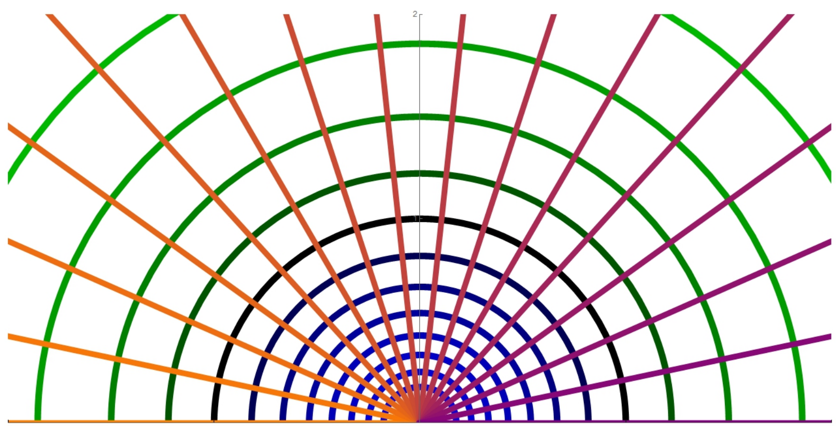
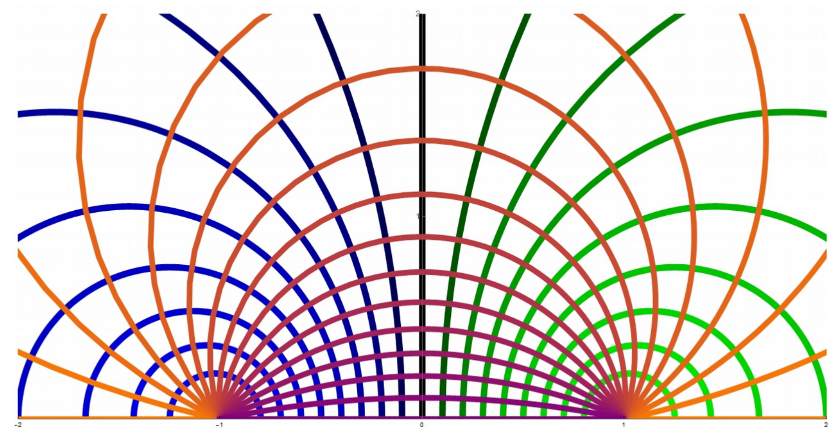
Symmetries and Dualities of Dirac Fermions in 2+1 Dimensions

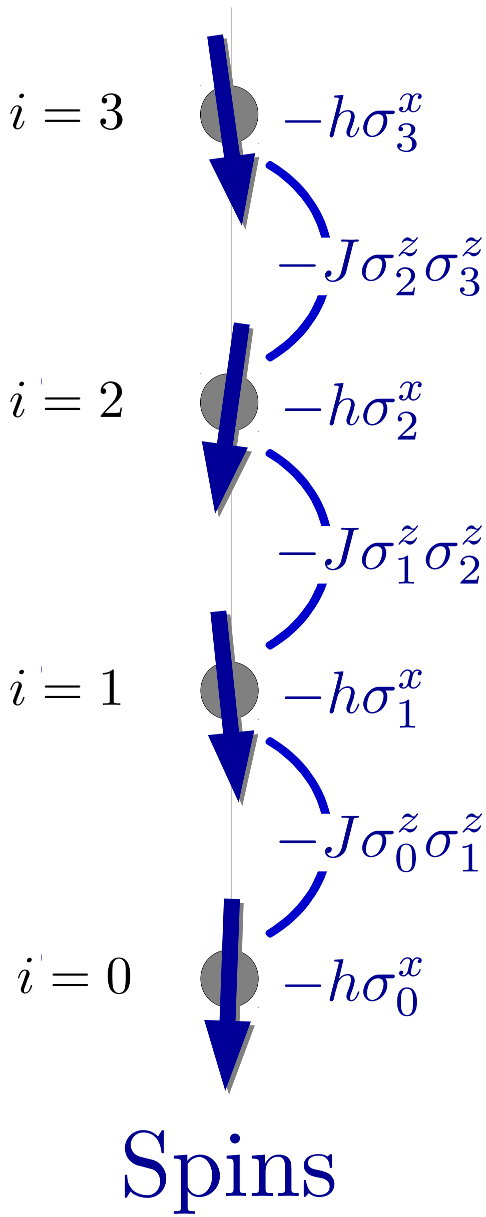
David F. Mross
(with Jason Alicea and Olexei I. Motrunich)

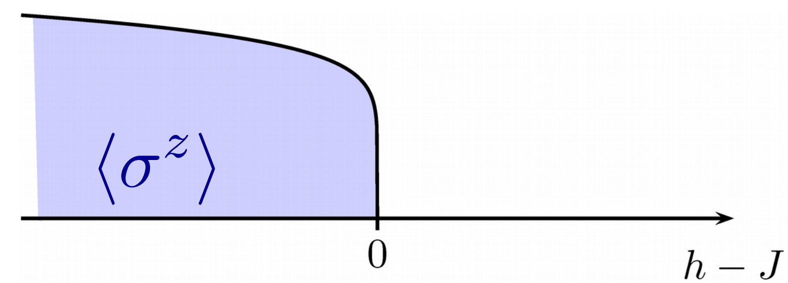




DFM, Essin, Alicea, PRX 5, 011011 (2015)
DFM, Alicea, Motrunich, PRL 117, 016802 (2016)
DFM, Alicea, Motrunich, cond-mat/1706.01106 (2017)

Warm up in 1+1 dimensions: Quantum Ising model





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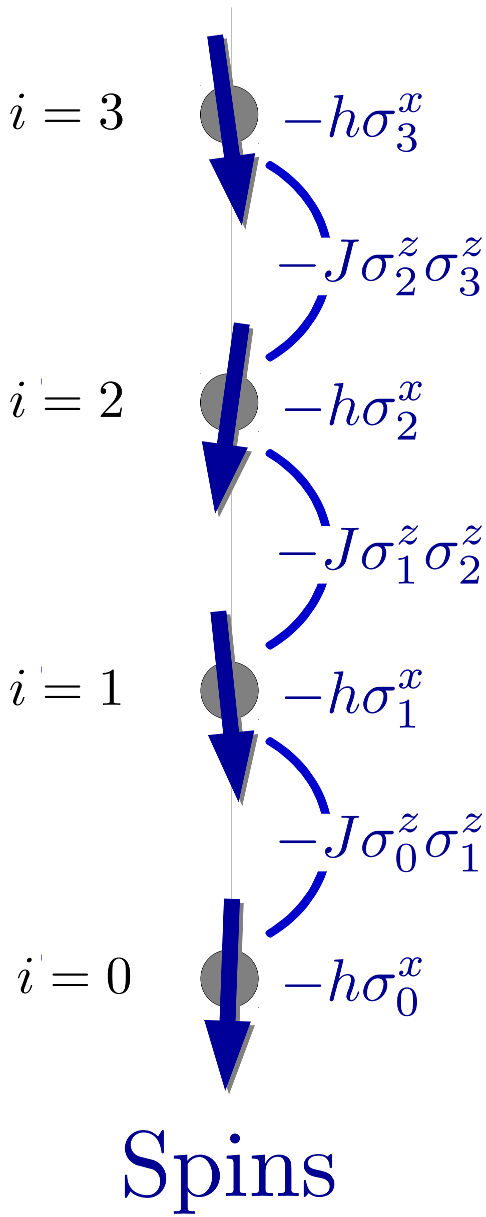

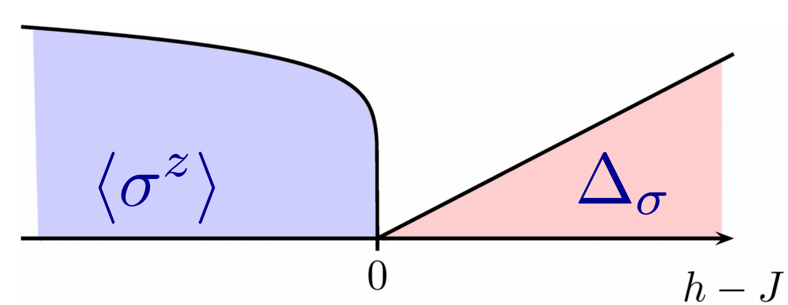

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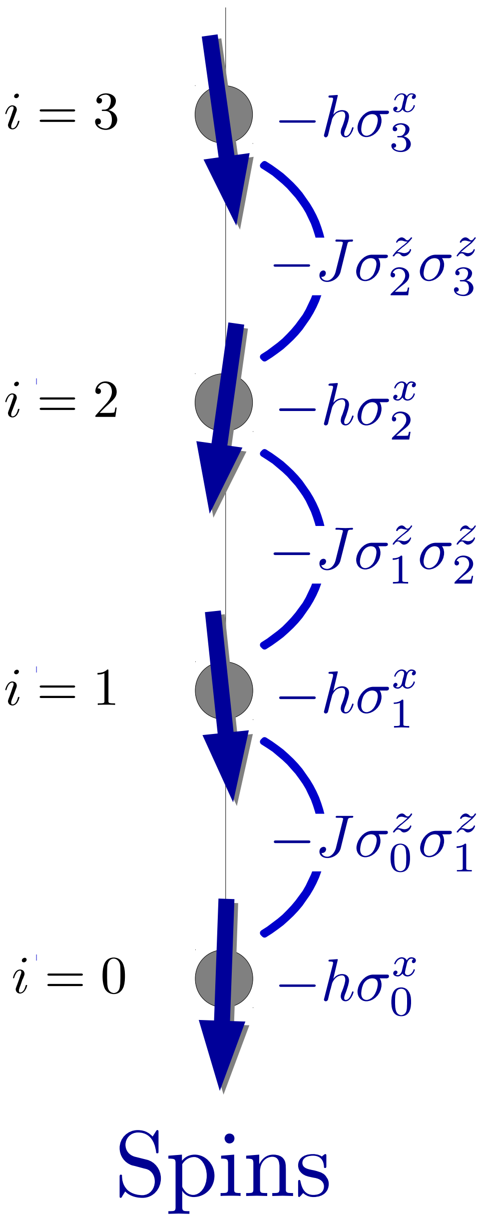
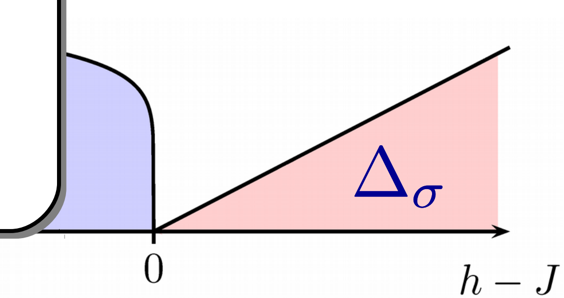


Duality Transformation

$$\sigma_i^z \sigma_{i+1}^z = \tau_{i+\frac{1}{2}}^x$$

$$\sigma_i^x = \tau_{i-\frac{1}{2}}^z \tau_{i+\frac{1}{2}}^z$$



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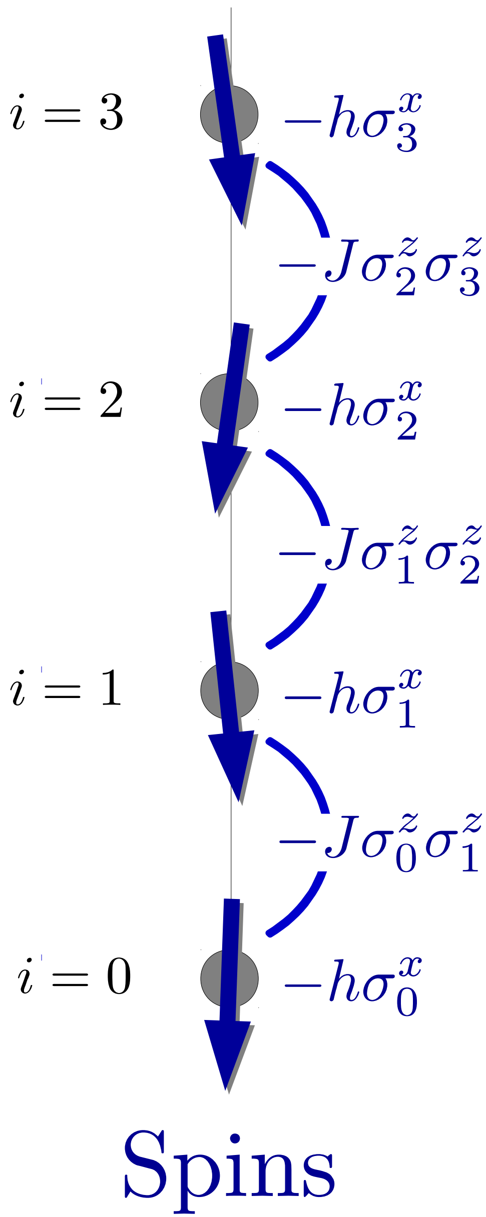


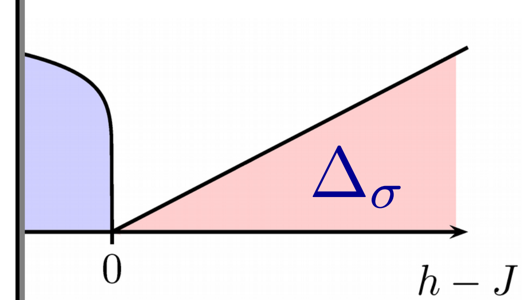
Duality Transformation

$$\sigma_i^z\sigma_{i+1}^z = \tau_{i+\frac{1}{2}}^x$$

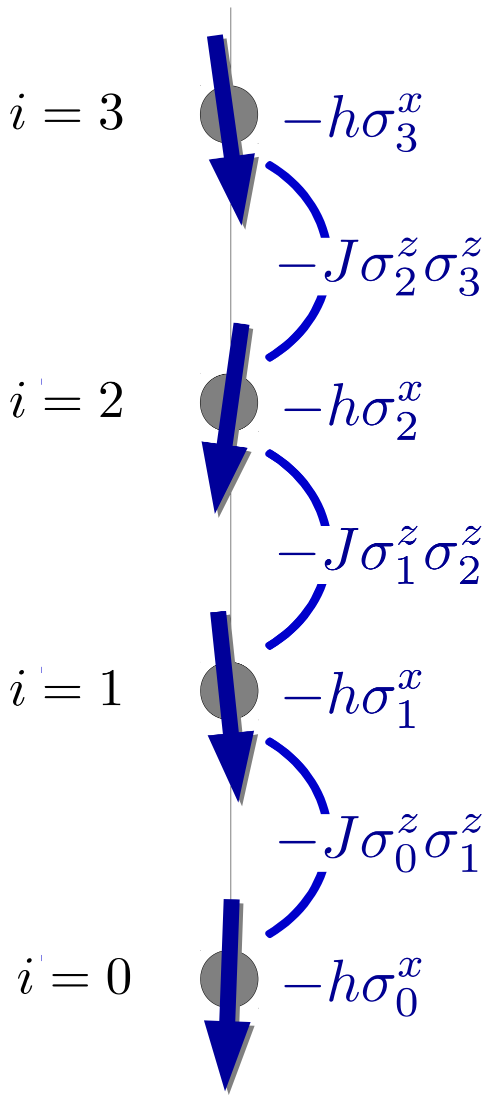
$$\sigma_i^x = \tau_{i-\frac{1}{2}}^z\tau_{i+\frac{1}{2}}^z$$

non-local:

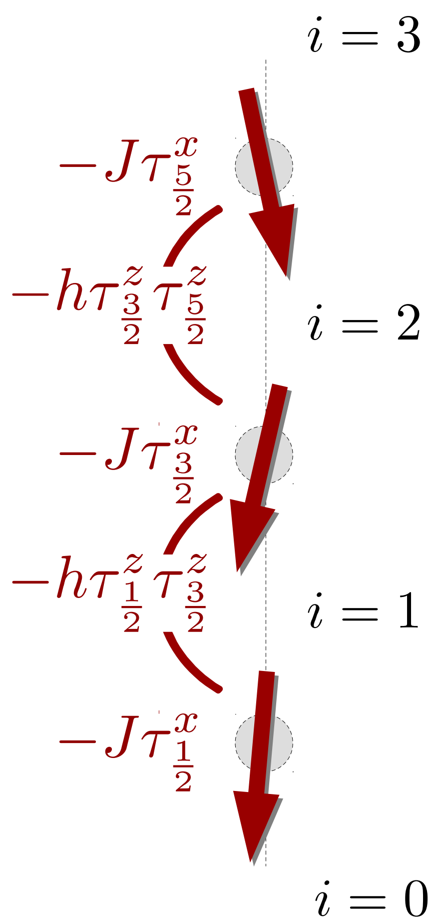
$$\sigma_i^z = \prod_{j<i}\tau_{j+\frac{1}{2}}^x$$



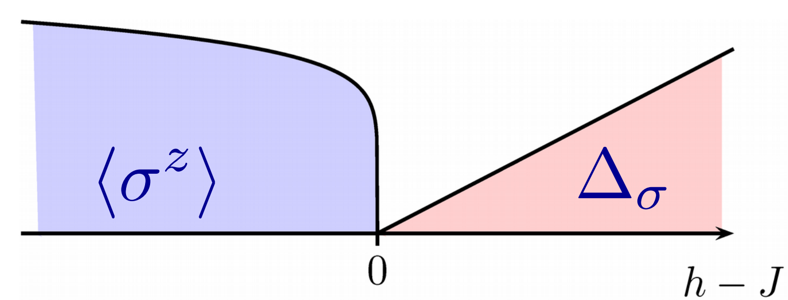
Warm up in 1+1 dimensions: Quantum Ising model



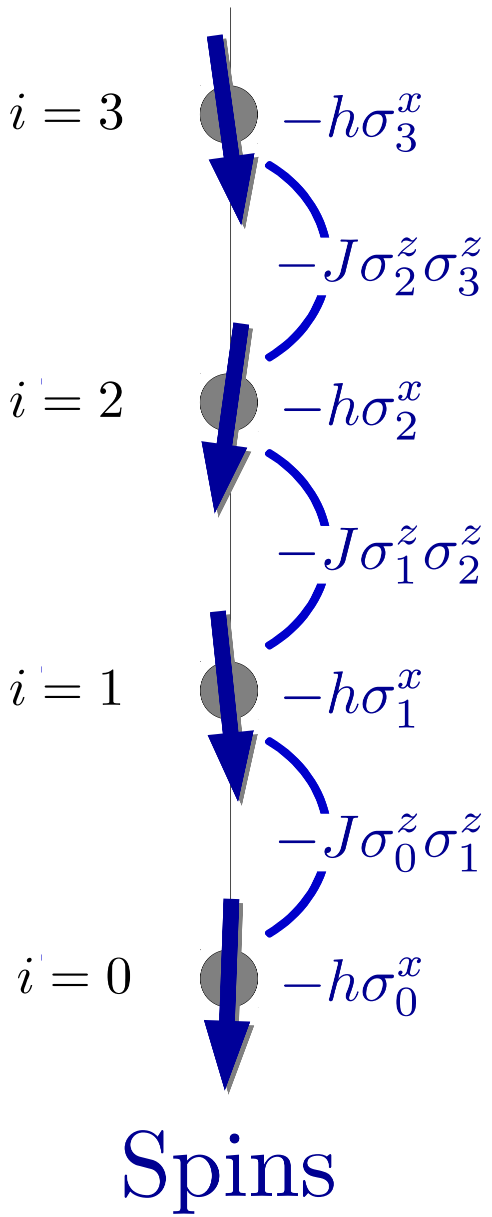
Spins

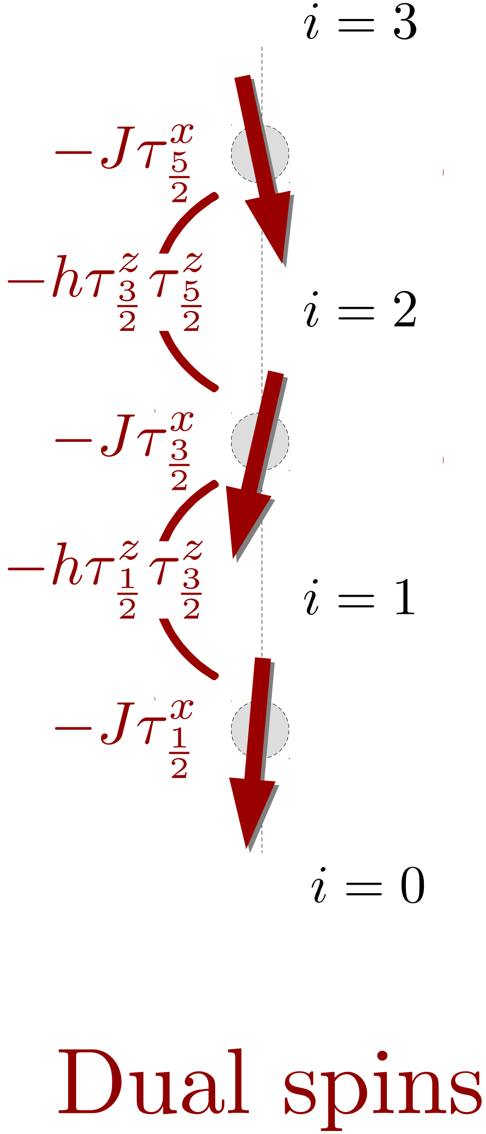


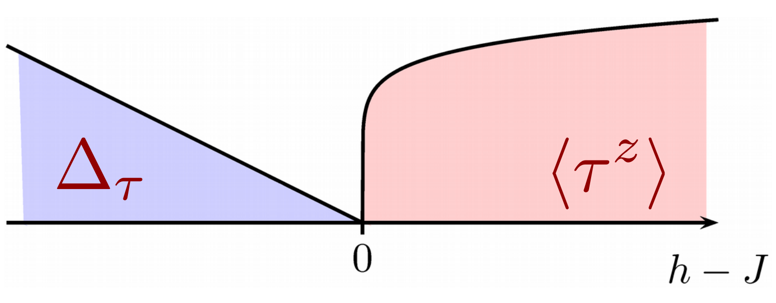
Dual spins



Warm up in 1+1 dimensions:
Quantum Ising model







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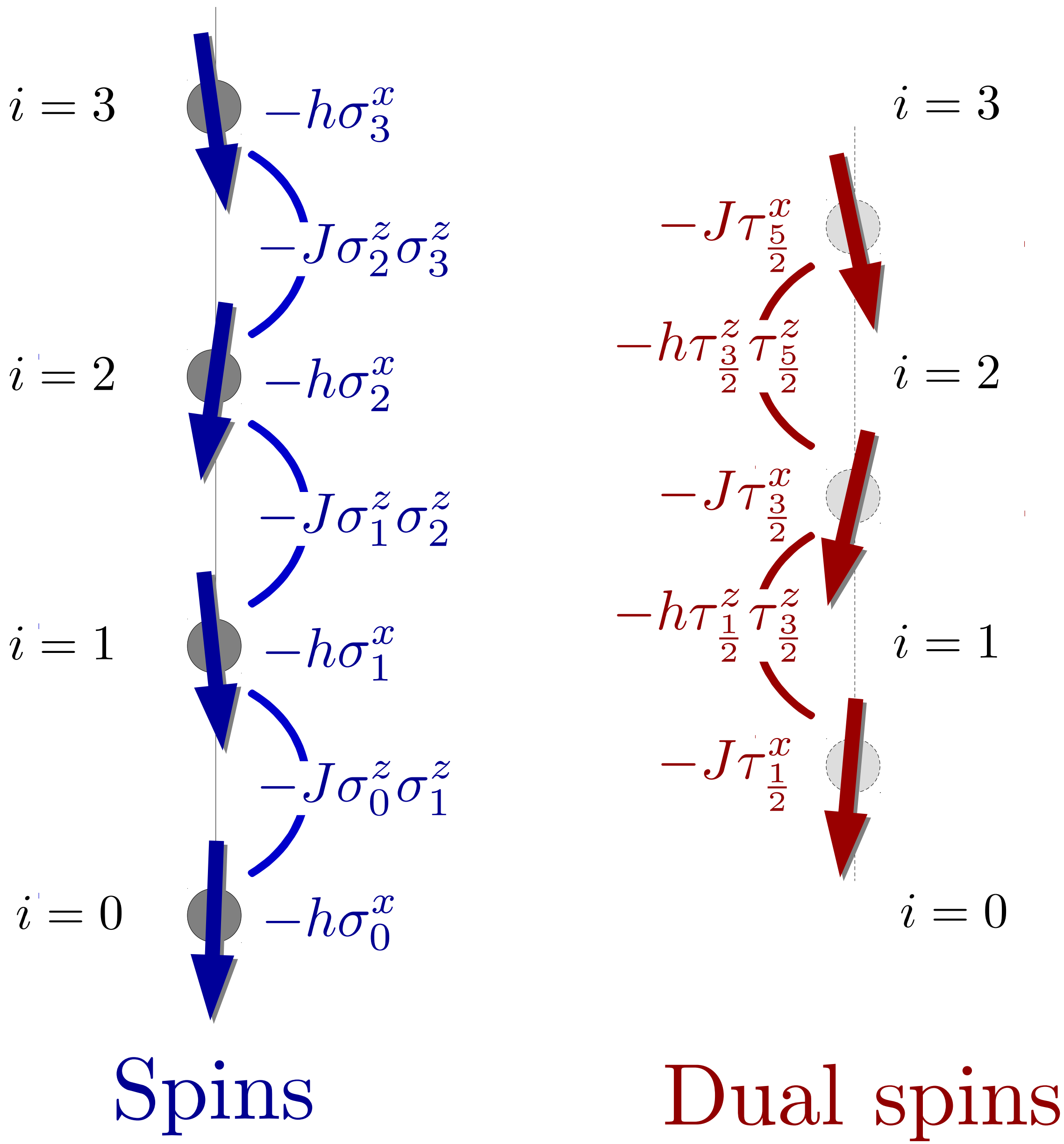

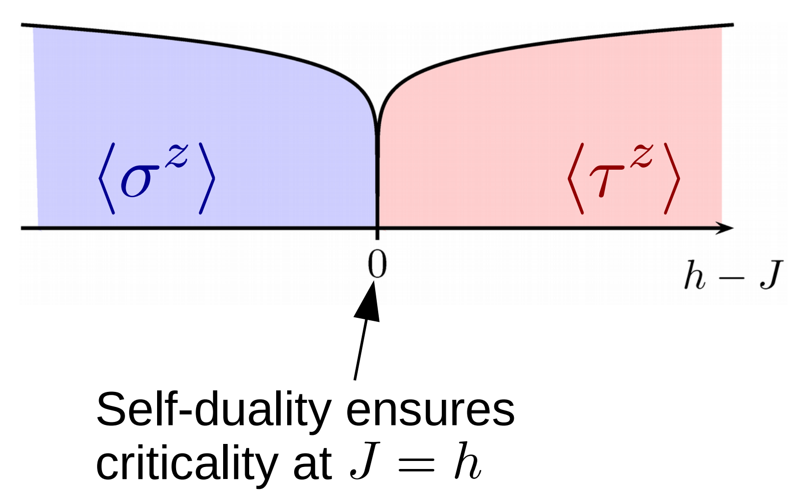

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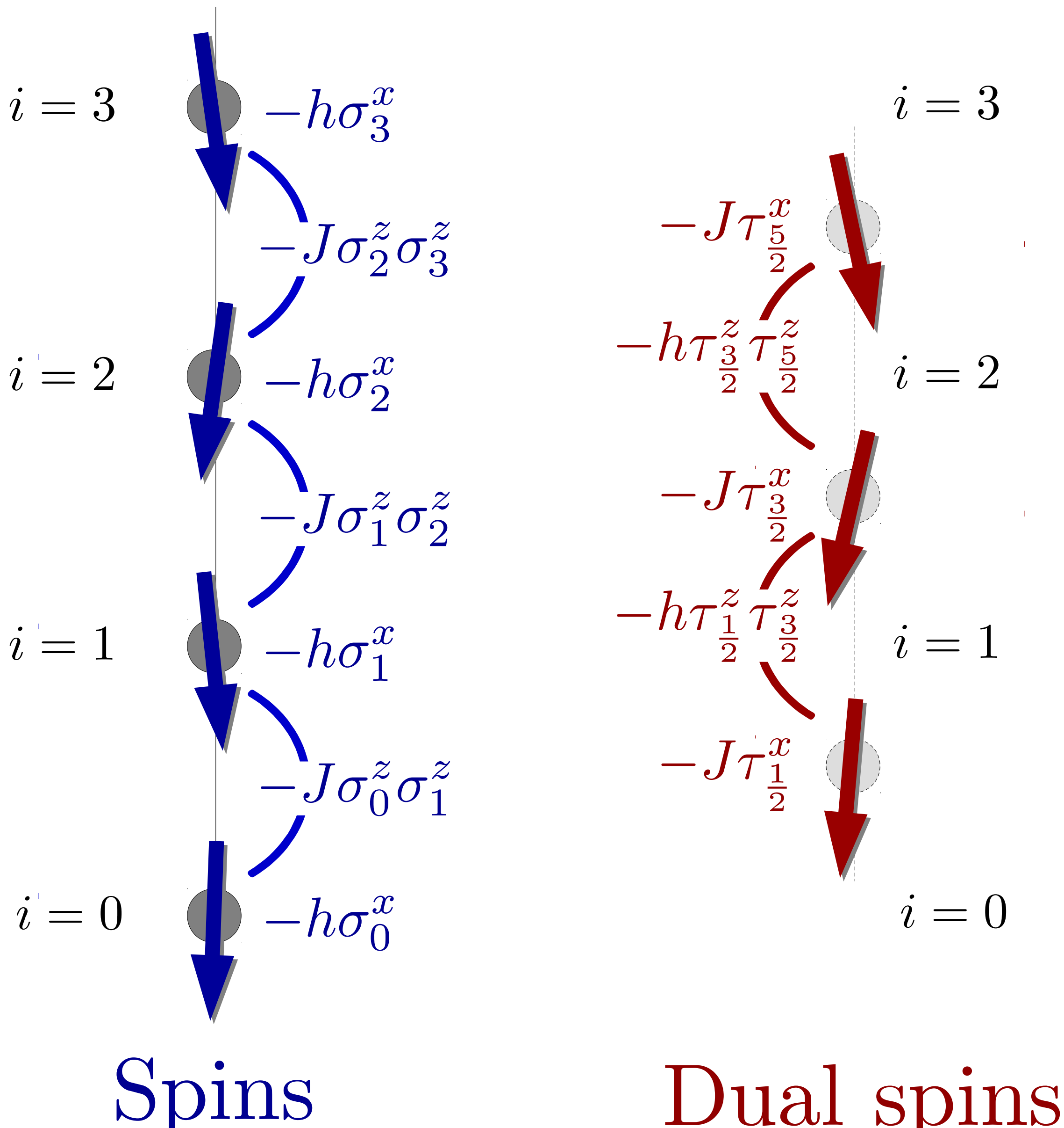
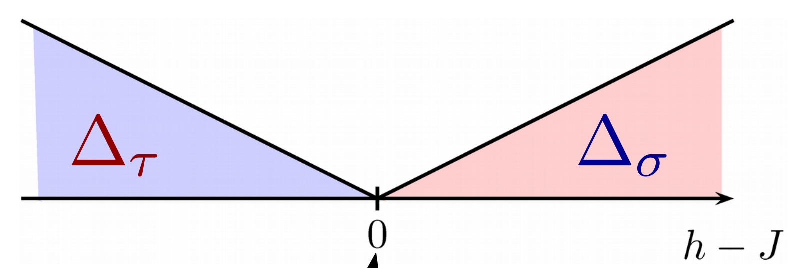
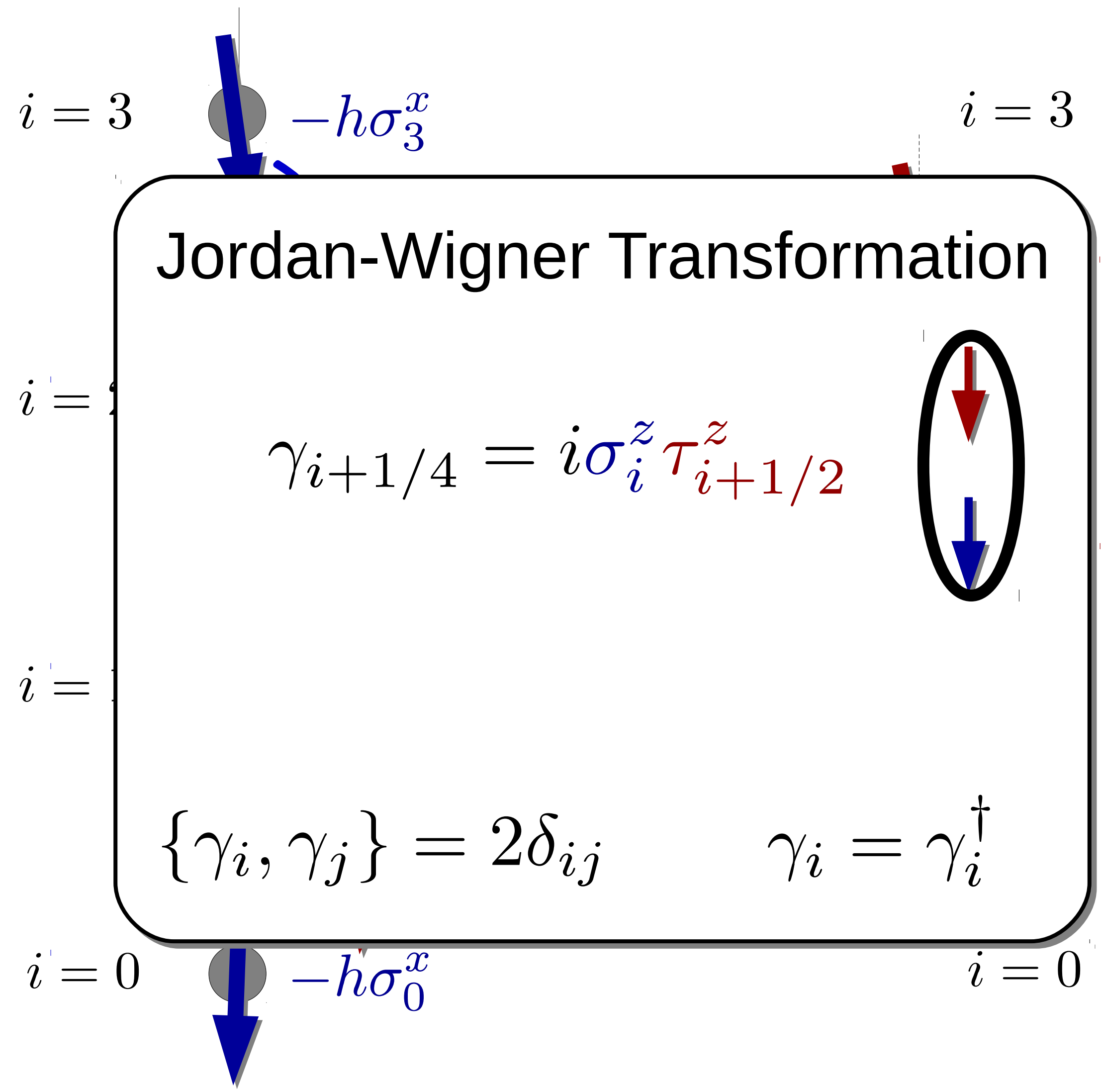




Self-duality ensures criticality at $J=h$

Warm up in 1+1 dimensions: Quantum Ising model



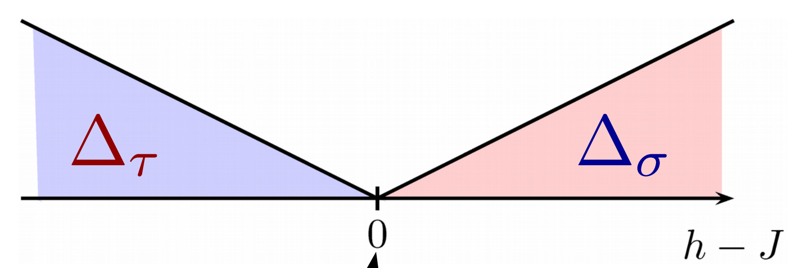
Jordan-Wigner Transformation

$$\gamma_{i+1/4} = i\sigma_i^z \tau_{i+1/2}^z$$

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij} \qquad \gamma_i = \gamma_i^\dagger$$

