Magneto-transport in Weyl metals.

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classical magnetoconductance of conventional metals and semiconductors

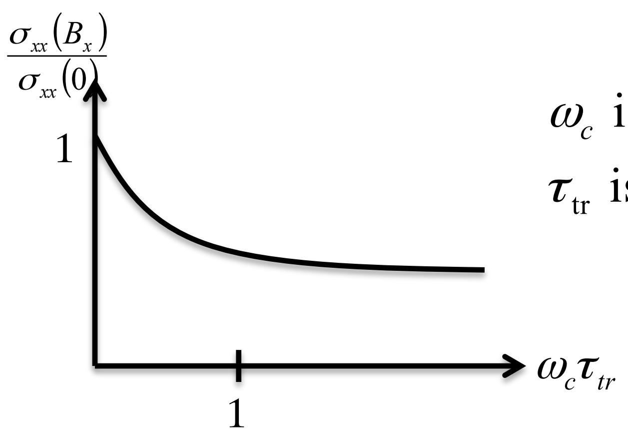
quasiclassical magnetoconductance is always negative

$$\mathbf{E} \perp \mathbf{B}$$

$$\sigma_{xx}(\mathbf{B}) = \frac{\sigma(0)}{1 + (\omega_c \tau_{tr})^2}$$

in the relaxation time approximation the longitudinal magnetoconductance is absent. Beyond this approximation it is negative and it saturates at $\omega_c \tau_{tr} > 1$

$$\mathbf{E} \parallel \mathbf{B}$$



ω_c is the cyclotron frequency

τ_{tr} is the transport relaxation time

a generalization of the quasi-classical equations of motion for non centrosymmetric or not time reversal conductors. (Luttinger; Sundaram and Niu)

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} + \left[\mathbf{\Omega}_{\mathbf{p}} \times \frac{d\mathbf{p}}{dt}\right]; \qquad \frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}\left[\frac{d\mathbf{r}}{dt} \times \mathbf{B}\right]$$

$$\frac{d\mathbf{r}}{dt} = \left(1 + \frac{e}{c}\mathbf{B}\cdot\mathbf{\Omega}_{\mathbf{p}}\right)^{-1}\left(\mathbf{v} + e\mathbf{E}\times\mathbf{\Omega}_{\mathbf{p}} + \frac{e}{c}\left(\mathbf{\Omega}_{\mathbf{p}}\cdot\mathbf{v}\right)\mathbf{B}\right)$$

$$\frac{d\mathbf{p}}{dt} = \left(1 + \frac{e}{c}\mathbf{B}\cdot\mathbf{\Omega}_{\mathbf{p}}\right)^{-1}\left(e\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B} + \frac{e^2}{c}\left(\mathbf{E}\cdot\mathbf{B}\right)\mathbf{\Omega}_{\mathbf{p}}\right)$$

$\mathbf{\Omega}_{\mathbf{p}} = \nabla_{\mathbf{p}}\mathbf{A}_{\mathbf{p}}; \qquad \mathbf{A}_{\mathbf{p}} = i\left\langle u_{\mathbf{p}} \,|\, \nabla_{\mathbf{p}} u_{\mathbf{p}}\right\rangle$

$\mathbf{\Omega}_{\mathbf{p}} = -\mathbf{\Omega}_{-\mathbf{p}}$ in timereversal invariant systems

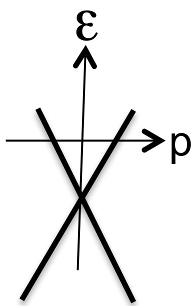
$\mathbf{\Omega}_{\mathbf{p}} = \mathbf{\Omega}_{-\mathbf{p}}$ in centosymmetric systems

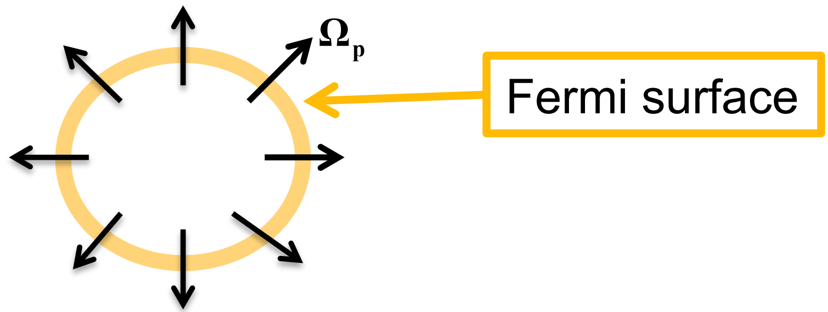
$\mathbf{\Omega}_{\mathbf{p}} = 0$ in centro – and time reversal symmetric systems

$u_{\mathbf{p}}(\mathbf{r})$ are Bloch wave functions

Berry curvature is divergence-free except at isolated points in the P-space, which are associated with band degeneracies.

$$\mathbf{\Omega}_{\mathbf{p}} = curl\,\mathbf{A}_{\mathbf{p}};$$





$$k^{(i)} = \frac{1}{2\pi\hbar}\int \mathbf{\Omega}_{\mathbf{p}}^{(i)} d\mathbf{S} = 0, \pm 1, \ldots$$

$$r_s = \frac{e^2}{\hbar v} \approx \frac{1}{10} << 1$$

at low magnetic field interactional corrections to the quasiclassical equations are small

Quasiclassic description of electron transport in conventional metals and semiconductors

$$\frac{\partial f_{\mathbf{p}}(\mathbf{r},t)}{\partial t}+\dot{\mathbf{r}}\frac{\partial f_{\mathbf{p}}}{\partial \mathbf{r}}+\dot{\mathbf{p}}\frac{\partial f_{\mathbf{p}}}{\partial \mathbf{p}}=I_{st}\left\{f_{\mathbf{p}}\right\}$$

