



Frustration from topology in distorted kagome antiferromagnets

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See arXiv:1705.00015

Acknowledgements



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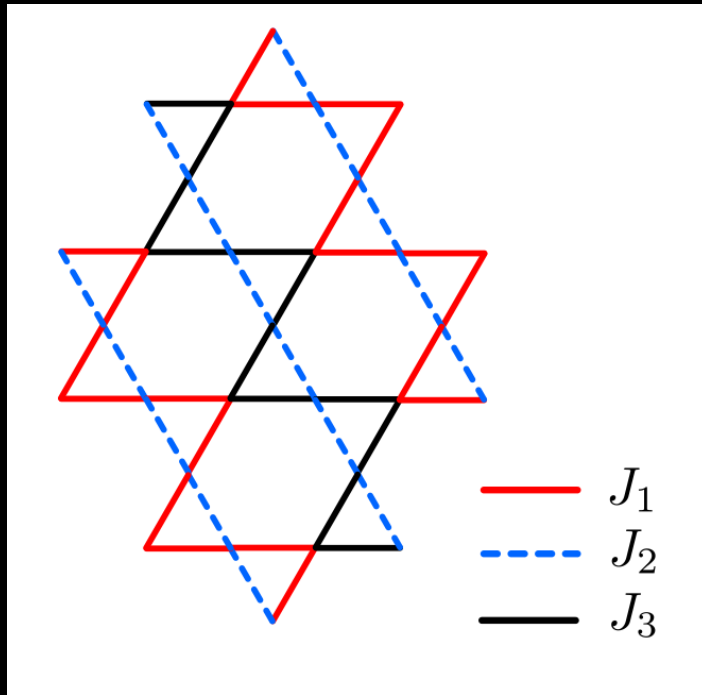


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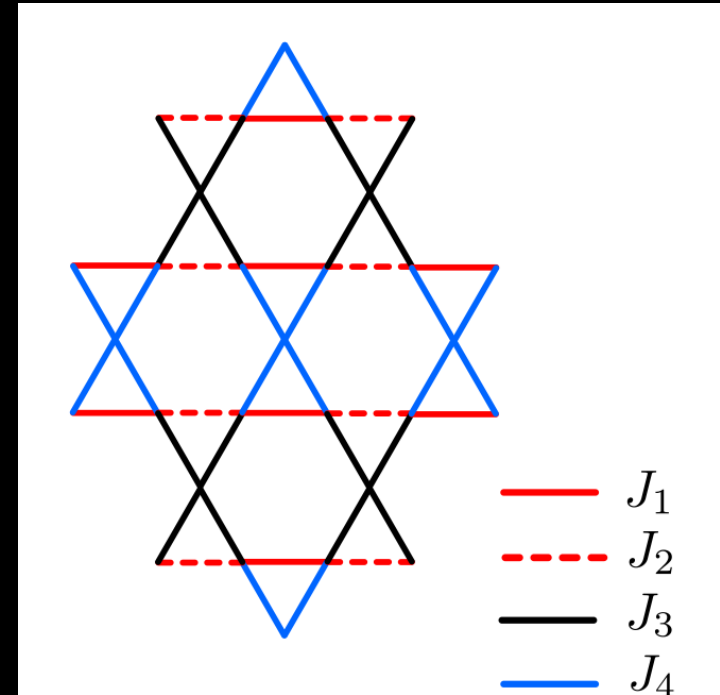
See [arXiv:1705.00015](https://arxiv.org/abs/1705.00015)

