Slow operators

and low frequency transport

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Stanford Institute for Theoretical Physics

August 23, 2017

Slow operators

Strong coupling $\rightarrow$ τ_{ee} small $\rightarrow$ most operators are 'fast'

So what controls observables at low frequencies $\omega \ll \dfrac{1}{\tau_{\rm ee}}$?

Pollock (1952)

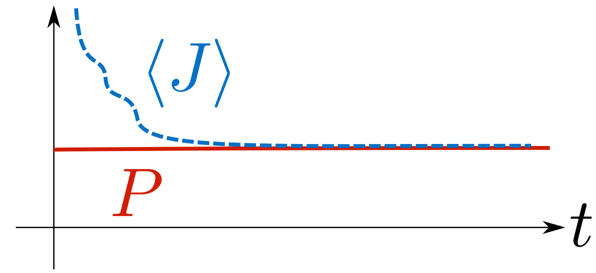
Slow operators

Strong coupling $\rightarrow$ τ_{ee} small $\rightarrow$ most operators are 'fast'

So what controls observables at low frequencies $\omega \ll \dfrac{1}{\tau_{\rm ee}}$?

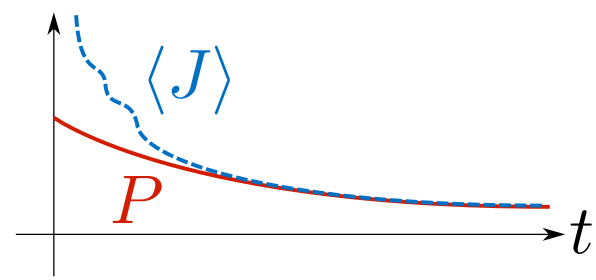
Some operators stay slow because of conservation laws or broken symmetries (Goldstones)

$$\sigma(\omega) = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{i}{\omega} + \ldots$$

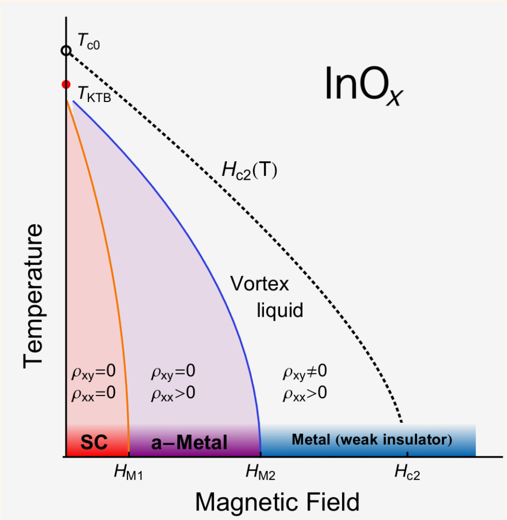


A single almost conserved operator leads to a sharp Drude peak

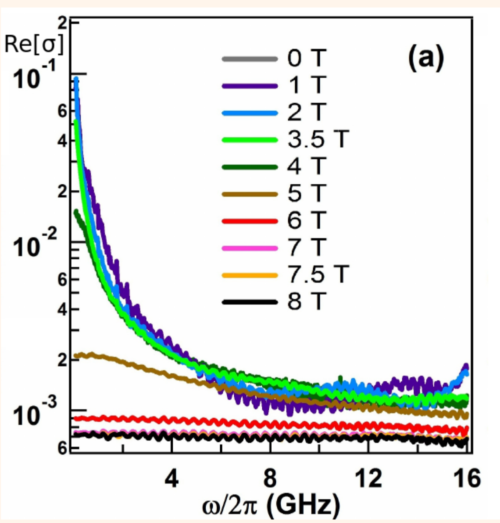
$$\sigma(\omega) = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{i}{\omega + i\Gamma} + \ldots$$



Fluctuating SC



Breznay Kapitulnik (2017)



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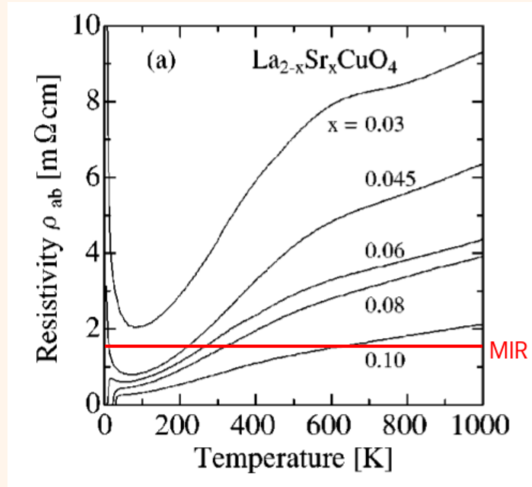
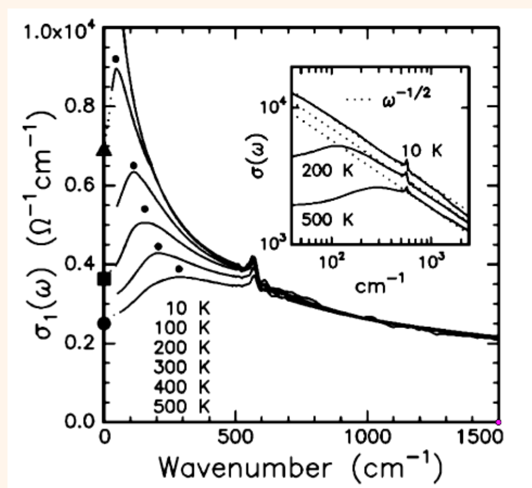
Bad Metals



Sun et al. (2001), ...

Peaks in $\sigma(\omega)$



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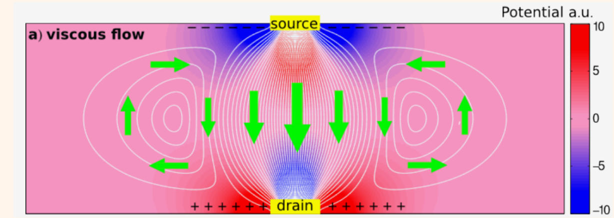

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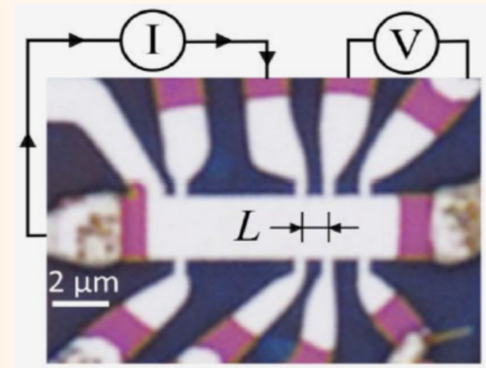
$$\sigma(k) = \ldots + \eta k^2 + \ldots$$



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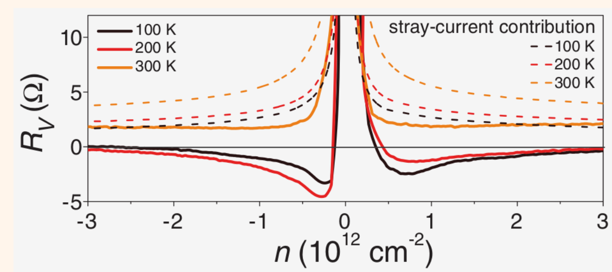
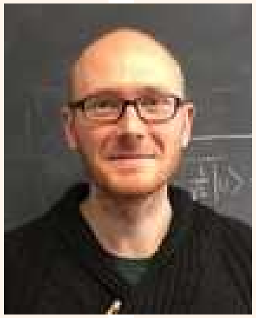


$\rightsquigarrow$ η_H in FQH?

Fluctuating SC

Sean Hartnoll

Richard Davison



Blaise Goutéraux

Bad Metals

Anna Karlsson

Non-local Resistance

Andrey Gromov

Theory tool: memory function

Write conductivities as

$$\sigma_{JJ}(\omega) = \sum_{A,B} \chi_{JA} \left(\frac{1}{-i\omega\chi + M(\omega)} \right)_{AB} \chi_{BJ} \,.$$

If A, B run over all slow operators that overlap with J, then M is small. More specifically,*

$$M_{AB} \simeq \lim_{\omega \to 0} \frac{\operatorname{Im} G^R_{\dot{A}\dot{B}}(\omega)}{\omega} \,.$$

If there is a single slow operator A in the sum, then M_{AA} dominates the denominator and one recovers a Drude conductivity

$$\sigma_{JJ}(\omega) \simeq \frac{\chi_{JA}^2}{-i\omega\chi_{AA} + M_{AA}} = \frac{\chi_{JA}^2}{\chi_{AA}} \cdot \frac{1}{\frac{M_{AA}}{\chi_{AA}} - i\omega} \,.$$

* terms and conditions apply

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Susceptibilities measure overlap between operators

Memory matrix

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Spectral density $\sim$ # of states that A, B can relax into

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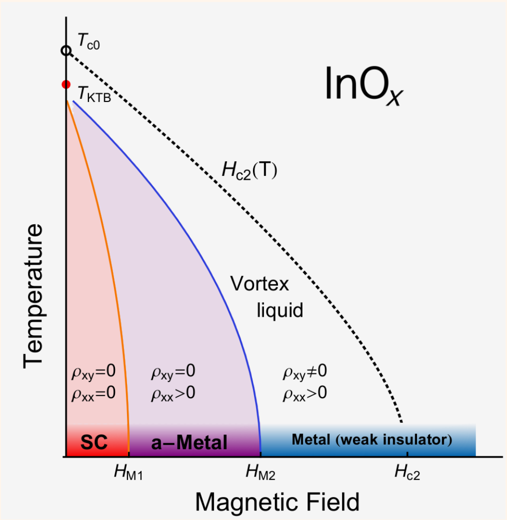