I_{st} is the scattering integral characterized by intervalley scattering time τ and intravalley scattering time τ_{intra}

a generalization of the kinetic equation

$$\frac{df_{\mathbf{p}}(\mathbf{r},t)}{dt}+\left(1+\frac{e}{c}\mathbf{B}\cdot\mathbf{\Omega_p}\right)^{-1}\left[\left(\mathbf{v}+e\mathbf{E}\times\mathbf{\Omega_p}+\frac{e}{c}(\mathbf{\Omega_p}\cdot\mathbf{v})\mathbf{B}\right)\frac{df_{\mathbf{p}}}{d\mathbf{r}}+\left(e\mathbf{E}+\frac{e}{c}\mathbf{v}\times\mathbf{B}+\frac{e^2}{c}(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega_p}\right)\frac{df_{\mathbf{p}}}{d\mathbf{p}}\right]=I_{st}(f_{\mathbf{p}})$$

$$N^{(i)}=\int d\mathbf{p}\left(1+\frac{e}{c}\mathbf{B}\cdot\mathbf{\Omega}_{\mathbf{p}}^{(i)}\right)f_{\mathbf{p}}^{(i)};\qquad \mathbf{j}^{(i)}=\int d\mathbf{p}\left[\mathbf{v}+e\mathbf{E}\times\mathbf{\Omega}_{\mathbf{p}}^{(i)}+\frac{e}{c}(\mathbf{\Omega}_{\mathbf{p}}^{(i)}\cdot\mathbf{v})\mathbf{B}\right]f_{\mathbf{p}}^{(i)}$$

conservation law of number of particles

$$\frac{dN^{(i)}}{dt}+\nabla\mathbf{j}^{(i)}+\frac{\delta N^{(i)}}{\tau}=k^{(i)}\frac{e^2}{4\pi^2\hbar^2 c}[\mathbf{E}\cdot\mathbf{B}]$$

Chiral anomaly related term
Adler Bell Jackiw

$$k^{(i)}=\frac{1}{2\pi\hbar}\int\mathbf{\Omega}_{\mathbf{p}}^{(i)}d\mathbf{S}=0,\pm1,\ldots$$

Quasiclassical kinetic equation captures effects of chiral anomaly

There are equilibrium valley currents in Weyl's metals

$$\int d\mathbf{p} \frac{d\varepsilon_{\mathbf{p}}}{d\mathbf{p}} f_{\mathbf{p}}\left(\varepsilon_{\mathbf{p}}\right) = 0$$

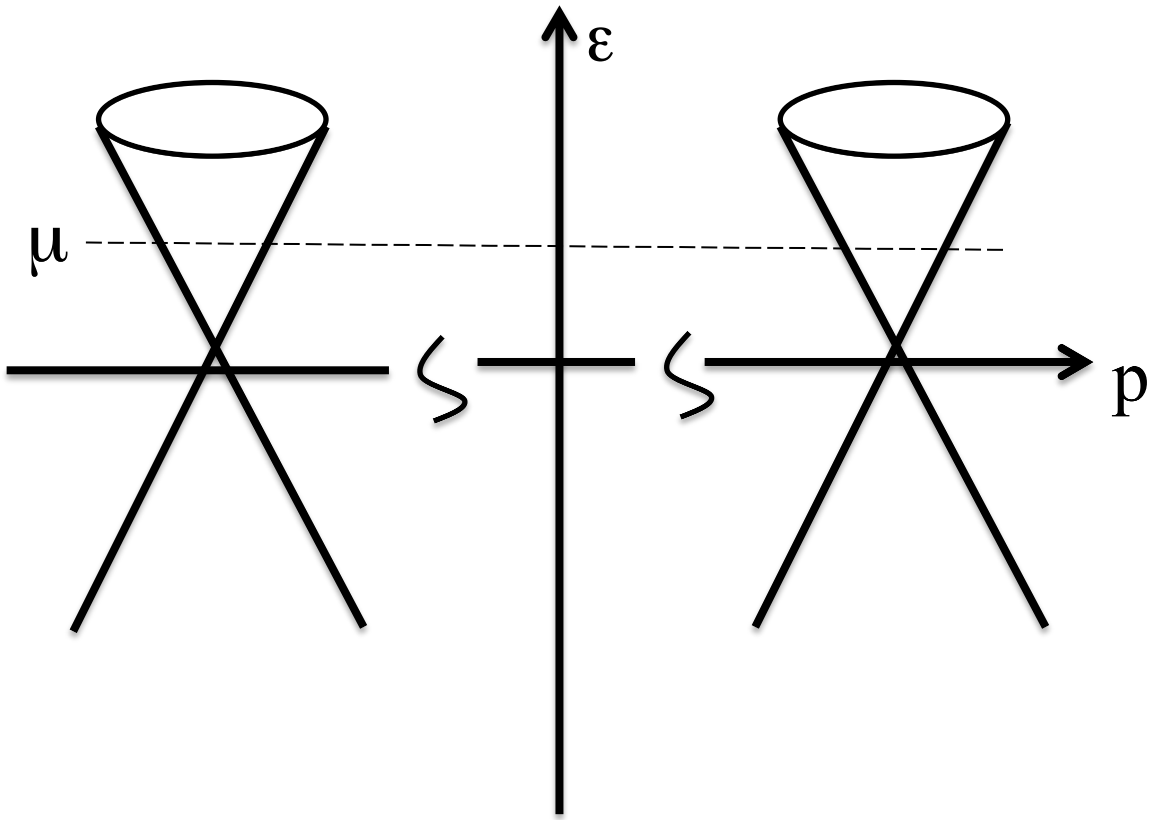
$$\mathbf{j}^{(i)} = \int d\mathbf{p} \, \frac{e}{c}\left(\mathbf{\Omega}_{\mathbf{p}}^{(i)} \cdot \mathbf{v}\right)\mathbf{B} f_{\mathbf{p}}^{(F)} == \mu \, k^{(i)}\mathbf{B};$$

Vilenkin

$$\mu^{(i)} = \mu, \qquad \sum_i k^{(i)} = 0; \qquad \mathbf{j} = \sum_i \mathbf{j}^{(i)} = 0;$$

Nielsen and Ninomiya theorem

Number of Weyl (massless Dirac) points should be even.
(Nielsen and Ninomiya theorem)



Anomaly-related contribution to the conductivity corresponds to positive magnetoconductance

$$\delta\mu^{(i)} \propto \mathbf{E}\cdot\mathbf{B}\tau$$

$$\sigma_{zz}^{(an)} = N_D \frac{e^2}{4\pi^2\hbar c}\frac{v}{c}\frac{(eB)^2 v^2}{\mu^2}\tau \qquad \mu > T$$

$$\sigma_{ij}^{(an)} = 0 \qquad i, j \neq z$$

τ is the intervaley relaxation time

The current flows only in z-direction parallel to the magnetic field.
It responds only to z-component of the electric field.