Distorted kagome antiferromagnets

See arXiv:1705.00015



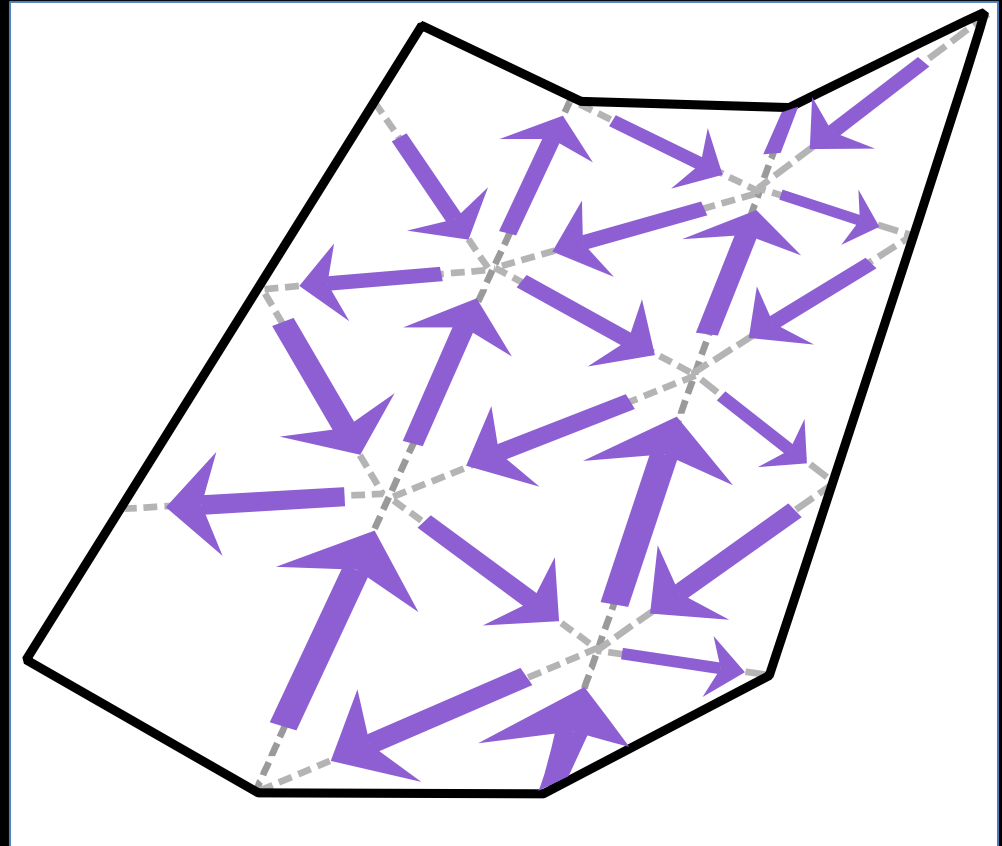
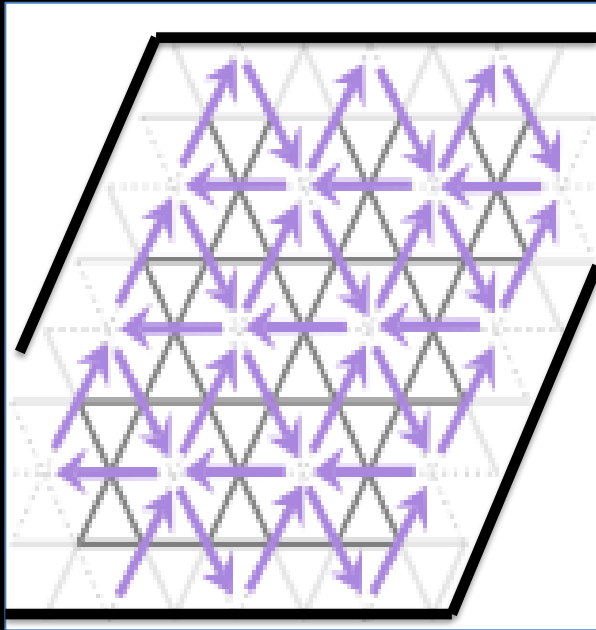
Ono et. al,
PRB 79, 174407 (2009)



Amemiya et. al.,
PRB 80, 100406R (2009)

