Chiral liquid phase of simple XXZ quantum magnets

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arXiv:1708.02980

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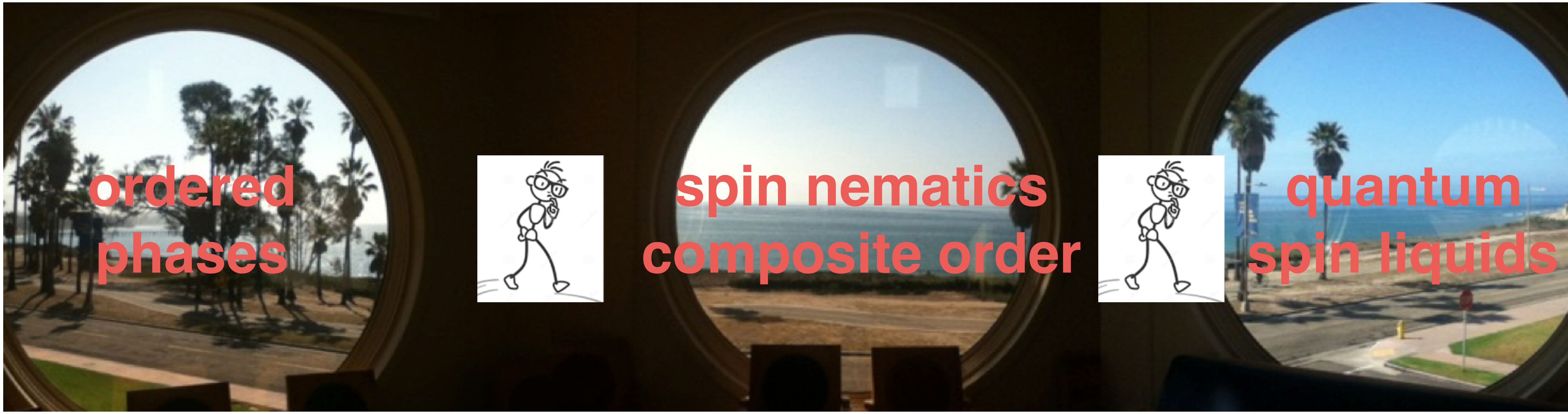
KITP "Intertwined order and fluctuations in quantum materials", October 4, 2017

Exotic ordered phases, emergent (Ising) orders



composite order parameter

$$\mathcal{O}^{\alpha\beta}(\mathbf{r}_i, \mathbf{r}_j) = \tfrac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \tfrac{1}{3}\delta^{\alpha\beta}\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$$

LE JOURNAL DE PHYSIQUE TOME 38, AVRIL 1977, PAGE 385

Classification
Physics Abstracts
7.480 — 8.514

A MAGNETIC ANALOGUE OF STEREOISOMERISM : APPLICATION TO HELIMAGNETISM IN TWO DIMENSIONS

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(*Reçu le 13 juillet 1976, révisé le 8 novembre 1976, accepté le 4 janvier 1977*)

Spin nematics

A. F. Andreev and I. A. Grishchuk

Institute of Physics Problems, USSR Academy of Sciences

(Submitted 6 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 467–475 (August 1984)

We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid ^{3}He is discussed.

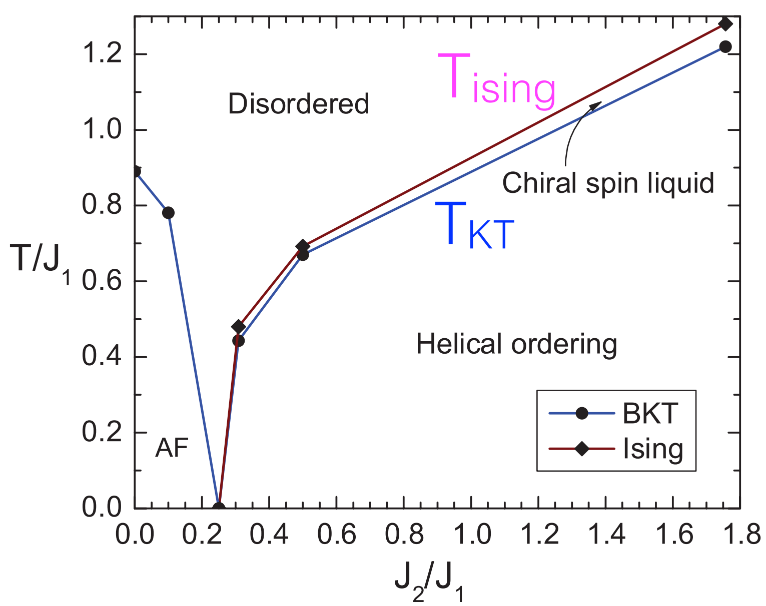
Emergent Ising order parameters

PHYSICAL REVIEW B **85**, 174404 (2012)

Chiral spin liquid in two-dimensional XY helimagnets

A. O. Sorokin[1,*] and A. V. Syromyatnikov[1,2,†]

$$H = \sum_{\mathbf{x}} (J_1 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x+a}}) + J_2 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x+2a}}) - J_b \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x+b}})),$$



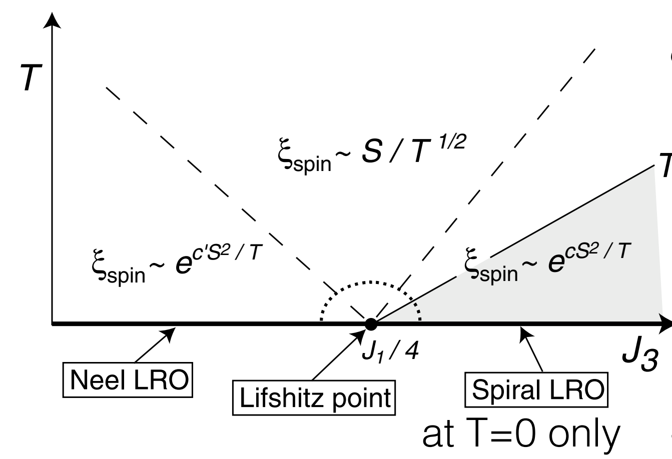
PRL **93**, 257206 (2004) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2004

Low-Temperature Broken-Symmetry Phases of Spiral Antiferromagnets

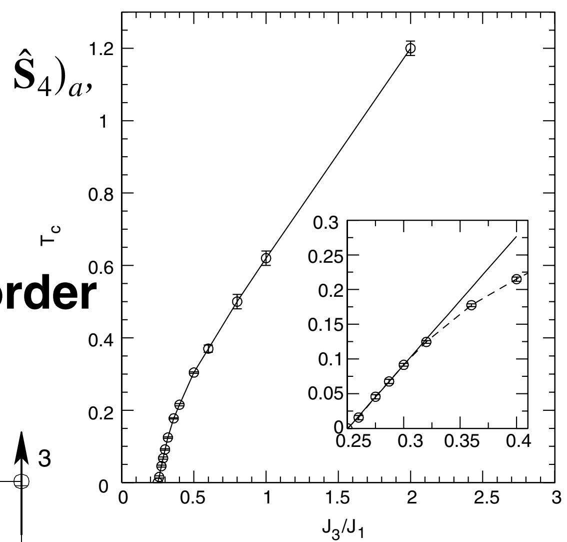
Luca Capriotti[1,2] and Subir Sachdev[2,3]

$$\hat{H} = J_1 \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j + J_3 \sum_{\langle\langle i,j \rangle\rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j,$$

$$\sigma_a = (\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_3 - \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{S}}_4)_a,$$