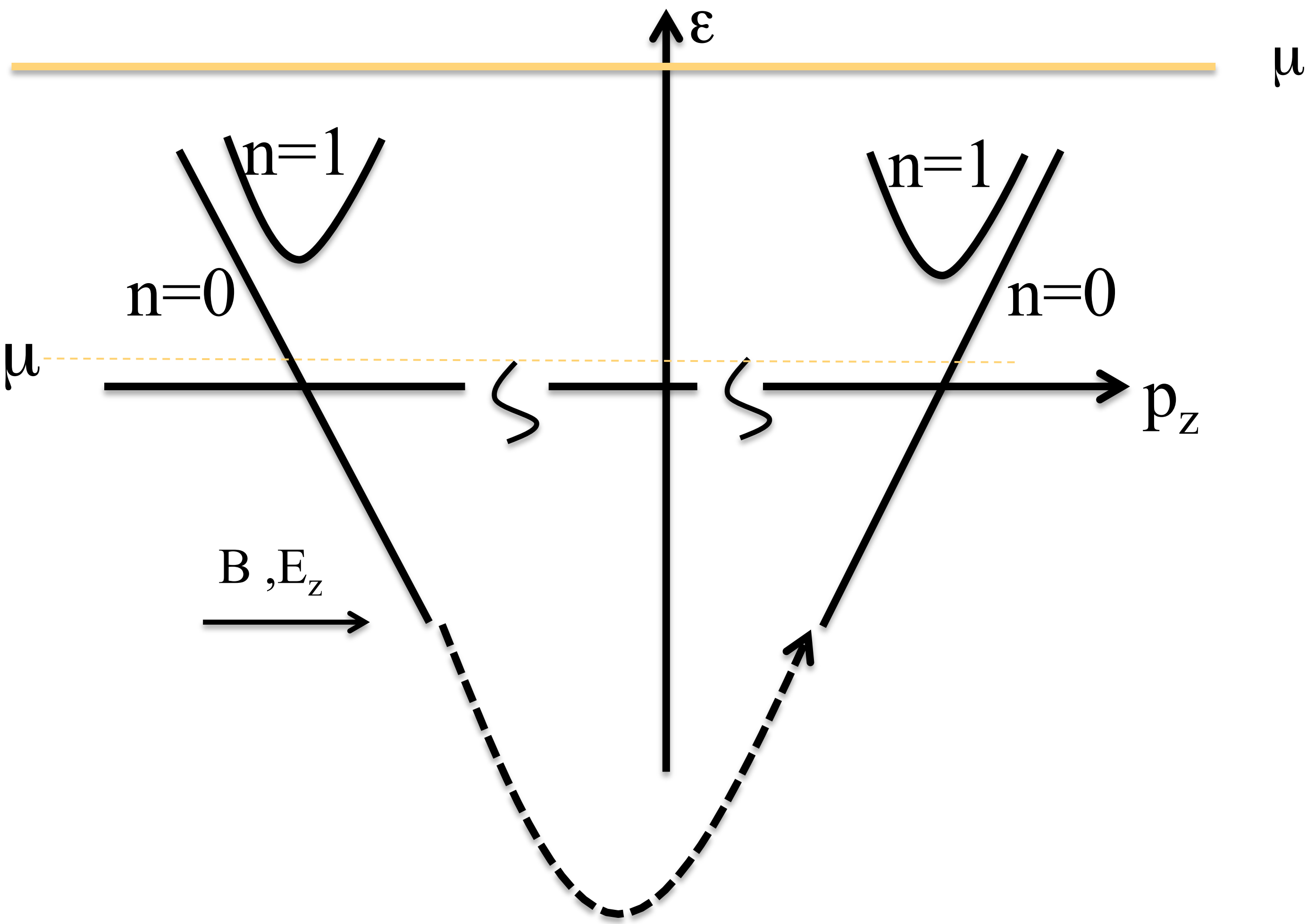
Anomaly related contribution dominates magnetoresistance if the ratio between the inter- and intra-valley relaxation times is sufficiently large!

$$\frac{\tau}{\tau_{\text{intra}}} \frac{1}{\left(\mu\tau_{\text{intra}}\right)^2} > 1 \qquad (\mu > T)$$

τ and τ_{intra} are the intervaley and intravaley relaxation times resepctively

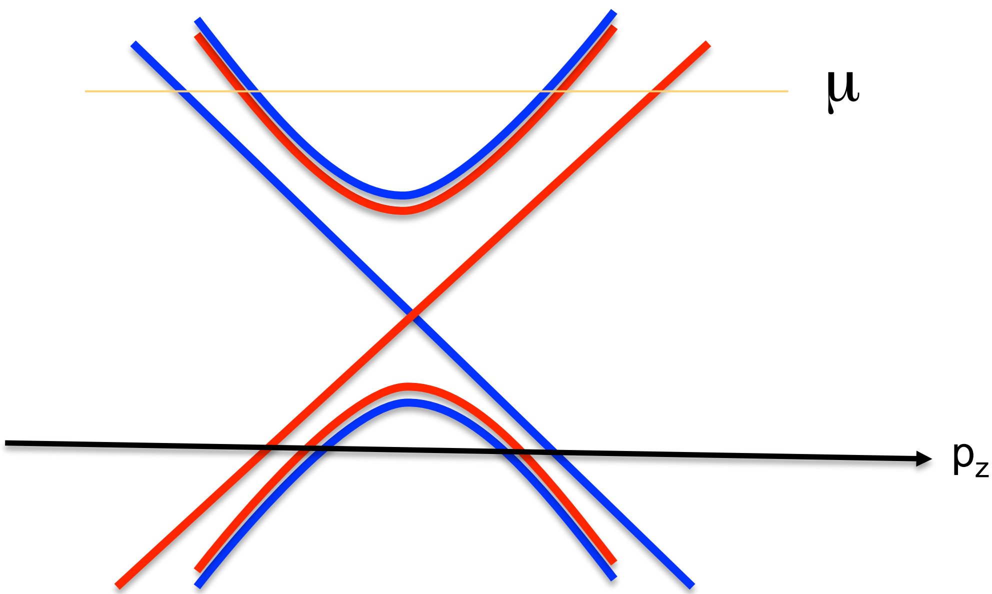
Electron spectrum in ultra-quantum limit

$$H = \pm v\boldsymbol{\sigma}\left(\frac{\nabla}{i} + \frac{e}{c}\vec{\mathbf{A}}\right)$$



