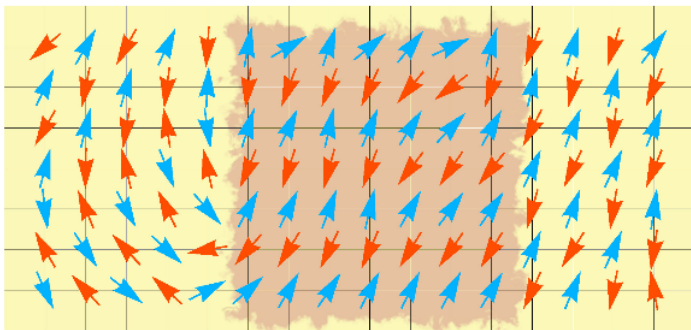


Enhanced nematic fluctuations near the Mott insulating phase of high- T_c cuprates

Peter P. Orth (Iowa State University)

In collaboration with B. Jeevanesan (Karlsruhe Institute of Technology, KIT), J. Schmalian (KIT) and R. M. Fernandes (University of Minnesota)

“Intertwined Order and Fluctuations in Quantum Materials”, KITP, 5 Sept 2017



PPO, B. Jeevanesan, R. M. Fernandes, J. Schmalian, arXiv:1703:02210 (2017).

B. Jeevanesan



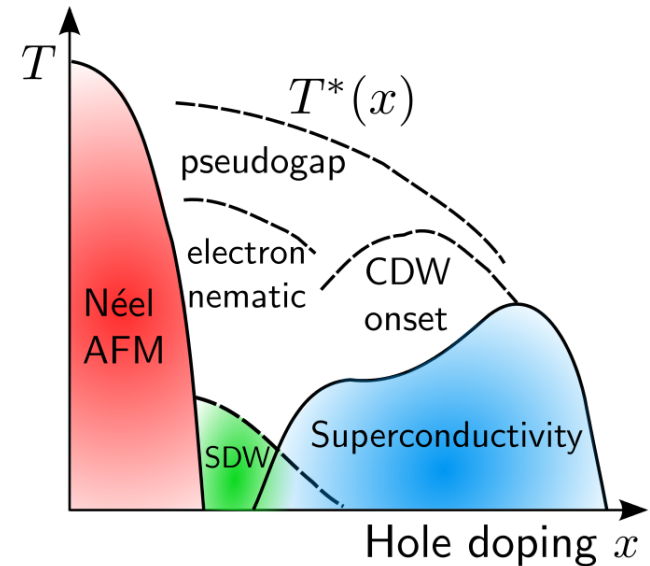
J. Schmalian

R. M. Fernandes



Generic features of cuprate phase diagram

- Various features/phases are present among different families
- Much progress at experimental front
 - Neel AF (Mott/charge transfer insulator zero doping $x=0$)
 - Incommensurate SDW (small x) [1]
 - (fluctuating) charge order (NMR [2], RXS [3], X-Ray [4])
 - Pseudogap in quasi-particle spectrum [5]
 - Nematicity (resistivity anisotropy [6], Nernst [7], STM [8], neutron scattering [1], torque magnetometry [9])
 - Superconductivity (at sufficiently large x)



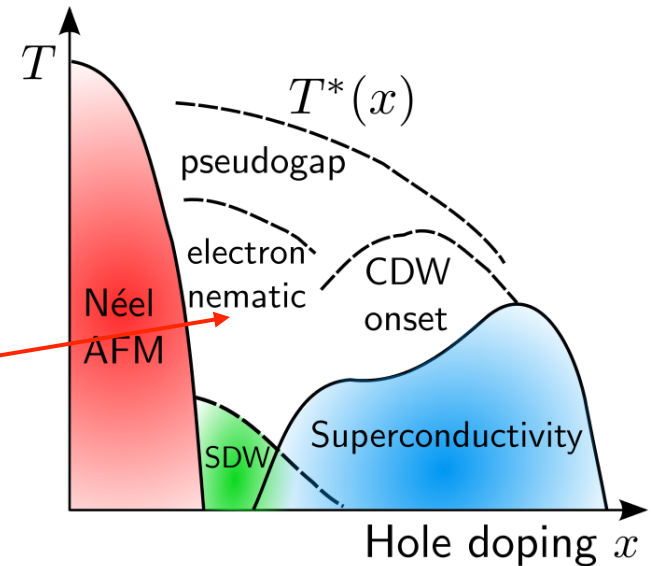
[1] D. Haug *et al.*, NJP **12**, 205006 (2010); [2] W. Wu *et al.*, Nature **477**, 191 (2011); [3] G. Ghiringhelli *et al.*, Science **337**, 821 (2012); [4] J. Chang *et al.*, Nat. Phys. **8**, 871 (2012); [5] T. Kondo *et al.*, Nature **457**, 296 (2009); [6] Y. Ando *et al.*, PRL **88**, 137005 (2002); [7] R. Daou *et al.*, Nature **463**, 519 (2010); [8] Y. Kohsaka *et al.*, Science (2007); [9] Y. Sato *et al.*, Nat. Phys. (2017).

Guiding principles in cuprate phase diagram

- Two guiding principles:
 - Novel phases other than SC only appear below $T^*(p)$
 - $T^*(p)$ increases monotonically as $p \rightarrow 0$ reaching $T^*(p) \rightarrow T_{\downarrow N}$
 - Importance of short-range AF order
 - Here, we want to focus on **electronic nematicity** (in YBCO)



Connection of **nematicity** and short-range **AF correlations**?

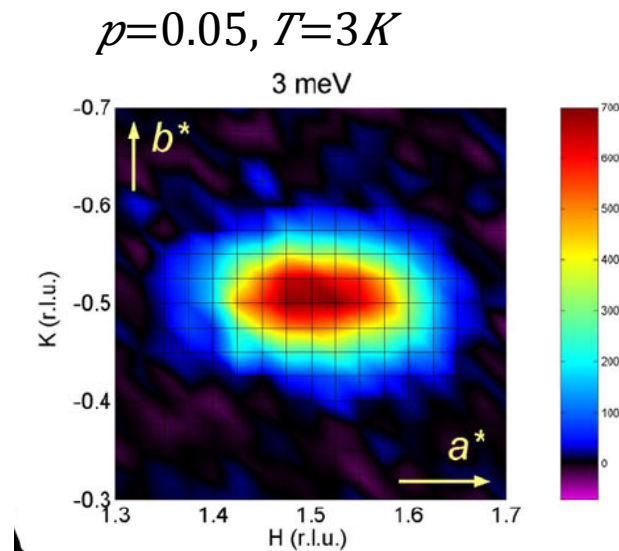
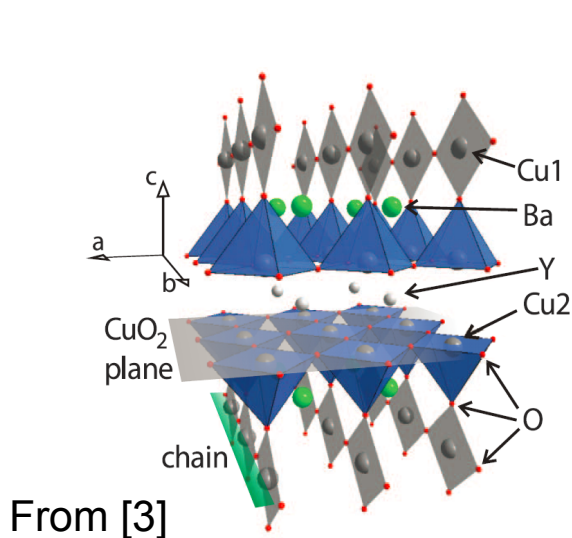


[1] D. Haug *et al.*, NJP **12**, 205006 (2010); [2] W. Wu *et al.*, Nature **477**, 191 (2011); [3] G. Ghiringhelli *et al.*, Science **337**, 821 (2012); [4] J. Chang *et al.*, Nat. Phys. **8**, 871 (2012); [5] T. Kondo *et al.*, Nature **457**, 296 (2009); [6] Y. Ando *et al.*, PRL **88**, 137005 (2002); [7] R. Daou *et al.*, Nature **463**, 519 (2010); [8] Y. Kohsaka *et al.*, Science (2007); [9] Y. Sato *et al.*, Nat. Phys. (2017).

