0 SC-metal transitions?

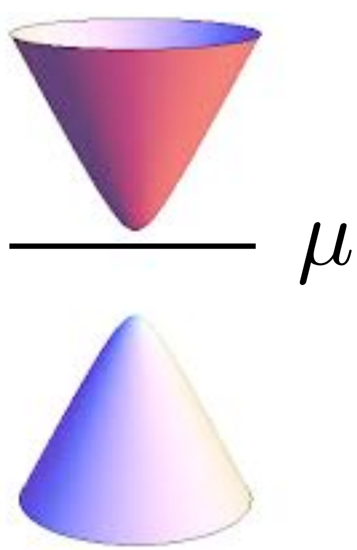
Yes.



BCS beta function for Fermi surface + U(1) gauge field

Summary

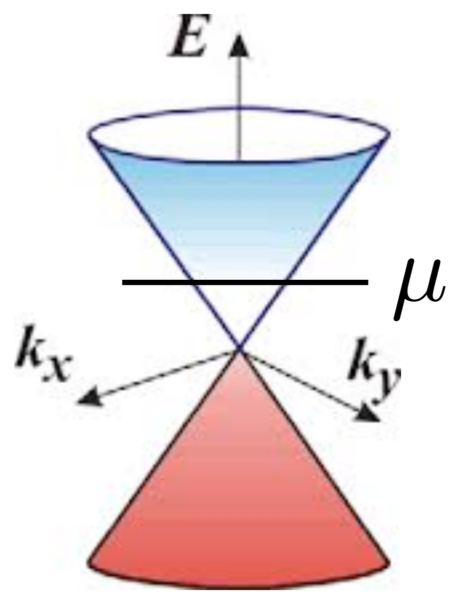
$m, \nabla \times c > 0$



superconductor

IQHE of fermions

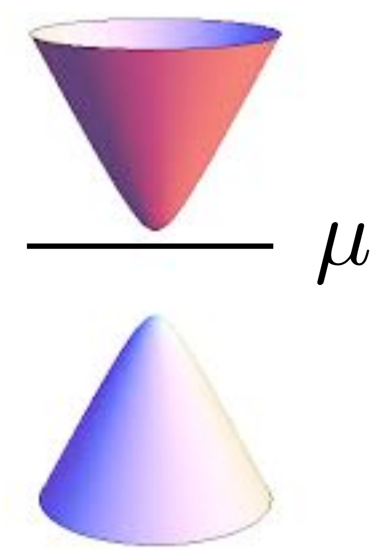
$m, \nabla \times c = 0$



SIT/metallic phases

Composite Fermi liquid (NFL)

$m, \nabla \times c < 0$



Insulator

Fermi insulator