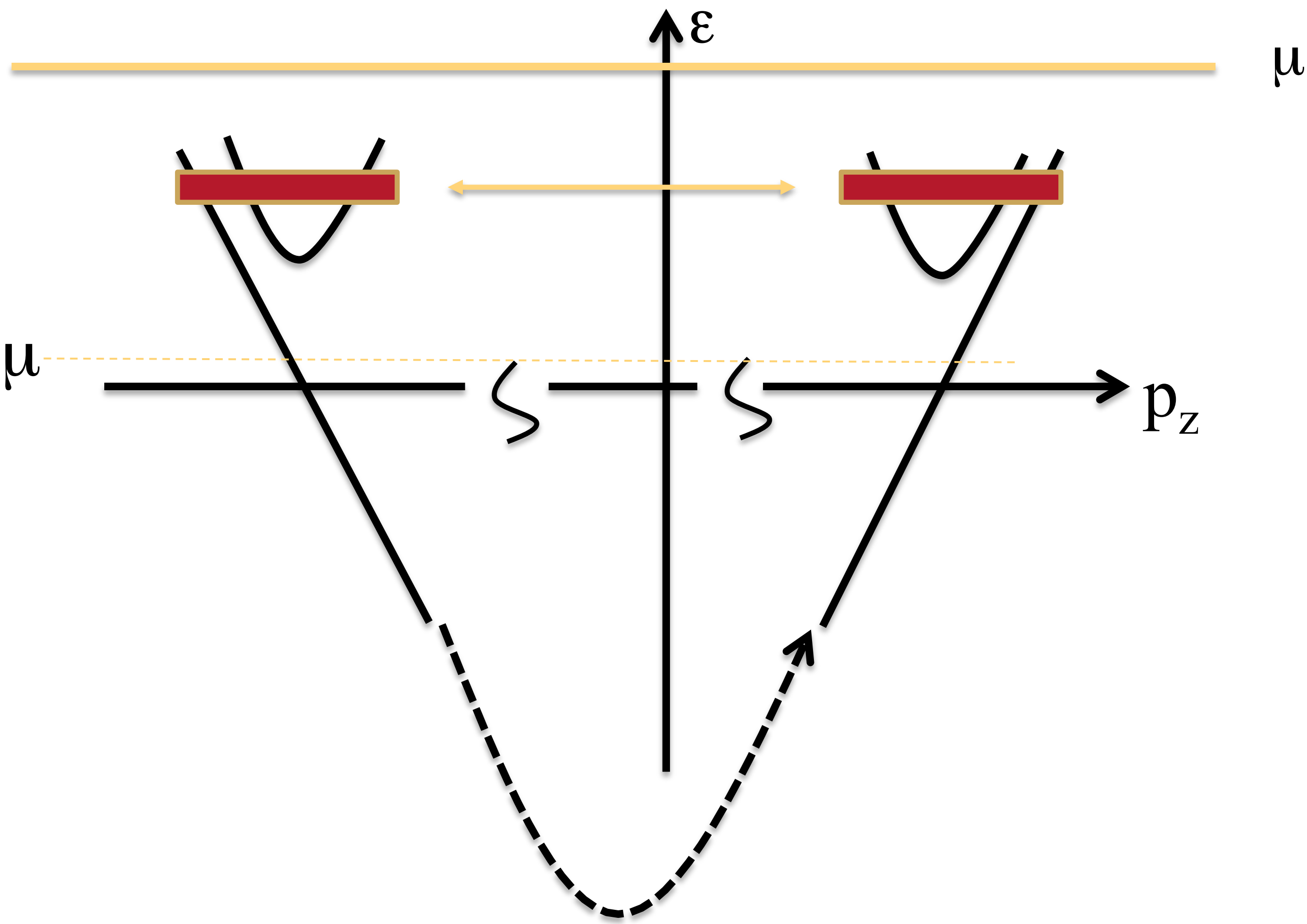
Electron spectrum of Dirac equation in magnetic field
Akhiezer, Berestetzkii, "Quantum electrodynamics"

$$E_n^2 = p_z^2 + m^2 + |e|B(2n+1)|e|B\sigma$$



At m=0 there are two non-degenerate chiral levels.
Red and blue lines correspond to branches of spectrum with different helicities.

Einstein's relation between conductivity and diffusion coefficient



Signature of the chiral anomaly in a Dirac semimetal – a current plume steered by a magnetic field*

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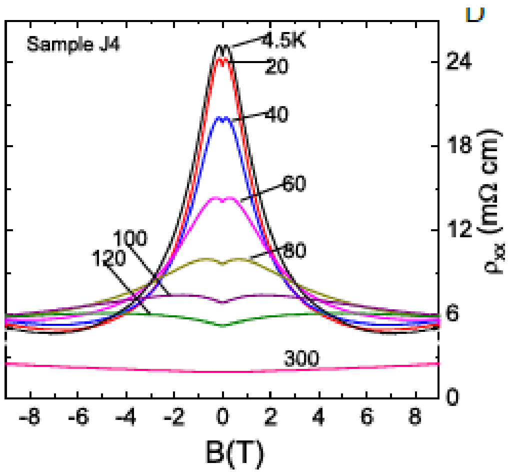
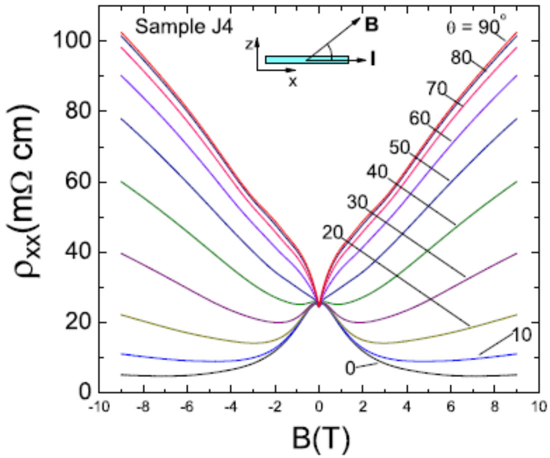
(Dated: March 30, 2015)