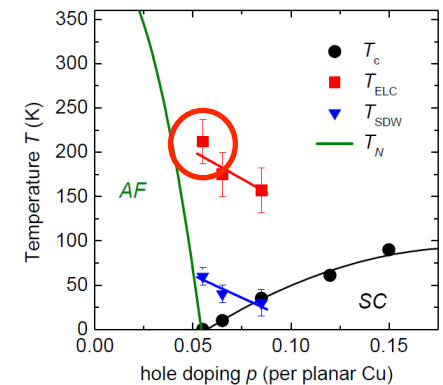
Experimental signatures of (spin) nematicity

Experimental signatures of nematicity in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

- **Nematic order** = electronic state **breaking C_{4v} symmetry** of CuO_2 plane
- $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ has **orthorhombic crystal structure** for $\delta \leq 0.7$: $a \neq b$; $\delta = 0.7, 0.65, 0.55$ ($p \geq 0.05$)
- **Inelastic neutron scattering peak** close to Neel ordering wavevector $Q = (\pi, \pi)$ is **elliptic** [1]



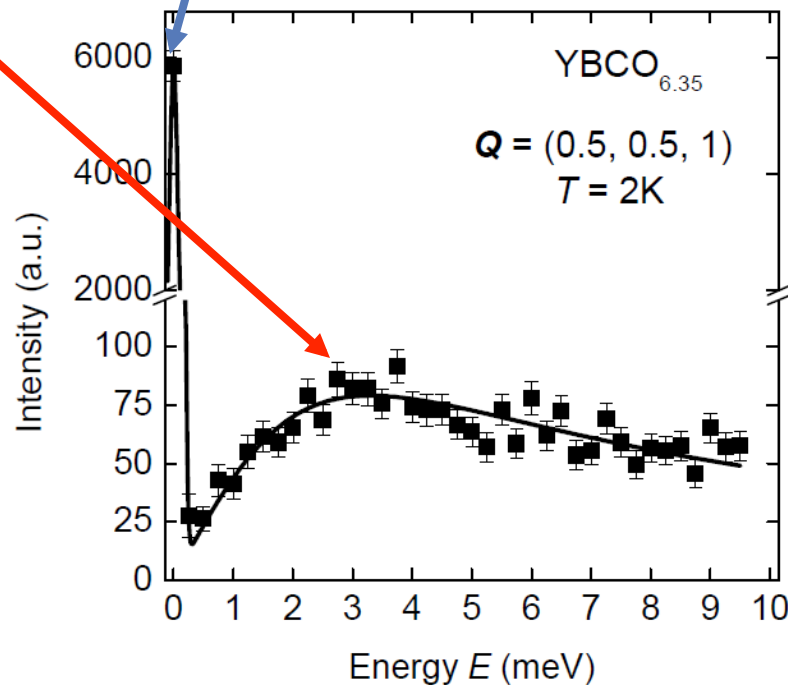
Location in PD:
Hole underdoped



[1] D. Haug *et al.*, New J. Phys. **12**, 105006 (2010); [2] S. Kivelson *et al.* Nature 1998; [3] G. Ghiringhelli *et al.*, Science **337**, 821 (2012); W. Metzner *et al.* PRL 2003, S. Sachdev *et al.* PRL 2013, Nie *et al.* PNAS 2014, Wang *et al.* PRB 2014, Fang *et al.* PRB 2008, Sun *et al.* PRL 2010, Kivelson *et al.* PRB 2004, Fischer *et al.* PRB 2011

Inelastic neutron scattering at low temperatures $T=3K$

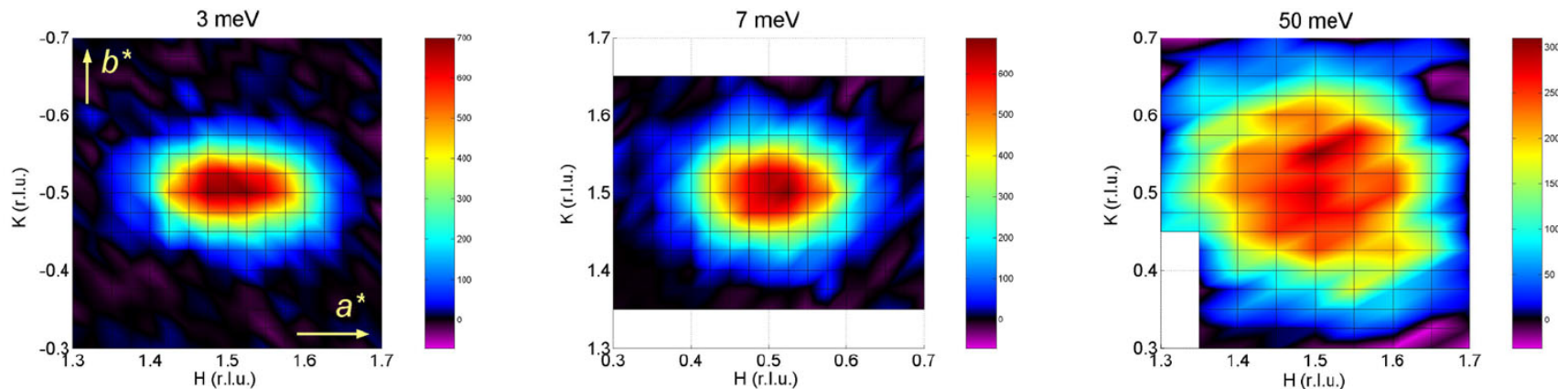
- At lowest $T=3K$: (Quasi-)elastic peak close to $Q=(\pi, \pi)$ describing static **incommensurate** spin-density wave (**SDW**) order
- **Inelastic contribution** peaked at energy of 3 meV remains present at larger temperatures T



From [1]

Experimental signatures of nematicity in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

- Scattering peaks at $Q=(\pi, \pi)$ and energy $E \leq 7 \text{ meV}$ show **in-plane anisotropy** ($a \neq b$)
- **Elliptical shape** with major axis along a direction