Ising nematic order



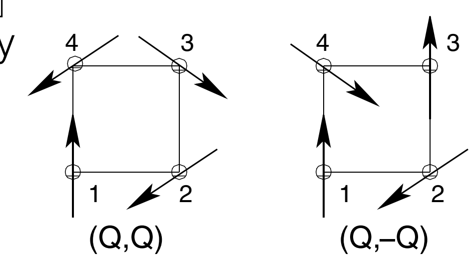


FIG. 2. The two different minimum energy configurations with magnetic wave vectors $\vec{Q} = (Q, Q)$ and $\vec{Q}^\star = (Q, -Q)$ with $Q = 2\pi/3$, corresponding to $J_3/J_1 = 0.5$.

PRL **95**, 137206 (2005) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2005

Two-Step Restoration of SU(2) Symmetry in a Frustrated Ring-Exchange Magnet

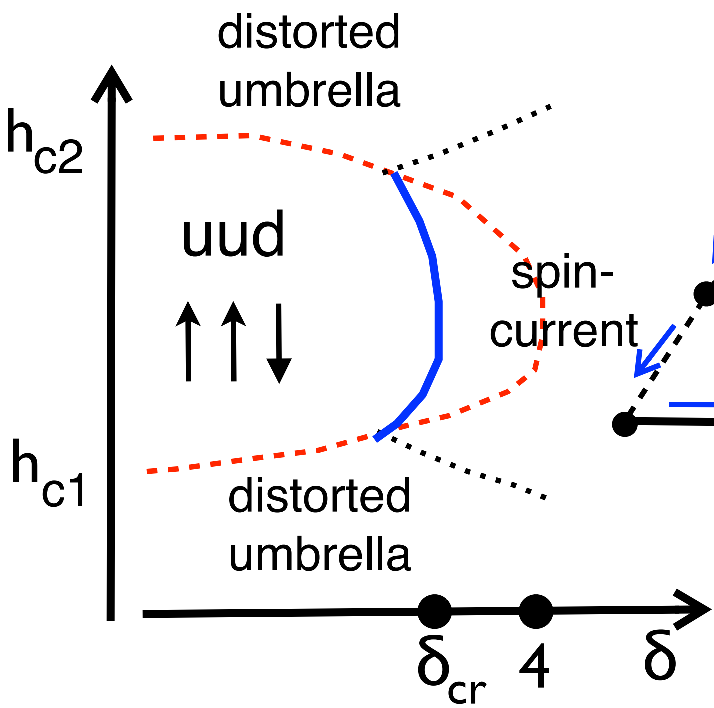
A. Läuchli,[1] J. C. Domenge,[2] C. Lhuillier,[2] P. Sindzingre,[2] and M. Troyer[3]
[1]*Institut Romand de Recherche Numérique en Physique des Matériaux (IRRMA), PPH-Ecublens, CH-1015 Lausanne, Switzerland*
[2] *Laboratoire de Physique Théorique des Liquides, Université P. et M. Curie, UMR 7600 of CNRS, case 121, 4 Place Jussieu, 75252 Paris Cedex, France*
[3]*Institut für Theoretische Physik, ETH Hönggerberg, CH-8093 Zürich, Switzerland*
(Received 23 December 2004; published 22 September 2005)

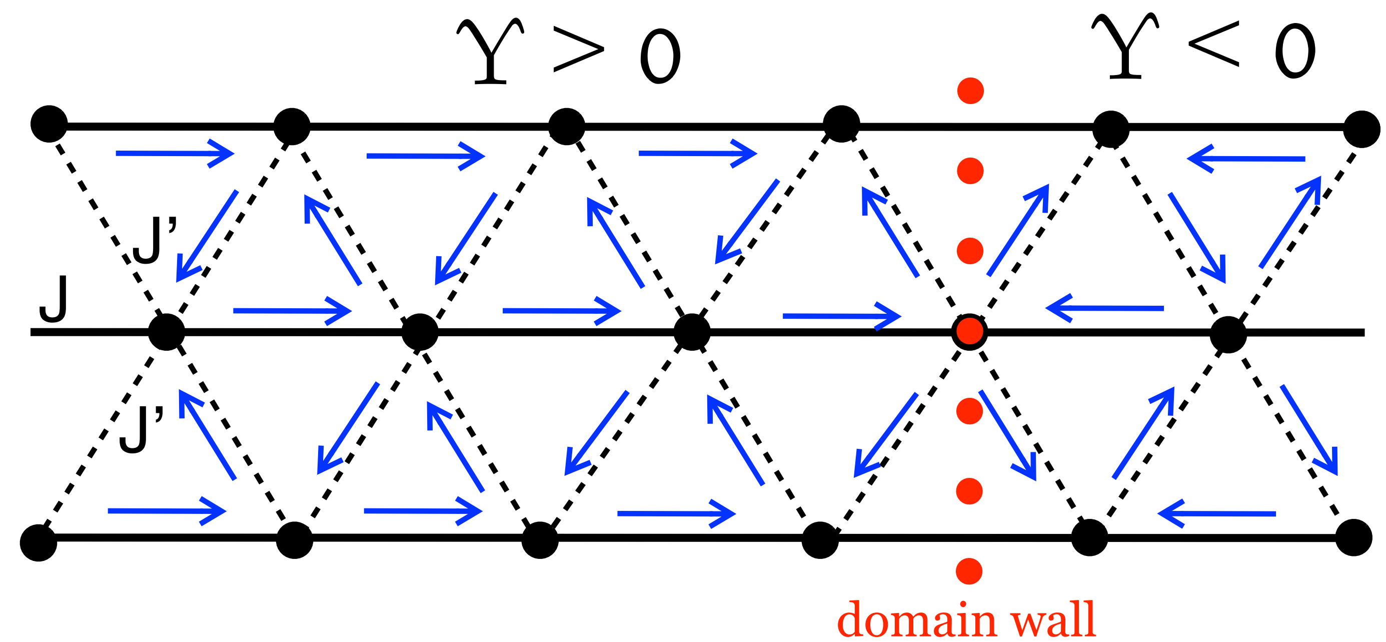
We demonstrate the existence of a spin-nematic, moment-free phase in a quantum four-spin ring-exchange model on the square lattice. This unusual quantum state is created by the interplay of frustration and quantum fluctuations that lead to a partial restoration of SU(2) symmetry when going from a four-sublattice orthogonal biaxial Néel order to this exotic uniaxial magnet. A further increase of frustration drives a transition to a fully gapped SU(2) symmetric valence bond crystal.