Spins

Dual spins



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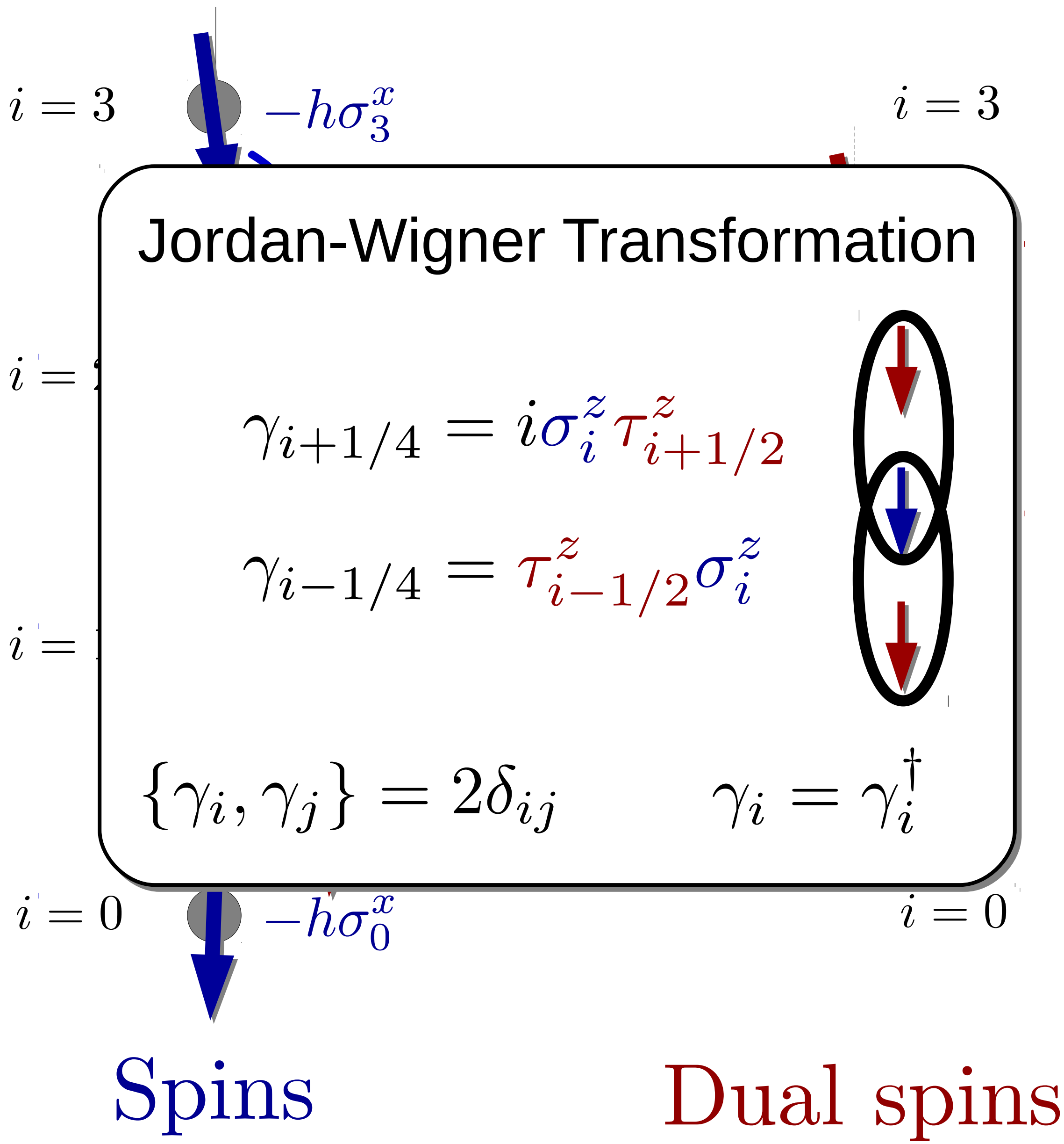
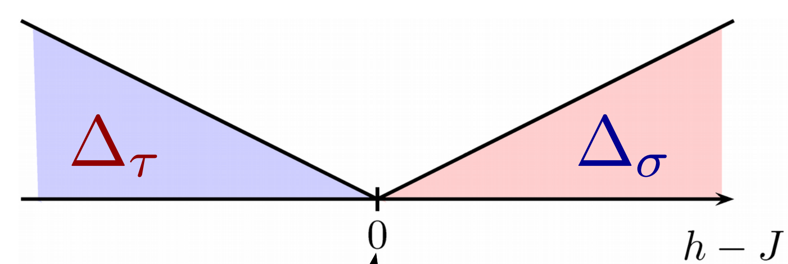


Jordan-Wigner Transformation

$$\gamma_{i+1/4} = i\sigma_i^z \tau_{i+1/2}^z$$

$$\gamma_{i-1/4} = \tau_{i-1/2}^z \sigma_i^z$$

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij} \qquad \gamma_i = \gamma_i^\dagger$$



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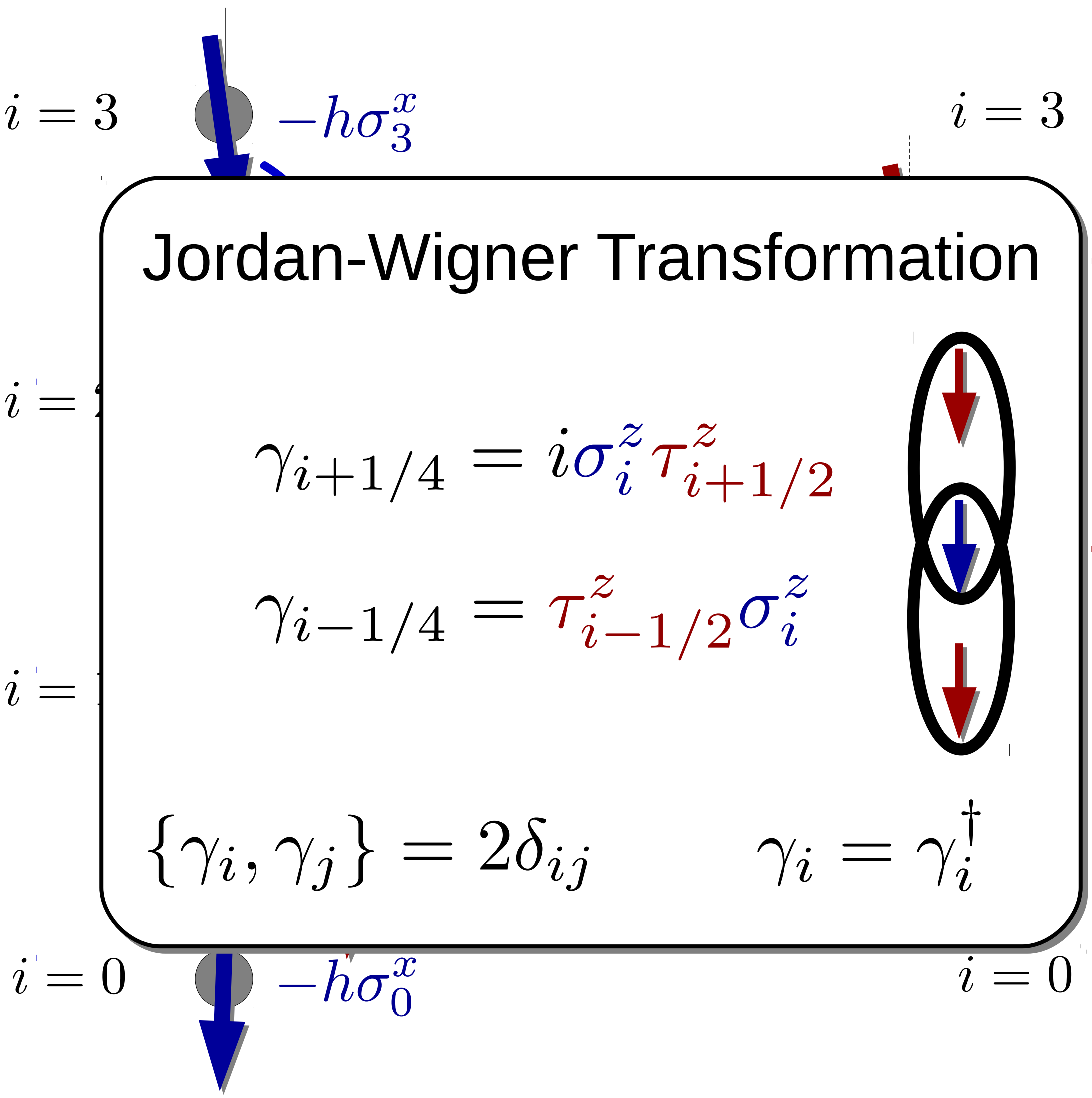
Warm up in 1+1 dimensions: Quantum Ising model



Jordan-Wigner Transformation

$$\gamma_{i+1/4} = i\sigma_i^z \tau_{i+1/2}^z$$

$$\gamma_{i-1/4} = \tau_{i-1/2}^z \sigma_i^z$$

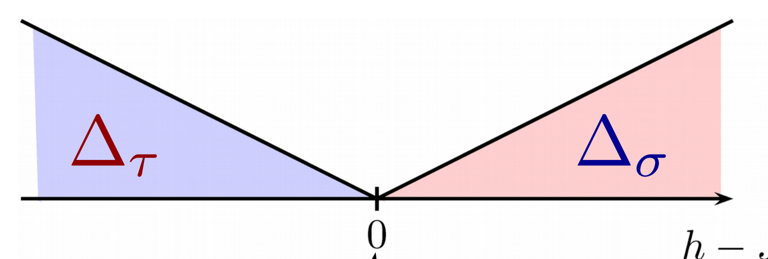
$$\{\gamma_i, \gamma_j\} = 2\delta_{ij} \qquad \gamma_i = \gamma_i^\dagger$$

Spins

Dual spins

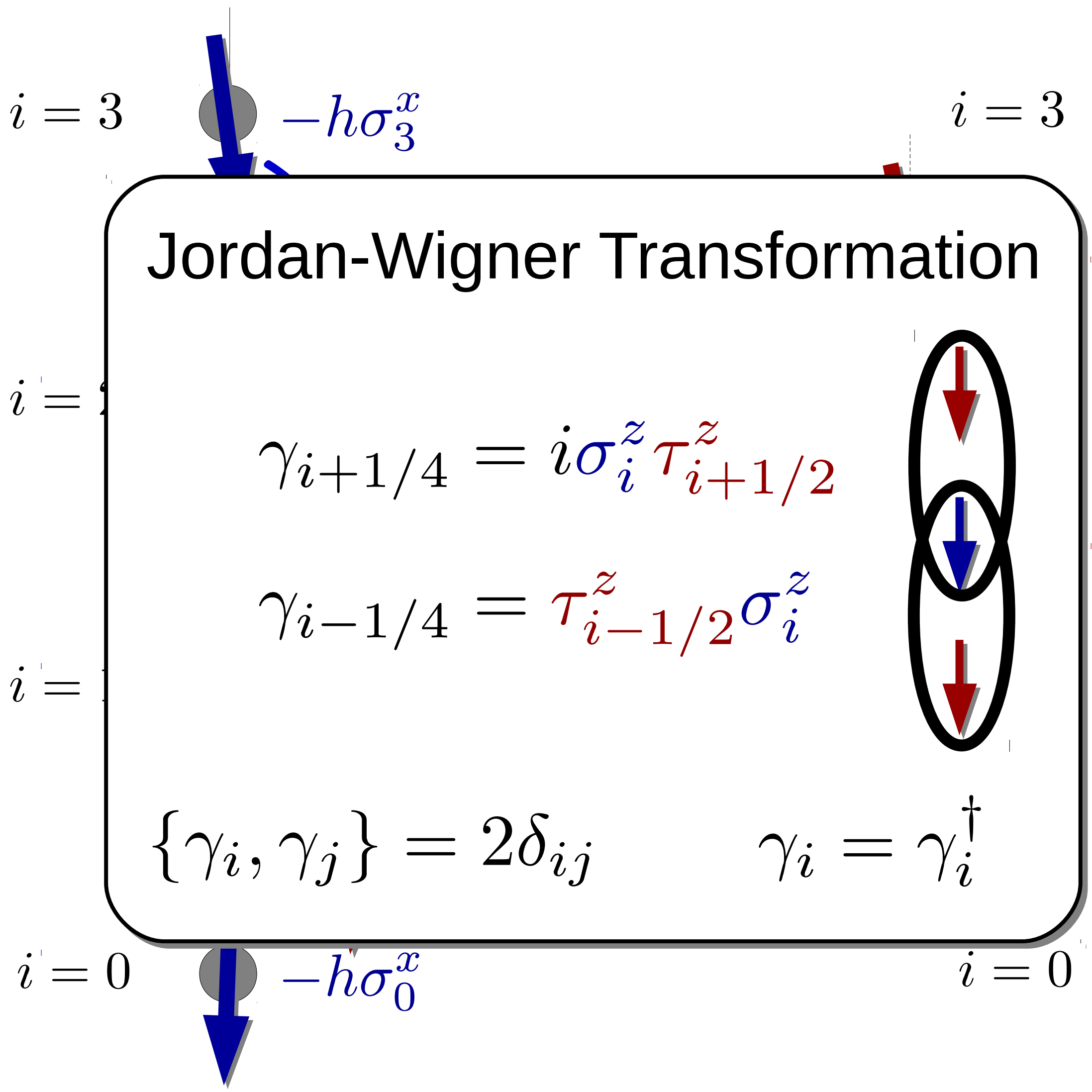
Gapped fermions

$$m = \Delta_\sigma + \Delta_\tau$$



Self-duality ensures criticality at $J = h$

Warm up in 1+1 dimensions: Quantum Ising model



Jordan-Wigner Transformation

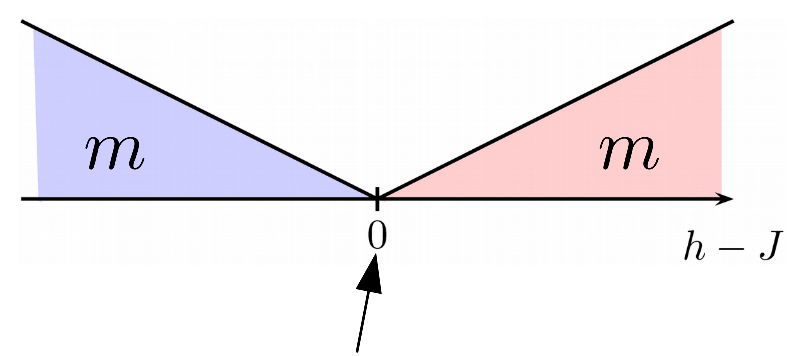
$$\gamma_{i+1/4} = i\sigma^z_i \tau^z_{i+1/2}$$

$$\gamma_{i-1/4} = \tau^z_{i-1/2}\sigma^z_i$$

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij} \qquad \gamma_i = \gamma_i^\dagger$$

Gapped fermions

$$m = \Delta_\sigma + \Delta_\tau$$

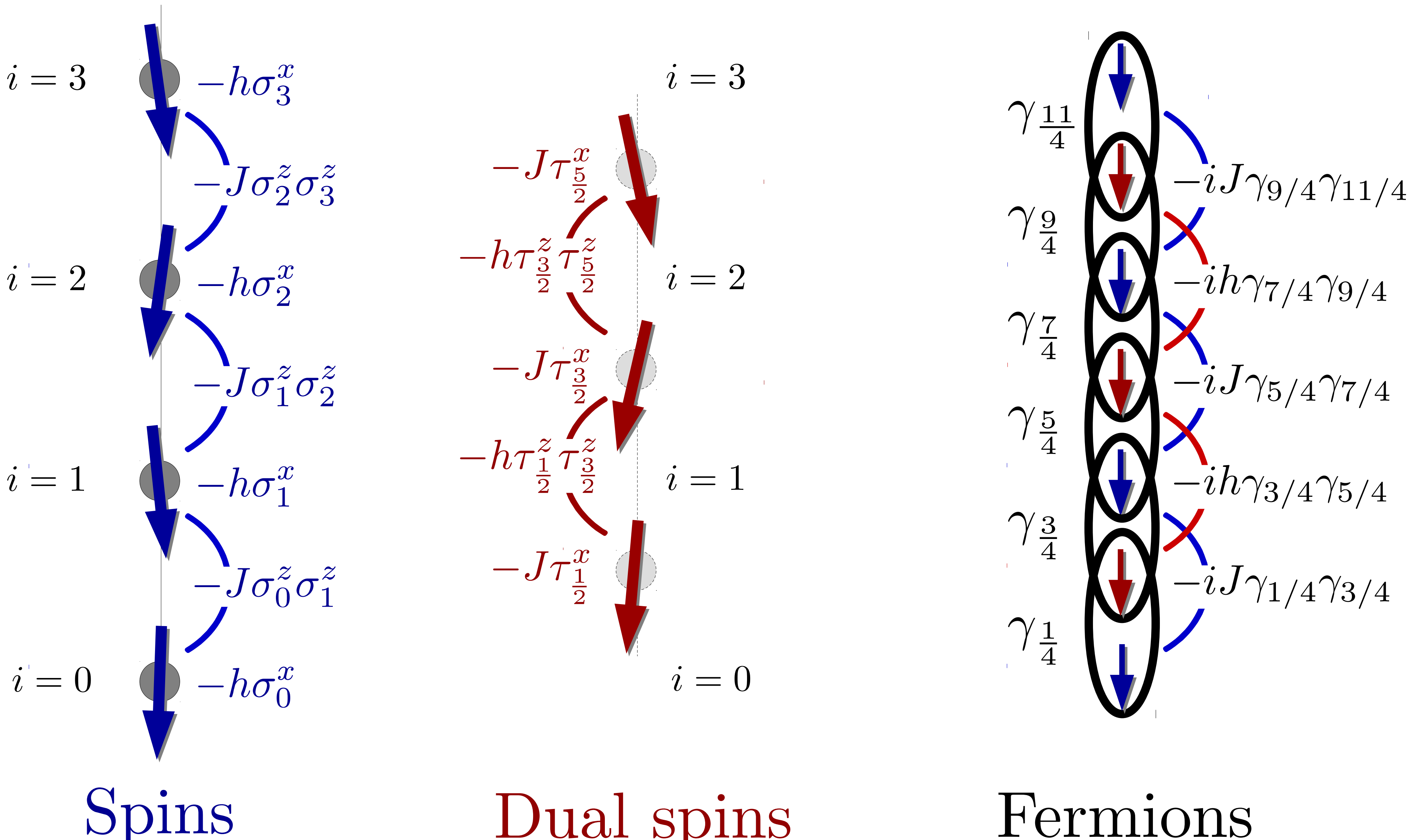


Self-duality ensures criticality at $J = h$

Spins

Dual spins

Warm up in 1+1 dimensions: Quantum Ising model



Warm up in 1+1 dimensions:
...odel

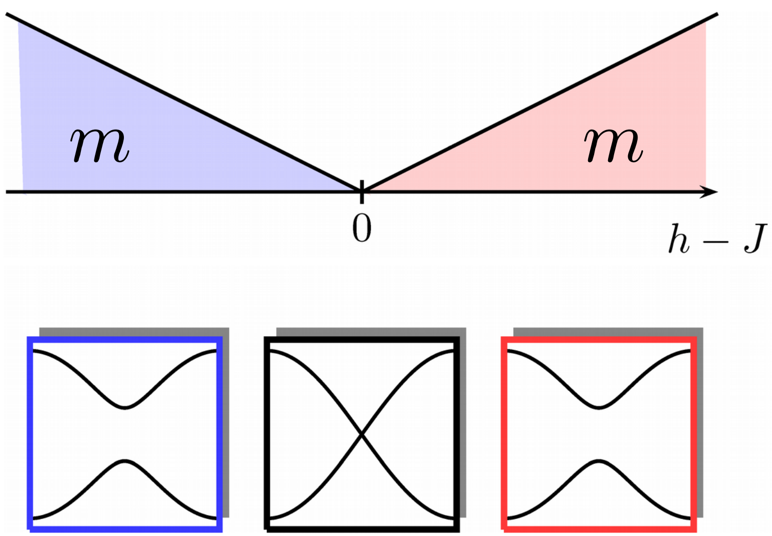
$i = 3$

$i = 2$

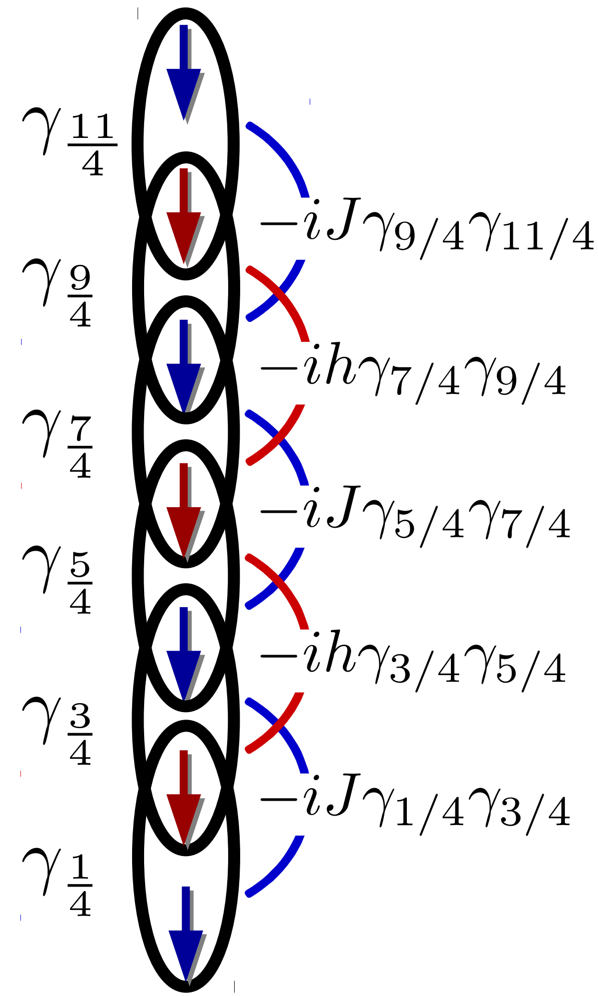
$i = 1$

$i = 0$

Self-duality ensures criticality at $J = h$



Translation symmetry yields gapless spectrum for $J = h$

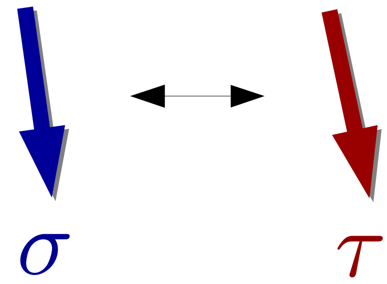


Sp...

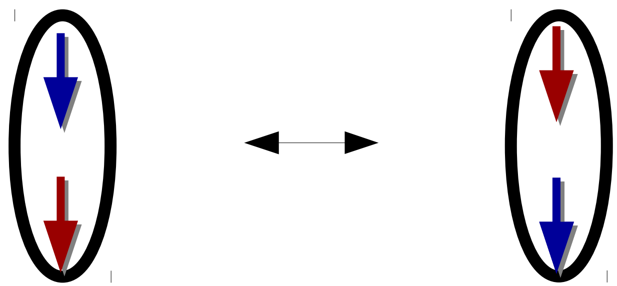
Fermions

Warm up in 1+1 dimensions:

- Duality Transformation on spin variables



- Translation by ½ of Majorana fermions



$$\gamma_{i-1/4} = \tau^z_{i-1/2}\sigma^z_i \qquad \gamma_{i+1/4} = i\sigma^z_i \tau^z_{i+1/2}$$

- Can interpret **duality** as a **symmetry** of new degrees of freedom
- Symmetry is **anomalous** (impossible in microscopic 1D system).

$i=3$

$i=2$

$i=1$

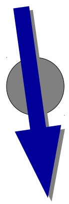
$i=0$

$\gamma_{11/4}$ $\gamma_{9/4}$ $\gamma_{7/4}$ $\gamma_{5/4}$ $\gamma_{3/4}$

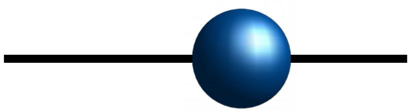
2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i

Bosons on wires labeled by i

 i

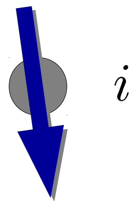
$$\sigma_i^z \longleftrightarrow b_i(x)$$

i 

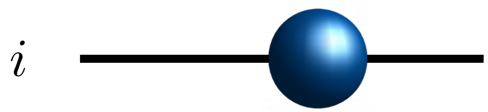
2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i

Bosons on wires labeled by i

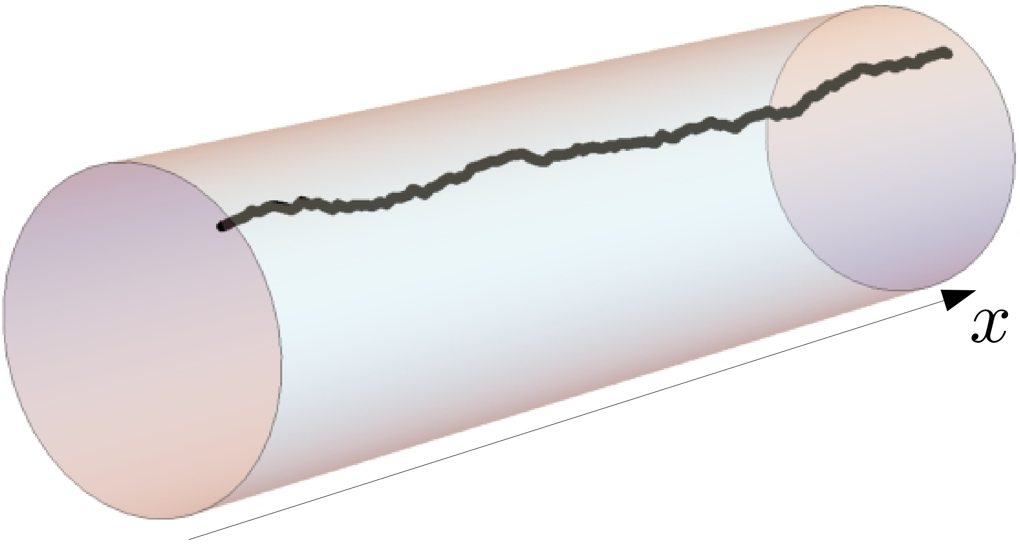


$\sigma_i^z \longleftrightarrow b_i(x)$



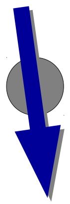
$\langle \sigma_i^z \rangle \neq 0$: Ferromagnet

$\langle b_i \rangle \neq 0$: Superfluid



2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i



i

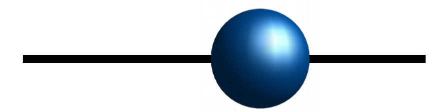
σ_i^z $\longleftrightarrow$ $b_i(x)$

σ_i^x

Creates defects (spin flips)
in ferromagnet

Bosons on wires labeled by i

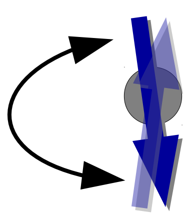
i

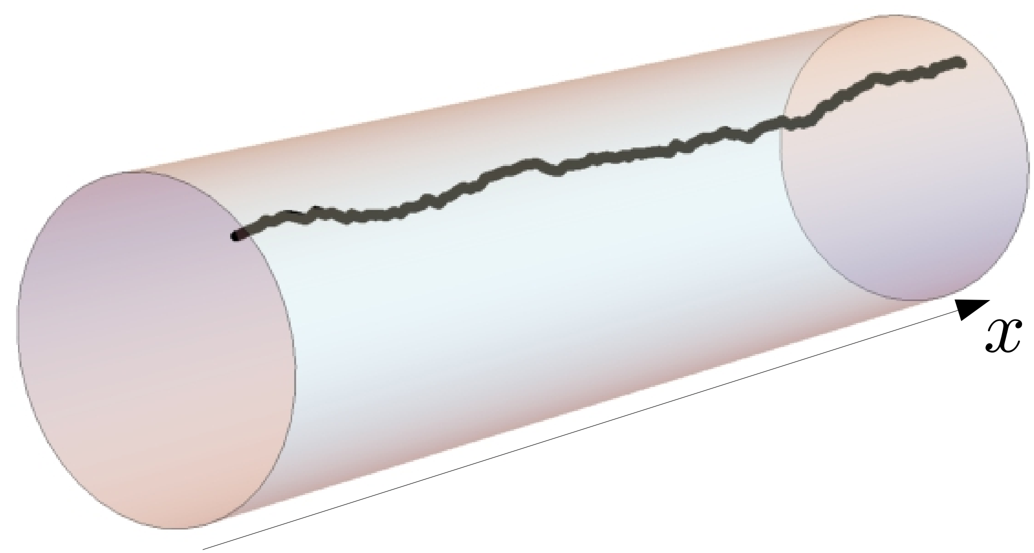


$\langle \sigma_i^z \rangle \neq 0$: Ferromagnet

$\langle b_i \rangle \neq 0$: Superfluid

σ_i^x:

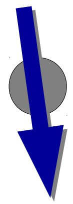




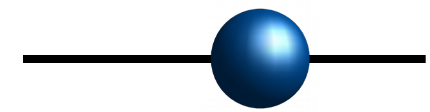
2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i

Bosons on wires labeled by i



$\sigma_i^z \longleftrightarrow b_i(x)$



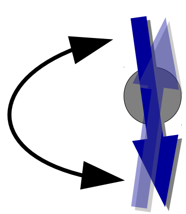
σ_i^x

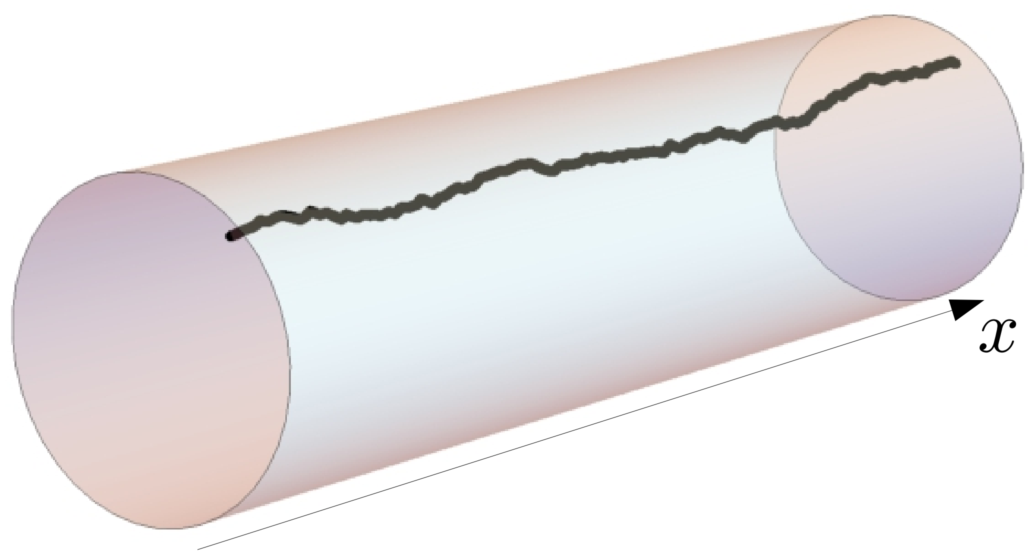
Creates defects (spin flips)
in ferromagnet

$\langle \sigma_i^z \rangle \neq 0$: Ferromagnet

$\langle b_i \rangle \neq 0$: Superfluid

σ_i^x:



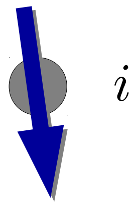


$\langle \sigma_i^z \rangle = 0$: Paramagnet

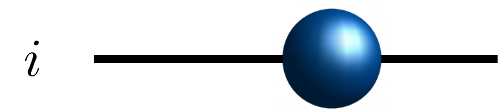
2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i

Bosons on wires labeled by i



$\sigma_i^z \longleftrightarrow b_i(x)$



$\sigma_i^x \longleftrightarrow p_i(x) = e^{2\pi i \int^x dx' \rho_i(x')}$

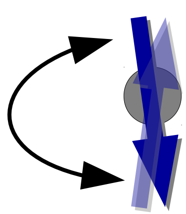
Creates defects (spin flips) in ferromagnet

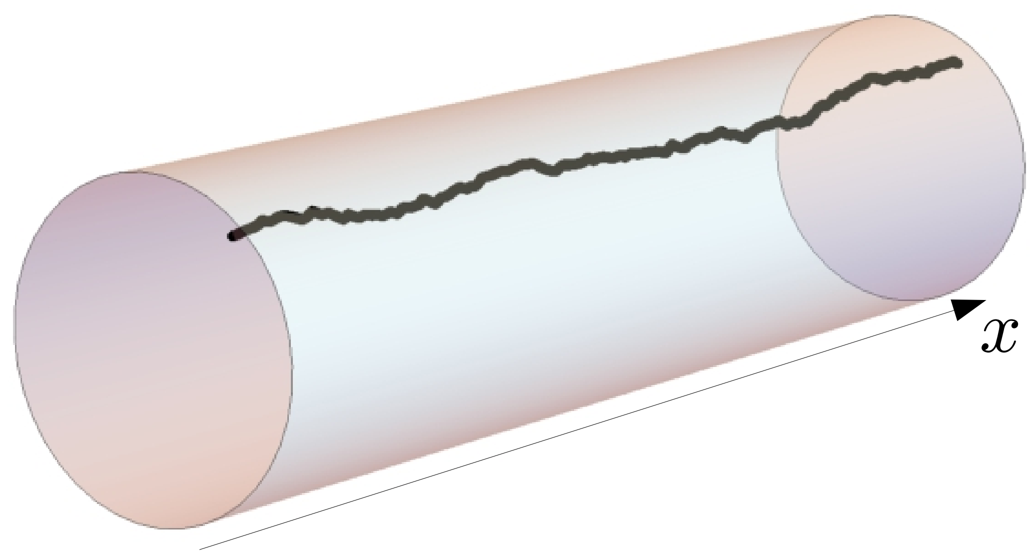
Creates defects (phase-slips) in superfluid

$\langle \sigma_i^z \rangle \neq 0$: Ferromagnet

$\langle b_i \rangle \neq 0$: Superfluid

σ_i^x:





$\langle \sigma_i^z \rangle = 0$: Paramagnet

2+1 dimensions: Particle-vortex duality

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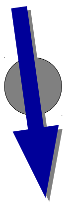
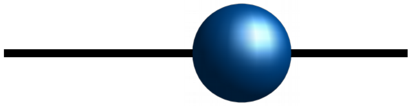


Bosons on wires labeled by i



$$\sigma_i^z \longleftrightarrow b_i(x)$$

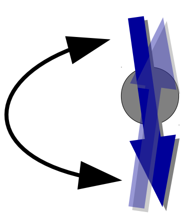
$$\sigma_i^x \longleftrightarrow p_i(x) = e^{2\pi i \int^x dx' \rho_i(x')}$$

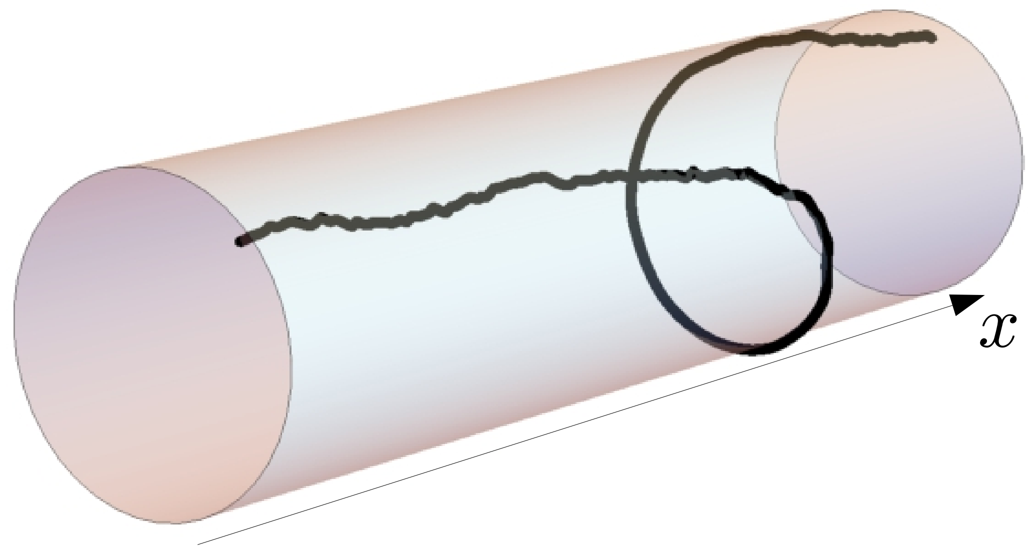
Creates defects (spin flips) in ferromagnet

Creates defects (phase-slips) in superfluid

$\langle \sigma_i^z \rangle \neq 0$: Ferromagnet

$\langle b_i \rangle \neq 0$: Superfluid

σ_i^x: 

$p_i(x)$: 

$\langle \sigma_i^z \rangle = 0$: Paramagnet

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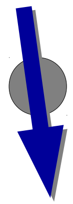
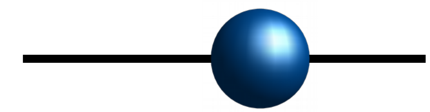


i

$$\sigma_i^z \longleftrightarrow b_i(x)$$

i



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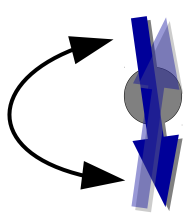
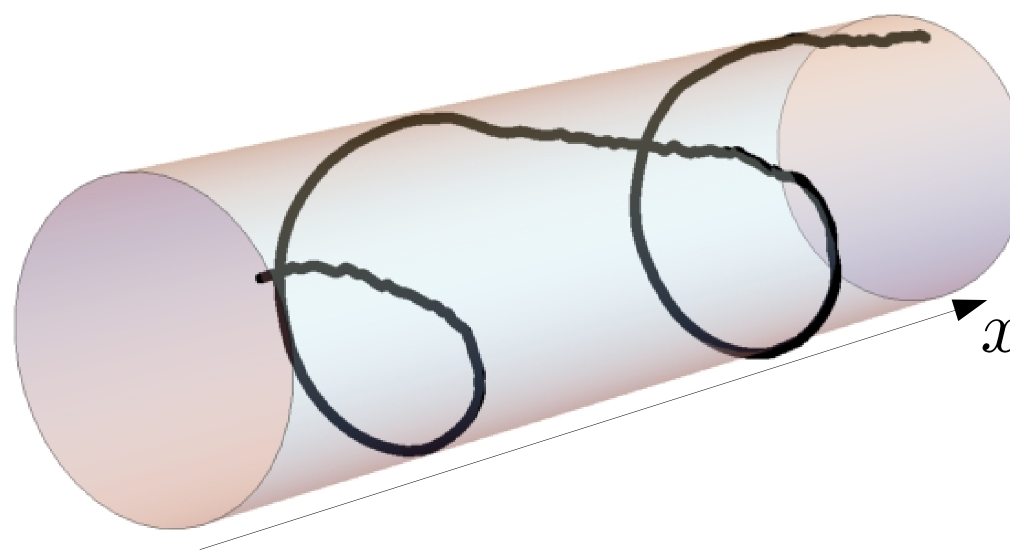


$p_i(x)$:

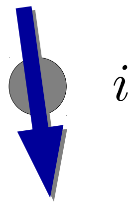


x

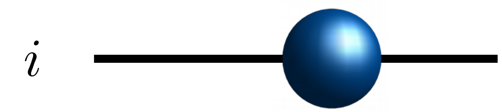
$\langle \sigma_i^z \rangle = 0$: Paramagnet

2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i — Bosons on wires labeled by i



$$\sigma_i^z \longleftrightarrow b_i(x)$$



$$\sigma_i^x \longleftrightarrow p_i(x) = e^{2\pi i \int^x dx' \rho_i(x')}$$

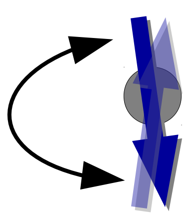
Creates defects (spin flips) in ferromagnet