Increasing energy E

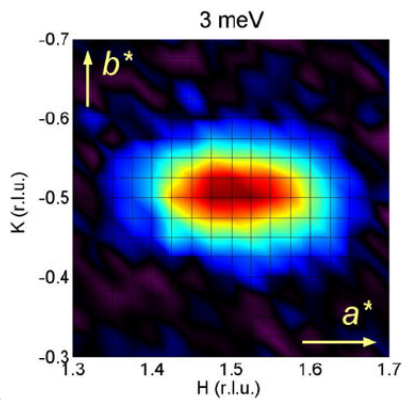
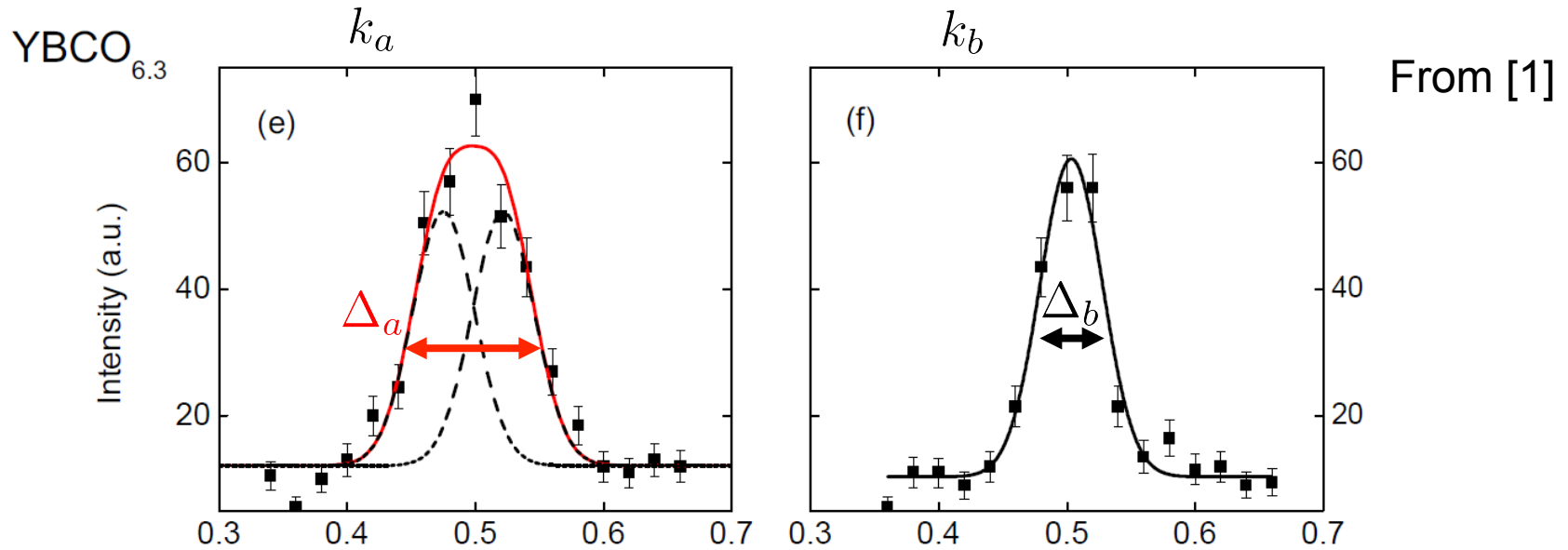


From [1]

Lowest doping $p=0.05$

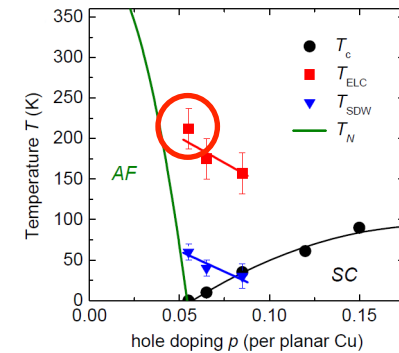
$$\Delta_a > \Delta_b$$

Experimental signatures of nematicity in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$



Lowest doping $p=0.05$

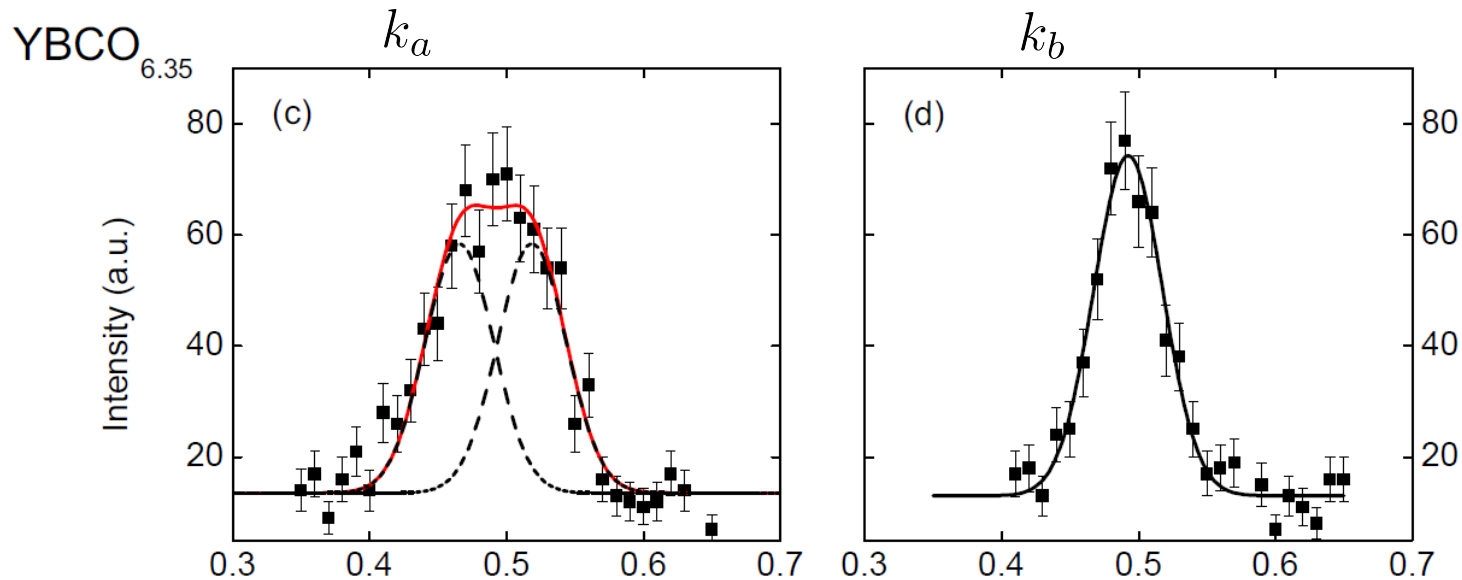
$$\Delta_a > \Delta_b$$



$$T=3\text{K}, E=3\text{meV}, k=1.55/A$$

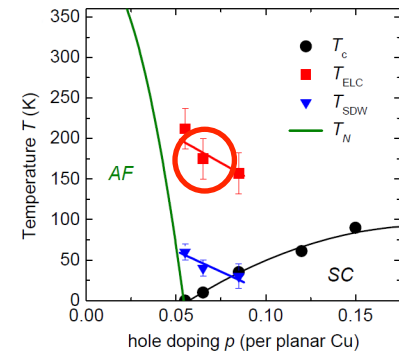
Experimental signatures of nematicity in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

- Ellipticity ($\Delta\downarrow a - \Delta\downarrow b$) increases with hole doping p