Today: vector chirality without magnetic order

Cheshire Cat's smile

Spin-current nematic state near the end-point of the 1/3 magnetization plateau (large-S analysis)





no transverse magnetic order $\langle \mathbf{S}_r^{x,y} \rangle = 0$ $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$ is not affected

Finite **vector chirality**

$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

Spontaneously broken Z_2 -- spatial inversion [in addition to broken Z_3 inherited from the UUD state]

Chubukov, OS PRL 2013

End-point of the plateau on kagome lattice

Semiclassical analysis of a magnetization plateau in a 2D frustrated ferrimagnet

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Leon Balents
Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106

(Dated: November 9, 2016)

PRB 2017

Kagome geometry

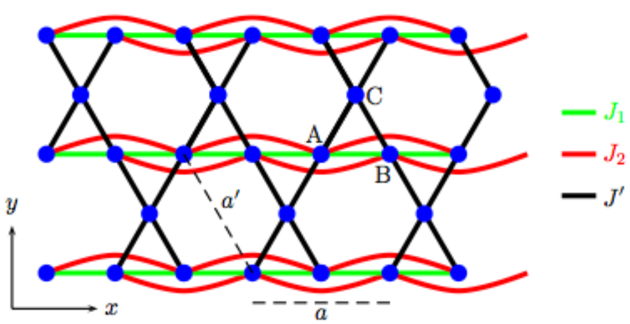


FIG. 1. Proposed Hamiltonian for volborthite. The blue dots represent spin-1/2 copper ions and the line segments represent Heisenberg couplings. $J_1 < 0$ is ferromagnetic while $J_2 > 0$ and $J' > 0$ are antiferromagnetic. The distances between adjacent unit cells is slightly anisotropic, with $a = 5.84$ Å and $a' = 6.07$ Å [10]. Capital letters label the sublattices.

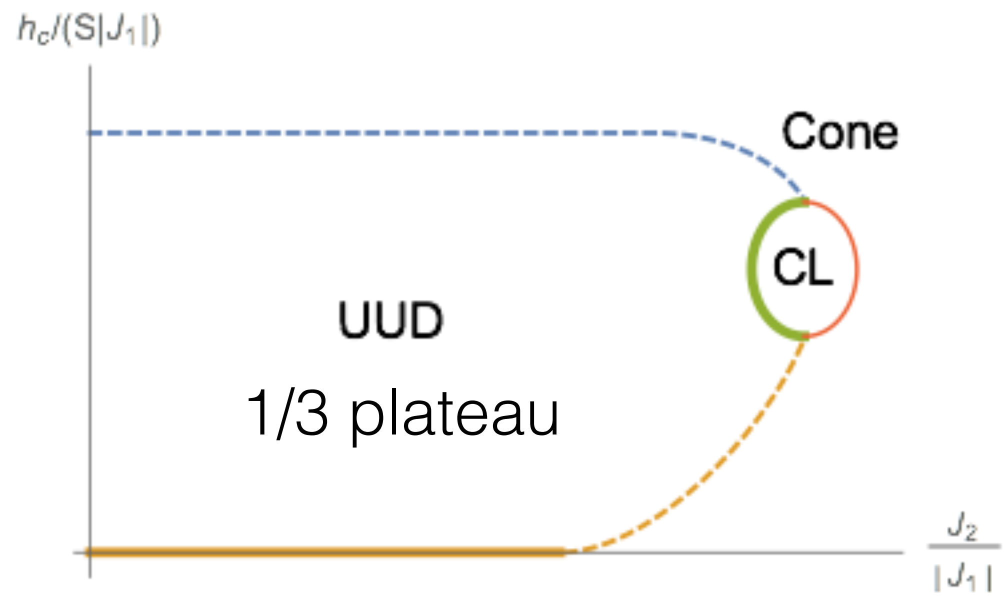


FIG. 6. Schematic quantum phase diagram at $J' = 0.5|J_1|$. The UUD state breaks no symmetries, the gapped chiral liquid (CL) phase only breaks chiral symmetry, and the gapless cone state breaks both chiral and a $U(1)$ symmetry combining translation and spin rotation. The thick solid lines and dashed lines represent first- and second-order transitions respectively. We did not investigate the nature of the transition between the chiral liquid and cone phases represented by the red line. In a 3D phase diagram like that of Fig. 4 that includes the applied field, the chiral liquid phase would appear as a thin tube around the stabilization curve where the two sheets meet.

Spin-current pattern

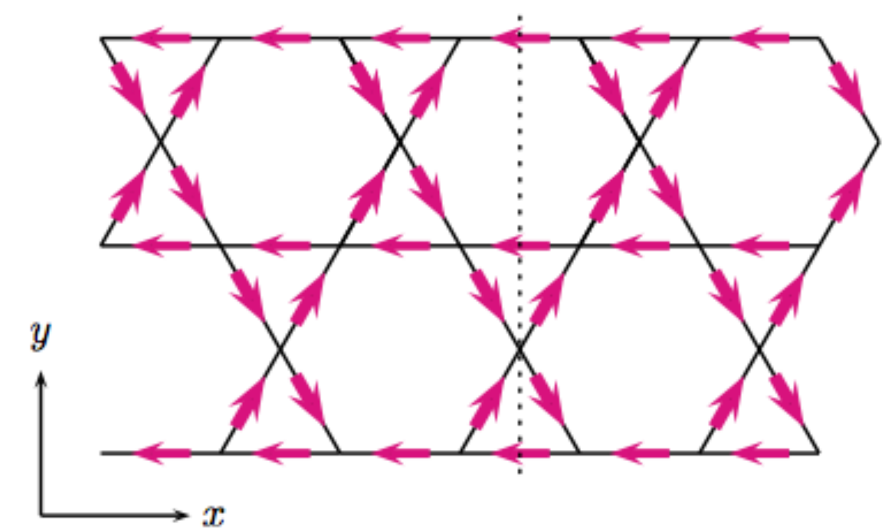
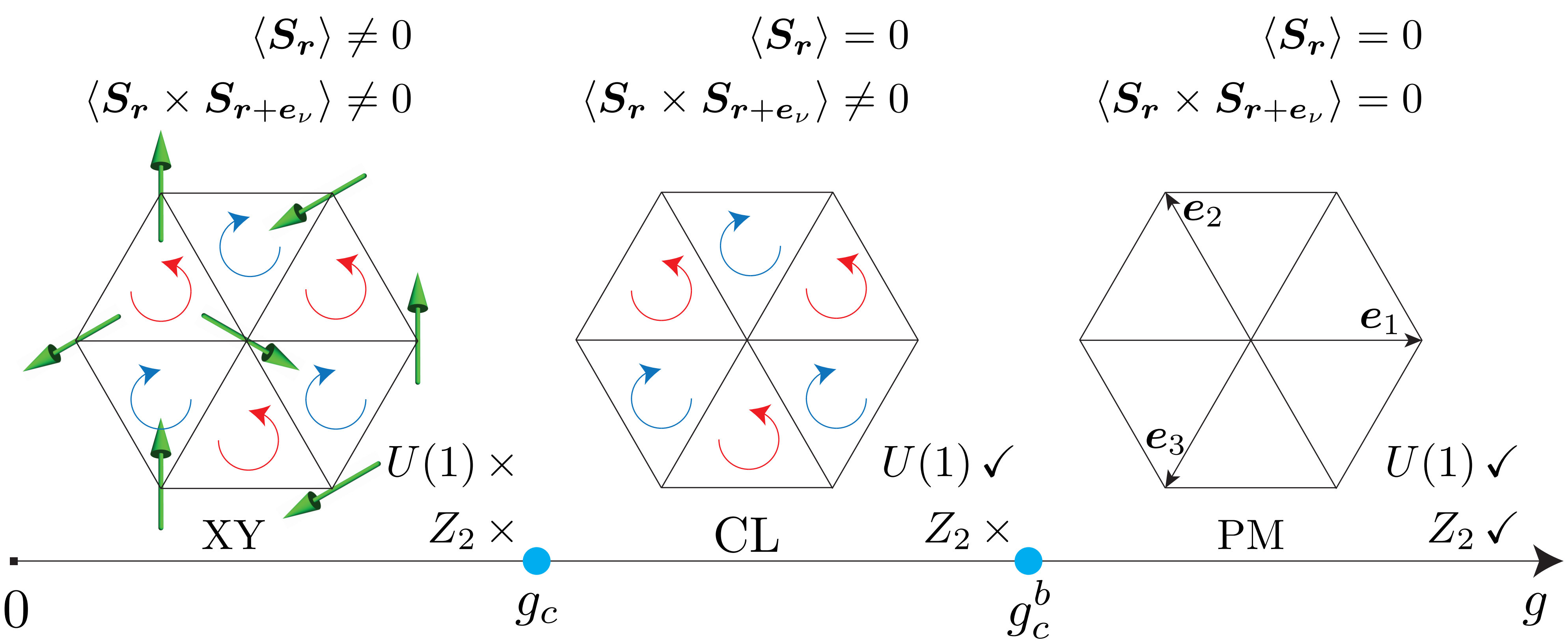