Undistorted kagome: Spin origami

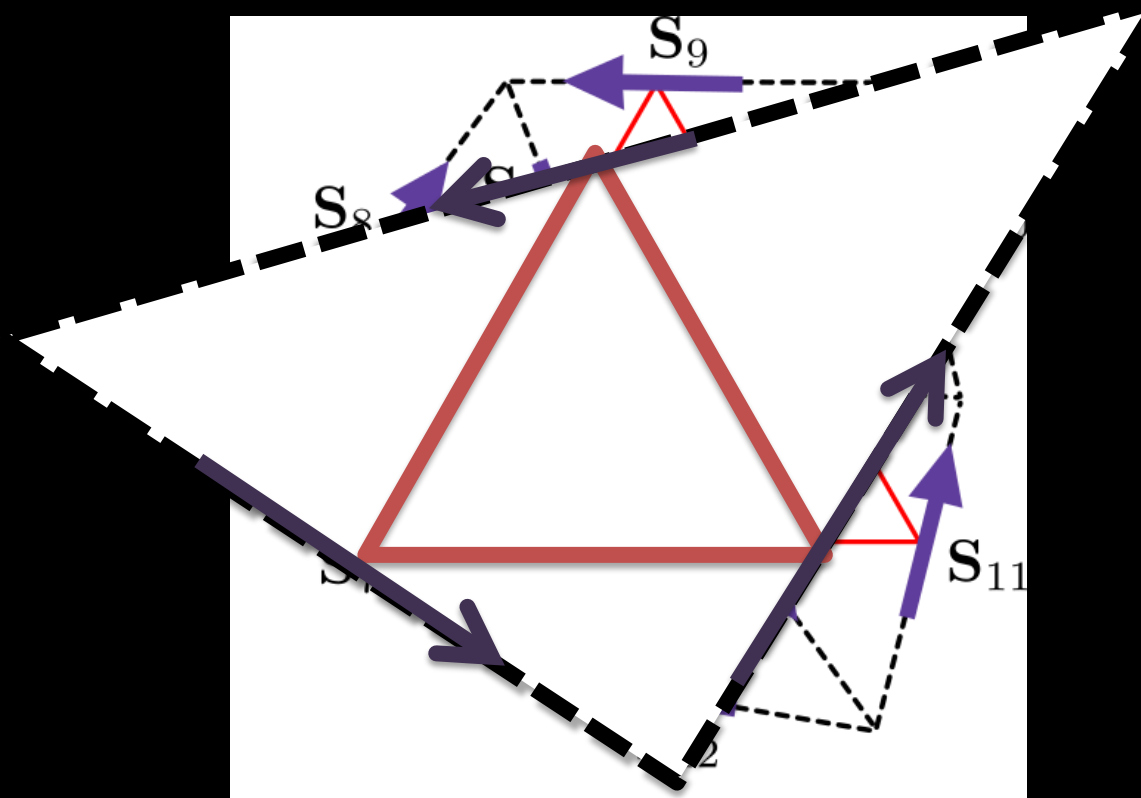
Shender et. al, 1993



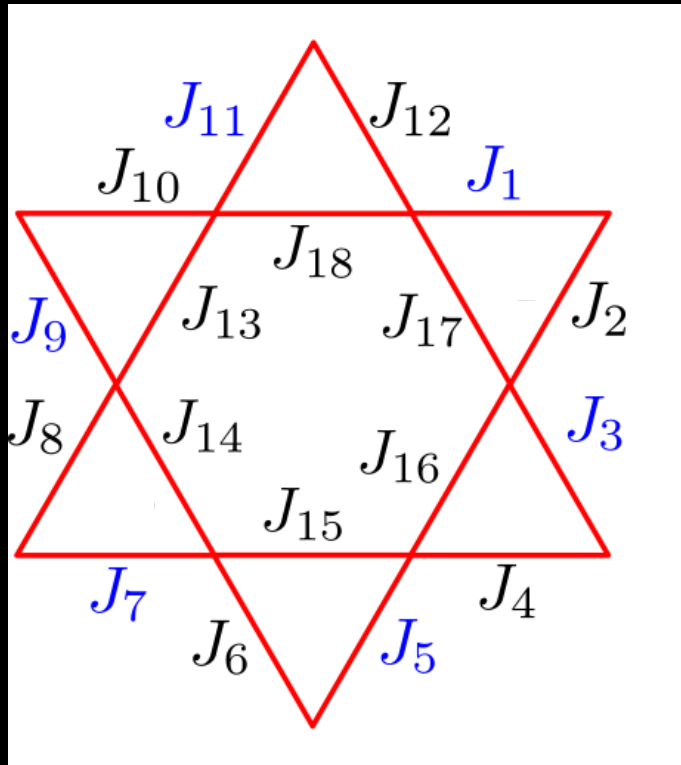
Ground states parameterized by an origami sheet!

Generalized spin origami

$$\vec{S}_{\Delta} \equiv a_i \vec{S}_i + a_j \vec{S}_j + a_k \vec{S}_k = 0 \quad \text{"triangle" constraint}$$

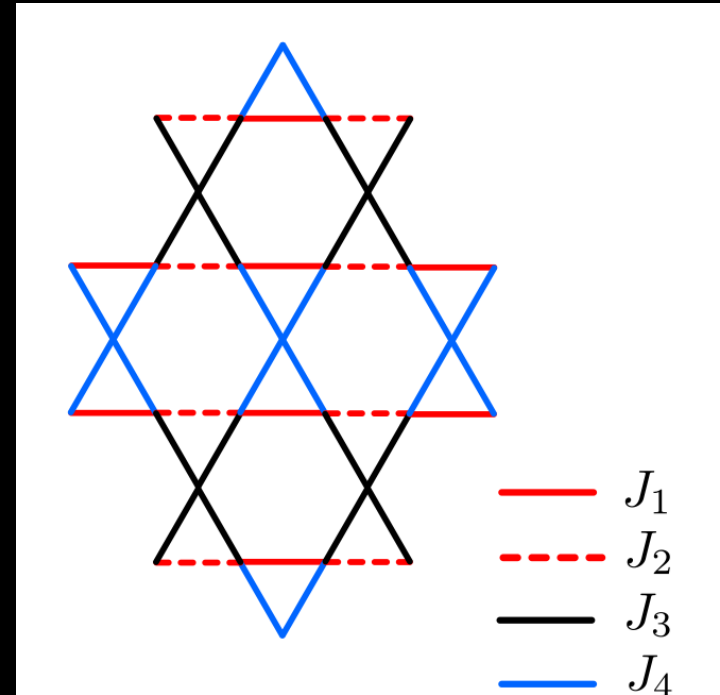
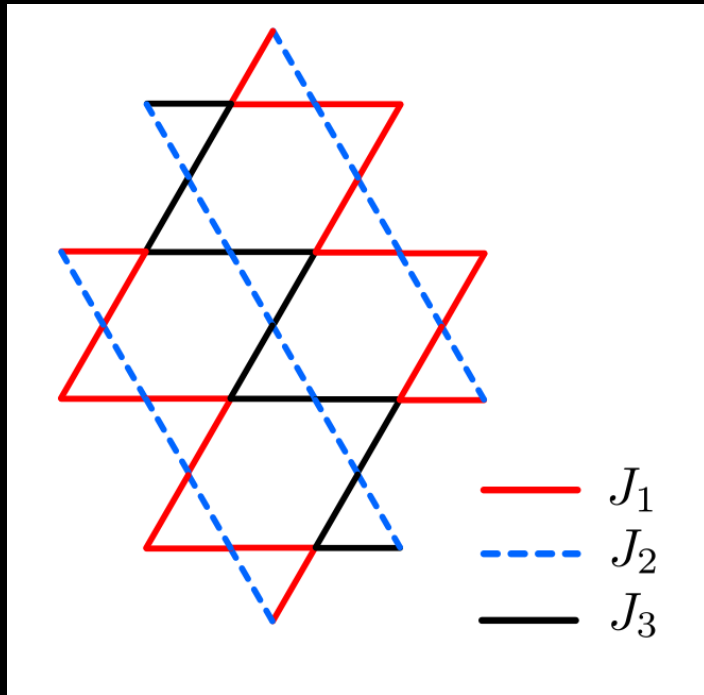