Na_3Bi





Observation of the chiral magnetic effect in $ZrTe_5$

Qiang Li,[1] Dmitri E. Kharzeev,[2,3] Cheng Zhang,[1] Yuan Huang,[4] I. Pletikosić,[1,5]
A. V. Fedorov,[6] R. D. Zhong,[1] J. A. Schneeloch,[1] G. D. Gu,[1] and T. Valla[1]

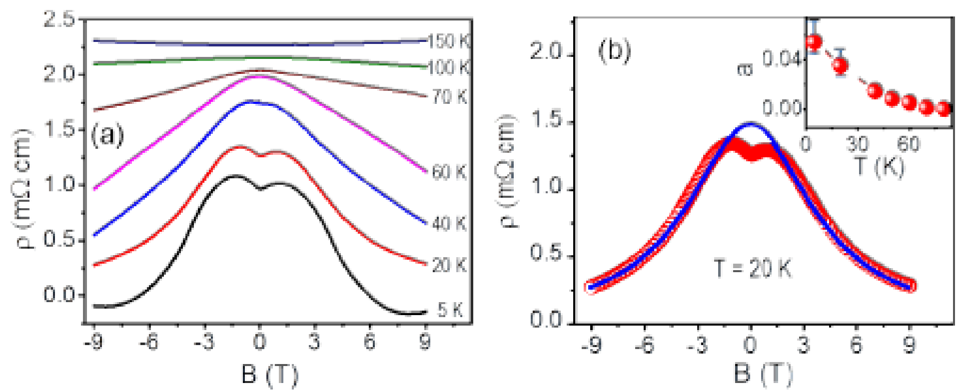


FIG. 2: Magnetoresistance in field parallel to current ($\vec{B} \parallel a$) in $ZrTe_5$. (a) MR at various temperatures. For clarity, the resistivity curves were shifted by 1.5 mΩcm (150 K), 0.9 mΩcm (100 K), 0.2 mΩcm (70 K) and −0.2 mΩcm (5 K). (b) MR at 20K (red symbols) fitted with the CME curve (blue line); inset: temperature dependence of the fitting parameter $a(T)$ in units of S/(cm T^2).

bservation of Negative Magnetoresistance and nontrivial π Berrys phase in 3D Weyl semi-metal NbAs

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(Dated: June 9, 2015)

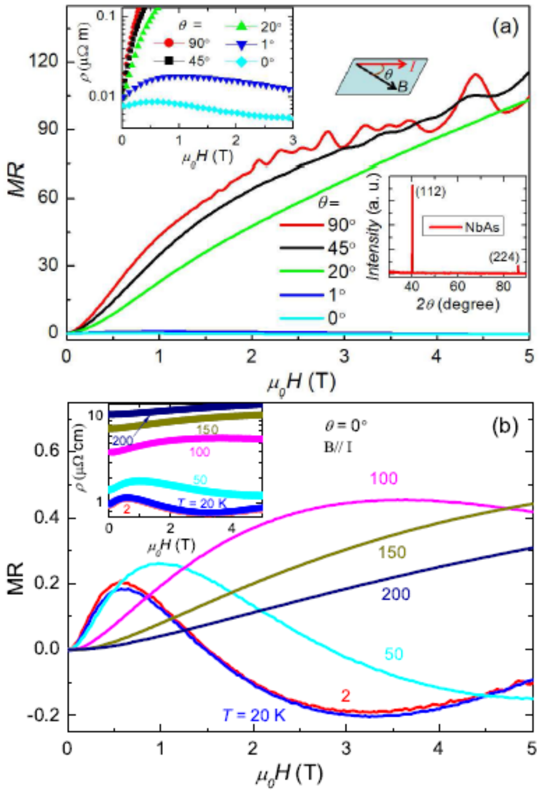

FIG. 1: (Color online) (a), Magnetic field dependence of Magnetoresistance with magnetic field ($\mu_0 H$) from perpendicular ($\theta = 90°$) to parallel ($\theta = 0°$) to the electric current (I) at $T = 2$ K. The upper left inset displays the original resistivity data plotted on logarithmic scale, emphasizing the contrast between extremely large positive MR for magnetic field perpendicular to current ($\theta = 90°$) and negative MR for field parallel to current ($\theta = 0°$). The upper right inset depicts the corresponding measurement configurations. The lower inset present the single crystal XRD data. (b), Magnetic field dependence of MR under various temperatures for magnetic field ($\mu_0 H$) parallel ($\theta = 0°$) to the electric current (I). The inset display the original resistivity data.

Observation of the chiral anomaly induced negative magneto-resistance in 3D Weyl semi-metal TaAs