FIG. 7. Spin-current configuration in the chiral liquid phase. The magenta arrows indicating the direction of spin current flow; in the orthogonal ground state the flow is reversed. The spin current on the diagonal bonds, represented by thicker arrows, is larger by a factor of $\sqrt{2}$ and determines the net current flow. The ground state is chiral and spontaneously breaks the lattice symmetry of reflection about the dotted line.

The minimal 2d quantum spin model

- Spin-1 model with featureless Mott ground state at large D > 0 [$S_r^z = 0$]
- Triangular lattice: two-fold degenerate spectrum, at +**Q** and -**Q**

$$H = \sum_{\langle r,r' \rangle} J(S_r^x S_{r'}^x + S_r^y S_{r'}^y + \Delta S_r^z S_{r'}^z) + D \sum_r (S_r^z)^2$$



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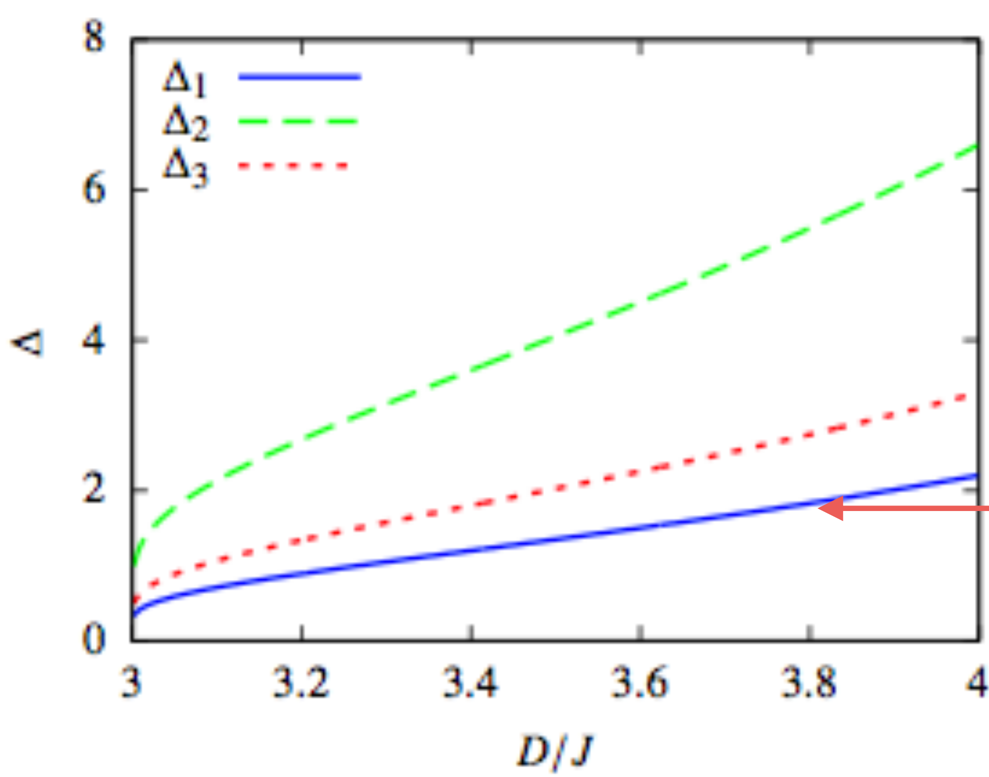
1. Toy problem of two-spin exciton. Derive Schrodinger eqn for the pair wave function ψ

$$|\mathrm{ex}\rangle = \sum_{n \neq m} \psi_{n,m} |n,m\rangle \text{ where } |n,m\rangle = \frac{1}{2} S_n^+ S_m^- \Pi_j |0\rangle_j \longrightarrow B_{\mathbf{g}} = \Delta J \sum_{\mathbf{g}'} M_{\mathbf{g}\mathbf{g}'} B_{\mathbf{g}'}$$

+ charge, — charge

where

$$M_{\mathbf{g}\mathbf{g}'} = \frac{1}{N} \sum_{\mathbf{q}} \frac{e^{i\mathbf{q}\cdot(\mathbf{g}'-\mathbf{g})}}{4J \sum_{j=1}^{3} \cos[\frac{K_j}{2}] \cos[q_j] + 2D - E}$$



Solution which is **odd** under inversion
is the first instability when approaching from large-D limit.
Indicates chiral Mott phase.
[Single-particle condensation occurs at D=3J.]