Creates defects (phase-slips) in superfluid

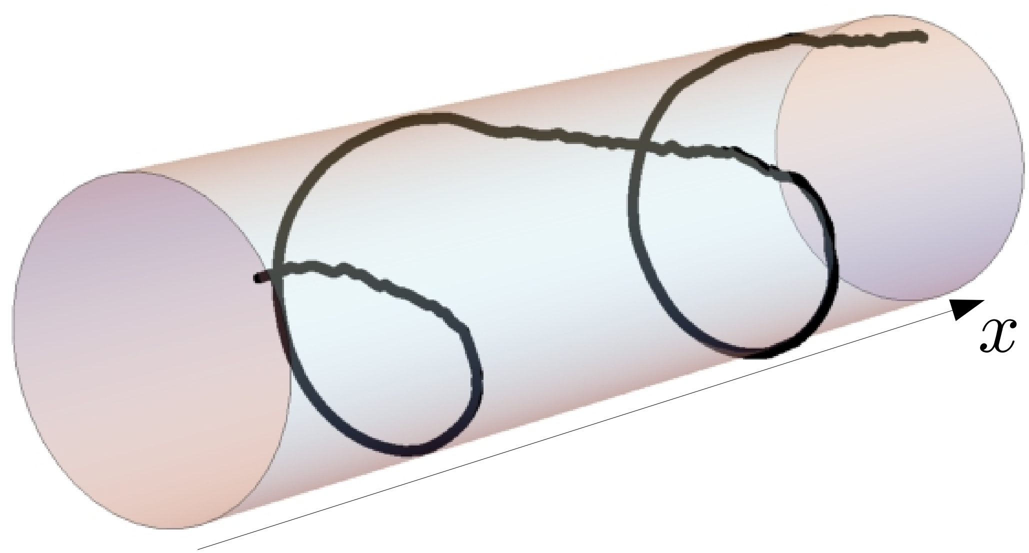
$\langle \sigma_i^z \rangle \neq 0$: Ferromagnet

$\langle b_i \rangle \neq 0$: Superfluid

σ_i^x:



$p_i(x)$:



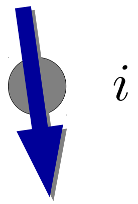
$\langle \sigma_i^z \rangle = 0$: Paramagnet

$\langle b_i \rangle = 0$: Mott Insulator

2+1 dimensions: Particle-vortex duality

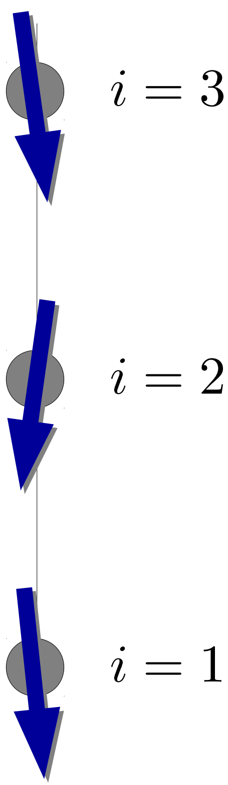
Spins on sites labeled by i

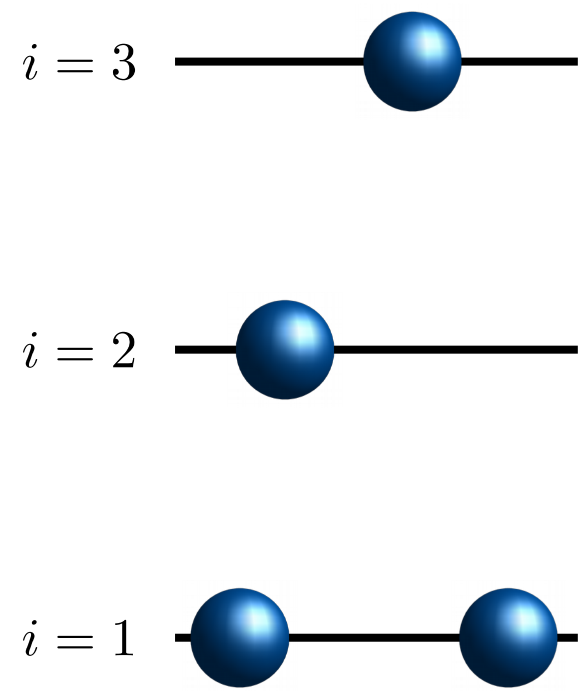
Bosons on wires labeled by i



$$\sigma_i^z \longleftrightarrow b_i(x)$$

$$\sigma_i^x \longleftrightarrow p_i(x) = e^{2\pi i \int^x dx' \rho_i(x')}$$

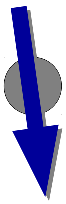




2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i

Bosons on wires labeled by i

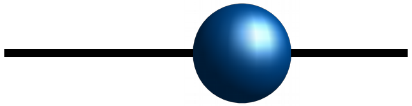


i

$$\sigma_i^z \longleftrightarrow b_i(x)$$

$$\sigma_i^x \longleftrightarrow p_i(x) = e^{2\pi i \int^x dx' \rho_i(x')}$$

i

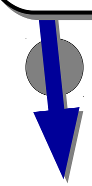


Ising duality

$$\sigma_i^z \sigma_{i+1}^z = \tau_{i+\frac{1}{2}}^x$$

$$\sigma_i^x = \tau_{i-\frac{1}{2}}^z \tau_{i+\frac{1}{2}}^z$$

Spin-flip is dual-spin exchange

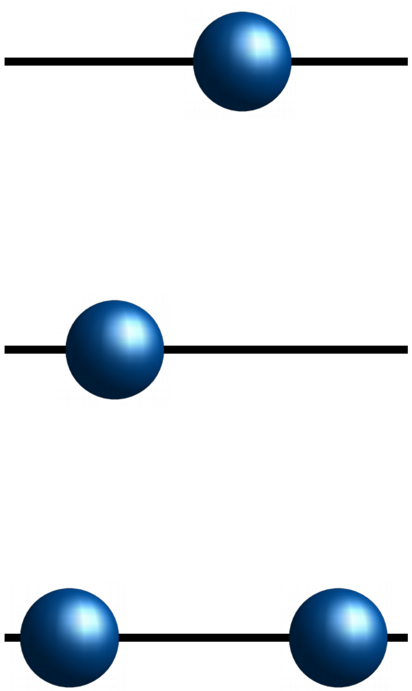


$i = 1$

$i = 3$

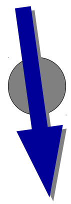
$i = 2$

$i = 1$

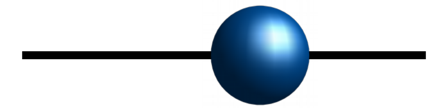


2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i



Bosons on wires labeled by i



$$\sigma_i^z \longleftrightarrow b_i(x)$$

$$\sigma_i^x \longleftrightarrow p_i(x) = e^{2\pi i \int^x dx' \rho_i(x')}$$

Ising duality

$$\sigma_i^z \sigma_{i+1}^z = \tau_{i+\frac{1}{2}}^x$$

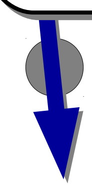
$$\sigma_i^x = \tau_{i-\frac{1}{2}}^z \tau_{i+\frac{1}{2}}^z$$

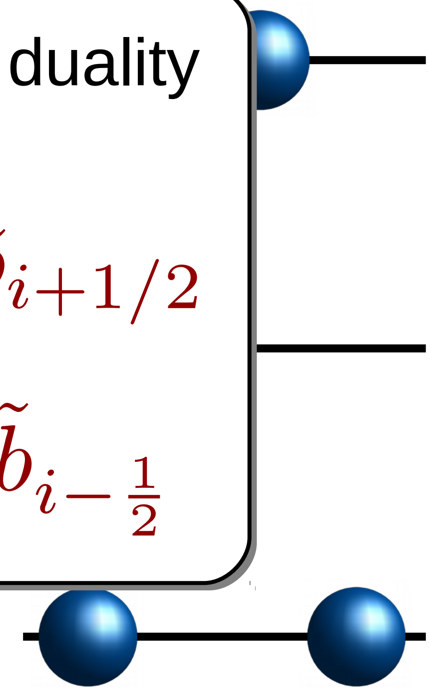
Spin-flip is dual-spin exchange

Boson-vortex duality

$$b_i^\dagger b_{i+1} = \tilde{p}_{i+1/2}$$

$$p_i = \tilde{b}_{i+\frac{1}{2}}^\dagger \tilde{b}_{i-\frac{1}{2}}$$

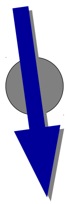


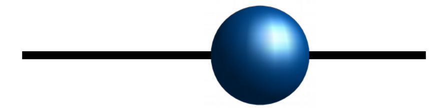


2+1 dimensions: Particle-vortex duality

Spins on sites labeled by i

Bosons on wires labeled by i





$$\sigma_i^z \longleftrightarrow b_i(x)$$

$$\sigma_i^x \longleftrightarrow p_i(x) = e^{2\pi i \int^x dx' \rho_i(x')}$$

Ising duality

$$\sigma_i^z \sigma_{i+1}^z = \tau_{i+\frac{1}{2}}^x$$

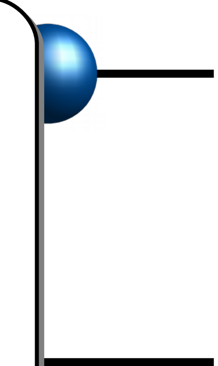
$$\sigma_i^x = \tau_{i-\frac{1}{2}}^z \tau_{i+\frac{1}{2}}^z$$

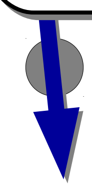
Boson-vortex duality

Phase-slip is vortex hopping

$$b_i^\dagger b_{i+1} = \tilde{p}_{i+1/2}$$

$$p_i = \tilde{b}_{i+\frac{1}{2}}^\dagger \tilde{b}_{i-\frac{1}{2}}$$

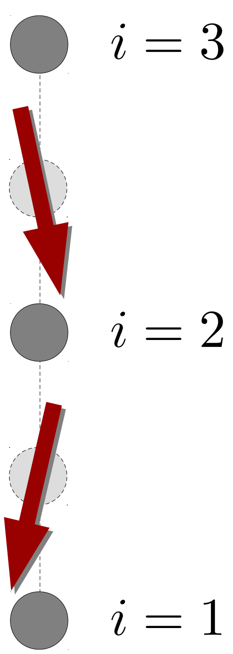


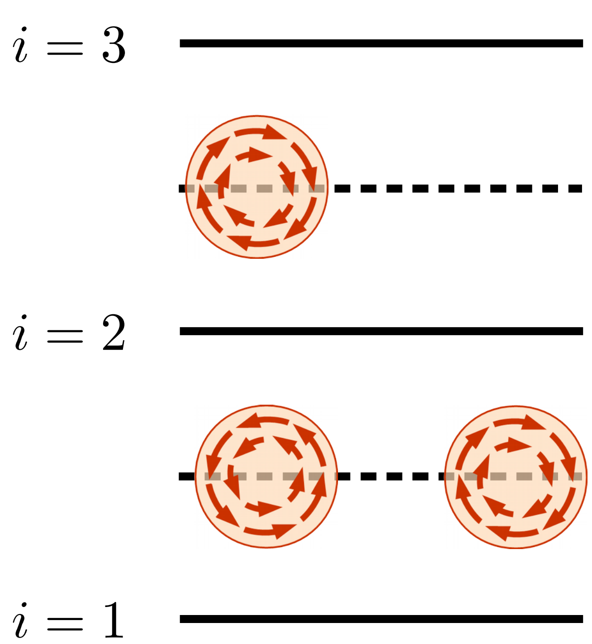


2+1 dimensions: Particle-vortex duality

Spins on dual sites labeled by $i+\frac{1}{2}$ Bosons on dual wires labeled by $i+\frac{1}{2}$

$$\tau^z_{i+\frac{1}{2}} \longleftrightarrow \tilde{b}_{i+\frac{1}{2}}(x)$$





2+1 dimensions: Particle-vortex duality

Spins on dual sites labeled by $i+\frac{1}{2}$

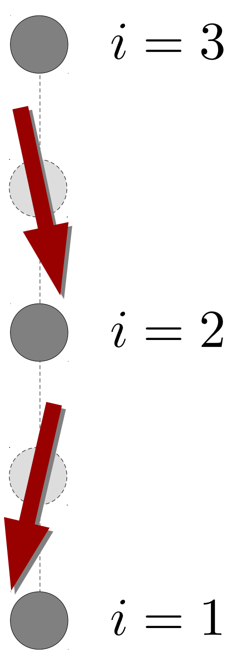
Bosons on dual wires labeled by $i+\frac{1}{2}$

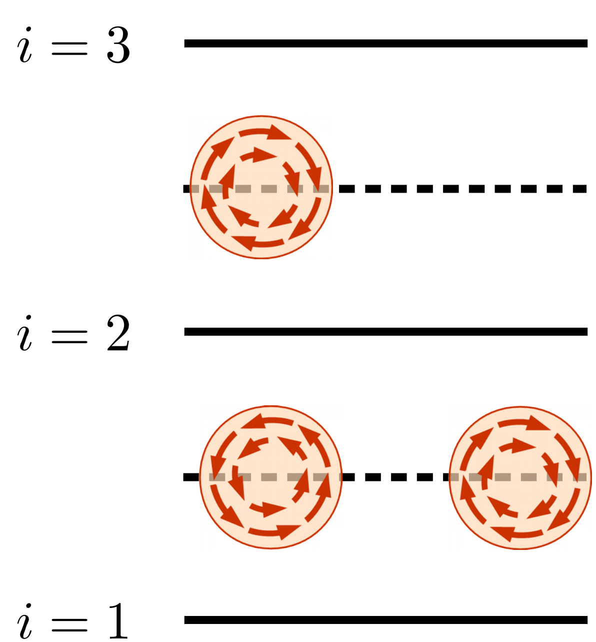
$$\tau^z_{i+\frac{1}{2}} \longleftrightarrow \tilde{b}_{i+\frac{1}{2}}(x)$$

Creates defects (spin flips) in dual ferromagnet →

$$\tau^x_{i+\frac{1}{2}} \longleftrightarrow \tilde{p}_{i+\frac{1}{2}}(x) = e^{2\pi i \int^x \tilde{\rho}_{i+1/2}}$$

Creates defects (phase-slips) in dual superfluid





2+1 dimensions: Particle-vortex duality

Spins on dual sites labeled by $i + \frac{1}{2}$

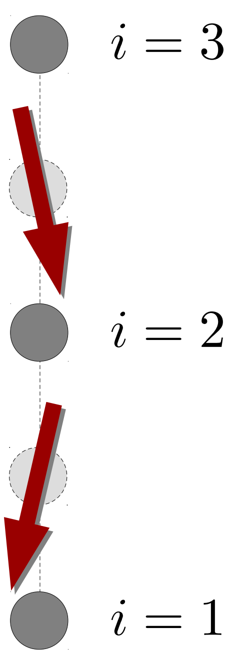
Bosons on dual wires labeled by $i + \frac{1}{2}$

$$\tau^z_{i+\frac{1}{2}} \longleftrightarrow \tilde{b}_{i+\frac{1}{2}}(x)$$

Creates defects (spin flips) in dual ferromagnet →

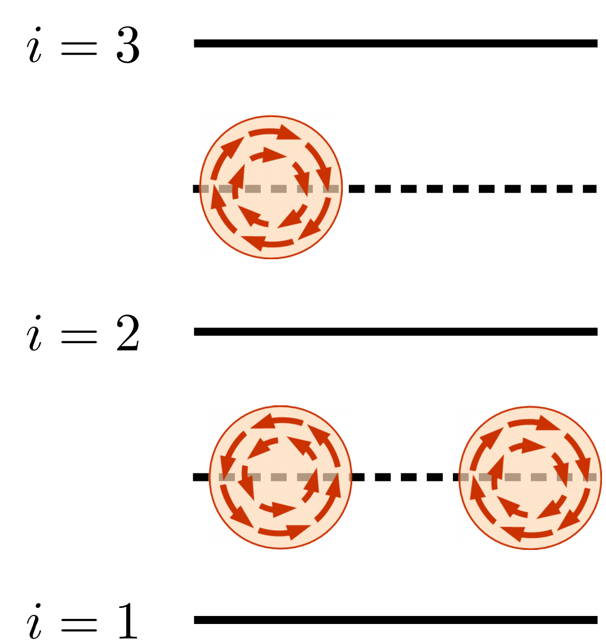
$$\tau^x_{i+\frac{1}{2}} \longleftrightarrow \tilde{p}_{i+\frac{1}{2}}(x) = e^{2\pi i \int^x \tilde{\rho}_{i+1/2}}$$

Creates defects (phase-slips) in dual superfluid



$$\langle \tau^z \rangle = 0: \text{ Ferromagnet}$$

$$\langle \tilde{b} \rangle = 0: \text{ Superfluid}$$



2+1 dimensions: Particle-vortex duality

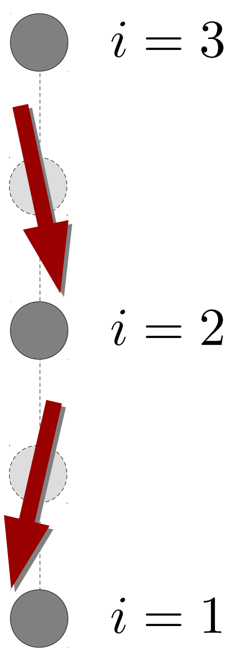
Spins on dual sites labeled by $i+\frac{1}{2}$ — Bosons on dual wires labeled by $i+\frac{1}{2}$

$$\tau^z_{i+\frac{1}{2}} \longleftrightarrow \tilde{b}_{i+\frac{1}{2}}(x)$$

Creates defects (spin flips) in dual ferromagnet →

$$\tau^x_{i+\frac{1}{2}} \longleftrightarrow \tilde{p}_{i+\frac{1}{2}}(x) = e^{2\pi i \int^x \tilde{\rho}_{i+1/2}}$$

Creates defects (phase-slips) in dual superfluid

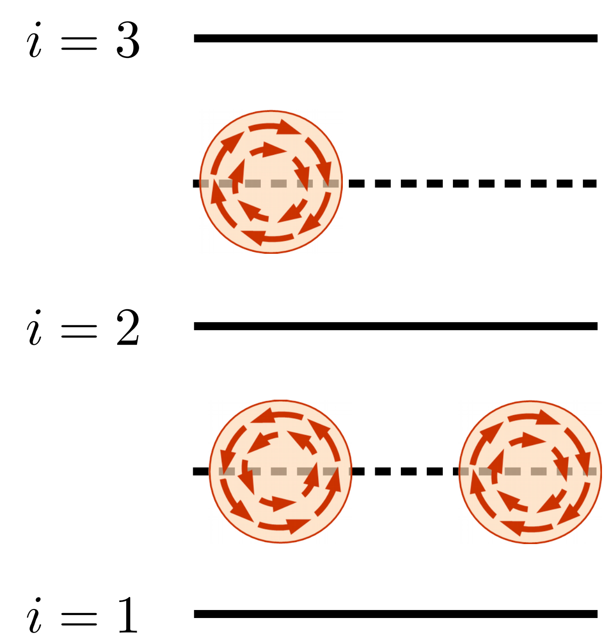


$$\langle \tau^z \rangle = 0 \text{: Ferromagnet}$$

$$\langle \tilde{b} \rangle = 0 \text{: Superfluid}$$

$$\langle \tau^z \rangle \neq 0 \text{: Paramagnet}$$

$$\langle \tilde{b} \rangle \neq 0 \text{: Mott Insulator}$$



2+1 dimensions: Particle-vortex duality

- Ising duality: Only spin-exchange and transverse field

$$H_{\text{Ising}} = -J\sigma_i^z\sigma_{i+1}^z - h\sigma_i^x$$

$$\updownarrow \quad \text{duality} \quad \updownarrow$$

$$H_{\text{Dual}} = -J\tau^x_{i+\frac{1}{2}} \quad - h\tau^z_{i-\frac{1}{2}}\tau^z_{i+\frac{1}{2}}$$

2+1 dimensions: Particle-vortex duality

- Ising duality: Only spin-exchange and transverse field

$$H_{\text{Ising}} = -J\sigma_i^z\sigma_{i+1}^z - h\sigma_i^x$$

$$\updownarrow \quad \text{duality} \quad \updownarrow$$

$$H_{\text{Dual}} = -J\tau_{i+\frac{1}{2}}^x \quad -h\tau_{i-\frac{1}{2}}^z\tau_{i+\frac{1}{2}}^z$$

- Boson-vortex duality: Kinetic energy within each wire

$$H_{\text{Boson}} = H_{\text{Boson}}^{\text{kin.}} - \int_x \left[-ub_i^\dagger b_{i+1} - vp_i + \text{H.c.}\right]$$

$$H_{\text{Vortex}} = H_{\text{Vortex}}^{\text{kin.}} - \int_x \left[-u\tilde{p}_{i+\frac{1}{2}} - \quad v\tilde{b}_{i-\frac{1}{2}}^\dagger \tilde{b}_{i+\frac{1}{2}} + \text{H.c.}\right]$$

2+1 dimensions: Particle-vortex duality

- Ising duality: Only spin-exchange and transverse field

$$H_{\text{Ising}} = -J\sigma_i^z \sigma_{i+1}^z - h\sigma_i^x$$

$$\updownarrow \quad \text{duality} \quad \updownarrow$$

$$H_{\text{Dual}} = -J\tau_{i+\frac{1}{2}}^x \quad -h\tau_{i-\frac{1}{2}}^z \tau_{i+\frac{1}{2}}^z$$

- Boson-vortex duality: Kinetic energy within each wire

$$H_{\text{Boson}} = H_{\text{Boson}}^{\text{kin.}} - \int_x \left[-u b_i^\dagger b_{i+1} - v p_i + \text{H.c.}\right]$$

$$\updownarrow \quad \text{duality} \quad \updownarrow$$

$$H_{\text{Vortex}} = H_{\text{Vortex}}^{\text{kin.}} - \int_x \left[-u\tilde{p}_{i+\frac{1}{2}} - \quad v\tilde{b}_{i-\frac{1}{2}}^\dagger \tilde{b}_{i+\frac{1}{2}} + \text{H.c.}\right]$$

2+1 dimensions: Particle-vortex duality

- Ising duality: Only spin-exchange and transverse field

$$H_{\text{Ising}} = -J\sigma_i^z\sigma_{i+1}^z - h\sigma_i^x$$

$$\updownarrow \quad \text{duality} \quad \updownarrow$$

$$H_{\text{Dual}} = -J\tau^x_{i+\frac{1}{2}} \quad -h\tau^z_{i-\frac{1}{2}}\tau^z_{i+\frac{1}{2}}$$

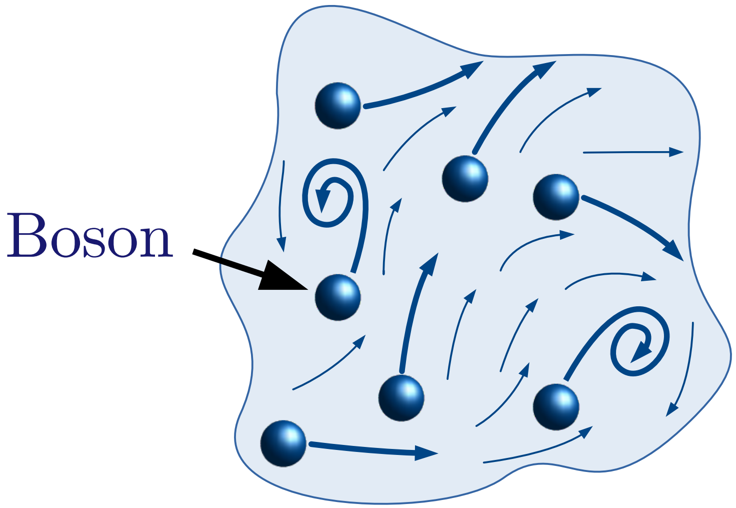
- Boson-vortex duality: Kinetic energy within each wire

$$H_{\text{Boson}} = H^{\text{kin.}}_{\text{Boson}} - \int_x \left[-ub_i^\dagger b_{i+1} - vp_i + \text{H.c.}\right]$$

~~duality~~ $\updownarrow$ $\qquad$ $\updownarrow$ duality $\updownarrow$

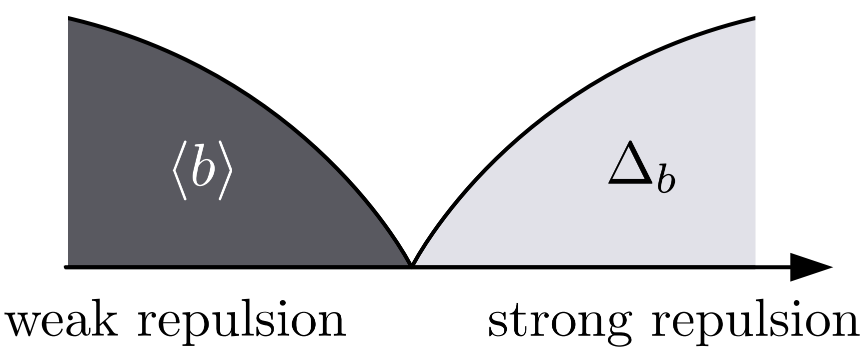
$$H_{\text{Vortex}} = H^{\text{kin.}}_{\text{Vortex}} - \int_x \left[-u\tilde{p}_{i+\frac{1}{2}} - \quad v\tilde{b}^\dagger_{i-\frac{1}{2}}\tilde{b}_{i+\frac{1}{2}} + \text{H.c.}\right]$$

2+1 dimensions: Particle-vortex duality

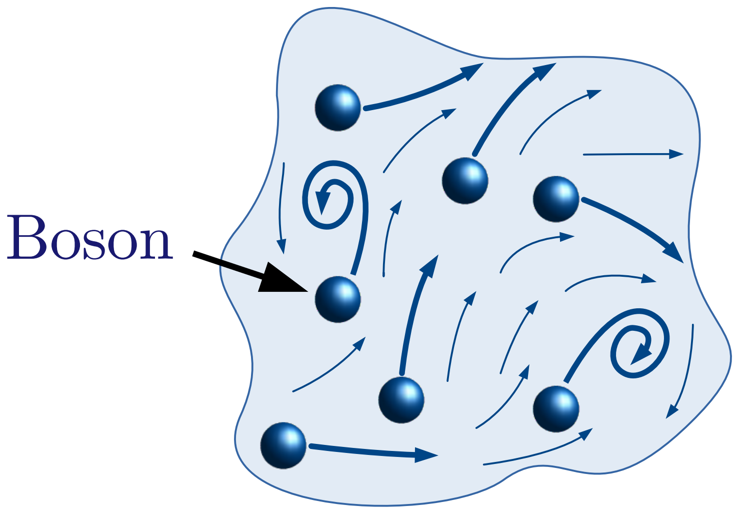


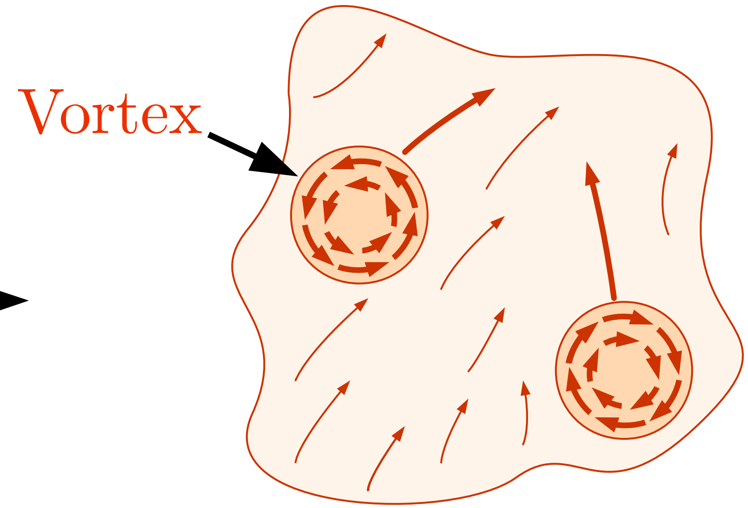
$$\mathcal{L}_{\text{Boson}} = |\partial_\mu \Psi_{\text{Boson}}|^2$$

Bosons with **short-range** interactions



2+1 dimensions: Particle-vortex duality



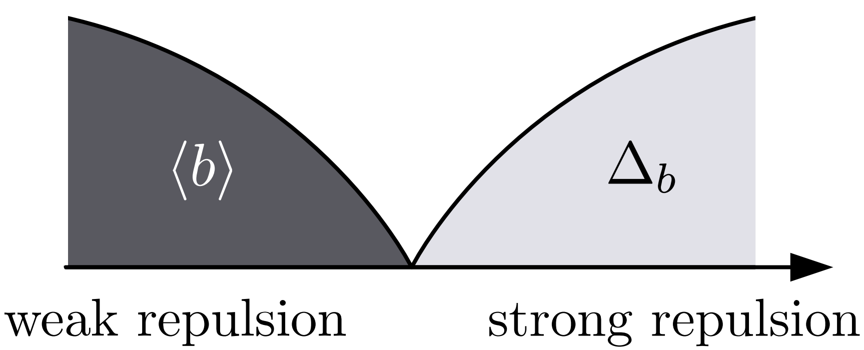


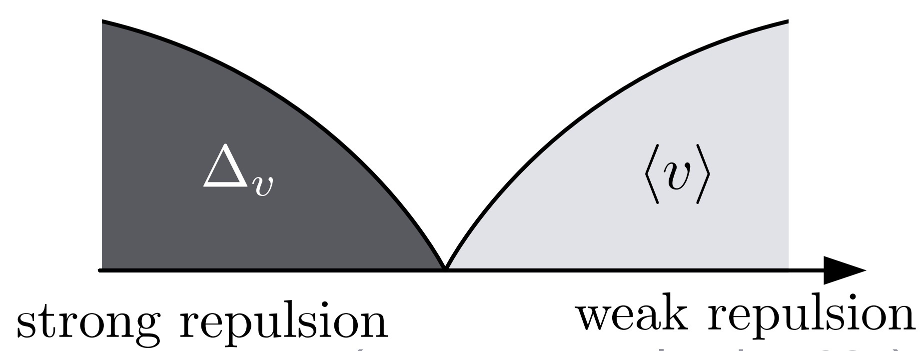
$$\mathcal{L}_{\text{Boson}} = |\partial_\mu \Psi_{\text{Boson}}|^2$$

$$\mathcal{L}_{\text{Vortex}} = |(\partial_\mu - i a_\mu)\Psi_{\text{Vortex}}|^2 + \frac{1}{2\kappa}(\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu)^2$$

Bosons with **short-range** interactions

Vortices with **long-range** interactions





(Dasgupta, Halperin 1981)

2+1 dimensions: Particle-vortex duality

$$H_{\text{Boson}} = H^{\text{kin.}}_{\text{Boson}} - \int_x \left[-u b_i^\dagger b_{i+1} - v p_i + \text{H.c.} \right]$$

~~duality~~ $\updownarrow$ $\qquad\qquad$ $\updownarrow$ duality $\updownarrow$

$$H_{\text{Vortex}} = H^{\text{kin.}}_{\text{Vortex}} - \int_x \left[-u \tilde{p}_{i+\frac{1}{2}} - v \tilde{b}^\dagger_{i-\frac{1}{2}} \tilde{b}_{i+\frac{1}{2}} + \text{H.c.} \right]$$

2+1 dimensions: Particle-vortex duality

$$\mathcal{L}^{\text{kin}}_{\text{Boson}}[b]$$

$$H_{\text{Boson}} = H^{\text{kin.}}_{\text{Boson}} - \int_x \left[-u b^\dagger_i b_{i+1} - v p_i + \text{H.c.}\right]$$

~~duality~~ $\updownarrow$ $\qquad\qquad$ $\updownarrow$ duality $\updownarrow$

$$H_{\text{Vortex}} = H^{\text{kin.}}_{\text{Vortex}} - \int_x \left[-u \tilde{p}_{i+\frac{1}{2}} - v \tilde{b}^\dagger_{i-\frac{1}{2}} \tilde{b}_{i+\frac{1}{2}} + \text{H.c.}\right]$$

2+1 dimensions: Particle-vortex duality

$$\mathcal{L}^{\text{kin}}_{\text{Boson}}[b] = \mathcal{L}_{\text{non-local}}[\tilde{b}]$$ (use operator mapping)

$$H_{\text{Boson}} = H^{\text{kin.}}_{\text{Boson}} - \int_x \left[-u b_i^\dagger b_{i+1} - v p_i + \text{H.c.} \right]$$

~~duality~~ $\updownarrow$ $\qquad$ $\updownarrow$ duality $\updownarrow$

$$H_{\text{Vortex}} = H^{\text{kin.}}_{\text{Vortex}} - \int_x \left[-u \tilde{p}_{i+\frac{1}{2}} - \quad v \tilde{b}^\dagger_{i-\frac{1}{2}} \tilde{b}_{i+\frac{1}{2}} + \text{H.c.} \right]$$

2+1 dimensions: Particle-vortex duality

$$\mathcal{L}^{\text{kin}}_{\text{Boson}}[b] = \mathcal{L}_{\text{non-local}}[\tilde{b}] \qquad \text{(use operator mapping)}$$

$$\mathcal{L}_{\text{gauge}}[\tilde{b}, a]$$

$$H_{\text{Boson}} = H^{\text{kin.}}_{\text{Boson}} - \int_x \left[-u b_i^\dagger b_{i+1} - v p_i + \text{H.c.} \right]$$

~~duality~~ $\updownarrow$ $\qquad\qquad \updownarrow$ duality $\updownarrow$

$$H_{\text{Vortex}} = H^{\text{kin.}}_{\text{Vortex}} - \int_x \left[-u \tilde{p}_{i+\frac{1}{2}} - v \tilde{b}^\dagger_{i-\frac{1}{2}} \tilde{b}_{i+\frac{1}{2}} + \text{H.c.} \right]$$

2+1 dimensions: Particle-vortex duality

$$\mathcal{L}^{\text{kin}}_{\text{Boson}}[b] = \mathcal{L}_{\text{non-local}}[\tilde{b}]$$ (use operator mapping)

$$\mathcal{L}_{\text{gauge}}[\tilde{b}, a] \to \mathcal{L}'_{\text{non-local}}[\tilde{b}]$$ (integrate out a)

$$H_{\text{Boson}} = H^{\text{kin.}}_{\text{Boson}} - \int_x \left[-u b^\dagger_i b_{i+1} - v p_i + \text{H.c.} \right]$$

~~duality~~ $\updownarrow$ $\qquad$ $\updownarrow$ duality $\updownarrow$

$$H_{\text{Vortex}} = H^{\text{kin.}}_{\text{Vortex}} - \int_x \left[-u \tilde{p}_{i+\frac{1}{2}} - v \tilde{b}^\dagger_{i-\frac{1}{2}} \tilde{b}_{i+\frac{1}{2}} + \text{H.c.} \right]$$

2+1 dimensions: Particle-vortex duality

$$\mathcal{L}^{\text{kin}}_{\text{Boson}}[b] = \mathcal{L}_{\text{non-local}}[\tilde{b}]$$ (use operator mapping)

$$\mathcal{L}_{\text{gauge}}[\tilde{b}, a] \to \mathcal{L}'_{\text{non-local}}[\tilde{b}]$$ (integrate out a)

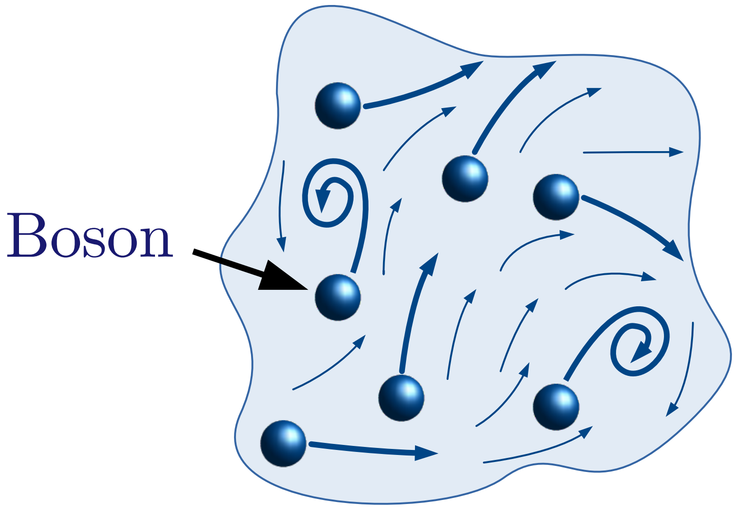
$$\to \text{find } \mathcal{L}_{\text{non-local}} = \mathcal{L}'_{\text{non-local}}$$

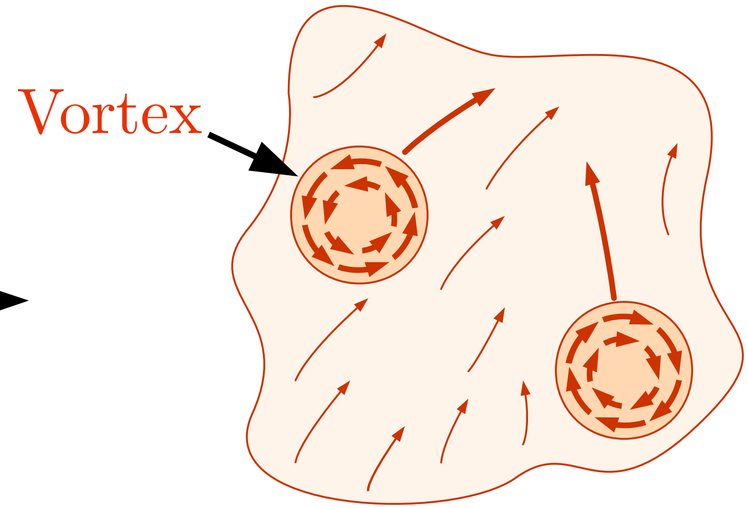
$$H_{\text{Boson}} = H^{\text{kin.}}_{\text{Boson}} - \int_x \left[-u b_i^\dagger b_{i+1} - v p_i + \text{H.c.} \right]$$