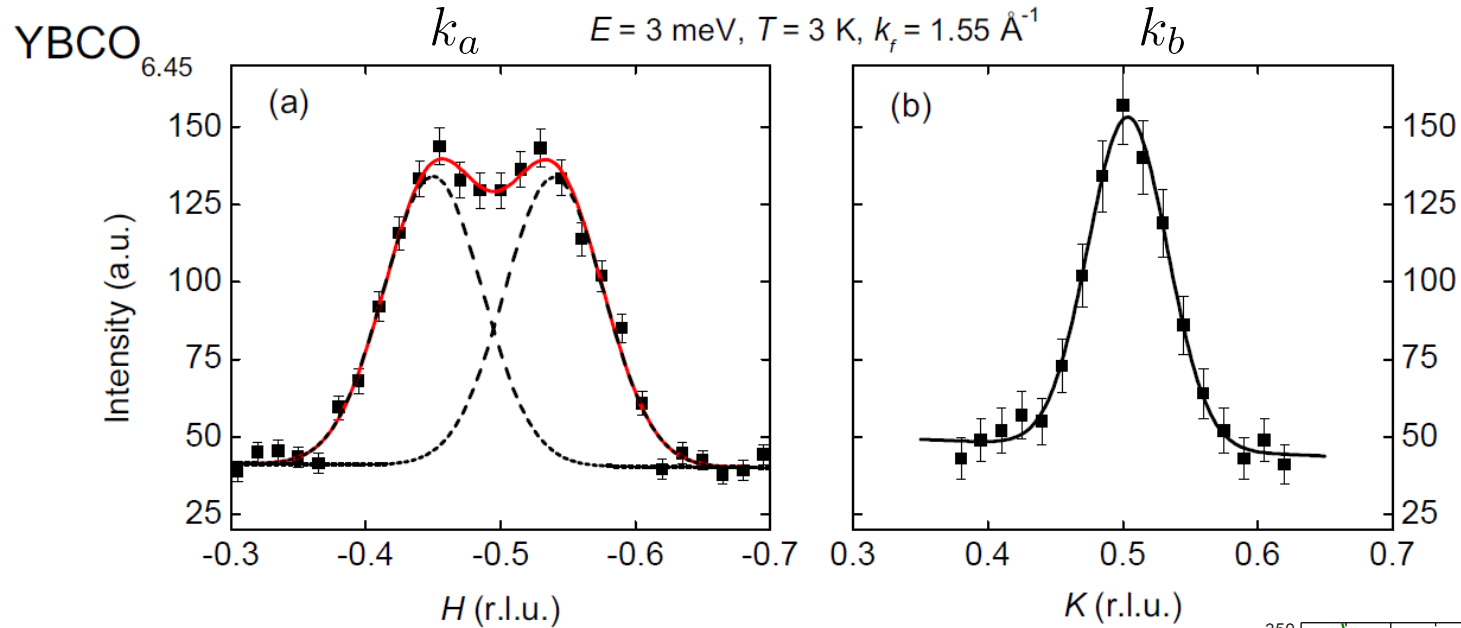
From [1]

Larger doping $p=0.06$



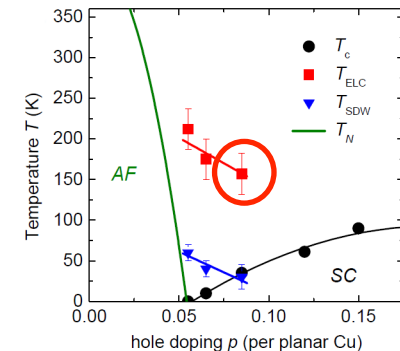
Experimental signatures of nematicity in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

- Scattering peaks eventually split: **incommensurate wavevectors along a**



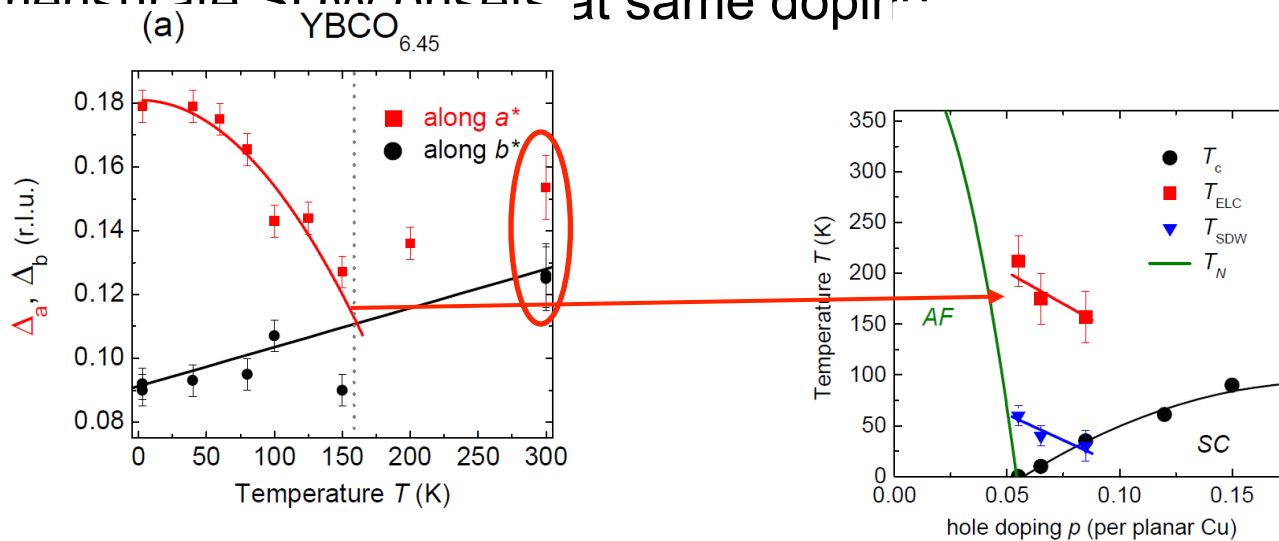
From [1]

Largest doping $p=0.08$



Summary of nematicity signatures in inelastic neutron

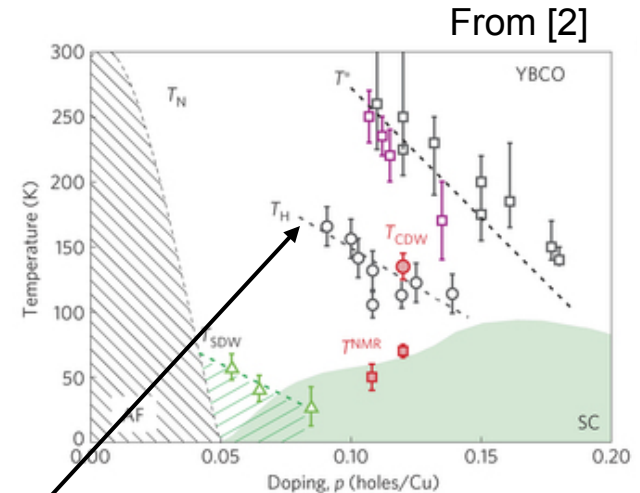
- Small ellipticity present even above $T \downarrow nem$ due to orthorhombic distortion of YBCO
- T -dependent ellipticity of inelastic peaks appears at large $T \downarrow nem \approx 150 K$
- Ellipticity increases with lowering T
- Doping-dependent onset temperature $T \downarrow nem$ (p)
- Incommensurate SDW onsets at same doping p



From [1]

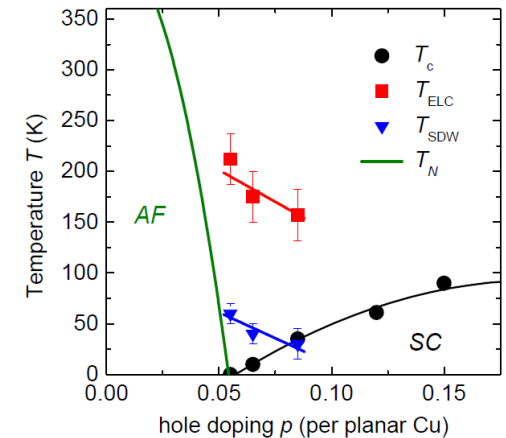
Summary of nematicity signatures in inelastic neutron