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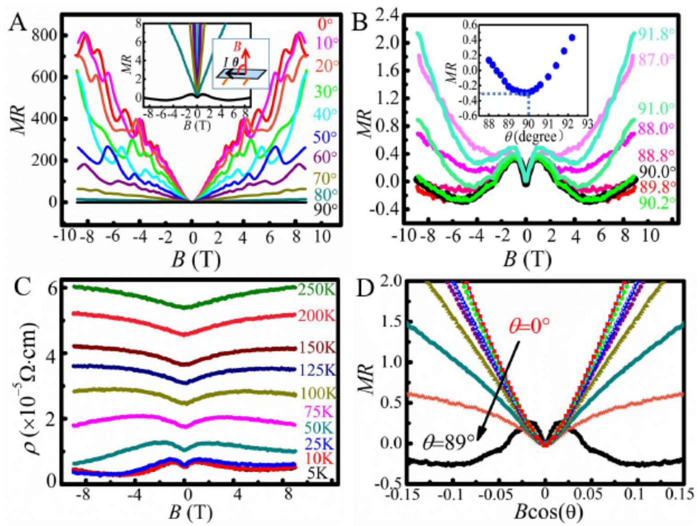
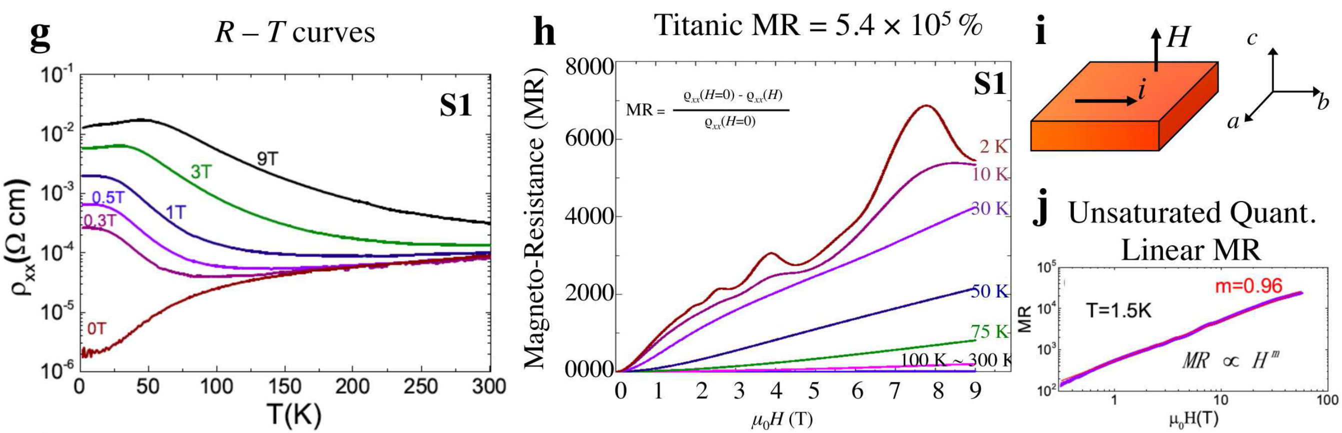


Figur 2: Angular and field dependence of MR in TaAs single crystal at 1.8 K. (**A**) Magneto-resistance with magnetic field (*B*) from perpendicular (θ=0°) to parallel (θ=90°) to the electric current (*I*). The inset zooms in on the lower MR part, showing negative MR at θ=90° (longitudinal negative MR), and depicts the correspondingly measurement configurations. (**B**) Magneto-resistance measured in different rotating angles around θ=90° with the interval of every 0.2°. The negative MR appeared at a narrow region around θ=90°, and most obviously when *B*//*I*. Either positive or negative deviations from 90° would degenerate and ultimately kill the negative MR in

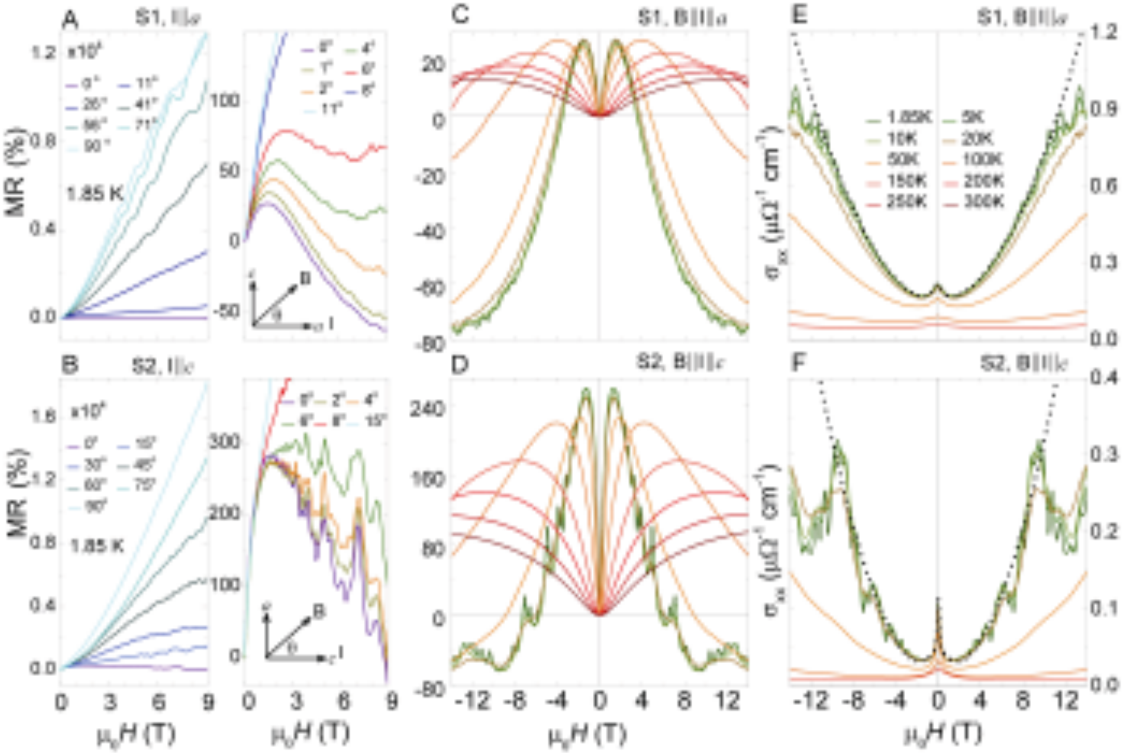
Observation of the Adler-Bell-Jackiw chiral anomaly in a Weyl semimetal

Chenglong Zhang*,[1] Su-Yang Xu*,[2,3] Ilya Belopolski*,[2,3] Zhujun Yuan*,[1] Ziquan Lin,[4] Bingbing Tong,[1] Nasser Alidoust,[2] Chi-Cheng Lee,[5,6] Shin-Ming Huang,[5,6] Hsin Lin,[5,6] Madhab Neupane,[2,7] Daniel S. Sanchez,[2] Hao Zheng,[2] Guang Bian,[2] Junfeng Wang,[4] Chi Zhang,[1,8] Titus Neupert,[2,9] M. Zahid Hasan†,[2,3] and Shuang Jia†[1,8]



Large and unsaturated negative magnetoresistance induced by the chiral anomaly in the Weyl semimetal TaP

