Pseudo-spin skyrmions in the under-doped cuprates



K.B. Efetov (Bochum)
H. Meier
M. Einenkel
T. Kloss
V. S. de Carvalho



X. Montiel









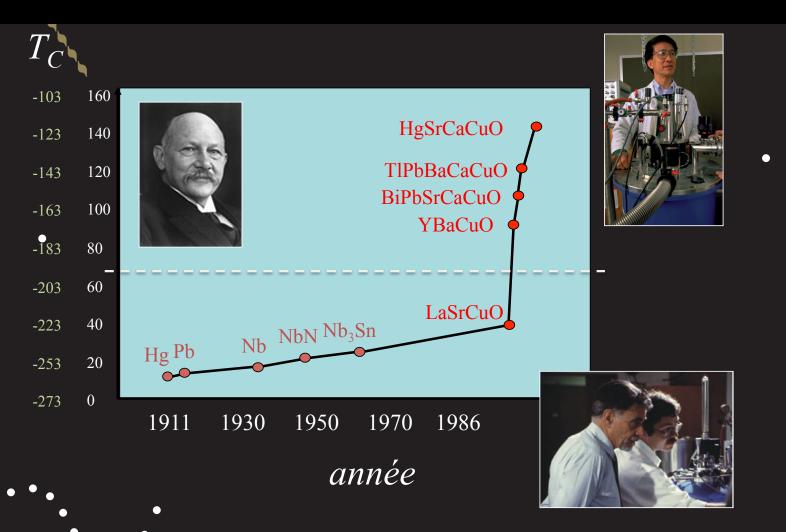
C. Morice & D.Chakraborty

Catherine Pépin (IPhT, CEA-Saclay)

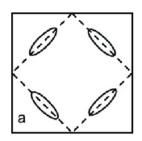
Intertwined 17, Santa Barbara, Sept. 12th, 2017

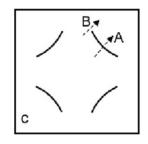
X.Montiel, T. Kloss and CP, PRB 2017

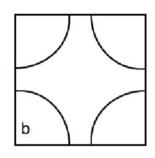
C.Morice, D. Chakraborty, X.Montiel and CP, preprint

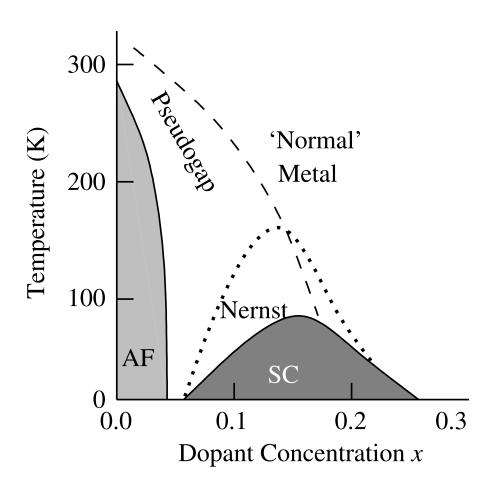


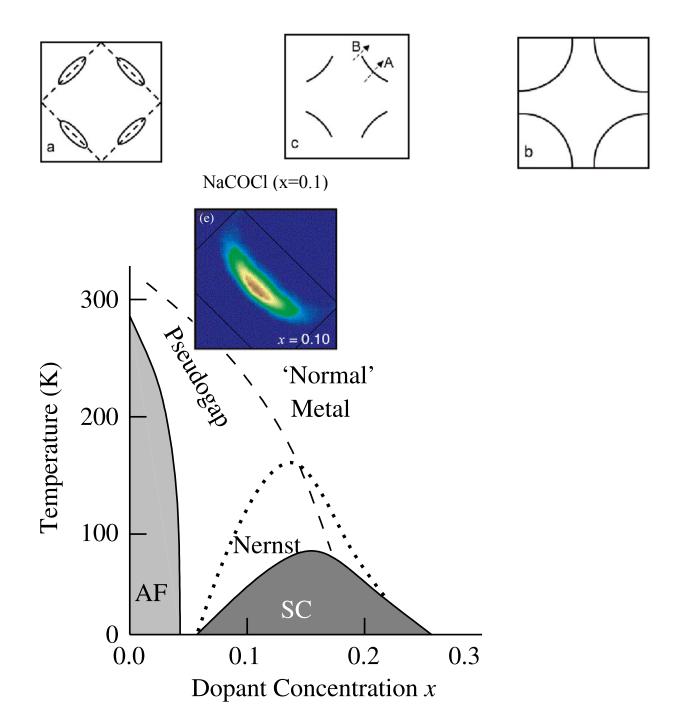
1987...

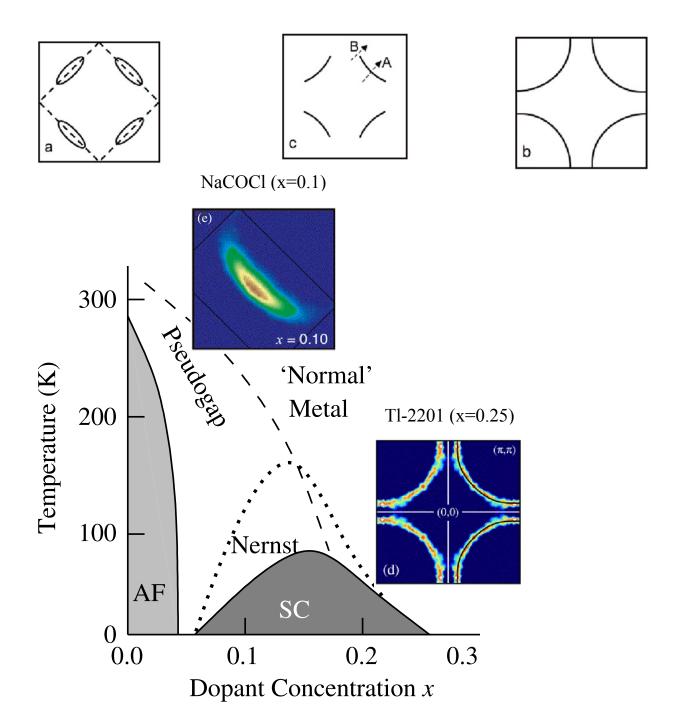












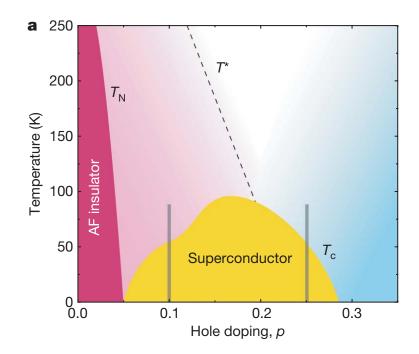
⁸⁹Y NMR Evidence for a Fermi-Liquid Behavior in YBa₂Cu₃O_{6+x}

H. Alloul, T. Ohno, (a) and P. Mendels

Physique des Solides, Université de Paris-Sud, 91405 Orsay, France
(Received 15 May 1989)

We report NMR shift ΔK and T_1 data of ⁸⁹Y taken from 77 to 300 K in YBa₂Cu₃O_{6+x} for 0.35 < x < 1, from the insulating to the metallic state. A Korringa law and therefore a Fermi-liquid picture is found to apply for the spin part K_s of ΔK . The spin contribution $\chi_s(x,T)$ to χ_m is singled out, as the T variation of ΔK scales linearly with the macroscopic susceptibility χ_m . This implies that Cu(3d) and O(2p) holes do not have independent degrees of freedom. Their hybridization, which has a σ character, hardly varies with doping. These results put severe constraints on theoretical models of high- T_c cuprates.

PACS numbers: 74.70.Vy, 75.20.En, 76.60.Cq, 76.60.Es



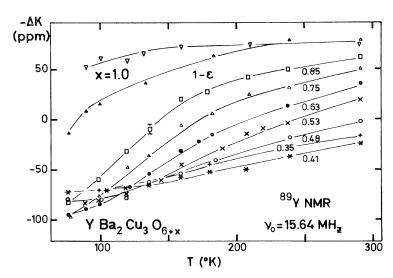


FIG. 1. The shift ΔK of the ⁸⁹Y line, referenced to YCl₃ plotted vs T, from 77 to 300 K. The lines are guides to the eye.

