A strongly correlated metal built from Sachdev-Ye-Kitaev models

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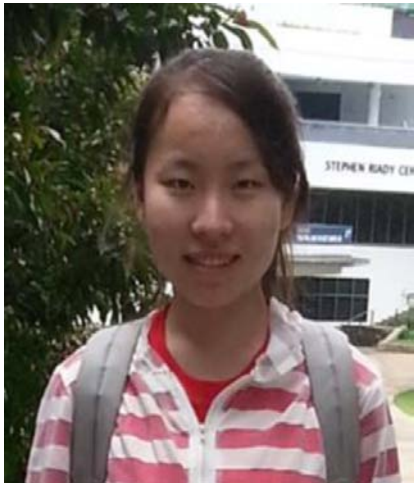
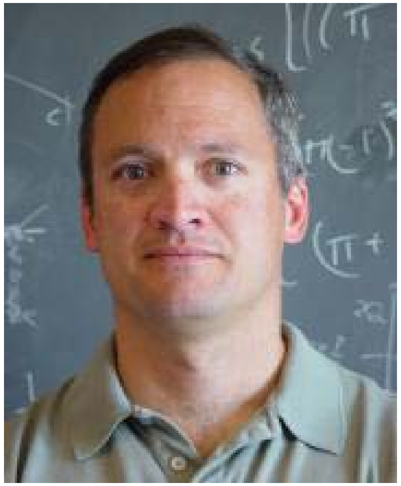
Acknowledgements

- Collaborators:



Xue-Yang Song (Harvard)



Leon Balents (KITP)

- Reference:

X.-Y. Song, CMJ, and L. Balents, arXiv: 1705.00117 (2017)

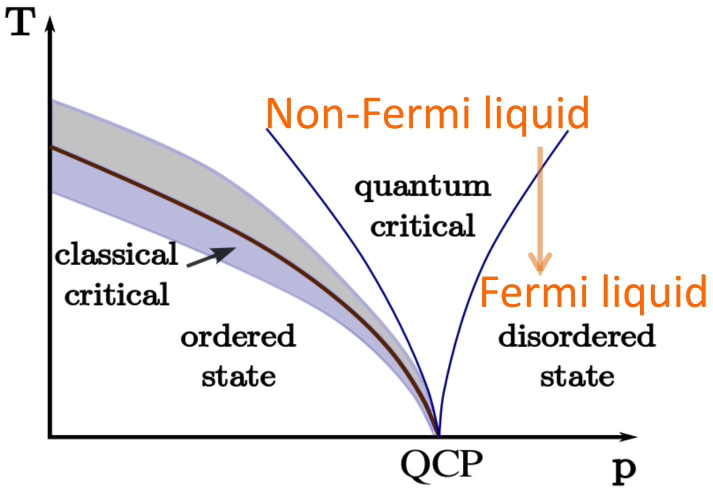
Introduction

Correlated metals:

- Commonly a highly renormalized heavy fermion liquid appears below a coherent scale. Above the coherent scale, the quasi-particle description fails.

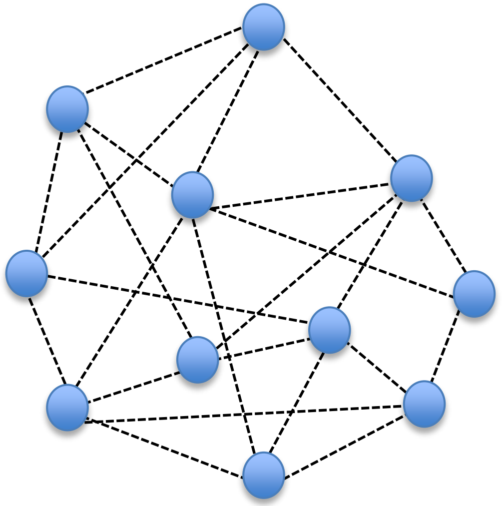
- Cuprates, heavy fermions,etc

- Fermi surface coupled to quantum critical point



Introduction

Sachdev-Ye-Kitaev (SYK) model:



- a “spherical-cow” exactly solvable correlated metal

- 0+1D quantum dot with q-fermion random all-to-all interaction among N fermion modes (**SYK$_q$ model**)

$$H_{SYK_2} = \sum_{ij} t_{ij} C_i^\dagger C_j \qquad H_{SYK_4} = \sum_{ijkl} U_{ijkl} C_i^\dagger C_j^\dagger C_k C_l$$

t_{ij} and U_{ijkl} are Gaussian random variables

Sachdev & Ye (1993); Sachdev (2015); Kitaev (2015); Maldacena & Stanford (2016)

Introduction

$\mathbf{SYK_2}$ Model: $$H_{SYK_2} = \sum_{ij} t_{ij} C_i^\dagger C_j$$

Fermi liquid: finite d.o.s. at Fermi level, linear-T heat capacity at low T…

$\mathbf{SYK_4}$ Model: $$H_{SYK_4} = \sum_{ijkl} U_{ijkl} C_i^\dagger C_j^\dagger C_k C_l$$

Exact solution at Large N solution: **Incoherent metal & $\mathbf{CFT_1}$**

Green's function: $\langle C_i(\tau) C_j^\dagger(0) \rangle \sim \delta_{ij} \tau^{-1/2}$ Self energy: $\Sigma(\omega) \sim \omega^{1/2}$

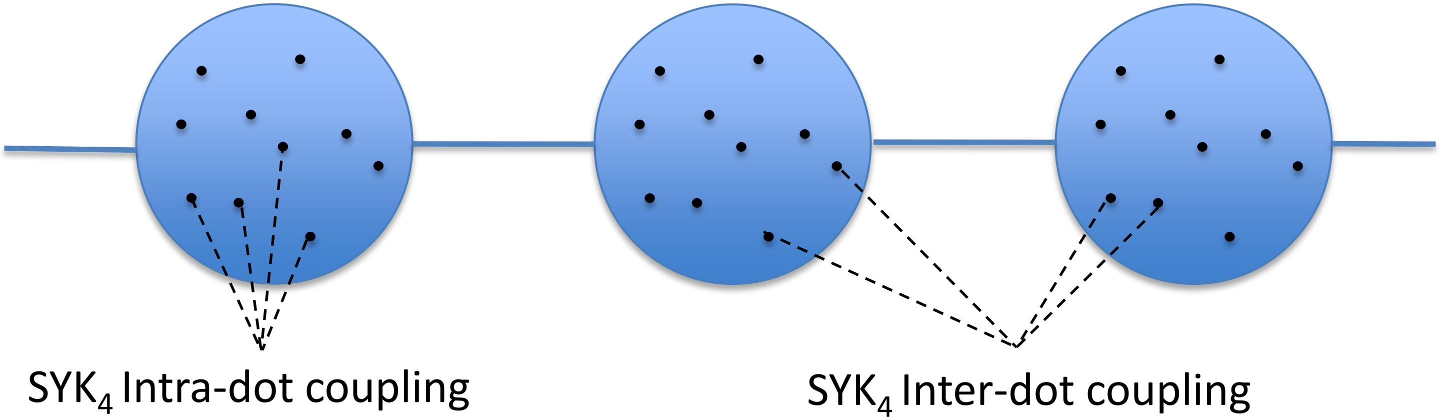
Finite zero-temperature entropy

Sachdev & Ye (1993); Sachdev (2015);
Kitaev (2015); Maldacena & Stanford (2016)