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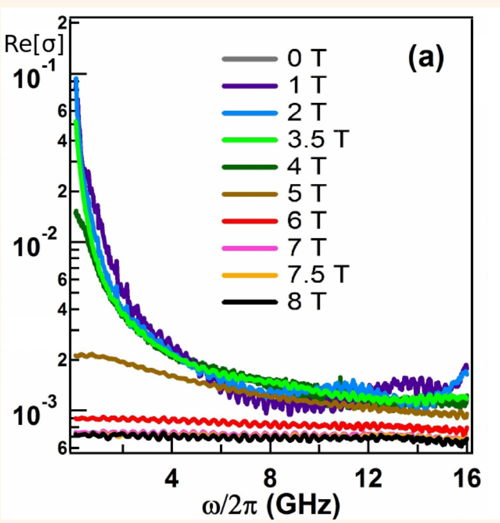

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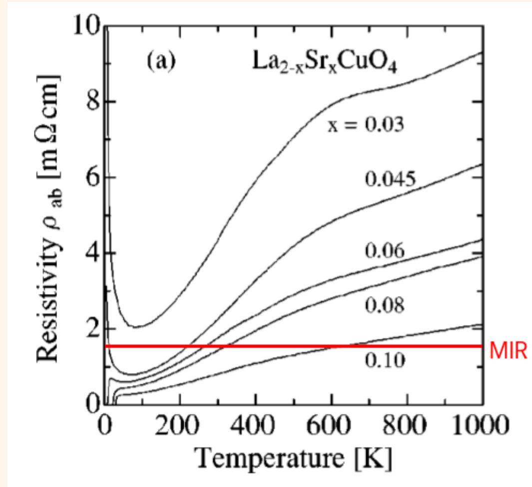

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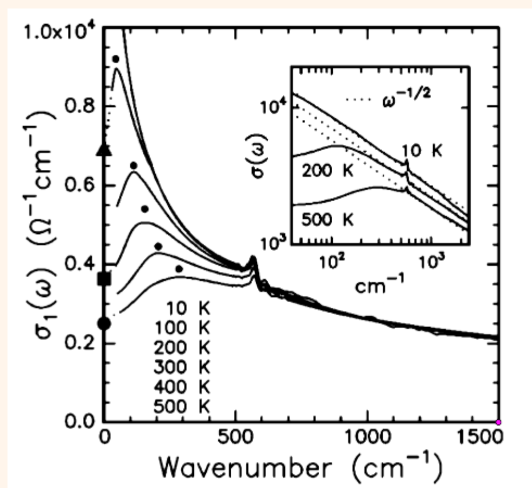

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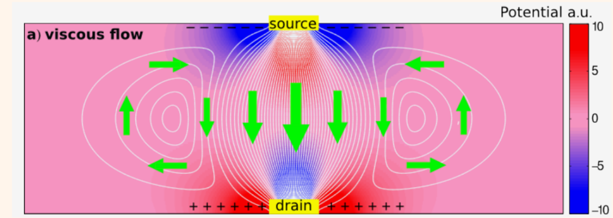

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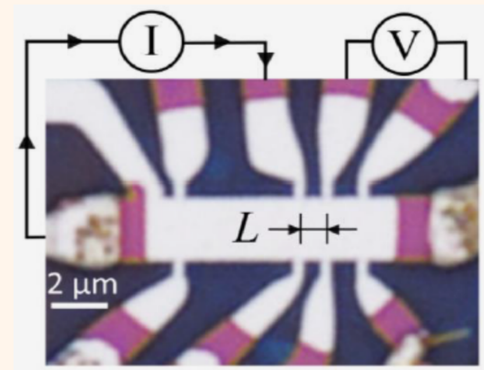

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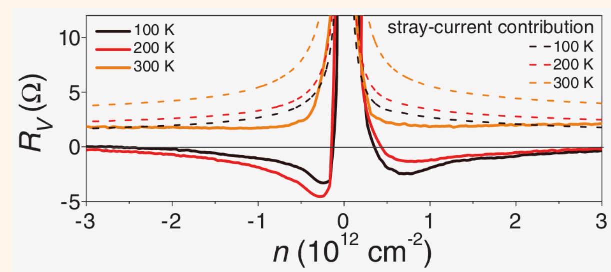

$\rightsquigarrow \eta_H$ in FQH?

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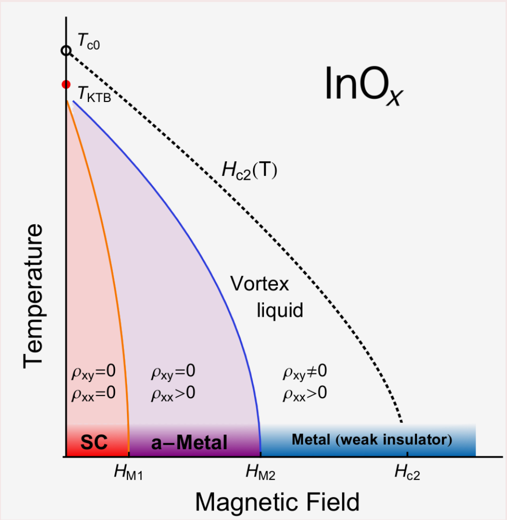

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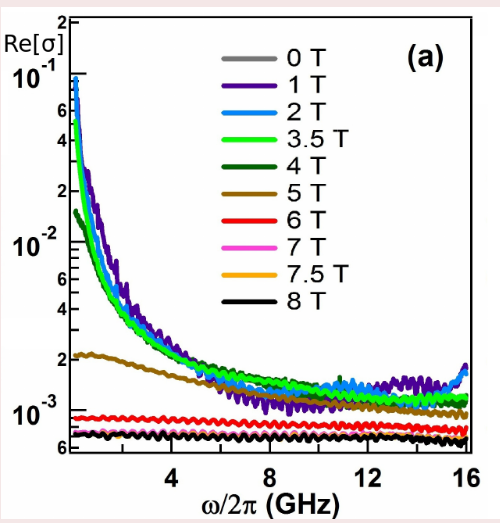

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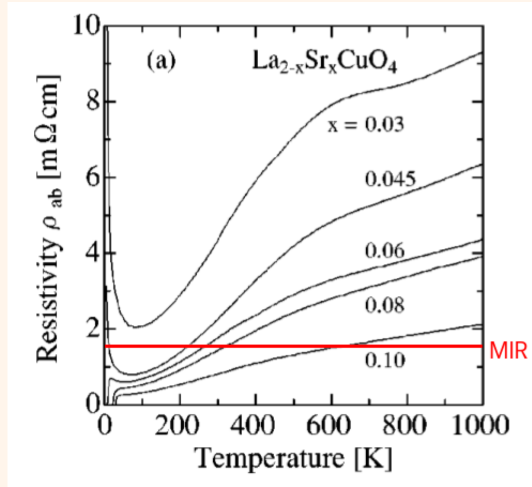

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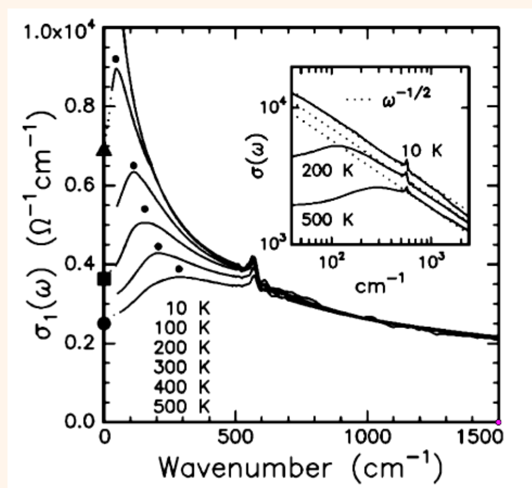

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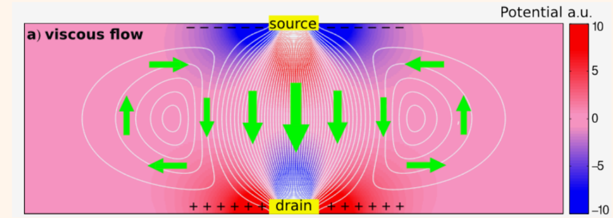

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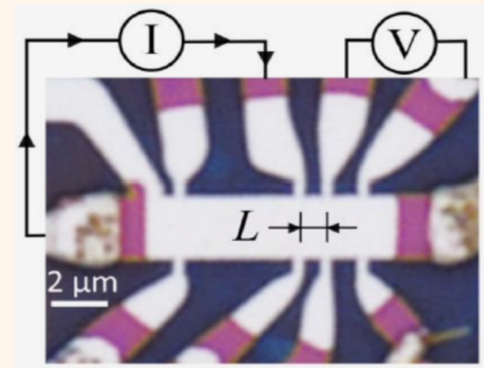

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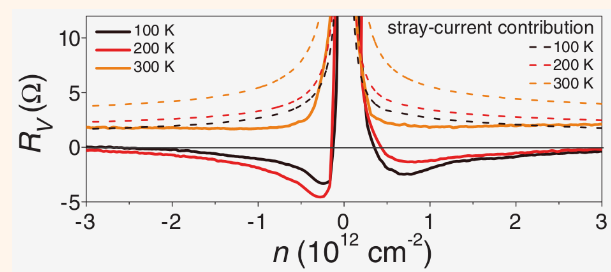

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Supercurrent relaxation

Dirty superconductor: only long-lived vector operator is the supercurrent

$$J^{\phi} = \int d^2x \, \nabla \phi$$

Topologically conserved. What relaxes it? Vortices: $\dot{J}^{\phi}_i = \frac{1}{2\pi} \epsilon_{ij} J^v_j$.