- Small ellipticity present even above $T \downarrow n_{em}$ due to orthorhombic distortion of YBCO
- T -dependent ellipticity of inelastic peaks appears at large $T \downarrow n_{em} \approx 150 K$
- Ellipticity increases with lowering T
- Doping-dependent onset temperature $T \downarrow n_{em}$ (p)



Other signatures of nematic "stripe" correlations

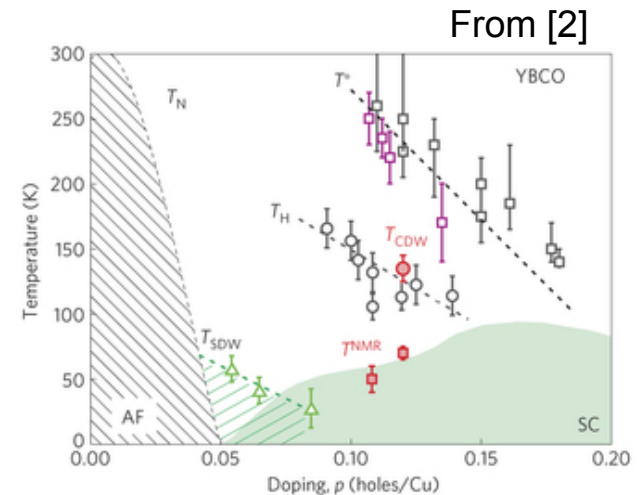
- Appears only for hole doping
- Appears only when tetragonal symmetry is explicitly broken (new data by Matsuda *et al.*?)
- (Fluctuating) stripe CDW nearby
- Appears at "large" temperatures scales $T \sim J$



From [1]

Summary of nematicity signatures in inelast. neutron

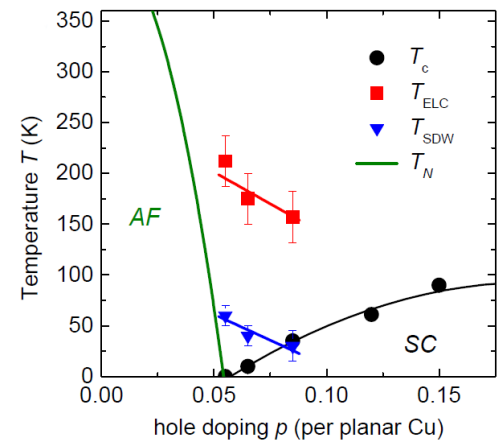
- T -dependent ellipticity of inelastic peaks appears at large $T \downarrow n_{em} \approx 150$ K
- Doping-dependent onset temperature $T \downarrow n_{em}(p)$
- Ellipticity increases with lowering T
- Incommensurate SDW onsets at same doping
- Small ellipticity present even above $T \downarrow n_{em}$ due to orthorhombic distortion of YBCO



We now describe microscopic mechanism by which nematicity appears close to AF Neel order.

Theory program:

- Microscopic derivation of biquadratic spin exchange K term from 3-band Hubbard model
- Analysis of biquadratic K term including classical and quantum fluctuations



Microscopic derivation of biquadratic spin exchange

Microscopic starting point: 3-orbital Hubbard model

- **Three-band Hubbard model:** Cu $d \downarrow x^2 - y^2$, O $p \downarrow x, p \downarrow y$ orbitals

$$H = H_\epsilon + H_t + H_U + H_V$$

Non-interacting part $H_t + H_\epsilon$

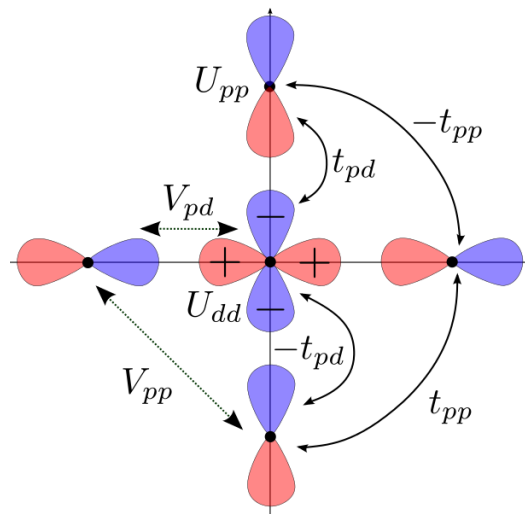
- hopping $t_{pd} > t_{pp}$
- orbital energy difference $\Delta = \epsilon_p - \epsilon_d > 0$.

On-site interactions

$$H_U = \sum_{\mathbf{R}_i} \left[U_{dd} n_{i\uparrow}^d n_{i\downarrow}^d + \frac{U_{pp}}{2} (n_{xi\uparrow}^p n_{xi\downarrow}^p + n_{yi\uparrow}^p n_{yi\downarrow}^p) \right]$$

Nearest-neighbor interactions

$$H_V = V_{pp} \sum_{\mathbf{R}_i, \sigma, \sigma'} \sum_{\delta'} n_{xi}^p n_{yi+\delta'}^p + V_{pd} \sum_{\delta} n_{xi}^d n_{i+\delta}^p$$



Undoped “vacuum state”: One hole per Cu, filled O orbitals

- ➡ Charge-transfer (Mott) insulator: $t_{ij} \ll U_{dd}, \Delta$
- ➡ Strong-coupling expansion in $t_{ij} / U_{dd} - \Delta, t_{ij} / \Delta$ yields t - J -model describing mobile holes coupled to localized Cu spins. **Role of oxygen density fluctuations?**

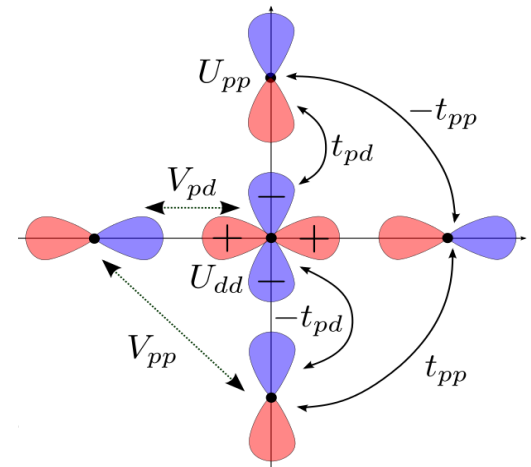
Relevant terms in strong-coupling expansion

- Focus on subspace of singly occupied Cu spins first
- At second order
 - effective hole hopping between oxygens renormalizing t_{pp}
 - Cu-O Kondo-like exchange coupling $\propto \mathbf{S}_i \cdot \mathbf{s}_{jk}$

$$\text{Cu spins: } \mathbf{S}_j = \frac{1}{2} \sum_{\alpha, \beta} \tilde{d}_{j\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} \tilde{d}_{j\beta}$$

$$\tilde{d}_{j\alpha} = (1 - n_{j\bar{\alpha}}) d_{j\alpha}^\dagger$$

$$\text{(Non-)local oxygen spins: } \mathbf{s}_{jk} = \frac{1}{2} \sum_{\alpha, \beta} p_{j\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} p_{k\beta}$$