Introduction

Coupled SYK Clusters

Couple an array of SYK_4 quantum dots with SYK_4 inter-dot coupling



Incoherent metal & local quantum criticality

No quasi-particles; Transport given only by diffusive modes

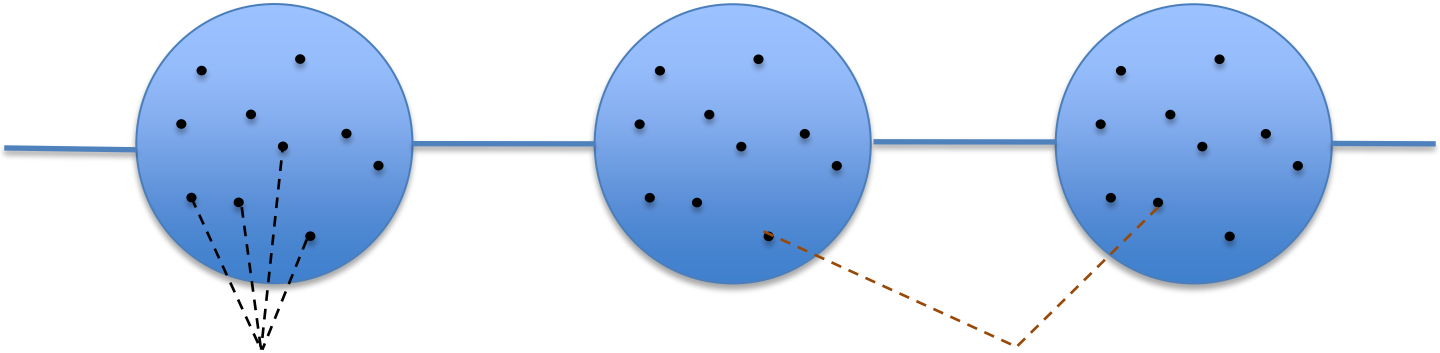
Conductivities almost temperature independent

Gu et al (2016)； Davison et al (2017)

Model

Our model

Couple a d-dimensional array of SYK_4 quantum dots with SYK_2 inter-dot couplings



$$H = \sum_x \sum_{jklm} U_{jklm,x} C_{jx}^\dagger C_{kx}^\dagger C_{lx} C_{mx} + \sum_{\langle xx' \rangle} \sum_{jk} t_{jk,xx'} C_{jx}^\dagger C_{kx'}$$

$t_{ij,xx'}$ and $U_{jklm,x}$ are Gaussian random variables with zero mean and variance

$$\overline{\left| U_{jklm,x} \right|^2} = 2U_0^2/N^3 \qquad \overline{\left| t_{jk,xx'} \right|^2} = t_0^2/N$$

Regime of interest: $T, t_0 << U_0$ & half filling ($\mu = 0$)

Imaginary time formalism

Hamiltonian：

$$H=\sum_{x}\sum_{jklm}U_{jklm,x}C^{\dagger}_{jx}C^{\dagger}_{kx}C_{lx}C_{mx}+\sum_{\langle xx'\rangle}\sum_{jk}t_{jk,xx'}C^{\dagger}_{jx}C_{kx'}$$

Disorder-averaged imaginary-time action：

$$S_c=\sum_{x}\int_0^{\beta}d\tau C^{\dagger}_{ix\tau}\partial_{\tau}C_{ix\tau}$$
$$+\int_0^{\beta}d\tau_1 d\tau_2\left[\frac{U_0^2}{4N^3}\sum_{x}C^{\dagger}_{ix\tau_1}C^{\dagger}_{jx\tau_1}C_{kx\tau_1}C_{lx\tau_1}C^{\dagger}_{ix\tau_2}C^{\dagger}_{jx\tau_2}C_{kx\tau_2}C_{lx\tau_2}+\frac{t_0^2}{N}\sum_{\langle xx'\rangle}C^{\dagger}_{ix\tau_1}C_{jx'\tau_1}C^{\dagger}_{ix\tau_2}C_{jx'\tau_2}\right]$$

Scaling analysis:

Start at t_0=0 with local criticality $[C]=1/4$

"U_0-term" marginal; "∂_τ-term" irrelevant; "t_0-term" relevant

Crossover at coherence scales $E_c=t_0^2/U_0$ to a SYK_2 Fermi liquid

Imaginary time formalism

Disorder average imaginary-time action：

$$S_c = \sum_x \int_0^\beta d\tau C^\dagger_{ix\tau} \partial_\tau C_{ix\tau}$$

$$+ \int_0^\beta d\tau_1 d\tau_2 \left[\frac{U_0^2}{4N^3} \sum_x C^\dagger_{ix\tau_1} C^\dagger_{jx\tau_1} C_{kx\tau_1} C_{lx\tau_1} C^\dagger_{ix\tau_2} C^\dagger_{jx\tau_2} C_{kx\tau_2} C_{lx\tau_2} + \frac{t_0^2}{N} \sum_{\langle xx' \rangle} C^\dagger_{ix\tau_1} C_{jx'\tau_1} C^\dagger_{ix\tau_2} C_{jx'\tau_2} \right]$$

Introduce dynamical mean-field $G_x(\tau,\tau')$ via a multiplier $\Sigma_x(\tau,\tau')$:

$$\Sigma_x(\tau,\tau')\left(G_x(\tau',\tau) + \frac{1}{N} \sum_i C_{ix\tau} C^\dagger_{ix\tau'} \right)$$

Large-N exact saddle point: $G_x(\tau,\tau') = G(\tau - \tau')$ $\Sigma_x(\tau,\tau') = \Sigma(\tau - \tau')$

$$G(i\omega_n)^{-1} = i\omega_n - \Sigma(i\omega_n)$$

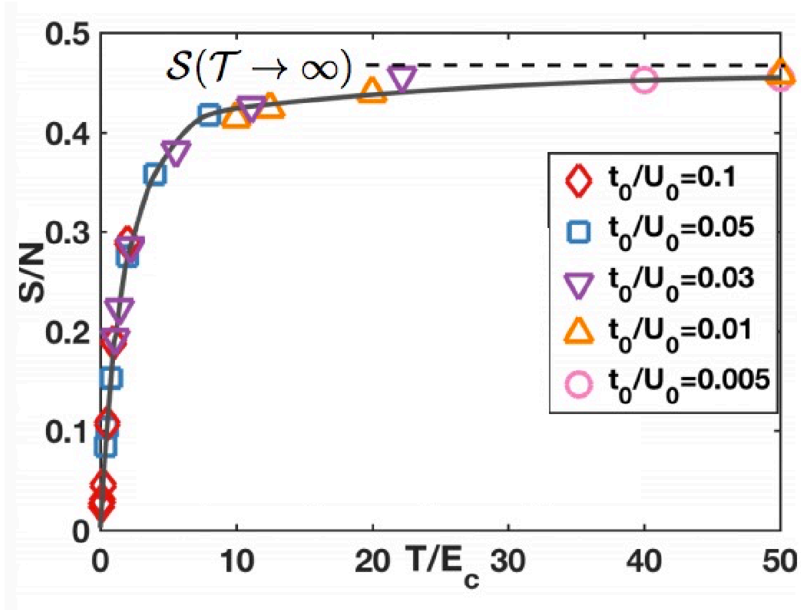
$$\Sigma(\tau) = \underline{-U_0^2 G(\tau)^2 G(-\tau)} - \underline{t_0^2 G(\tau)}$$

Thermodynamical Properties

Thermodynamics from saddle point solution:

Universal behavior of entropy:

$$S(T) = N\mathcal{S}(T/E_c)$$



Sommerfeld coefficient: $$\gamma \equiv \lim_{T\to 0}\frac{dS}{dT} = \frac{N}{E_c}\mathcal{S}'(0)$$

