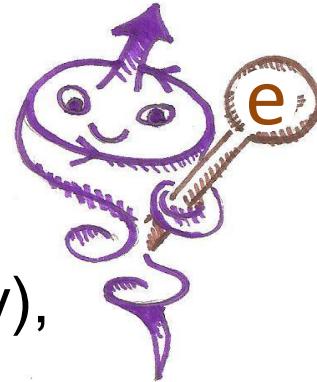


# Bilayer Graphene as a Playground for Exotic Magnetic Phases

↳  
bases



## Collaborators

Ganpathy Murthy (U Kentucky),  
Herb Fertig (Indiana U)

Useful discussions:  
Jun Zhu, Jing Li,  
Andrea Young,  
Mike Zaletel



# The Quantum Hall Effect in graphene

$$E_n = \frac{\sqrt{2}h\nu_F}{l_B} \sqrt{|n|}$$

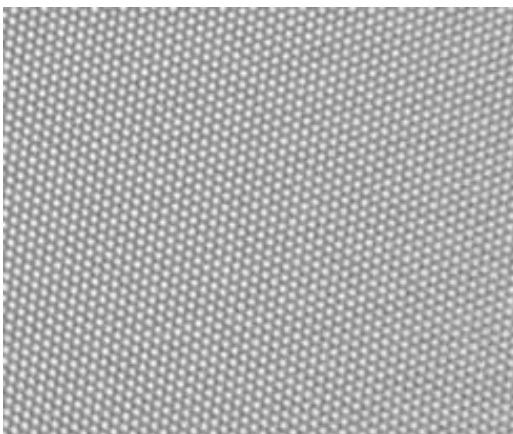
n=2

n=1

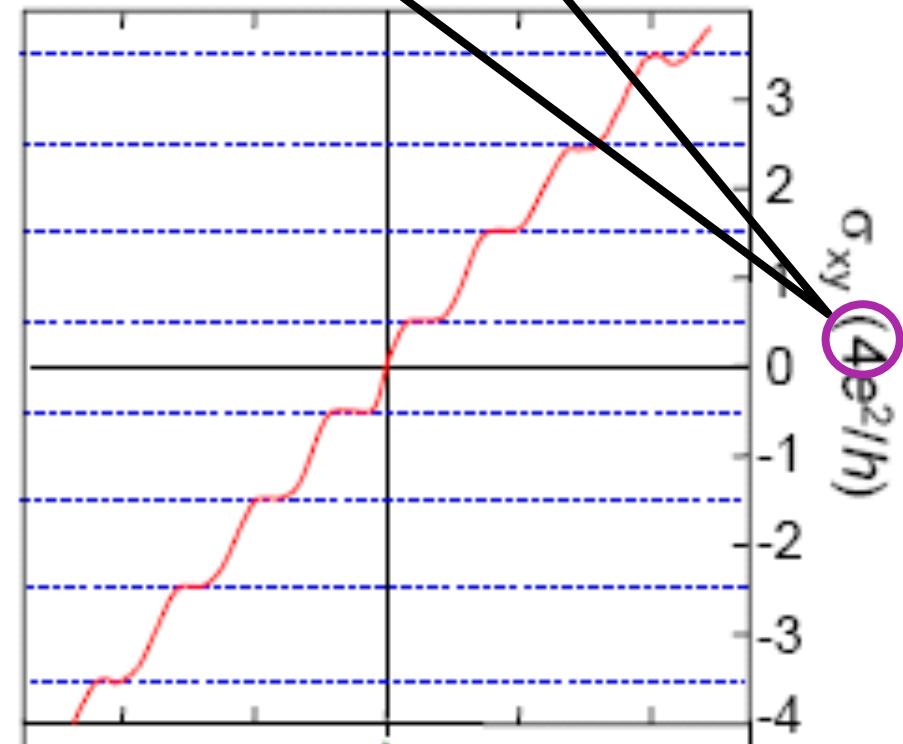
n=0

n=-1

n=-2

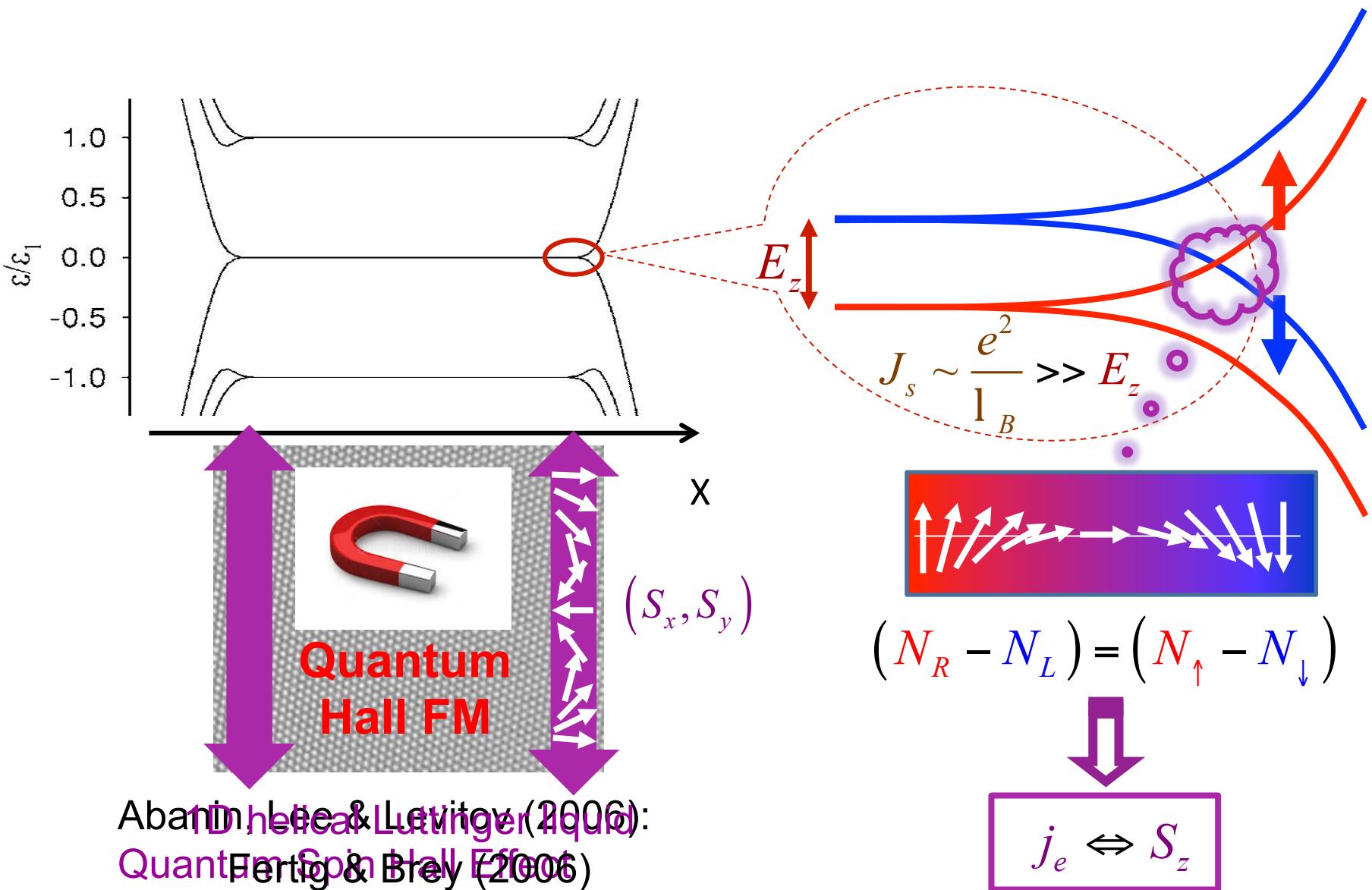


$\uparrow\kappa'$   $\uparrow\kappa$   $\downarrow\kappa'$   $\downarrow\kappa$  SU(4)



Novoselov, Geim et al., Nature 438, 197 (05)