Schwinger boson representation of S=1

$$S^z_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^z \mathbf{b}_{\mathbf{r}} = b^\dagger_{\mathbf{r}\uparrow} b_{\mathbf{r}\uparrow} - b^\dagger_{\mathbf{r}\downarrow} b_{\mathbf{r}\downarrow},$$
$$S^+_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^+ \mathbf{b}_{\mathbf{r}} = \sqrt{2}(b^\dagger_{\mathbf{r}\uparrow} b_{\mathbf{r}0} + b^\dagger_{\mathbf{r}0} b_{\mathbf{r}\downarrow}),$$
$$S^-_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^- \mathbf{b}_{\mathbf{r}} = \sqrt{2}(b^\dagger_{\mathbf{r}\downarrow} b_{\mathbf{r}0} + b^\dagger_{\mathbf{r}0} b_{\mathbf{r}\uparrow}).$$

Large-**D** limit: b_0 is condensed, $\langle b_{\mathbf{r}0} \rangle = s$, $b_{\mathbf{r}\uparrow,\downarrow}$ are excitations about the vacuum.

$$\bar{\mathcal{H}}_{sw} = \sum_{\boldsymbol{k},\sigma} (\mu + s^2 \epsilon_{\boldsymbol{k}}) b^\dagger_{\boldsymbol{k}\sigma} b_{\boldsymbol{k}\sigma} + N(\mu - D)(s^2 - 1)$$
$$+ \sum_{\boldsymbol{k},\sigma} \frac{s^2 \epsilon_{\boldsymbol{k}}}{2} (b^\dagger_{\boldsymbol{k}\sigma} b^\dagger_{-\boldsymbol{k}\bar{\sigma}} + h.c.),$$

$$b_{\boldsymbol{k}\sigma} = u_{\boldsymbol{k}} \gamma_{\boldsymbol{k}\sigma} + v_{\boldsymbol{k}} \gamma^\dagger_{-\boldsymbol{k}\bar{\sigma}},$$

$$u_{\boldsymbol{k}} = (\mu + \omega_{\boldsymbol{k}}) / \left(2\sqrt{\mu \omega_{\boldsymbol{k}}}\right),$$
$$v_{\boldsymbol{k}} = (\mu - \omega_{\boldsymbol{k}}) / \left(2\sqrt{\mu \omega_{\boldsymbol{k}}}\right),$$
$$\omega_{\boldsymbol{k}} = \sqrt{\mu^2 + 2\mu s^2 \epsilon_{\boldsymbol{k}}}.$$

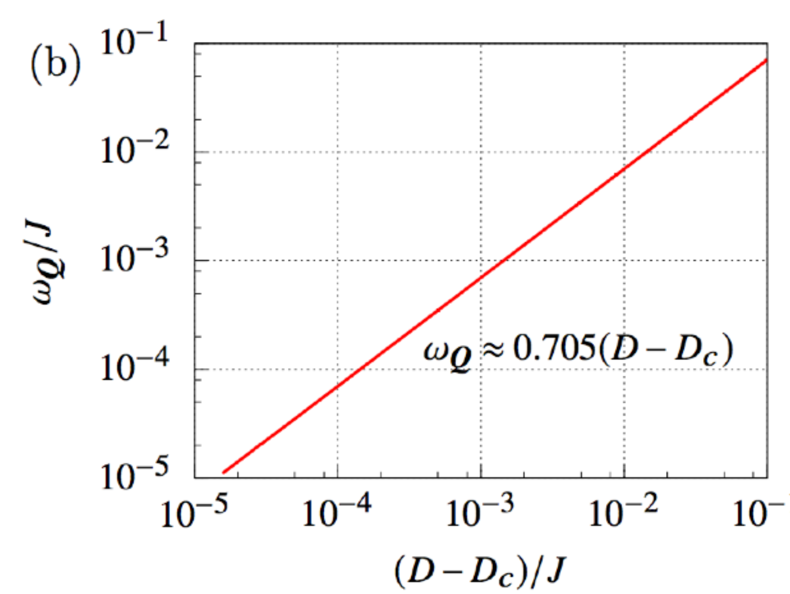
This accounts for quantum fluctuations

$$\bar{\mathcal{H}}_{sw} = N(\mu - D)(s^2 - 1) + \sum_{\boldsymbol{k}\sigma} \left[\omega_{\boldsymbol{k}} (\gamma^\dagger_{\boldsymbol{k}\sigma} \gamma_{\boldsymbol{k}\sigma} + \frac{1}{2}) - \frac{\mu}{2} \right]$$

Magnon interaction comes from Ising part of the exchange

$$\mathcal{H}^{(4)}_{\mathrm{I}} = \zeta J \sum_{\boldsymbol{r},\nu} (n_{\boldsymbol{r}\uparrow} - n_{\boldsymbol{r}\downarrow})(n_{\boldsymbol{r}+\boldsymbol{e}_\nu\uparrow} - n_{\boldsymbol{r}+\boldsymbol{e}_\nu\downarrow})$$

$$\zeta = J_z / J$$



Interaction between magnons

$$\mathcal{H}_{\mathrm{I}}^{(4)} = \frac{1}{N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q}} V_{\boldsymbol{q}}^{22o}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma^{\dagger}_{\boldsymbol{k}_1+\boldsymbol{q}\uparrow}\gamma^{\dagger}_{\boldsymbol{k}_2-\boldsymbol{q}\downarrow}\gamma_{\boldsymbol{k}_2\downarrow}\gamma_{\boldsymbol{k}_1\uparrow}$$

$$+\frac{1}{N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q},\sigma} V_{\boldsymbol{q}}^{22s}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma^{\dagger}_{\boldsymbol{k}_1+\boldsymbol{q}\sigma}\gamma^{\dagger}_{\boldsymbol{k}_2-\boldsymbol{q}\sigma}\gamma_{\boldsymbol{k}_2\sigma}\gamma_{\boldsymbol{k}_1\sigma}$$

$$+\frac{1}{N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q},\sigma}\Big[V_{\boldsymbol{q}}^{31}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma^{\dagger}_{\boldsymbol{k}_1+\boldsymbol{q}\sigma}\gamma_{\boldsymbol{k}_1\sigma}\gamma_{\boldsymbol{k}_2\sigma}\gamma_{-\boldsymbol{k}_2+\boldsymbol{q}\bar{\sigma}}$$

$$+V_{\boldsymbol{q}}^{40}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma_{-\boldsymbol{k}_1-\boldsymbol{q}\uparrow}\gamma_{-\boldsymbol{k}_2+\boldsymbol{q}\downarrow}\gamma_{\boldsymbol{k}_2\downarrow}\gamma_{\boldsymbol{k}_1\uparrow}+h.c.\Big]$$

Number conserving
2 -> 2

Non-conserving
3 -> 1, 1 -> 3
4 -> 0, 0 -> 4

Chiral order parameter

$$\kappa = \frac{1}{N} \sum_{\boldsymbol{r} \in \nabla} \hat{z} \cdot \sum_{j=1}^{3} \langle \boldsymbol{S}_{\boldsymbol{r}} \times \boldsymbol{S}_{\boldsymbol{r}+\boldsymbol{e}_j} \rangle$$

Vector chirality

$$\kappa = -\frac{1}{N} \sum_{\boldsymbol{q}} \sum_{j=1}^{3} \sin[\boldsymbol{q} \cdot \boldsymbol{e}_j] \langle S^{+}_{\boldsymbol{q}} S^{-}_{-\boldsymbol{q}} \rangle$$

q-space

Total spin S^z=0,
Odd under **Q** -> - **Q**

$$\kappa = -\frac{3\sqrt{3}\mu s^2}{N\omega_{\boldsymbol{Q}}} \langle \gamma^{\dagger}_{-\boldsymbol{Q}\uparrow} \gamma^{\dagger}_{\boldsymbol{Q}\downarrow} - \gamma^{\dagger}_{\boldsymbol{Q}\uparrow} \gamma^{\dagger}_{-\boldsymbol{Q}\downarrow} + \text{h.c.} \rangle$$