Star condition



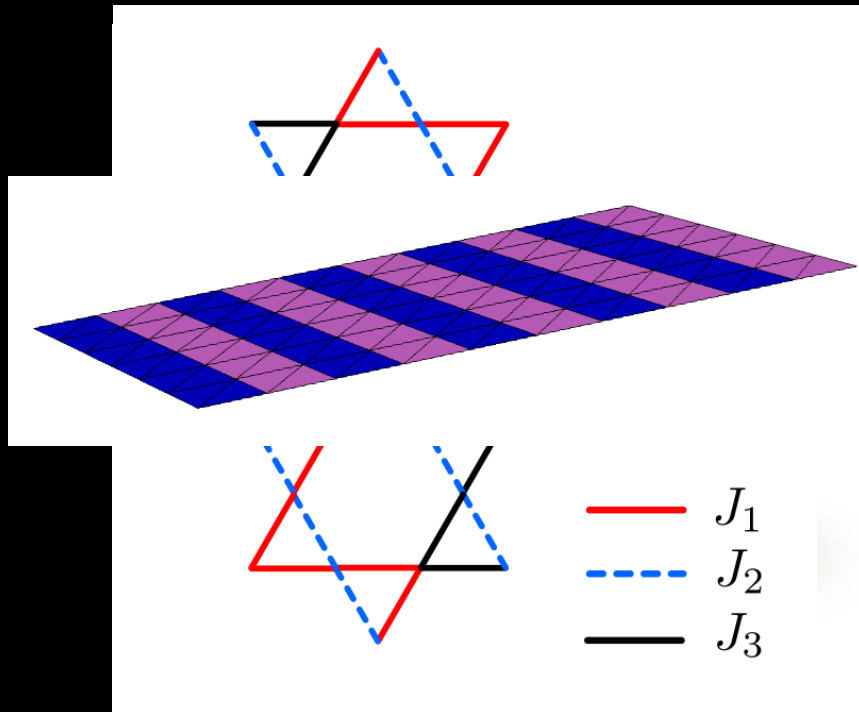
$$J_1 J_3 J_5 J_7 J_9 J_{11} = J_2 J_4 J_6 J_8 J_{10} J_{12}$$