# The $\nu = 0$ QH State in Graphene

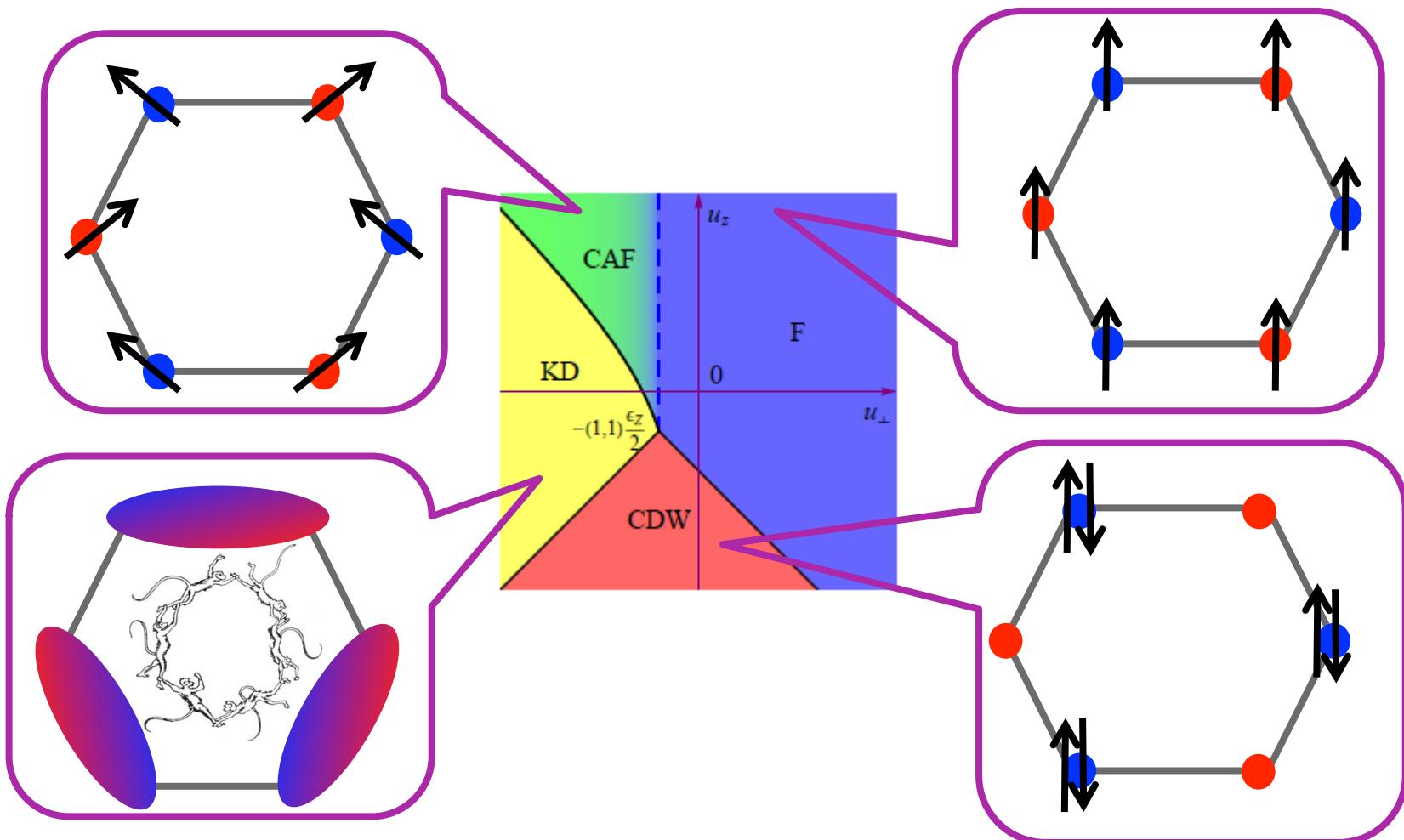


# Experimentally: $\nu = 0$ an INSULATOR!

Jung & MacDonald, Nandkishore & Levitov (2009):

Variety of SU(4) symmetry-broken ground states in the bulk

Kharitonov (2012): Coulomb + lattice-scale interactions

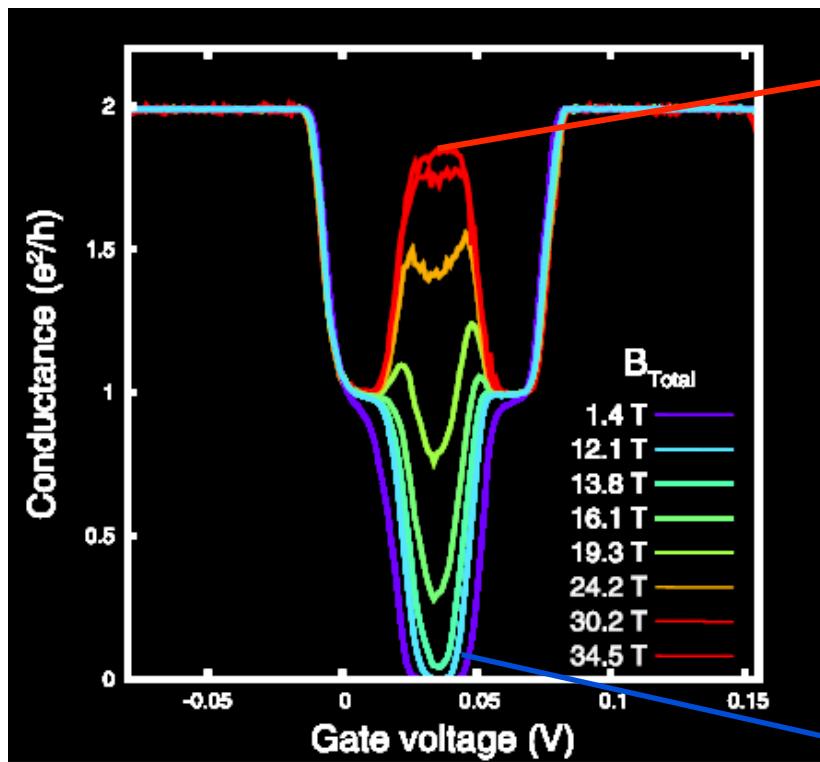


