Superconductivity, non-Fermi liquid behavior, and Raman scattering anomalies near a metallic quantum critical point

Samuel Lederer

KITP "Intertwined," 21 September 2017

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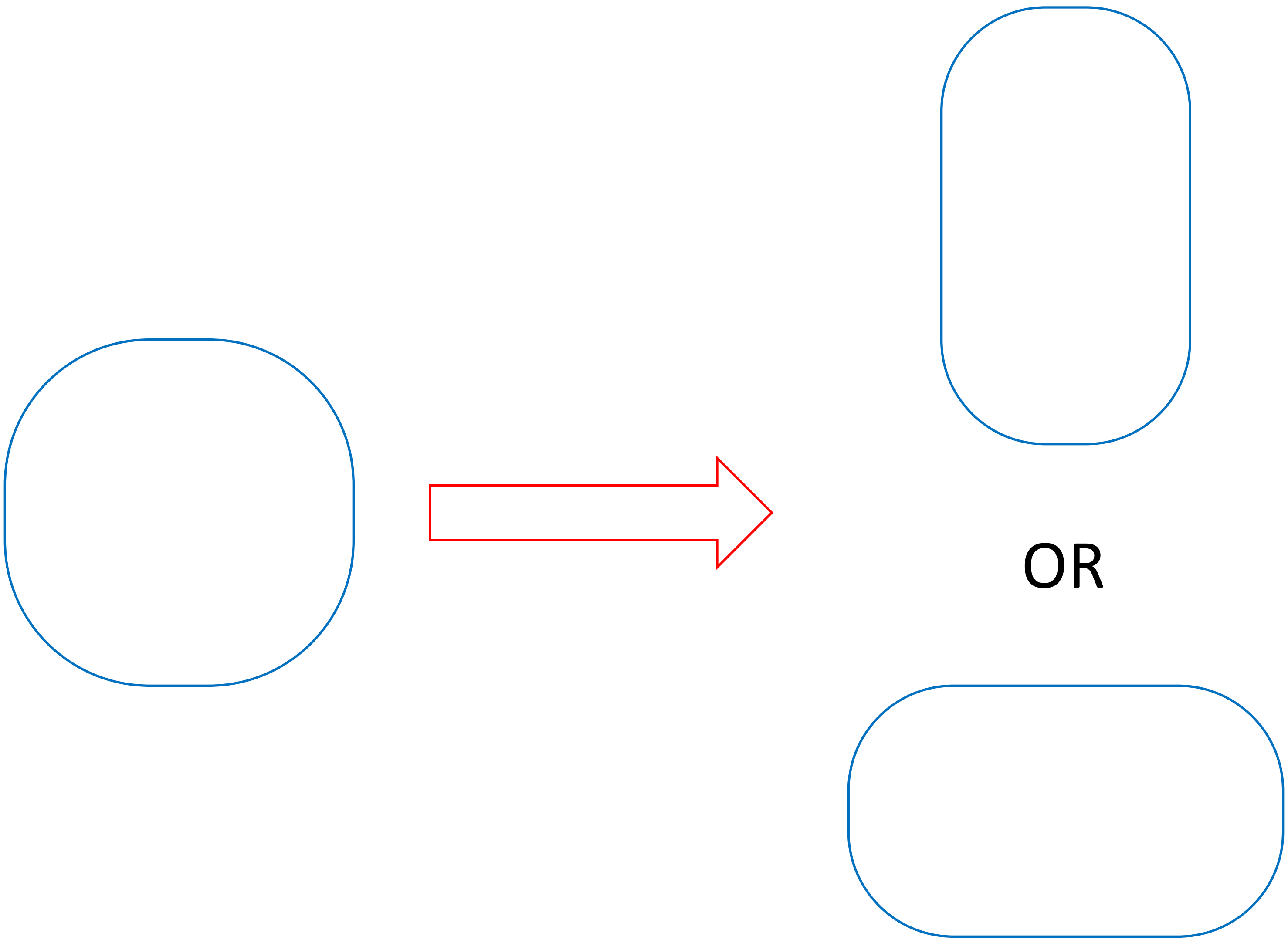
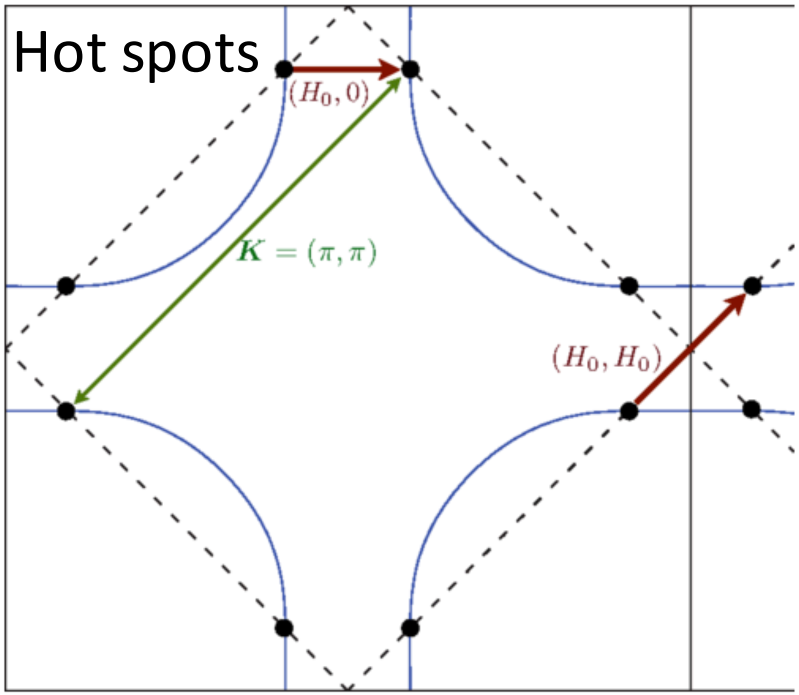
Ising nematic order + Fermi surface

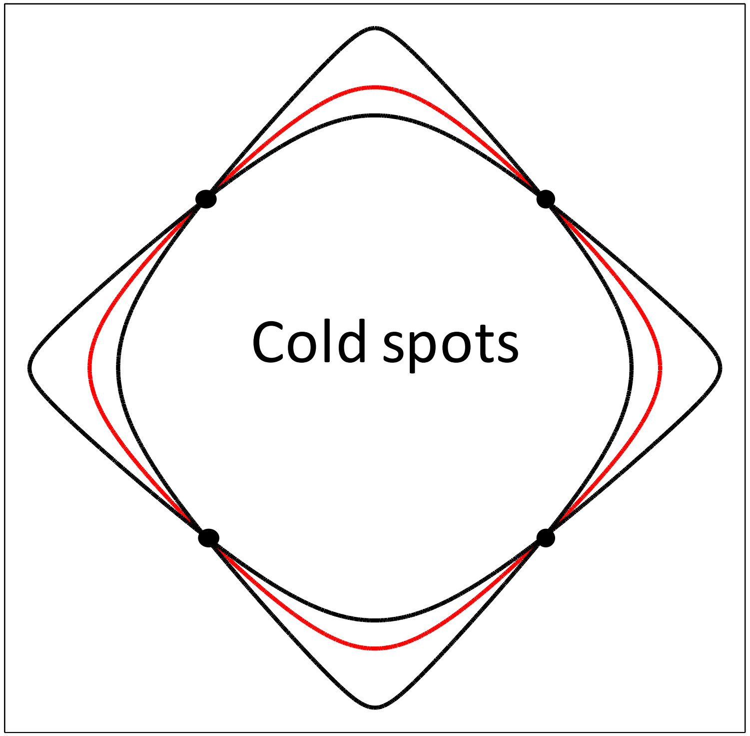


Density wave vs. Q=0 quantum critical fluctuations

Antiferromagnetic QCP



Ising nematic QCP



J. Sau and S. Sachdev, Phys. Rev. B 89, 075129 (2014)

The questions

- Can an Ising nematic QCP lead to high T_C superconductivity? Non-Fermi liquid?

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- Can an Ising nematic QCP lead to high T_C superconductivity? Non-Fermi liquid?
  - Monte Carlo studies, with Yoni Schattner (Stanford), Erez Berg (Chicago), Steve Kivelson (Stanford)
  - PRX **6**, 031028 (2016); PNAS **114**, 4905 (2017)
- What are some distinctive experimental consequences of an Ising nematic QCP?

Pairing vs non-Fermi liquid

- Ising nematic quantum critical fluctuations incompatible with Fermi liquid ground state

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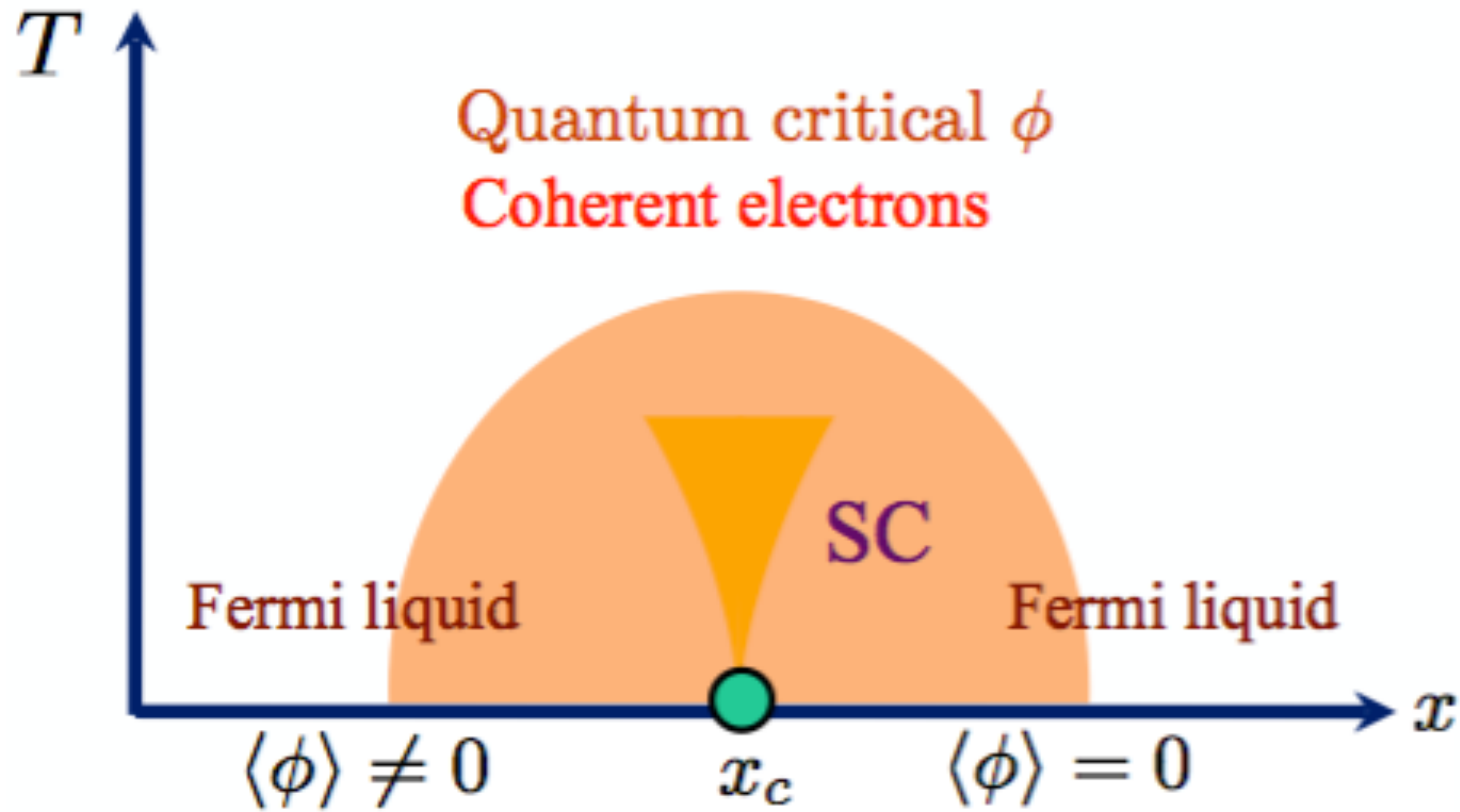
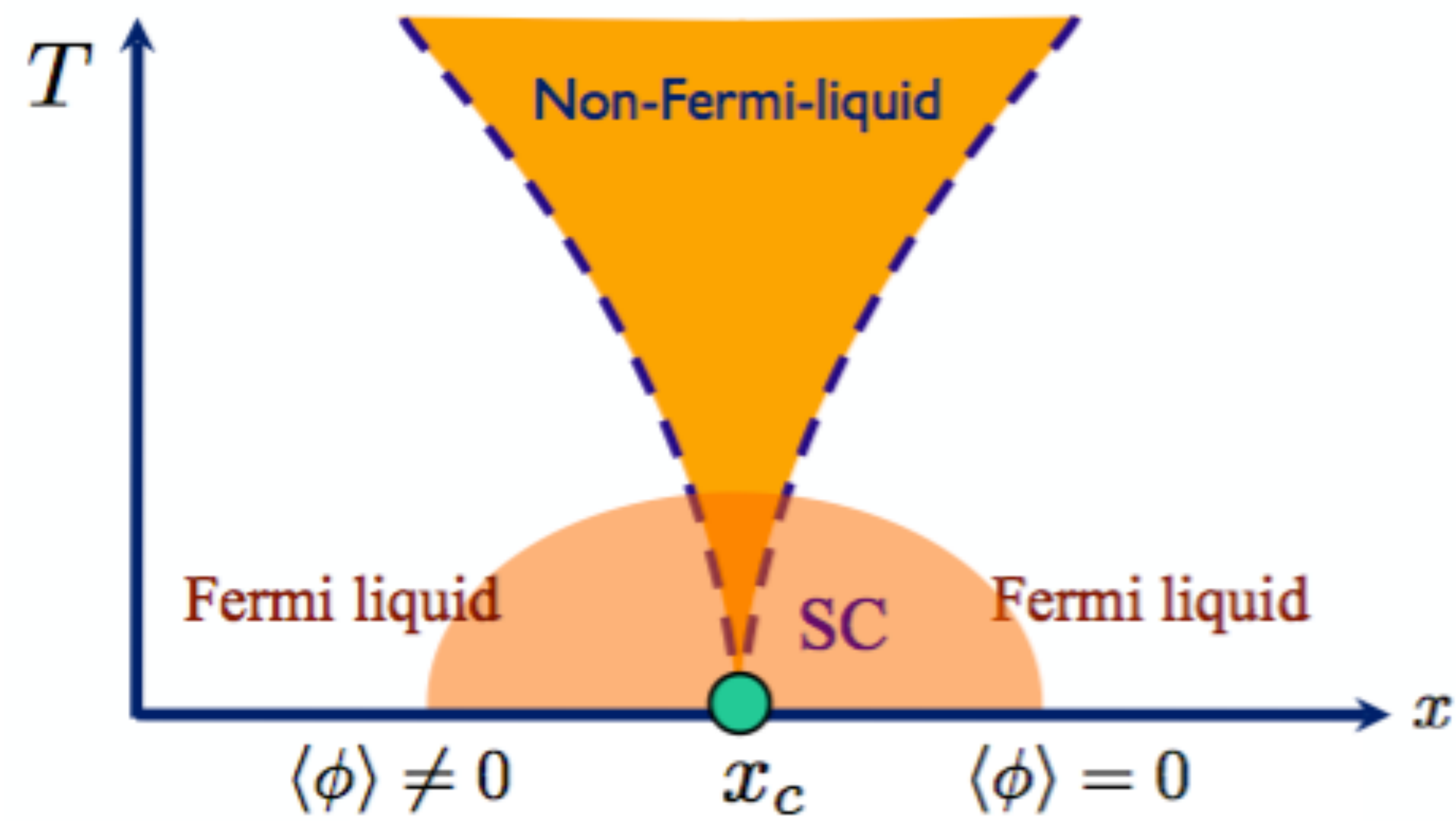
So who wins?