~~duality~~ $\updownarrow$ $\qquad$ $\updownarrow$ duality $\updownarrow$

$$H_{\text{Vortex}} = H^{\text{kin.}}_{\text{Vortex}} - \int_x \left[-u \tilde{p}_{i+\frac{1}{2}} - \quad v \tilde{b}^\dagger_{i-\frac{1}{2}} \tilde{b}_{i+\frac{1}{2}} + \text{H.c.} \right]$$

2+1 dimensions: Particle-vortex duality



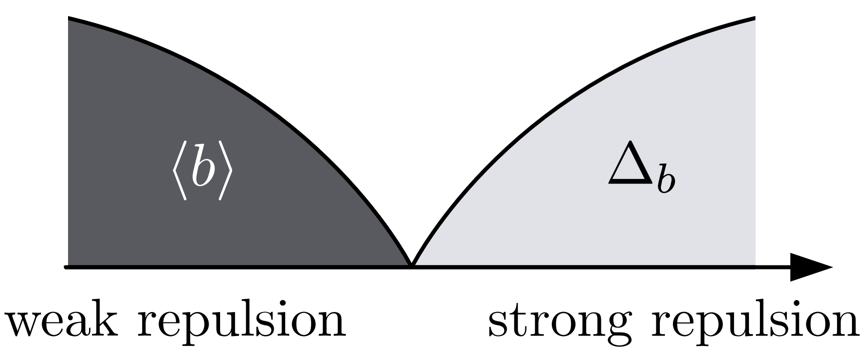


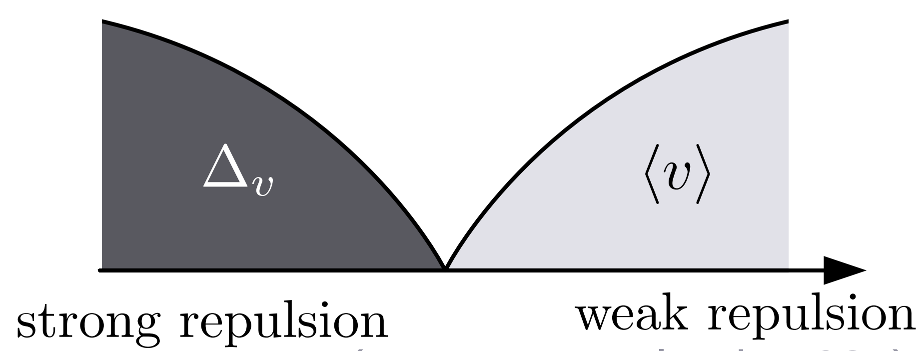
$$\mathcal{L}_{\text{Boson}} = |\partial_\mu \Psi_{\text{Boson}}|^2$$

$$\mathcal{L}_{\text{Vortex}} = |(\partial_\mu - i a_\mu)\Psi_{\text{Vortex}}|^2 + \frac{1}{2\kappa}(\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu)^2$$

Bosons with **short-range** interactions

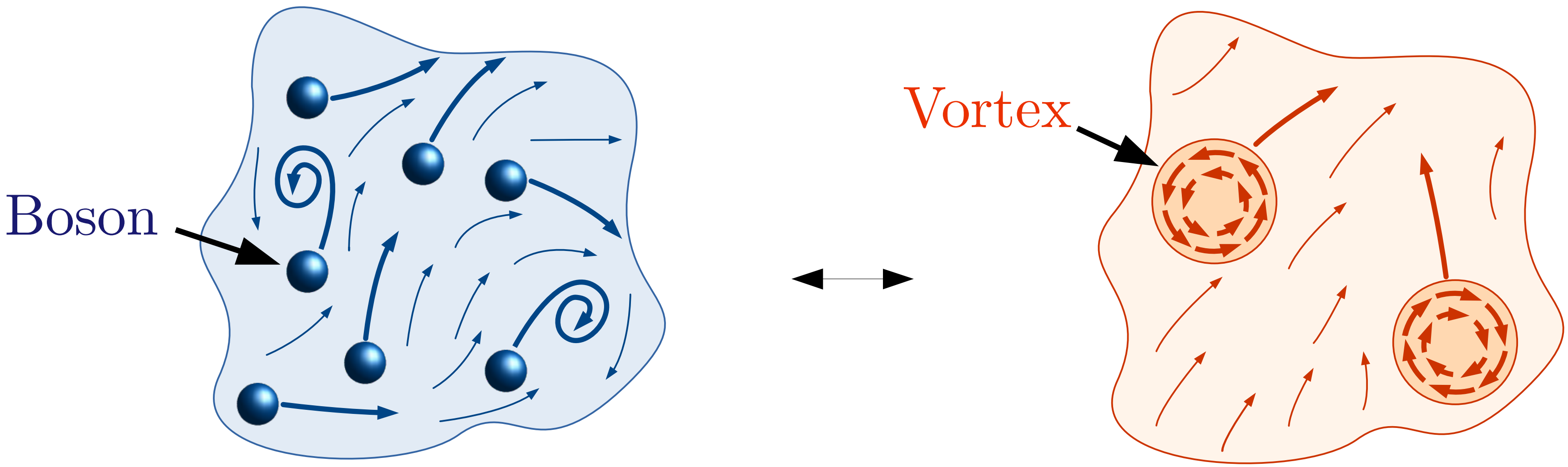
Vortices with **long-range** interactions





(Dasgupta, Halperin 1981)

2+1 dimensions: Particle-vortex duality

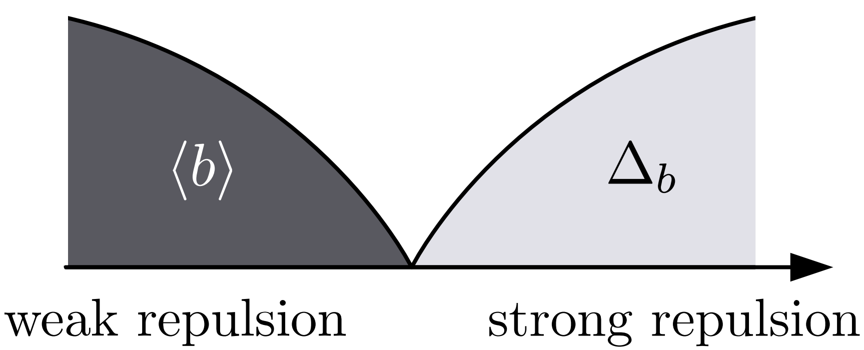


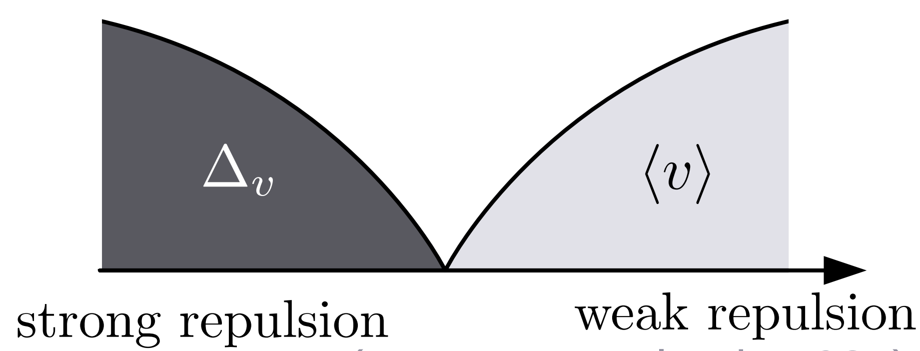
$$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\text{Boson}}|^2 + \frac{1}{2K}(\epsilon_{\lambda\mu\nu}\partial_\mu A_\nu)^2$$

$$\mathcal{L}_{\text{Vortex}} = |\partial_\mu \Psi_{\text{Vortex}}|^2$$

Bosons with **long-range** interactions

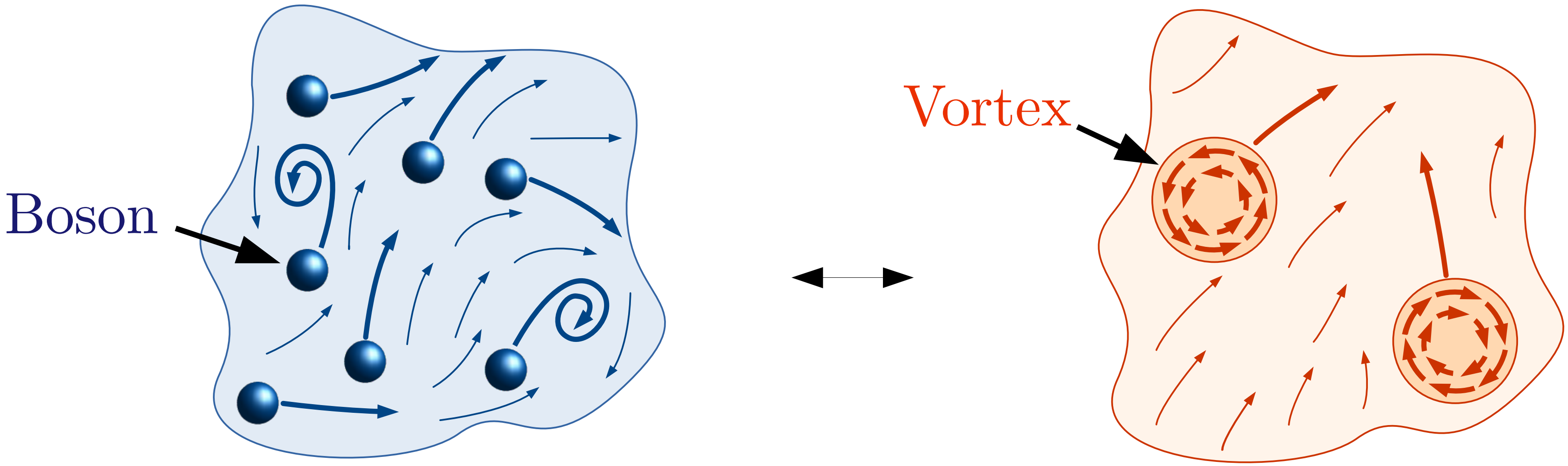
Vortices with **short-range** interactions





(Dasgupta, Halperin 1981)

2+1 dimensions: Particle-vortex duality

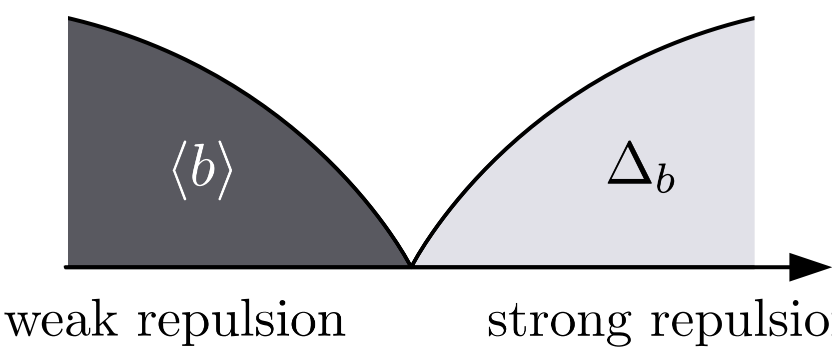


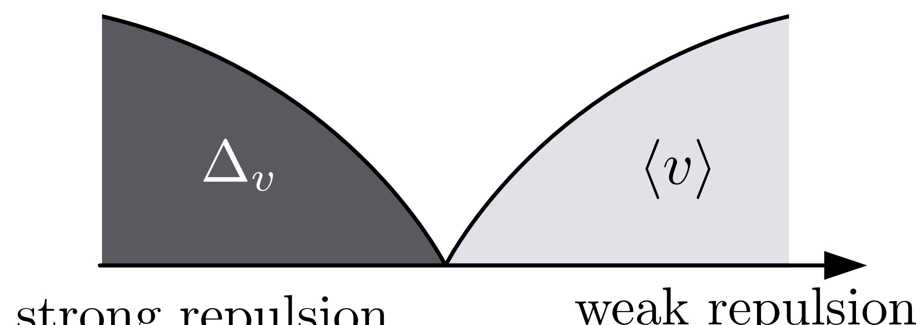
$$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\text{Boson}}|^2 + \frac{i}{4\pi} A_\lambda \epsilon_{\lambda\mu\nu} \partial_\mu A_\nu$$

$$\mathcal{L}_{\text{Vortex}} = |(\partial_\mu - ia_\mu)\Psi_{\text{Vortex}}|^2 - \frac{i}{4\pi} a_\lambda \epsilon_{\lambda\mu\nu} \partial_\mu a_\nu$$

Bosons with **intermediate-range** interactions

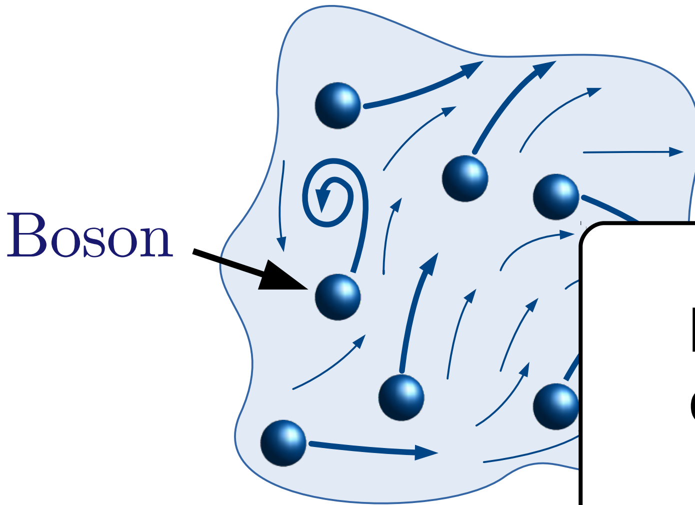
Vortices with **intermediate-range** interactions

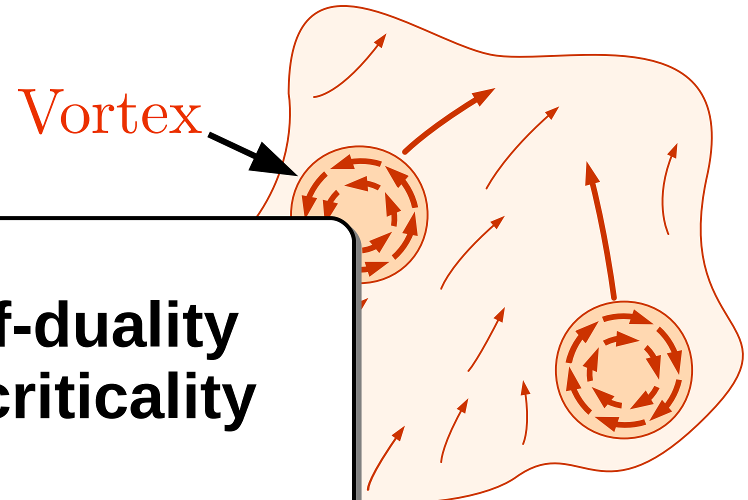




(Fradkin, Kivelson 1996)

2+1 dimensions: Particle-vortex duality





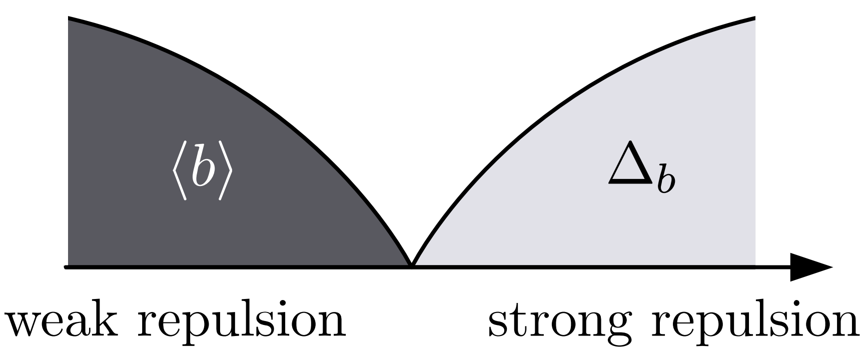
Exact self-duality ensures criticality

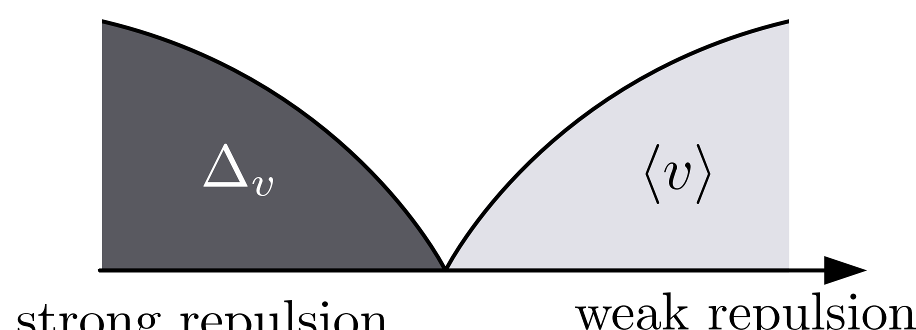
$$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\text{Boson}}|^2 + \frac{i}{4\pi} A_\lambda \epsilon_{\lambda\mu\nu}\partial_\mu A_\nu$$

$$\mathcal{L}_{\text{Vortex}} = |(\partial_\mu - ia_\mu)\Psi_{\text{Vortex}}|^2 - \frac{i}{4\pi} a_\lambda \epsilon_{\lambda\mu\nu}\partial_\mu a_\nu$$

Bosons with **intermediate-range** interactions

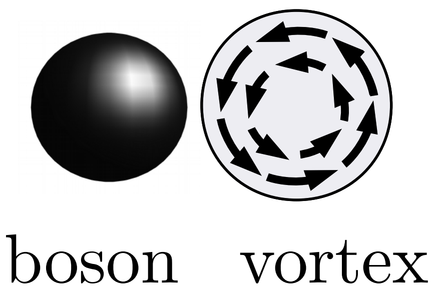
Vortices with **intermediate-range** interactions

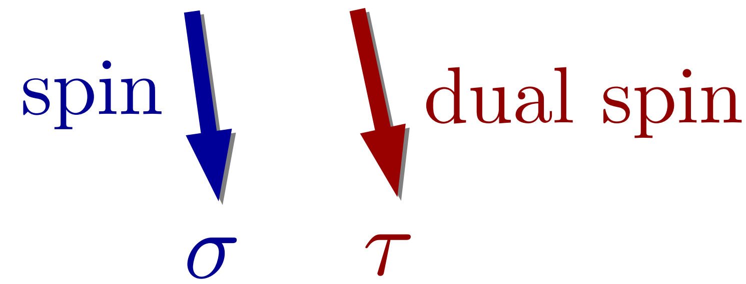


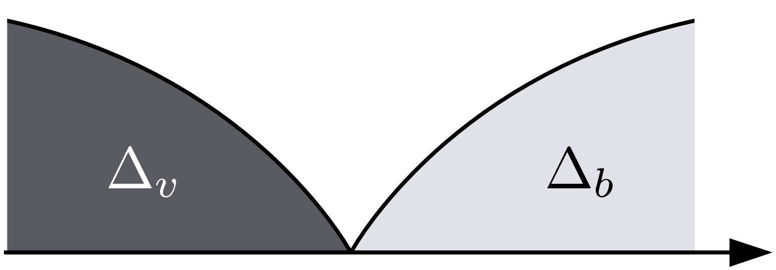


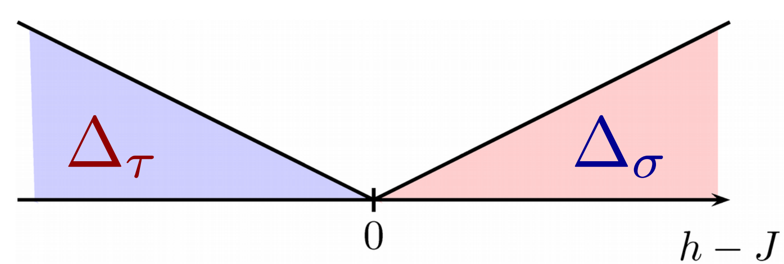
(Fradkin, Kivelson 1996)

Particle-vortex duality as a symmetry

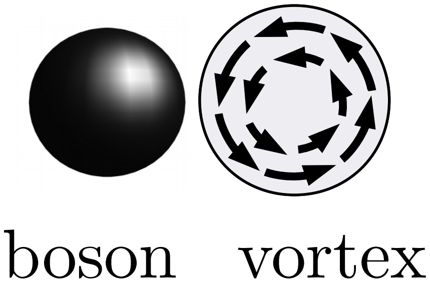


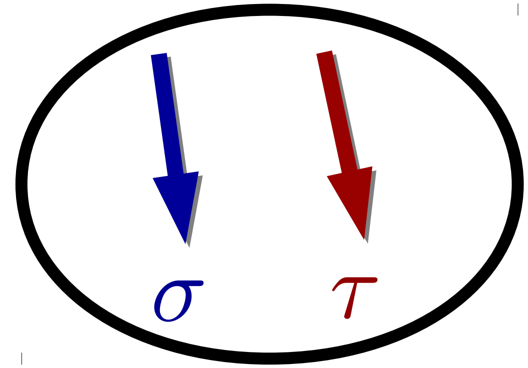






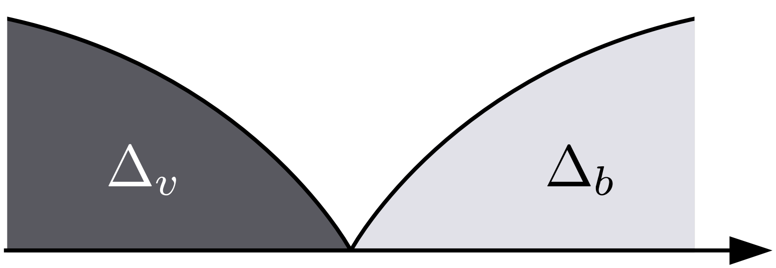
Particle-vortex duality as a symmetry

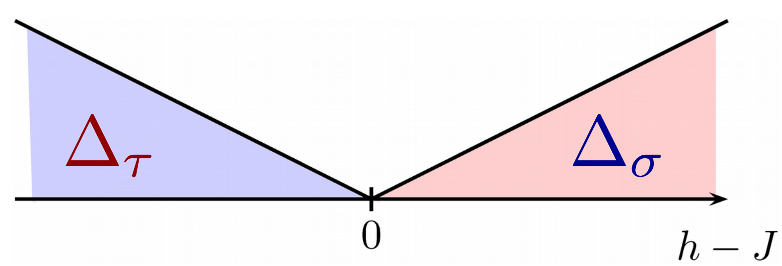




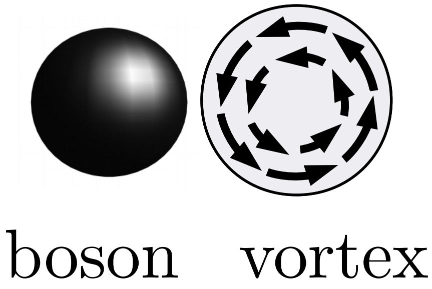
$$\gamma_{i+1/4} = i\sigma_i^z \tau_{i+1/2}^z$$

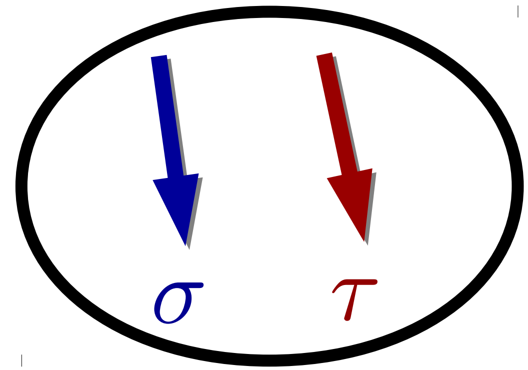
Jordan-Wigner fermion





Particle-vortex duality as a symmetry

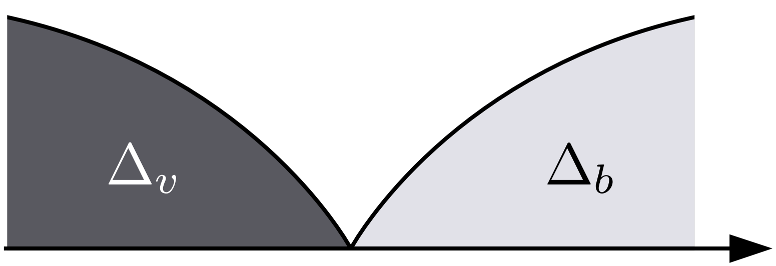


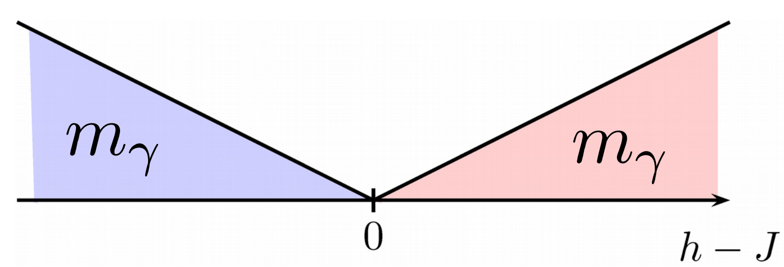


$$\gamma_{i+1/4} = i\sigma_i^z \tau_{i+1/2}^z$$

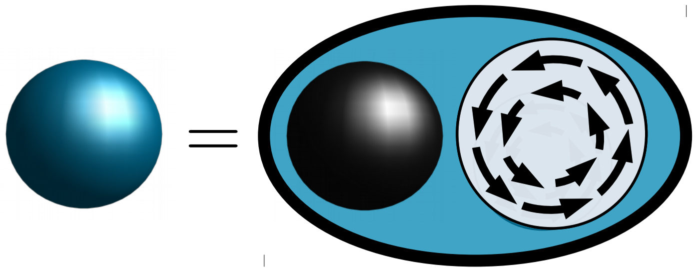
Jordan-Wigner fermion

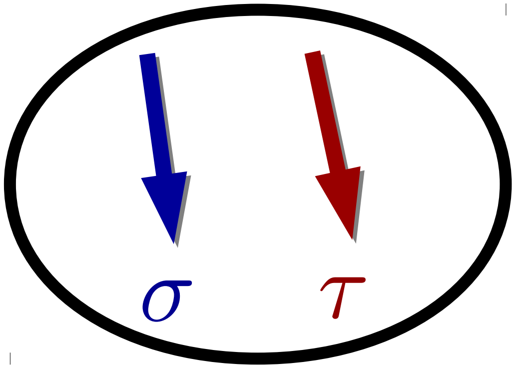
$$m_\gamma = \Delta_\sigma + \Delta_\tau$$





Particle-vortex duality as a symmetry





$\psi = \Psi_{\text{Boson}}\Psi_{\text{Vortex}}$

$\gamma_{i+1/4} = i\sigma_i^z \tau_{i+1/2}^z$

composite fermion

Jordan-Wigner fermion

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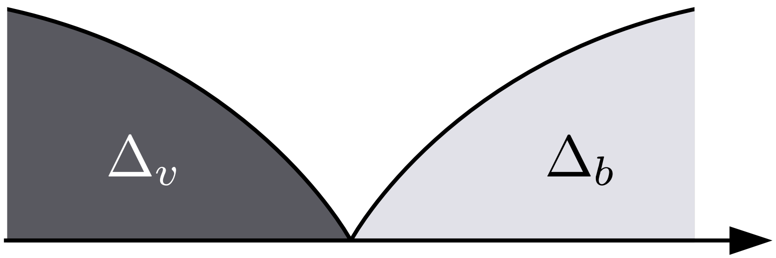
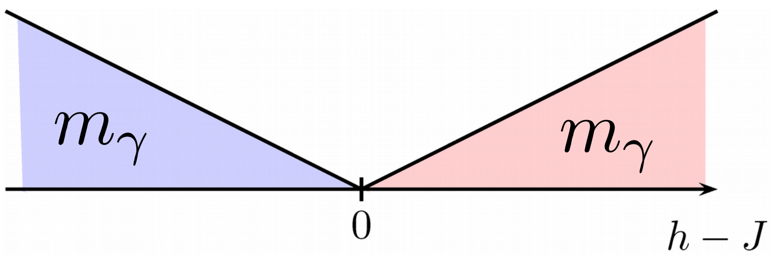
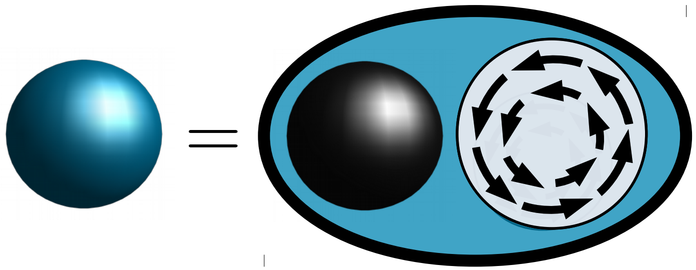
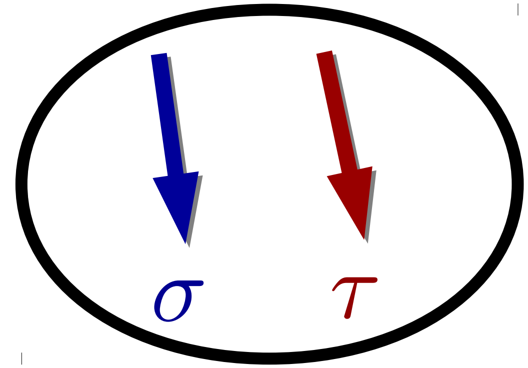




Particle-vortex duality as a symmetry





$$\psi = \Psi_{\text{Boson}}\Psi_{\text{Vortex}}$$

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Jordan-Wigner fermion

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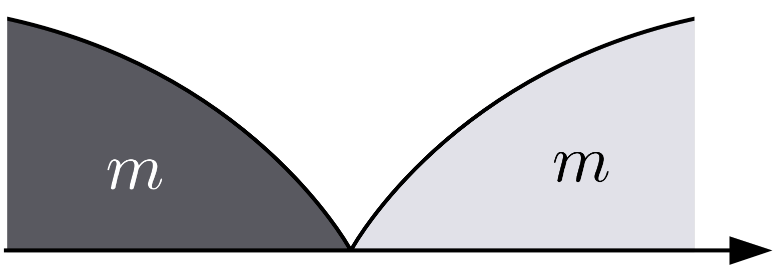

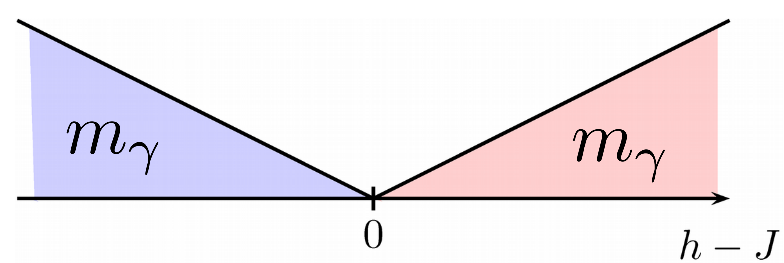

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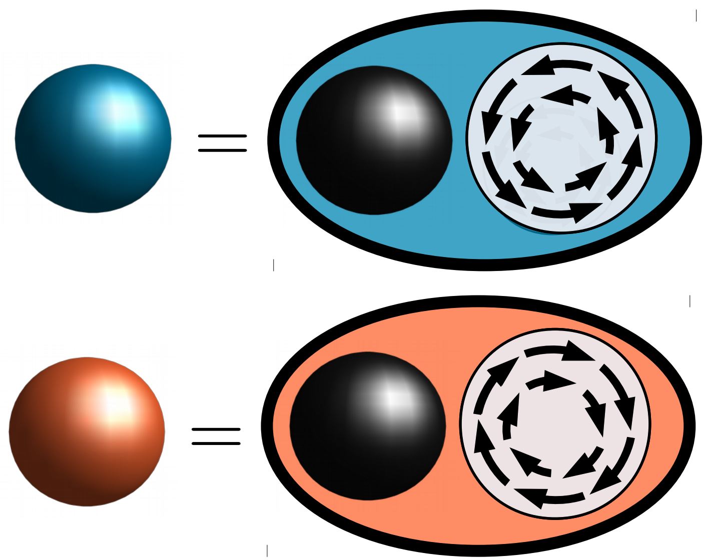

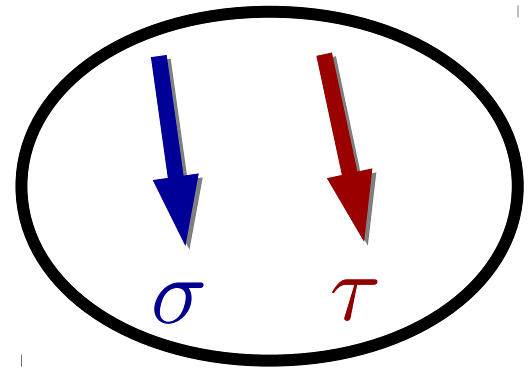

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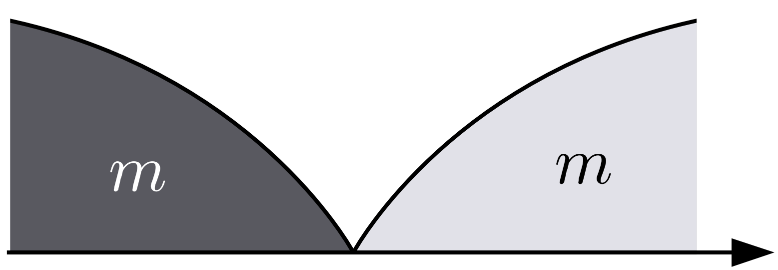
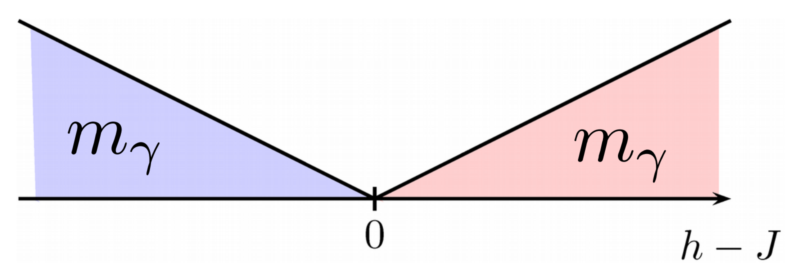
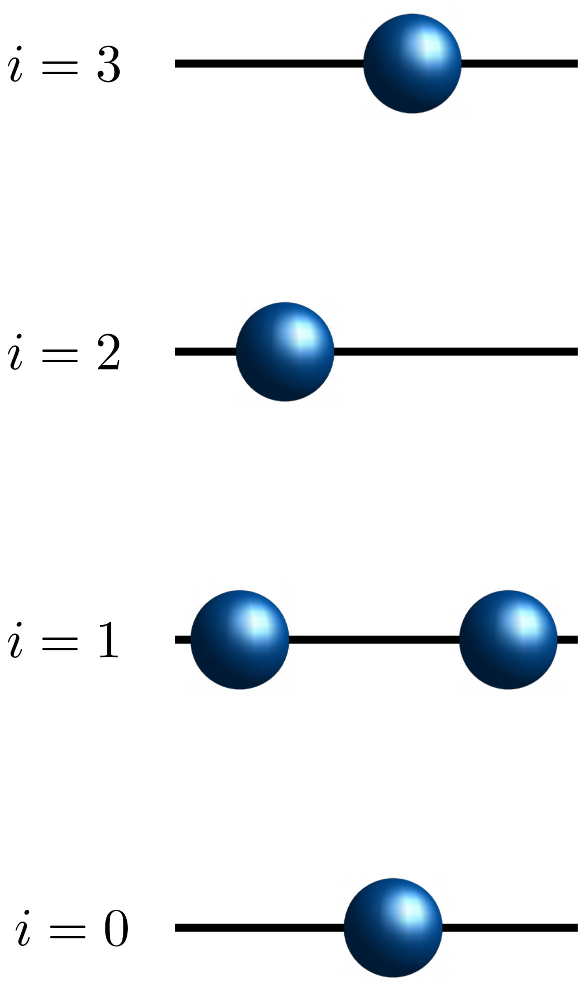
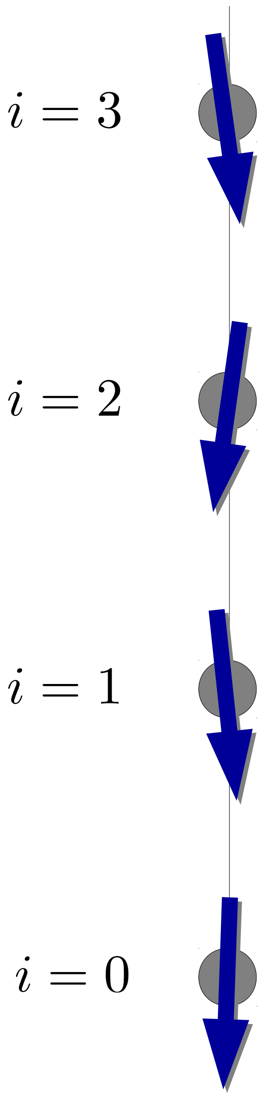




Particle-vortex duality as a symmetry

- Make analogy precise: Wire models





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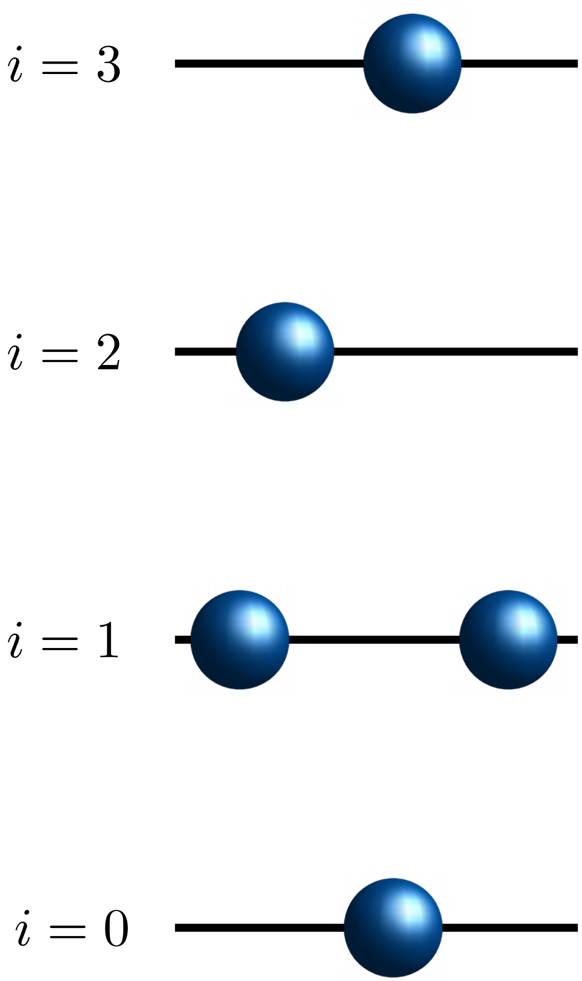