# Experimentally: $\nu = 0$ an INSULATOR!

Jung & MacDonald, Nandkishore & Levitov (2009):

Variety of SU(4) symmetry-broken ground states in the bulk

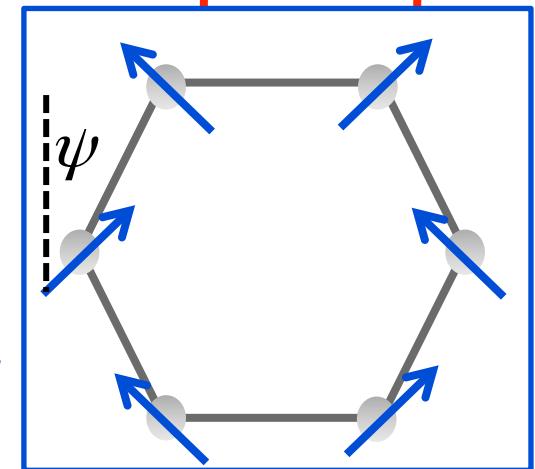
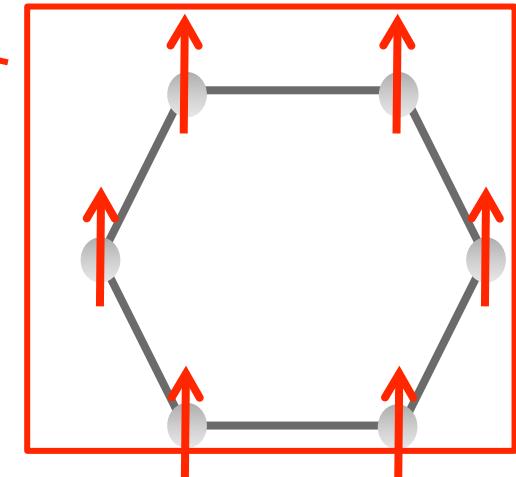
Kharitonov (2012): Canted Anti-Ferromagnet  $\rightarrow$  Ferromagnet