Single long-lived operator $\Rightarrow$ memory matrix gives Drude conductivity

$$\sigma(\omega) = \frac{\rho_s}{\Omega - i\omega} + \ldots$$

with

$$\Omega = \rho_s \lim_{\omega \to 0} \frac{\text{Im}\, G^R_{J^v J^v}(\omega)}{\omega} = \rho_s \sigma^v \,.$$

In particular $\sigma_{\text{dc}} = \dfrac{1}{\sigma^v}$ Bardeen Stephen (1965)

Seen in optical data

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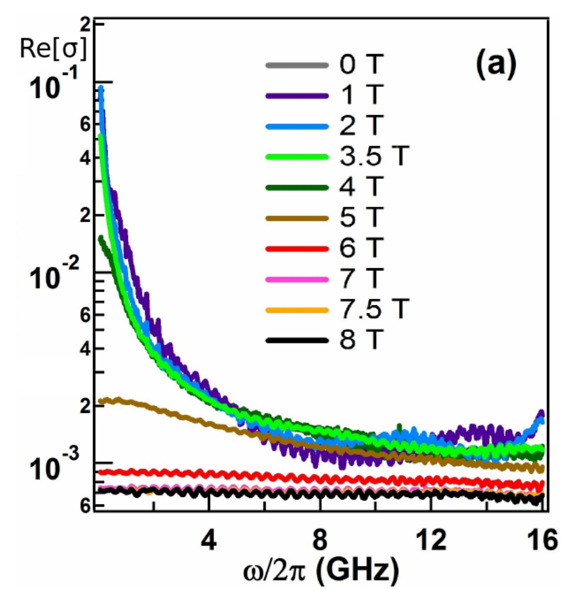

Liu Armitage et al. (2013)

SC in a magnetic field

Since B breaks parity, both J_x^ϕ and J_y^ϕ can 'overlap' with the current J_x

Memory matrix formula for the conductivity has more structure

Davison LVD Goutéraux Hartnoll (2016)

$$\sigma_{xx}(\omega) = \rho_s \frac{(1-\rho_v^2)(-i\omega+\Omega)+2\rho_v\Omega_H}{(-i\omega+\Omega)^2+\Omega_H^2} + \ldots$$

$$\sigma_{xy}(\omega) = \rho_s \frac{(1-\rho_v^2)\Omega_H+2\rho_v(-i\omega+\Omega)}{(-i\omega+\Omega)^2+\Omega_H^2} + \ldots$$

With two new parameters, $\Omega_H = \rho_s \sigma_H^v$ and ρ_v.

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With two new parameters, $\Omega_H = \rho_s \sigma_H^v$ and ρ_v.

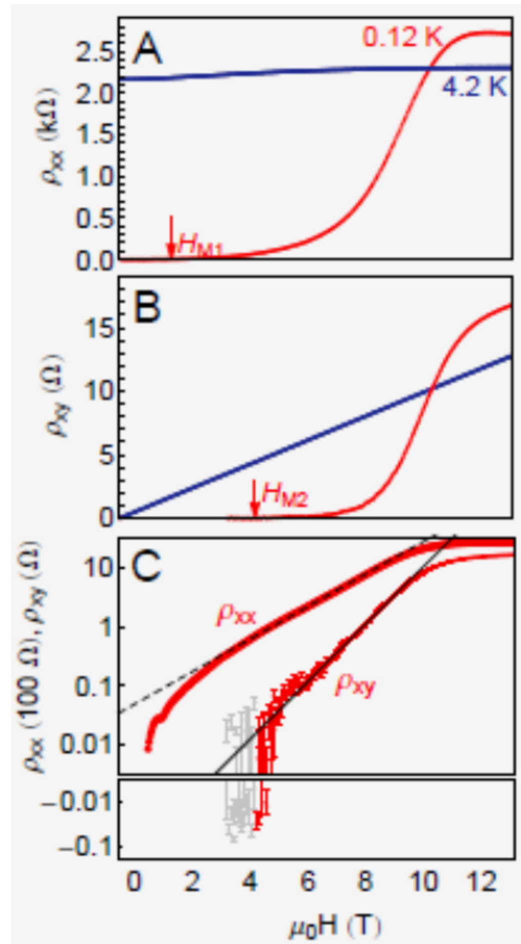
Hall sector is most sensitive to new parameters

extract them from dc data Breznay Kapitulnik (2017)

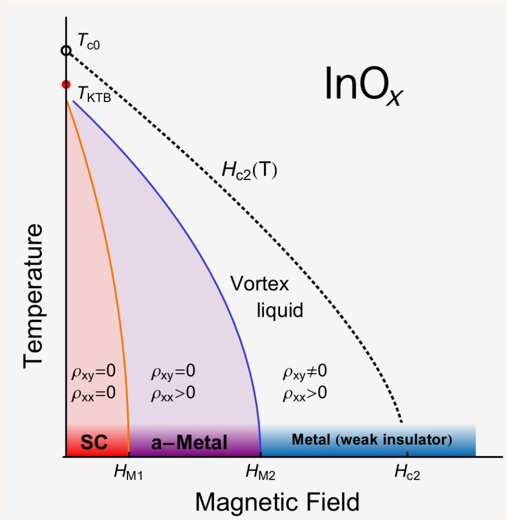
$\rightarrow$ constrains optical data Wang Armitage et al. (2017)

Is particle-hole symmetry required to understand data?

Sharper characterization of 2D anomalous metals



Fluctuating SC



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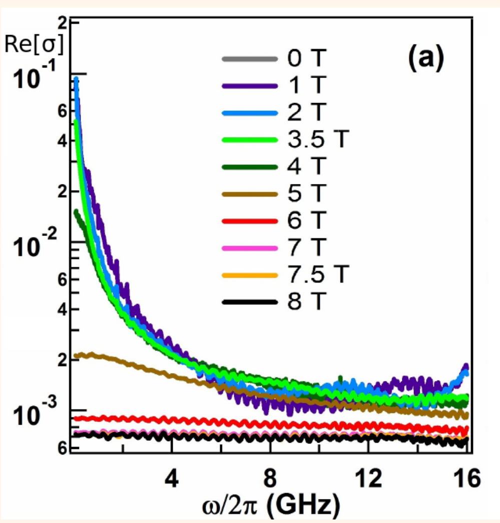

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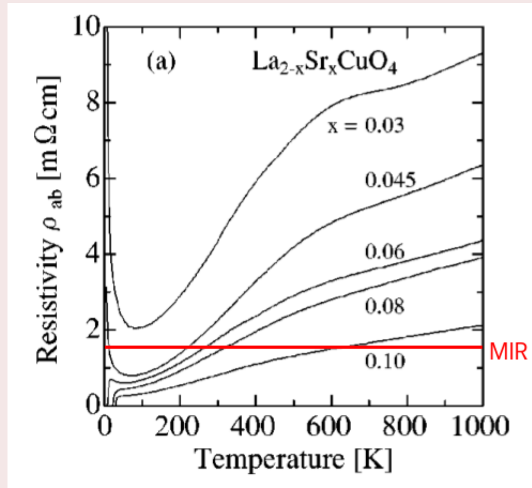

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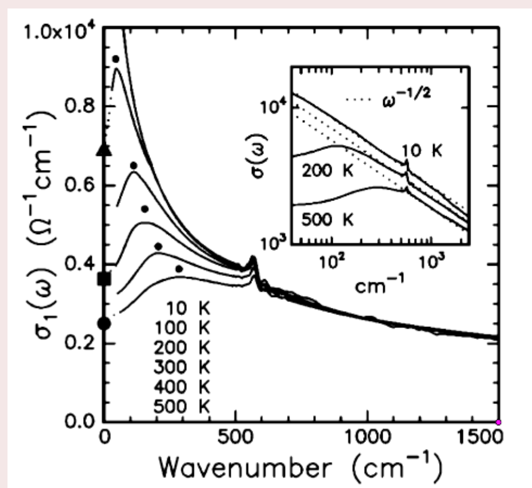

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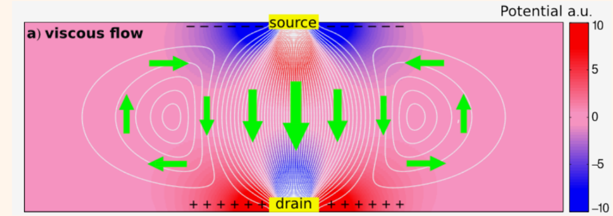

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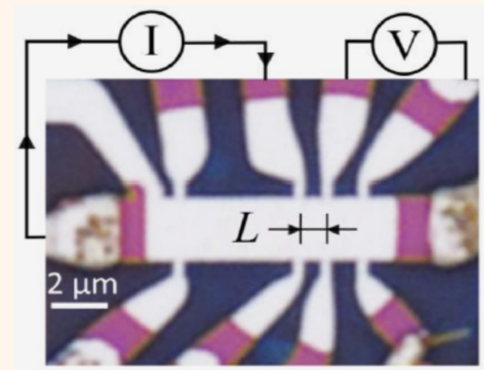

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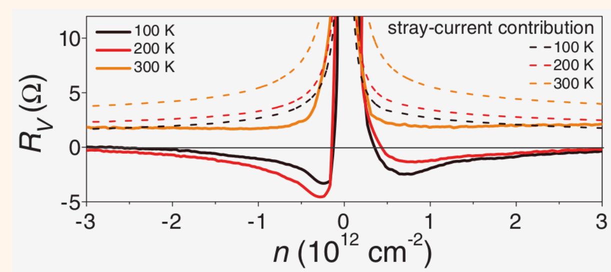


$\rightsquigarrow$ η_H in FQH?