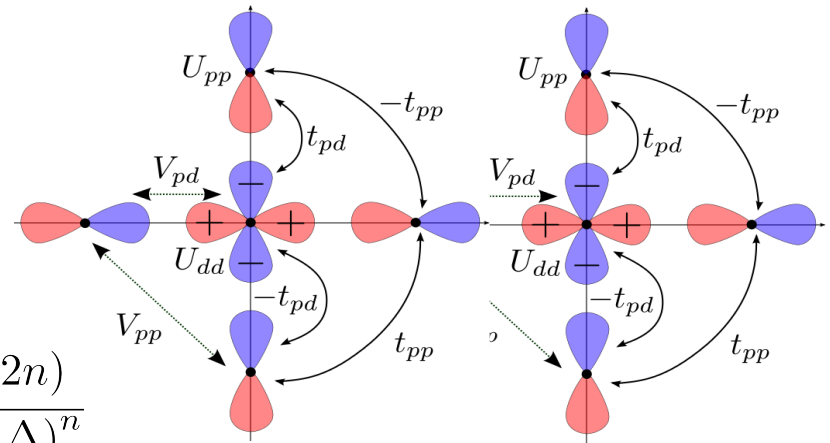
Relevant terms in Cu spin model

- At second order
 - effective hole hopping between oxygens renormalizing t_{pp}
 - Cu-O Kondo-like exchange coupling $\propto \mathbf{S}_i \cdot \mathbf{s}_{jk}$
- At fourth order
 - Heisenberg exchange of Cu spins
 - Non-local exchange of oxygen spins $H_J = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$
 - Cu-O Kondo-like exchange couplings
 - Oxygen density $n_{\uparrow p}$ dependent Heisenberg exchange of Cu spins

$$H_{J'} = -J' \sum_{i,\delta} n_{i+\frac{\delta}{2}}^p \mathbf{S}_i \cdot \mathbf{S}_{i+\delta}$$

Exchange constants are of the same order:

$$J = \sum_{n=0}^2 \frac{t_{pd}^4 (4 - n^2 - \delta_{n,2})}{2\Delta^{3-n} (U_{dd} - \Delta)^n} \quad J' = \sum_{n=0}^3 \frac{t_{pd}^4 \text{sign}(3 - 2n)}{\Delta^{3-n} (U_{dd} - \Delta)^n}$$



Effect of oxygen charge fluctuations

- Oxygen density $n\uparrow p$ dependent Heisenberg exchange of Cu spins
 - Effect of quadrupolar oxygen charge fluctuations

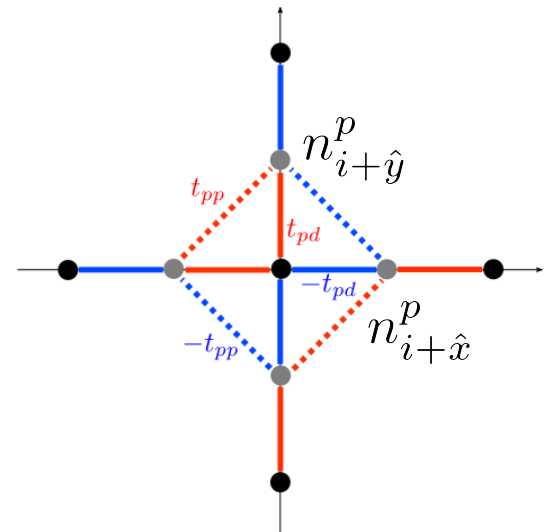
$$H_{J'} = -J' \sum_{i,\delta} n_{i+\frac{\delta}{2}}^p \mathbf{S}_i \cdot \mathbf{S}_{i+\delta} = -J' \sum_i \left(n_{i+\frac{\hat{x}}{2}}^p \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}} + n_{i+\frac{\hat{y}}{2}}^p \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}} \right)$$

$$n_{i+\hat{x}}^p = (\bar{n}_i^p + \eta_i)/2$$

$$n_{i+\hat{y}}^p = (\bar{n}_i^p - \eta_i)/2$$



$$-J' \sum_i \frac{\eta_i}{2} \left(\mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}} \right)$$



Biquadratic spin exchange term K

$$-J' \sum_i \frac{\eta_i}{2} \left(\mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}} - \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}} \right)$$

Effect of quadrupolar oxygen charge fluctuations

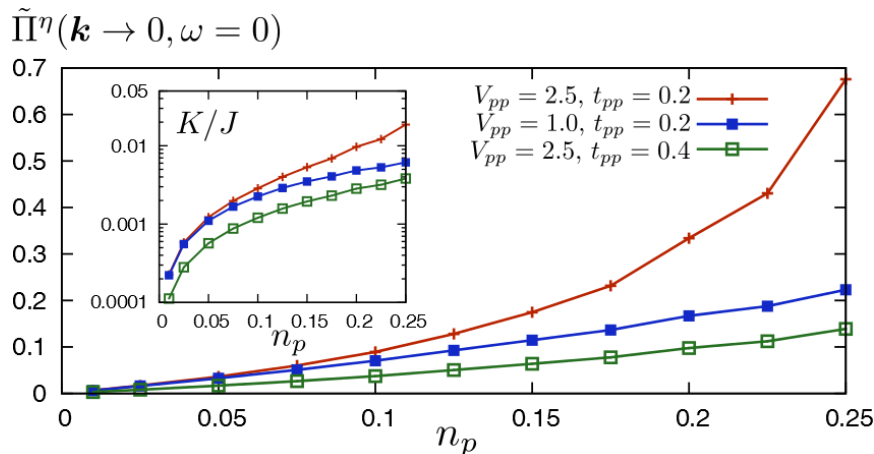


- Decoupling oxygen interactions in the nematic channel η
- Integrating out oxygen degrees of freedom

$$H_K = -K \sum_i \left[\mathbf{S}_i \cdot \left(\mathbf{S}_{i+\hat{x}} - \mathbf{S}_{i+\hat{y}} \right) \right]^2 \quad \text{with} \quad K = \frac{J'^2}{2} \lim_{\mathbf{k} \rightarrow 0} \frac{\Pi_{\mathbf{k}}^\eta}{1 - U_{\mathbf{k}} \Pi_{\mathbf{k}}^\eta} > 0$$

Biquadratic spin exchange term, enhanced by O-O repulsion

$$U_{\mathbf{k}=0} = V_{pp} - \frac{U_{pp}}{8}$$



$\Pi_{\mathbf{k}}^\eta$: Bare oxygen charge susceptibility in quadrupolar channel

- $\propto n \downarrow p$ at small doping
- Enhanced by large $V \downarrow p p$ and small $t \downarrow p p$
- K/J up to a few percent

Analysis of t-J-K model at half-filling (first step)