Does SC preempt non-Fermi liquid?



M. Metlitski et al, arxiv: 1403.3694

Or non-Fermi liquid above T_C (like cuprates)?



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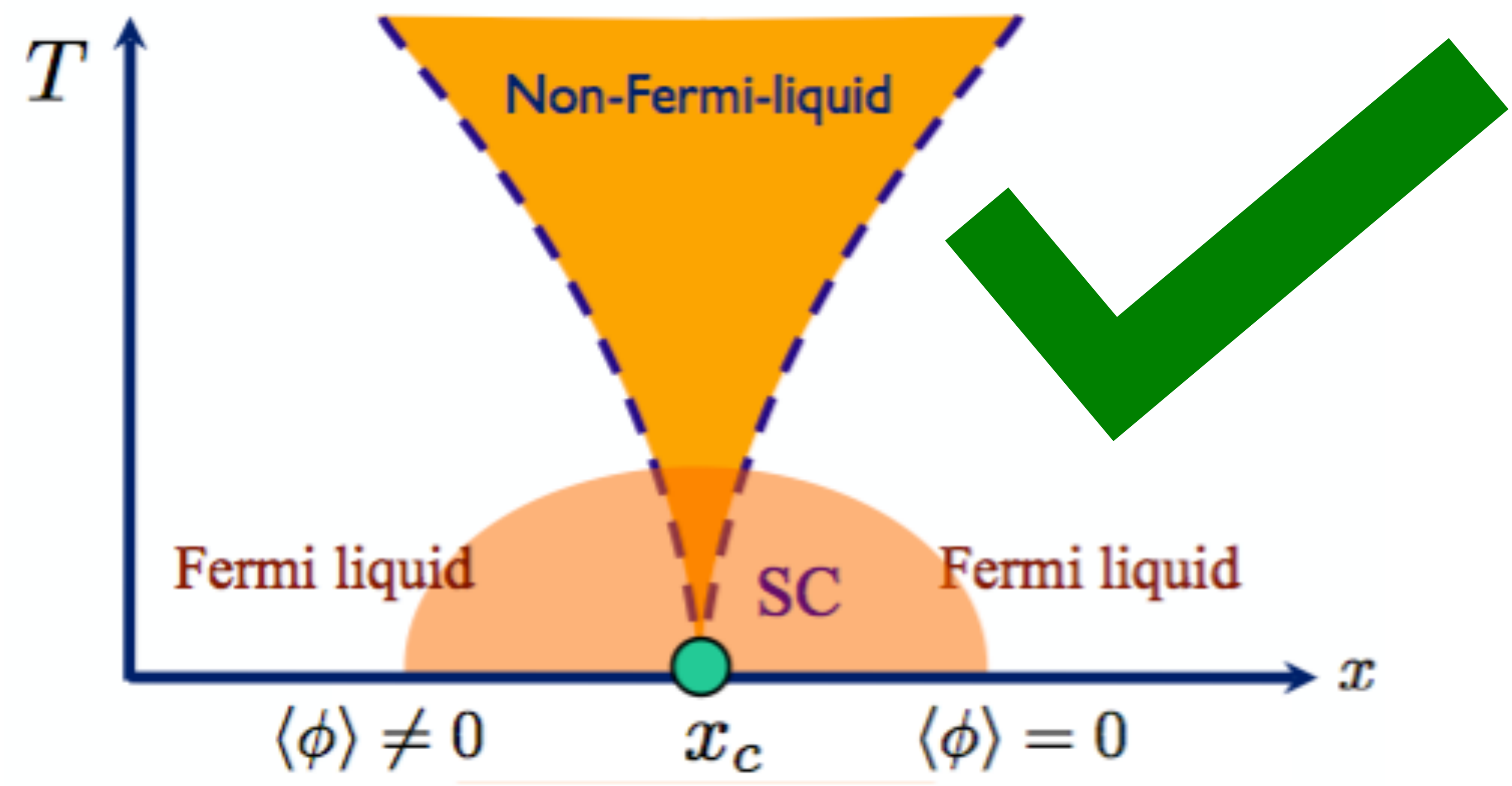

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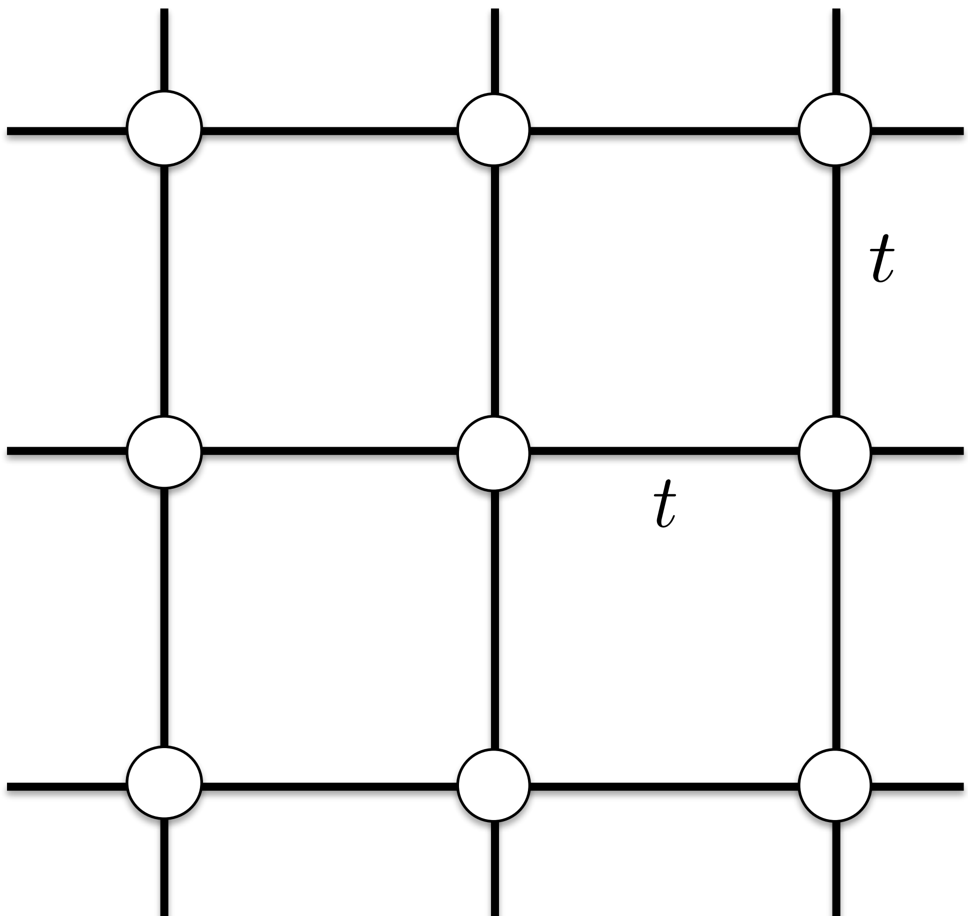
Microscopic Hamiltonian

$$H = H_f + H_{nem} + H_{int}$$

$$H_f = -t \sum_{\langle i,j \rangle, s} c^\dagger_{i,s} c_{j,s} + h.c. - \mu \sum_i c^\dagger_{i,s} c_{i,s}$$



Microscopic Hamiltonian

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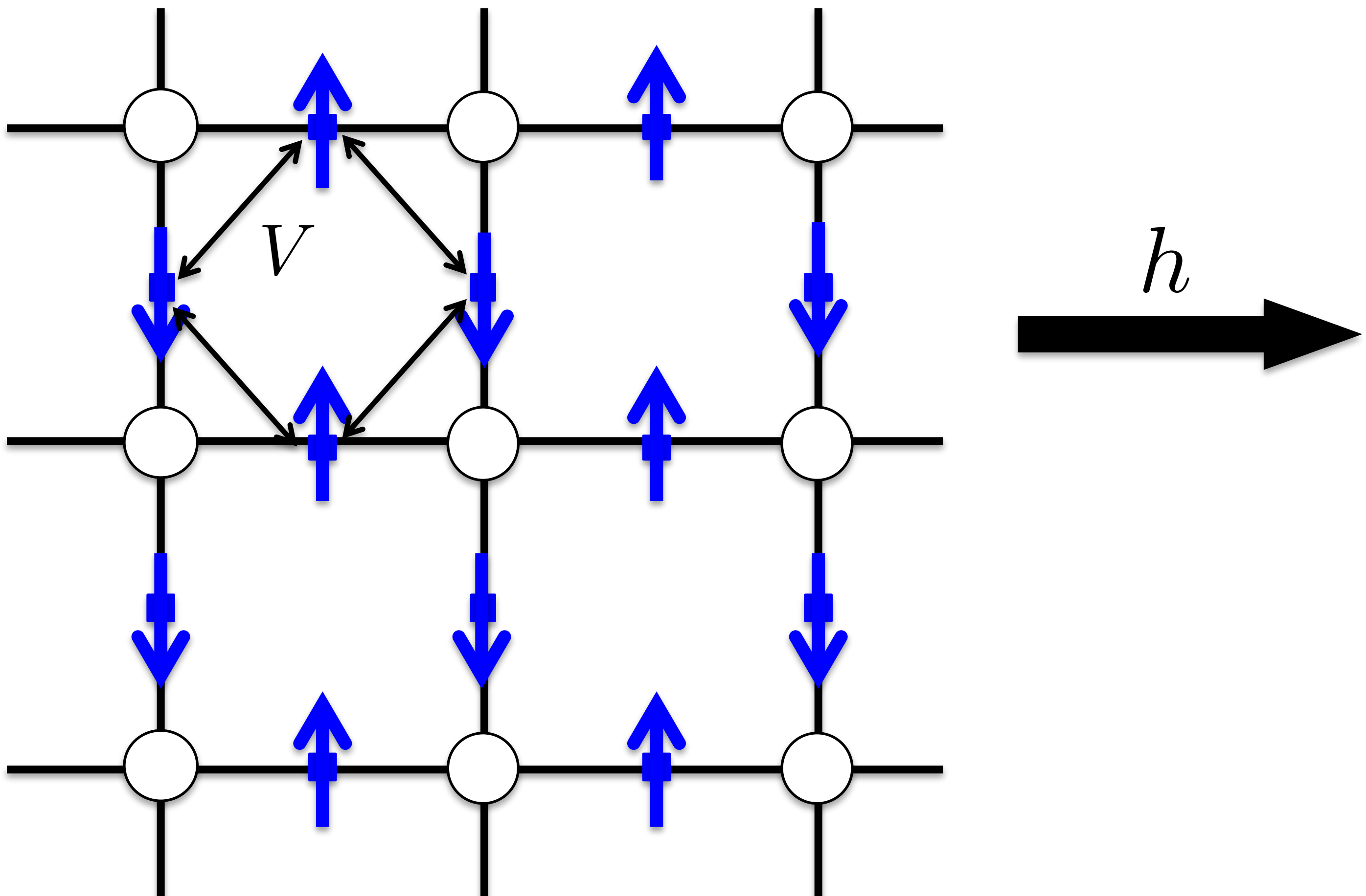
$$H_{nem} = V \sum_{\langle\langle i,j\rangle,\langle k,l\rangle\rangle} \sigma^z_{\langle i,j\rangle}\sigma^z_{\langle k,l\rangle} + h\sum_{\langle i,j\rangle}\sigma^x_{\langle i,j\rangle}$$