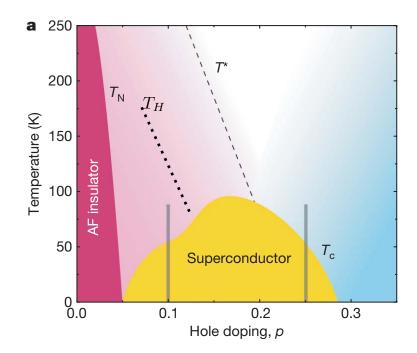
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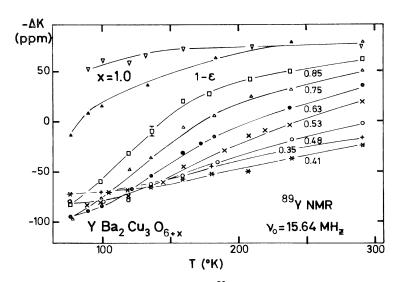
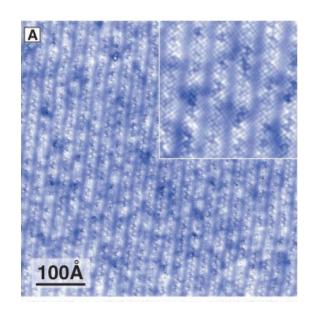
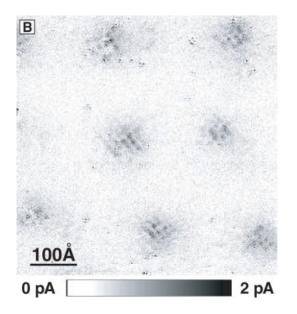


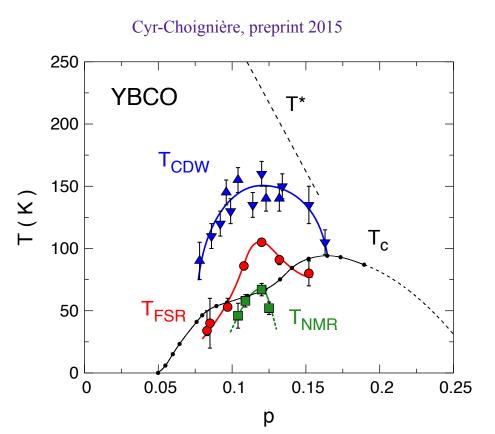
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Charge modulations ...

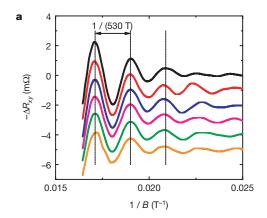




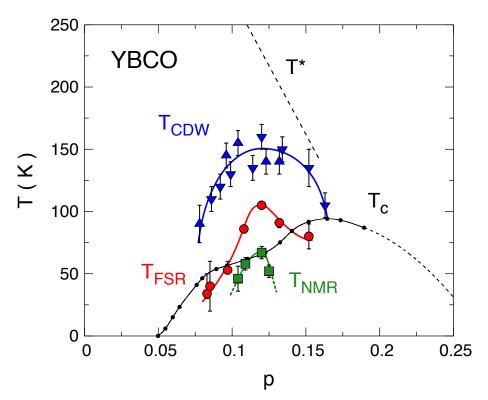
Hoffman, 2002



Doiron-Leyraud et al. (2007) Sebastian et al. (2011)



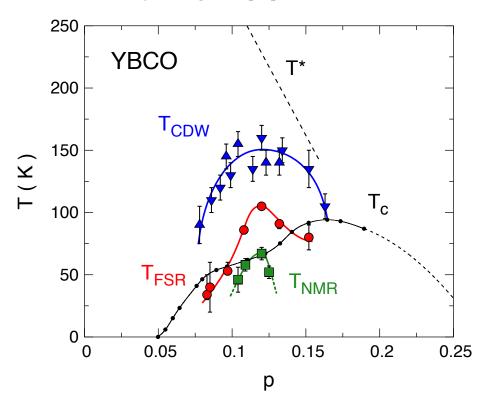
Cyr-Choignière, preprint 2015

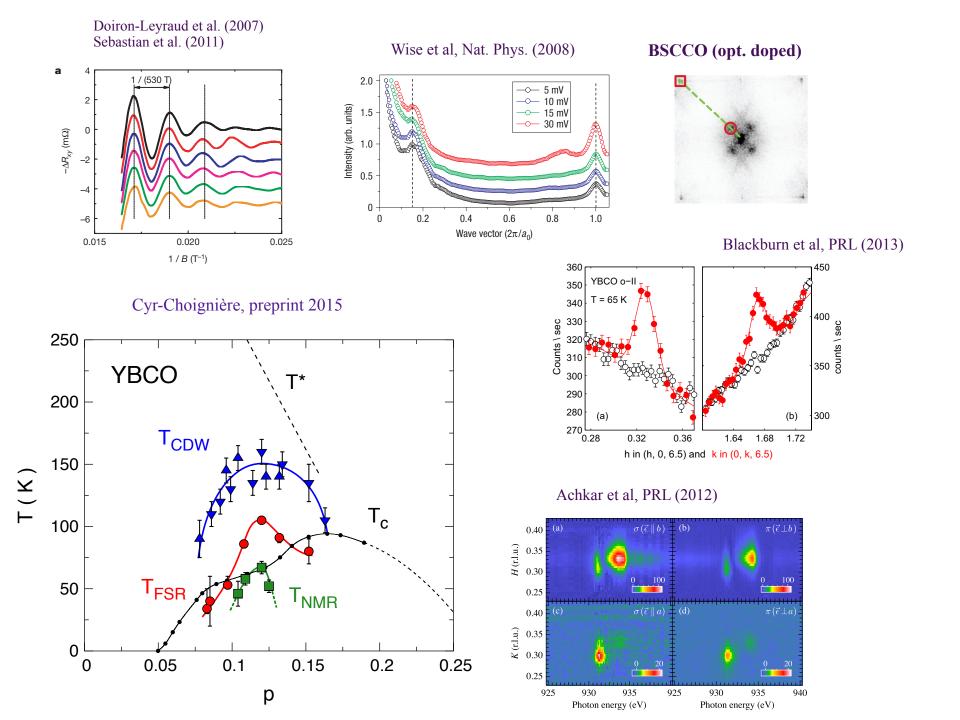


Doiron-Leyraud et al. (2007) Sebastian et al. (2011) Wise et al, Nat. Phys. (2008) **BSCCO** (opt. doped) а 2.0 Intensity (arb. units) $-\Delta R_{xy}$ (m Ω) 0 -0.4 0.6 0.8 1.0 0.2 Wave vector $(2\pi/a_0)$ 0.015 0.020 0.025

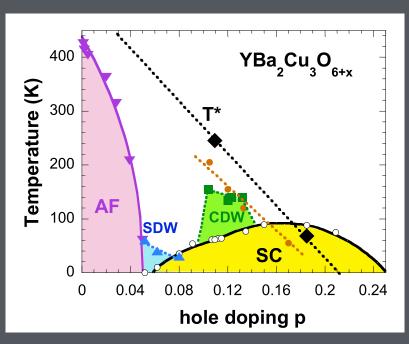
Cyr-Choignière, preprint 2015

1 / B (T⁻¹)

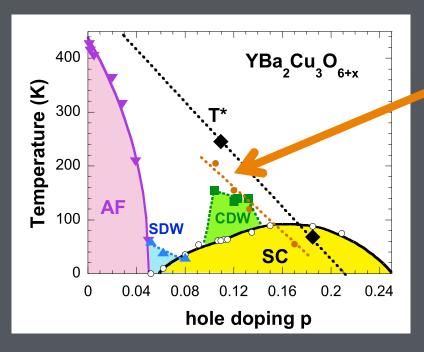








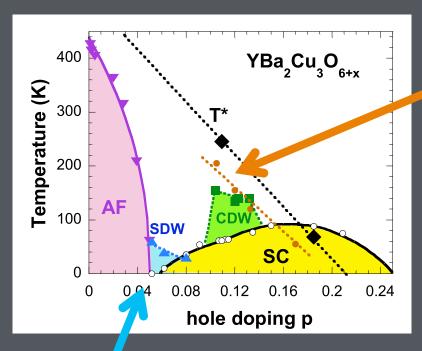
YBa₂Cu₃O_{6+x}



anomalous Kerr effect $T_k < T^*$

Xia, PRL 2008

YBa₂Cu₃O_{6+x}



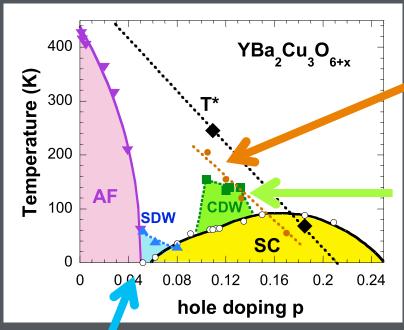
anomalous Kerr effect $T_k < T^*$

Xia, PRL 2008

glassy SDW : $T_{SDW} \ll T^*$ (neutron, μ SR, RMN)

