Emergent Dirac fermions and broken symmetries in confined and deconfined phases of Ising gauge theories

Snir Gazit, Mohit Randeria and Ashvin Vishwanath

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see also – Assaad, Grover, Phys. Rev. X 6, 041049





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Phase transitions in a model w/o symmetries

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Tupitsyn (2010)

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Phase transitions in a model w/o symmetries

Emergent Gauge theory Higgs (confinement) transition

Tupitsyn (2010)

Dimer covering on a triangular lattice



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The dimer hardcore constraint translates into an Ising "Gauss law"

$$\prod_{b \in r} \sigma_b^x = -1$$

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- Symmetry breaking and confinement coincide.

Ising Lattice Gauge Theory

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Ising Gauge Theory



 h_c

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$$\mathcal{H}_{\mathbb{Z}_2} = -J \sum_{\Box} \prod_{b \in \Box} \sigma_b^z - h \sum_b \sigma_b^x$$






$$\mathcal{H}_{f} = -t \sum_{b = \langle i,j \rangle} \sigma_{b}^{z} c_{i,\alpha}^{\dagger} c_{j,\alpha} + h.c$$
$$-\mu \sum_{i,\alpha} c_{i,\alpha}^{\dagger} c_{i,\alpha} \qquad \sigma_{b}^{z} t \left(\begin{array}{c} \mathbf{f}_{i,\alpha} \mathbf{f}_{i,\alpha}$$





Ising "Gauss law"





<u>Key observation</u> – for real gauge theories the fermion determinant is also real

 $\det(\uparrow) \det(\downarrow) = \det(\uparrow)^2 > 0$

















Affleck, Marston (1988) Arovas, Auerbach (1988) Leib (1994)

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What happens at large hopping amplitude t?

Find
$$\{\sigma_b^z\}$$
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Two Dirac nodes

$$E_k = \pm 2t\sqrt{\cos(k_x)^2 + \cos(k_y)^2}$$



Phase Diagram at $\mu = 0$





Frozen gauge field limit $h \rightarrow 0$

 $\mathcal{H} = -t \sum \sigma_b^z c_{i,\alpha}^\dagger c_{j,\alpha} - J \sum \prod \sigma_b^z$ $\Box b \in \Box$ $b = \langle i, j \rangle$



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 $t \ll J$ $% T_{a} = 1$ $% T_{a} = 1$ $T_{a} = 1$ T_{a



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- $t \ll J$ Zero flux phase
 - $\sigma_b^z = 1$ Large FS
- $t \gg J$ π flux phase

Dirac Nodes



$$\mathcal{H} = -t \sum_{b = \langle i,j \rangle} \sigma_b^z c_{i,\alpha}^{\dagger} c_{j,\alpha} - J \sum_{\Box} \prod_{b \in \Box} \sigma_b^z$$



Phase transition at t/J = 6.77







Weak coupling $h \ll J$











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Orbital magnetic susceptibility

$$\chi(q) = \frac{\partial M(q)}{\partial B(q)} = -\frac{v_F}{4q}$$



Koshino, Arimura, Ando (2009)

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$$\frac{\tilde{\chi}}{L} = -\frac{4\pi}{Lt}\chi(q = 2\pi/L) \to \frac{v_F}{2t} \to 1$$





Koshino, Arimura, Ando (2009)




























Peak at $t_c = 1.8(1)$



SU(2) Pseudo-spin symmetry breaking





Strong coupling

Strong coupling
$$h \gg J$$

SU(2) Pseudo-spin symmetry breaking
SU(2) order parameter

0.02

1.0

1.5

1

2.0

2.5

h/.J

0.00







Same critical coupling!

SU(2) symmetry breaking and confinement coincide.

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Without fine tuning:

- 1. 1st order phase transition
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new universality class?

Pairing susceptibility

$$\tilde{P}_{\rm CDW} = L^{1-\eta} P_{\rm CDW}(\delta h L^{1/\nu})$$

$$h_c = 0.71(2), \nu = 0.58(5), \eta = 1.0(3)$$



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Obeys hyper-scaling $2-\alpha = d\nu$









Field theory description of the confinement transition. How to force confinement and symmetry breaking coincide?



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- Can we introduce "physical", i.e gauge neutral, fermions to realize Fermi liquids?
- Other Gauge groups?



Sign problem free lattice gauge theory at

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BCS* to BEC transition driven by confinement



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- Emergent Dirac fermion at zero chemical

potential and large hopping amplitude





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- BCS* to BEC transition driven by confinement
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- New universality class where symmetry

breaking and confinement coincide



Deconfined / Dirac

t/J



 $\mu = 0$

Confined / BEC+CDW

h/J