Microscopic Hamiltonian

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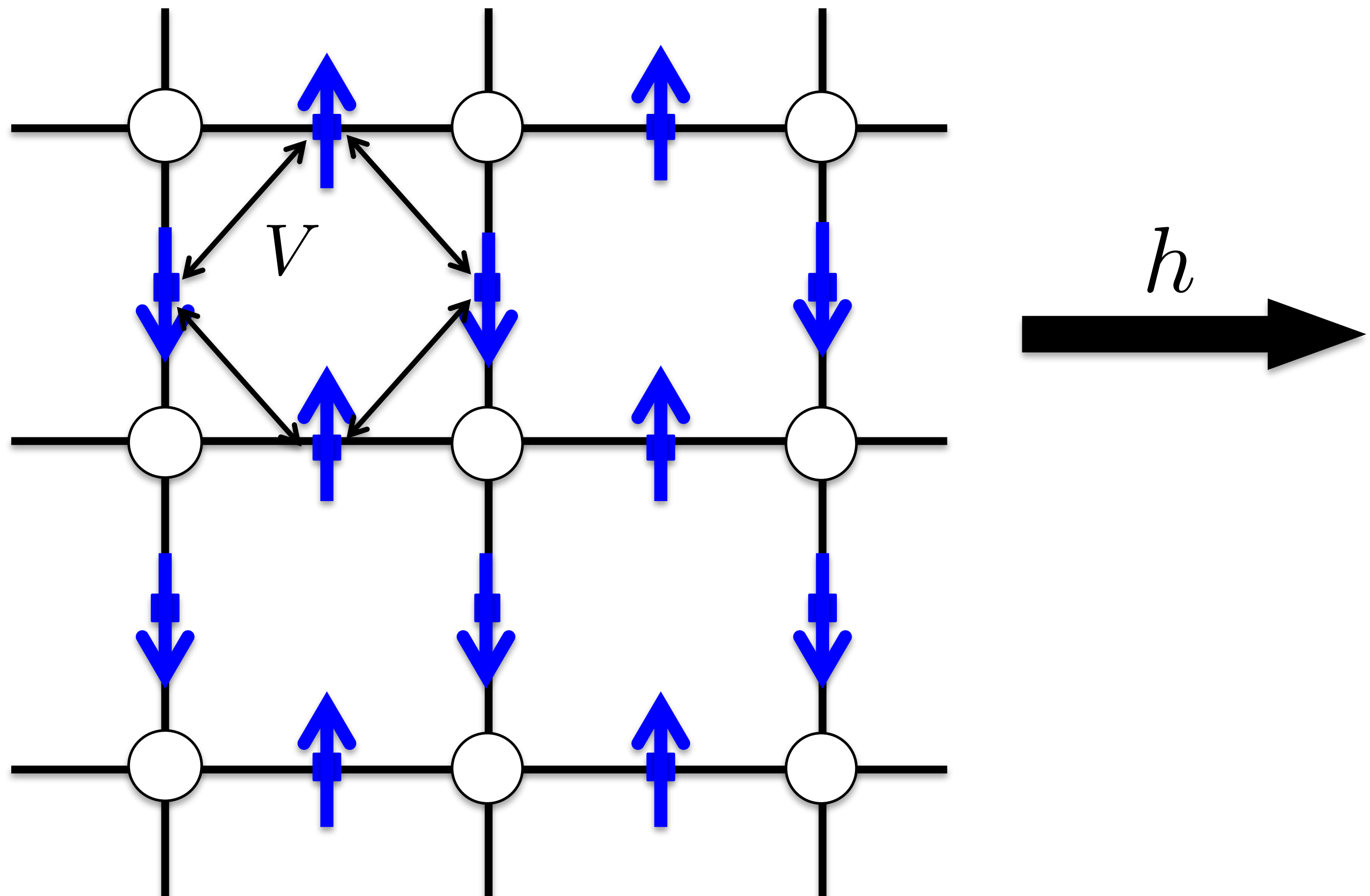
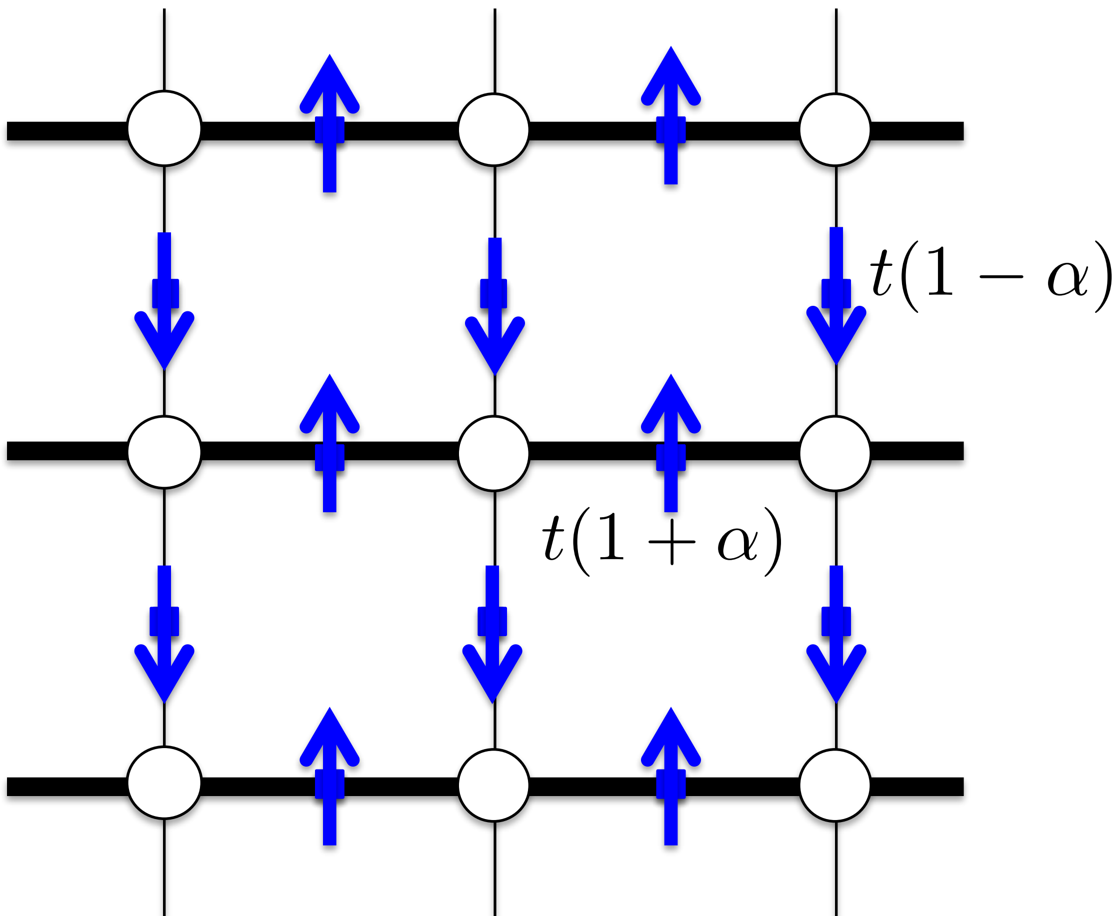
$$S_{nem}[\phi] = \int d\tau d^2 r \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + m^2 \phi^2 + \phi^4 \right]$$



Microscopic Hamiltonian

$$H = H_f + H_{nem} + H_{int}$$

$$H_{int} = -\alpha t \sum_{\langle i,j \rangle, s} \sigma^z_{\langle i,j \rangle} \left(c^\dagger_{i,s} c_{j,s} + h.c. \right)$$



Determinant Quantum Monte Carlo

- Maps 2-dimensional quantum model to (2+1)-dimensional classical model with size $\beta = T^{-1}$ in imaginary time direction

- Numerically exact algorithm:
  expensive but correct!

- Model is free of fermion sign problem

R. Blankenbecler, D. J. Scalapino, and R. L. Sugar, Phys. Rev. D **24**, 2278 (1981)

Parameter choices

Temperatures $T = E_F$ to $T < E_F/100$

System sizes 6×6 to 24×24

Imaginary time step $\Delta\tau = 0.05t^{-1}$

$$
\begin{aligned}
t &= 1 \\
V &= 0.5 \\
\mu &= 1.0 \\
h &= \text{tunable parameter} \\
\alpha &= 1.5
\end{aligned}
$$

Basic results

- Transition remains continuous

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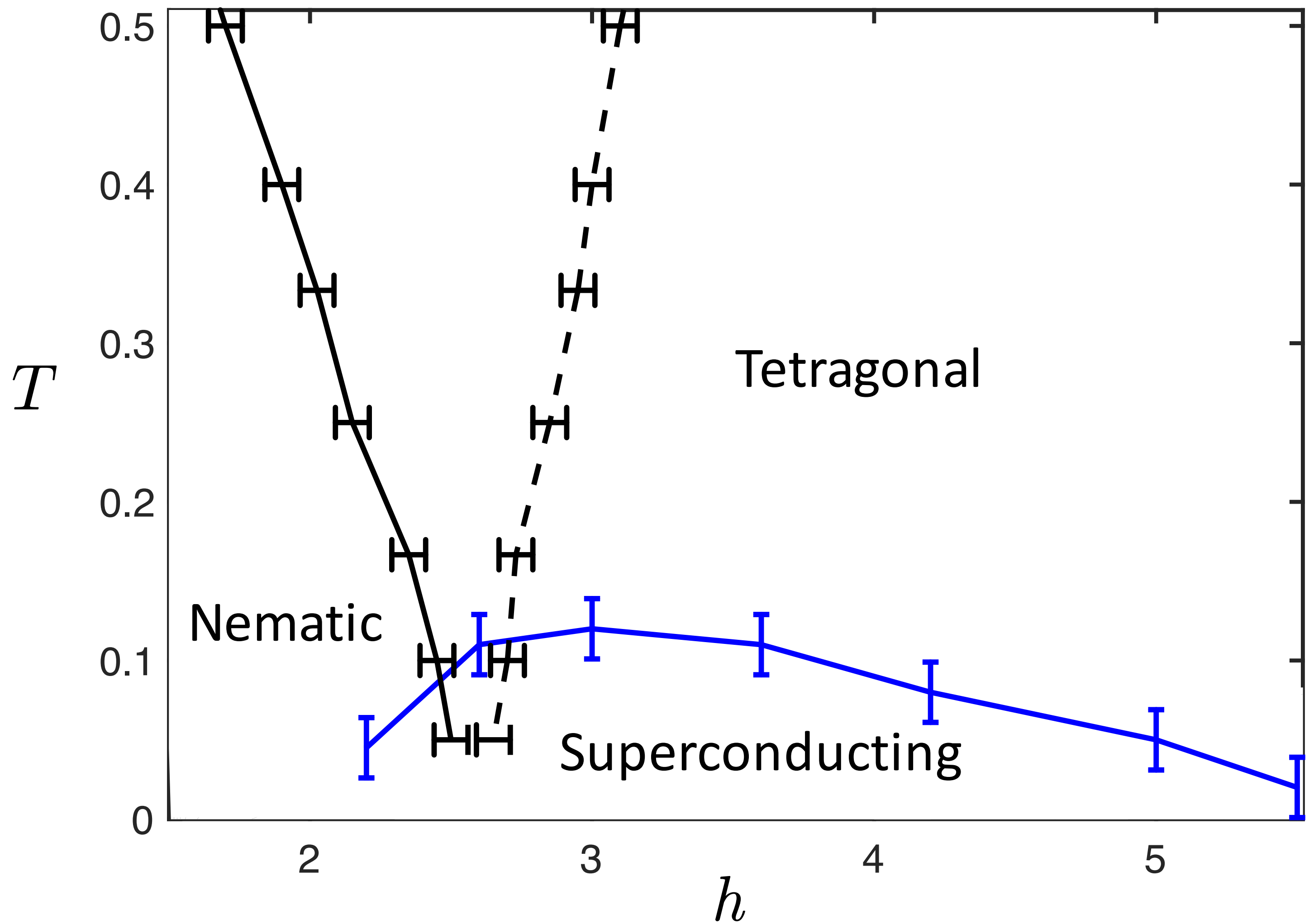
- Transition remains continuous
- Bosons strongly renormalized by coupling to fermions
- Fermions strongly renormalized by coupling to bosons, superconductivity emerges at large enough α