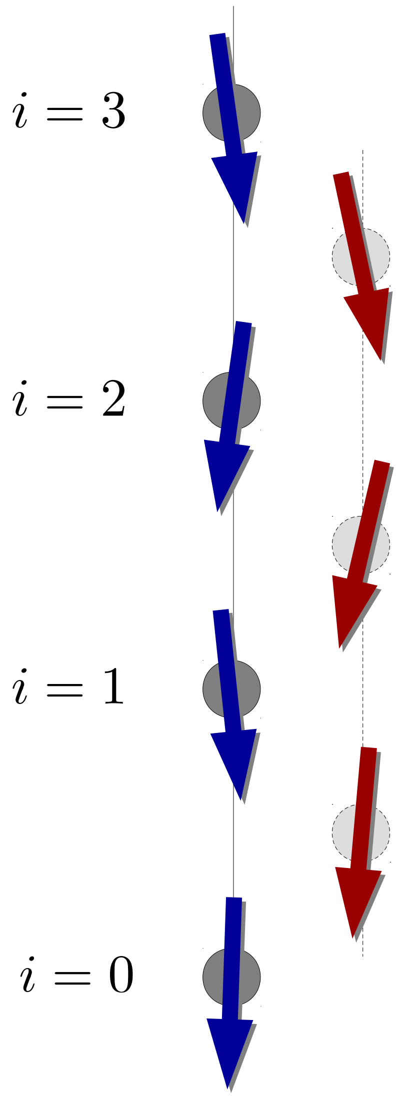

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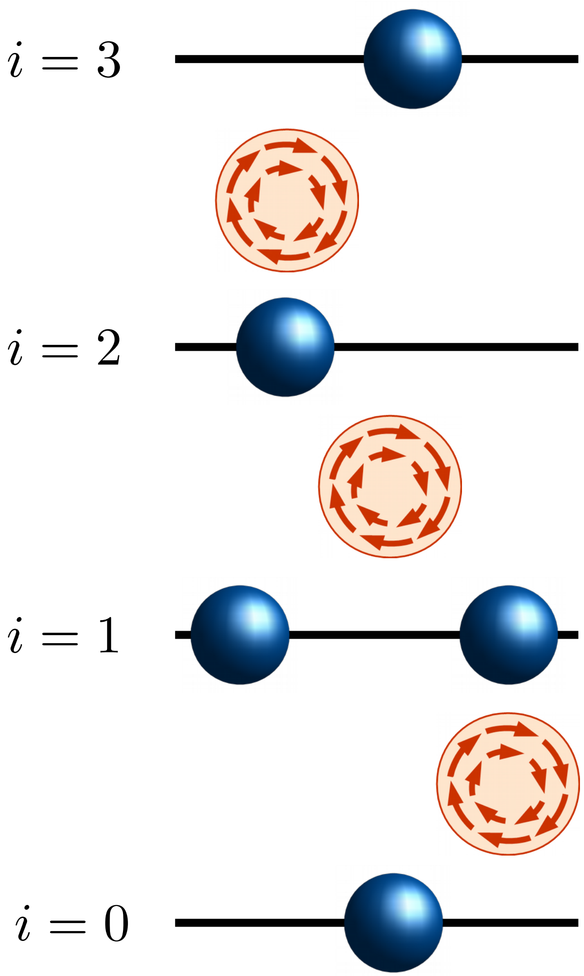

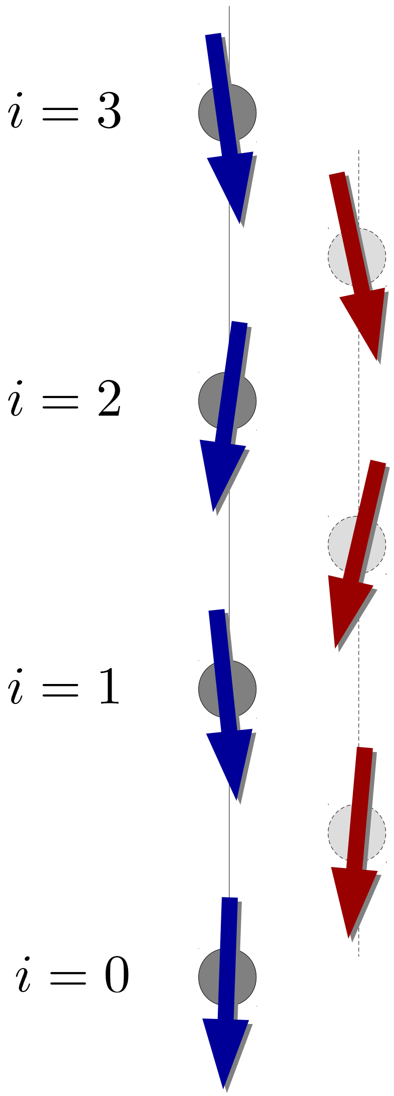

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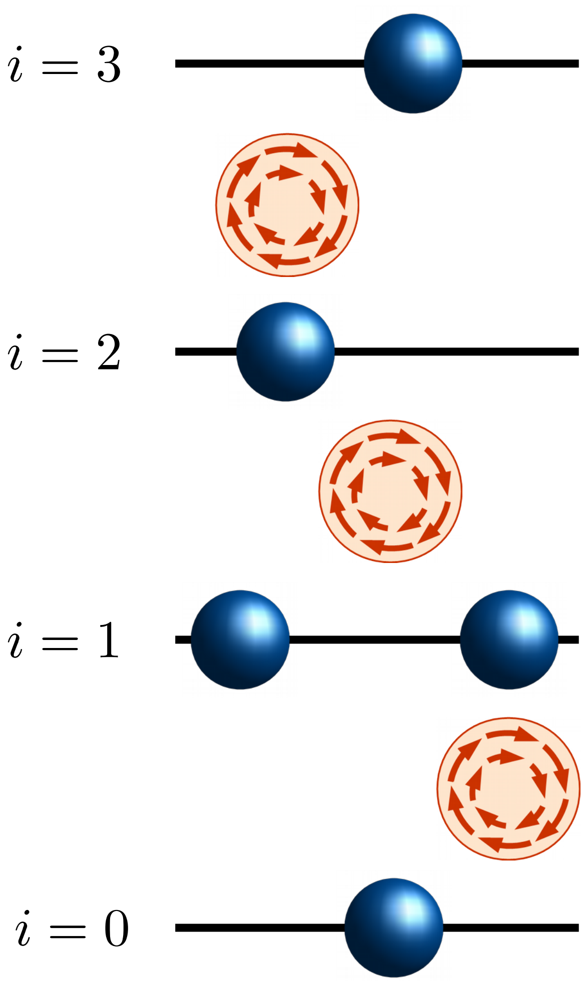

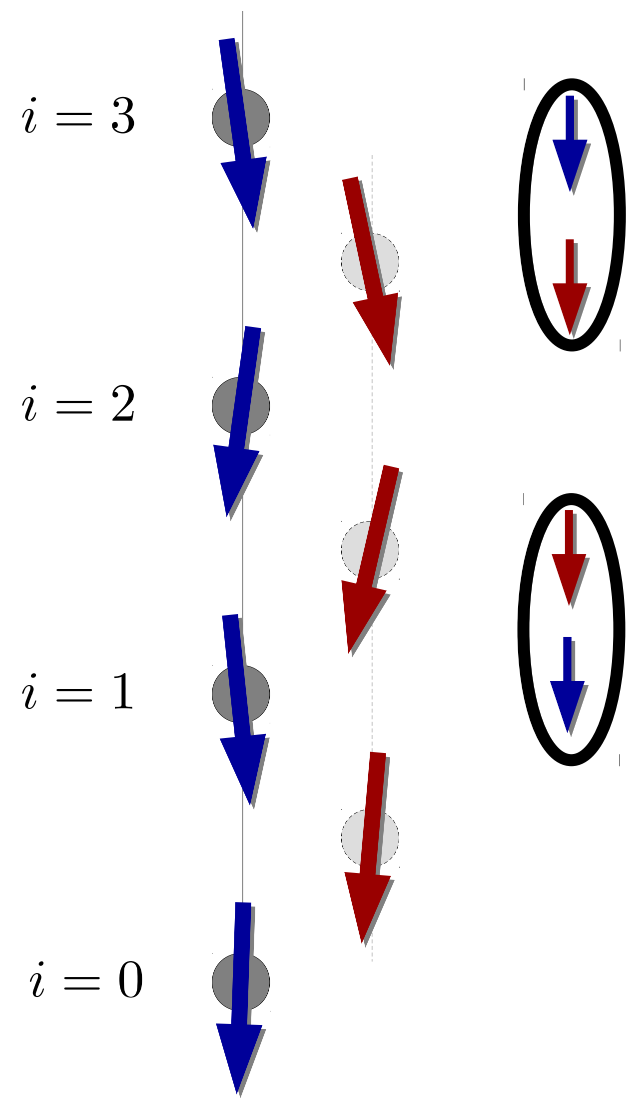

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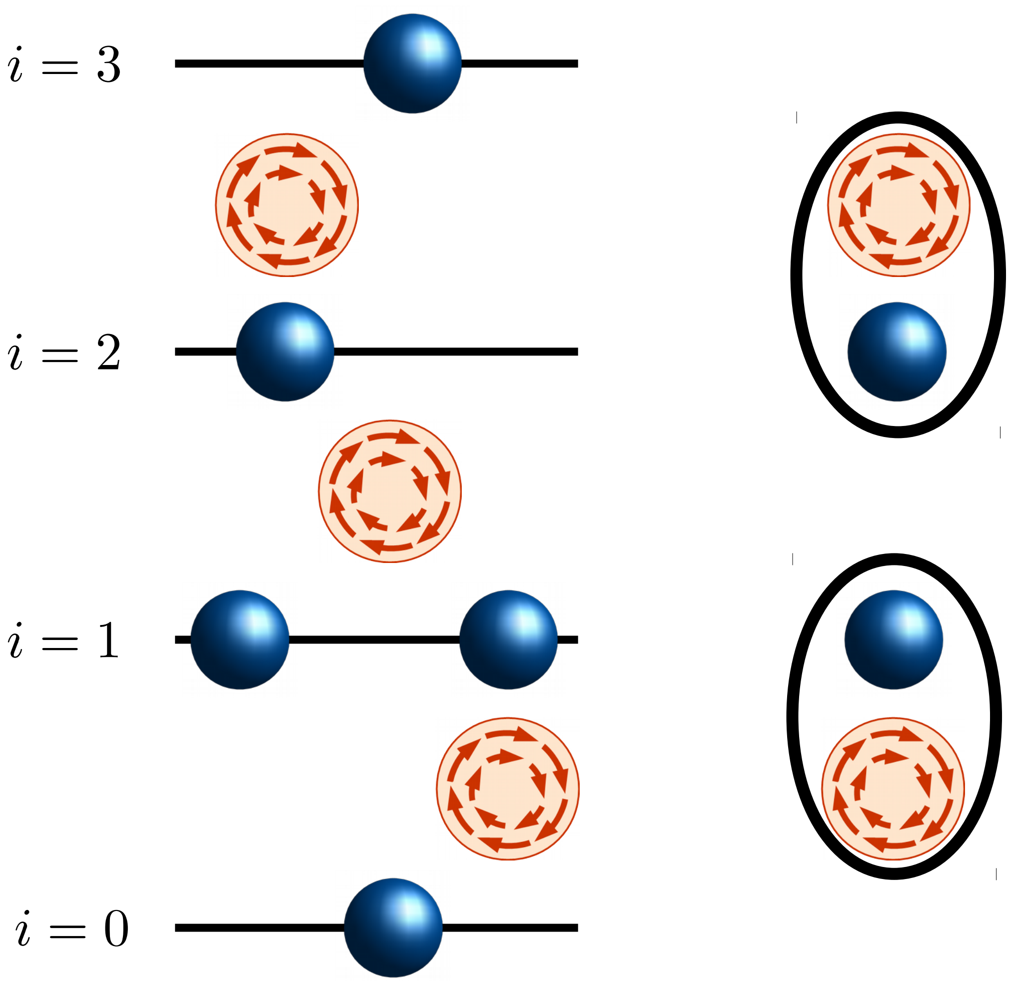

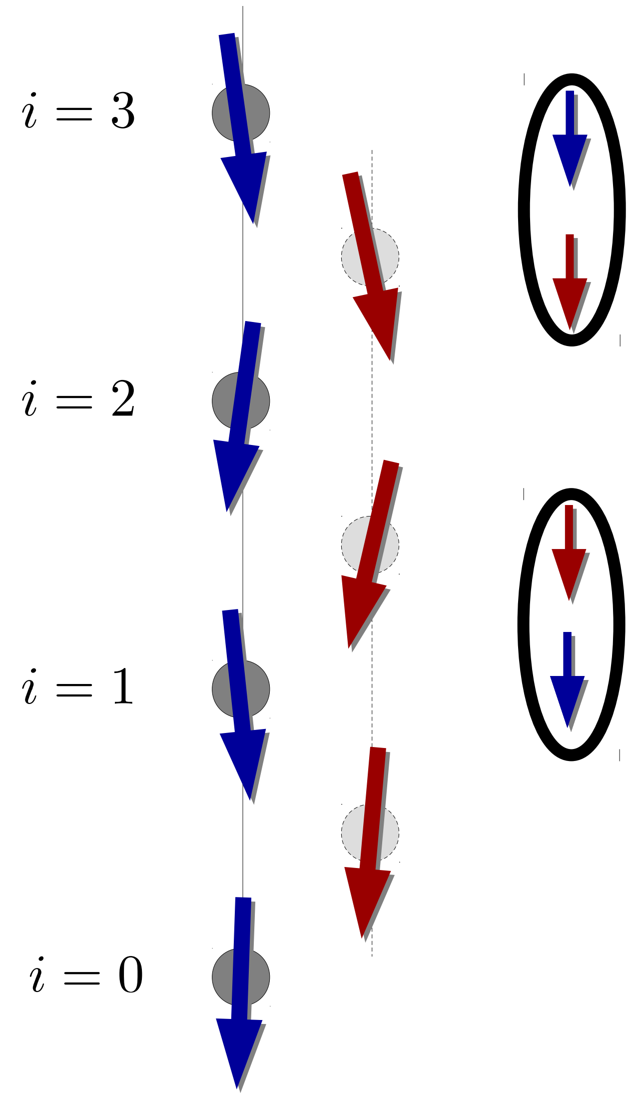

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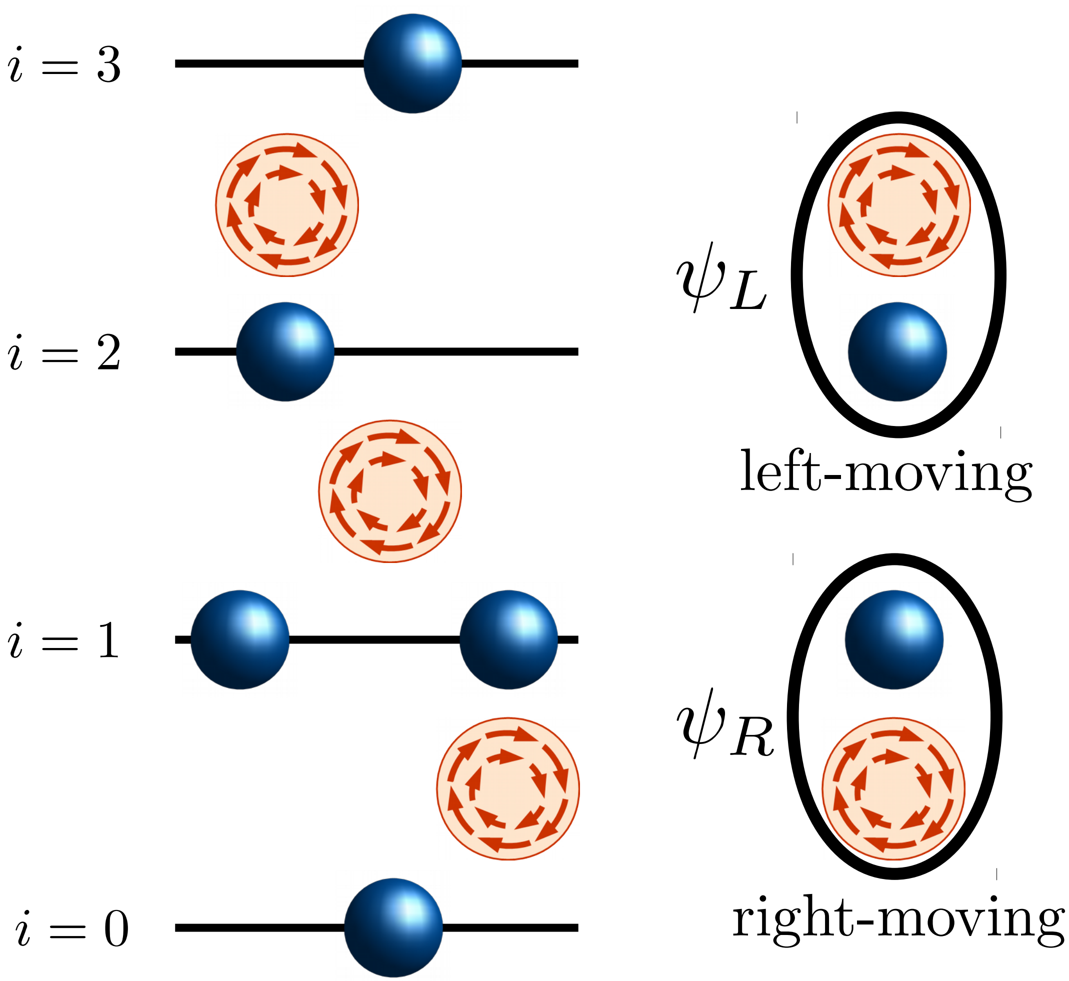

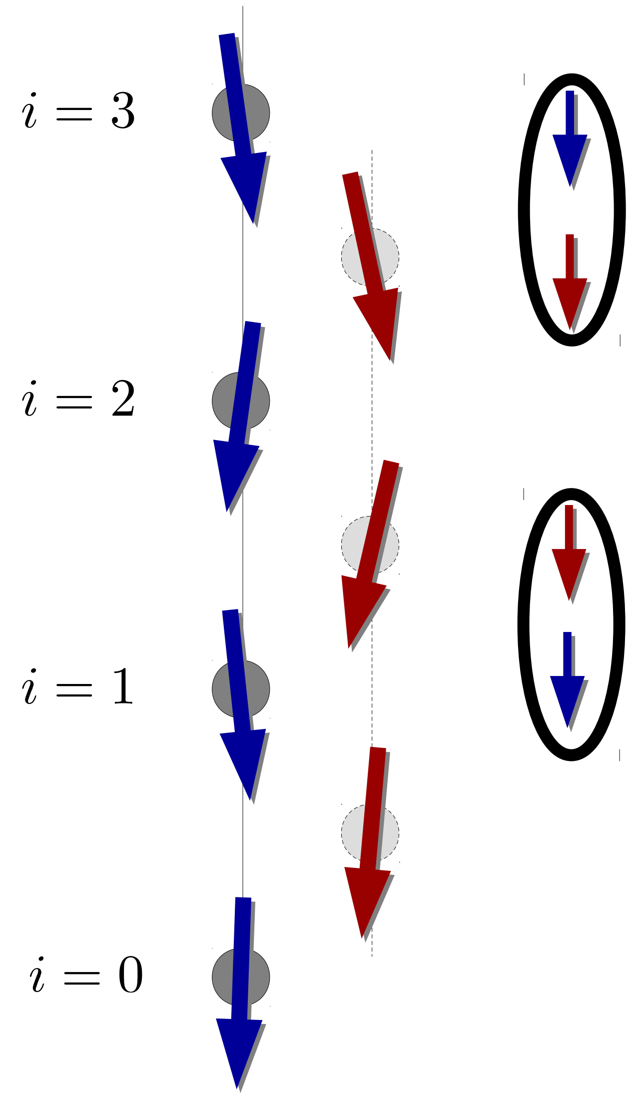

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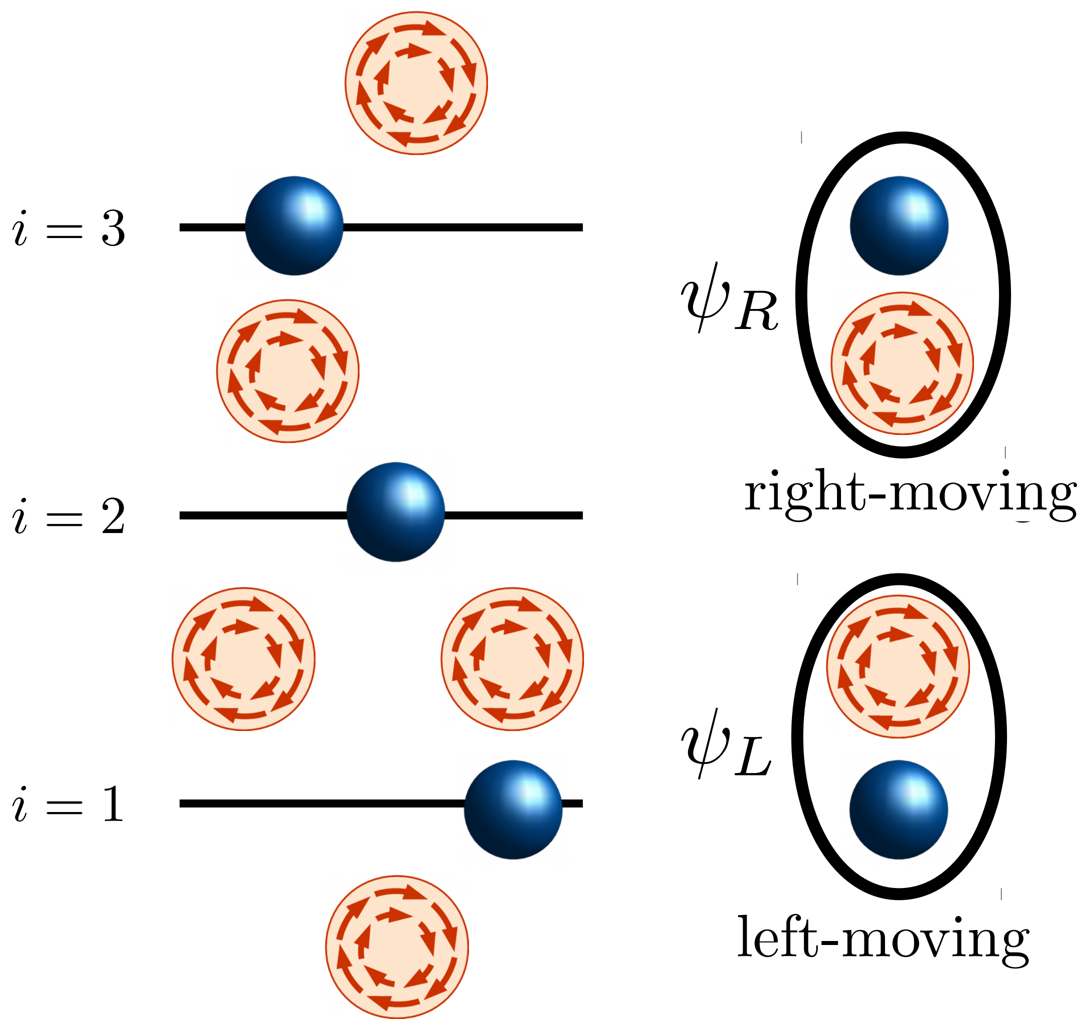

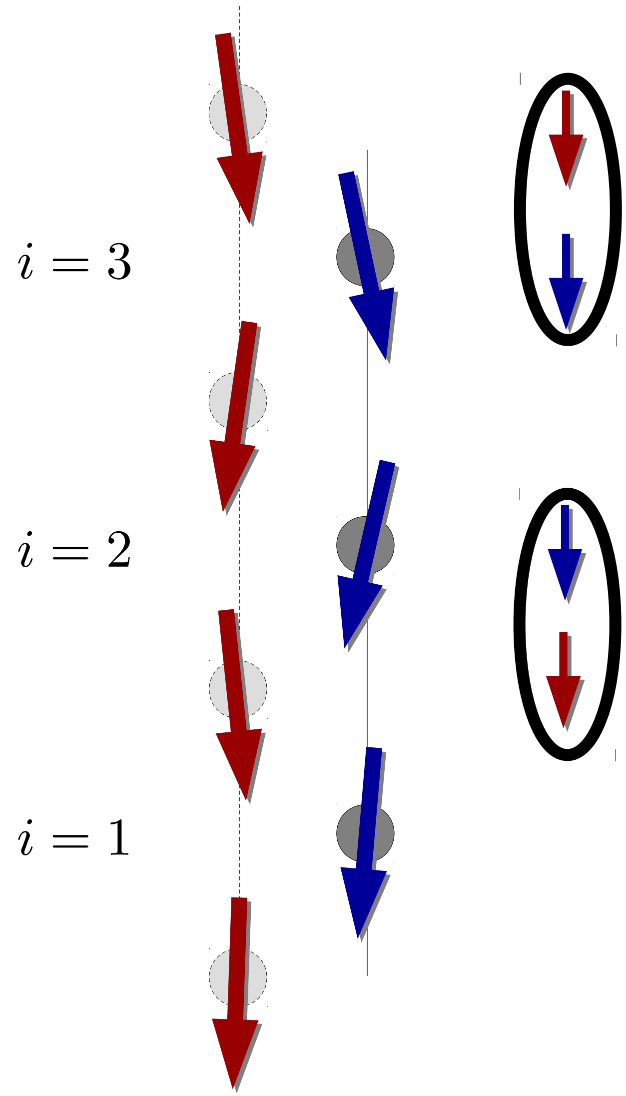

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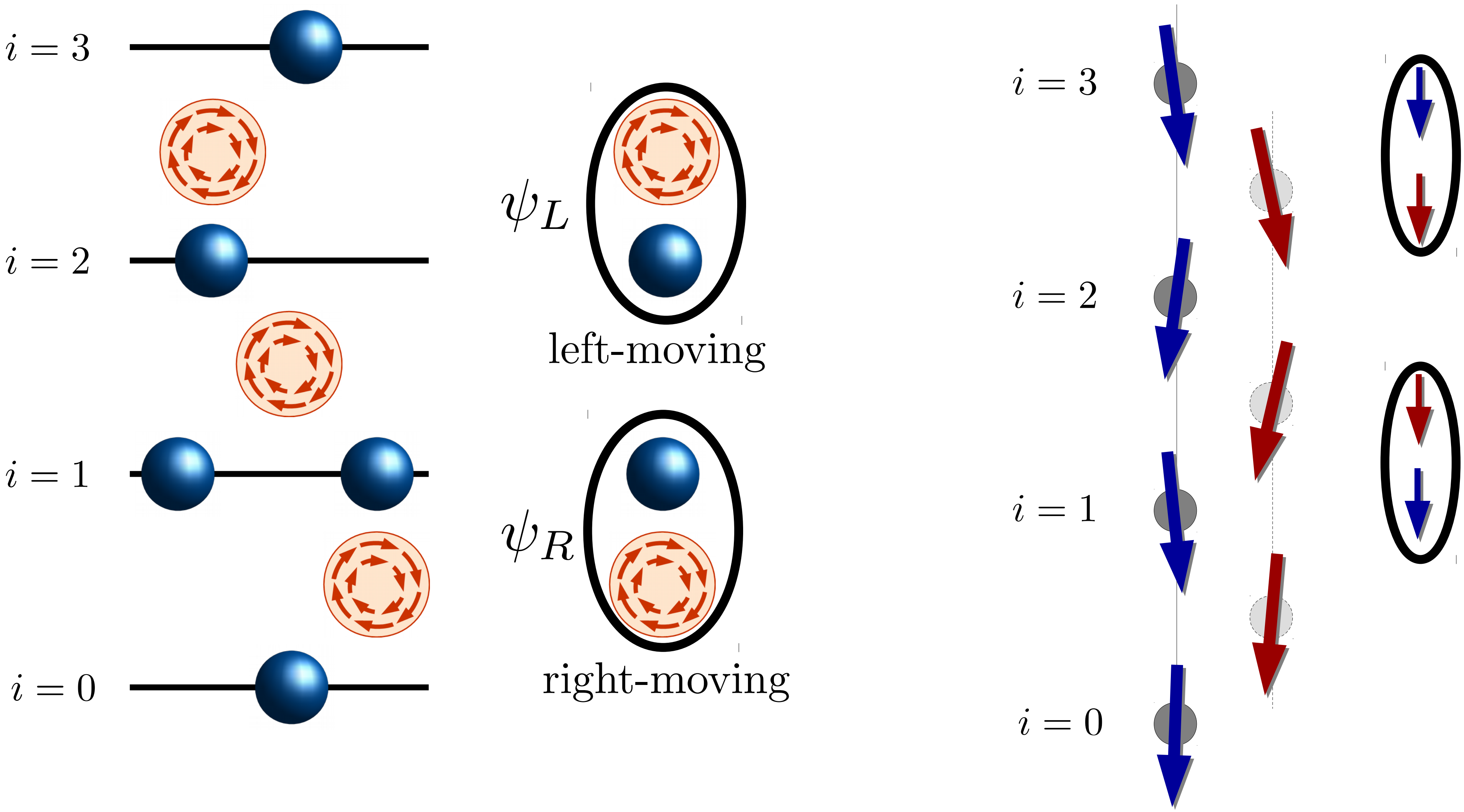

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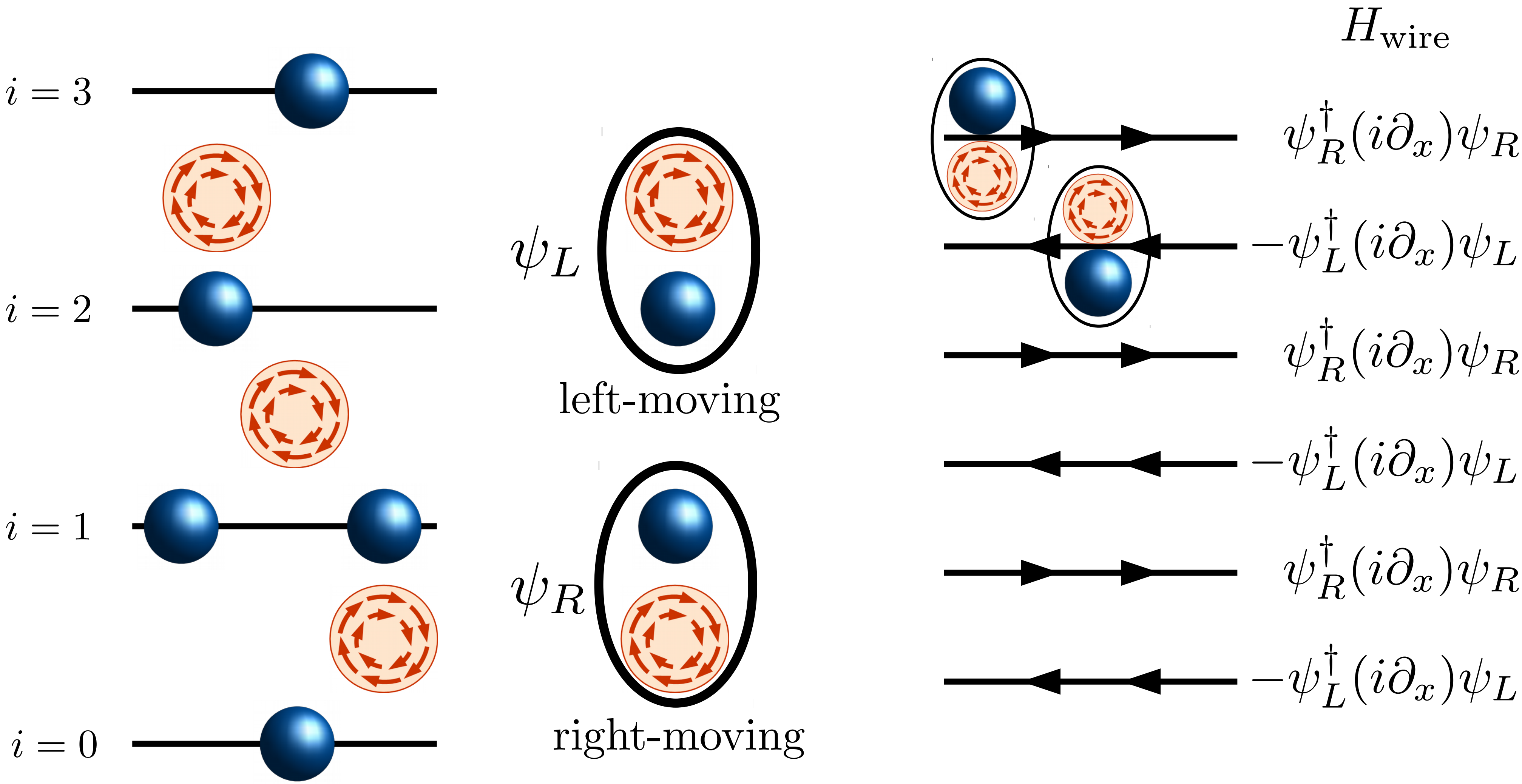

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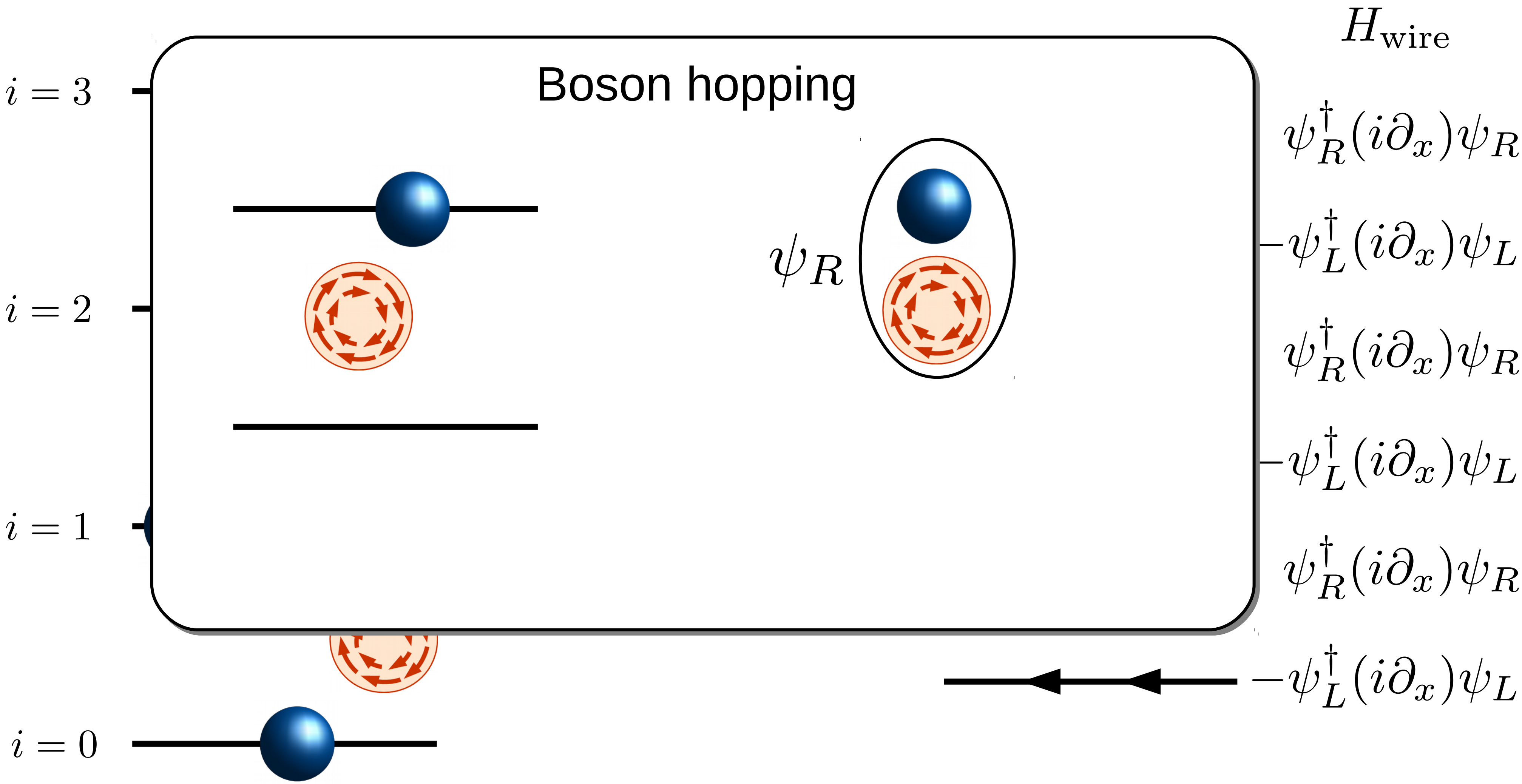