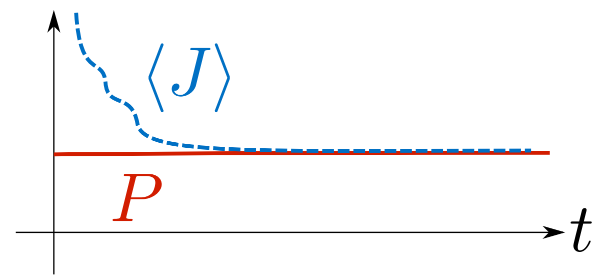
How can a dc conductivity be 'small'?

(Bad metals and violation of the Mott-Ioffe-Regel bound $\sigma \gtrsim \hbar/e^2$)

Fermi Liquid theory: infinitely many conserved quasiparticle excitations can carry current $\rightarrow$ $\sigma_{\text{dc}} = \infty$.

Strong interactions are not enough to make σ_{dc} small: if translation is a symmetry then

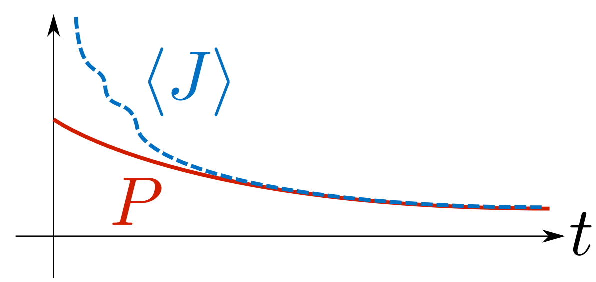
$$\sigma(\omega) = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{i}{\omega} + \ldots$$



where $\chi_{JP} = ne$ and $\chi_{PP} = nm_\star$.
Weakly breaking translations leads to a sharp Drude peak

$$\sigma(\omega) = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{i}{\omega + i\Gamma} + \ldots$$

$$\Rightarrow \quad \sigma_{\text{dc}} = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{1}{\Gamma} + \ldots$$

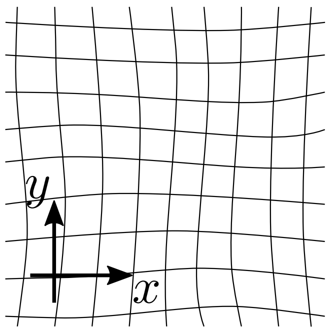


How can a DC conductivity be 'small'?

$$\sigma(\omega) = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{1}{\Gamma - i\omega}$$

- Break translations strongly (Γ large)
- Emergent particle-hole symmetry ($\chi_{JP} = 0$)
- **Tendency towards spatial order:** the Drude pole is now also carried by a Goldstone boson: $[\phi_i, P_j] = i\delta_{ij}$.

$\delta\phi$



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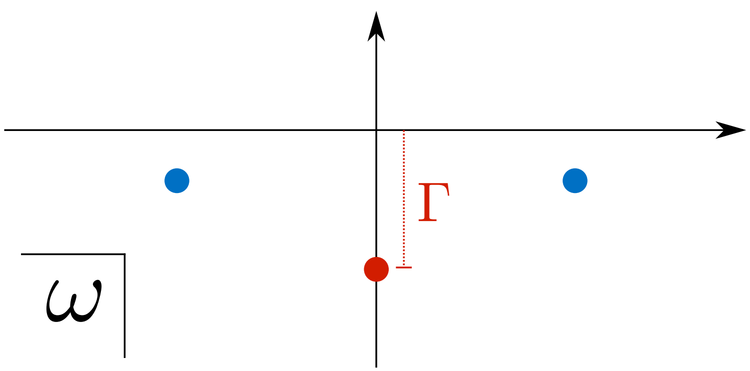

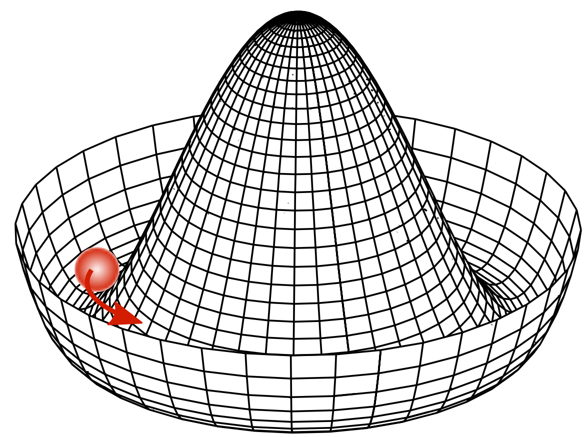

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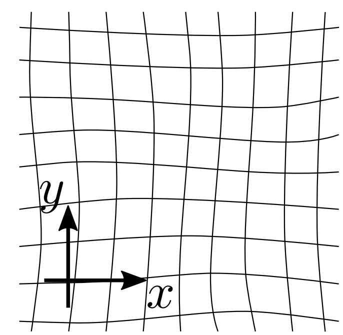

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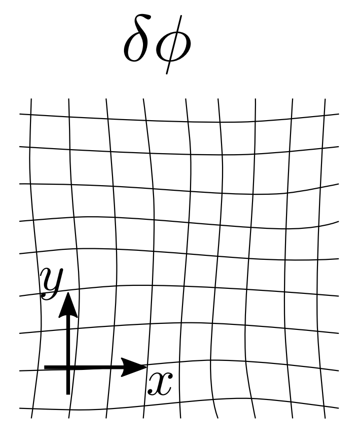

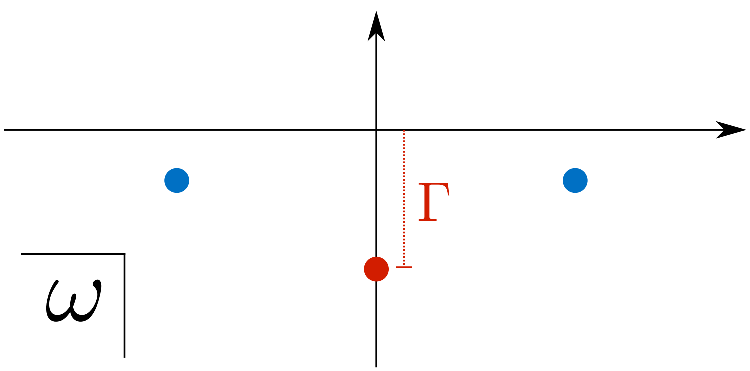

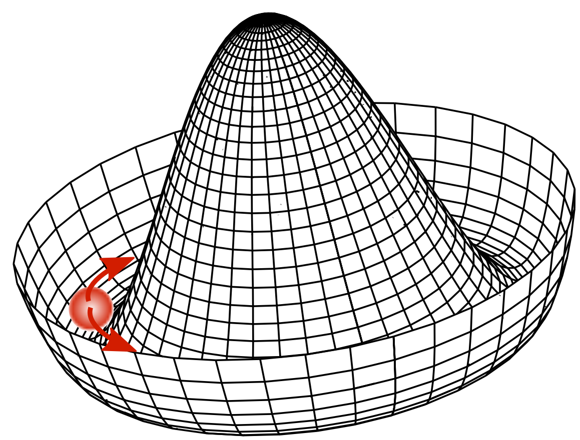

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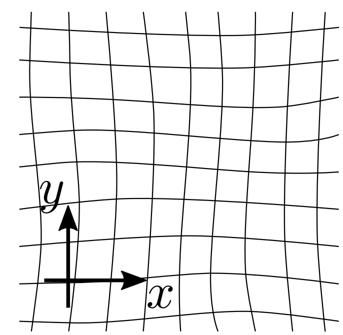
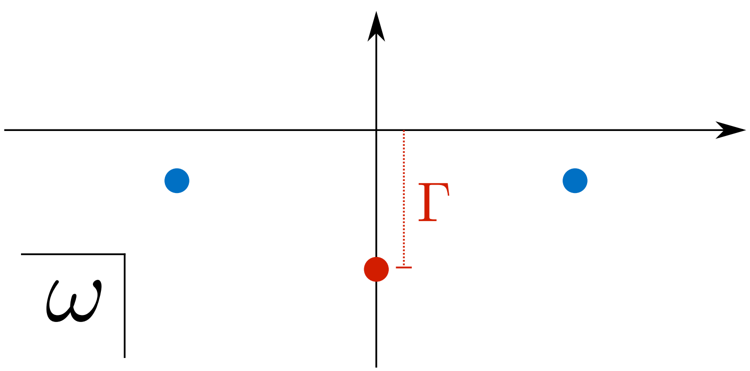
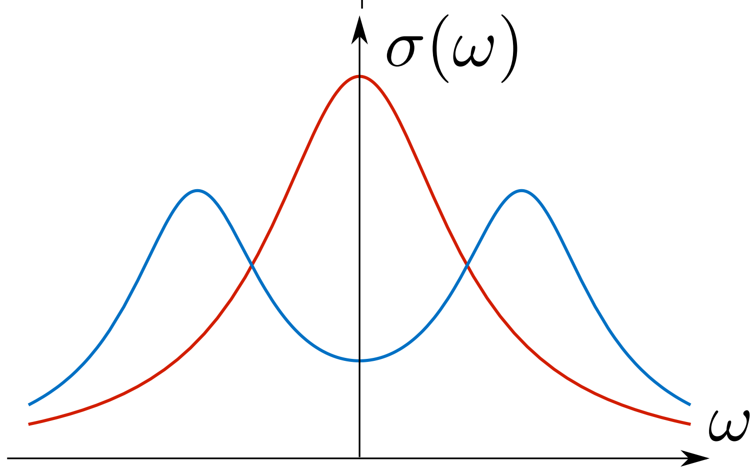




$$\sigma(\omega) = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{(\Omega - i\omega)}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$