Phase diagram



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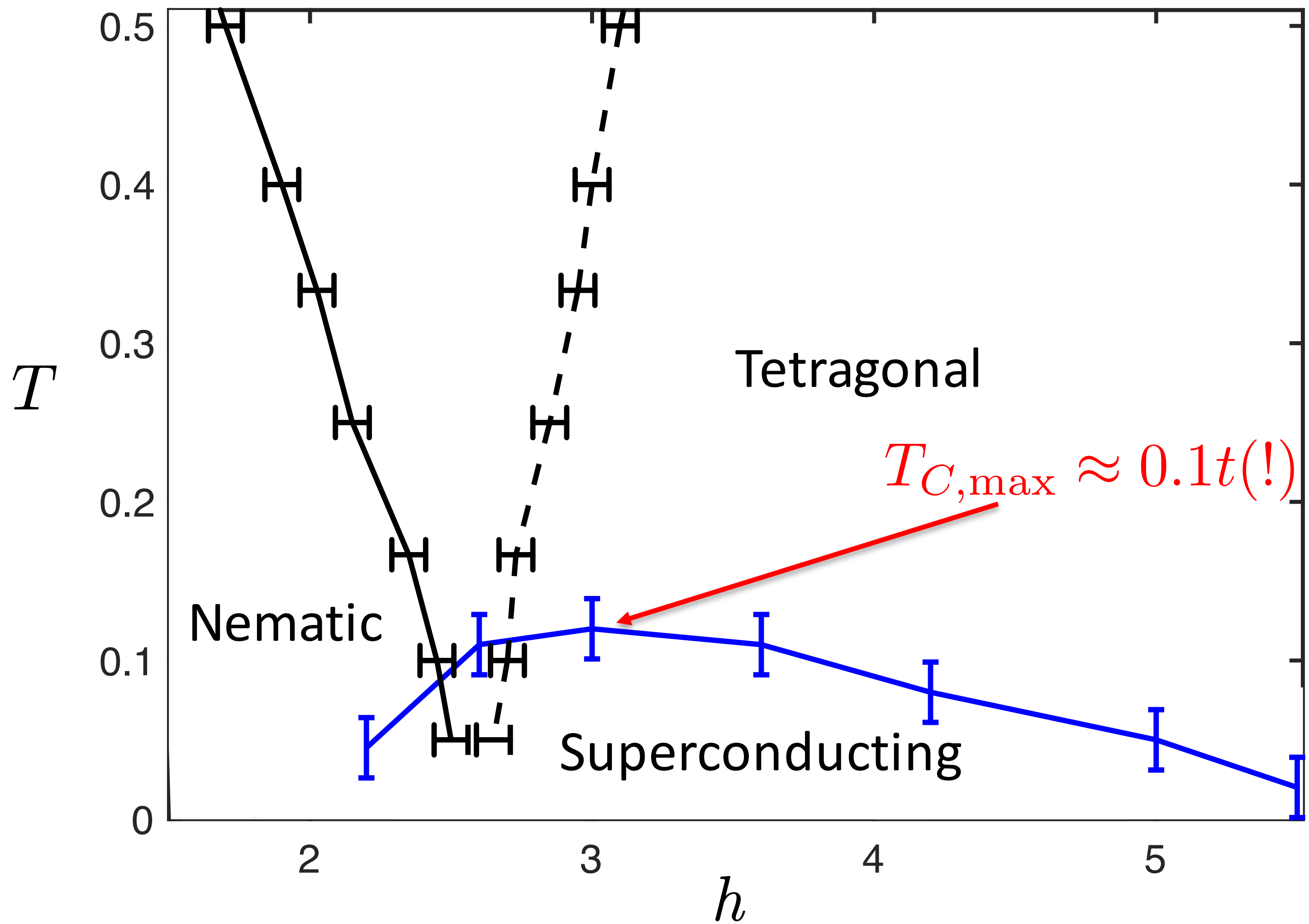
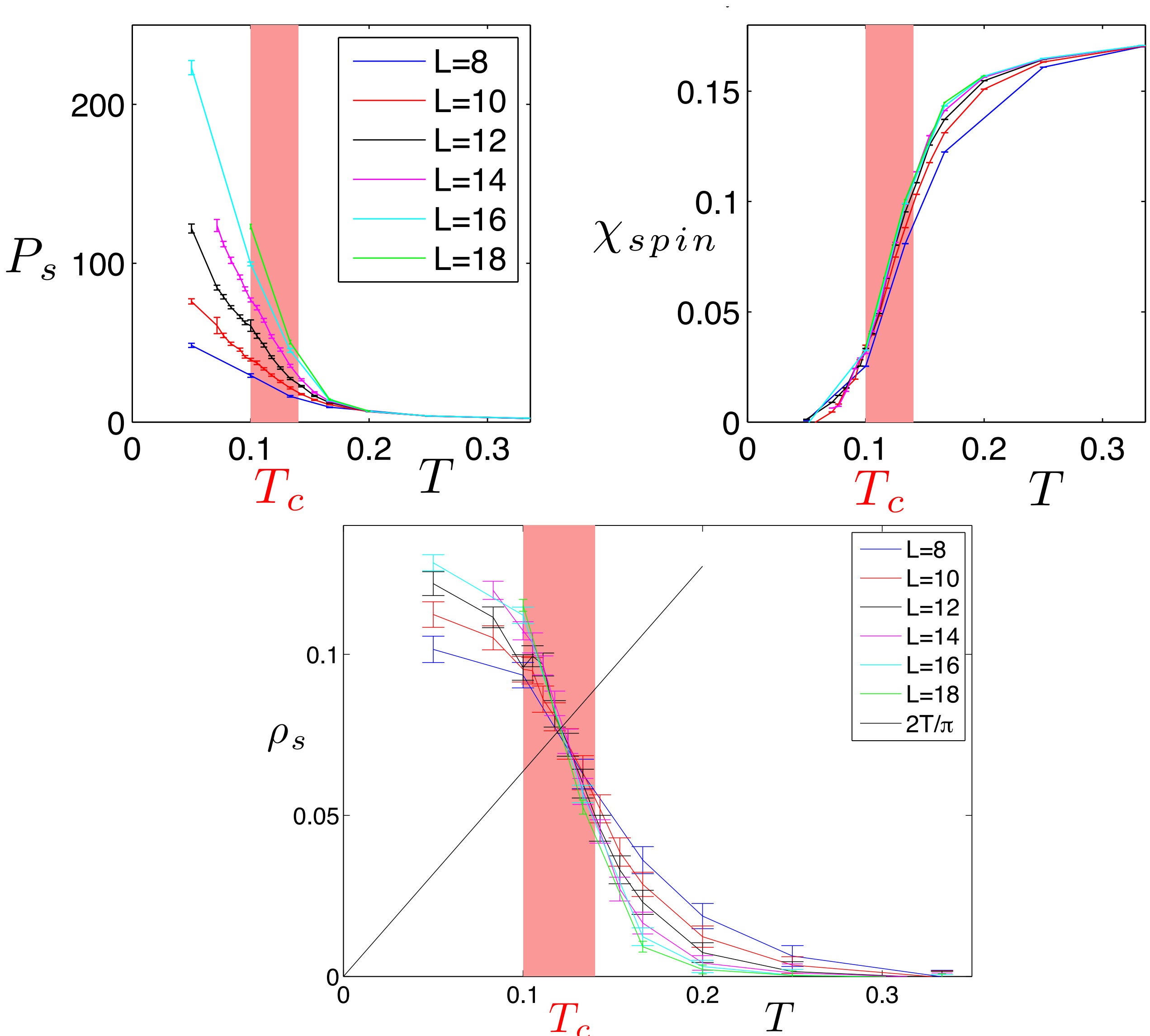


Determining T_C ($h \approx h_C$ shown)



An important caveat for dynamics

- Algorithm is in imaginary time, measurements in real time

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- Algorithm is in imaginary time, measurements in real time
- Need analytic continuation $i\omega_n \rightarrow \omega + i0^+$ to compute measurable quantities
- Difficult for numerical data; introduces uncontrolled errors

A probe of spectral weight at at frequencies ~ T

$$G\left(k, \frac{\beta}{2}\right) = \int d\omega \left(\frac{A(k,\omega)}{2\cosh\left[\frac{1}{2}\beta\omega\right]} \right)$$

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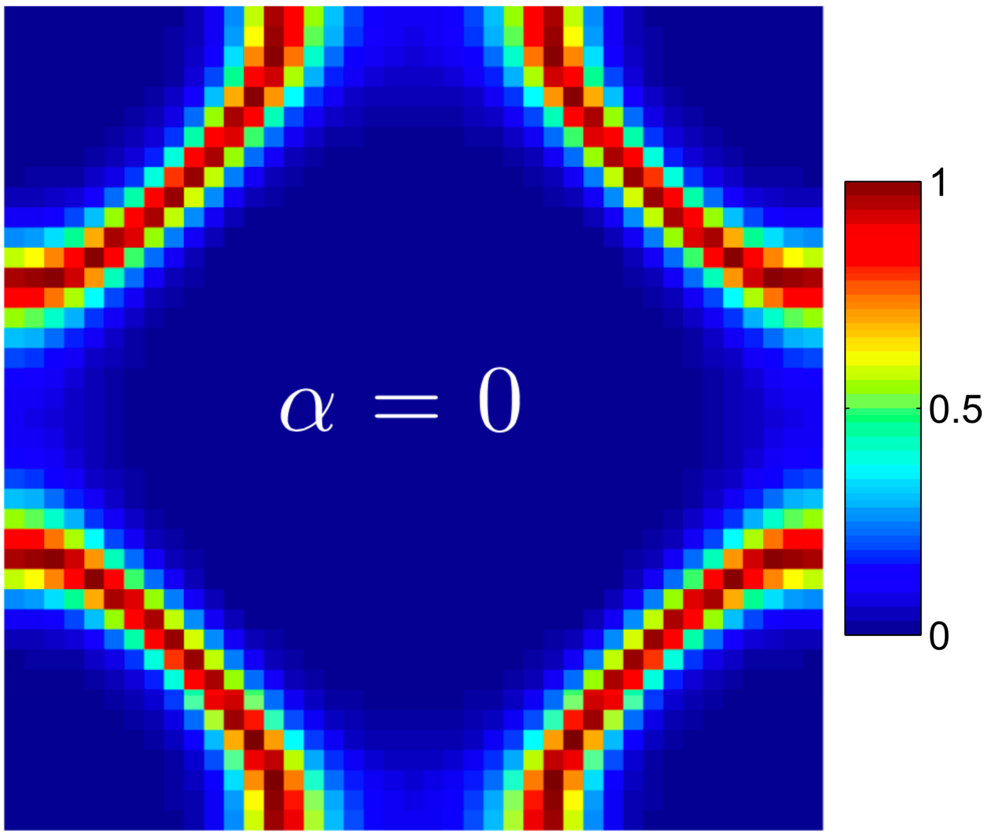
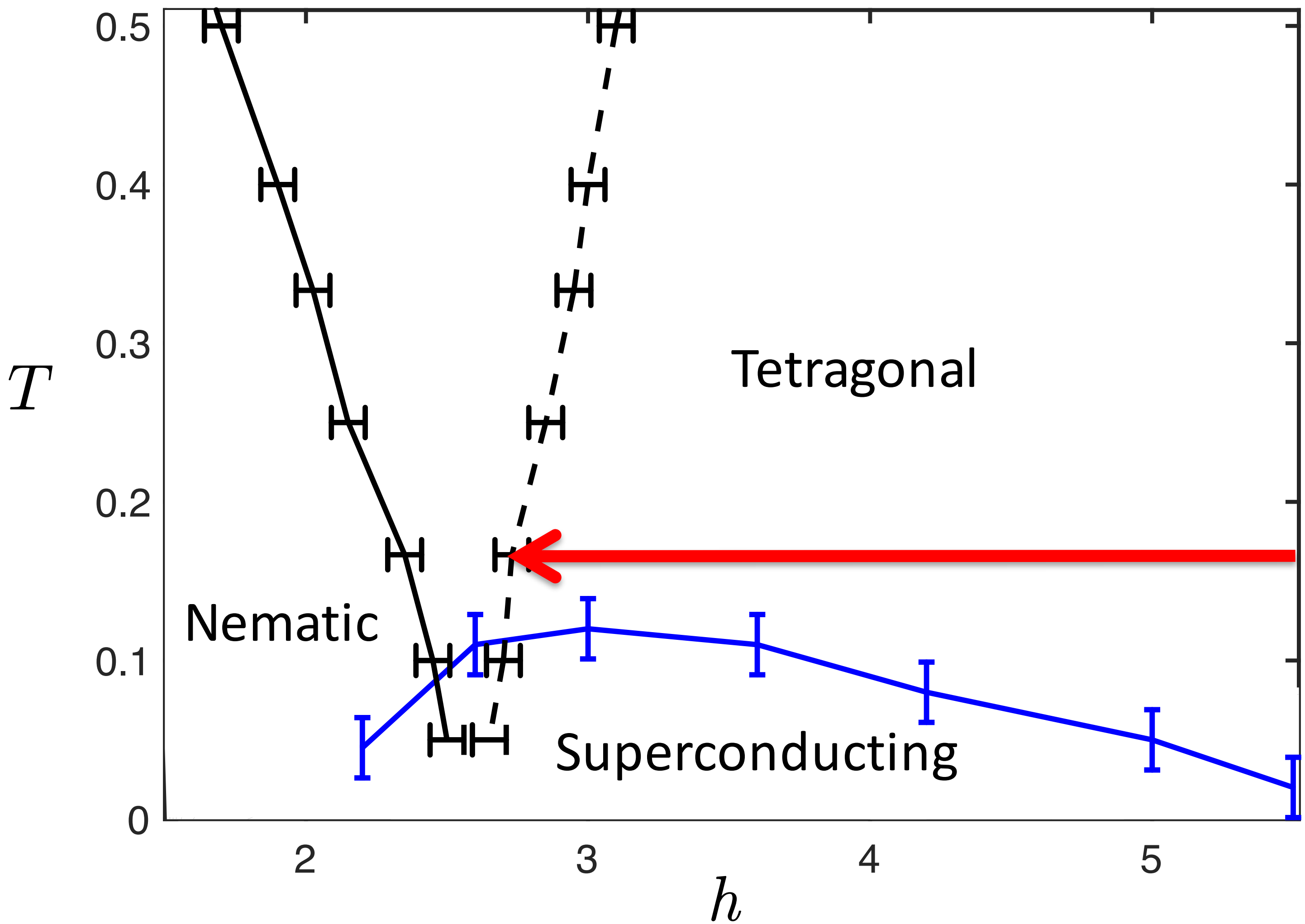
- In a Fermi liquid regime, 2G(k,β/2) is the effective quasiparticle residue Z