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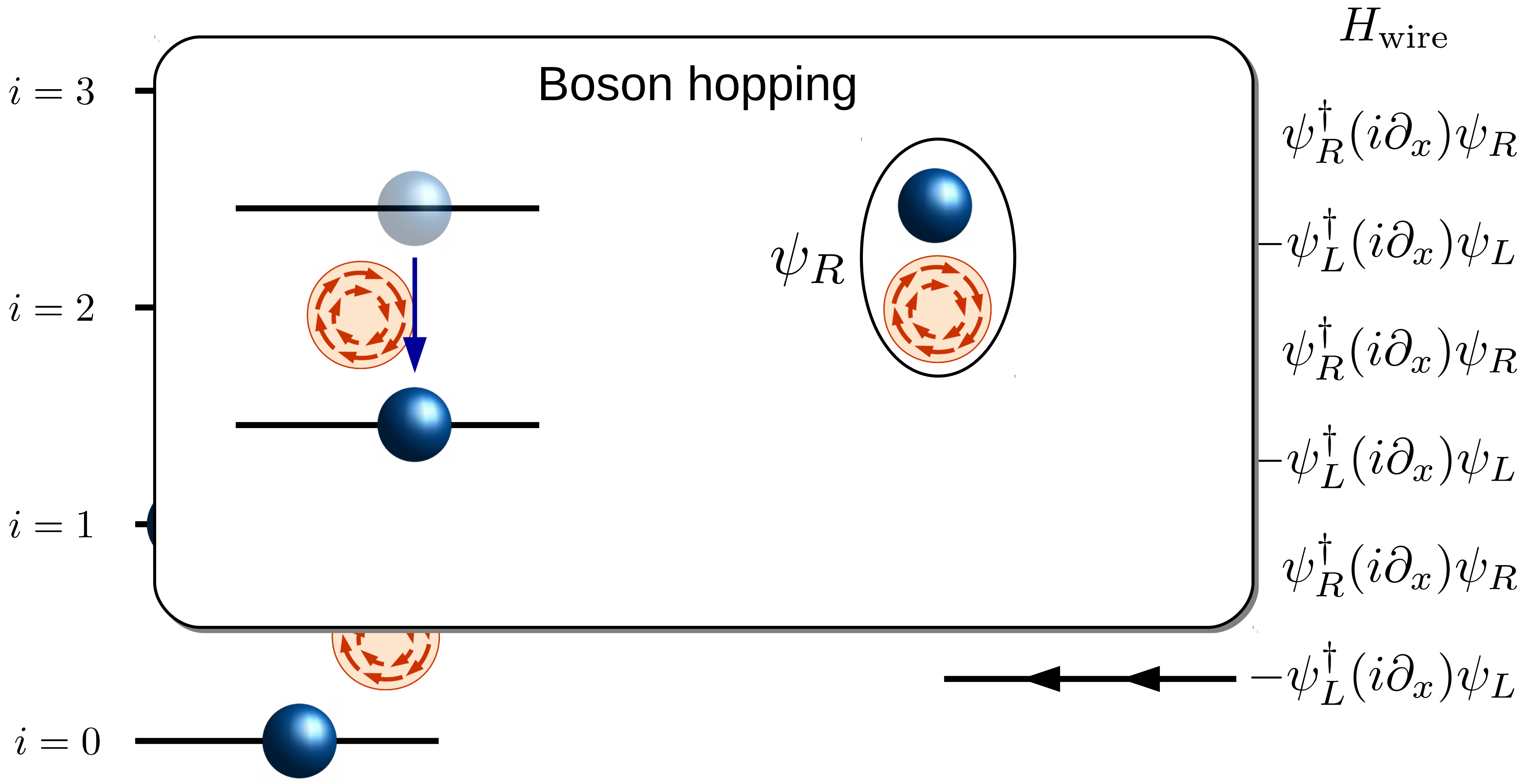

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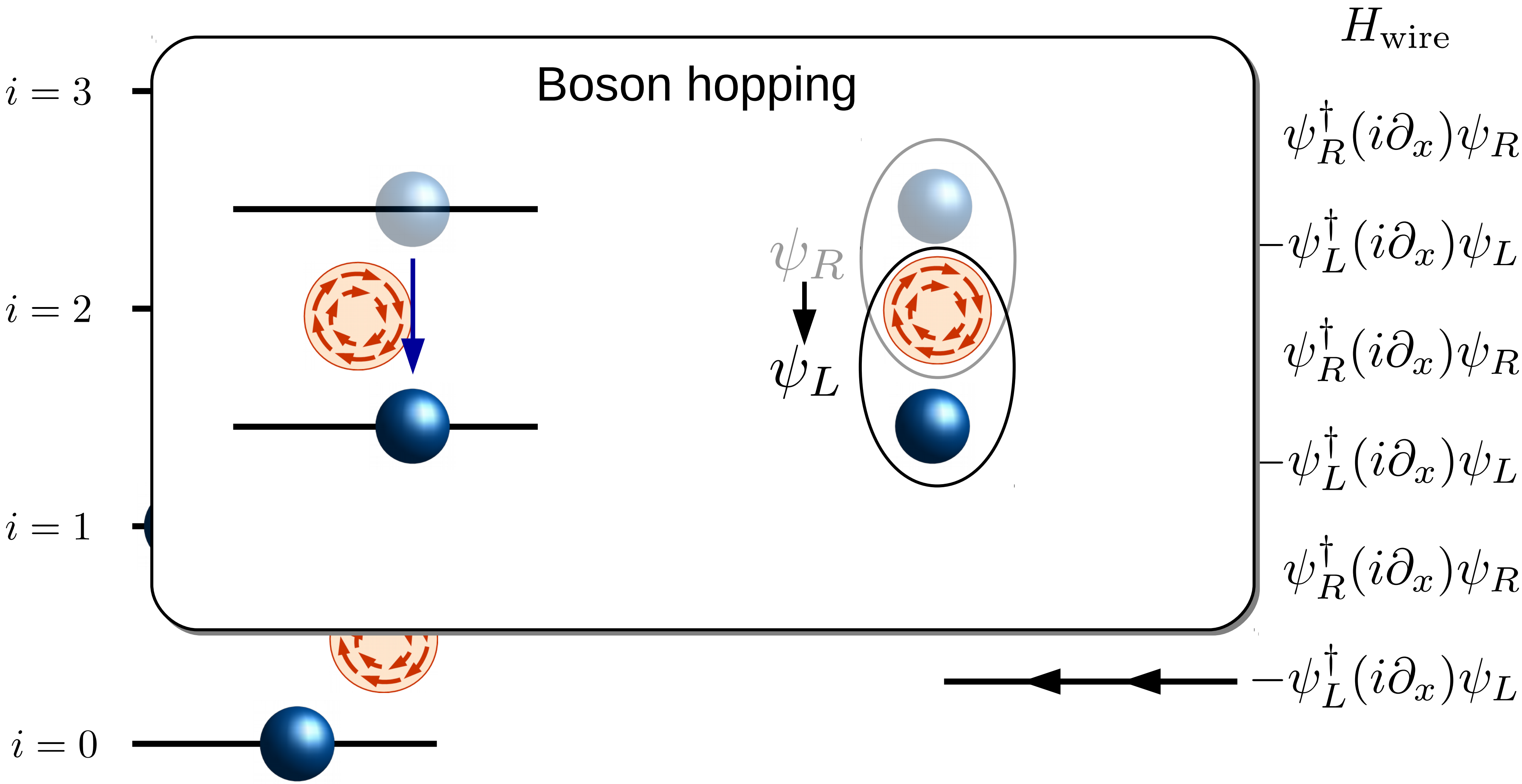

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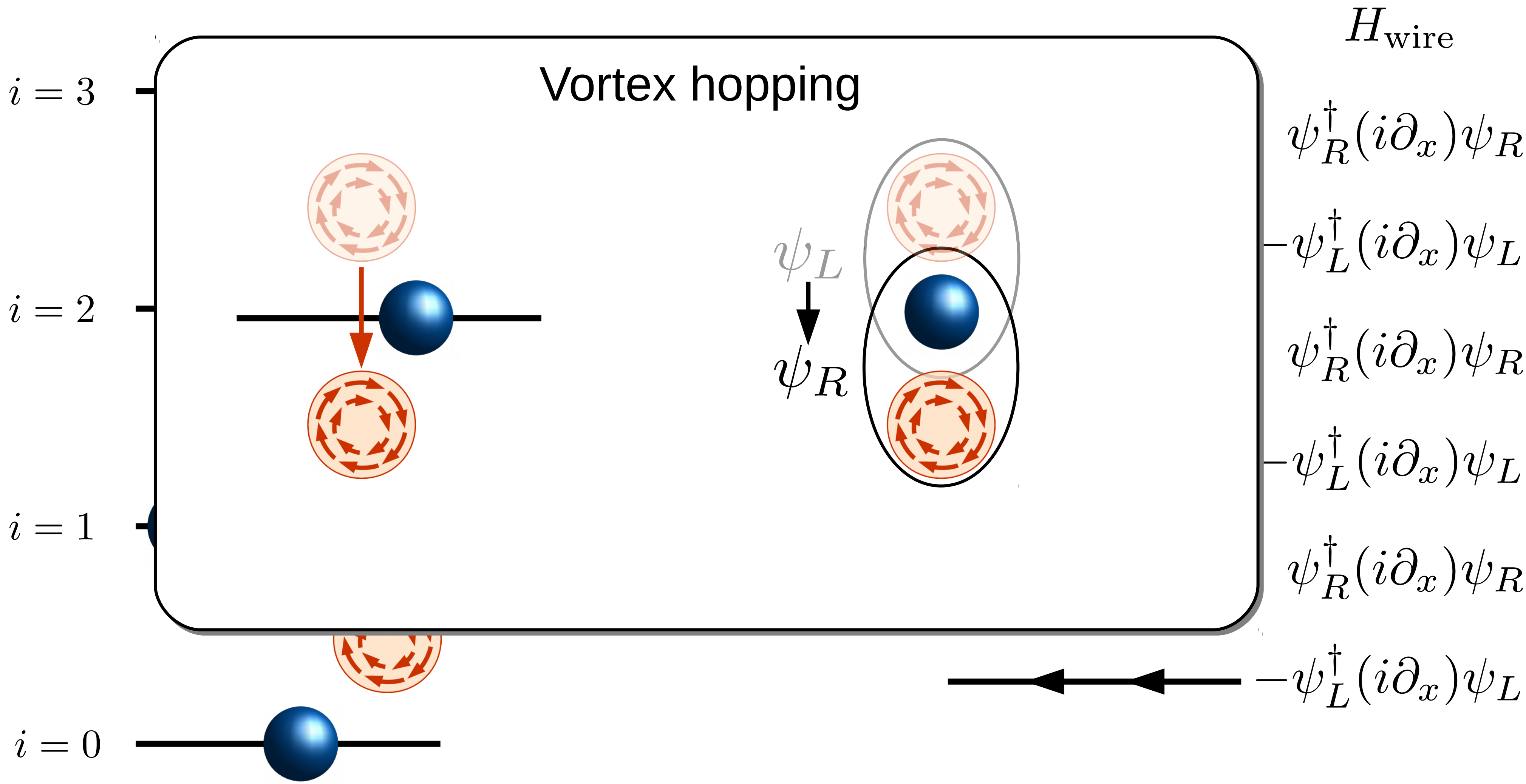

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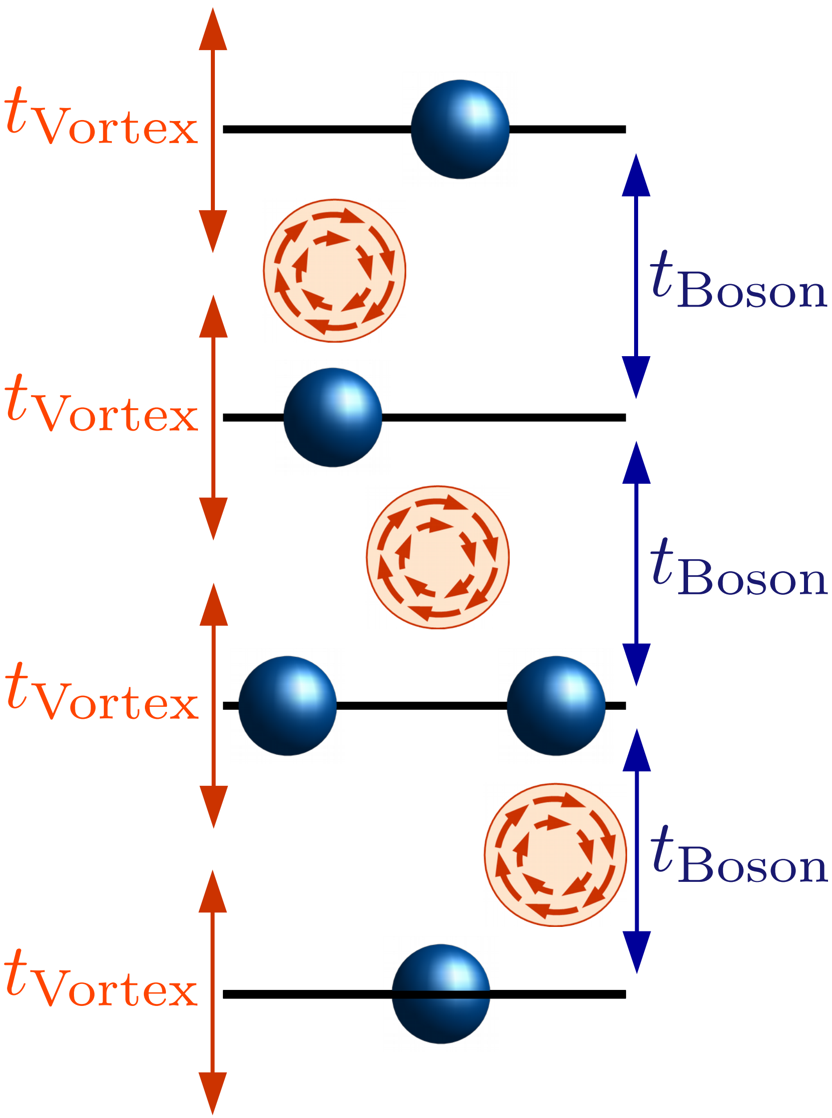
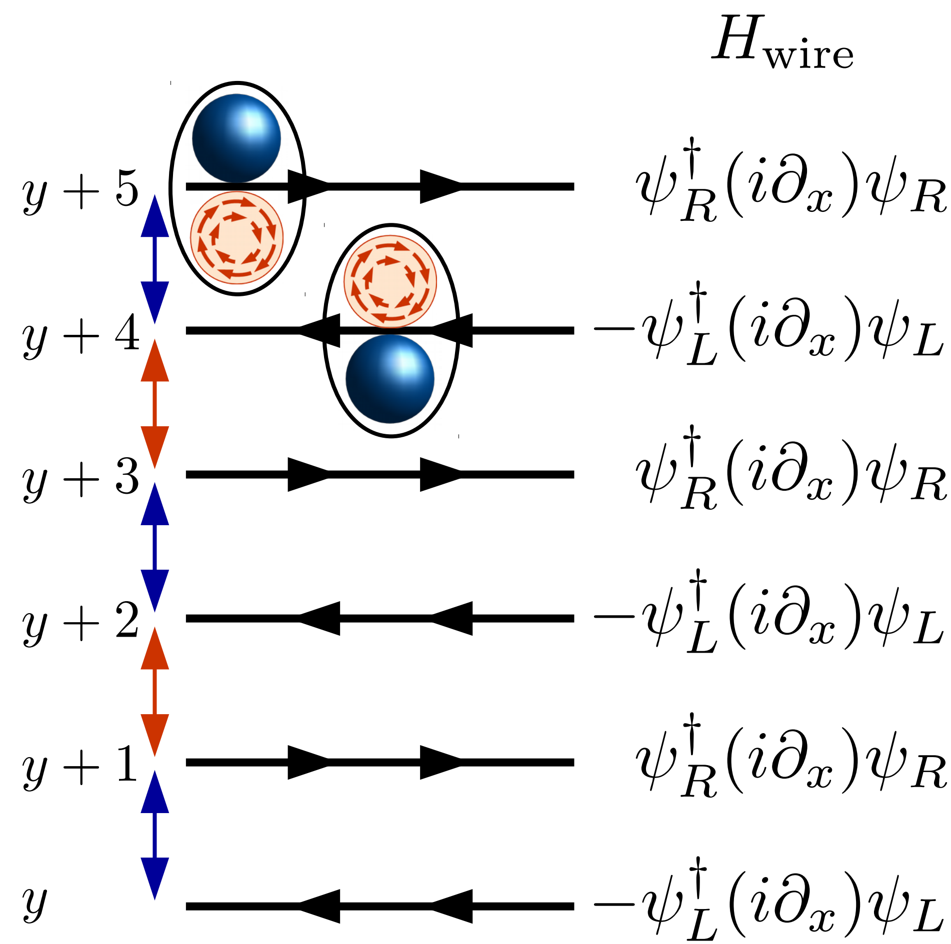


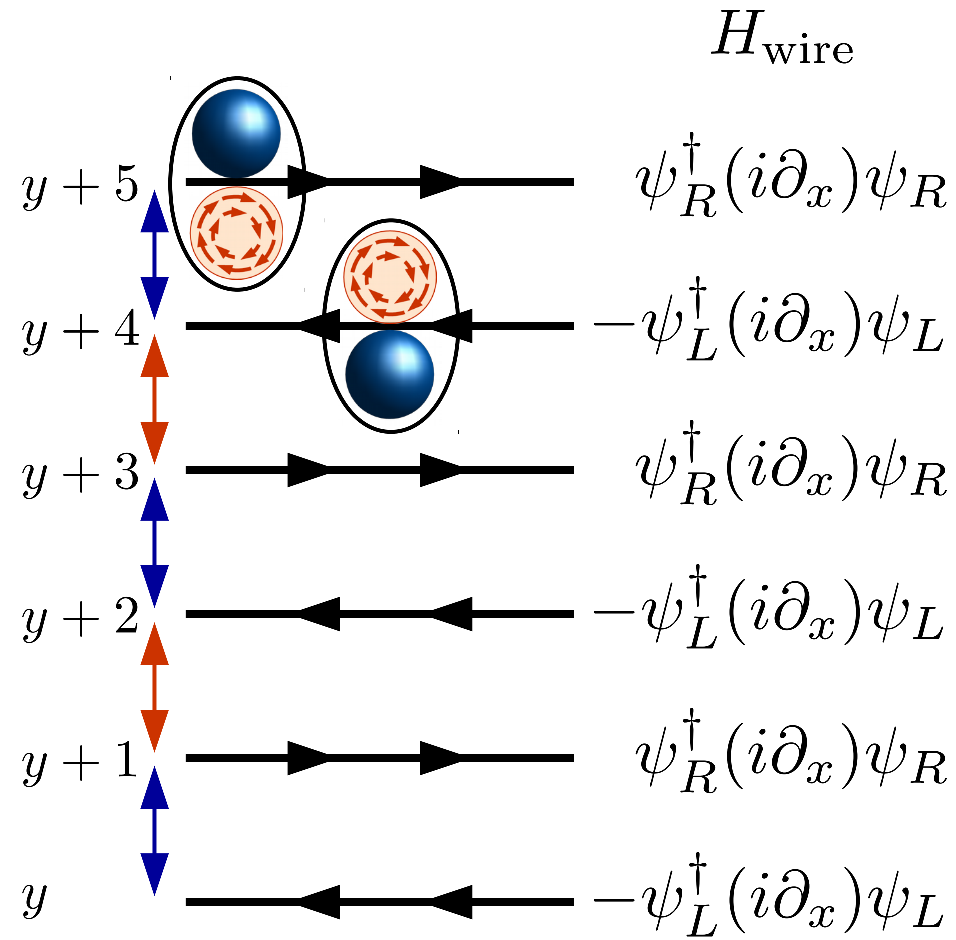


Particle-vortex duality as a symmetry

- Make analogy precise: Wire models

when $t_{\text{Vortex}} = t_{\text{Boson}}$

$$H_{\text{hop}} = -t \sum_y \psi_y^\dagger \psi_{y+1} + \text{H.c.}$$



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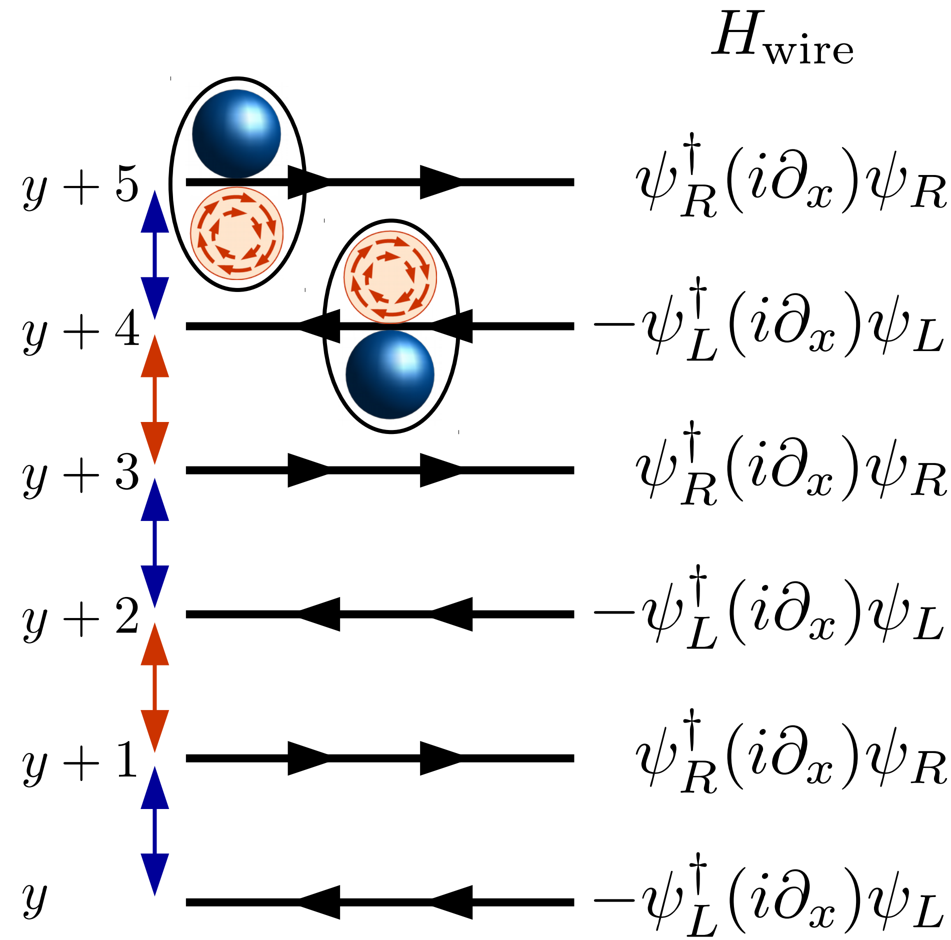
2-component spinor $\quad \Psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$

$$H_{\text{wire}} = \Psi^\dagger k_x \tau_z \Psi$$



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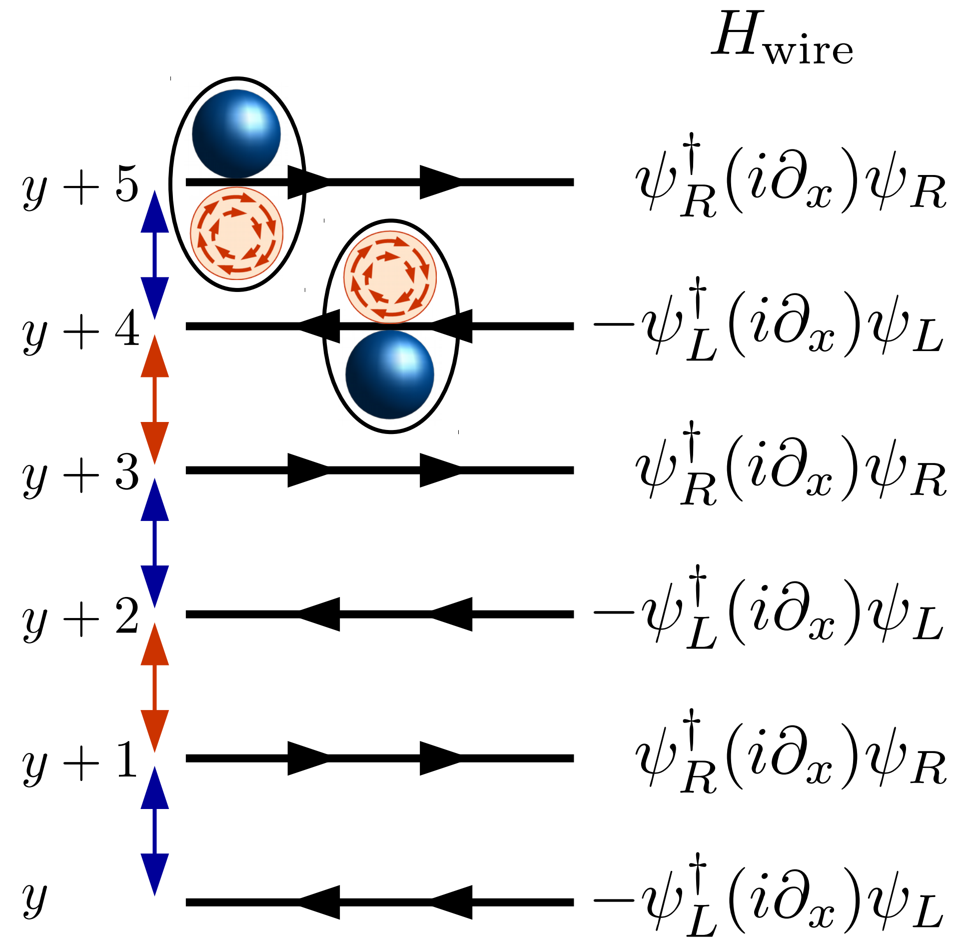
$$H_{\text{wire}} = \Psi^\dagger k_x \tau_z \Psi$$

$$H_{\text{hop}} = \Psi^\dagger \begin{pmatrix} 0 & \sin k_y \\ \sin k_y & 0 \end{pmatrix} \Psi$$



Particle-vortex duality as a symmetry

- Make analogy precise: Wire models

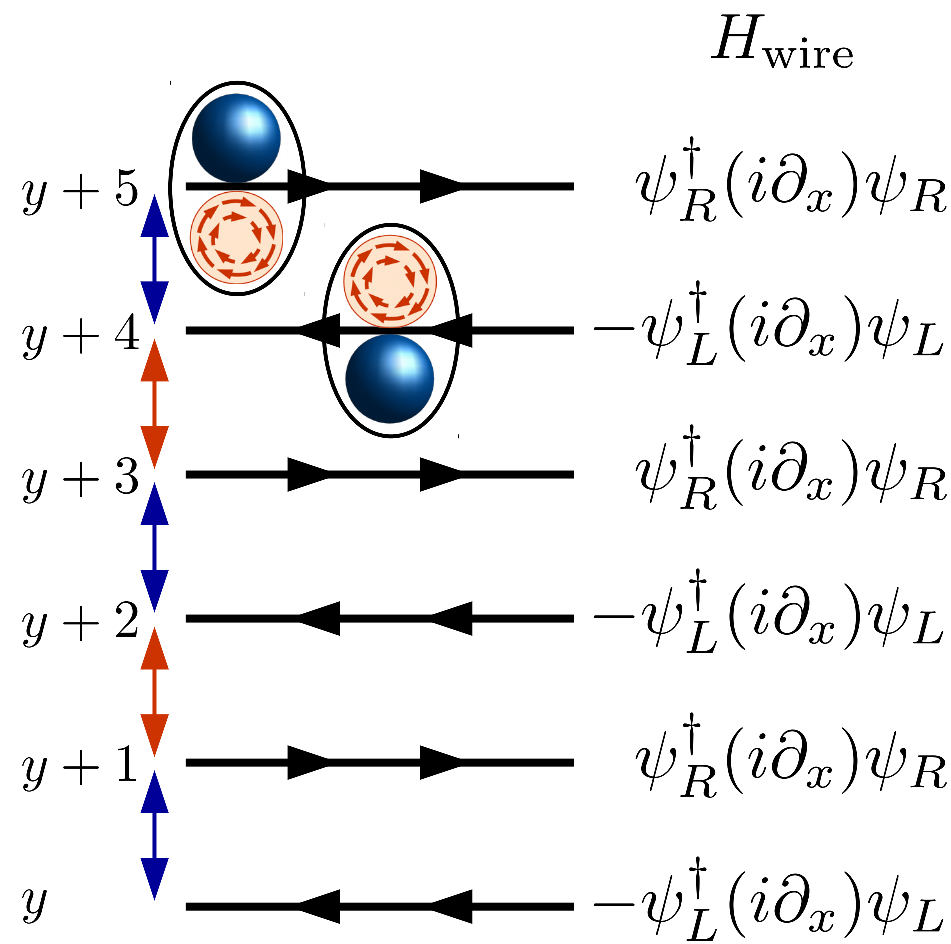
when $t_{\text{Vortex}} = t_{\text{Boson}}$

$$H_{\text{hop}} = -t \sum_{y} \psi_y^\dagger \psi_{y+1} + \text{H.c.}$$

2-component spinor $\quad \Psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$

$$H_{\text{wire}} = \Psi^\dagger k_x \tau_z \Psi$$

$$H_{\text{hop}} = \Psi^\dagger \begin{pmatrix} 0 & k_y \\ k_y & 0 \end{pmatrix} \Psi$$



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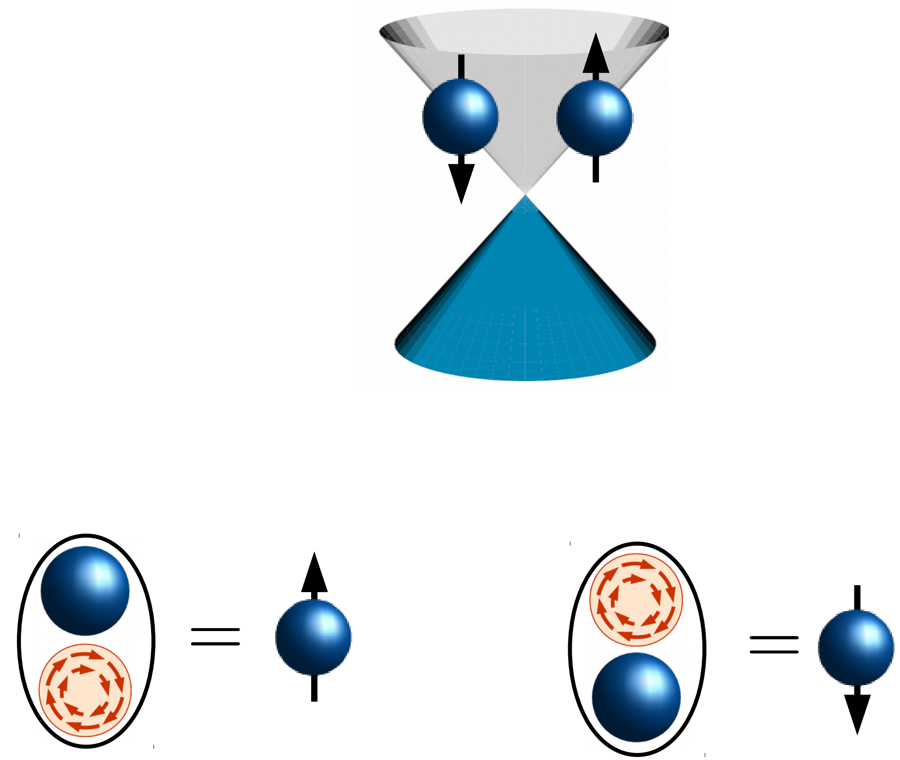
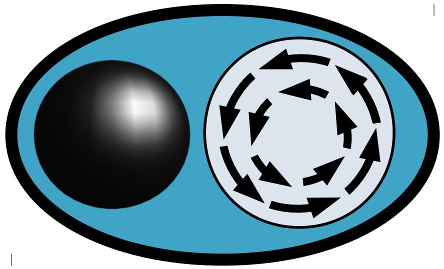
$$H_{\text{hop}} + H_{\text{wire}} = H_{\text{Dirac}}$$

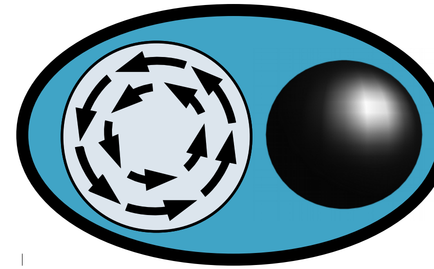


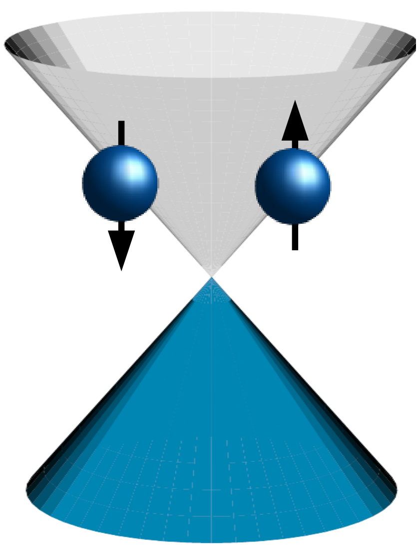
Particle-vortex duality as a symmetry

$\psi_{A,\uparrow}$

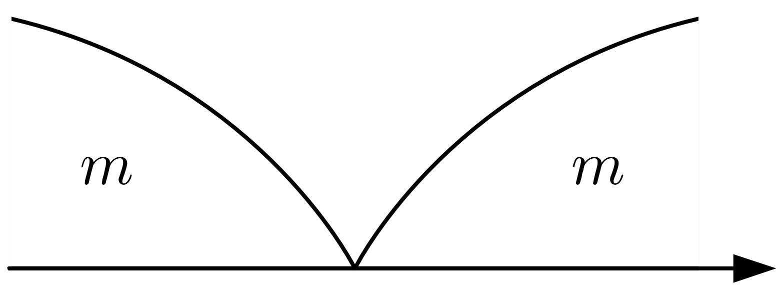


$\psi_{A,\downarrow}$

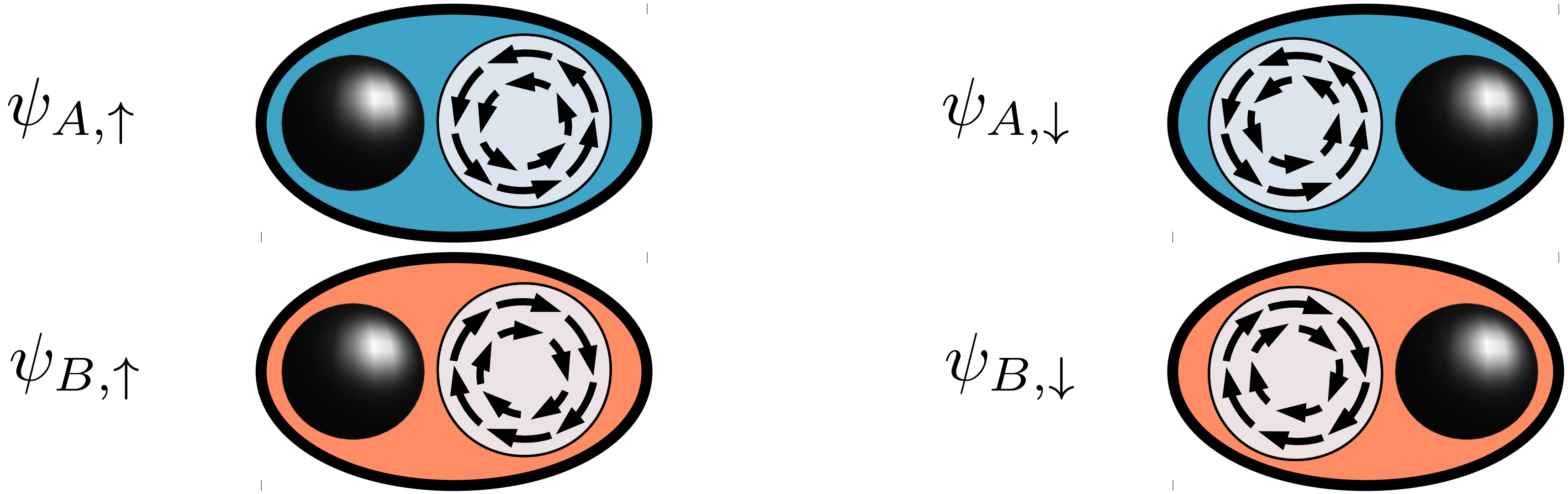


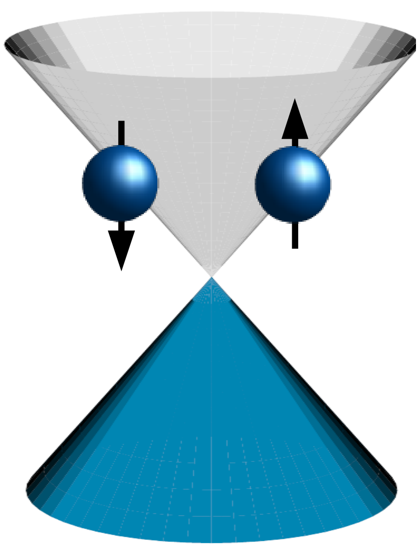


massless Dirac fermions
at critical point



Particle-vortex duality as a symmetry





massless Dirac fermions

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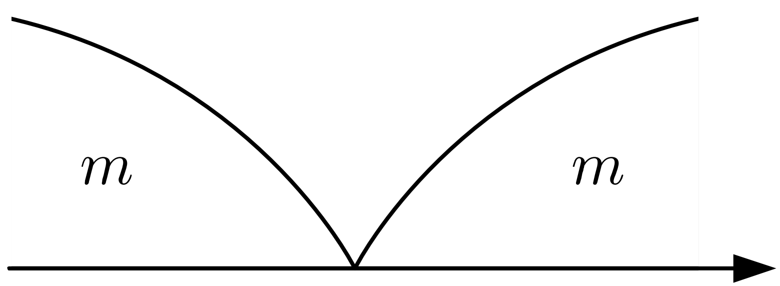
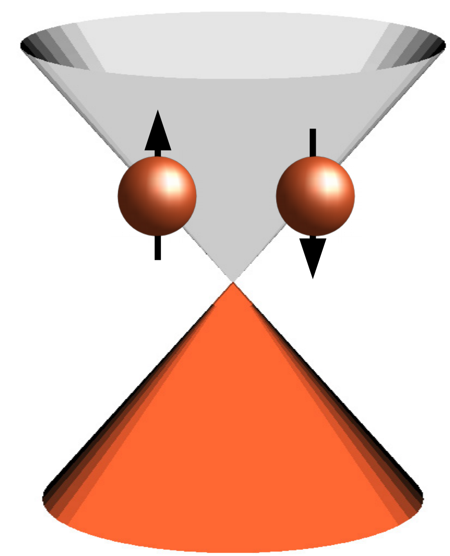
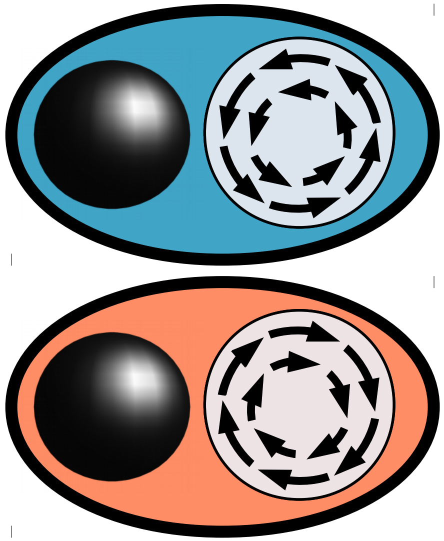