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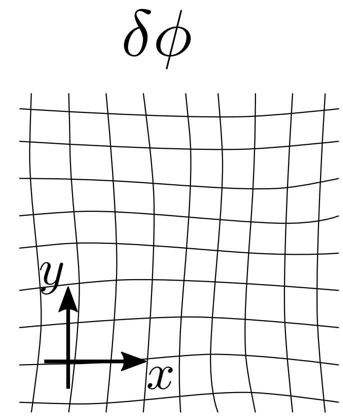
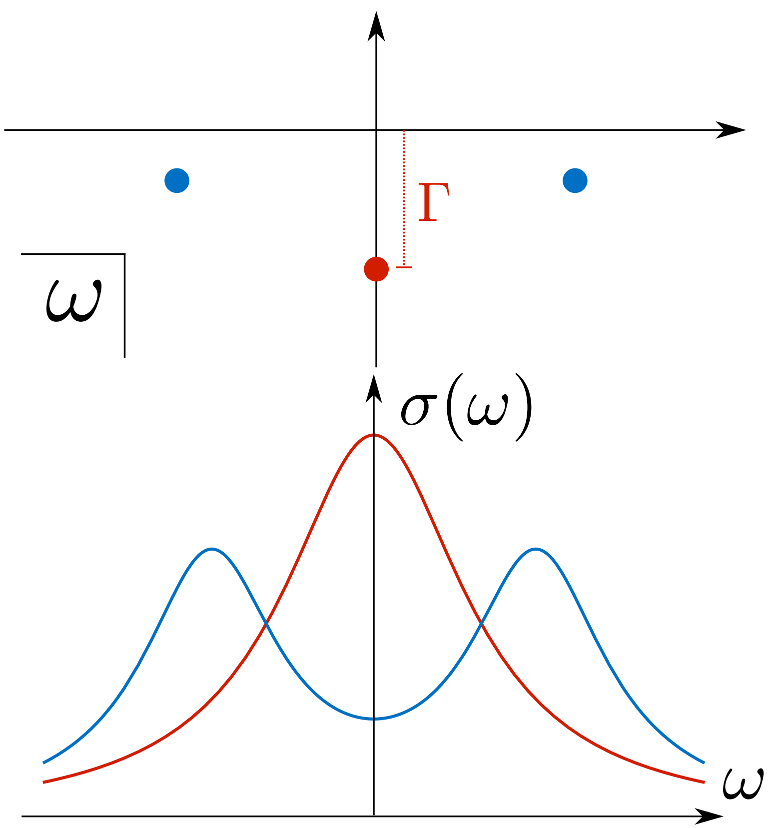
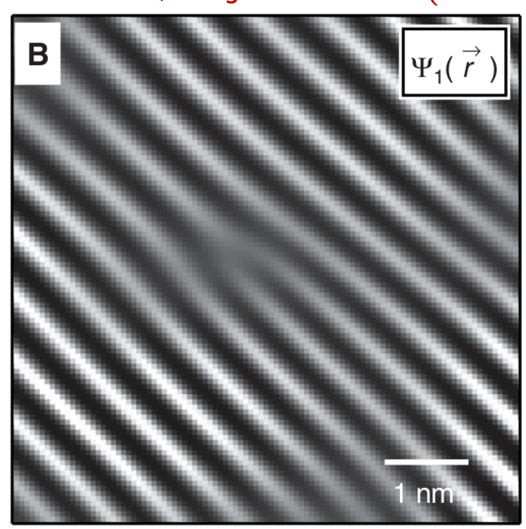




$$\sigma(\omega) = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{(\Omega - i\omega)}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

Mesaros, Fujita et. al. (2011)

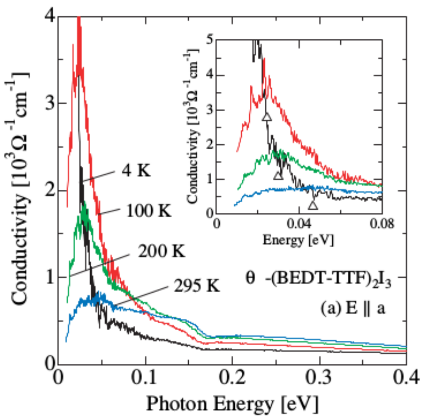


LVD Goutéraux Hartnoll Karlsson (2016)