Haug, New J. Phys. 2010 T. Wu et al., PRB 2013

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Xia, PRL 2008

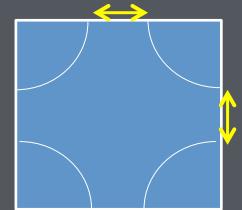
Incipient CDW $-T_m < T^*$

 $Q^* = (\delta, 0)$ and $(0, \delta)$ with $\delta \sim 0.3$

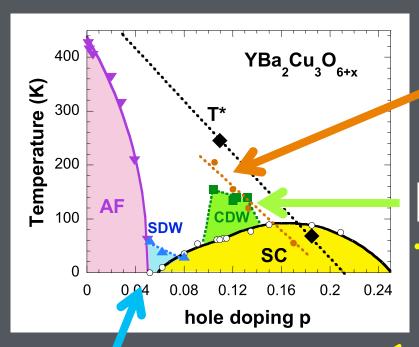
Chang , Nature Phys. 2012 Ghiringhelli, Science 2012

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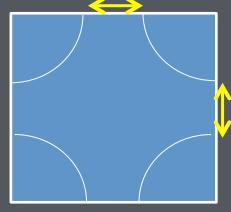
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Stable CDW under magnetic field & Fermi surface reconstruction (NMR, quantum oscillation, ultrasound)

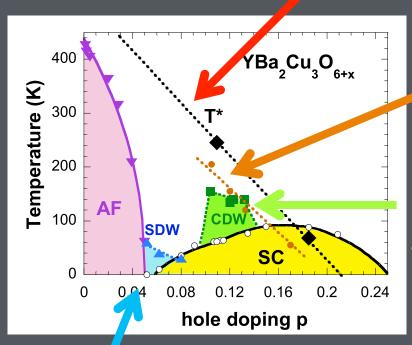
D. Lebocui, Malare 2001.

T. Wu et al., *Nature* 2011.

Nematicity

Charge order Landscape

YBa₂Cu₃O_{6+x}



anomalous Kerr effect $T_k < T^*$

Xia, PRL 2008

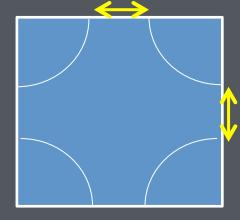
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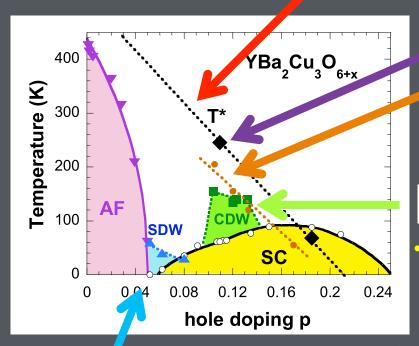
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T. Wu et al., *Nature* 2011.

Nematicity

YBa₂Cu₃O_{6+x}



loop currents

anomalous Kerr effect $T_k < T^*$

Xia, PRL 2008

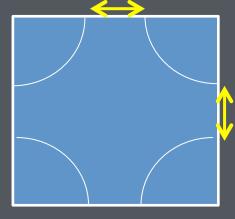
Incipient CDW – $T_m < T^*$

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Nematicity

Inversion symmetry

loop currents

anomalous Kerr effect $T_k < T^*$

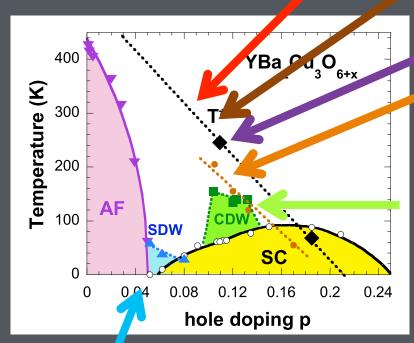
Xia, PRL 2008

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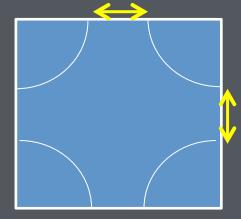
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Stable CDW under magnetic field & Fermi surface reconstruction (NMR, quantum oscillation, ultrasound)

D. Leboeui, Nature 2001.

T. Wu et al., *Nature* 2011.

Why is there so many competing orders occurring around T*?

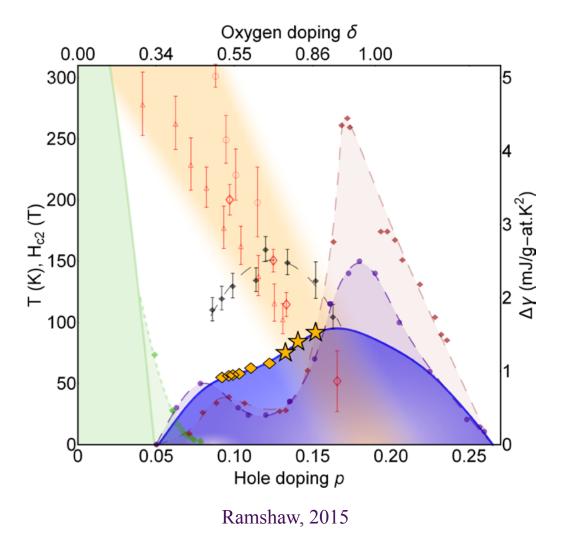
Why is there so many competing orders occurring around T*?

Orders at q=0 don't open a gap in the electron's density of states.

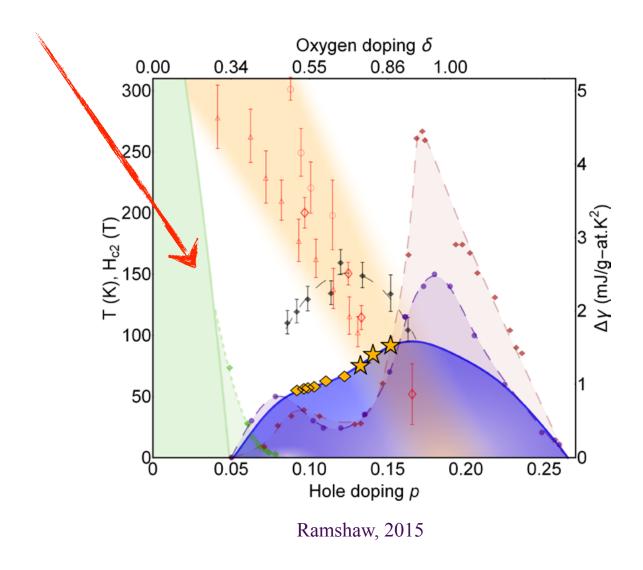
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Is the Pseudo-Gap a phase transition or a cross over?

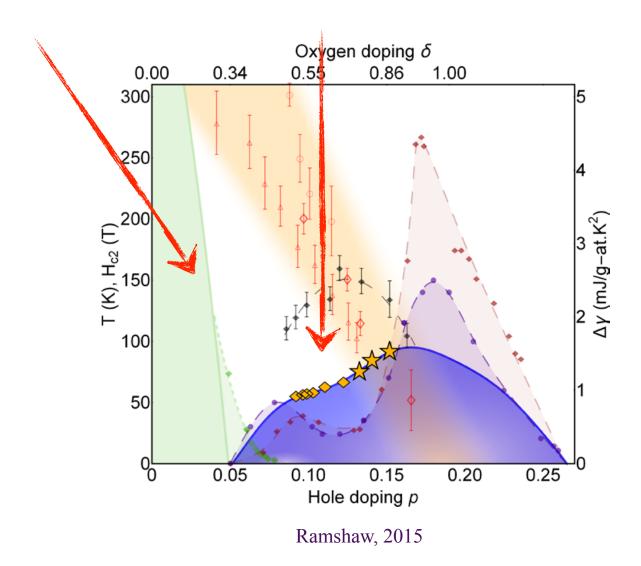


Mott transition



Mott transition

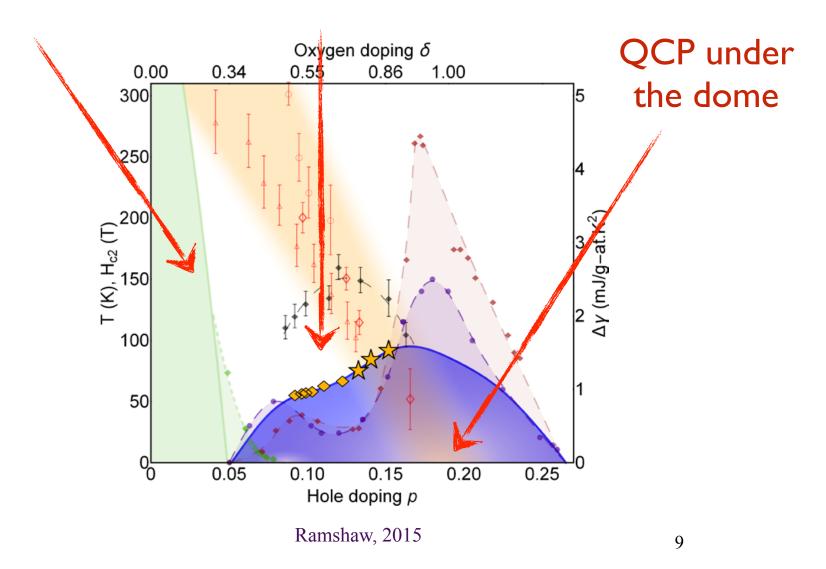
Fluctuations



9

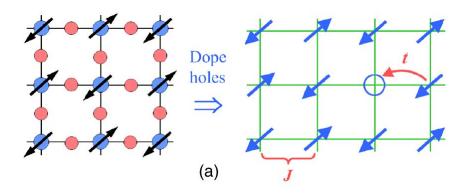
Mott transition

Fluctuations



The context: doping a Mott insulator

Resonating Valence Bond (RVB)