$E_z$

$$E_z^c = 2g_{xy}$$

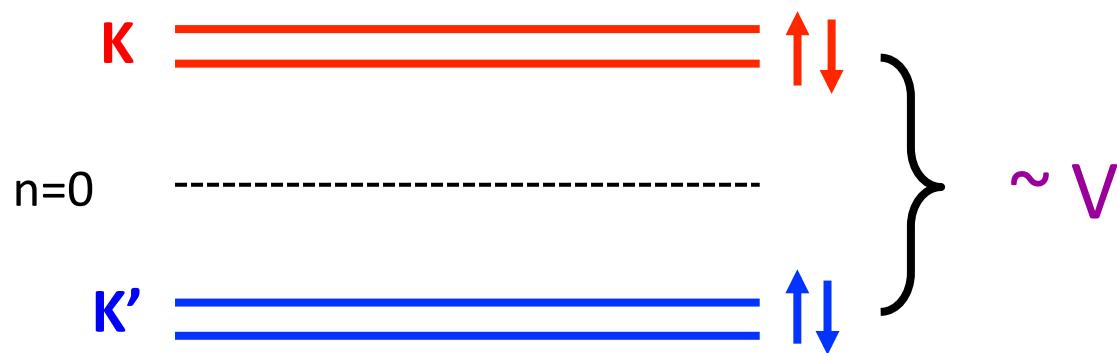
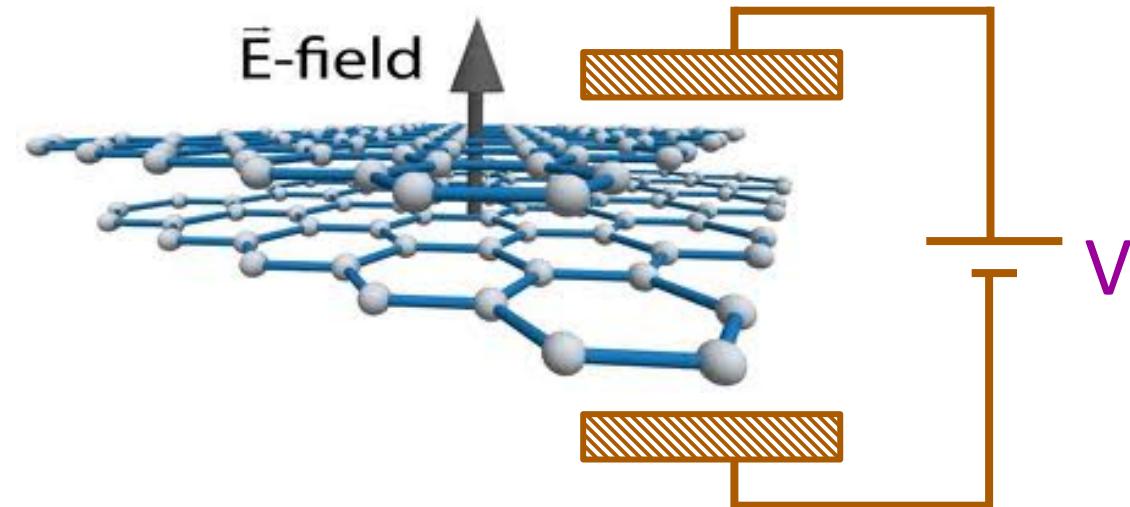
$$\cos \psi = \frac{E_z}{2g_{xy}}$$



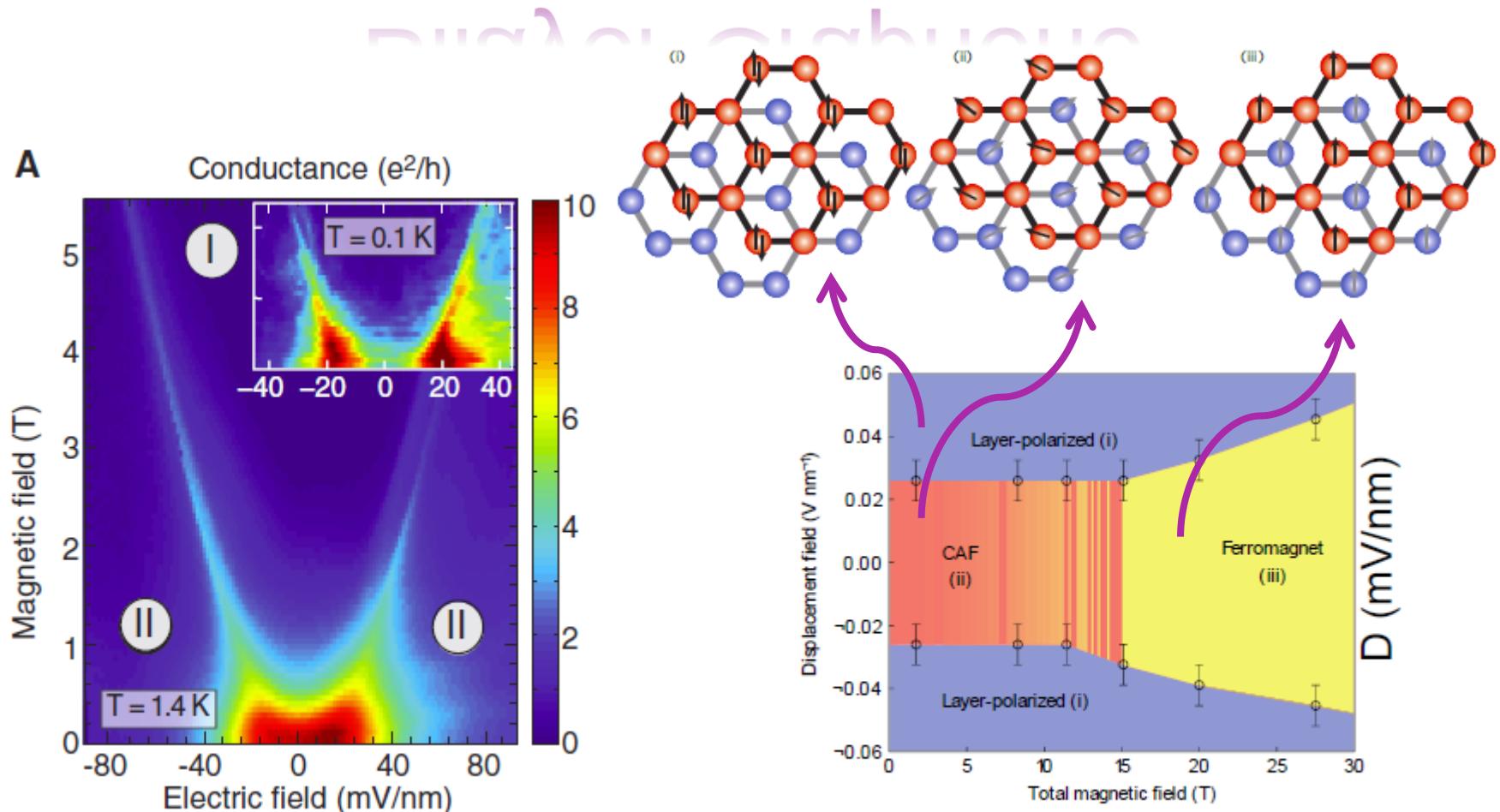
A. F. Young *et al.*, *Nature* 505 (2014):  
tilted magnetic field