Particle-vortex duality as a symmetry

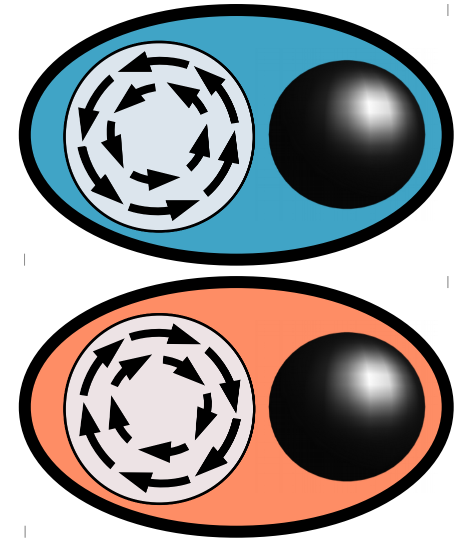
$\psi_{A,\uparrow}$

$\psi_{B,\uparrow}$

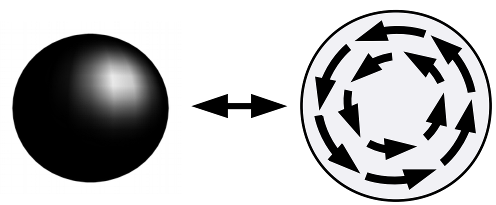


$\psi_{A,\downarrow}$

$\psi_{B,\downarrow}$



Duality

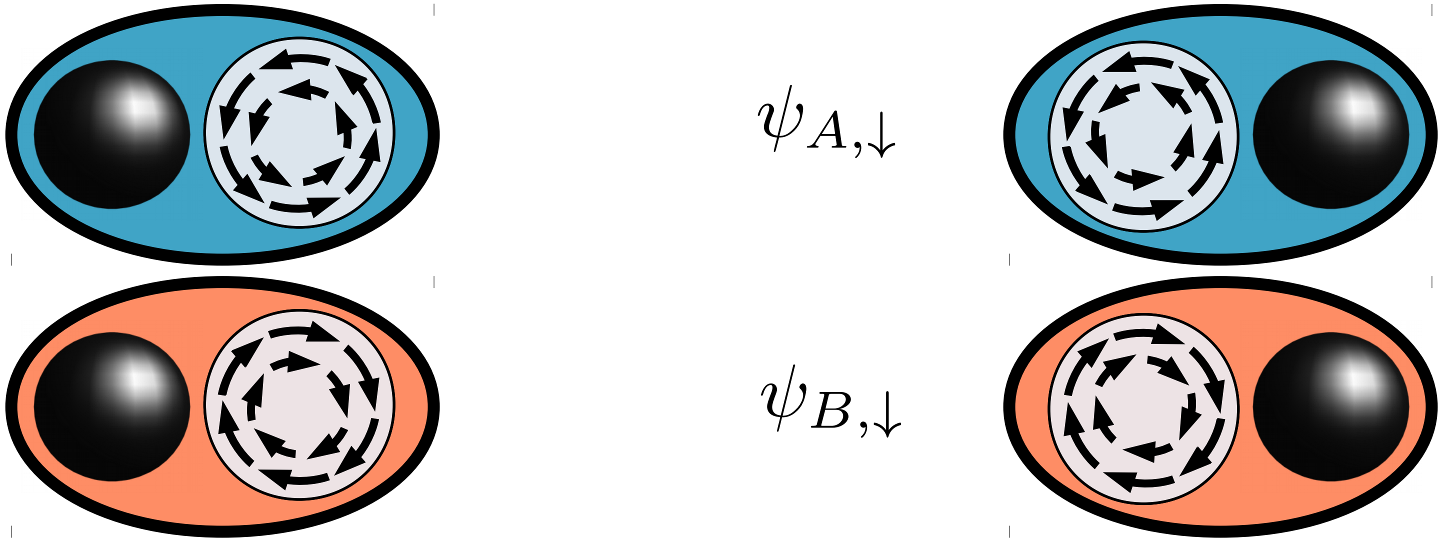


$$\psi_{A,\sigma} \leftrightarrow \psi_{A,-\sigma}$$
$$\psi_{B,\sigma} \leftrightarrow \psi^{\dagger}_{B,-\sigma}$$

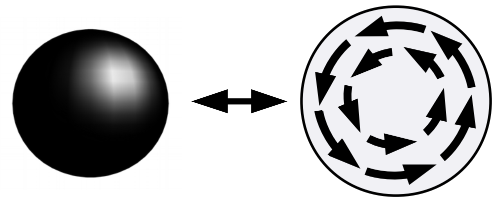
Particle-vortex duality as a symmetry

$\psi_{A,\uparrow}$ $\psi_{A,\downarrow}$

$\psi_{B,\uparrow}$ $\psi_{B,\downarrow}$



Duality



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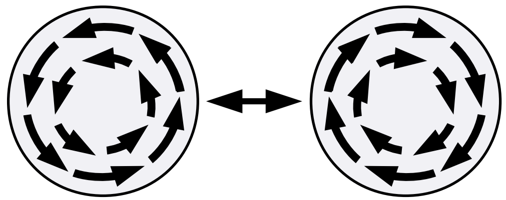
Time reversal

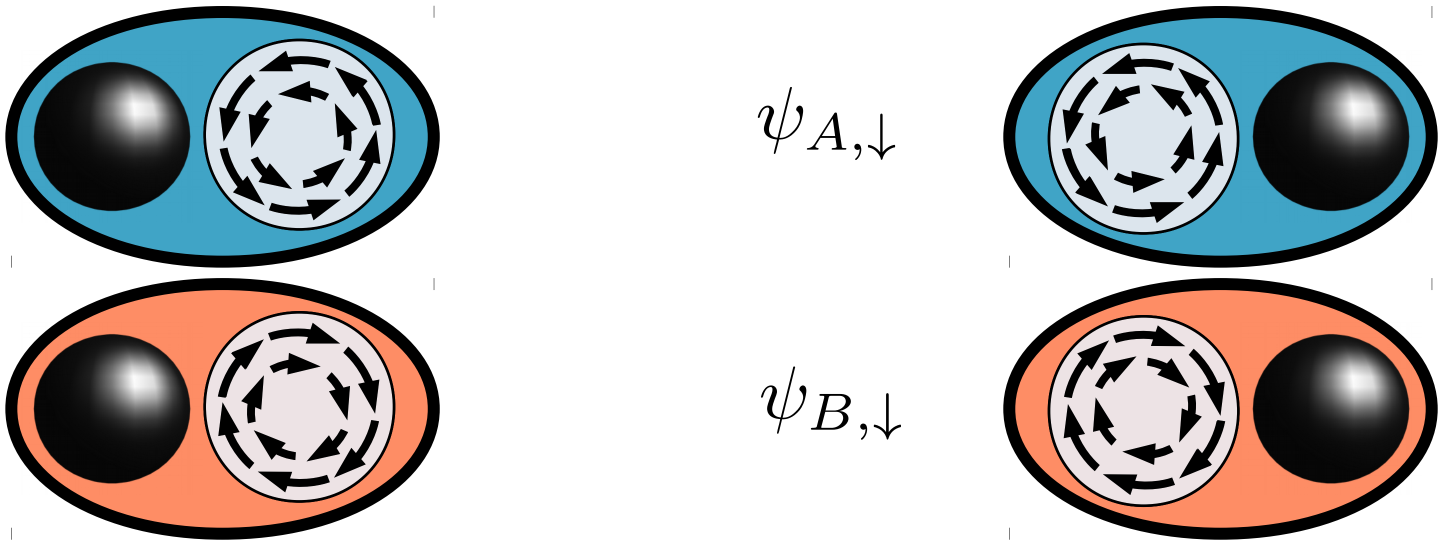


$$\psi_{A,\sigma} \leftrightarrow \psi_{B,\sigma}$$

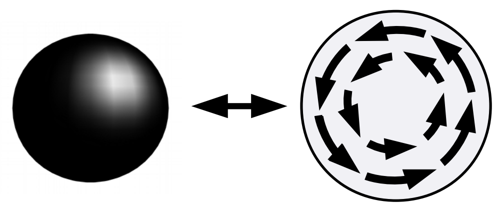
Particle-vortex duality as a symmetry

$\psi_{A,\uparrow}$ $\psi_{A,\downarrow}$

$\psi_{B,\uparrow}$ $\psi_{B,\downarrow}$



Duality



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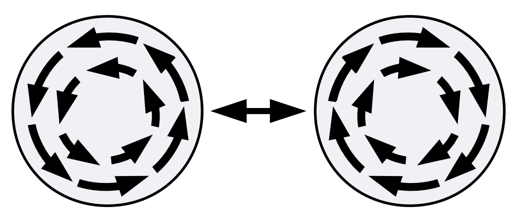
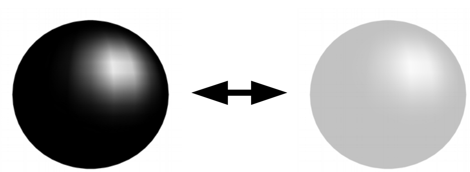
Time reversal



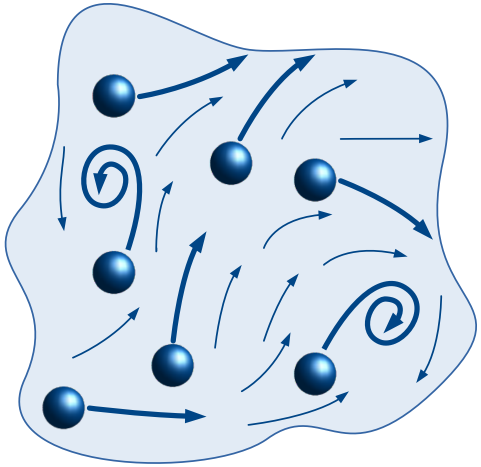
$$\psi_{A,\sigma} \leftrightarrow \psi_{B,\sigma}$$

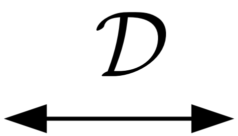
Charge conjugation

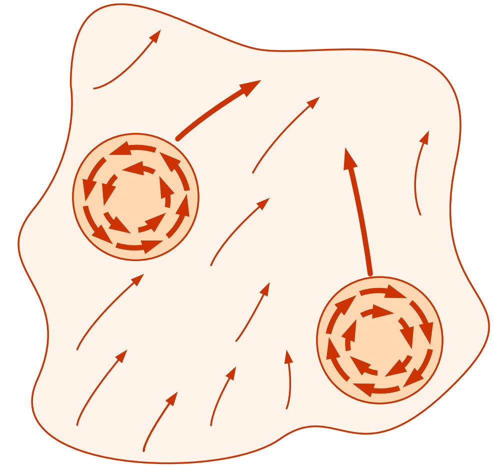


$$\psi_{A,\sigma} \leftrightarrow \psi^{\dagger}_{B,\sigma}$$

Symmetries and dualities I: bosonic duality

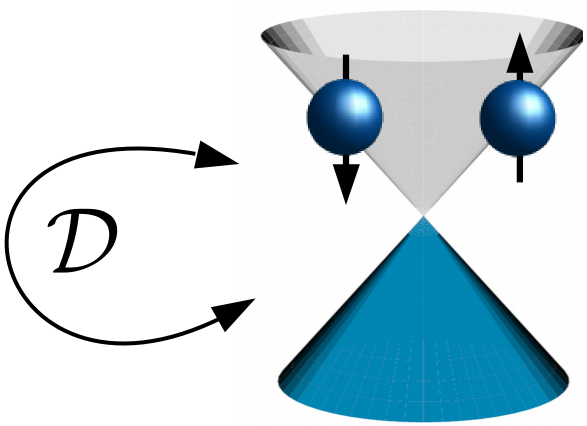






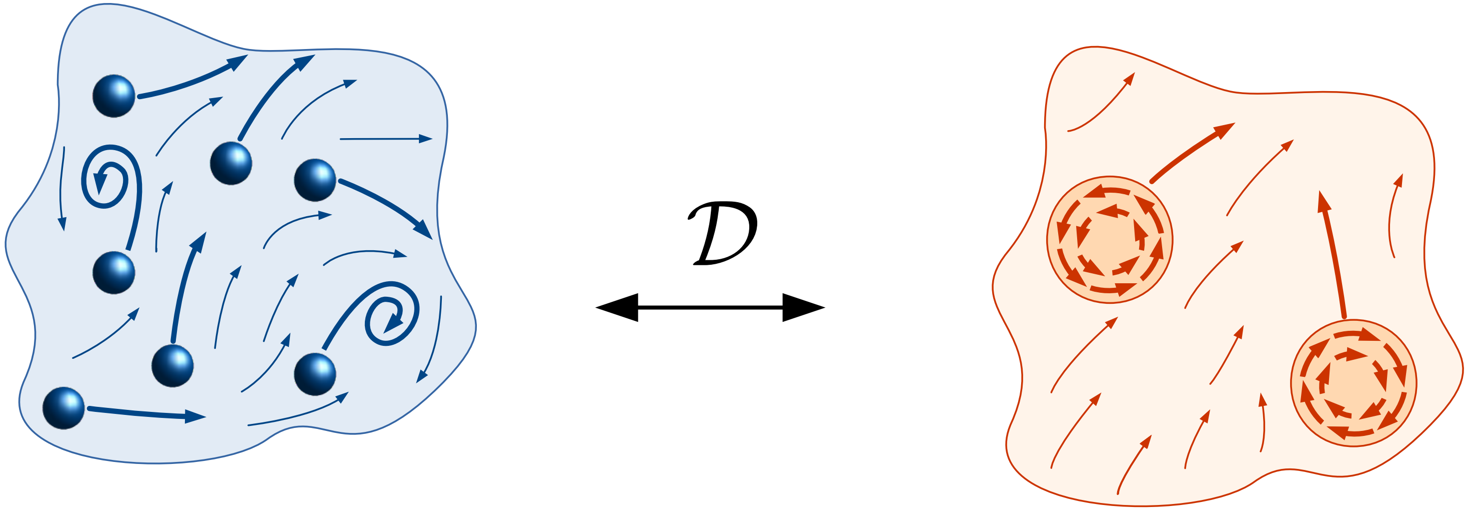
$$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\text{Boson}}|^2 + \frac{i}{4\pi} A_\lambda \epsilon_{\lambda\mu\nu}\partial_\mu A_\nu$$

$$\mathcal{L}_{\text{Vortex}} = |(\partial_\mu - ia_\mu)\Psi_{\text{Vortex}}|^2 - \frac{i}{4\pi} a_\lambda \epsilon_{\lambda\mu\nu}\partial_\mu a_\nu$$



$$\mathcal{L}_{\text{Dirac}} = i\bar{\Psi}_{\text{F}}\partial_\mu\gamma^\mu\Psi_{\text{F}}$$

Symmetries and dualities I: bosonic duality



$$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\text{Boson}}|^2 + \frac{i}{4\pi} A_\lambda \epsilon_{\lambda\mu\nu}\partial_\mu A_\nu$$

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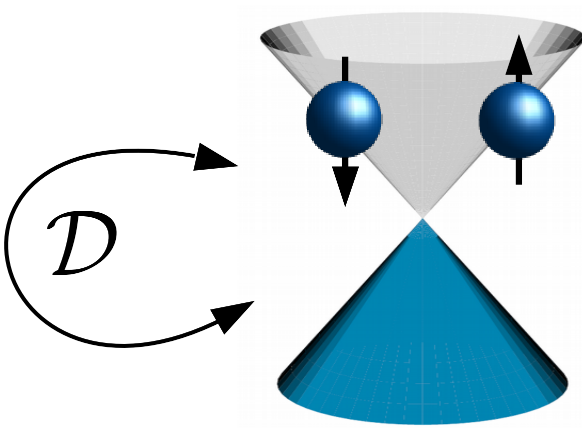
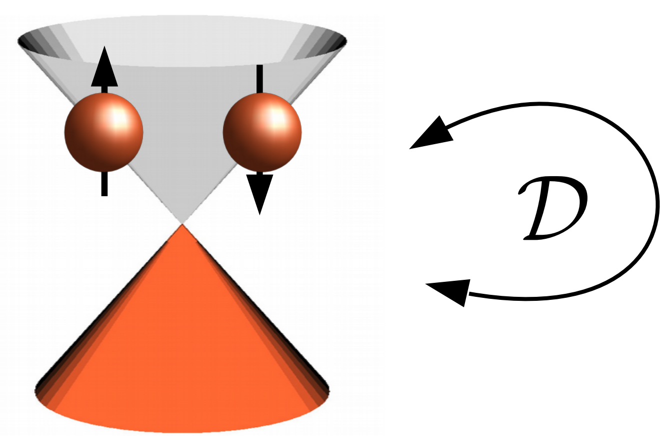
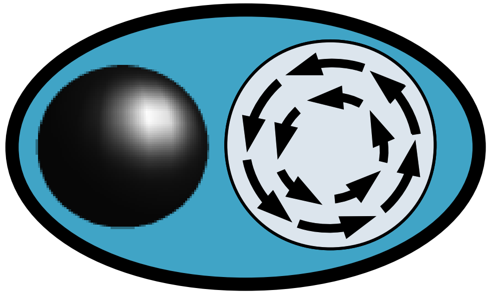


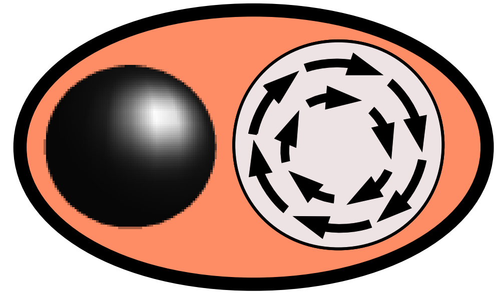


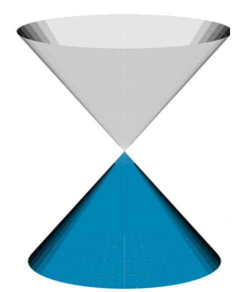
$$\mathcal{L}_{\text{Dirac}} = i\bar{\Psi}_{\text{F}}\partial_\mu\gamma^\mu\Psi_{\text{F}}$$

$$\mathcal{L}_{\text{QED}_3} = i\bar{\Psi}_{\text{DF}}(\partial_\mu - ia_\mu)\gamma^\mu\Psi_{\text{DF}} + \frac{1}{2\kappa}(\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu)^2$$

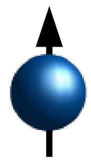
Particle-vortex duality of fermions





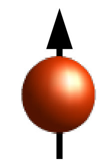


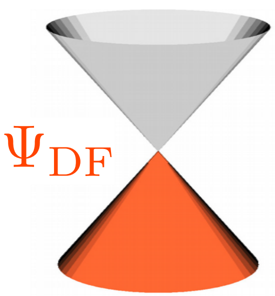
fermion



$$\mathcal{L}_{\mathrm{Dirac}} = i\bar{\Psi}_{\mathrm{F}}\partial_\mu\gamma^\mu\Psi_{\mathrm{F}}$$

dual fermion





$$\mathcal{L}_{\mathrm{QED}_3} = i\bar{\Psi}_{\mathrm{DF}}(\partial_\mu - ia_\mu)\gamma^\mu\Psi_{\mathrm{DF}} + \frac{1}{2\kappa}(\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu)^2$$

Fermions with **short-range** interactions

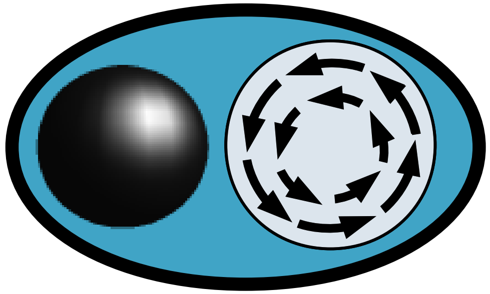
Dual fermions with **long-range** interactions

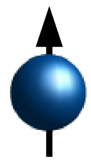
(Wang, Senthil 2015; Metlitski, Vishwanath 2015; Kachru, Mulligan, Torroba, Wang 2015; Mross, Alicea, Motrunich 2015; Karch, Tong 2016, Seiberg, Senthil, Wang, Witten 2016)

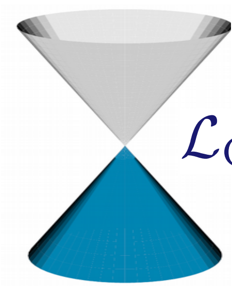
- Duality can be implemented as exact transformation in wire models

(Mross, Alicea, Motrunich 2015)

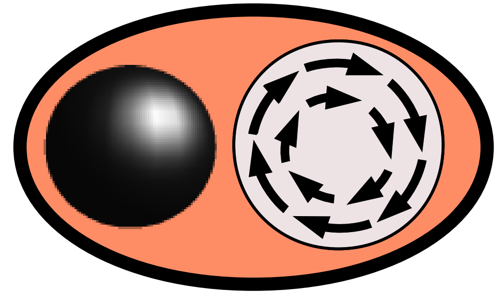
Particle-vortex duality of fermions



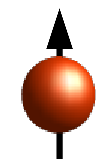




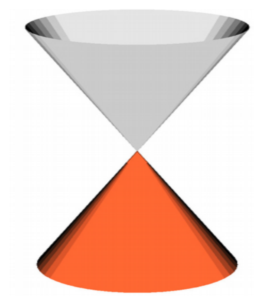
fermion



$$\mathcal{L}_{\mathrm{QED}_3} = i\bar{\Psi}_{\mathrm{F}}(\partial_\mu - iA_\mu)\gamma^\mu\Psi_{\mathrm{F}} + \frac{1}{2\kappa}(\epsilon_{\lambda\mu\nu}\partial_\mu A_\nu)^2$$



dual fermion



$$\mathcal{L}_{\mathrm{Dirac}} = i\bar{\Psi}_{\mathrm{DF}}\partial_\mu\gamma^\mu\Psi_{\mathrm{DF}}$$

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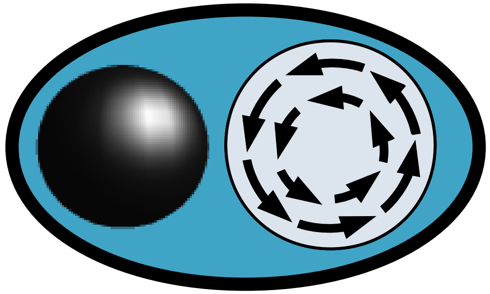
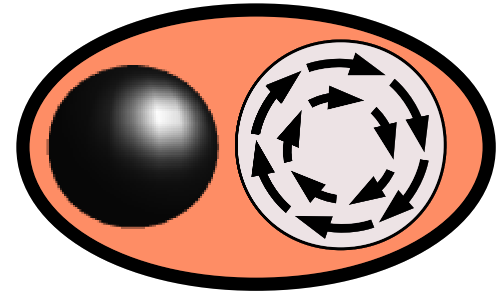
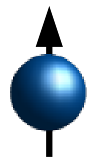
Particle-vortex duality of fermions

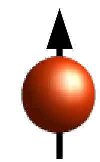


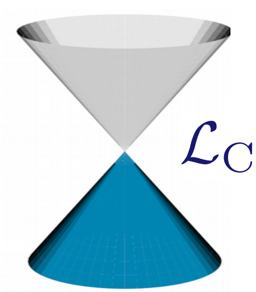


fermion



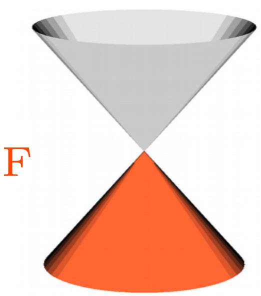
dual fermion





$$\mathcal{L}_{\mathrm{CS}} = i\bar{\Psi}_{\mathrm{F}}(\partial_\mu - iA_\mu)\gamma^\mu\Psi_{\mathrm{F}} + \tfrac{i}{8\pi}A_\lambda\epsilon_{\lambda\mu\nu}\partial_\mu A_\nu$$

$$\mathcal{L}_{\mathrm{CS}} = i\bar{\Psi}_{\mathrm{DF}}(\partial_\mu - ia_\mu)\gamma^\mu\Psi_{\mathrm{DF}} - \tfrac{i}{8\pi}a_\lambda\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu$$



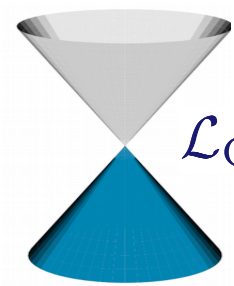
Fermions with **intermediate-range** interactions

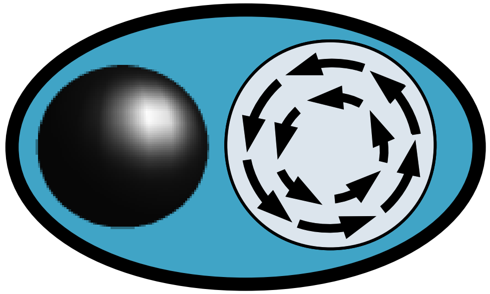
Dual fermions with **intermediate-range** interactions

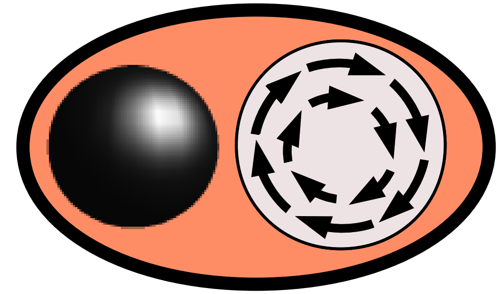
- Duality can be implemented as exact transformation in wire models

(Mross, Alicea, Motrunich 2015)

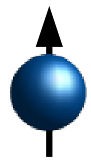
Particle-vortex duality of fermions





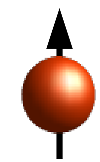


fermion



$$\mathcal{L}_{\rm CS} = i\bar{\Psi}_{\rm F}(\partial_\mu - iA_\mu)\gamma^\mu\Psi_{\rm F} + \frac{i}{8\pi}A_\lambda\epsilon_{\lambda\mu\nu}\partial_\mu A_\nu$$

dual fermion



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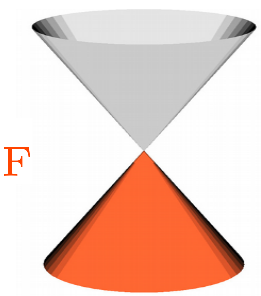
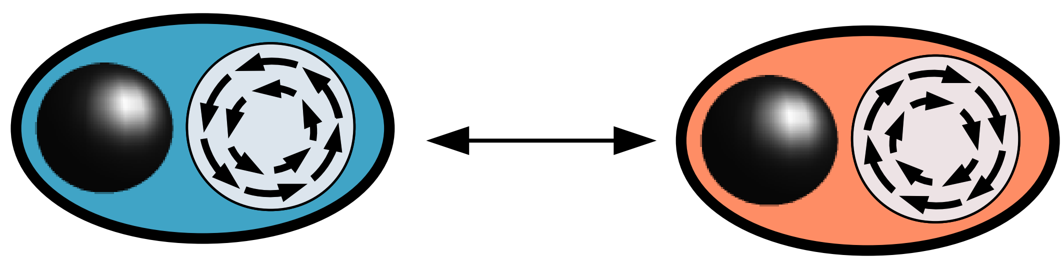
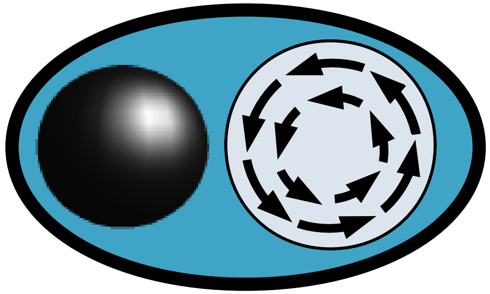
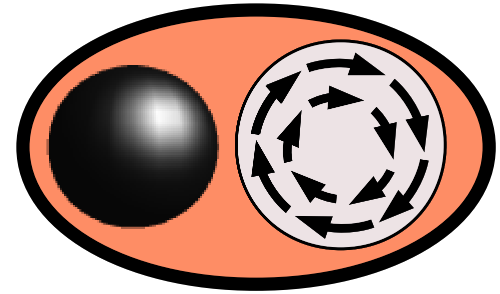


- Fermionic duality:

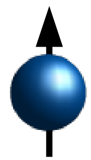


Particle-vortex duality of fermions

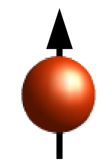


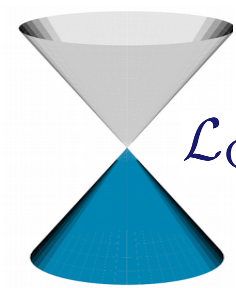


fermion



dual fermion

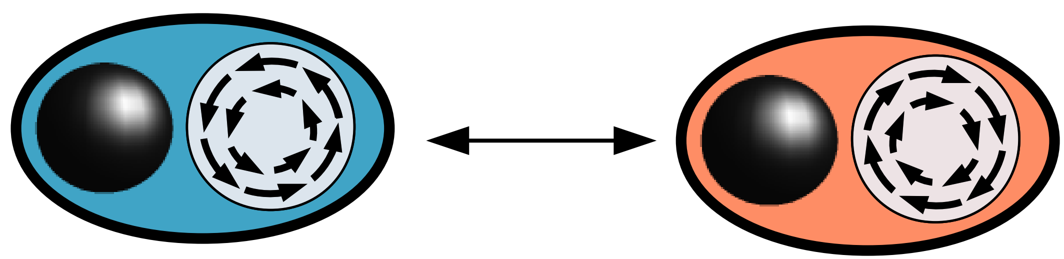




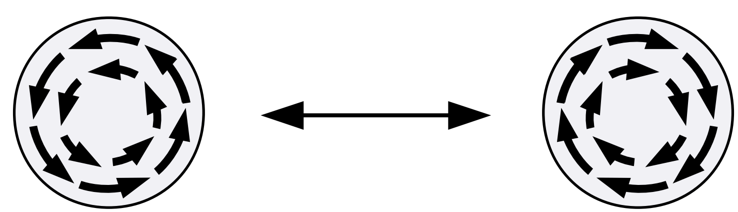
$$\mathcal{L}_{\text{CS}} = i\bar{\Psi}_{\text{F}}(\partial_\mu - iA_\mu)\gamma^\mu\Psi_{\text{F}} + \frac{i}{8\pi}A_\lambda\epsilon_{\lambda\mu\nu}\partial_\mu A_\nu$$

$$\mathcal{L}_{\text{CS}} = i\bar{\Psi}_{\text{DF}}(\partial_\mu - ia_\mu)\gamma^\mu\Psi_{\text{DF}} - \frac{i}{8\pi}a_\lambda\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu$$

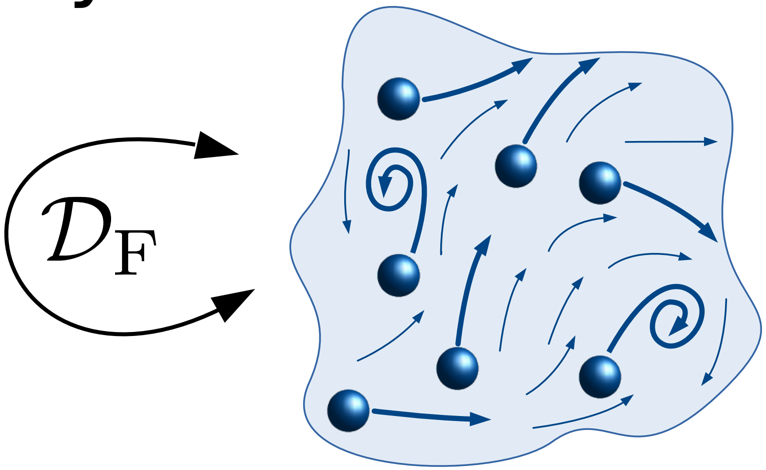
- Fermionic duality:

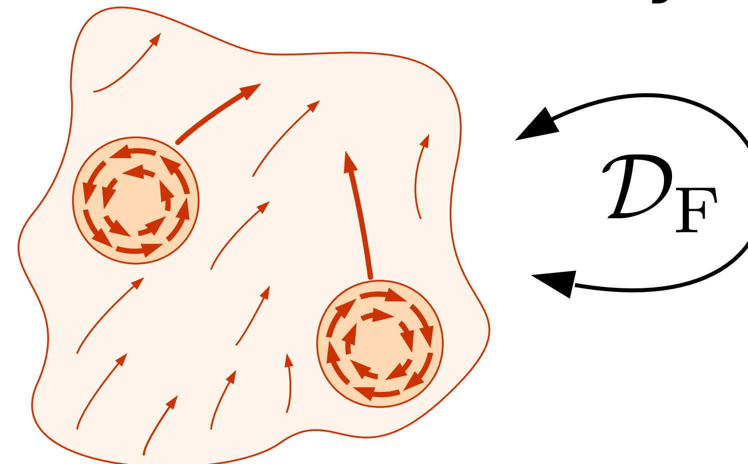


- Bosonic symmetry:



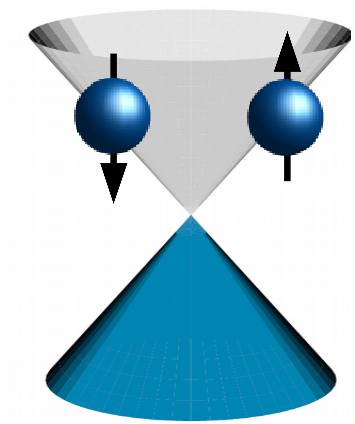
Symmetries and dualities II: fermionic duality

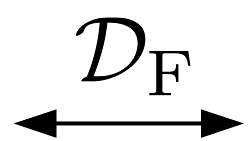


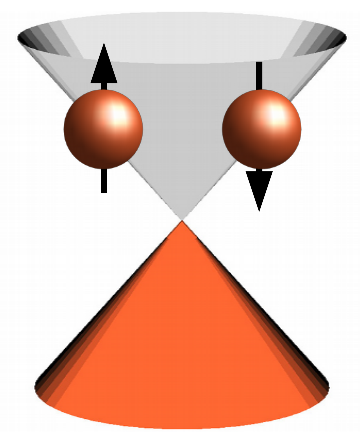


$$\mathcal{L}_\mathrm{Boson} = |\partial_\mu \Psi_\mathrm{Boson}|^2$$

$$\mathcal{L}_\mathrm{Vortex} = |(\partial_\mu - i a_\mu)\Psi_\mathrm{Vortex}|^2 + \frac{1}{2\kappa}(\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu)^2$$







$$\mathcal{L}_\mathrm{CS} = i\bar{\Psi}_\mathrm{F}(\partial_\mu - iA_\mu)\gamma^\mu\Psi_\mathrm{F} + \frac{i}{8\pi}A_\lambda\epsilon_{\lambda\mu\nu}\partial_\mu A_\nu$$

$$\mathcal{L}_\mathrm{CS} = i\bar{\Psi}_\mathrm{DF}(\partial_\mu - ia_\mu)\gamma^\mu\Psi_\mathrm{DF} - \frac{i}{8\pi}a_\lambda\epsilon_{\lambda\mu\nu}\partial_\mu a_\nu$$

(Karch, Tong 2016, Seiberg, Senthil, Wang, Witten 2016)

- Bosonic self-duality $\leftrightarrow$ fermionic time-reversal symmetry
- Fermionic self-duality $\leftrightarrow$ bosonic time-reversal symmetry

- Bosonic self-duality $\leftrightarrow$ fermionic time-reversal symmetry
- Fermionic self-duality $\leftrightarrow$ bosonic time-reversal symmetry

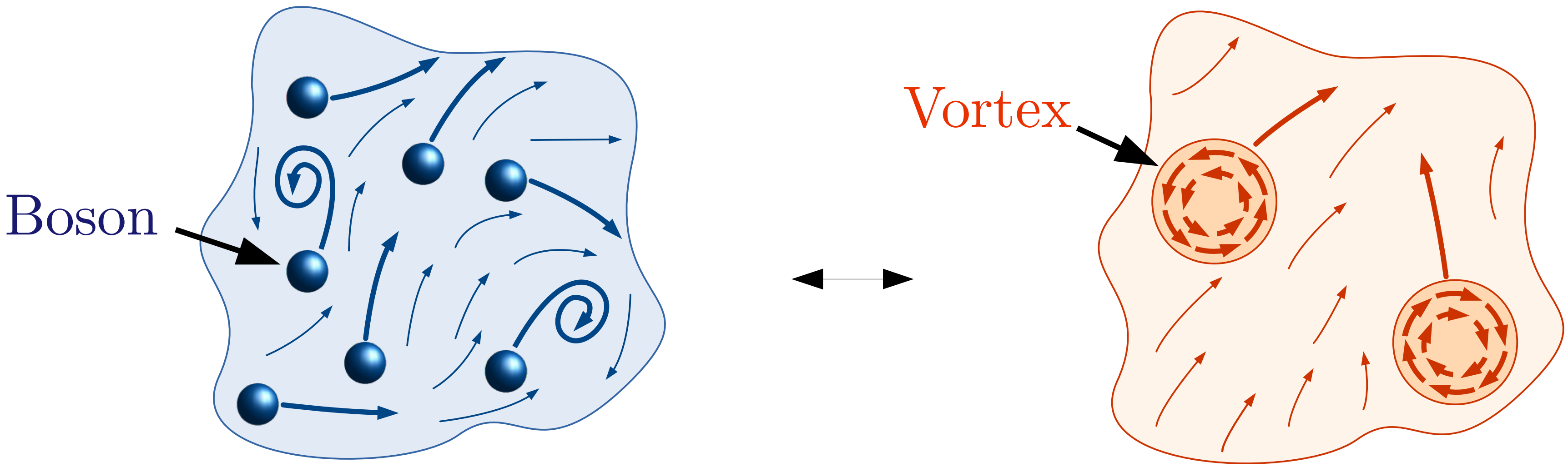
Can bosons and fermions be simultaneously self-dual?