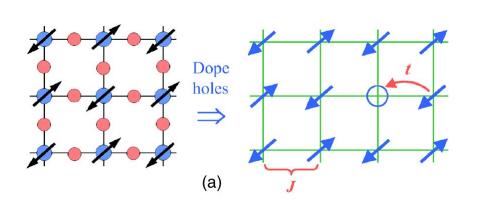
$$H = P \left[-\sum_{\langle ij \rangle, \sigma} t_{ij} c_{i\sigma}^{\dagger} c_{i\sigma} + J \sum_{\langle ij \rangle} \left(\mathbf{S}_{i} \cdot \mathbf{S}_{j} - \frac{1}{4} n_{i} n_{j} \right) \right] P$$

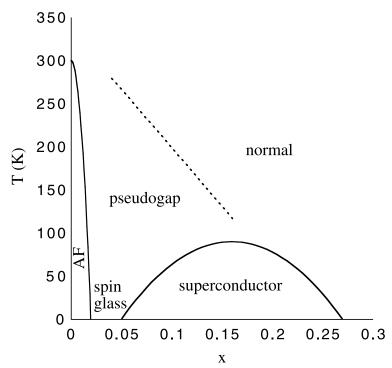
Anderson, Sachdev, Lee, Nagaosa, Rice ...

P: projection on no double occupancy

The context: doping a Mott insulator

Resonating Valence Bond (RVB)





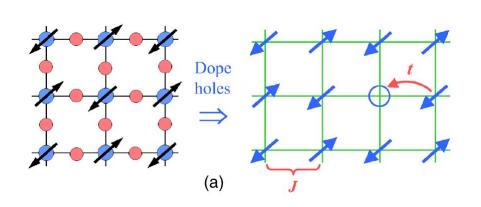
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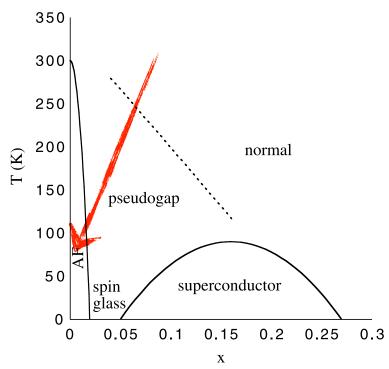
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Anderson, Sachdev, Lee, Nagaosa, Rice ...

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Fluctuations

Emery Kivelson 95

	TABLE 1 Phase stiffness and $T_{\theta}^{\sf max}$ for various superconductors							
Material	/ (Å)	λ (Å)	T _c (K)	V _o (K)	T_{θ}^{max}/T_{c}	Ref.		
Pb	830	390	7	6×10^5	2×10 ⁵	17		
Nb₃Sn	60	640	18	2×10^4	2×10^{3}	18		
UBe ₁₃	140	11,000	0.9	10 ²	3×10^{2}	19, 20		
LaMO ₆ S ₈	200	7,000	5	4×10^2	2×10^{2}	12, 21		
$B_{0.6}K_{0.4}BiO_3$	40	3,000	20	5×10^{2}	50	12		
K ₃ C ₆₀	30	4,800	19	10 ²	17	22, 23		
(BEDT) ₂ Cu(NCS) ₂	15.2	8,000	8	15	1.7	24		
$Nd_{2-x}Ce_{x}Cu_{2}O_{4+\delta}$	6.0	1,000	21	4×10^2	16	25		
$Tl_2Ba_2CuO_{6+\delta}$	11.6	2,000	80	2×10^2	2	26, 27		
	11.6	1,800	55	2×10^2	3.6	26, 27		
Bi ₂ Sr ₂ CaCu ₂ O ₈	7.5	1,850	84	140	1.5	28, 29		
Bi ₂ Pb _x Sr ₂ Ca ₂ Cu ₃ O ₁₀	5.9	1,850	105	110	0.9	28		
	8.9	1,850	105	160	1.4	28		
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	6.6	2,200	38	85	2	30		
YBa ₂ Cu ₃ O _{7-δ}	5.9	1,600	92	145	1.4	31		
YBa ₂ Cu ₄ O ₈	6.8	2,600	80	65	0.7	31		

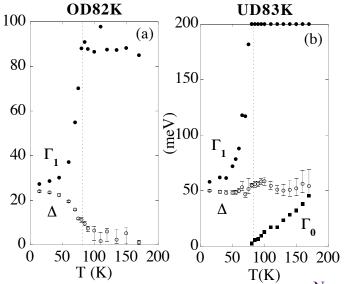
$$V_0 = \frac{(\hbar c)^2 a}{16\pi e^2 \lambda^2(0)}$$

$$T_{\theta}^{\max} \simeq V_0$$

Fluctuations

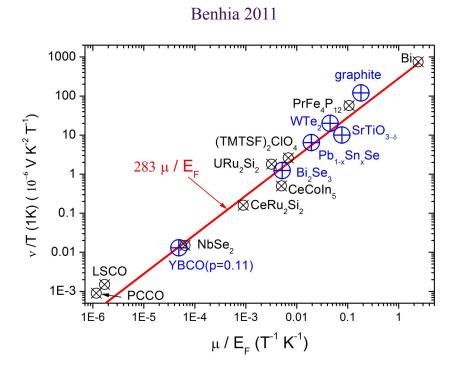
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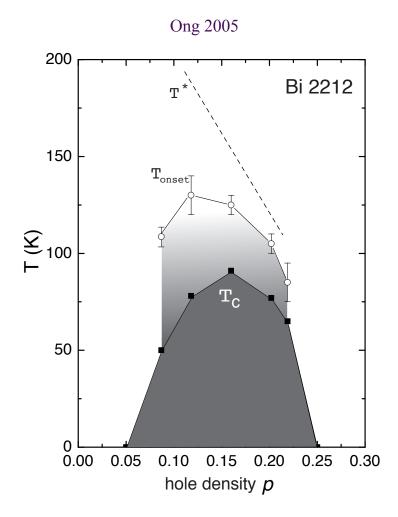


$$V_0 = \frac{(\hbar c)^2 a}{16\pi e^2 \lambda^2(0)}$$

$$T_{\theta}^{\max} \simeq V_0$$

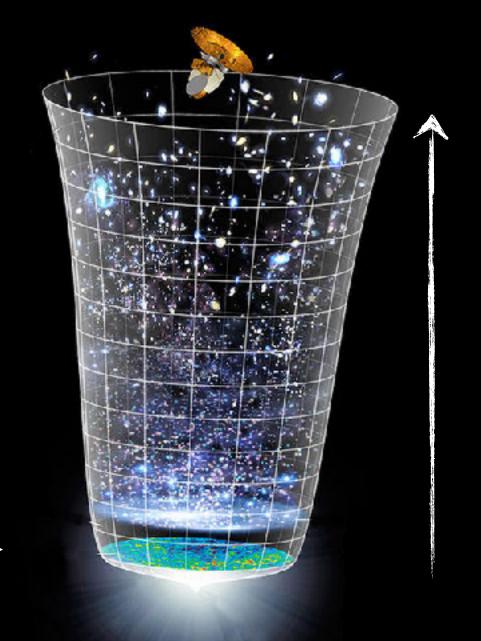


Fluctuations of phase, amplitude, and others...