# Bilayer Graphene

$B_{eff} \rightarrow$  Perpendicular electric field



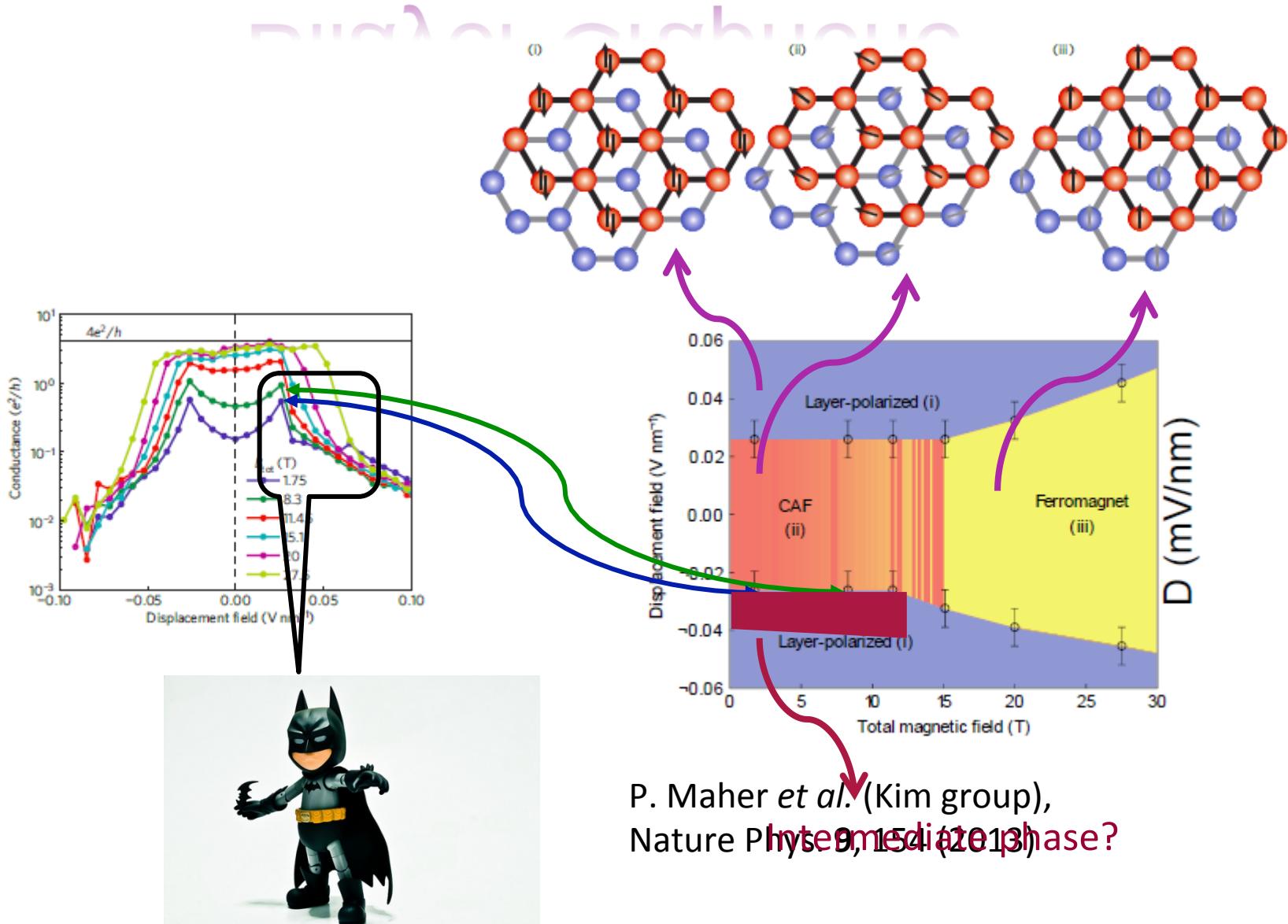
# Bilayer Graphene



R. T. Weitz *et al.* (Yacoby group),  
Science **330**, 812 (2010)

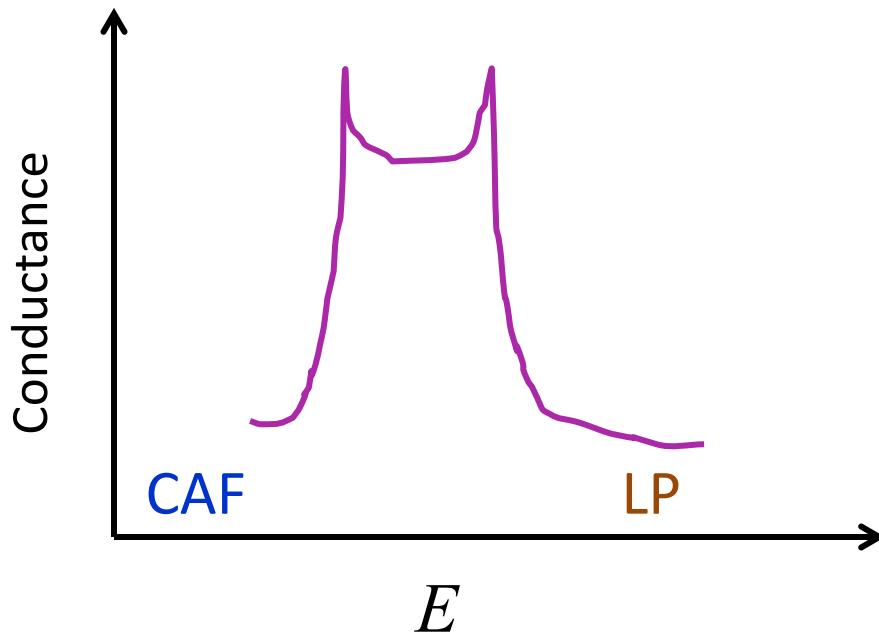
P. Maher *et al.* (Kim group),  
Nature Phys. **9**, 154 (2013)

# Bilayer Graphene

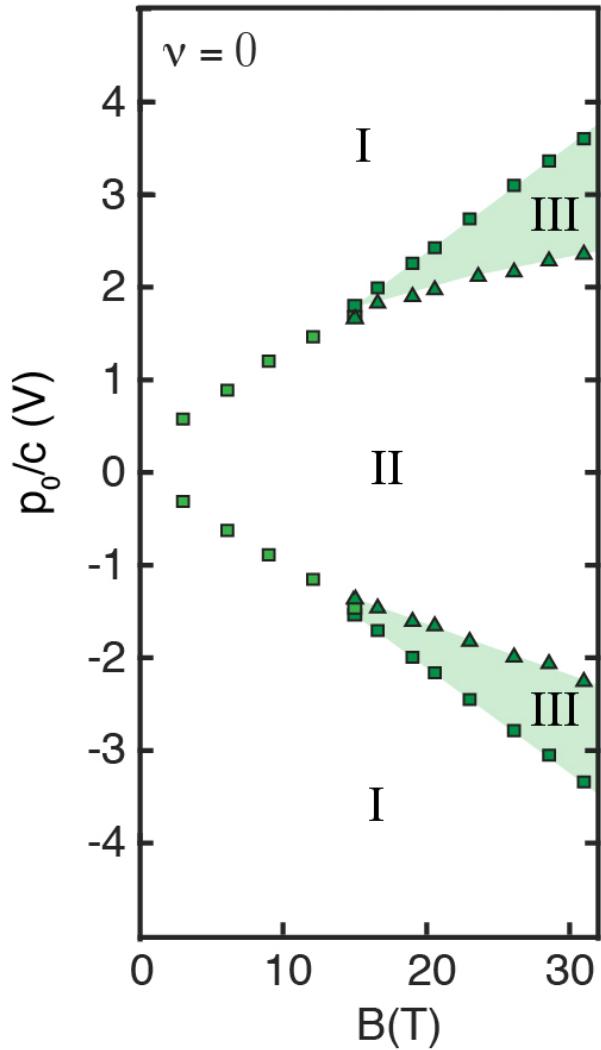


# Bilayer Graphene

Jun Zhu's lab, Penn State (preliminary data,  
in part of samples):



# Bilayer Graphene



Compressibility and  
relative capacitance  