Low-energy approximation

$$\phi_L(\boldsymbol{k}) \equiv \gamma_{\boldsymbol{Q}-\boldsymbol{k}\uparrow} \gamma_{\bar{\boldsymbol{Q}}+\boldsymbol{k}\downarrow}$$
$$\phi_R(\boldsymbol{k}) \equiv \gamma_{\bar{\boldsymbol{Q}}+\boldsymbol{k}\uparrow} \gamma_{\boldsymbol{Q}-\boldsymbol{k}\downarrow}$$

Boson pair operators

$$\kappa = -\frac{3\sqrt{3}\mu s^2}{N\omega_{\boldsymbol{Q}}} \left(\phi_R^* + \phi_R - \phi_L^* - \phi_L \right)$$

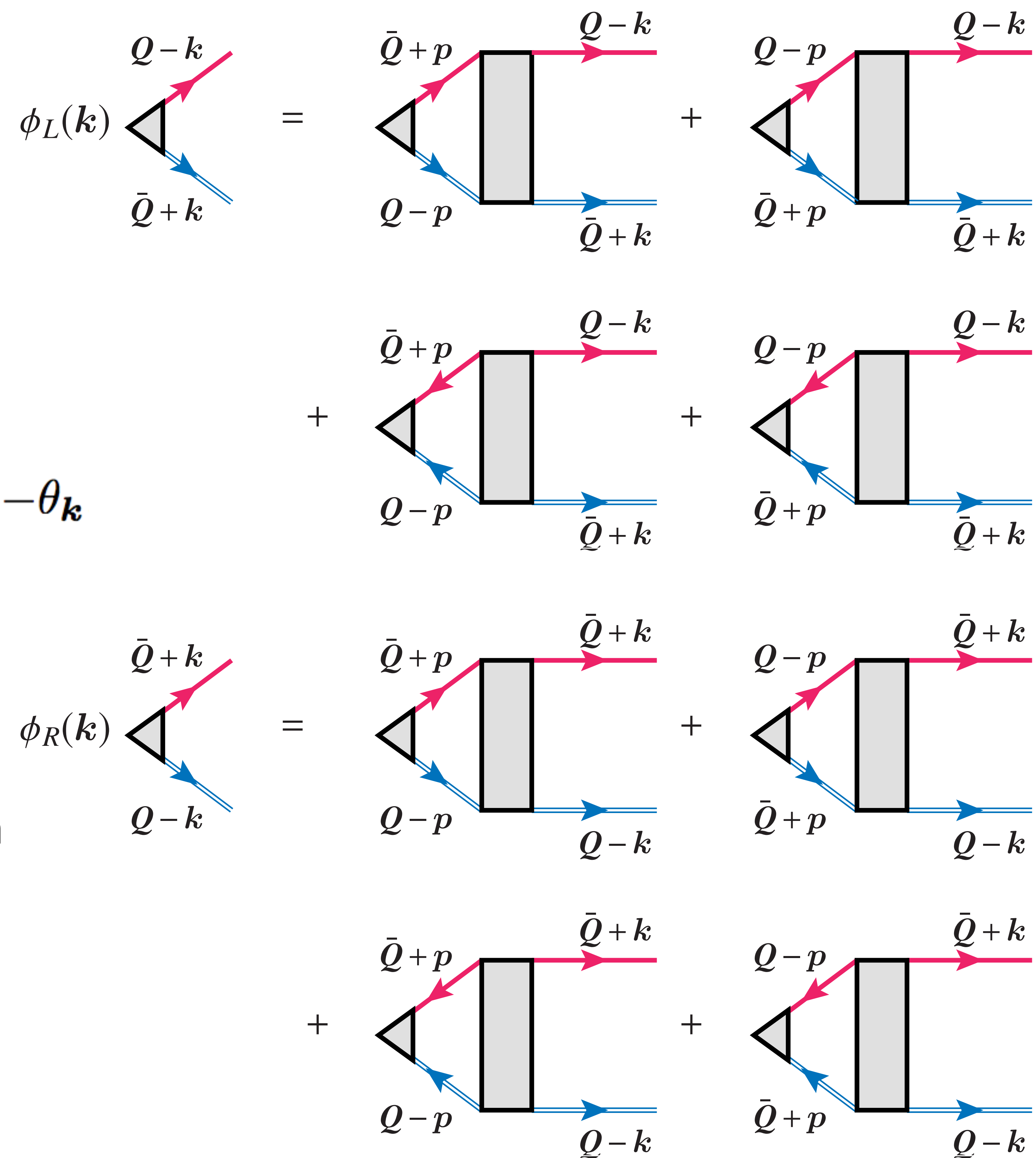
$$\theta_{\boldsymbol{k}} \equiv 2\omega_{\boldsymbol{Q}-\boldsymbol{k}} \langle \phi_R(\boldsymbol{k}) - \phi_L(\boldsymbol{k}) \rangle$$

Convenient parameterization

Integral equation for pair vertices

$$\frac{1}{N}\sum_{\boldsymbol{p}} \frac{F^{22o}_{\boldsymbol{k},\boldsymbol{p}}\,\theta_{\boldsymbol{p}} - 4F^{04}_{\boldsymbol{k},\boldsymbol{p}}\,\theta^{*}_{\boldsymbol{p}}}{2\omega_{\boldsymbol{Q}-\boldsymbol{p}}} = -\theta_{\boldsymbol{k}}$$

Shaded rectangles denote
fully dressed
Irreducible interactions between
Low-energy magnons



First order in $J_z = \zeta J$

Interaction is given by bare vertices $$F^{22o}_{\boldsymbol{k},\boldsymbol{p}} = -4F^{04}_{\boldsymbol{k},\boldsymbol{p}} \approx -\frac{9}{4} J_z \frac{(\omega_{\boldsymbol{Q}-\boldsymbol{p}} + \omega_{\boldsymbol{Q}-\boldsymbol{k}})^2}{\omega_{\boldsymbol{Q}-\boldsymbol{p}}\omega_{\boldsymbol{Q}-\boldsymbol{k}}}.$$

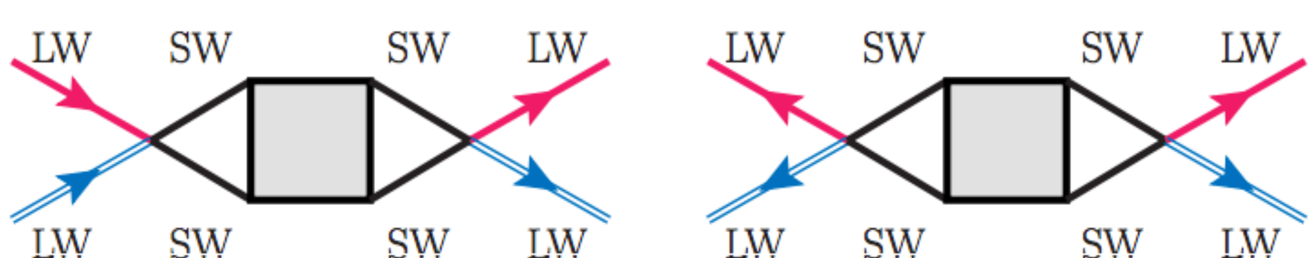
Obtain for the 2-magnon instability $$1 = a\frac{J_z}{N}\sum_{\boldsymbol{p}} \frac{1}{\omega_{\boldsymbol{Q}-\boldsymbol{p}}}.$$