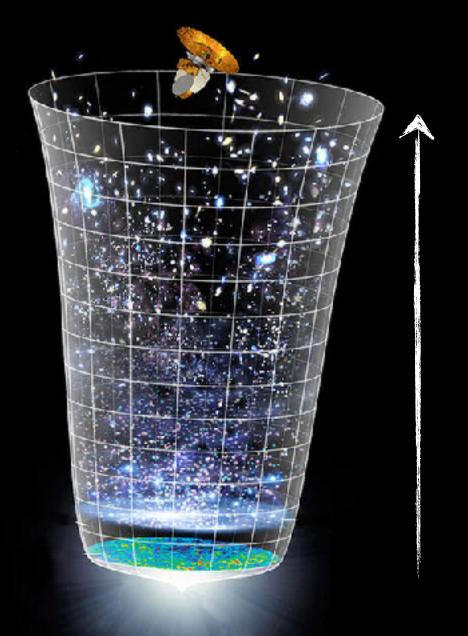




Condensate

Phase fluctuations

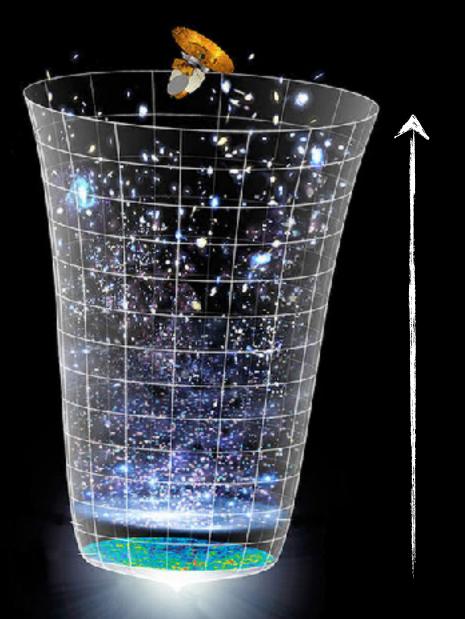
Condensate



Amplitude
Fluctuations ...>

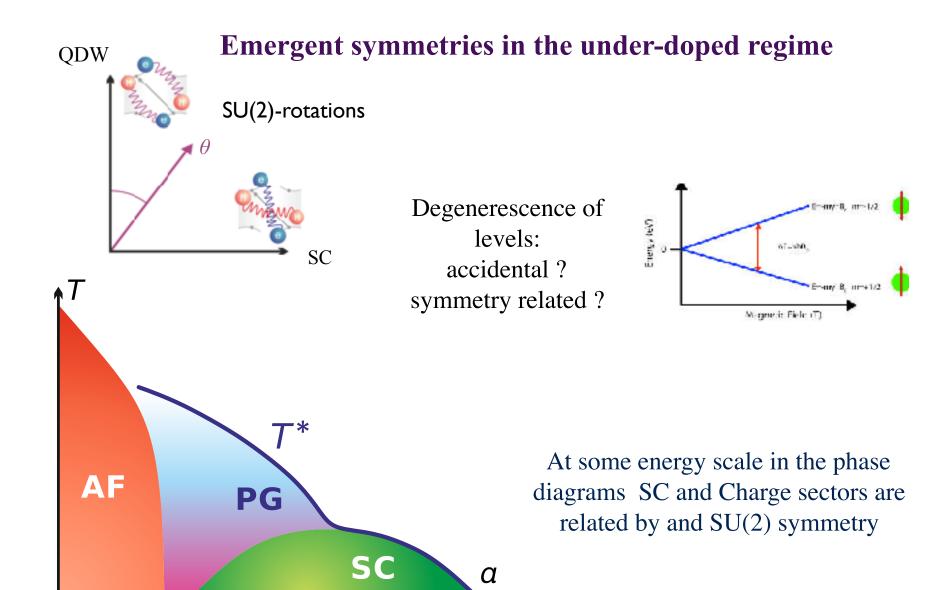
Phase fluctuations ...>

Condensate



« Schyzofrenic » SU(2) flucutations

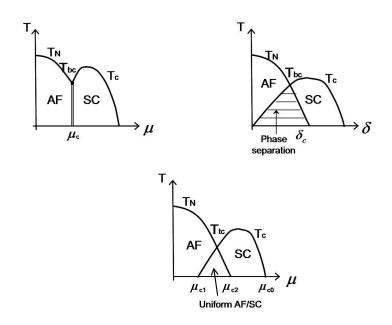
Emergent SU(2) flucutations



Sachdev et al (2013) Efetov, Meier, CP (2013)

SO(5)-group AF AF Type 1 SC Type 1.5 g or μ Uniform AF/SC

Fine-tuning condition?



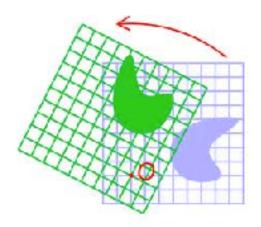
Demler, Zhang, Hanke (2005)

SU(2) symmetry related to the SU(2) symmetry of the superexchange hamiltonian and gauge SU(2) symmetry

$$U_{ij} = \begin{pmatrix} -\chi_{ij}^* & \Delta_{ij} \\ \Delta_{ij}^* & \chi_{ij} \end{pmatrix}$$

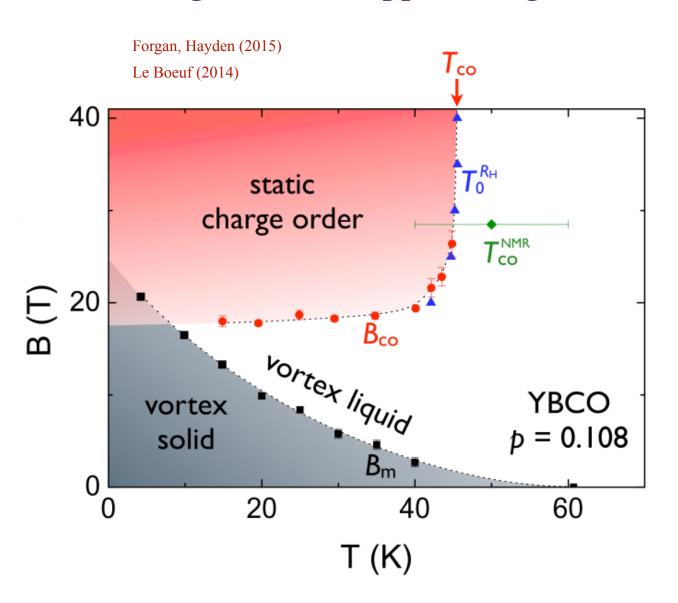
$$\chi_{ij}\delta_{\alpha\beta} = 2\langle f_{i\alpha}^{\dagger}f_{j\beta}\rangle, \quad \chi_{ij} = \chi_{ji}^{*},$$

$$\Delta_{ij}\epsilon_{\alpha\beta} = 2\langle f_{i\alpha}f_{j\beta}\rangle, \quad \Delta_{ij} = \Delta_{ji}.$$



Sachdev et al (2013) Kotliar and Liu (1988) Lee, Wen, Nagaosa, RMP (2006)

Phase diagram under applied magnetic field



The concept of SU(2) symmetry

C.N. Yang & S-C. Zhang (1989)

Pseudo-Spins

$$\eta^{+} = \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}+\mathbf{Q}\downarrow}^{\dagger}$$

$$\eta_{z} = \sum_{\mathbf{k}} \left(c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{k}\uparrow} + c_{\mathbf{k}+\mathbf{Q}\downarrow}^{\dagger} c_{\mathbf{k}+\mathbf{Q}\downarrow} - 1 \right)$$

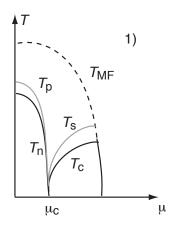
l=1 representation

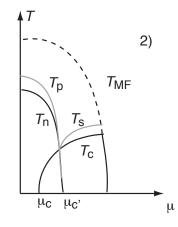
$$\Delta_{1} = -\frac{1}{\sqrt{2}} \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger},$$

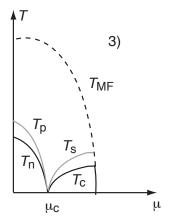
$$\Delta_{0} = \frac{1}{2} \sum_{\mathbf{k},\sigma} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}+\mathbf{Q}\sigma},$$

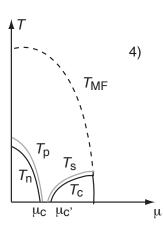
$$\Delta_{-1} = -\Delta_{1}^{\dagger},$$

$$\left[\eta^{\pm}, \Delta_{m}\right] = \sqrt{l\left(l+1\right) - m\left(m\pm1\right)} \Delta_{m\pm1},$$
$$\left[\eta_{z}, \Delta_{m}\right] = m\Delta_{m}.$$