Chandra Shekhar[1] *, Frank Arnold[1] *, Shu-Chun Wu[1], Yan Sun[1], Marcus Schmidt[1], Nitesh Kumar[1], Adolfo G. Grushin[2], Jens H. Bardarson[2], Ricardo Donizeth dos Reis[1], Marcel Naumann[1], Michael Baenitz[1], Horst Borrmann[1], Michael Nicklas[1], Elena Hassinger[1], Claudia Felser[1], & Binghai Yan[1,2] †



Anomalous magnetoresistance in $TaAs_2$

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N. J. Ghimire[†], E. D. Bauer, J. D. Thompson[‡], and F. Ronning[§]
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(Dated: January 22, 2016)

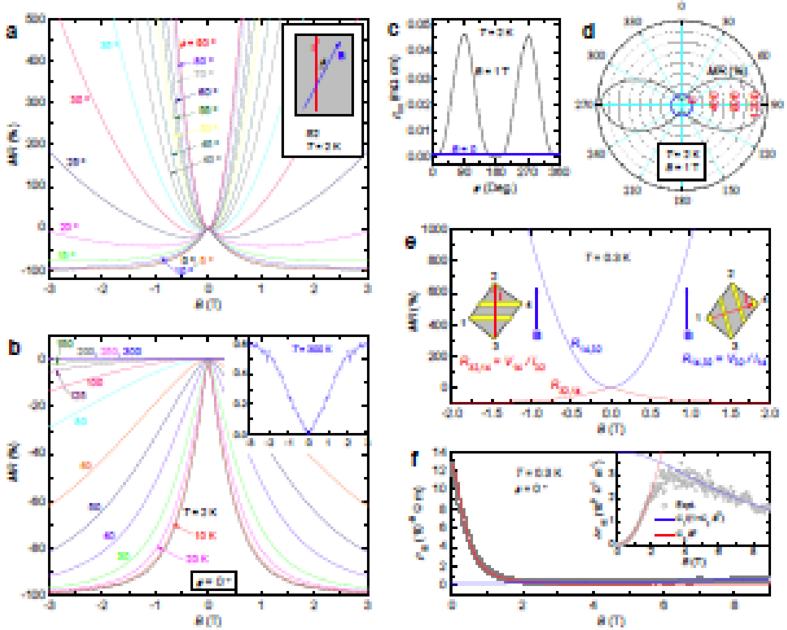


Figure 3 | Longitudinal magnetoresistance (LMR) of $TaAs_2$. a, Field-dependent MR of $TaAs_2$ with various angles ϕ at 2 K. The inset shows the configuration of the measurements. **b**, MR at different temperatures, measured at ϕ=0. The inset displays the data at 300 K. **c** shows the angular dependence of ρ_{xx} at 2 K and 1 T. **d** is a polar plot of MR at 2 K. **e** shows the results with two different geometries, $R_{32,14}=V_{14}/I_{32}$ (red) and $R_{14,32}=V_{32}/I_{14}$ (blue). Schematic sketches of the geometry are shown in the insets. **f** presents theoretical fits of $\rho_{xx}(B)$ and $\Delta\sigma_{xx}(B)$. The high-field part of $\Delta\sigma_{xx}(B)$ is fit to $C_1/(1+C_2B^2)$ (blue), while the low-field part is fit to C_3B^2 (red).

Why the parameter τ/τ_{intra} is so big in this experiments?

Conceptual and practical difficulties in ultra-quantum limit

1. B dependence of $\tau(B)$ is not universal.

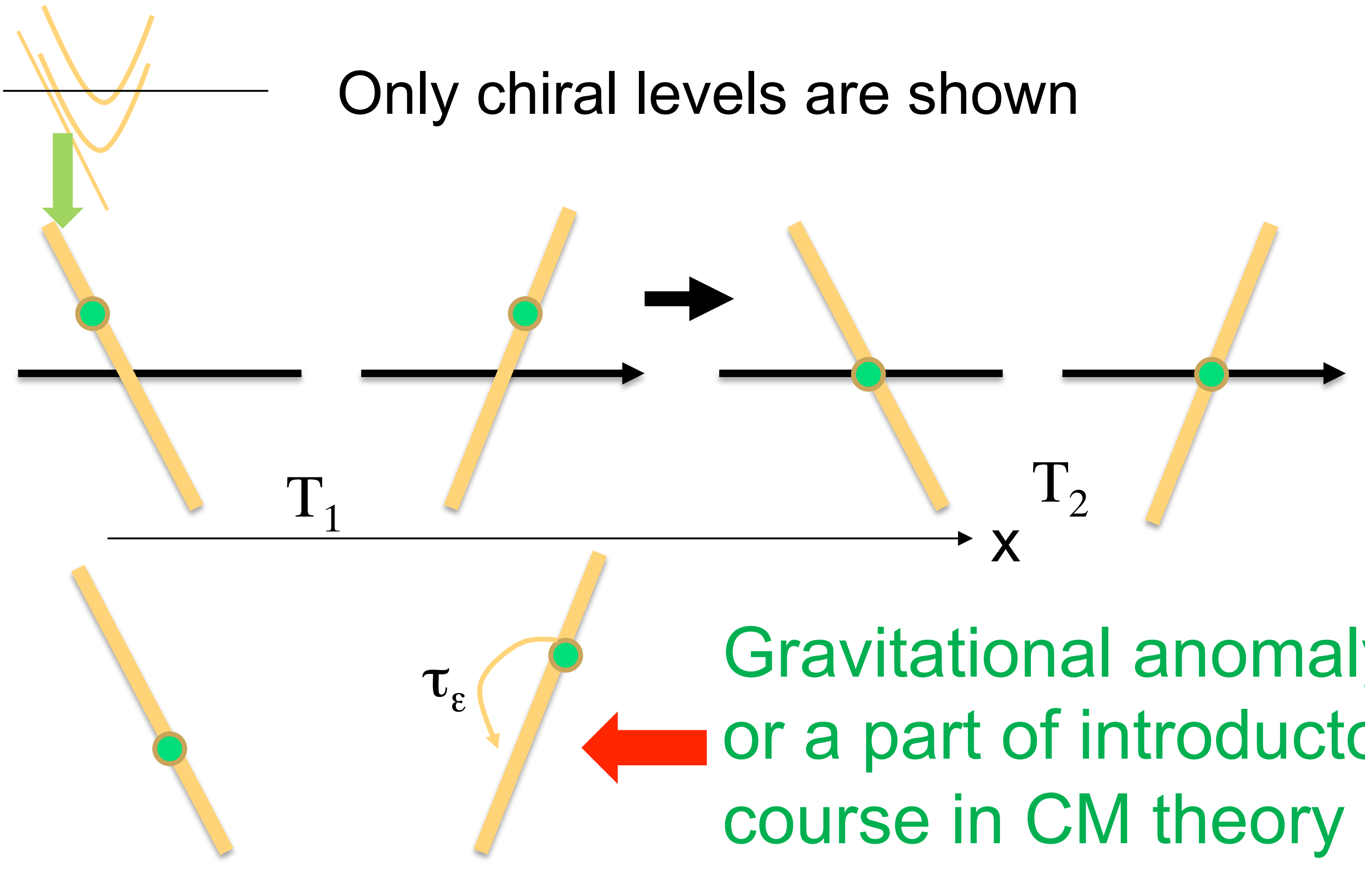
For example in the case of short range impurities $\frac{1}{\tau} \propto \frac{1}{L_B^2}$ is proportional to the density of states and σ_{zz} is indepenent on B