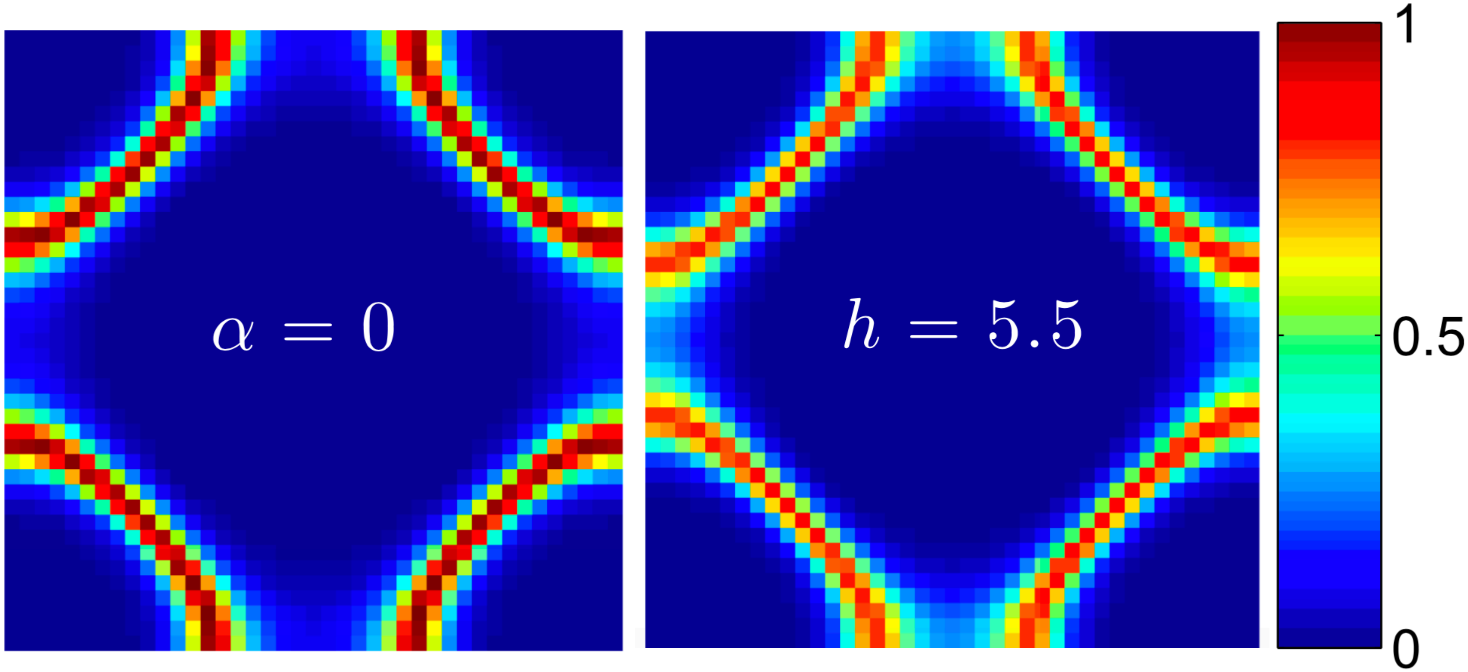
2G(k,β/2) for free fermions (β=6)



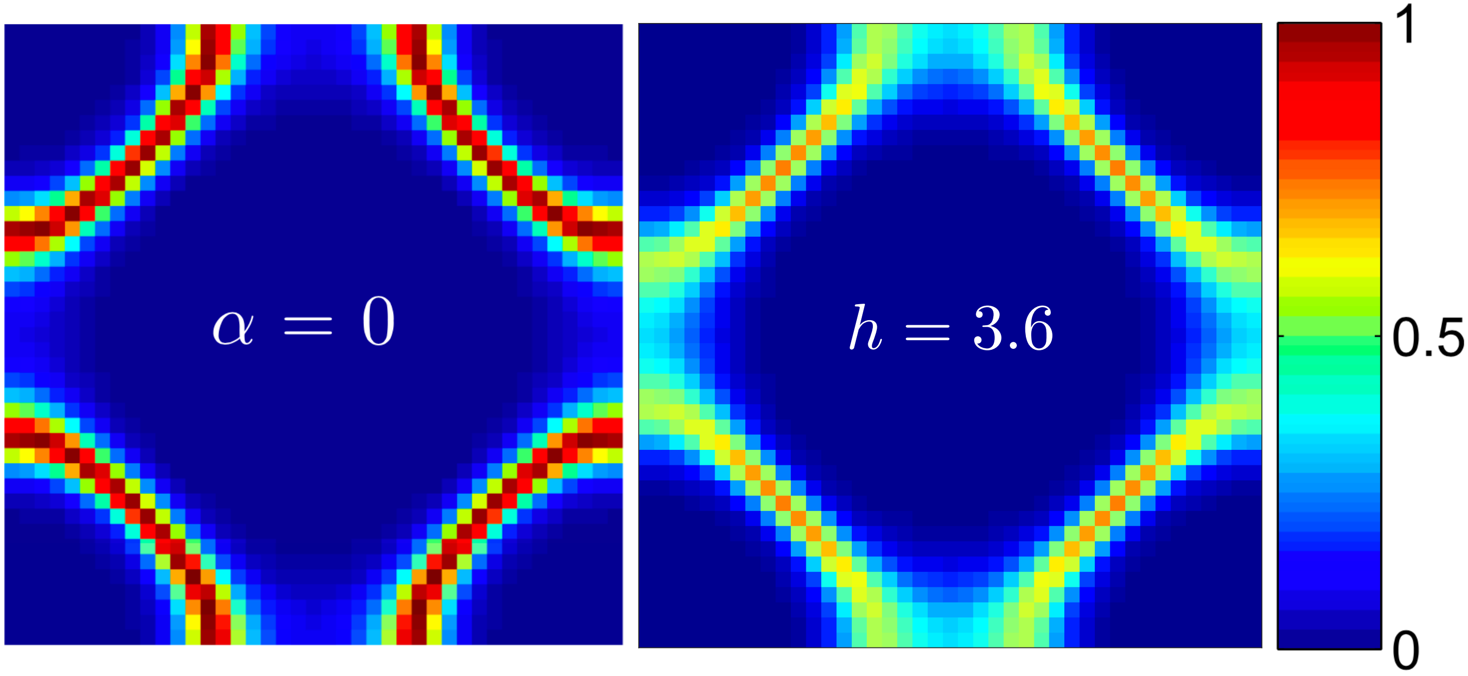
2 G(k,β/2), tuning towards h_c (β=6)



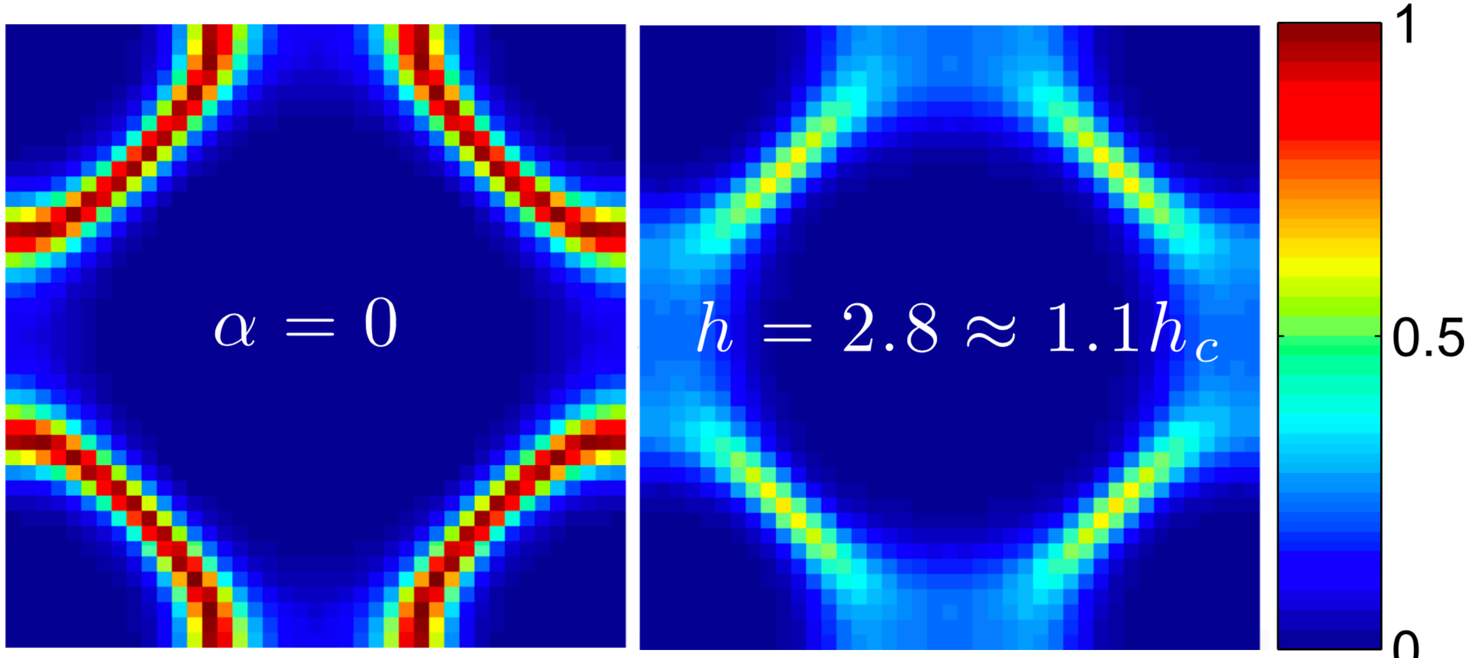
h>>h_c



h>h$_c$



$h \approx h_c$



Results from ω_n>>T regime at $h \approx h_c$

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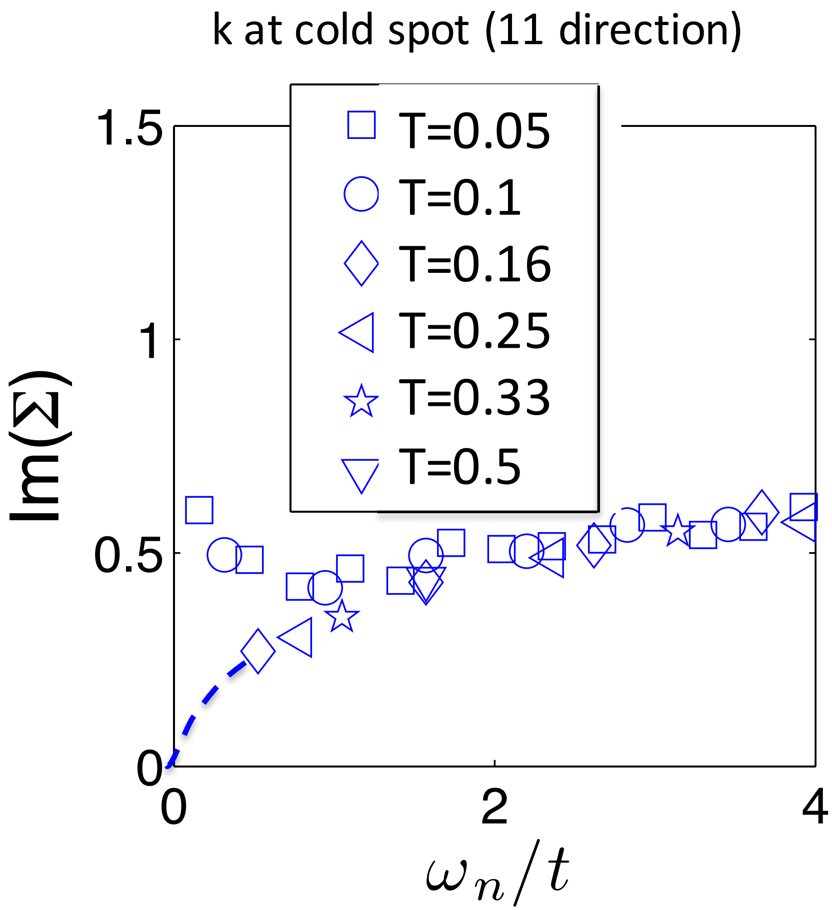
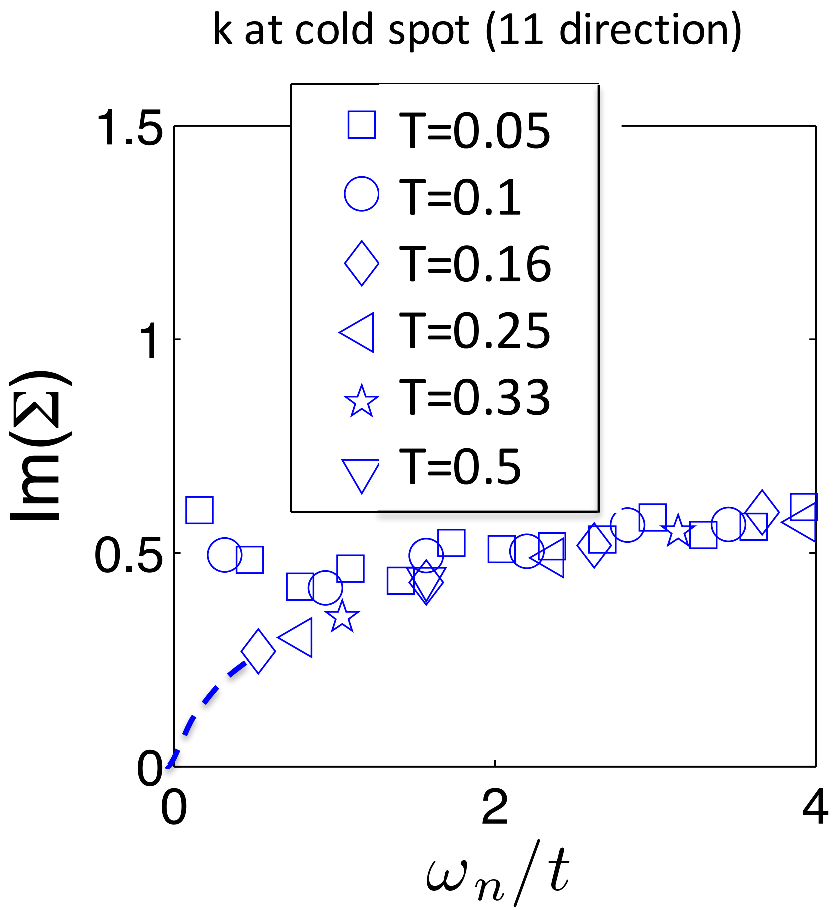
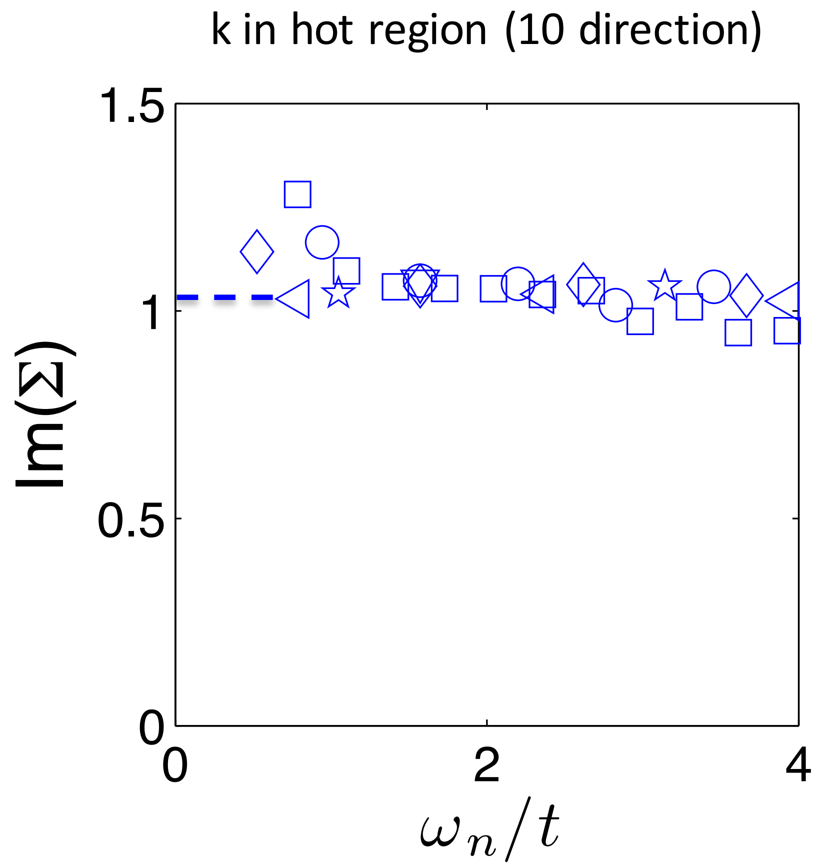
Self energy at h=h$_c$