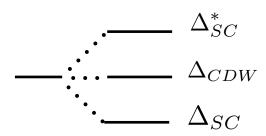


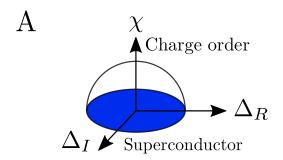




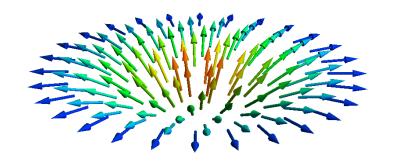
Topology and local structures

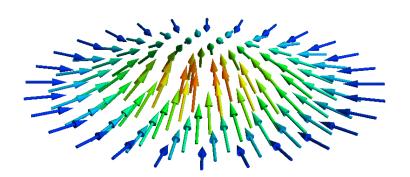
0(3) non linear σ -model



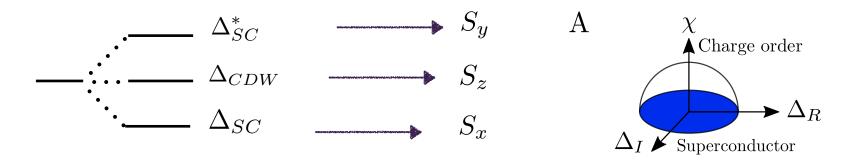


Topological structure: Skyrmions in the pseudo spin space

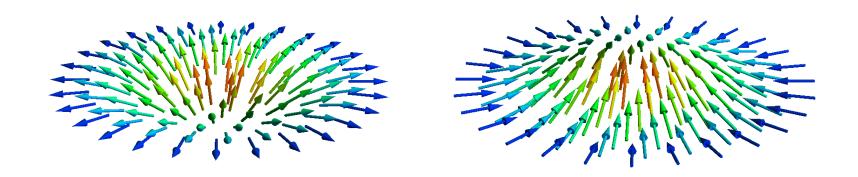




0(3) non linear σ -model

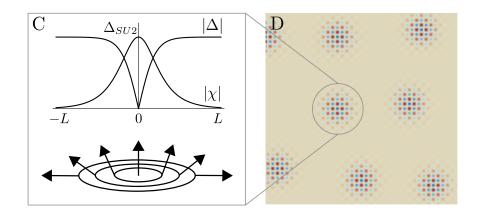


Topological structure: Skyrmions in the pseudo spin space

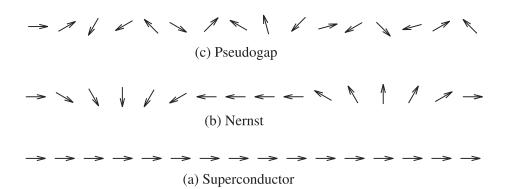


Homotopy classes

$$\Delta_{-,R}^2 + \Delta_{-,I}^2 + \Delta_{+,R}^2 + \Delta_{+,I}^2 = 1$$
$$\pi_2(S_3) = 0$$

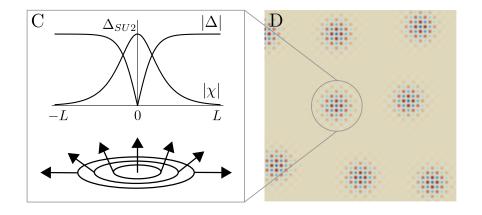


Vortex structure Phase diagram



Homotopy classes

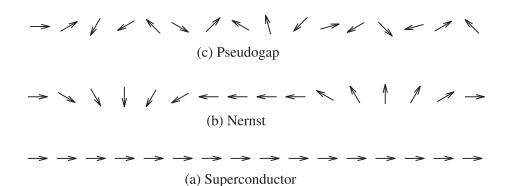
$$\Delta_{-,R}^2 + \Delta_{-,I}^2 + \Delta_{+,R}^2 + \Delta_{+,I}^2 = 1$$
$$\pi_2(S_3) = 0$$

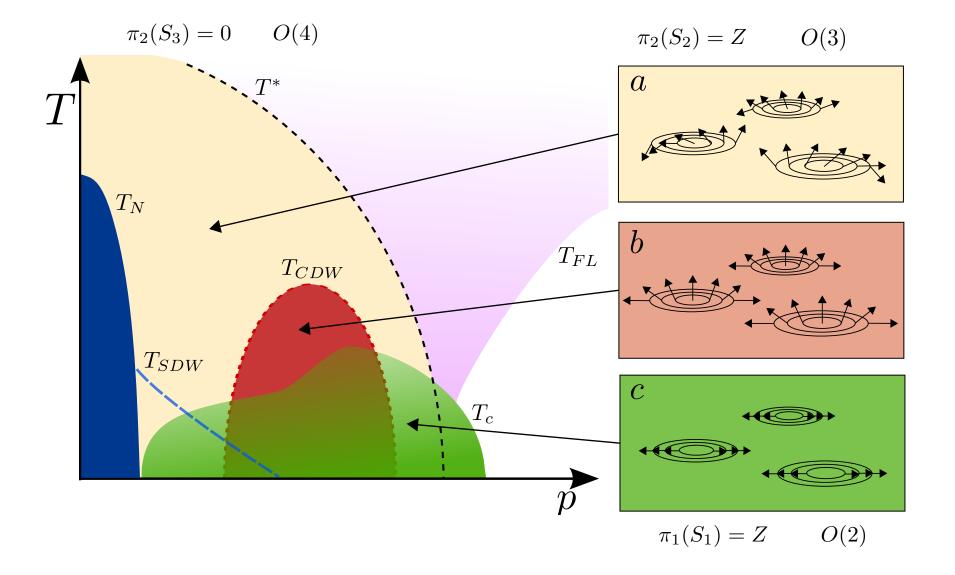


Phase of the CDW is frozen to an integer value

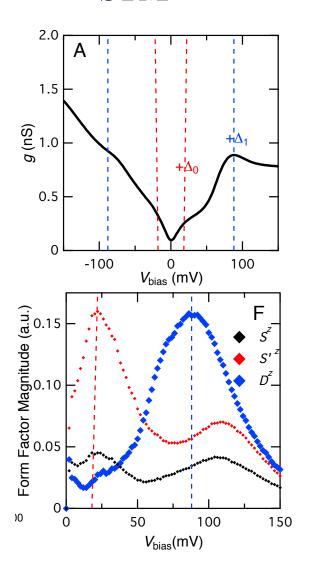
$$\Delta_{-}^{2} + \Delta_{+,R}^{2} + \Delta_{+,I}^{2} = 1$$
$$\pi_{2}(S_{2}) = Z$$

Vortex structure Phase diagram

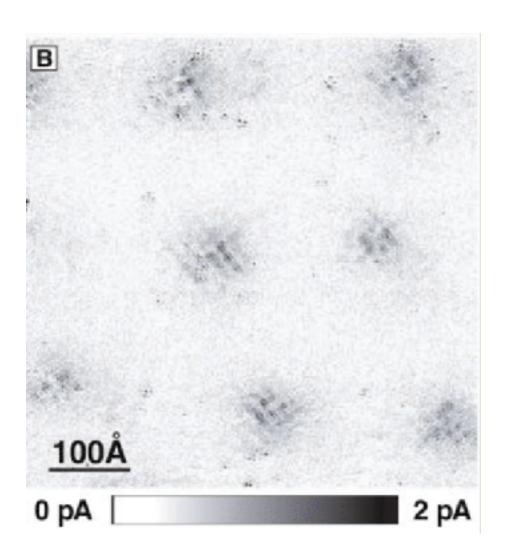




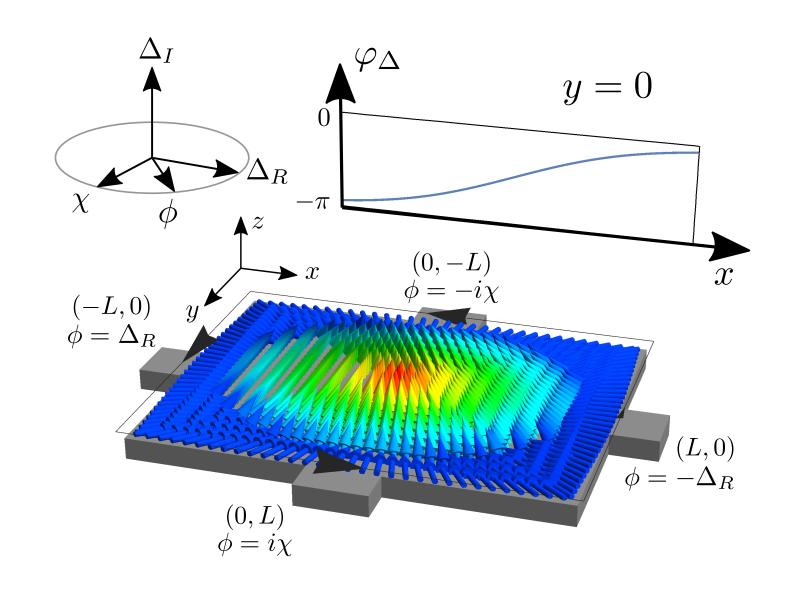
STM



Hamidian et al. (2015)