measurements:

B. M. Hunt *et al.* (Andrea Young's group  
at UCSB), arXiv:1607.06461

# Our Theory

$$H = H_0 + H_{\text{int}}$$

$$H_0 = \sum_X H_X - g\mu_B B_T \sigma_X^z$$

$$H_X = \begin{pmatrix} E + \Delta a^\dagger a & -\omega_c (a^2 + \lambda a^\dagger) \\ -\omega_c ((a^\dagger)^2 + \lambda a) & -E + \Delta a a^\dagger \end{pmatrix}$$

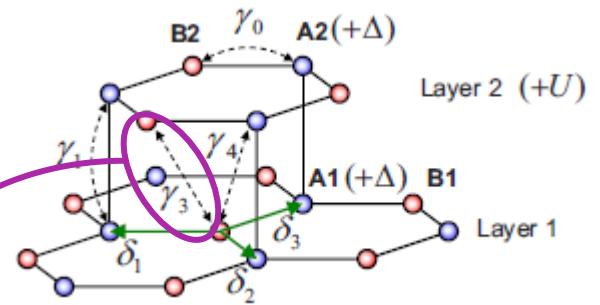
$$H_{\text{int}} = \sum_{X,X'} g_0 n_X n_{X'} + g_{xy} (\tau_X^x \tau_{X'}^x + \tau_X^y \tau_{X'}^y) + g_z \tau_X^z \tau_{X'}^z + g_{nz} O_X^z O_{X'}^z$$