Self energy at $h=h_c$





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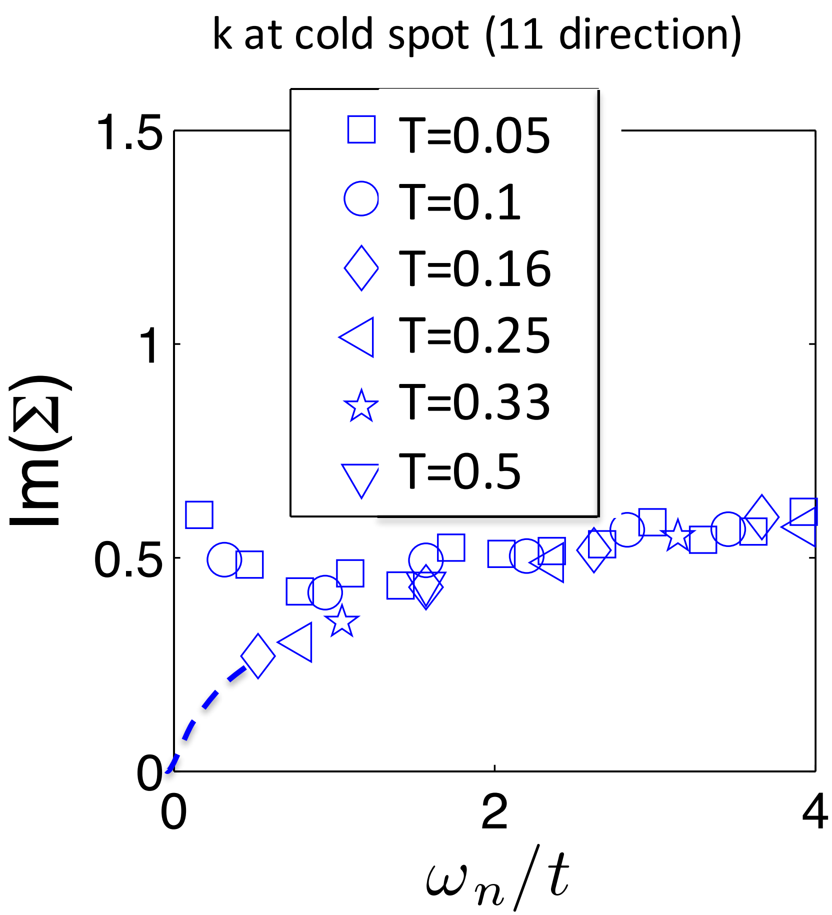
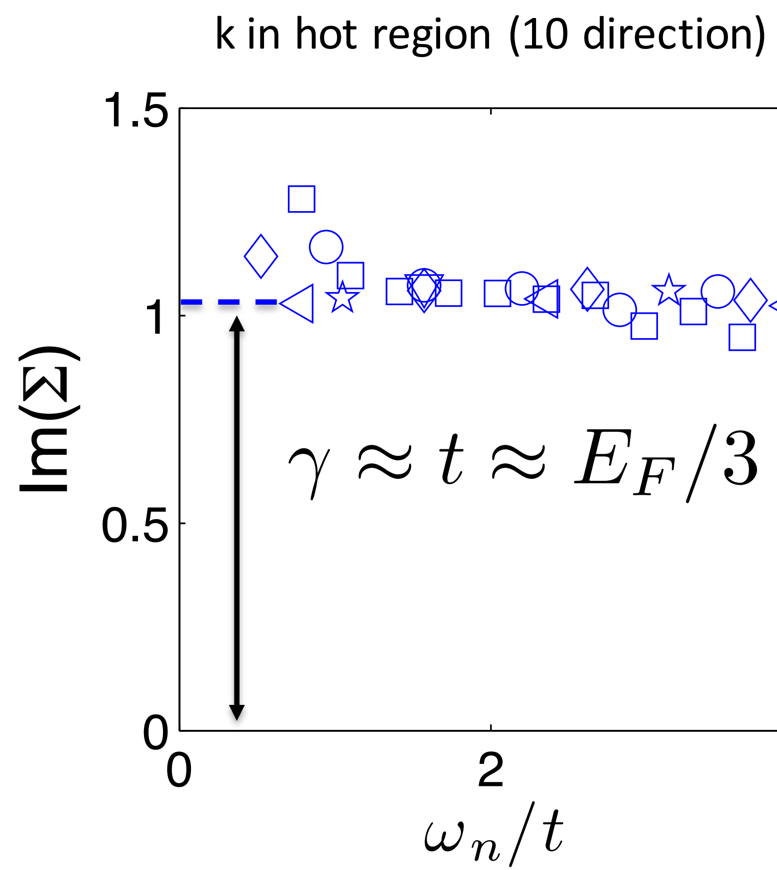
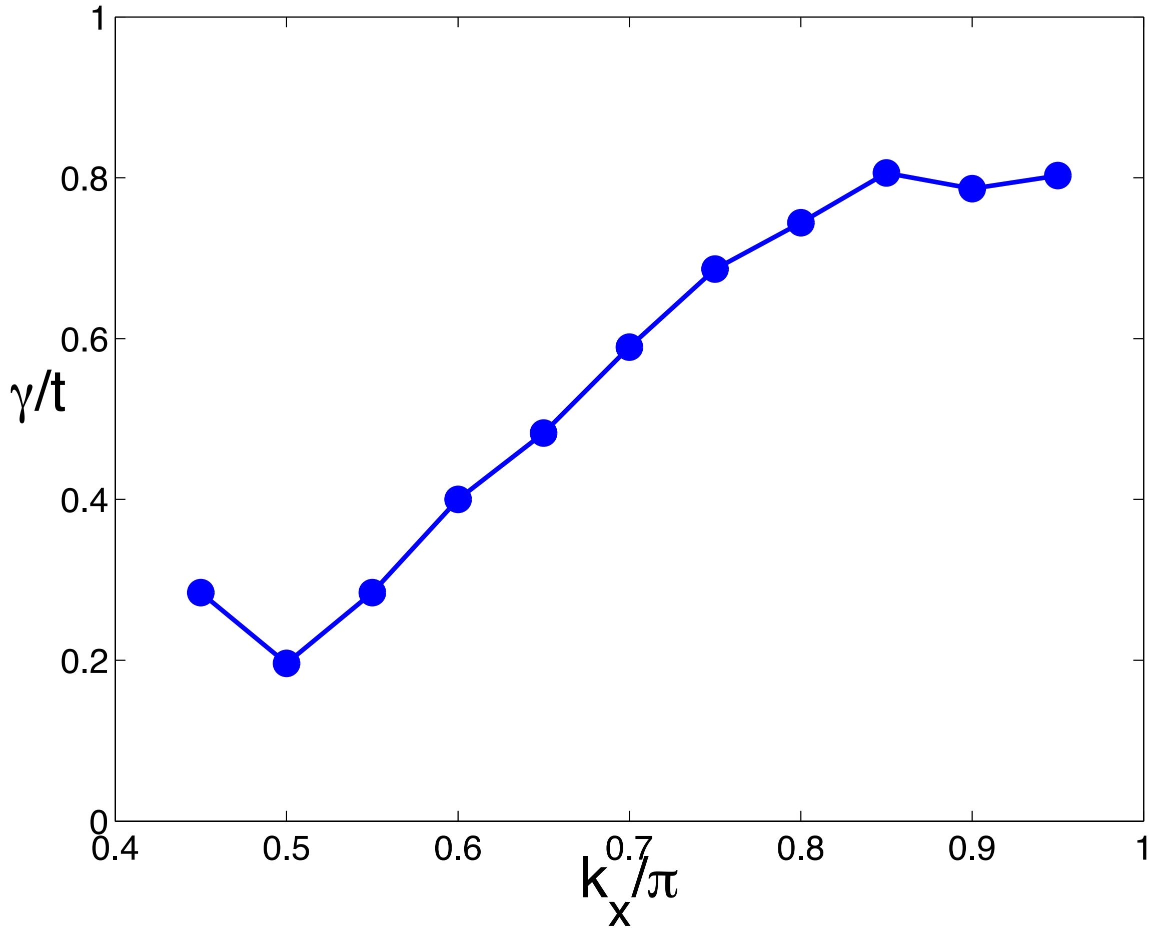
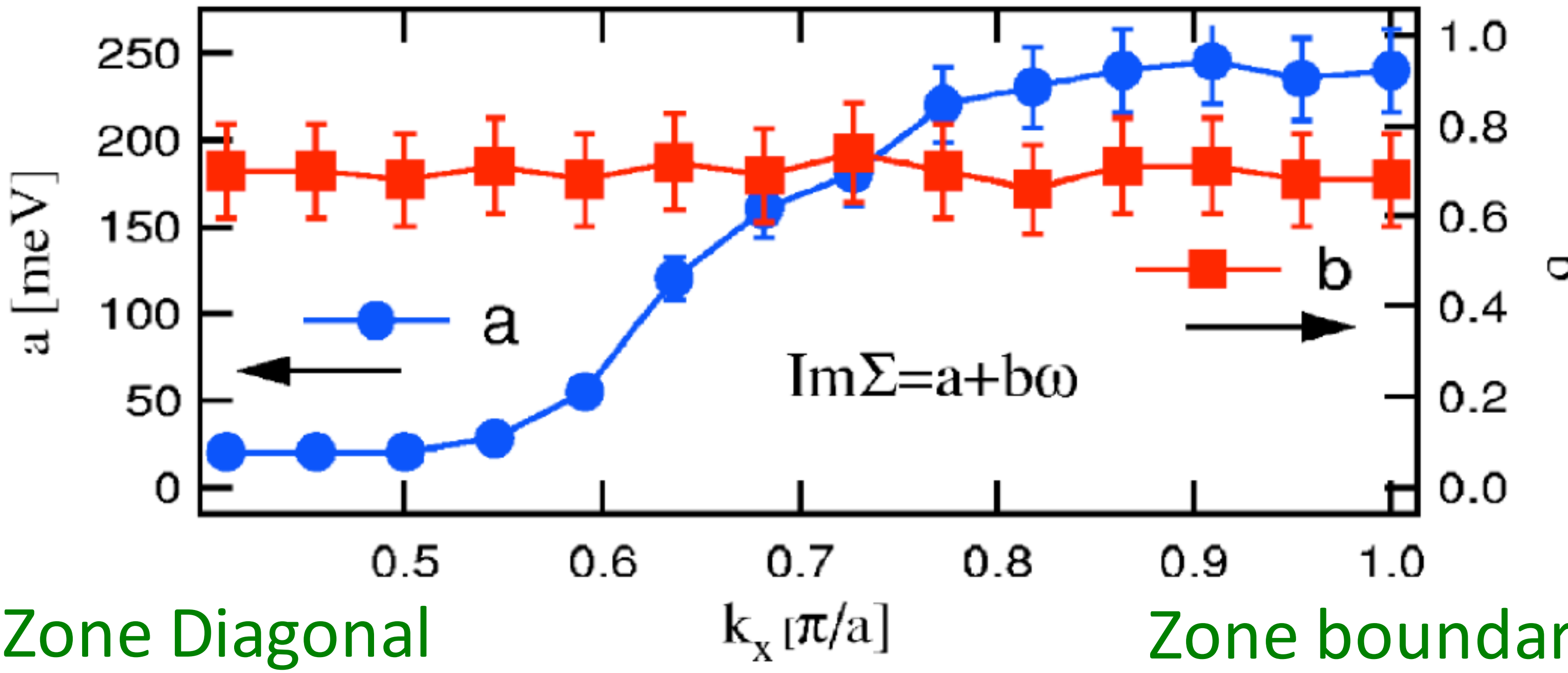