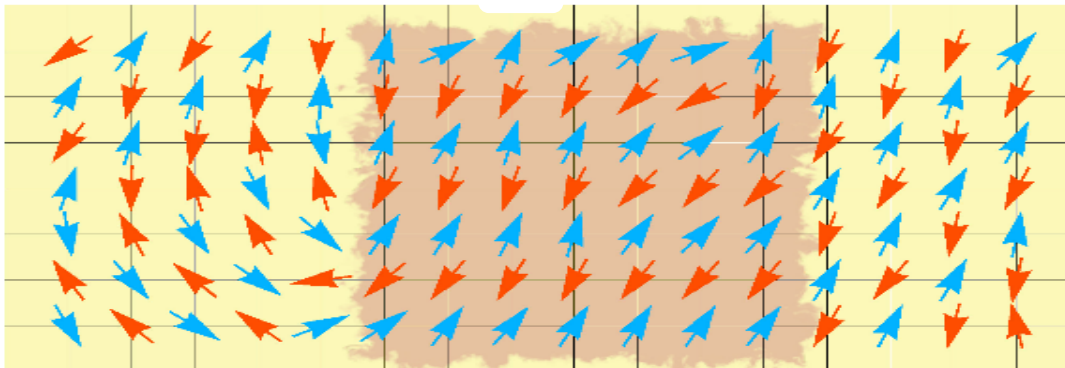
Generalized t-J-K model

- Must include biquadratic exchange K term in t-J model

$$H_{t-J-K} = \sum_{ij\alpha} t_{ij} \tilde{d}_{i\alpha}^\dagger \tilde{d}_{j\alpha} + J \sum_{\langle ij \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right) - K \sum_i [\mathbf{S}_i \cdot (\mathbf{S}_{i-\hat{x}} + \mathbf{S}_{i+\hat{x}} - \mathbf{S}_{i-\hat{y}} - \mathbf{S}_{i+\hat{y}})]^2$$

- Analyze K-term at half-filling of Cu sites

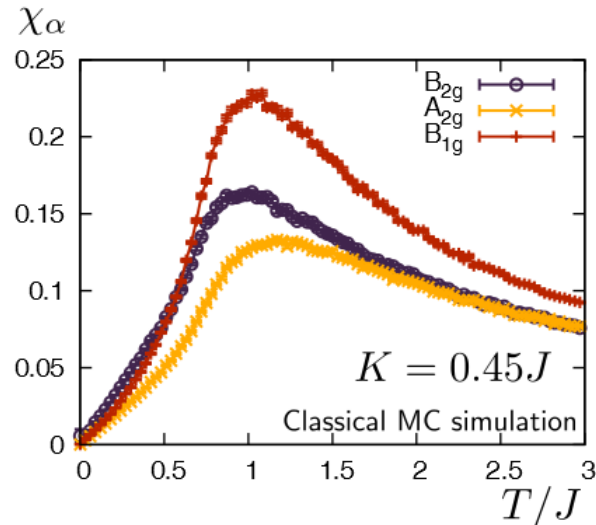
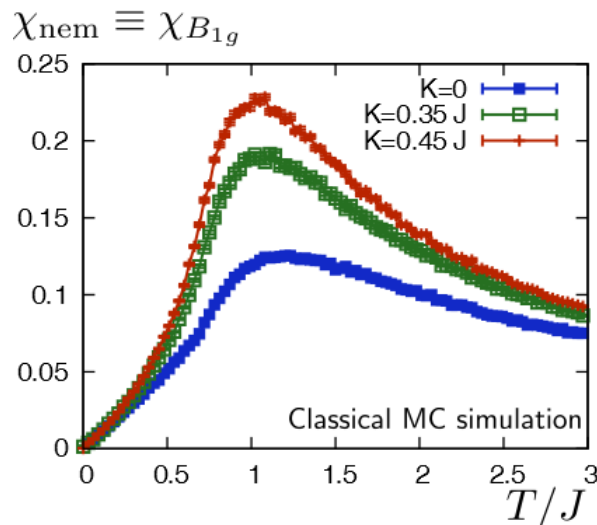
- Does not modify spin-wave spectrum [1], because $H_K |\text{Néel}\rangle = 0$
- Supports local “stripe” like fluctuations in Neel phase



Classical Monte-Carlo simulation of J-K model

- Composite “nematic” spin variable: $\varphi_i = \mathbf{S}_i \cdot (\mathbf{S}_{i-\hat{x}} + \mathbf{S}_{i+\hat{x}} - \mathbf{S}_{i-\hat{y}} - \mathbf{S}_{i+\hat{y}})$
- **Static nematic response**

$$\chi_{\text{nem}}(T) = \int_0^{1/T} d\tau \sum_i \langle \mathcal{T}_\tau \varphi_i(\tau) \varphi_0(0) \rangle$$
- Enhanced response in nematic B_{1g} channel
- **Peaks at large $T \approx J$**
- Enhancement $\propto K$
- **Vanishes for $T \rightarrow 0$** as thermal fluctuations are suppressed. Disagrees with experimental observation.

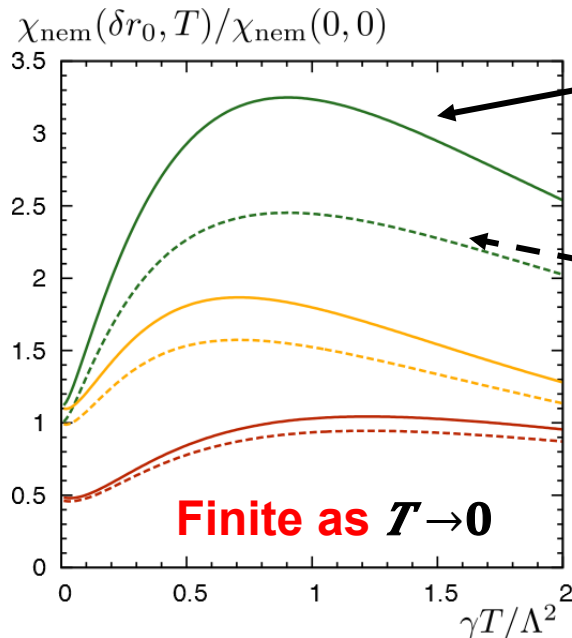


Quantum fluctuations within large-N approach

- Effective soft-spin action after decoupling in nematic channel

$$S = S_{\text{dyn}} + \int_r \left[(\nabla \mathbf{n}_r)^2 - \varphi_r \left((\partial_x \mathbf{n}_r)^2 - (\partial_y \mathbf{n}_r)^2 \right) \right] + \int_r \left[r_0 \mathbf{n}_r^2 + \frac{u}{2} (\mathbf{n}_r \cdot \mathbf{n}_r)^2 + \frac{\varphi_r^2}{2g} - h_r \varphi_r \right]$$

Neel order parameter $\mathbf{n} \downarrow r$
 $g \propto K/J$
 Orthorhombic distortion field $h \downarrow r$

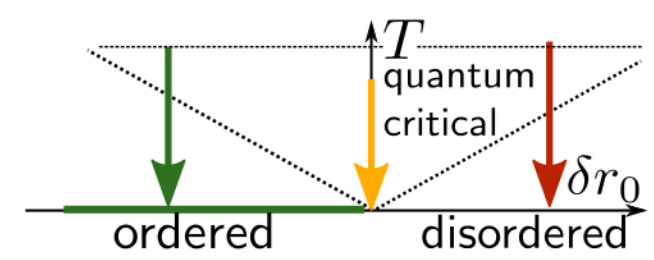


Full Susceptibility

$$\chi_{\text{nem}} = \frac{\chi_{\text{nem}}^{(0)}}{1 - \frac{g}{N} \chi_{\text{nem}}^{(0)}}$$