Bad metals as density waves

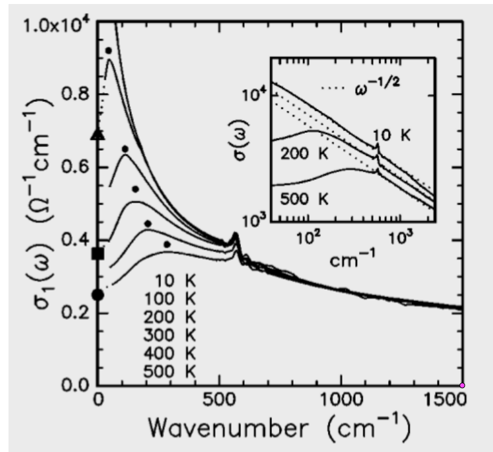
θ-(BEDT-TTF)$_2$I$_3$

Takenaka et al '05



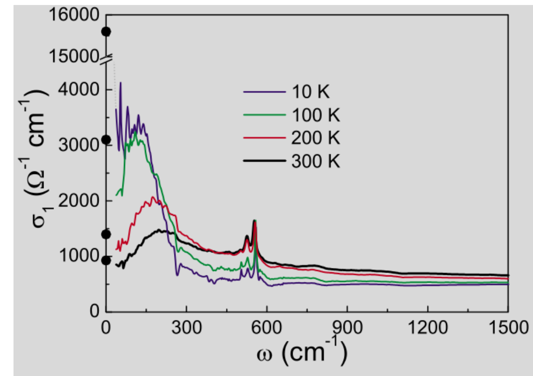
Ca$_2$RuO$_3$

Lee et al '02



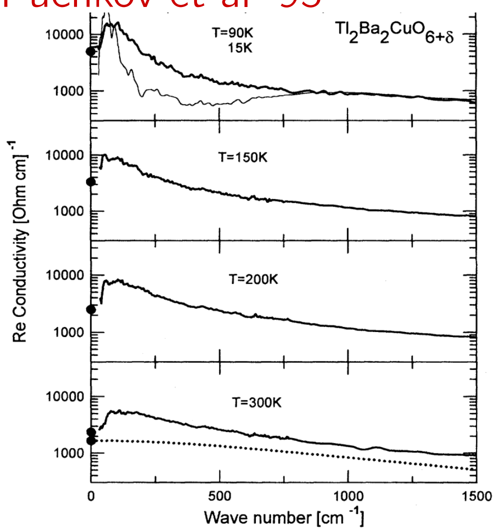
Na$_{0.7}$CoO$_2$

Wang et al '04



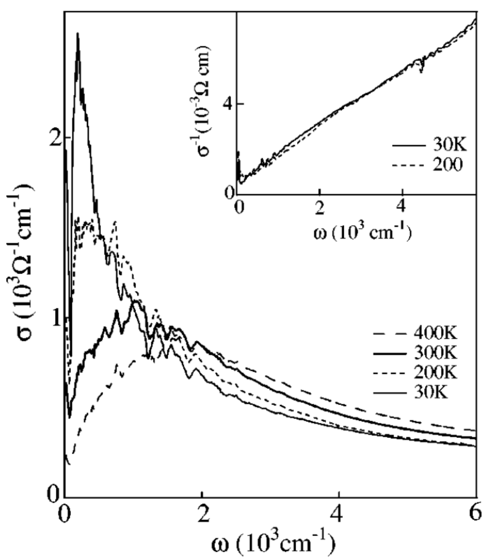
Ti$_2$Ba$_2$CuO$_{6+\delta}$

Puchkov et al '95



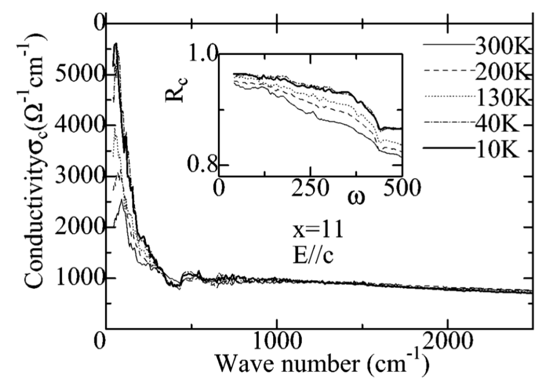
Bi$_2$Sr$_2$CuO$_6$

Lupi et al '00



Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$

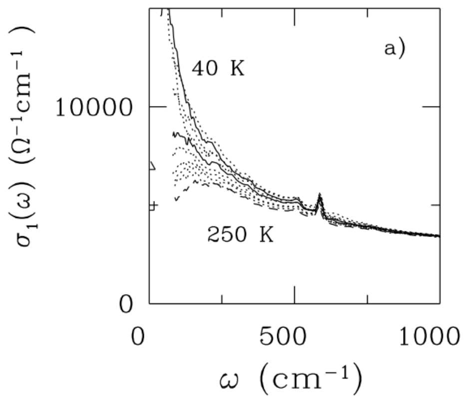
Osafune et al '99



Bad metals as density waves

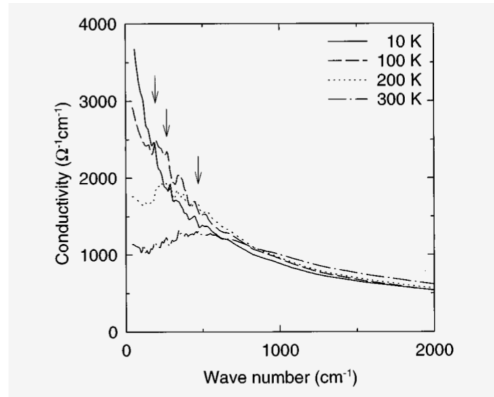
$SrRuO_3$

Kostic et al '98



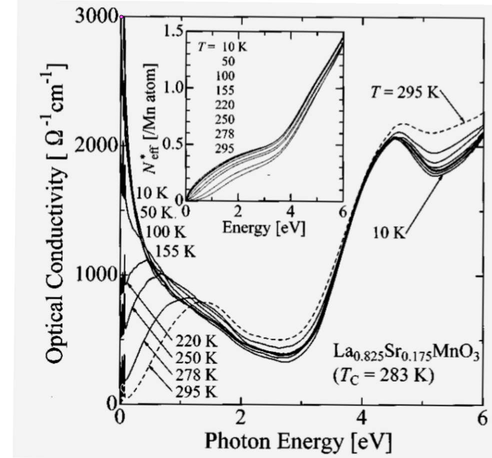
$Bi_2Sr_2CuO_6$

Tsvetkov et al '97



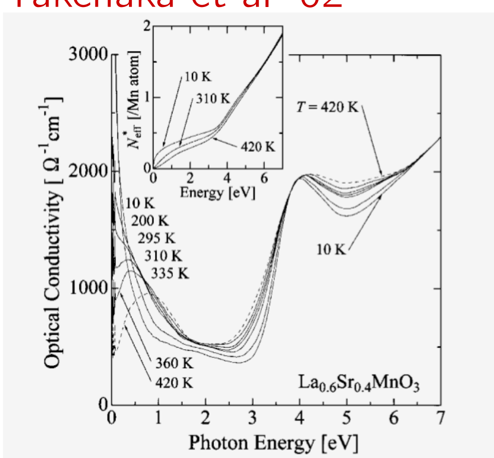
$La_{0.825}Sr_{0.175}MnO_3$

Takenaka et al '99



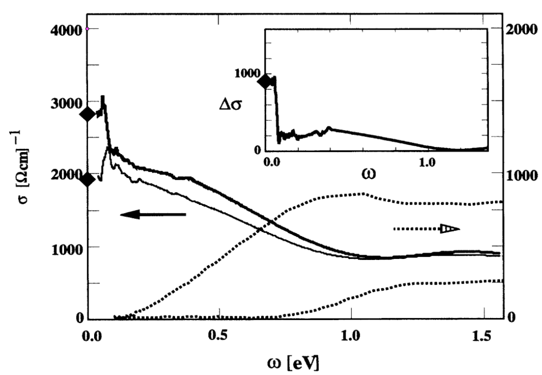
$La_{0.6}Sr_{0.4}MnO_3$

Takenaka et al '02



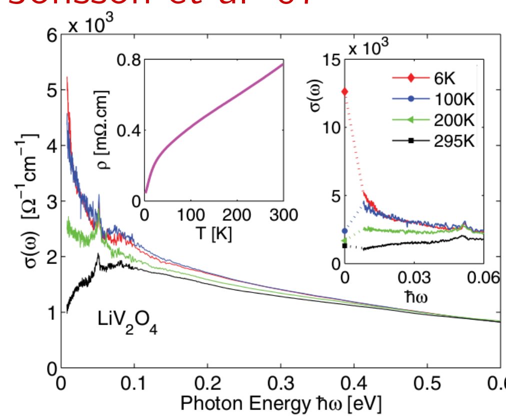
V_2O_3

Rozenberg et al '95



LiV_2O_4

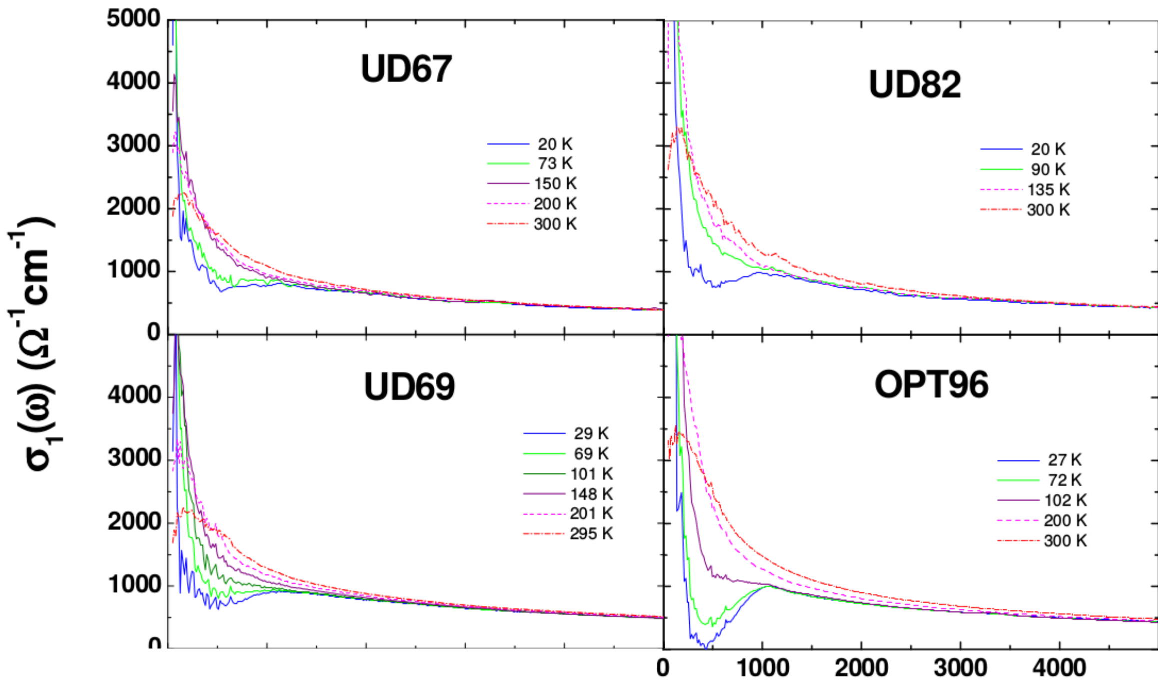
Jönsson et al '07



BAD METALS AS DENSITY WAVES

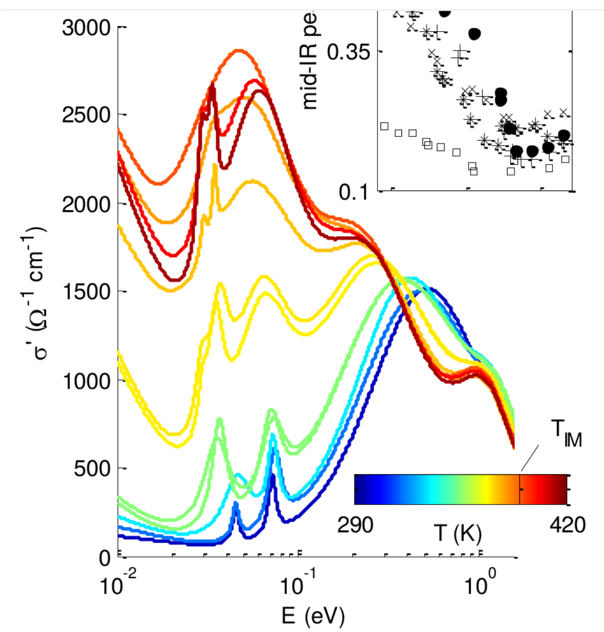
$Bi_2Sr_2CaCu_2O_{8+\delta}$

Hwang et al '07



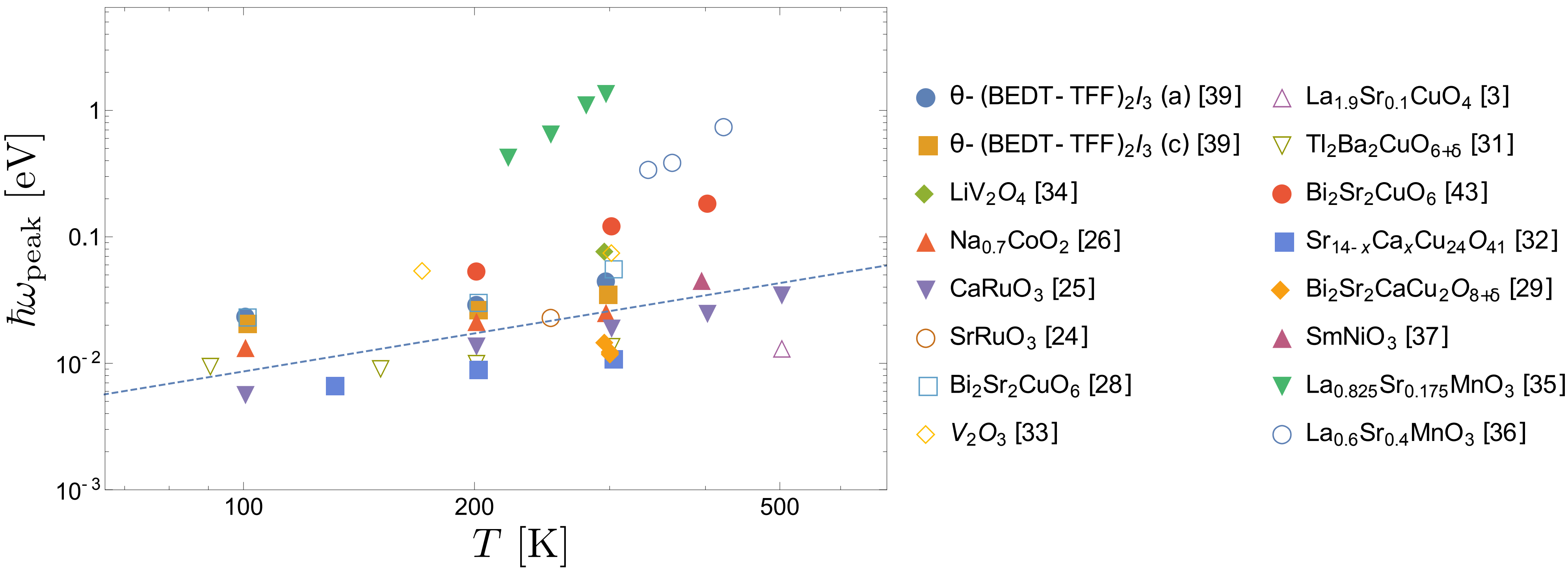