Hoffmann (2002)

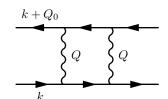


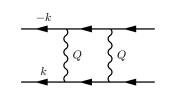
Key ingredient

Short range AF correlations: J strong enough

$$H = \sum_{i,j,\sigma} c_{i,\sigma}^{\dagger} t_{ij} c_{j,\sigma} + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\mathbf{S}_i = \sum_{\alpha,\beta} c_{i,\alpha}^{\dagger} \sigma_{\alpha\beta} c_{i\beta}$$



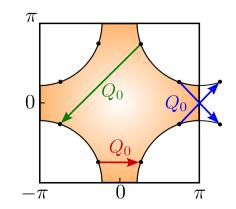


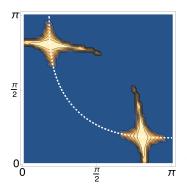
b)

Generalizes the 8 hot spots model close to AF QCP

Metlitski, Sachdev et al (2011) Efetov, Meier, CP (2013)

Degeneracy of wave vectors 0, (q,q), (q,0), (0,q) and SC at the hot spots

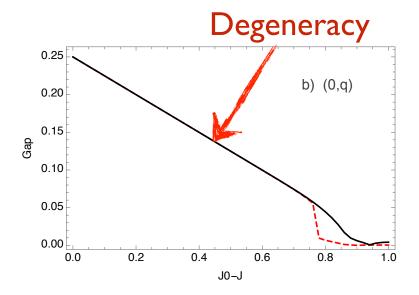




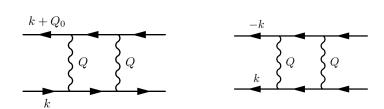
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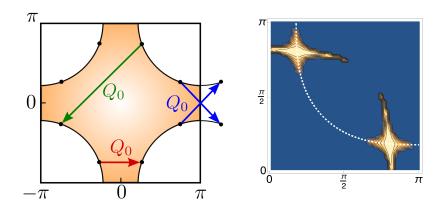


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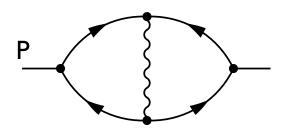
AF decreases as a function of doping $x \simeq (J_0 - J)^{\alpha}$



Order by disorder: SU(2) fluctuations lift the degeneracy

Large nematic response

40 a) 20 рх and
Response along the
crystal axes



Loop currents
OK with symmetries

Order by disorder: SU(2) fluctuations lift the degeneracy

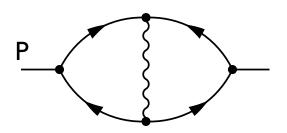
Qx 40 a) 20 рх

Large nematic response

and

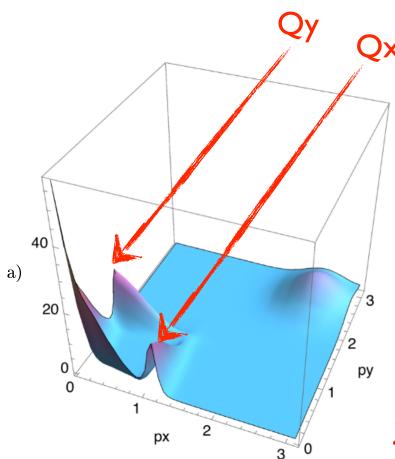
Response along the

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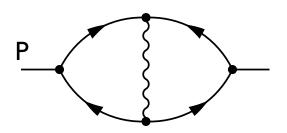
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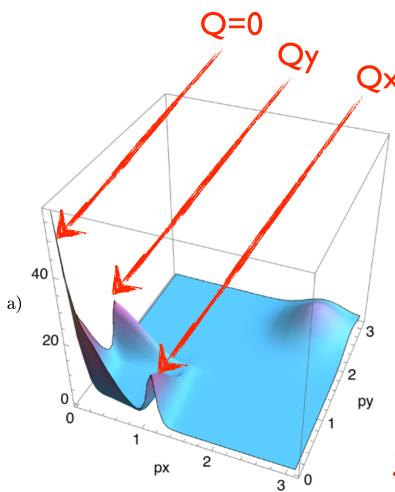
Large nematic response and Response along the

crystal axes



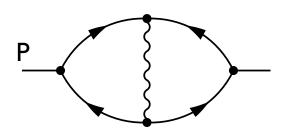
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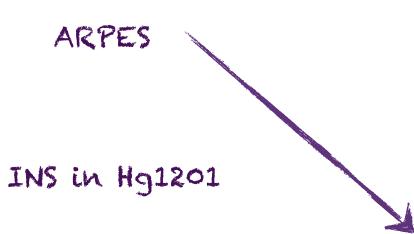
Experiments

ARPES

INS in Hg1201

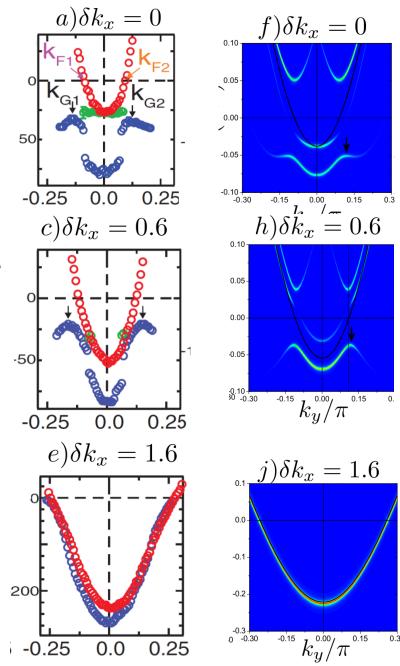
Anomalous transport

Raman: Alg resonance



Anomalous transport

Raman: A19 resonance

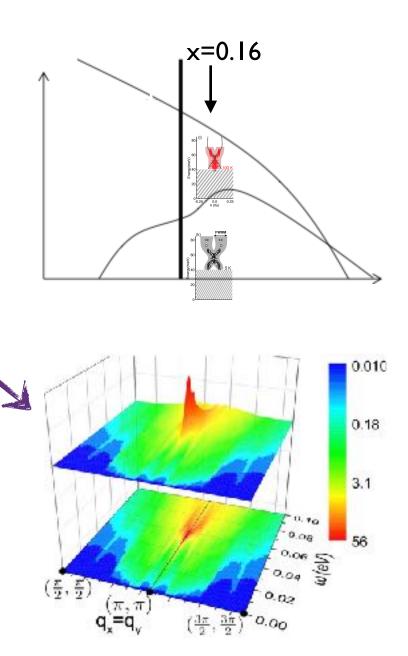




INS in Hg1201

Anomalous transport

Raman: Alg resonance

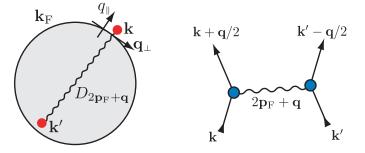


ARPES

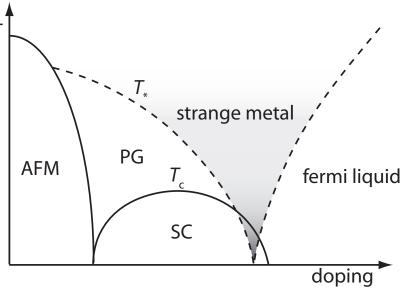
$$\rho \sim T/\log T$$

$$\Sigma \sim i\epsilon_n/\log|\epsilon_n|$$

INS in Hg1201



Anomalous transport



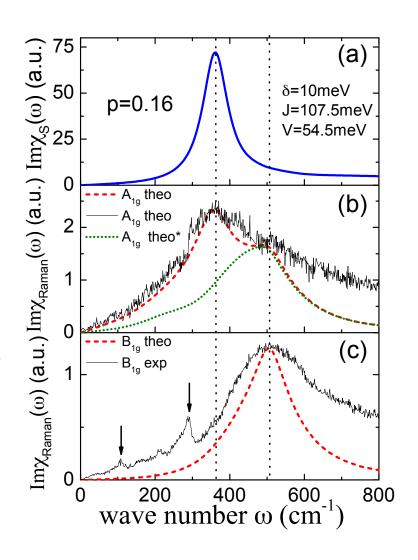
Raman: Alg resonance

ARPES

INS in H91201

Anomalous transport

Raman: A1g resonance



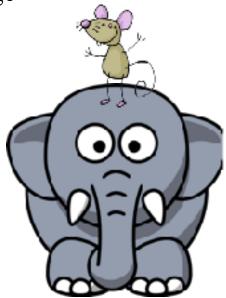
Conclusions

- Charge orders are a key players in cuprate physics: natural competitor of superconductivity.
- Quasi- degeneracy between charge and SC levels is treated within SU(2) rotations and non linear σ -model
 - Local structures, or skyrmions, are a signature of the model
 - Experiments looked at : ARPES, transport (strange metal phase), Raman spectroscopies, Hall effect (evolution of carriers # with doping)...