$$\omega_c, \Delta \sim B_\perp$$

$$g_\alpha \sim \frac{e^2 a_0}{l^2} \sim B_\perp$$

$$\lambda \sim \frac{1}{\sqrt{B_\perp}}$$

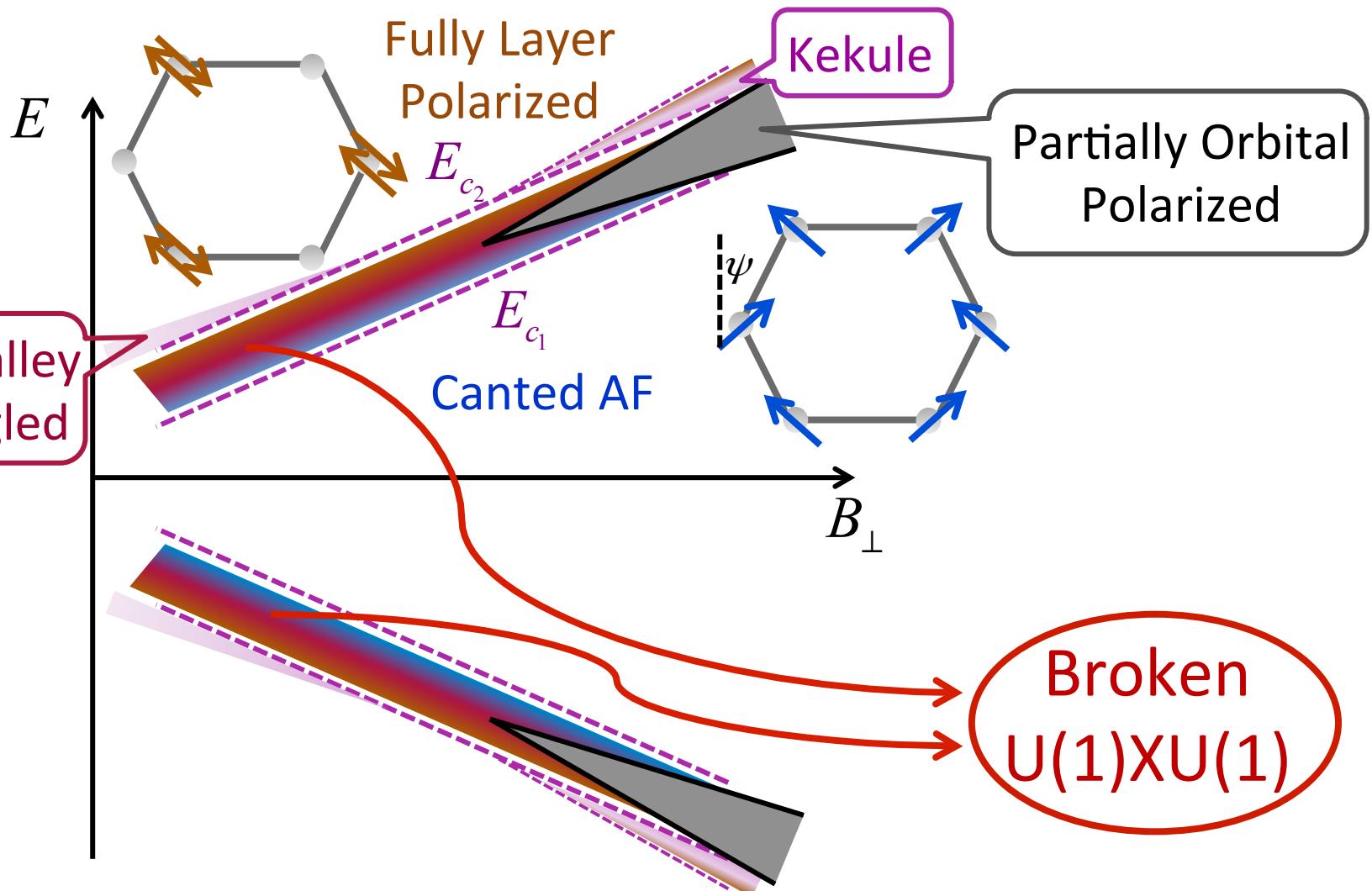
$\sim 1$  for  $B_\perp$   
A few Tesla



$$a = \frac{1}{\sqrt{2}} \left( 1 \frac{\partial}{\partial x} + \frac{(x - X)}{l} \right)$$

$$l = \sqrt{\frac{hc}{eB_\perp}} \quad X = l^2 k_y$$

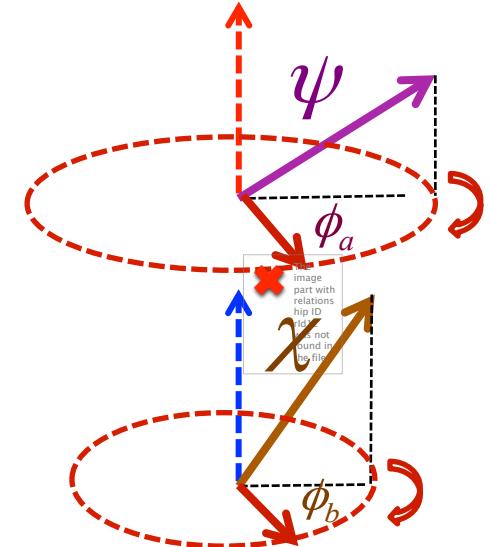
# Our Theory: Possible phase diagram



# Spin-Valley entangled

$$|a\rangle = \cos\left[\frac{\psi}{2}\right]|\uparrow, K\rangle - e^{i\phi_a} \sin\left[\frac{\psi}{2}\right]|\downarrow, K'\rangle$$

$$|b\rangle = -\cos\left[\frac{\chi}{2}\right]|\uparrow, K'\rangle + e^{i\phi_b} \sin\left[\frac{\chi}{2}\right]|\downarrow, K\rangle$$