$SmNiO_3$

Jaramillo et al '14



$$\hbar\omega_{\text{peak}} \simeq \alpha \cdot k_B T\,, \qquad \hbar\Delta\omega \simeq \beta \cdot k_B T\,,$$

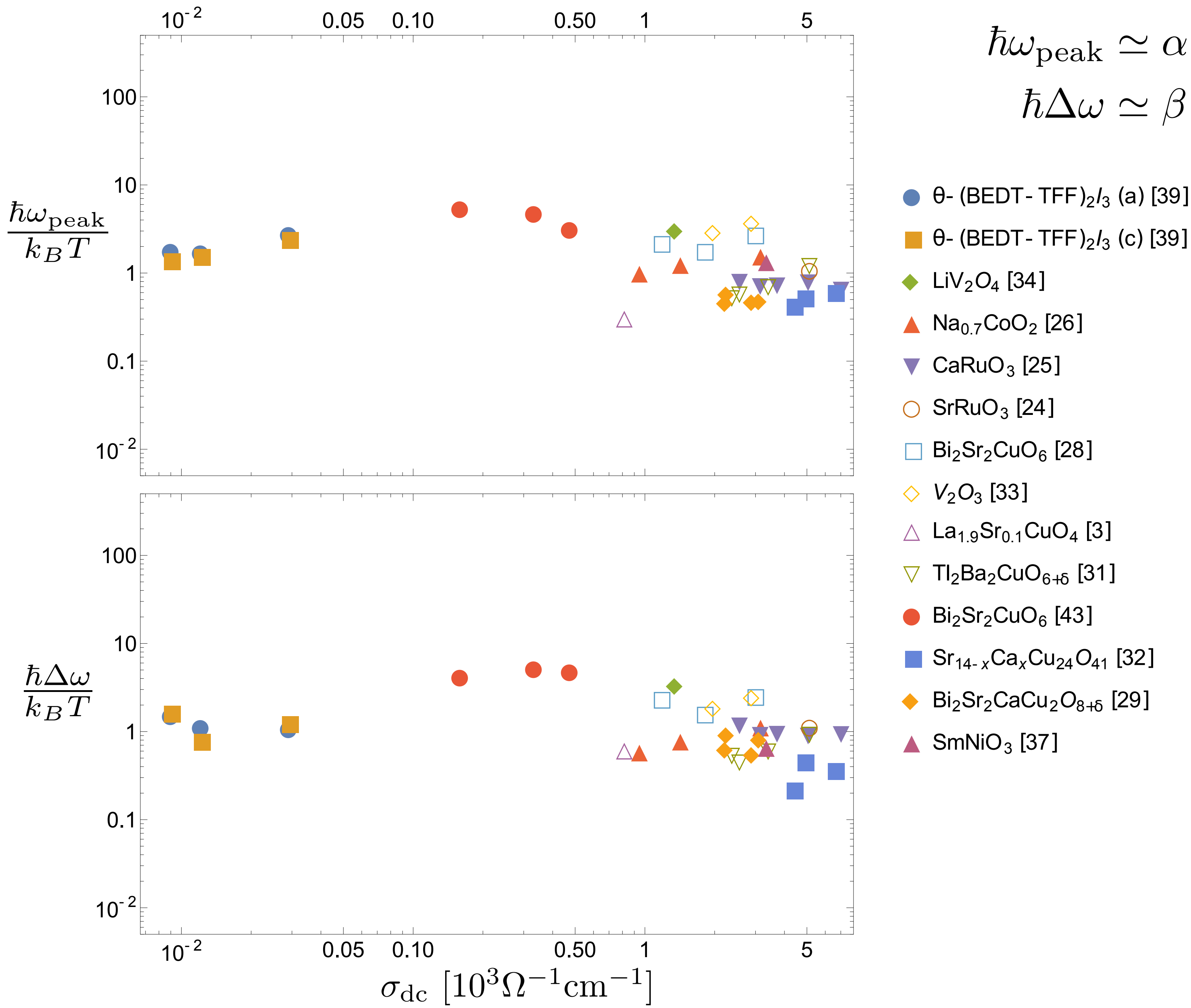
with $\alpha, \beta \sim 1$ and only mildly temperature dependent.



(dashed line is $\hbar\omega = k_B T$)

$$\hbar\omega_{\text{peak}} \simeq \alpha \cdot k_B T$$

$$\hbar\Delta\omega \simeq \beta \cdot k_B T$$



Linear in T resistivity

Hydrodynamics with translational order

$$\sigma(\omega) = \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2} + \ldots$$

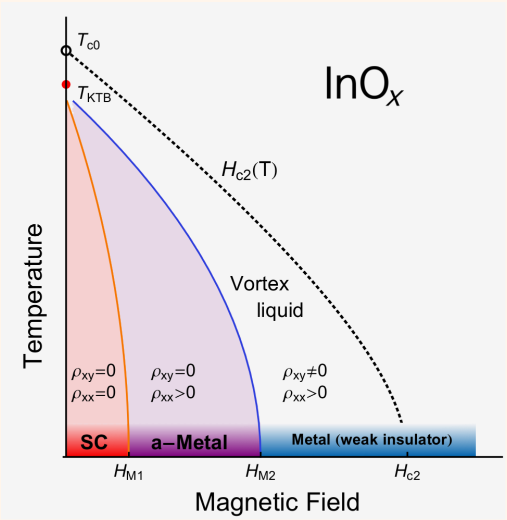
Data on bad metals suggests $\omega_o \sim \Gamma \sim \Omega \sim T$.

This in turn implies

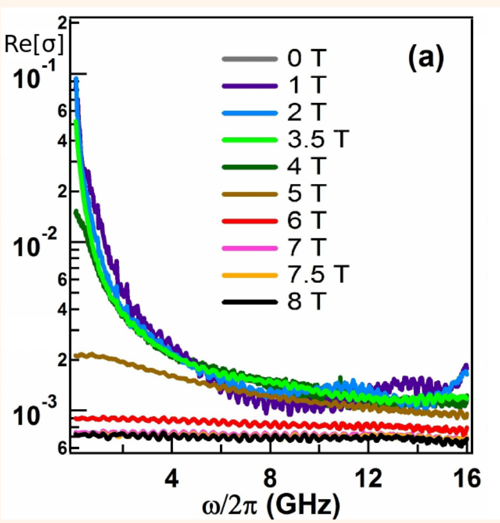
$$\sigma_{\mathrm{dc}} \simeq \frac{\rho^2}{\chi_{PP}} \frac{\Omega}{\Omega\Gamma + \omega_o^2} \sim \frac{\rho^2}{\chi_{PP}} \frac{1}{T} \, .$$

$\Rightarrow$ linear in T resistivity!

Fluctuating SC



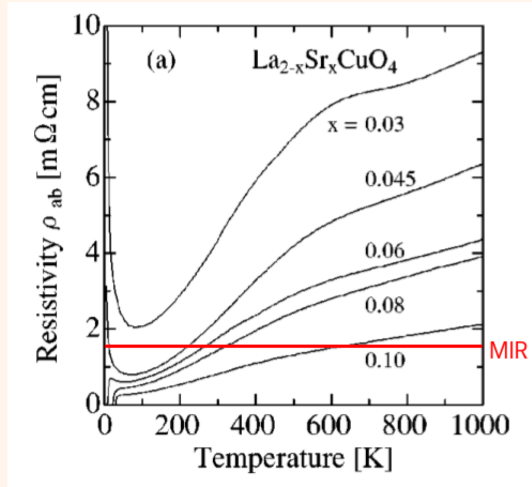
Breznay Kapitulnik (2017)



Liu Armitage et al. (2013)

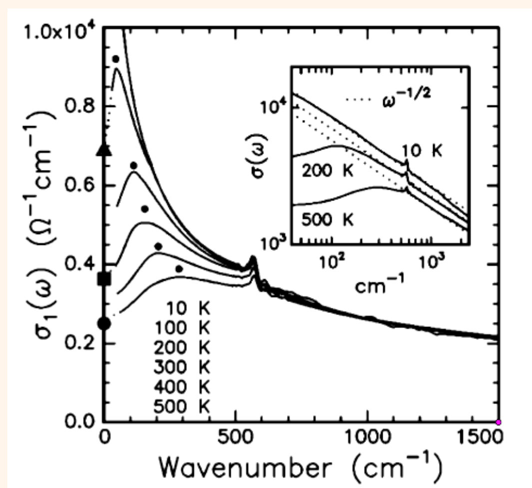
Wang Armitage et al. (2017)

Bad Metals



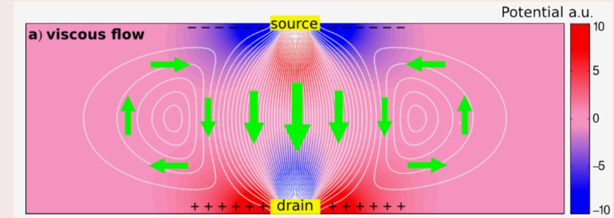
Sun et al. (2001), ...