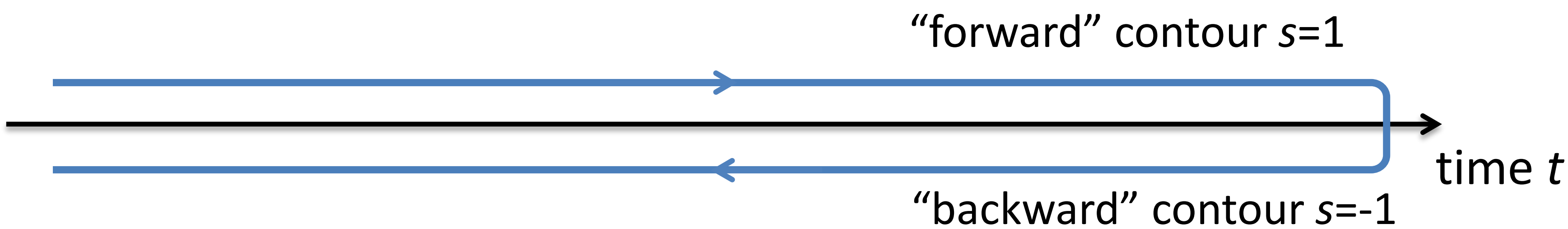
Charge compressibility: $$K \equiv \left.\frac{\partial Q}{\partial \mu}\right|_T \sim \frac{1}{U_0}$$

Heavy Fermi liquid at low T

Relation to Landau parameter: $$\gamma / K = \frac{\pi^2}{3}(1+F) \sim \left(\frac{U_0}{t_0}\right)^2 \gg 1$$

Real-time formalism

Keldysh formalism



After disorder average,we introduce dynamical mean-fields $G_{x,ss'}(t,t')$ multipliers $\Sigma_{x,ss'}(t,t')$,integrate out the fermions to obtain the effective action:

$$S_K/N = \sum_x \ln\det\left[\sigma^z i\partial_t \delta(t-t') - \Sigma(t,t')\right]$$

$$+\sum_{ss}\int_{t_i}^{t_f} dt\,dt'\left[\frac{U_0^2}{4}\sum_x ss' G_{x,ss'}(t,t')^2 G_{x,s's}(t',t)^2 - \sum_x \Sigma_{x,ss'}(t,t')G_{x,s's}(t',t) + t_0^2 \sum_{\langle xx'\rangle} ss' G_{x,ss'}(t,t')G_{x',s's}(t',t)\right]$$

Real-time formalism

Large-N exact solution of $G_{ss'}(t,t')$ and $\Sigma_{ss'}(t,t')$ can be obtained by numerically solving the saddle point equations

$G_{ss'}(t,t')$ can be written in terms of advanced, retarded and Keldysh Green's functions G_A, G_R and G_K, which are related to each other

Temperature dependence as expected from the scaling analysis

Real-time formalism

Fluctuation and Transport

Collective hydrodynamical modes capturing charge and energy fluctuations

For U(1) charge, we consider U(1) phase fluctuations around the saddle point $G_{x,ss'}(t,t') \to G_{x,ss'}(t-t') e^{-i(\varphi_s(x,t)-\varphi_{s'}(x,t'))}$

Effective hydro theory:

$$S_\varphi / N = -2K \sum_p \int d\omega \, \varphi_{c,\omega} \left(i\omega^2 - D_\varphi p^2 \omega \right) \varphi_{q,\omega} + \ldots \qquad \text{with} \quad \varphi_{c/q} = \varphi_+ \pm \varphi_-$$

$\varphi_{c/q}$ are conjugate to charge densities. *K* is the charge compressibility. D_φ is the charge diffusion constant

Einstein relation for electric conductivity: $\sigma = NKD_\varphi$

Real-time formalism

Fluctuation and Transport

For energy fluctuations, we consider the time-reparametrization:

$$t \to t + \varepsilon_{x,s}(t),$$

$$G_{x,ss'}(t,t') \to G_{x,ss'}\left(t + \varepsilon_s(t) - t' - \varepsilon_{s'}(t')\right),$$