2. It is dificult to distinguish chiral anomaly related magnetoresistance from magnetoresistance in conventional conductors in ultra – quantum magnetic field

3. At T = 0 the system is unstable with respect to charge density wave. In this case chiral anomaly does not exist

Temperature gradient does not pump either electrons or energy between valleys with different chiralities! So the magnetetic field dependence of the thermoelectric effect has nothing to do with any

Only chiral levels are shown



Gravitational anomaly or a part of introductor course in CM theory !

Thermal conductivity

$$\kappa_{zz} = \frac{\pi^2 \sigma_{zz}^{(an)} T}{3e^2} \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau}$$

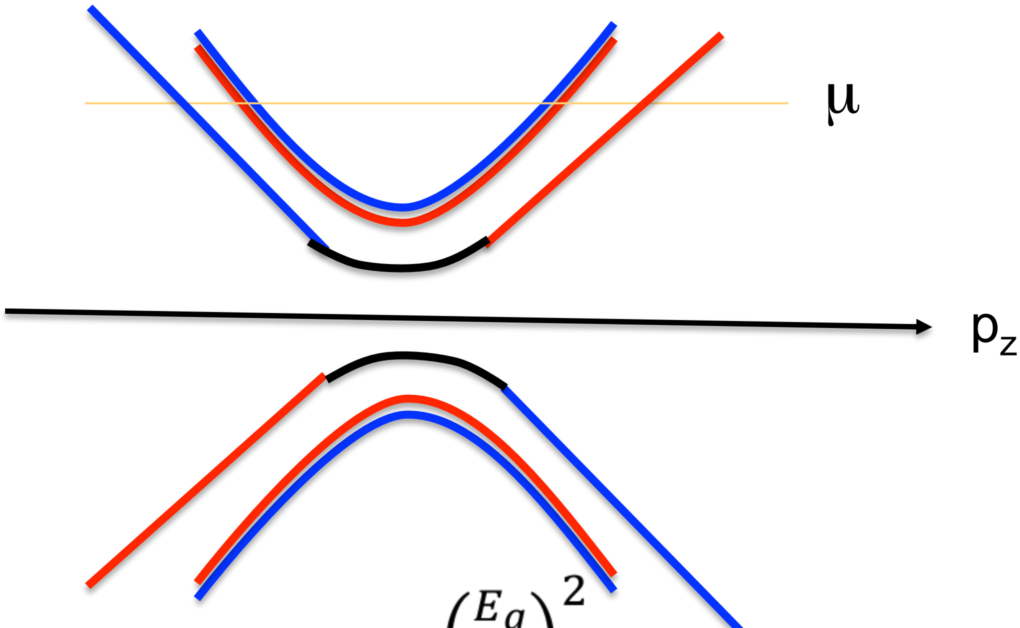
Thermoelectric coefficient

$$\eta_{zz} = \frac{\pi^2}{3e^2} T \frac{d\sigma_{zz}^{(an)}}{d\mu} \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau}$$

All effects which have been observed in Weyl metals are allowed by symmetry. So it is only a question of finding a proper relation between parameters which will lead to explanation of observables.

semiconductors with sufficiently small gaps and sufficiently large chemical potential should have magnetic field dependence of kinetic coefficients which are similar to those in Weyl and Dirac metals

Spectrum of Dirac equation at finite mass



$$\tau \sim \tau_{intra} \left(\frac{E_g}{E}\right)^2$$

what about other mechanisms of negative parallel magnetoresistance?

NEGATIVE LONGITUDINAL MAGNETORESISTANCE ASSOCIATED WITH SCATTERING FROM IONIZED IMPURITY CENTERS

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Semiconductor Institute, Academy of Sciences, U.S.S.R. Submitted October 28, 1968

Zh. Eksp. Teor. Fiz. 56, 801-812 (March, 1969)

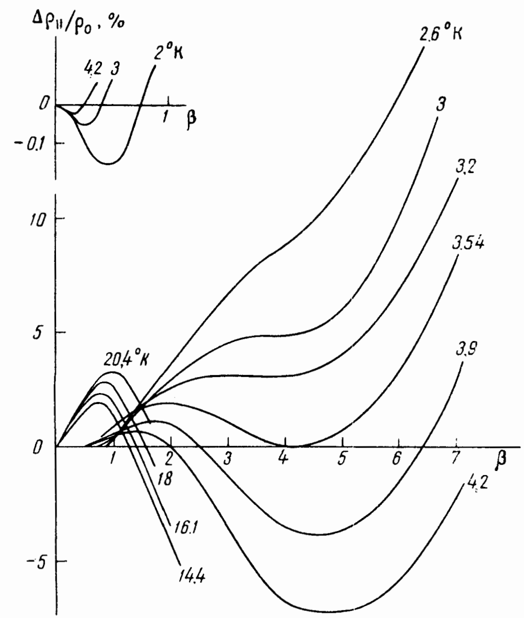

FIG. 3. Experimental dependences of the magnetoresistance $\Delta\rho_{\parallel}/\rho$ on β at different temperatures. Initial segments of some curves, representing amplified measurements, are shown in the upper left-hand corner.

conclusion