# “Broken U(1)XU(1)”:

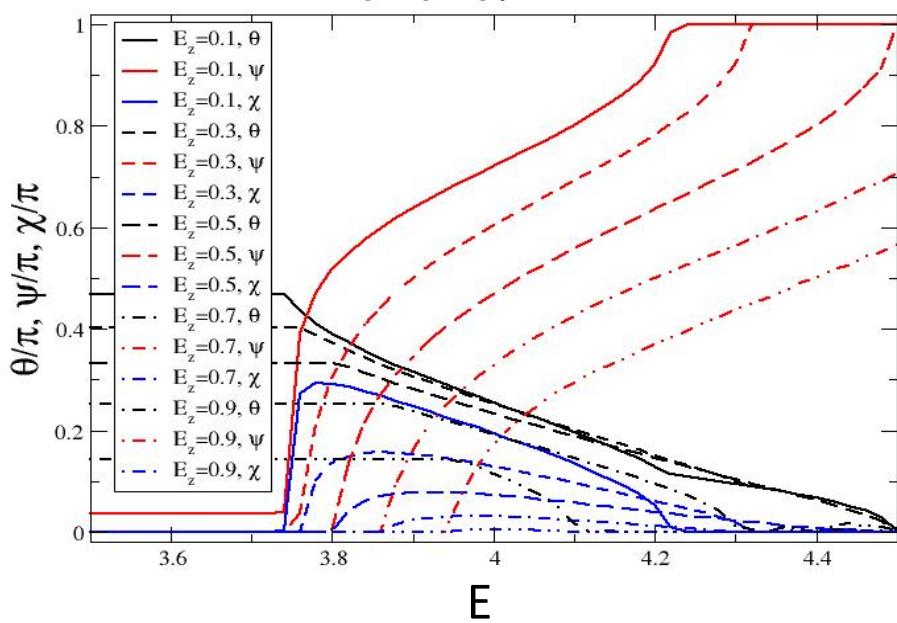
$$\left| a \right\rangle = \cos \left[ \frac{\psi}{2} \right] \left| \uparrow, K \right\rangle - e^{i\phi_a} \sin \left[ \frac{\psi}{2} \right] \left| \downarrow, K' \right\rangle$$

$$\left| b \right\rangle = -\cos \left[ \frac{\chi}{2} \right] \left| \uparrow, K' \right\rangle + e^{i\phi_b} \sin \left[ \frac{\chi}{2} \right] \left| \downarrow, K \right\rangle$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \cos \left[ \frac{\theta}{2} \right] \left| a \right\rangle + e^{i\phi_c} \sin \left[ \frac{\theta}{2} \right] \left| b \right\rangle$$

Angles in the 3-angle ansatz, various  $E_z$

$g_0=1, g_{z}=2, g_{xy}=-0.5, r^2=0.5$



Two independent  
U(1)-angles:

$$\phi_a, \phi_b, \phi_c \Rightarrow \varphi, \eta$$

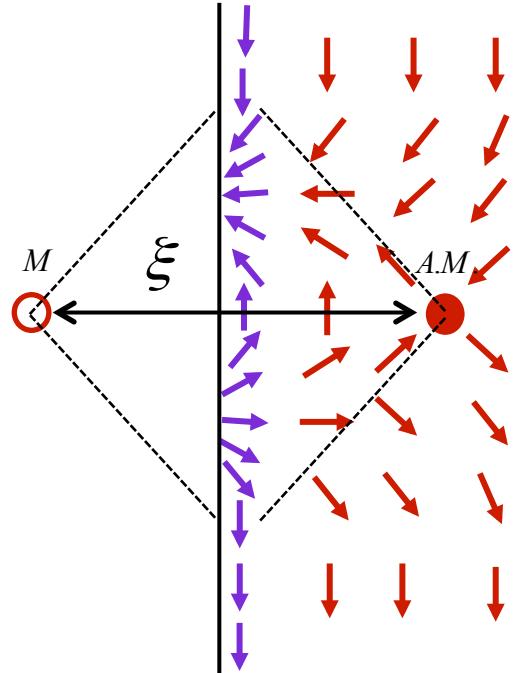
# Collective Charged Mode on the Edge

(Murthy, ES, Fertig 2014; Tikhonov, ES, Murthy, Fertig 2016)

Meron-Antimeron  
pair

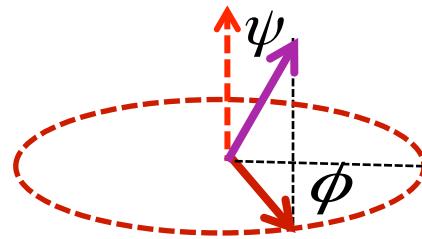
$$Q = 1e \Leftrightarrow \text{Pontryagin number}$$

$$\Delta_{\text{edge}} = E_M + E_{A.M.} \sim \rho_b \ln \rho_b$$



Bulk pseudospin stiffness:

$$\rho_b \propto \sin^2 \psi$$



# Summary

## SYNTHETIC METAMATERIALS

- The  $\nu = 0$  QH state in bilayer graphene has a rich phase diagram, dictated by interplay of  $H_{\text{int}}$ ,  $E$ ,  $B$  and the trigonal warping parameter  $\lambda$ .
- Possible spin-valley entangled intermediate phases between CAF and FLP: broken  $U(1) \times U(1)$  symmetry
- Low-energy charged excitations: edge spin-textures, softening near the transitions