Write the hydro theory in terms of $\varepsilon(t)$

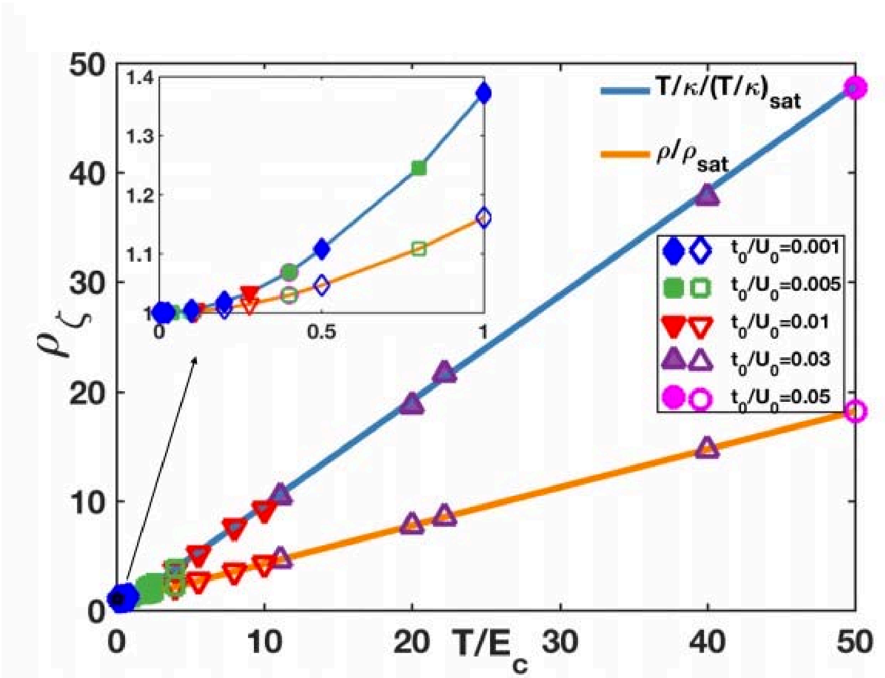
$$iS_\varphi / N = 2\gamma T^2 \sum_p \int d\omega\ \varepsilon_{c,\omega}\left(i\omega^2 - D_\varepsilon p^2 \omega\right)\varepsilon_{q,\omega} + \ldots$$

Obtain the thermal conductivity via the Einstein relation

$$\kappa / T = N\gamma D_\varepsilon$$

Real-time formalism

Electrical/thermal resistivity v.s. Temperature



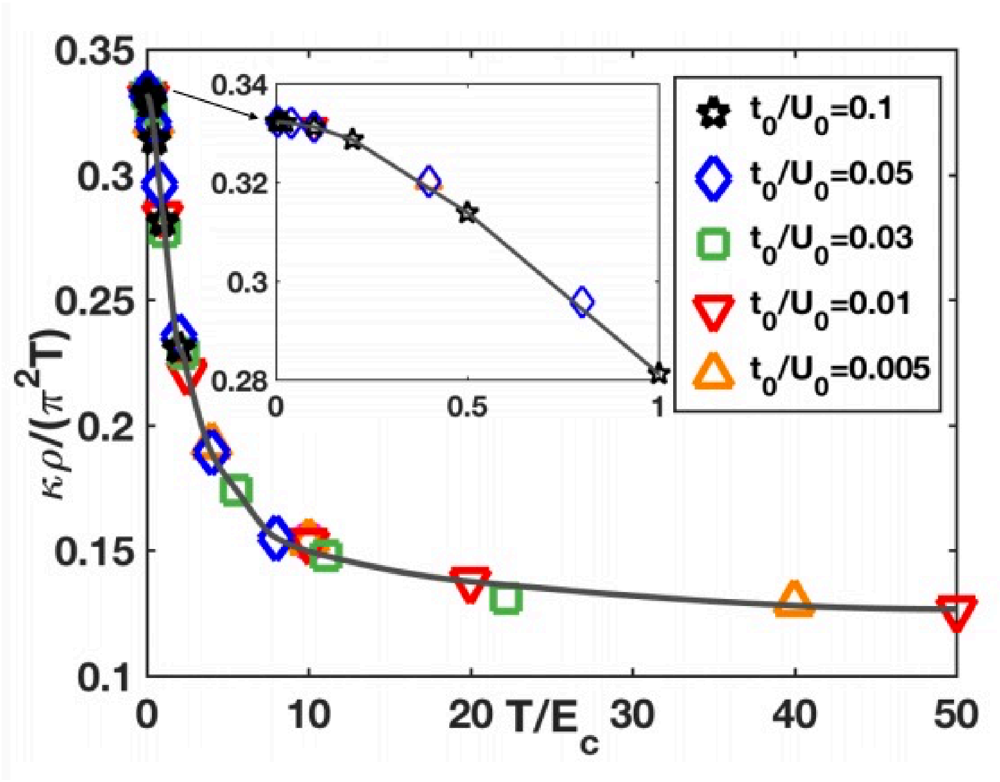
High temperature:Electrical/thermal resistivity ~ T