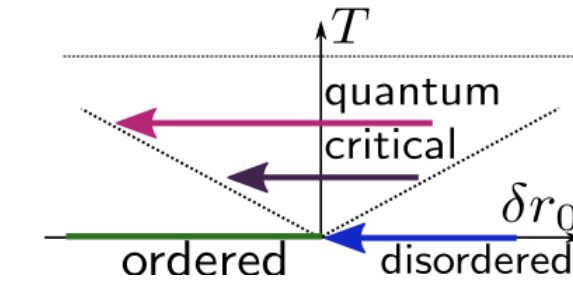
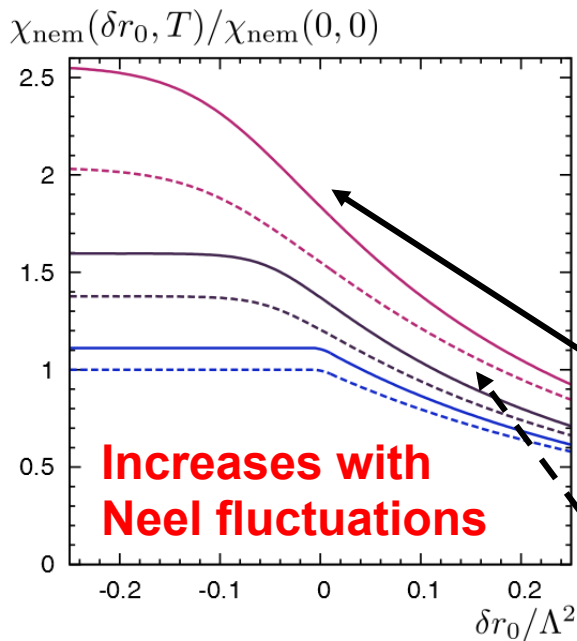
Bare susceptibility

$$\chi_{\text{nem}}^{(0)} = \frac{N}{2} \int_q \frac{|\mathbf{q}|^4 \cos^2(2\theta)}{(\xi^{-2} + |\mathbf{q}|^2 + f(\omega_n))^2}$$



Changing temperature at fixed doping

Quantum fluctuations within large-N approach



Changing doping or pressure at fixed T

Full Susceptibility $\chi_{\text{nem}} =$

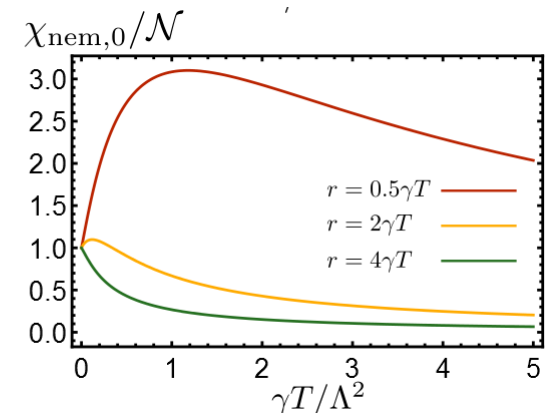
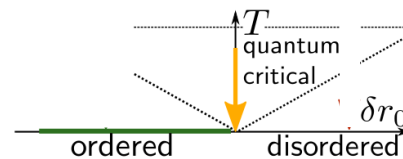
$$\frac{\chi_{\text{nem}}^{(0)}}{1 - \frac{g}{N} \chi_{\text{nem}}^{(0)}}$$

Bare susceptibility $\chi_{\text{nem}}^{(0)} =$

$$\frac{N}{2} \int_{\mathbf{q}} \frac{|\mathbf{q}|^4 \cos^2(2\theta)}{(\xi^{-2} + |\mathbf{q}|^2 + f(\omega_n))^2}$$

- Nematic susceptibility is not universal at Neel quantum critical point (QCP)
- Depends on microscopic details such as a in $r \downarrow 0 = a(T - T \downarrow c)$

Example: qualitative behavior above QCP depends on a

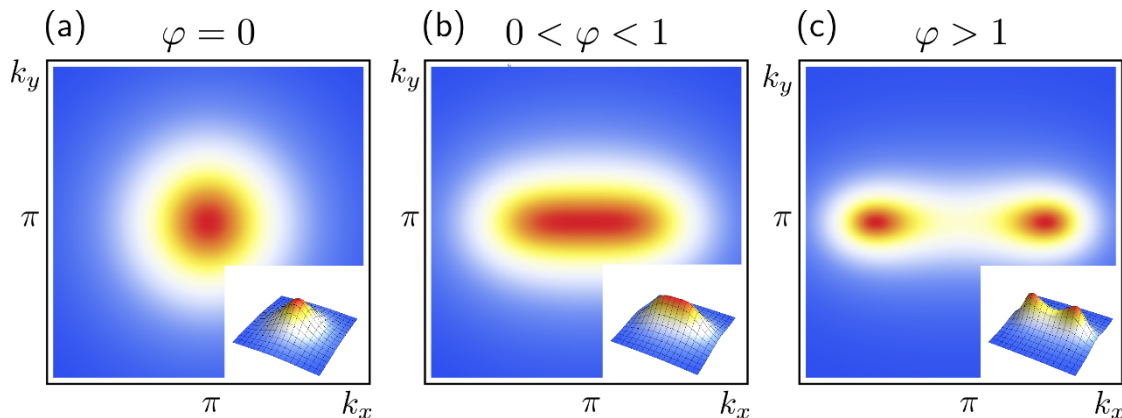


Finite nematic order in symmetry breaking field

- Nematic susceptibility large but not divergent
- Finite nematic order $\langle \varphi \rangle = \langle S_{li} (S_{li+x} - S_{li+y}) \rangle = \chi_{nem} h_{lr}$ only in presence of symmetry-breaking field
- Symmetry-breaking field given by CuO chains in YBCO.
- Prediction: similar behavior in tetragonal system under applied strain

Finite φ leads to elliptic magnetic scattering peaks

$$\chi_{AFM}(\mathbf{Q} + \mathbf{q}, \omega) = \frac{1}{\xi^{-2} + \mathbf{q}^2 - \varphi (q_x^2 - q_y^2) + f(\omega_n)}$$



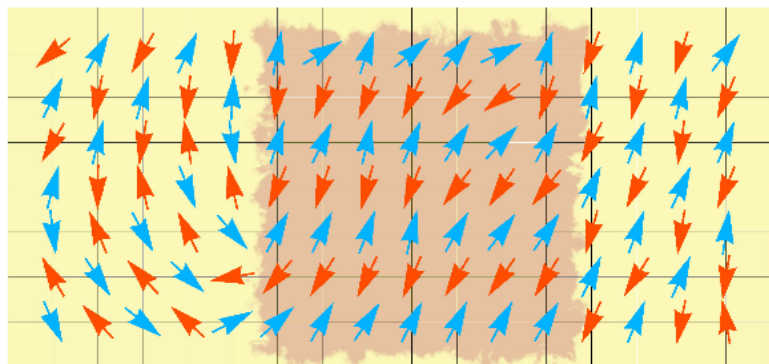
Incommensurability transition induced by nematic order. Explains common onset [1].

Alternative scenario: nematic order vestige to SDW [2]

Conclusion

Electronic nematicity in cuprates due to

- Short-range “stripe” AFM correlations due to **biquadratic spin exchange** K
- Large, but **non diverging nematic susceptibility** $\chi_{\downarrow nem}$ and presence of a **symmetry breaking field** $h_{\downarrow r}$ (e.g. oxygen chains or external strain)
- **Enhanced by** large quadrupolar oxygen density fluctuations due to repulsive interactions $V_{\downarrow pp}$.
- Include biquadratic exchange term in generalized $t-J-K$ model to describe lightly hole-doped cuprates



PPO, B. Jeevanesan, R. M. Fernandes, J. Schmalian, arXiv:1703:02210 (2017).

Explains:

- Ellipticity of neutron peaks
- Common onset of incommensurate SDW order
- Why nematicity is only observed on hole doped side
- Why only observed if tetragonal symmetry is explicitly broken