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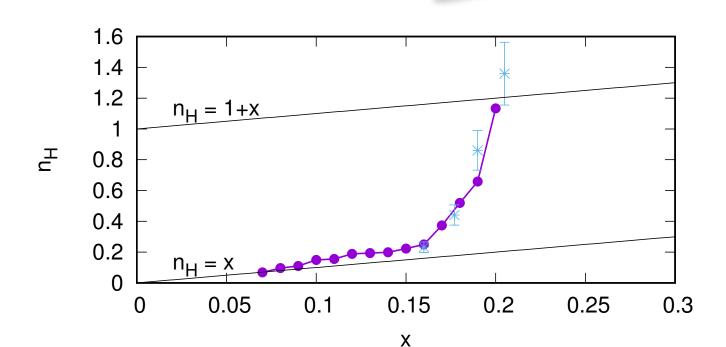
Hall resistivity

$$\sigma_{xx} = -\frac{2\pi e^2}{VN} \sum_{k} v_x(k)^2 \int d\omega \frac{\partial f(\omega)}{\partial \omega} A(k, \omega)^2$$

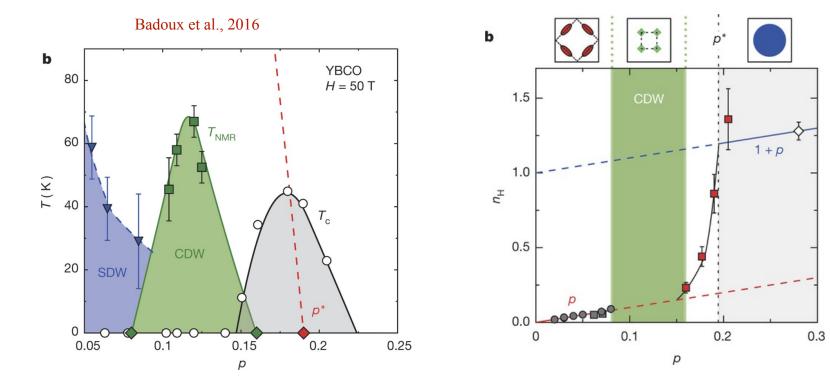
$$R_H = \frac{\sigma_{xy}}{(\sigma_{xx})^2}, n_H = \frac{V}{eR_H}$$

$$\sigma_{xy} = -\frac{4\pi^2 e^3}{3VN} \sum_{k} v_x(k) \left(v_x(k) \frac{\partial v_y(k)}{\partial k_y} - \frac{v_y(k) \left(\partial v_y(k) \right)}{\partial k_x} \right) \int d\omega \frac{\partial f(\omega)}{\partial \omega} A(k, \omega)^3$$

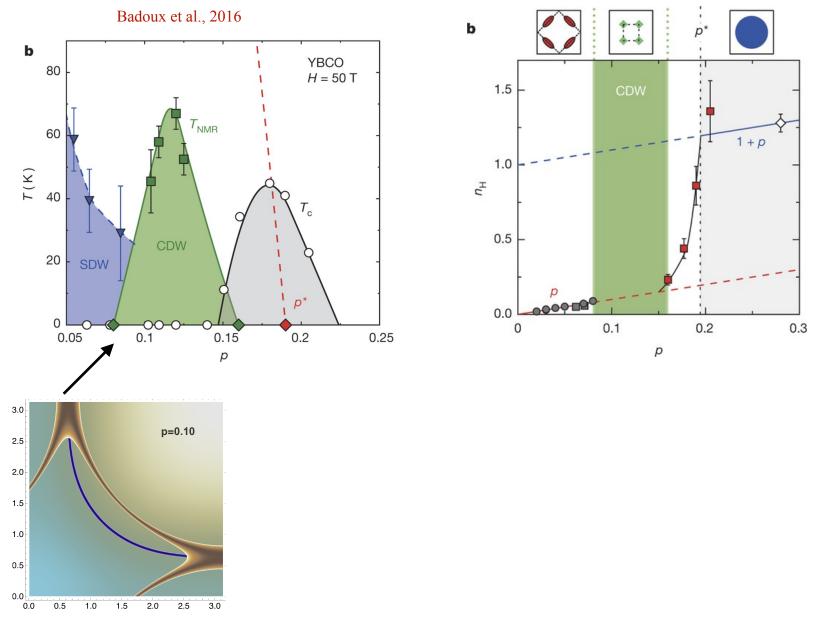
$$G(k, \omega) = \frac{1}{\omega - \xi_k - B \frac{M(k)^2}{\omega + \xi_k}}$$



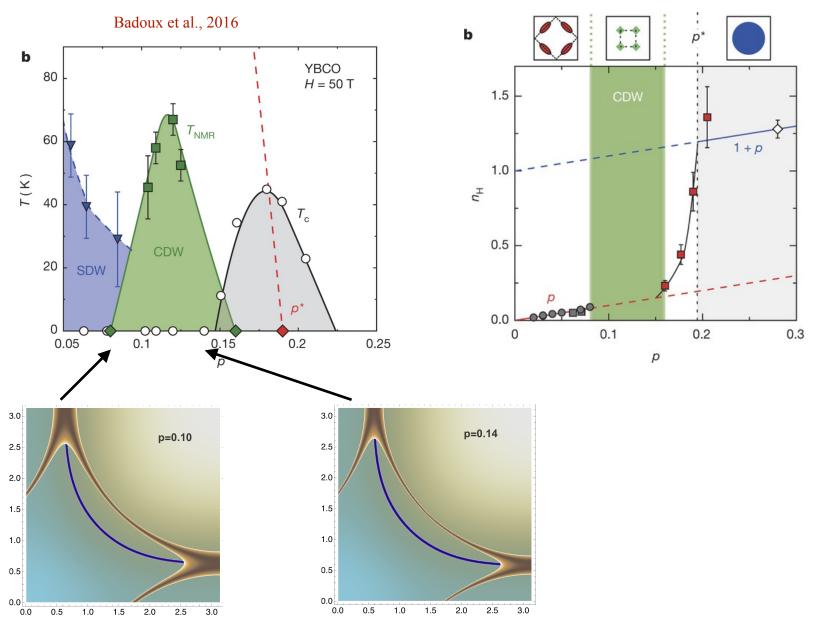
Badoux et al 2016



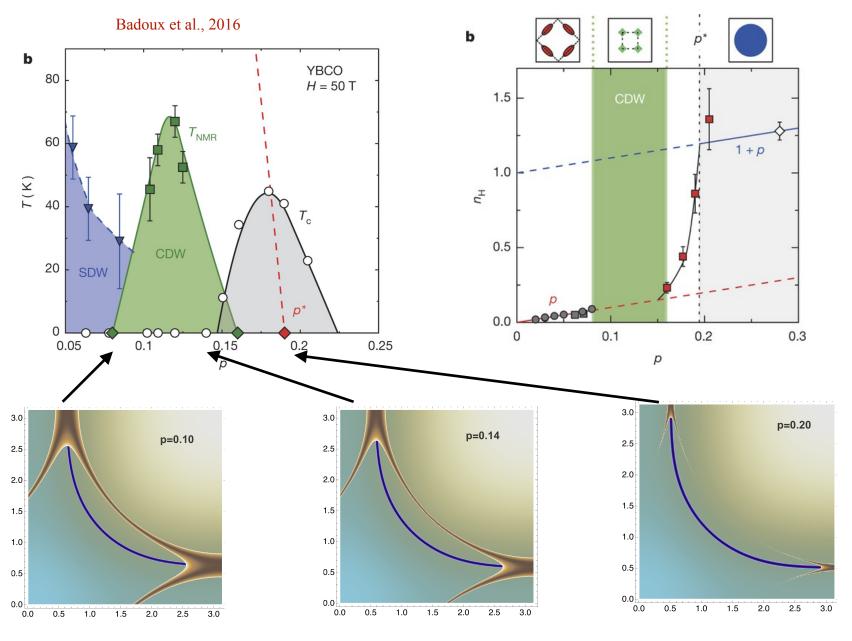
AF correlations -> Fermi Arcs -> p



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 $\rho \sim T/\log T$ $\Sigma \sim i\epsilon_n/\log|\epsilon_n|$