Distorted kagome antiferromagnets

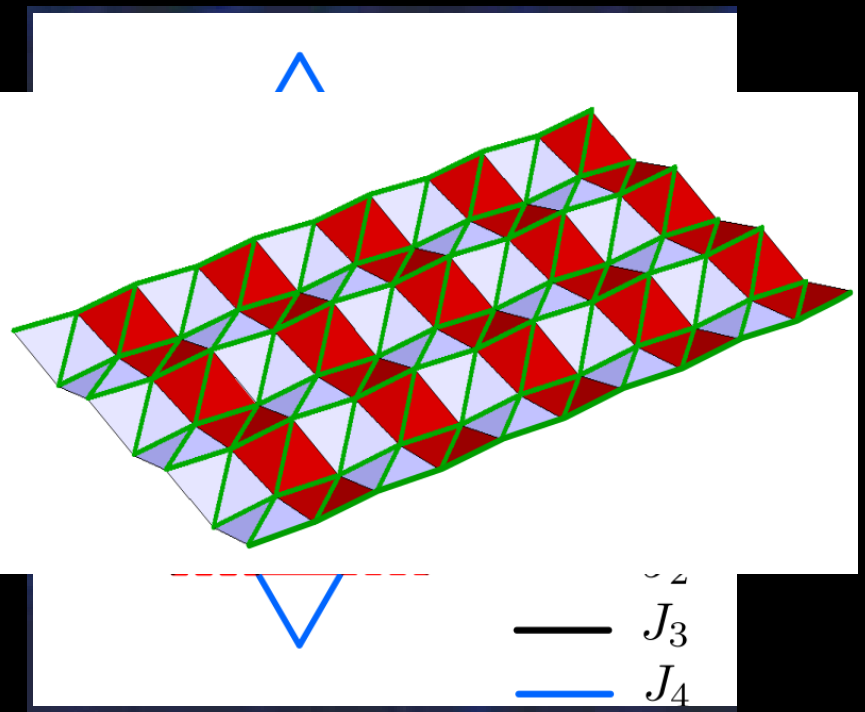


Both obey the star condition

Two kinds of origami



Flattenable

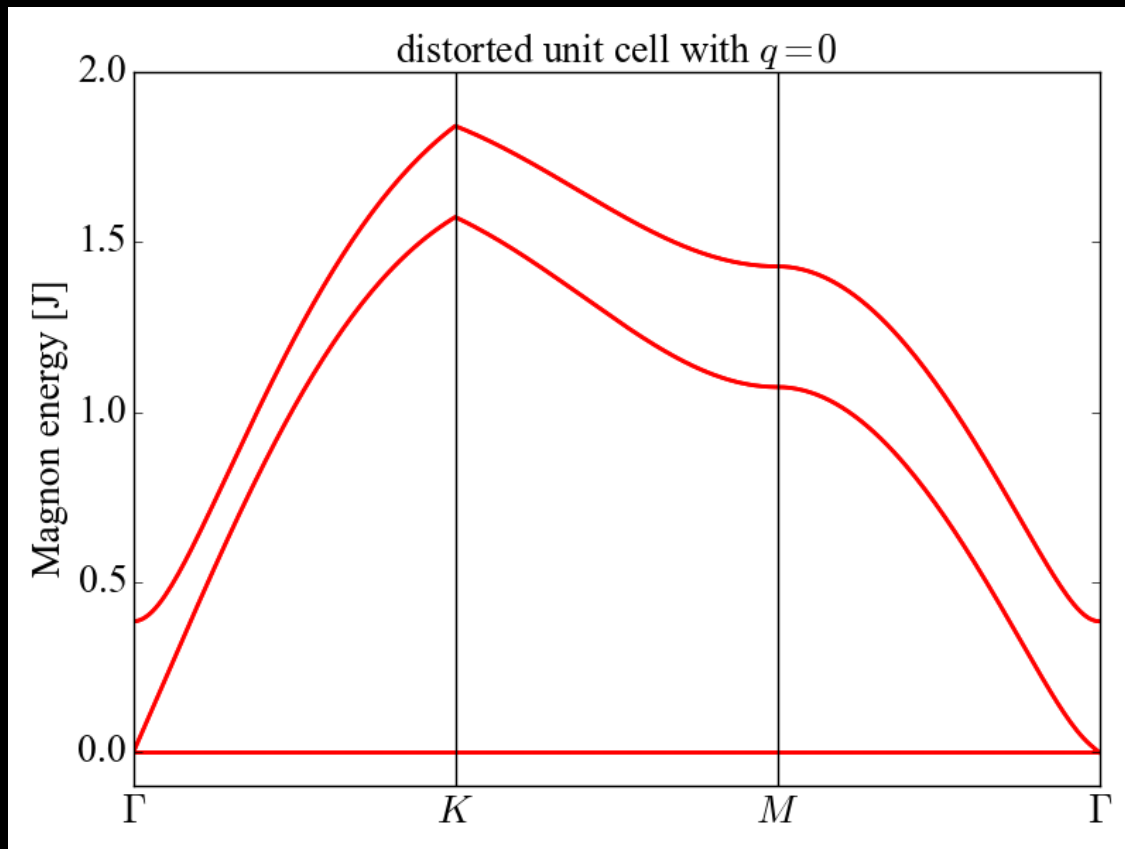


Crumpled



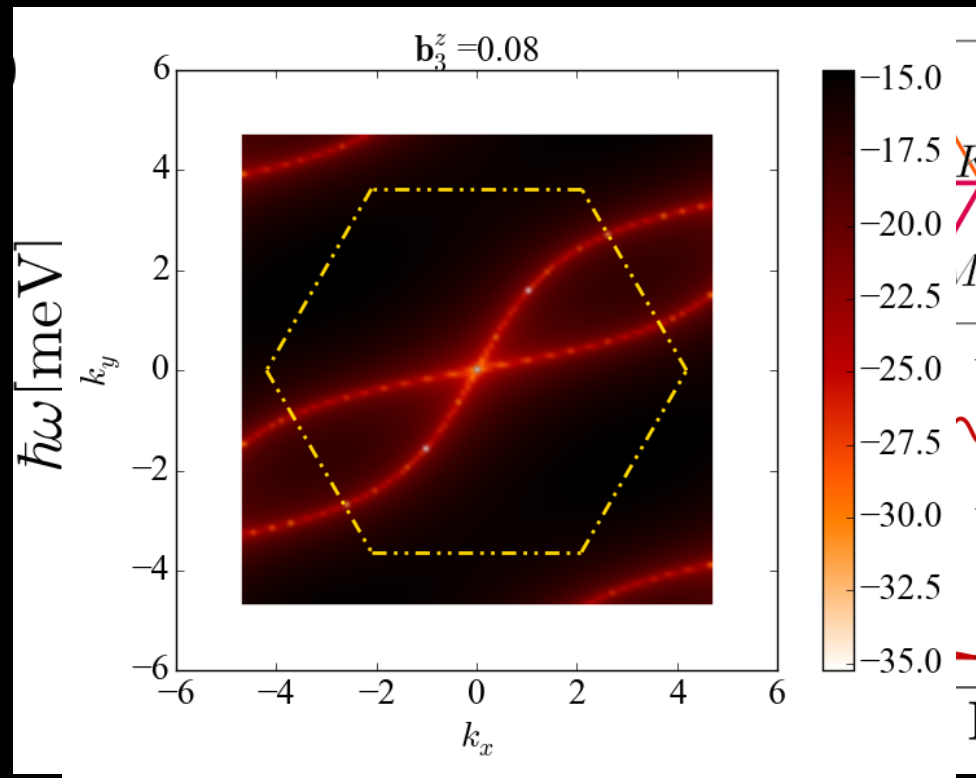
Spin wave dispersions

Flattenable spin origami systems



Spin wave dispersions

Crumpled spin origami systems



Lines of zero modes like a Fermi surface!

In both cases, distortion doesn't just
lift the accidental degeneracy!

Protected by a topological invariant?

Definition “topological invariant”:

A number that cannot change unless
the energy gap closes

Maxwell counting

See Moessner and Chalker, 1998



Number of vertices n_v per unit cell:

2

Number of edges n_e per unit cell:

6

Maxwell d.o.f. count per unit cell:

$$\nu = dn_v - n_e = 0, \quad d = 3$$

v is deformation invariant

$$\nu \left(\begin{array}{c} \text{Image of a teal origami bag in its upright, rectangular shape.} \end{array} \right) = \nu \left(\begin{array}{c} \text{Image of the same teal origami bag in a flattened, deformed shape.} \end{array} \right)$$

Maxwell counting as topological invariant

Kane and Lubensky, Nature Physics, 2014

Maxwell condition: $\nu=0$ (Origami)

Isostatic condition: $\nu=0, \Delta > 0$

$\nu = 0 \longrightarrow \nu = 1$ Gap closes ($\Delta = 0$)