Variation of damping rate γ on "Fermi surface"



ARPES linewidths for optimally doped Bi-2212



Zone Diagonal (cold spot)

Zone boundary (hot region)

Kaminski et al. Phys. Rev. B **71**, 014517 (2005)

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- (Nearly) coherent quasiparticles at the cold spot
- Elsewhere, no apparent Fermi surface:
  - Low frequency scattering rate essentially independent of temperature, extends to high ω_n
- Much like disorder scattering from a nematic glass; unlike any known theoretical treatment

Transport from the current-current correlator

$$\Lambda_{xx}(\tau) \equiv \mathcal{T}\langle J_x(\tau) J_x(0)\rangle$$

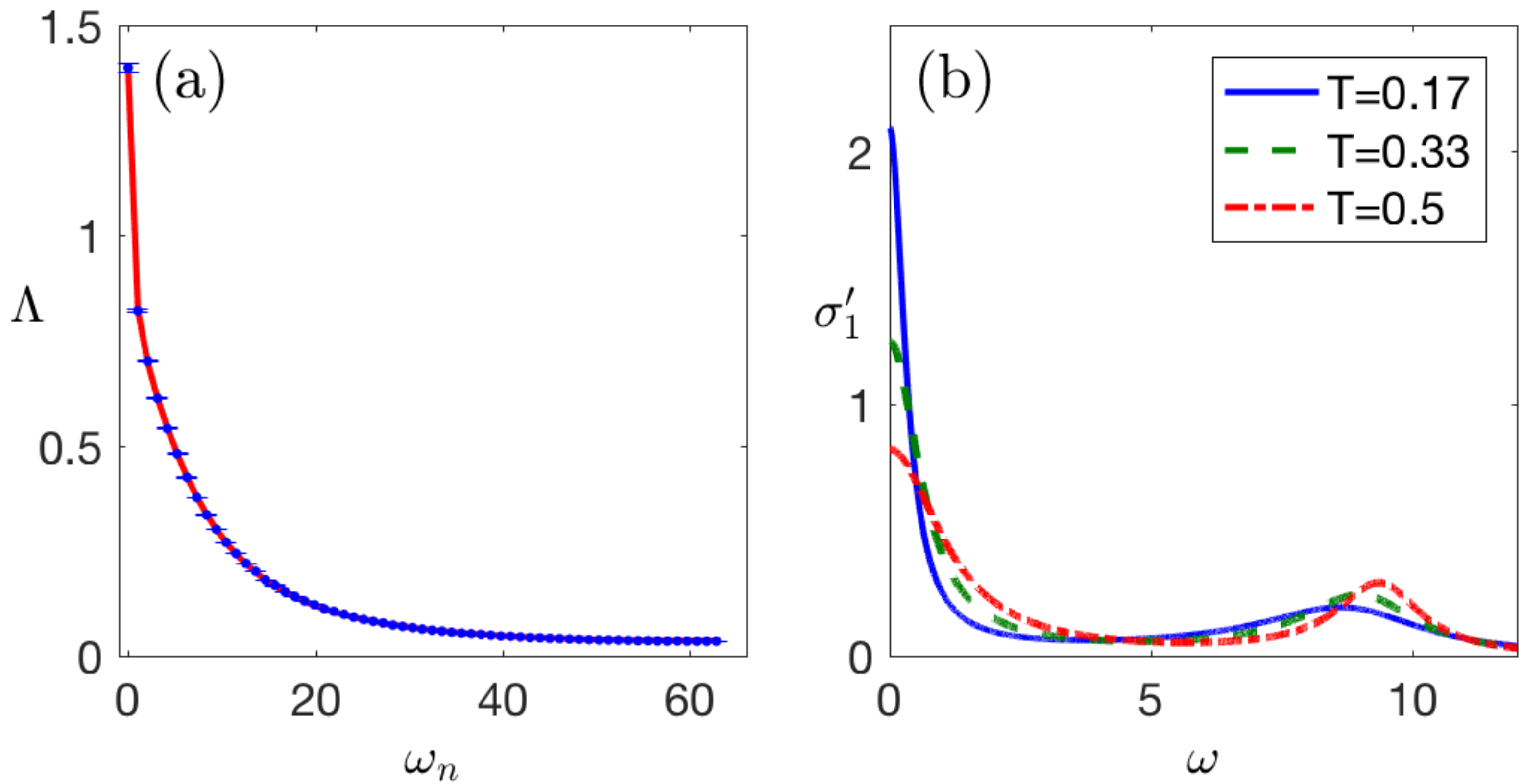
$$\Lambda_{xx}(\tau) = \int d\omega \left(\frac{\omega \sigma'_{xx}(\omega) e^{-\omega(\tau - \beta/2)}}{2\pi \sinh\left[\frac{1}{2}\beta\omega\right]} \right)$$

$$\Lambda_{xx}(i\omega_n) = \int \frac{d\omega}{\pi} \frac{\omega^2 \sigma'_{xx}(\omega)}{\omega^2 + \omega_n^2}$$

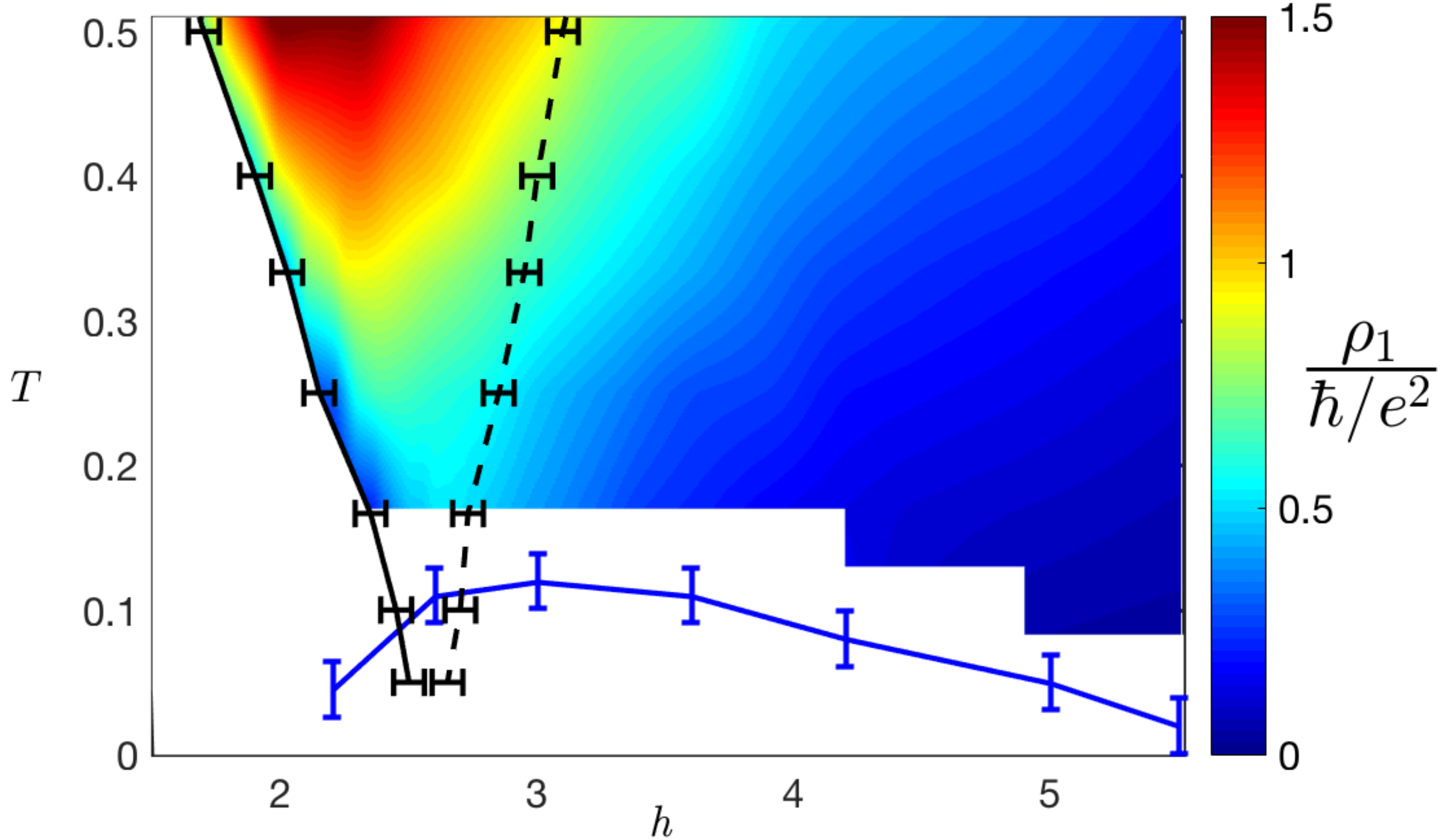
Methods to estimate DC resistivity

- Analytic continuation by maximum entropy
- Forming a "resistivity proxy" using $\Lambda(\tau)$ and its second derivative at $\tau=\beta/2$ (correct for narrow Lorentzian)
- Analytic continuation by curve fitting (assuming σ' is a sum of two Lorentz oscillators)

Analytic continuation from curve fitting



Phase diagram with transport



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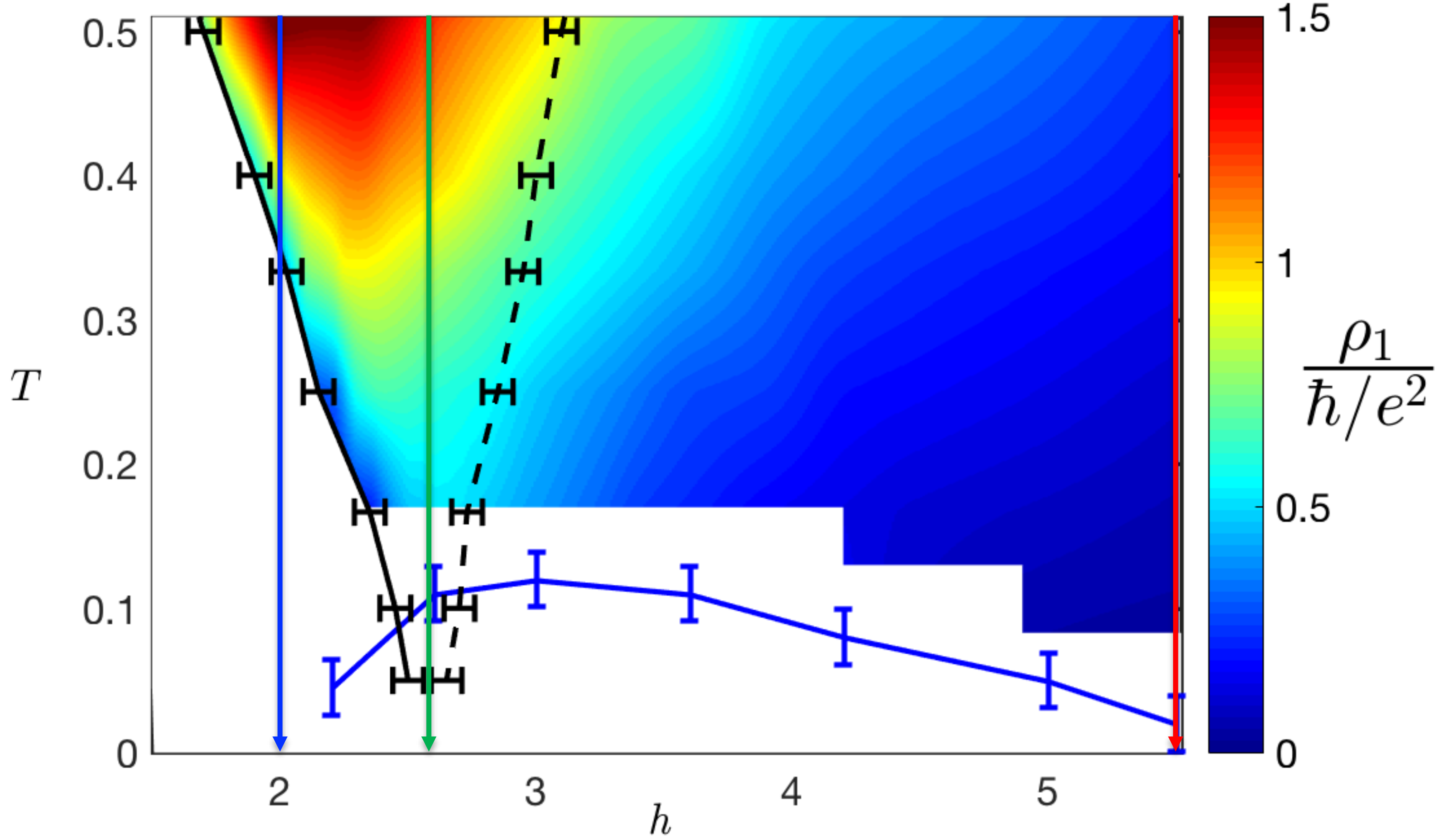
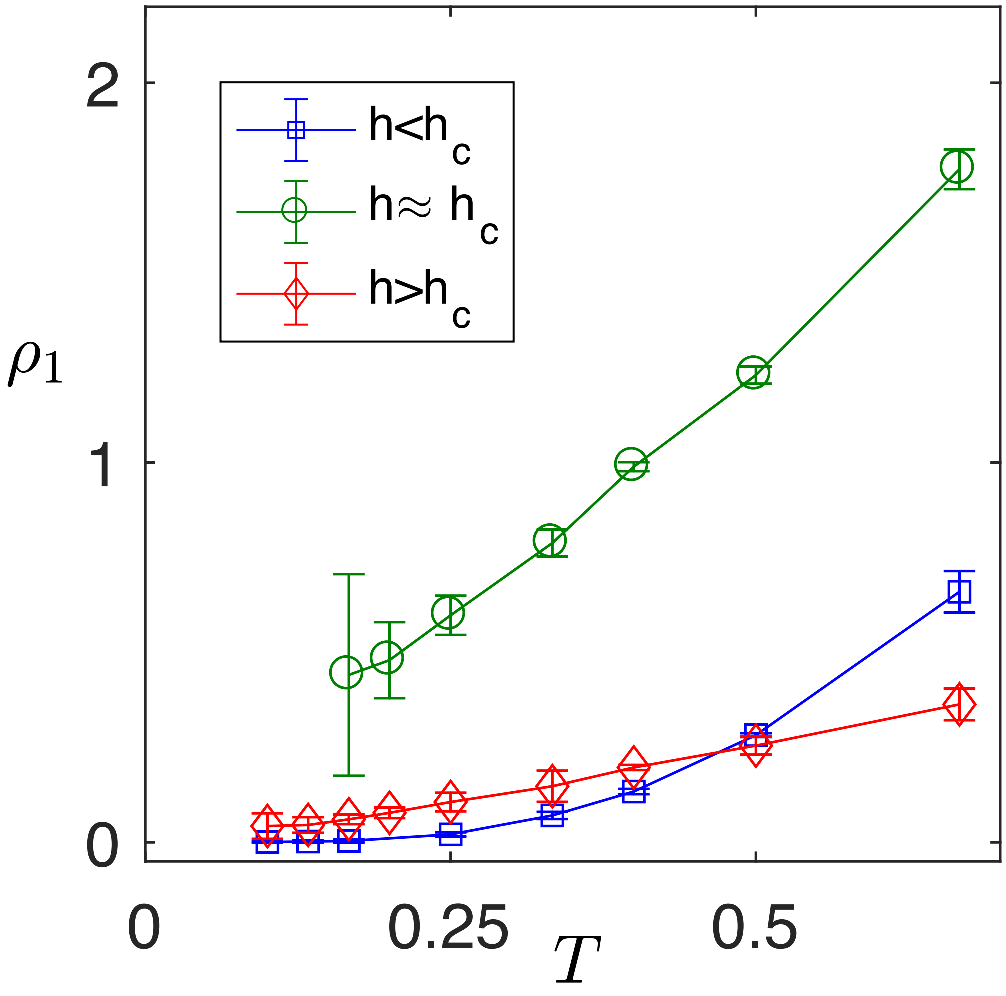