No weak-coupling instability !
Interaction vertices are of order 1 (in units of J_z) and are not singular :-(

$$A_{\boldsymbol{k}_1,\boldsymbol{k}_2} \equiv u_{\boldsymbol{k}_1}u_{\boldsymbol{k}_2} - v_{\boldsymbol{k}_1}v_{\boldsymbol{k}_2} = \frac{\omega_{\boldsymbol{k}_1} + \omega_{\boldsymbol{k}_2}}{2\sqrt{\omega_{\boldsymbol{k}_1}\omega_{\boldsymbol{k}_2}}},$$
$$B_{\boldsymbol{k}_1,\boldsymbol{k}_2} \equiv u_{\boldsymbol{k}_1}v_{\boldsymbol{k}_2} - v_{\boldsymbol{k}_1}u_{\boldsymbol{k}_2} = \frac{\omega_{\boldsymbol{k}_1} - \omega_{\boldsymbol{k}_2}}{2\sqrt{\omega_{\boldsymbol{k}_1}\omega_{\boldsymbol{k}_2}}}.$$

$\mathbf{k}_{1,2}$ near $\pm\mathbf{Q}$

Need to renormalize it!

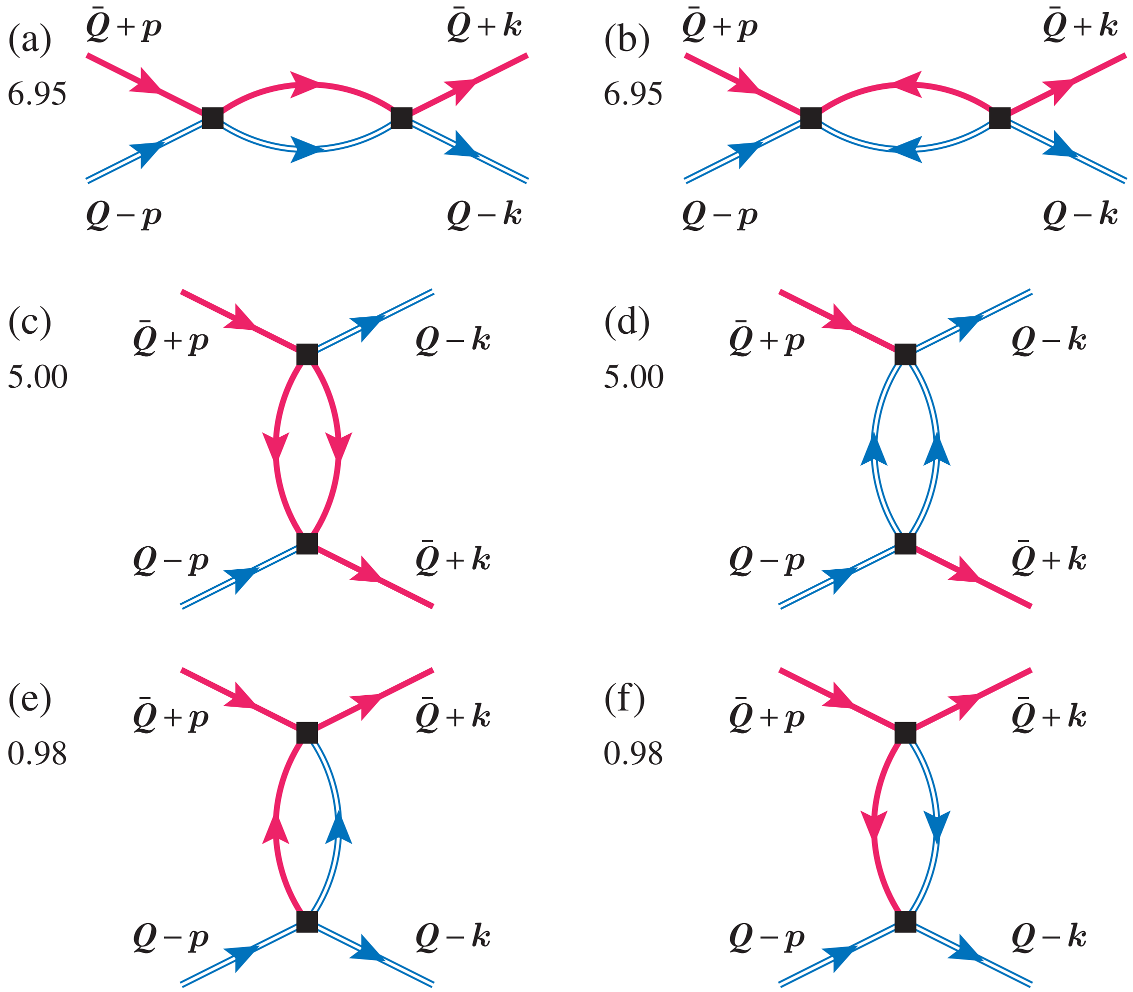


LW = Long wavelength, SW = short wavelength