Focus on the rigidity matrix

Kane and Lubensky, Nature Physics 2014

Expand the constraint functions in spin waves

$$\begin{aligned} S_{\Delta\alpha} &\equiv a_i S_{i\alpha} + a_j S_{j\alpha} + a_k S_{k\alpha} \\ &= R_{\Delta\alpha, i\mu} x^{i\mu} \end{aligned}$$

R determines everything $\rightarrow H = \frac{1}{2} \mathbf{x}^T \mathbf{R}^T \mathbf{R} \mathbf{x}$

$\nu = \# \text{ of cols of } R - \# \text{ of rows of } R$

$$= \text{rank} R + \text{nullity} R - \text{rank} R^T - \text{nullity} R^T$$

$$= \text{nullity} R - \text{nullity} R^T$$

Topological invariant of crumpled case

$R(\mathbf{k})$ is a complex square matrix but $v(\mathbf{k})=0$

Origami realness condition: $\det R(\mathbf{k})$ is real!

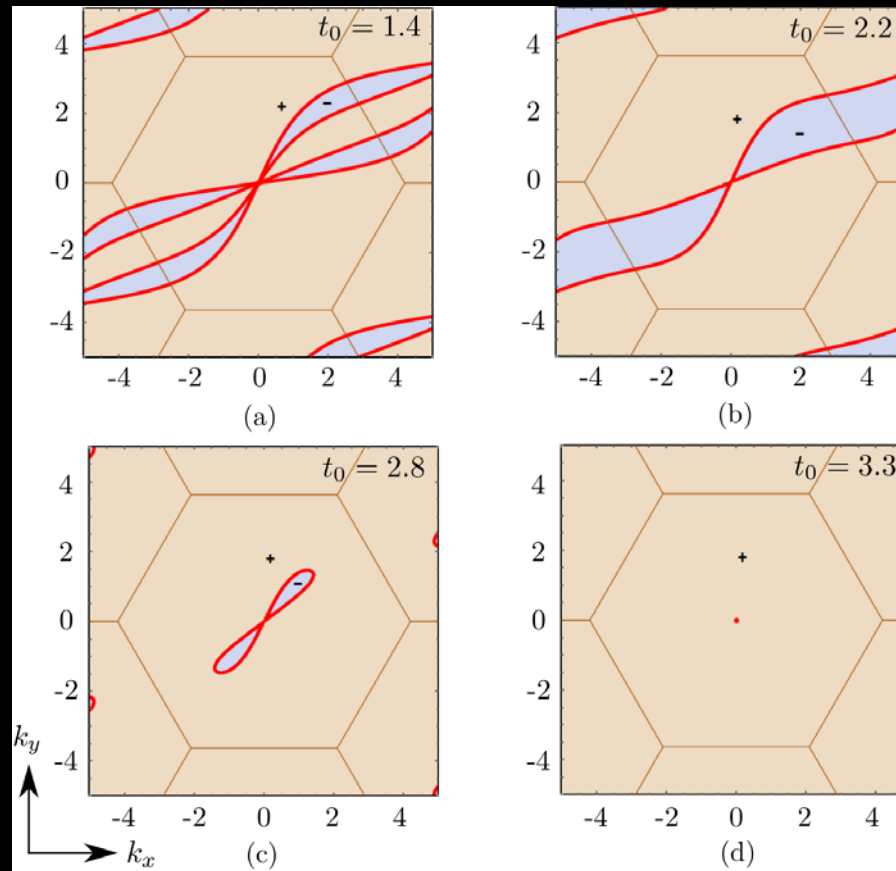
Chen et. al. PRL 116, 135501 (2016)

Define:

$$\eta = \text{sign } \det R(\mathbf{k})$$

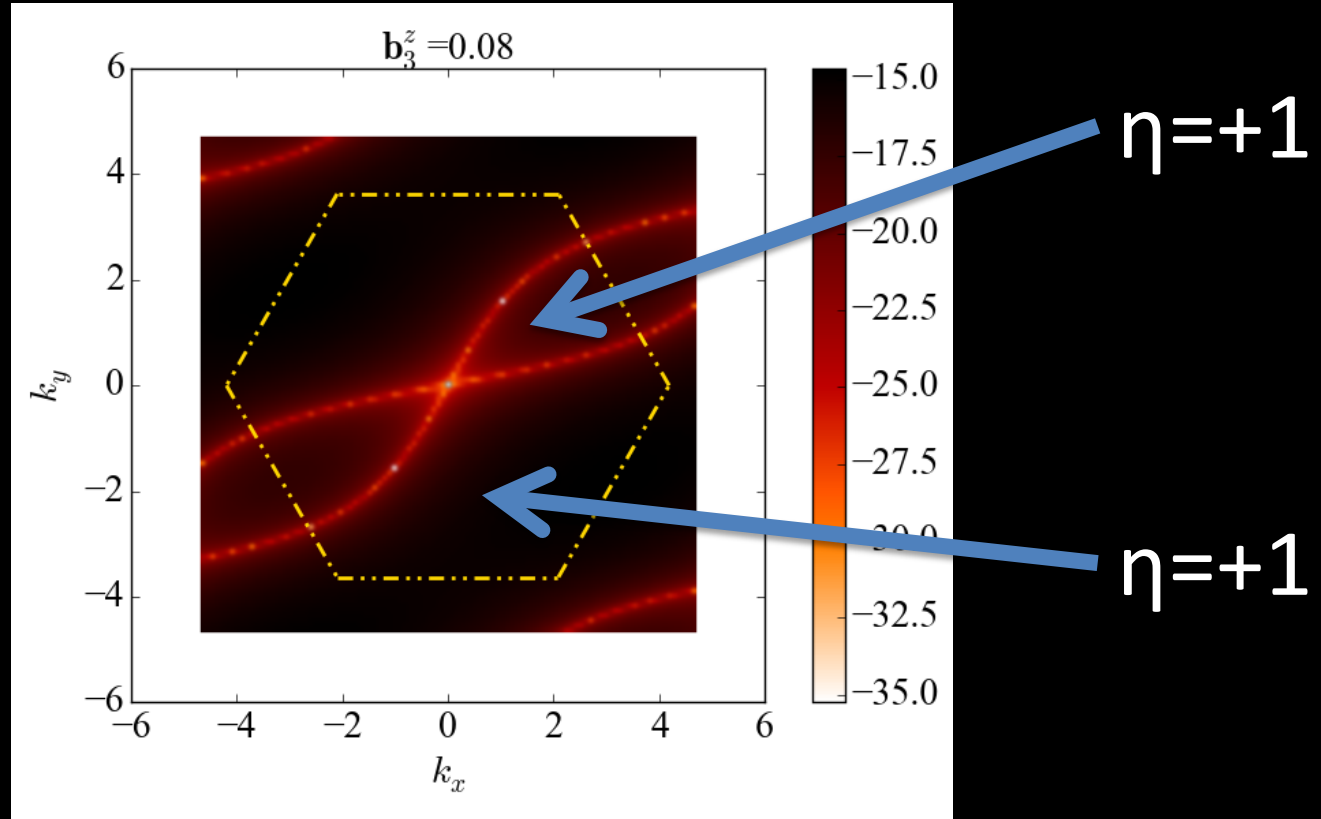
η is a “topological invariant”!

Weyl line nodes of generic origami



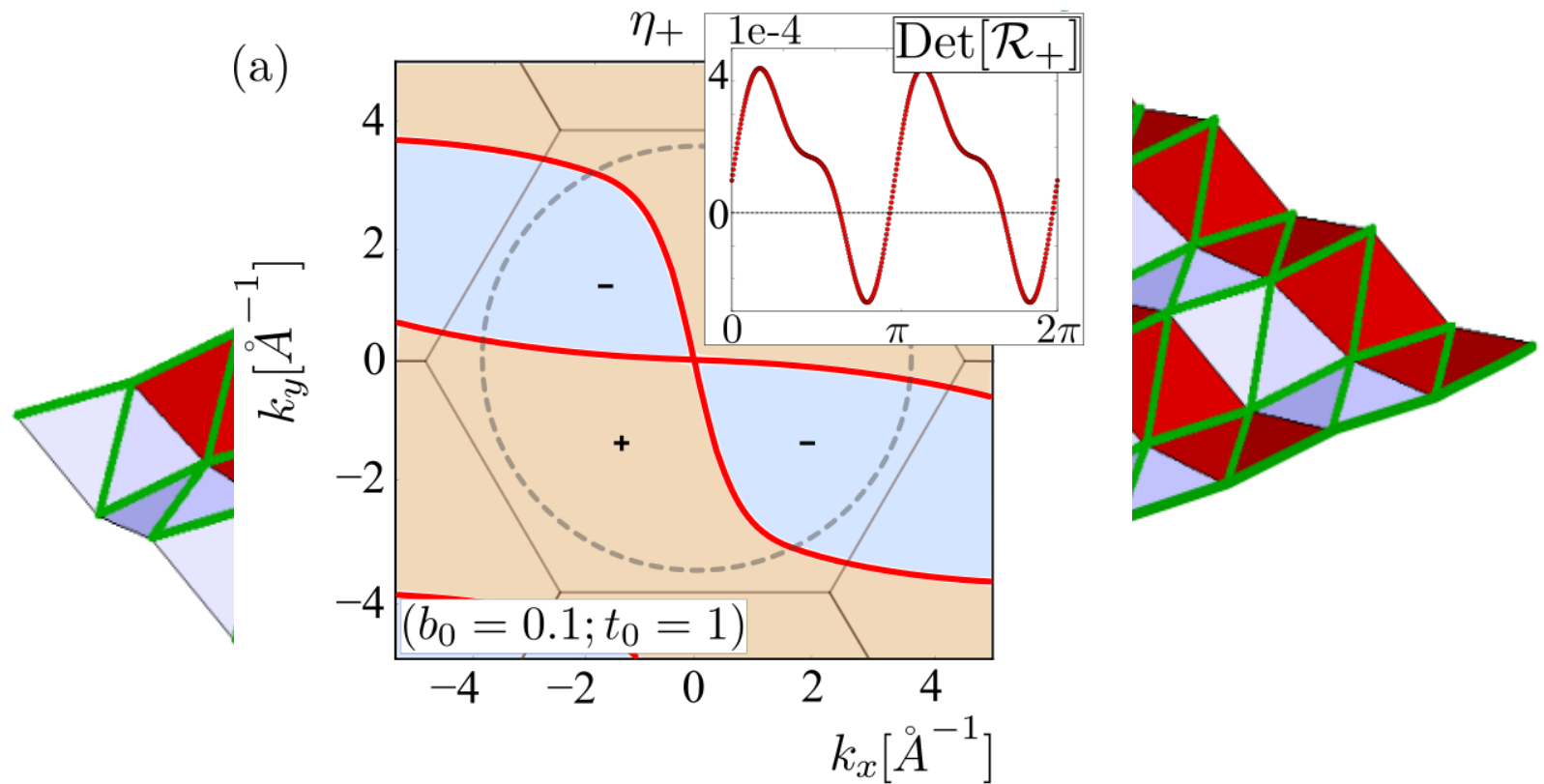
Changing ground states just
moves the line nodes!