Peaks in $\sigma(\omega)$



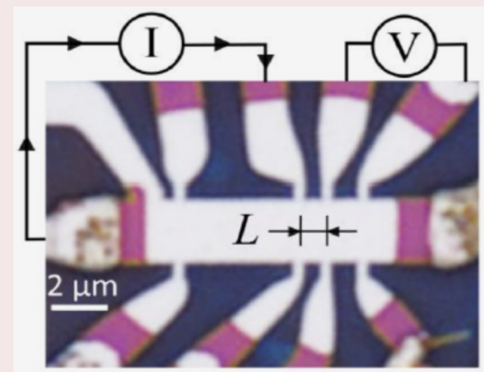
Lee et al. (2002), ...

Non-local Resistance

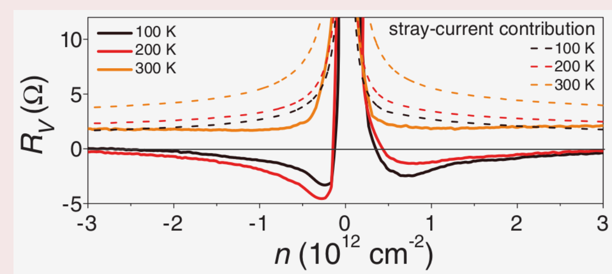


Levitov Falkovich (2015)

$$\sigma(k) = \ldots + \eta k^2 + \ldots$$



Bandurin Polini et al. (2015)

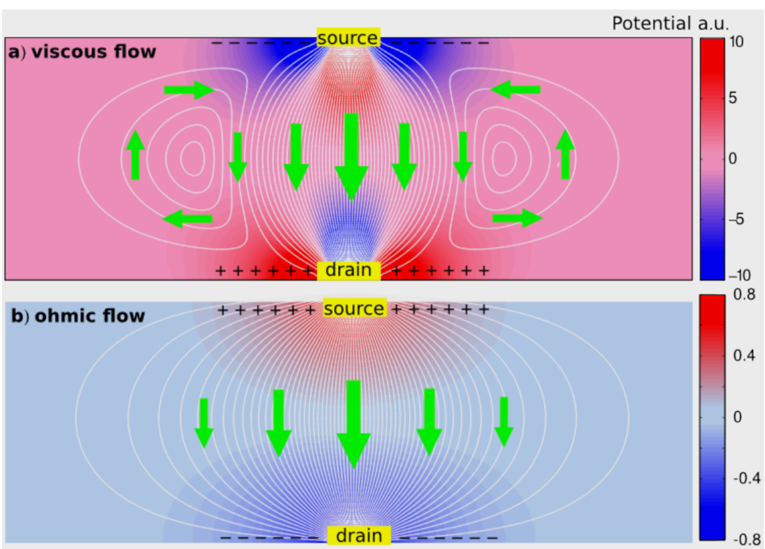


$\rightsquigarrow \eta_H$ in FQH?

Viscous transport

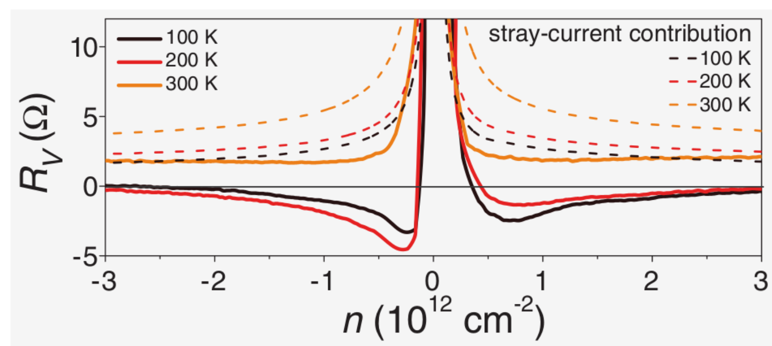
Finite wavelength dc transport $\sigma_{ij}(k)$

$$\sigma_{ij}(\omega) = \sum_{A,B} \chi_{J_i A} \left[\frac{1}{-i\omega\chi + M(\omega)} \right]_{AB} \chi_{B J_j} \qquad \Rightarrow \qquad \rho_{ij}^{\mathrm{dc}} \simeq \frac{1}{n^2} M_{P_i P_j} \, .$$



Levitov Falkovich (2015)

Torre Polini et al. (2015)



Graphene Bandurin Polini et al. (2015)
$PdCo_2$ Moll Mackenzie et al. (2015)
WP2 Gooth Gotsmann el al. (2017)

Viscous transport

Finite wavelength dc transport $\sigma_{ij}(k)$

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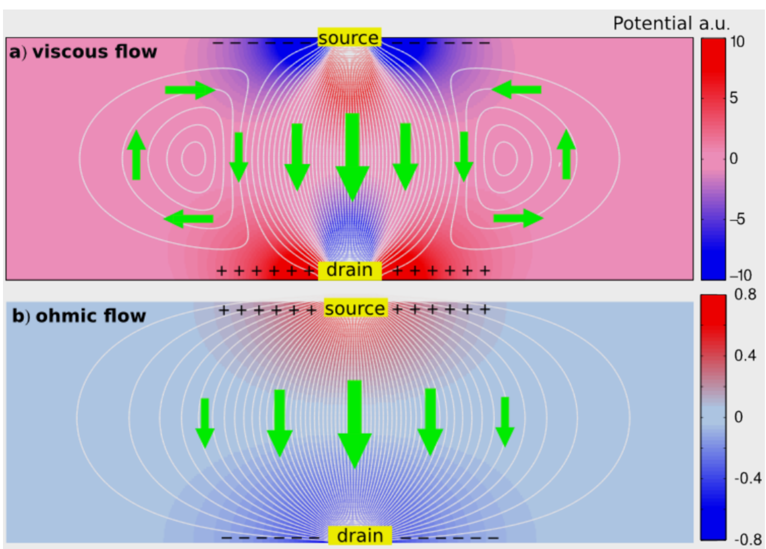
At finite wavelength $\dot{P}^i = -\nabla_j T^{ij} + \ldots$, so

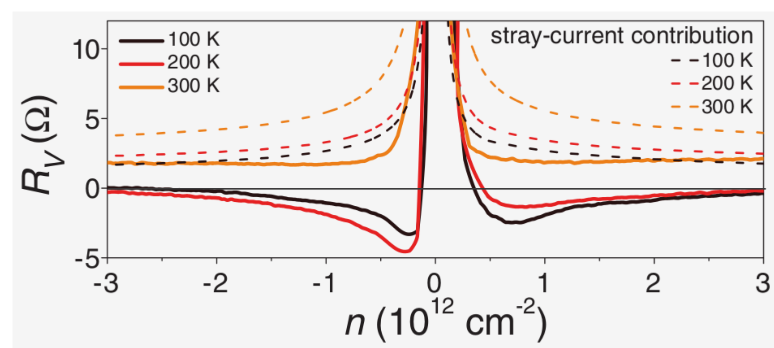
$$M_{P_i P_j} = k^a k^b \lim_{\omega \to 0} \frac{\text{Im}\, G^R_{T_{ia} T_{jb}}(\omega)}{\omega} = \eta_{iajb} k^a k^b$$

With momentum relaxation: $\rho^{\text{dc}}_{ij} = \frac{\Gamma m_\star}{n} \delta_{ij} + \frac{\eta_{iajb}}{n^2} k^a k^b + O(k^4)$



Levitov Falkovich (2015)

Torre Polini et al. (2015)



Graphene Bandurin Polini et al. (2015)
$PdCo_2$ Moll Mackenzie et al. (2015)
WP2 Gooth Gotsmann el al. (2017)

Hall viscosity

In the presence of a magnetic field $\dot{P}^i - \epsilon^{ij} B J_j = -\nabla_j T^{ij} + \ldots$ one finds LVD Gromov (2017)

$$\rho_{ij}^{\mathrm{dc}} = \frac{1}{n^2} \left(\epsilon_{ij} n B + \chi_\Omega k_i (\epsilon k)_j + n m_\star \Gamma \delta_{ij} + \eta_{iajb} k^a k^b + \ldots \right)$$

Hall viscosity $\eta_{ijkl} = \eta_H \delta_{ik} \epsilon_{jl}$ can characterize QH states, e.g. $\nu = 5/2$ distinguishes candidate states: Moore-Read, anti-Pfaffian, ...

QH: No finite size corrections to the boundary potential Niu Thouless (1987) $\to$ fixes no stress boundary conditions