- Bosonic self-duality $\leftrightarrow$ fermionic time-reversal symmetry
- Fermionic self-duality $\leftrightarrow$ bosonic time-reversal symmetry

Can bosons and fermions be simultaneously self-dual?

Need self-dual bosons with time-reversal symmetry (no Chern-Simons term)!

Symmetries and dualities III: simultaneous duality



$$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\text{Boson}}|^2 + \frac{\lambda}{4\pi|\mathbf{k}|}|\mathbf{k}\times A|^2$$

$$\mathcal{L}_{\text{Vortex}} = |(\partial_\mu - ia_\mu)\Psi_{\text{Vortex}}|^2 + \frac{1}{4\pi\lambda|\mathbf{k}|}|\mathbf{k}\times a|^2$$

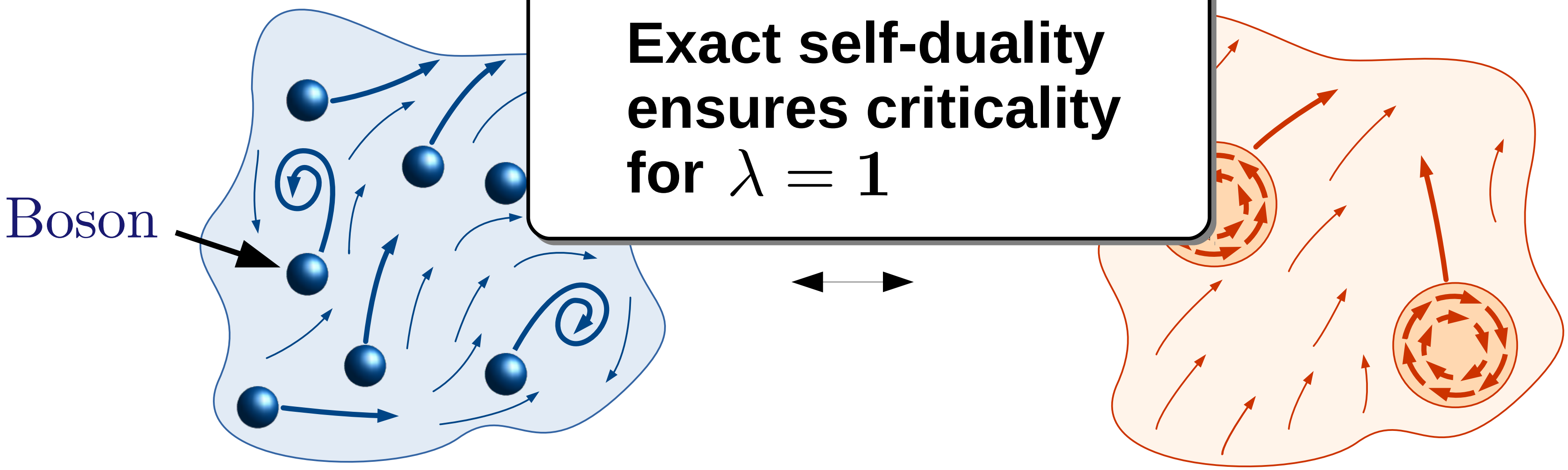
Bosons with time-reversal invariant **intermediate-range** interactions

Vortices with time-reversal invariant **intermediate-range** interactions

(Fradkin, Kivelson 1996)

Symmetries and dualities III: simultaneous duality

Exact self-duality ensures criticality for $\lambda = 1$



$$\mathcal{L}_{\mathrm{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\mathrm{Boson}}|^2 + \frac{\lambda}{4\pi|\mathbf{k}|}|\mathbf{k} \times A|^2$$

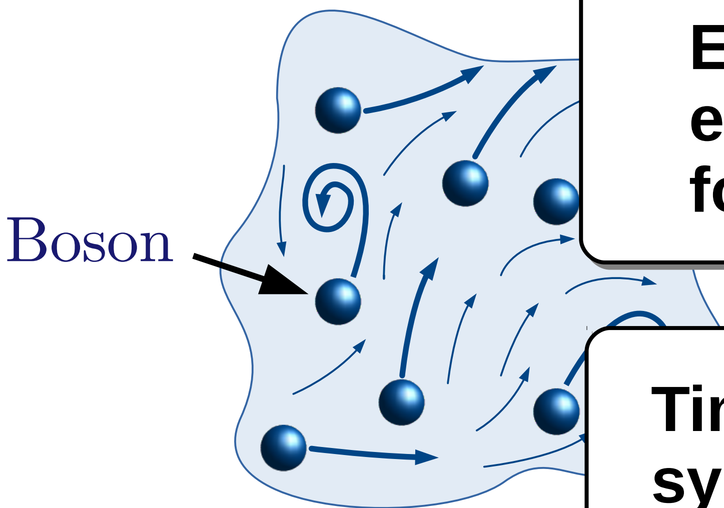
$$\mathcal{L}_{\mathrm{Vortex}} = |(\partial_\mu - ia_\mu)\Psi_{\mathrm{Vortex}}|^2 + \frac{1}{4\pi\lambda|\mathbf{k}|}|\mathbf{k} \times a|^2$$

Bosons with time-reversal invariant **intermediate-range** interactions

Vortices with time-reversal invariant **intermediate-range** interactions

(Fradkin, Kivelson 1996)

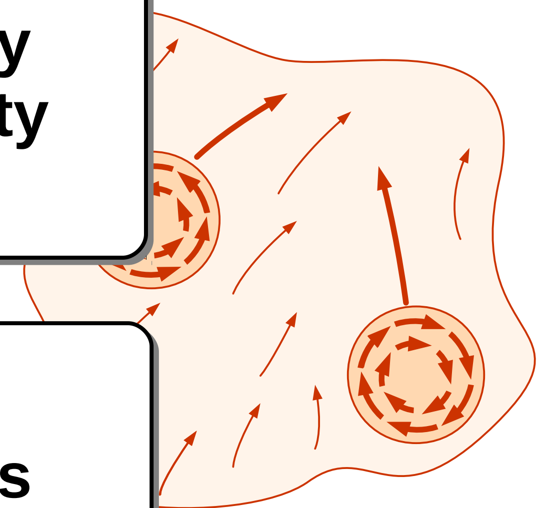
Symmetries and dualities III: simultaneous duality



Exact self-duality ensures criticality for $\lambda = 1$

$\longleftrightarrow$

Time-reversal symmetry ensures fermionic self-duality



$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA$ … $+ \frac{\lambda}{4\pi|\mathbf{k}|}|\mathbf{k} \times A|^2$

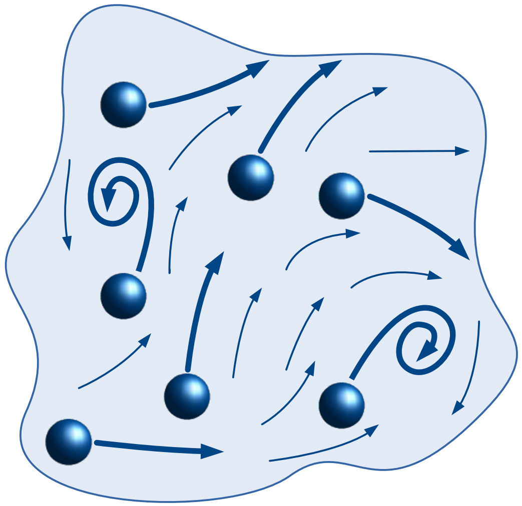
… $\partial_\mu - ia_\mu)\Psi_{\text{Vortex}}|^2 + \frac{1}{4\pi\lambda|\mathbf{k}|}|\mathbf{k} \times a|^2$

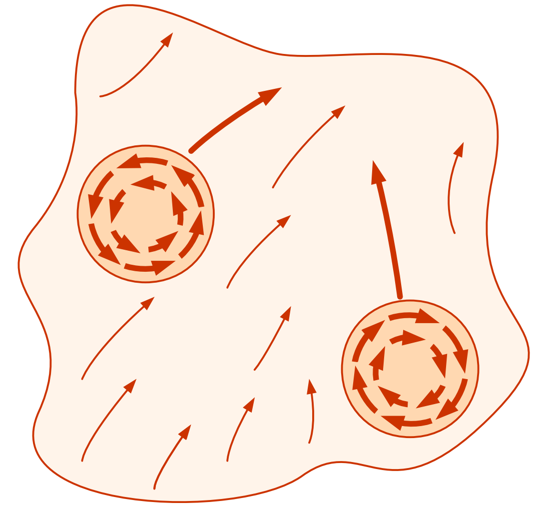
Bosons with time-reversal invariant **intermediate-range** interactions

Vortices with time-reversal invariant **intermediate-range** interactions

(Fradkin, Kivelson 1996)

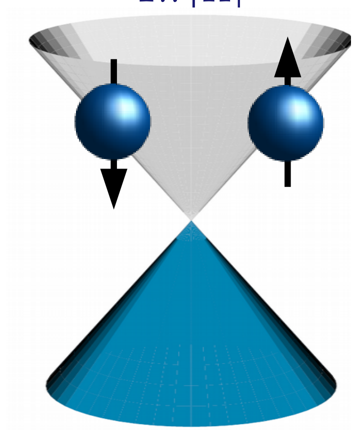
Symmetries and dualities III: simultaneous duality

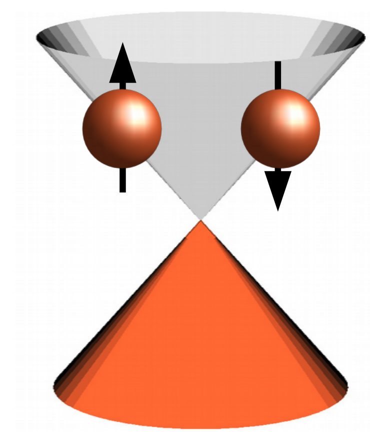




$$\mathcal{L}_{\text{Boson}} = |(\partial_\mu - iA_\mu)\Psi_{\text{Boson}}|^2 + \frac{1}{4\pi|\mathbf{k}|}|\mathbf{k} \times A|^2$$

$$\mathcal{L}_{\text{Vortex}} = |(\partial_\mu - ia_\mu)\Psi_{\text{Vortex}}|^2 + \frac{1}{4\pi|\mathbf{k}|}|\mathbf{k} \times a|^2$$





$$\mathcal{L}_{\text{Fermion}} = i\bar{\Psi}_{\text{F}}(\partial_\mu - iA_\mu)\gamma^\mu\Psi_{\text{F}} + \frac{1}{8\pi|\mathbf{k}|}|\mathbf{k} \times A|^2$$

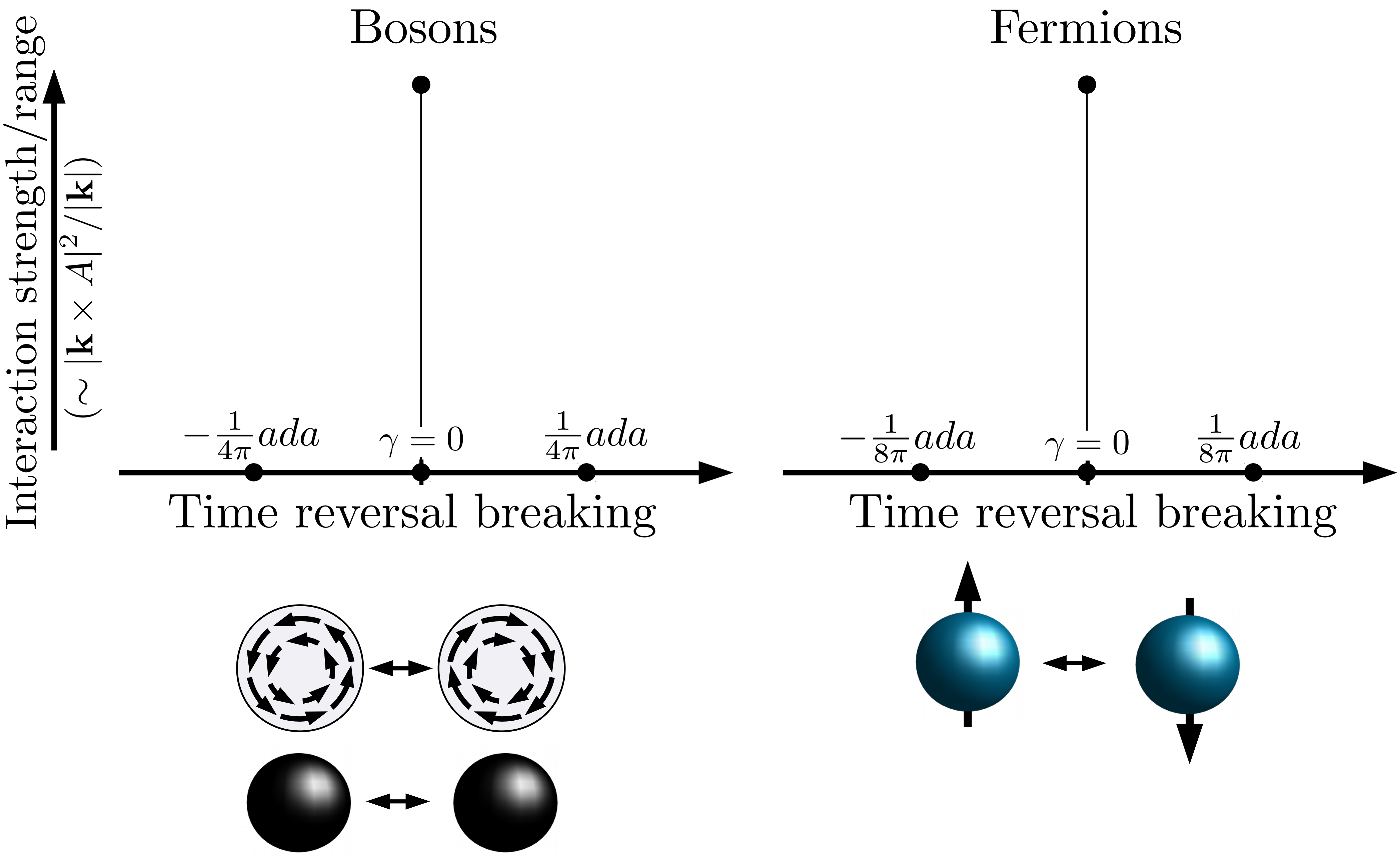
$$\mathcal{L}_{\text{Dual Fermion}} = i\bar{\Psi}_{\text{DF}}(\partial_\mu - ia_\mu)\gamma^\mu\Psi_{\text{DF}} + \frac{1}{8\pi|\mathbf{k}|}|\mathbf{k} \times a|^2$$

Generalized models

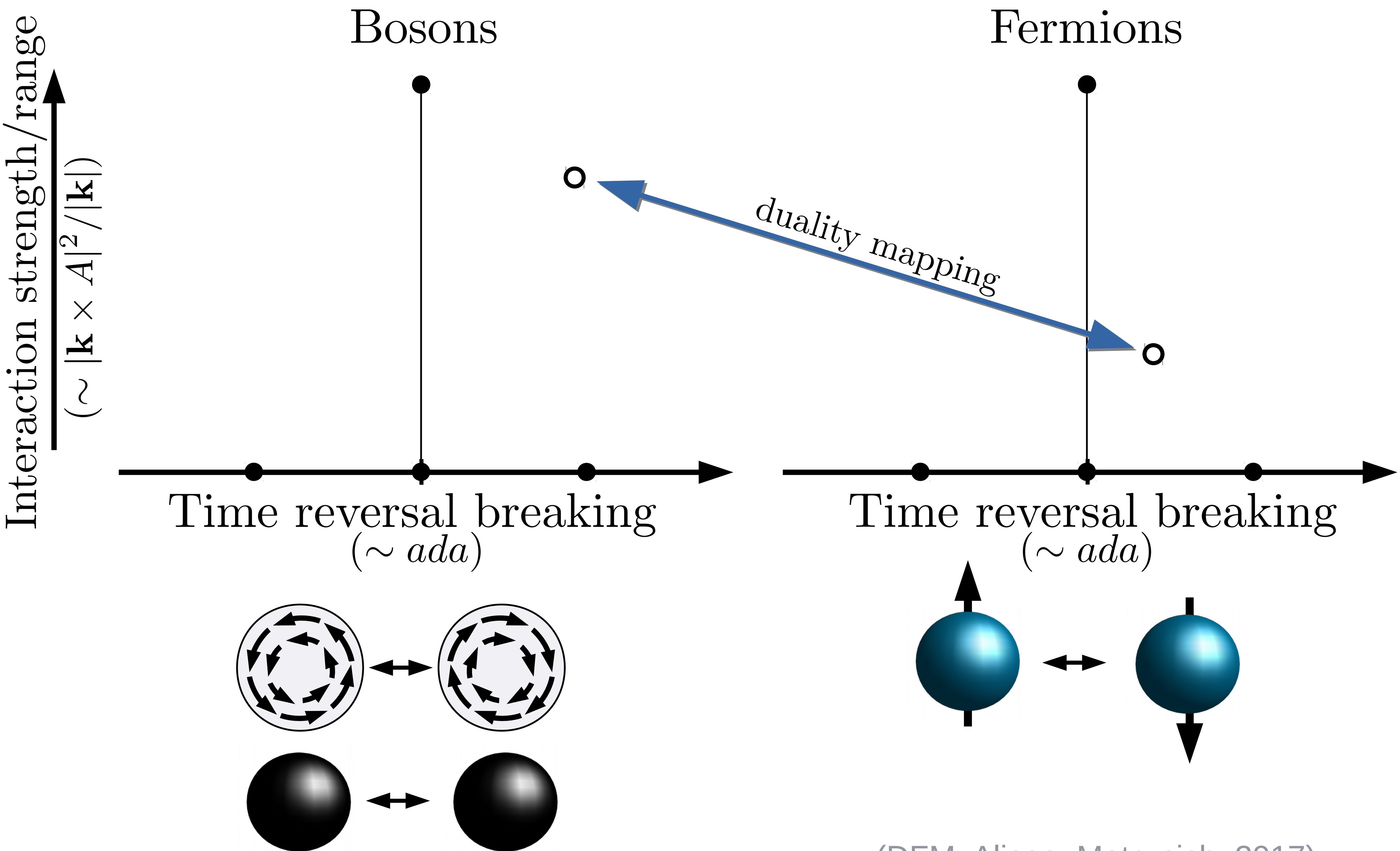
Interaction strength/range

Bosons	Fermions
$\lvert \mathbf{k} \times A \rvert^2$	$\lvert \mathbf{k} \times A \rvert^2$
$\frac{\lambda}{4\pi} \frac{\lvert \mathbf{k} \times A \rvert^2}{\lvert \mathbf{k} \rvert}$	$\frac{\lambda}{8\pi} \frac{\lvert \mathbf{k} \times A \rvert^2}{\lvert \mathbf{k} \rvert}$
$\rho_s A^2$	$\rho_s A^2$

Generalized models

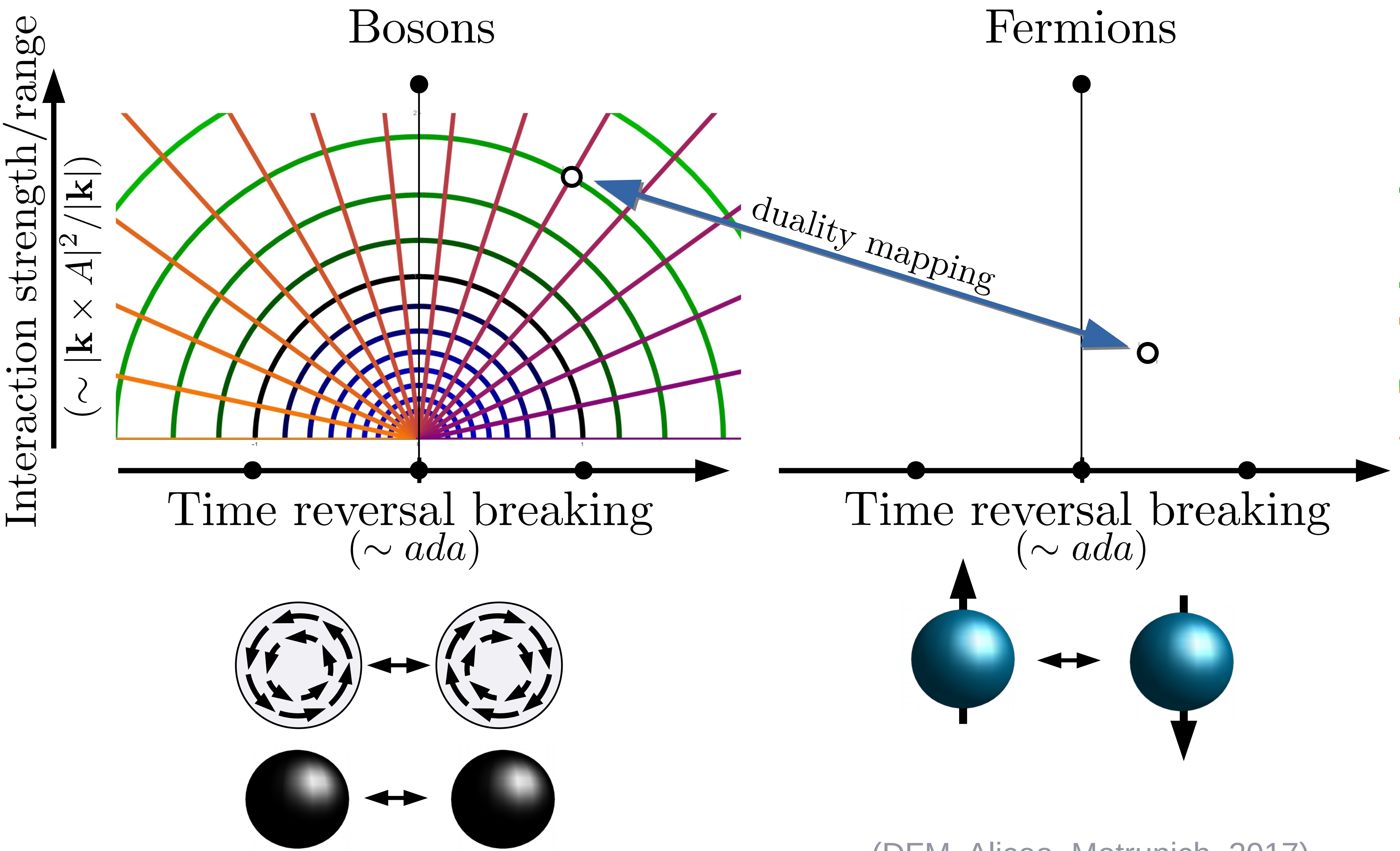


Generalized models



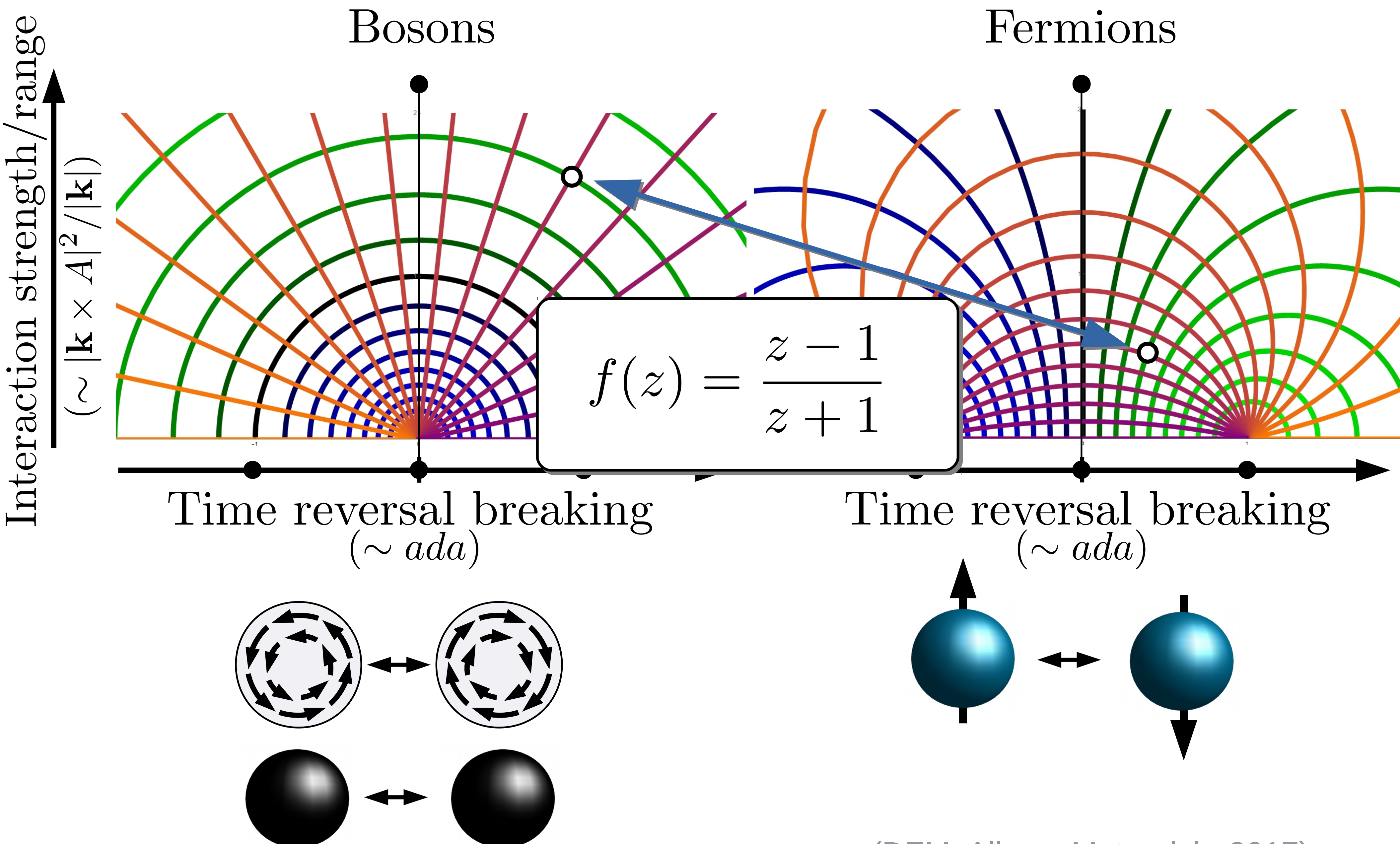
(DFM, Alicea, Motrunich, 2017)

Generalized models



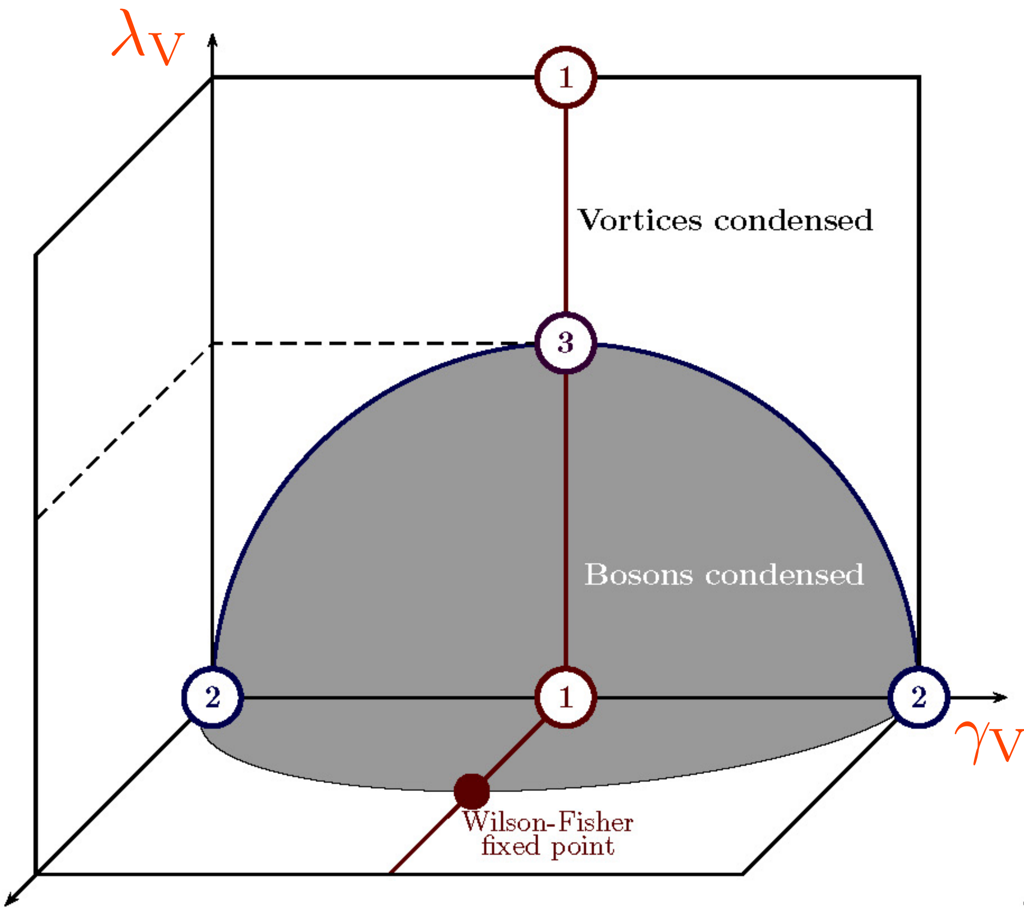
(DFM, Alicea, Motrunich, 2017)

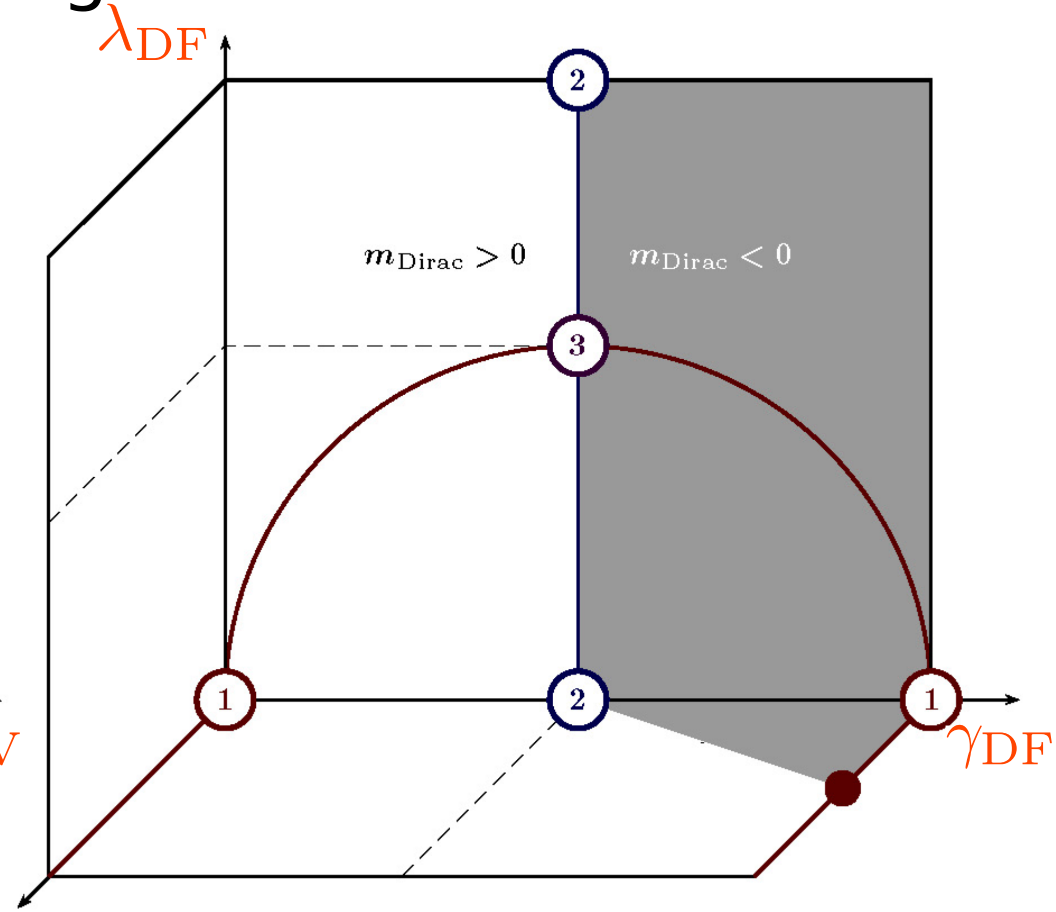
Generalized models



(DFM, Alicea, Motrunich, 2017)

Phase diagrams



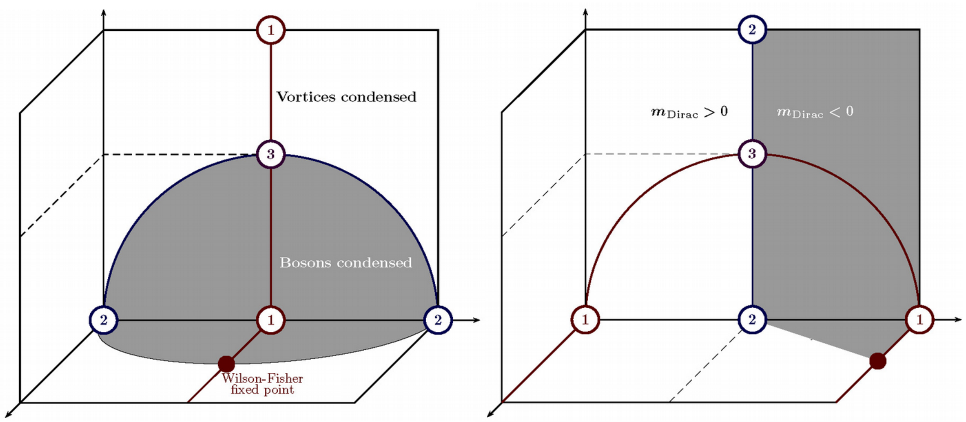


$$\mathcal{L}_{\rm Boson} = |(\partial_\mu - iA_\mu)\Psi_{\rm Boson}|^2 + \lambda_{\rm B}\frac{1}{4\pi|\mathbf{k}|}|\nabla \times A|^2 + \gamma_{\rm B}\frac{i}{4\pi}AdA$$

$$\mathcal{L}_{\rm Fermion} = i\bar{\Psi}_{\rm F}(\partial_\mu - iA_\mu)\gamma^\mu\Psi_{\rm F} + \lambda_{\rm F}\frac{1}{8\pi|\mathbf{k}|}|\nabla \times A|^2 + \gamma_{\rm F}\frac{i}{8\pi}AdA$$

$$\lambda_{\rm V} = \frac{\lambda_{\rm B}}{\gamma_{\rm B}^2 + \lambda_{\rm B}^2} \qquad \gamma_{\rm V} = \frac{-\gamma_{\rm B}}{\gamma_{\rm B}^2 + \lambda_{\rm B}^2} \qquad \lambda_{\rm DF} = \frac{\lambda_{\rm F}}{\gamma_{\rm F}^2 + \lambda_{\rm F}^2} \qquad \gamma_{\rm DF} = \frac{-\gamma_{\rm DF}}{\gamma_{\rm DF}^2 + \lambda_{\rm DF}^2}$$

Outlook



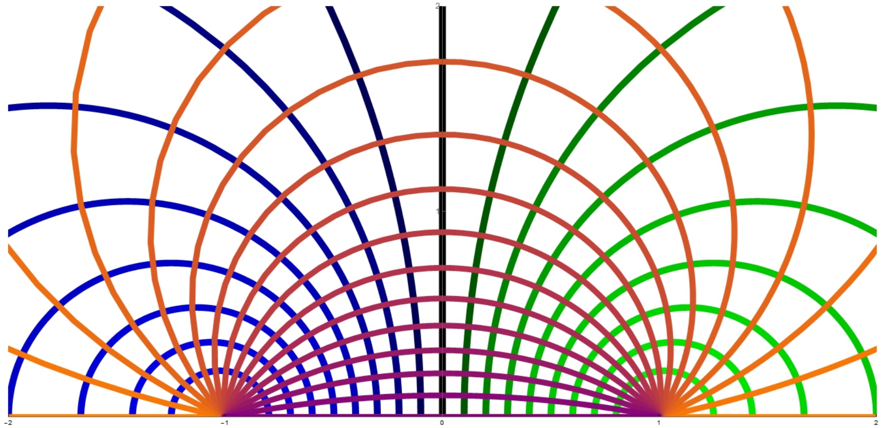
Strong or weak duality?

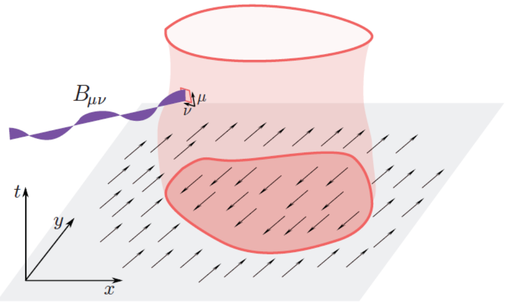
- Concrete predictions from strong duality
- Need numerical verification

Additional dualities

- Extra symmetries (e.g., N-flavor QED)
- Spin models (e.g, quantum spin liquids)
- Majorana fermions (bosonized theory proposed, but symmetries unclear)

(Metlitski, Vishwanath, Xu 2016)





(Beekman, Sadri, Zaanen 2011)

Different dimensions

- Known particle-vortex duality in 3+1 dimensions.