Line nodes of $\text{Cs}_2\text{CeCu}_3\text{F}_{12}$

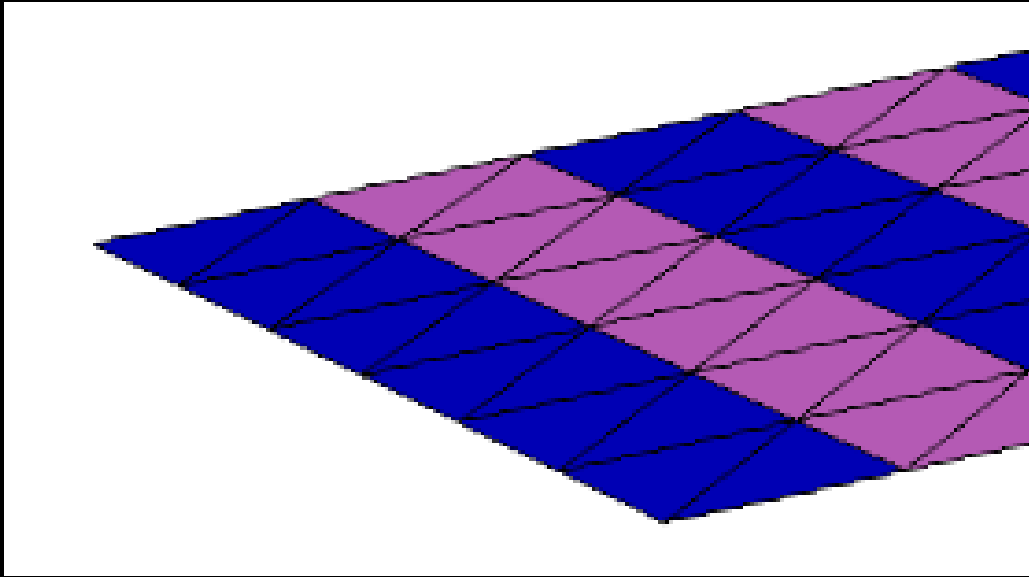


Doubly degenerate Dirac line nodes!

Diamond origami



Topological invariants of flattenable origami



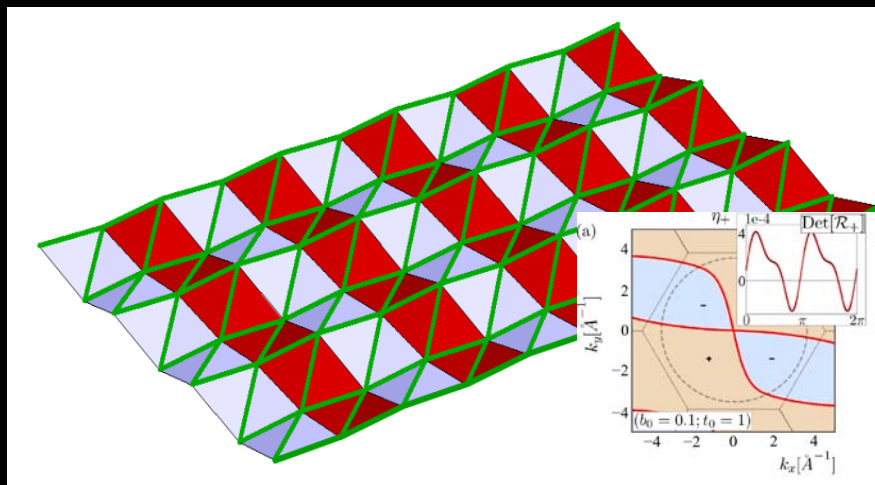
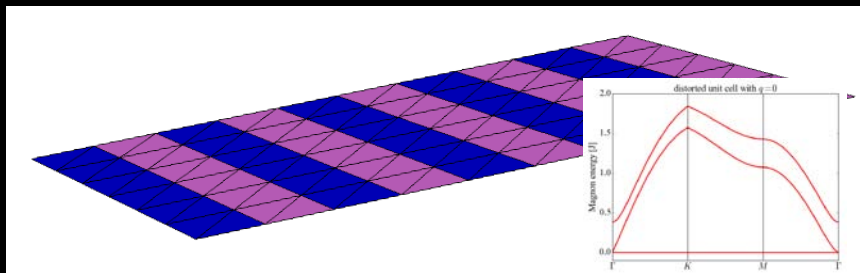
One zero mode per
flat vertex

Build a topological invariant from curvature?

Conclusions

Frustration characterized by
topological invariants

Spin origami and distorted
Kagome antiferromagnets



Generic frustration is an intertwined order!

Intertwined orders refers to the case in which $T^0_{\text{SC}} \sim T^0_{\text{CDW}}$ over a range of situations, i.e., in the case of the cuprates, over a range of doping concentrations and material families. Where this occurs...it has no natural explanation simply in terms of robust and generic features of coupled order parameters.

Fradkin, Kivelson, Tranquada, RMP 87, 457 (2015)