Low temperature:Electrical/thermal resistivity ~ constant + A T^2

Constant Kadowaki-Woods ratio: $A_{eletric}/\gamma^2$

Real-time formalism

Lorentz ratio $L = \kappa / (\sigma T)$ **v.s. Temperature**

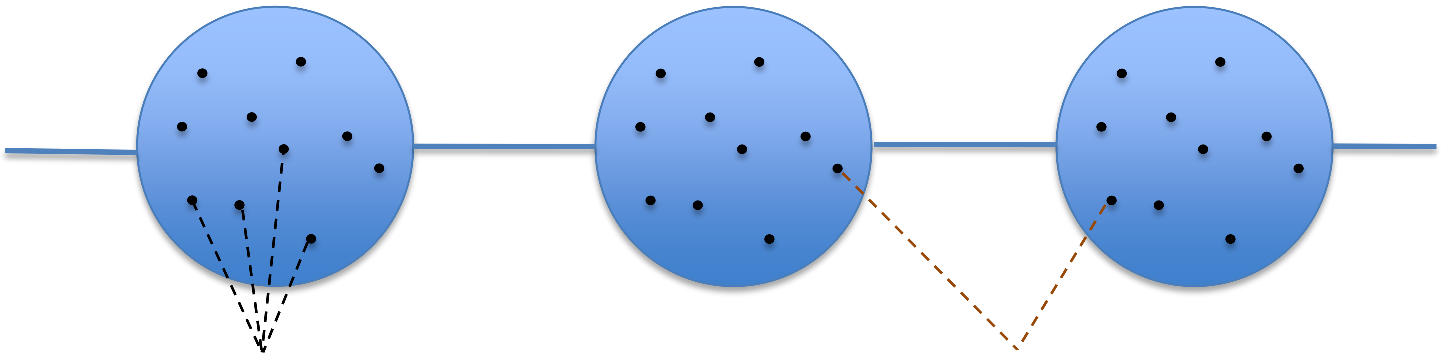


Low temperature: $L = \pi^2/3$, identical to Fermi liquid

High temperature: $L = \pi^2/8$

Summary

Charge conserved SYK_4 intra-dot interaction with quadratic hopping between dots



SYK_4 Intra-cluster coupling ***U***

SYK_2 Inter-cluster coupling ***t***

Heavy Fermi liquid for temperature *T* beneath the coherent scale E_c and incoherent metal for *T* above

Heavy Fermi liquid with large renormalized mass, T^2 low temperature (plus constant) resistivity,constant Kadowaki-Woods ratio, Lorentz ratio $L = \pi^2/3$

High temperature NFL with linear-T resistivity and Lorentz ratio $L = \pi^2/8$

Thank you for your attention