Temperature dependence



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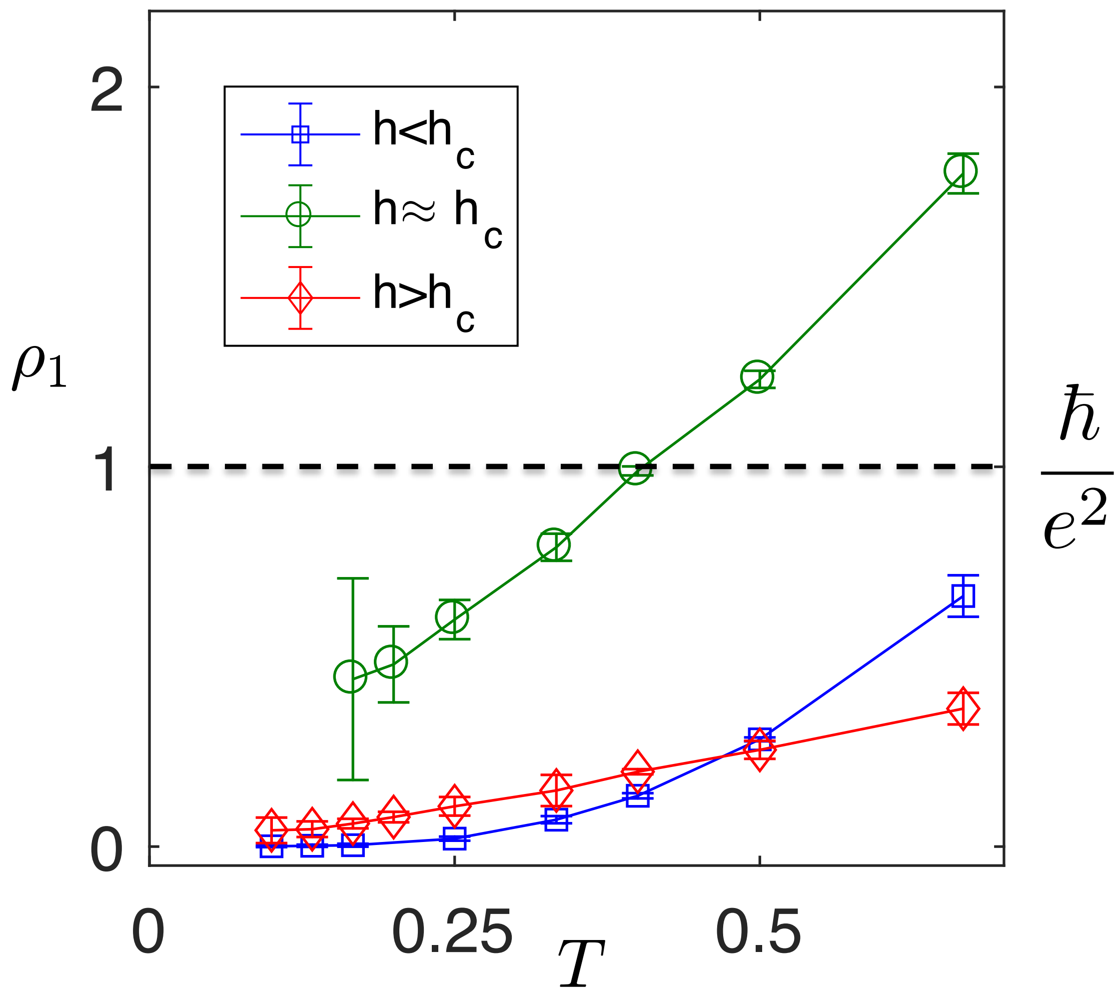

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  - High T_c superconductivity
  - Non-Fermi liquid normal state in a range of temperatures above T_C, with highly anisotropic single-particle lifetimes
  - Transport consistent with T-linear resistivity violating the Ioffe-Regel limit, i.e. “bad metal”

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- What are some distinctive experimental consequences of an Ising nematic QCP?
  - Diagrammatic calculations for Raman scattering, with Avi Klein (MN), Debanjan Chowdhury (MIT), EB, Andrey Chubukov (MN)
  - Arxiv:1708.05308

Raman scattering

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- $$I_{B_{1g}} \propto [1 + n_B(\omega)]\, \chi''(\omega), \text{ where}$$

$$\chi(\omega) = S_T(i\omega_n \to \omega + i0^+)$$

$$S_T(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n \tau} \langle \tilde{\rho}(\tau) \tilde{\rho}(0) \rangle$$

$$\tilde{\rho} \approx \sum_k [\cos(k_x) - \cos(k_y)] c_k^\dagger c_k$$

Quirks of q=0, ω>0

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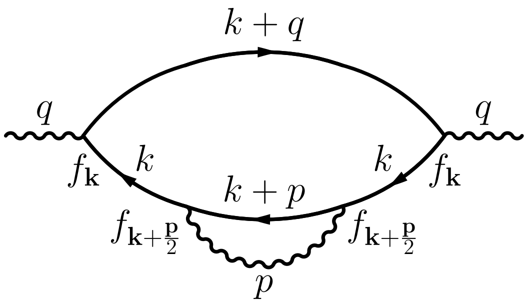
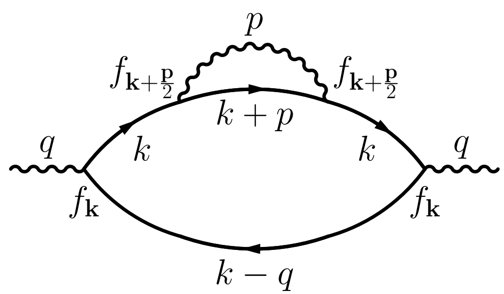
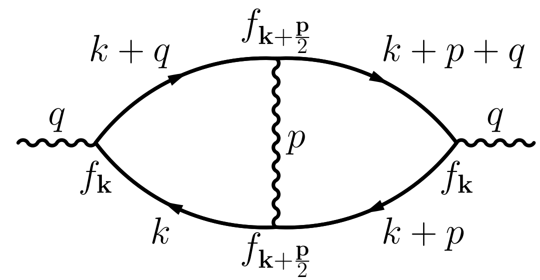
- If order parameter is conserved (total charge, total spin, etc), response must ***vanish***, since the uniform density is time-independent
  - Microscopically, nematicity/quadrupole density is **not conserved**, but it is conserved at one loop order…

$$\lim_{\mathbf{q}\to 0} \Pi_1(\mathbf{q}, iq_0 = 0) \sim \rho_0, \quad \Pi_1(\mathbf{q} = 0, iq_0) = 0$$

Two loop diagrams







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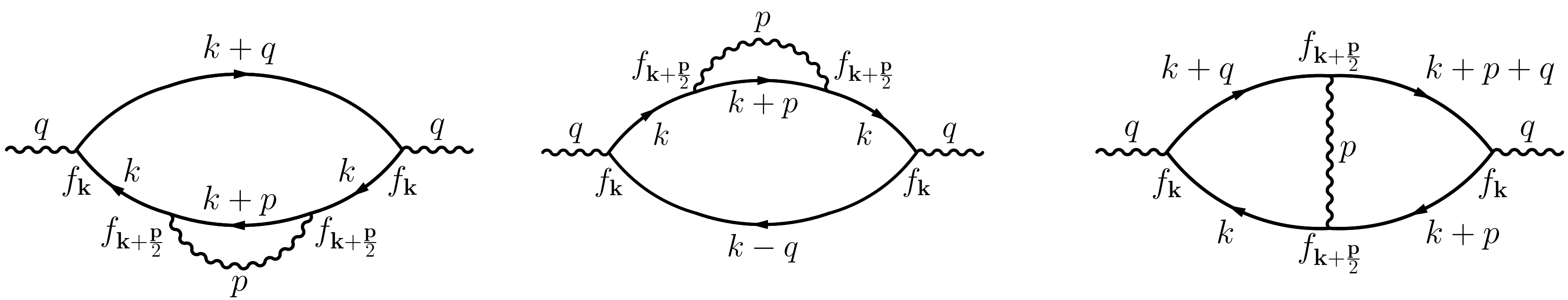
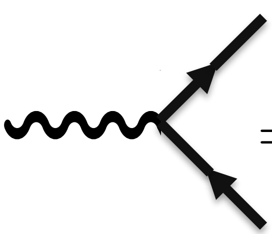


 $= D_0(p) = \dfrac{\chi_0}{|\mathbf{p}|^2 + p_0^2/c^2 + \xi_0^{-2}}$

 $= G_0(k) = \dfrac{1}{ik_0 - \epsilon_{\mathbf{k}}}$

 $= g \int dk dp \left[f_{\mathbf{k}} \phi_p \overline{\psi}_{k+p/2} \psi_{k-p/2} \right], \;\; f_{\mathbf{k}} = \cos(k_x) - \cos(k_y)$

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  III. $\omega_0 \gg \omega$: non-Fermi liquid regime

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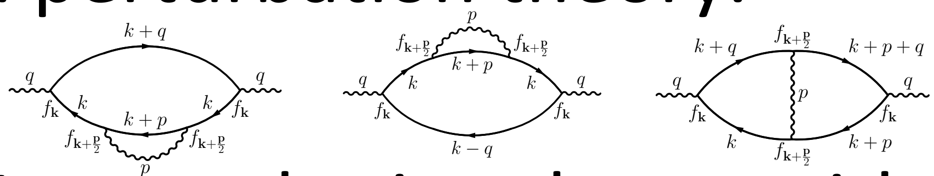
✔ II. $\omega_1 \gg \omega \gg \omega_0 \sim \bar{g}^2 / E_F$: boson dressed by Landau damping, fermions essentially free

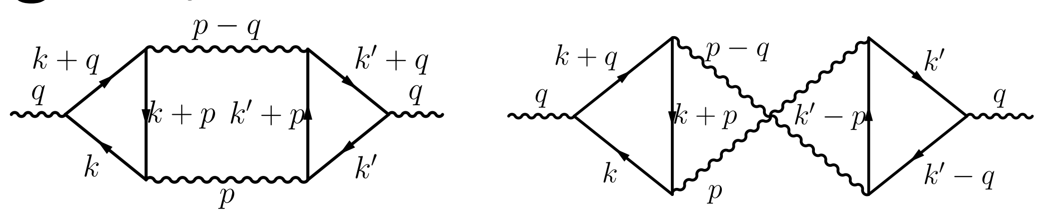
✘ III. $\omega_0 \gg \omega$: non-Fermi liquid regime

Methods

- Regime I: perturbation theory:



- Regime II: perturbation theory with dressed boson propagator, the above + Aslamazov-Larkin:



- Regime III: the above, with dressed boson, dressed fermions, ladder series of vertex corrections (uncontrolled)

Results

- Regime I:

$$\Pi(\mathbf{q}=0, iq_0) \approx \left(\frac{\bar{g}}{E_F}\right)\rho_0\left[A_I + B_I\frac{|q_0|}{E_F}\right],\ \omega_1 \ll |q_0| \ll E_F$$

- Regime II:

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- Regime III:

$$\Pi(\mathbf{q}=0, iq_0) \approx \left(\frac{\bar{g}}{E_F}\right)\rho_0\left[A_{II} + B_{II}\left(\frac{\bar{g}|q_0|}{E_F^2}\right)^{1/3}\right],\ |q_0| \ll \omega_0$$

Connecting to experiment

- Raman response related to nematic susceptibility:

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Part II

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