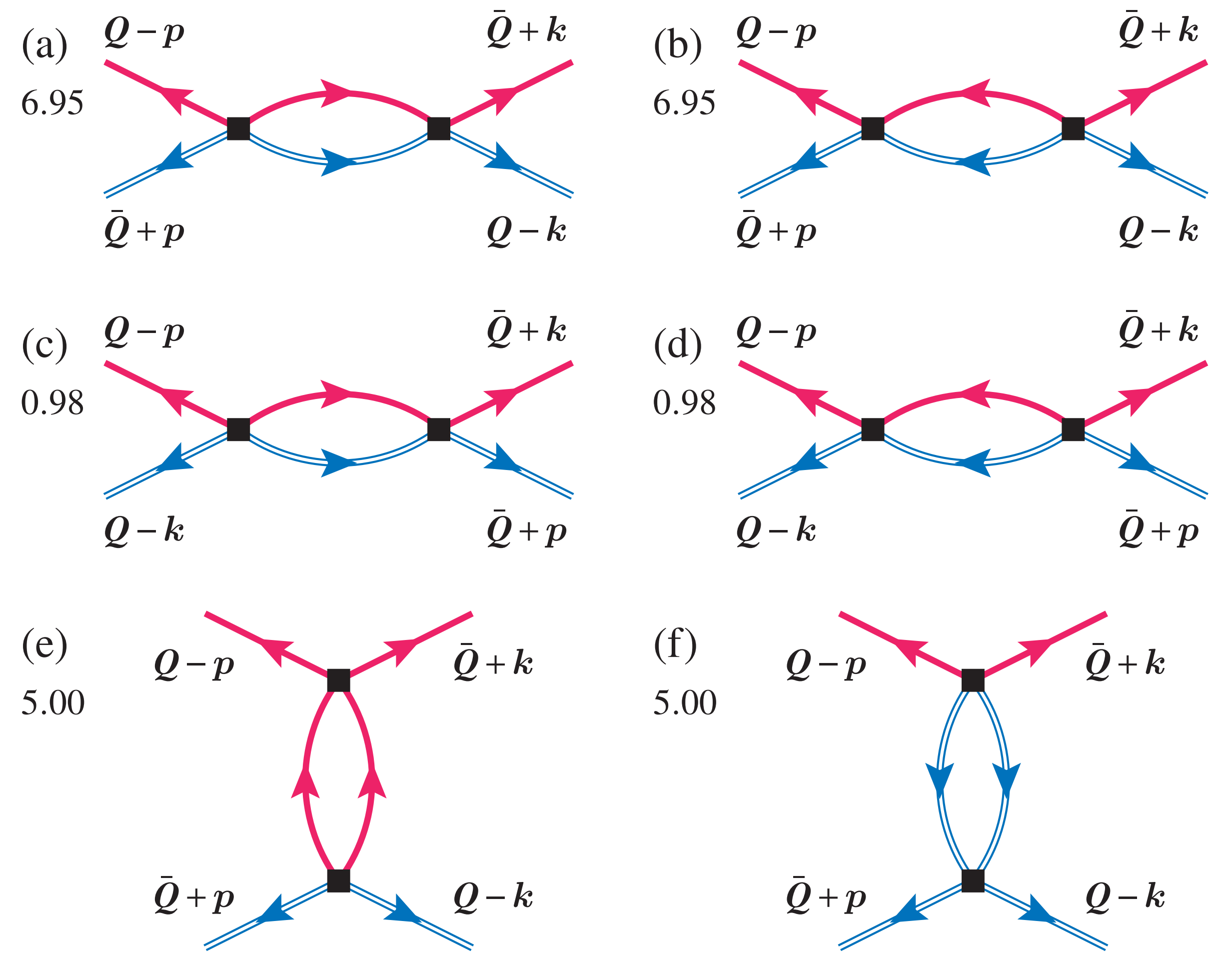
To find the dressed interaction, we have to go to the 2nd order in J_z …

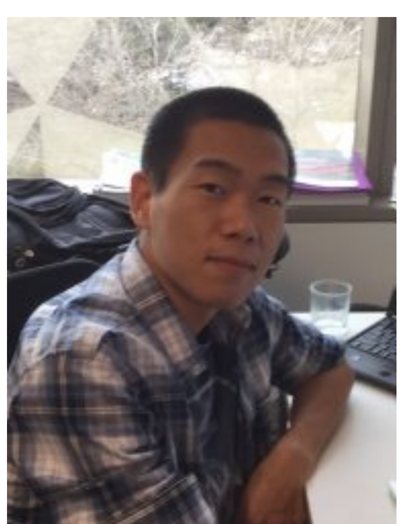
Kohn-Luttinger like mechanism but for bosons

F^{22}



F^{04}





The result

Pair vertex is **real**, renormalized interaction is **singular**

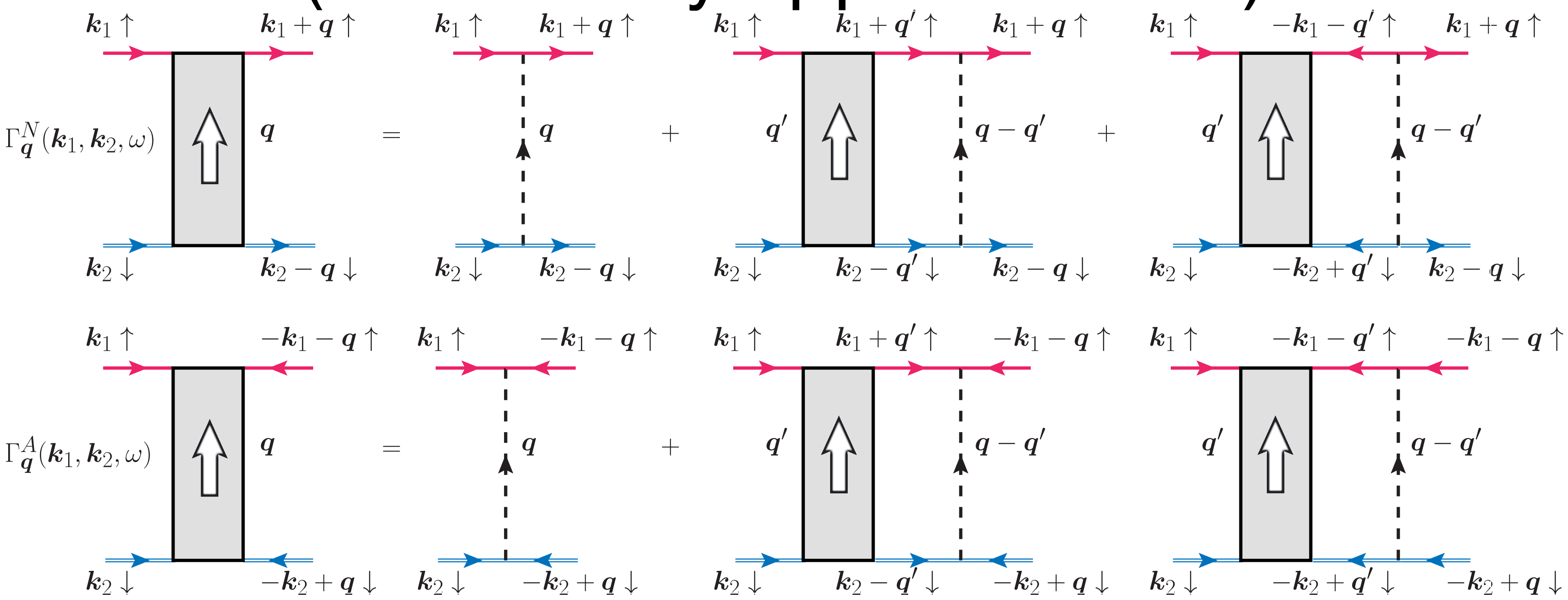
$$\frac{1}{N}\sum_{\boldsymbol{p}}\frac{\alpha\zeta^2 J^3}{\omega^2_{\boldsymbol{Q}-\boldsymbol{p}}\omega_{\boldsymbol{Q}-\boldsymbol{k}}}\theta_{\boldsymbol{p}} = \theta_{\boldsymbol{k}} \qquad \alpha = 2.49$$

$$\frac{1}{\alpha\zeta^2 J^3} = \frac{1}{N}\sum_{\boldsymbol{p}}\frac{1}{\omega^3_{\boldsymbol{Q}-\boldsymbol{p}}} \approx \frac{1}{N}\sum_{\boldsymbol{p}}\frac{1}{(\omega^2_{\boldsymbol{Q}} + 9J^2 s^4 p^2)^{3/2}}$$

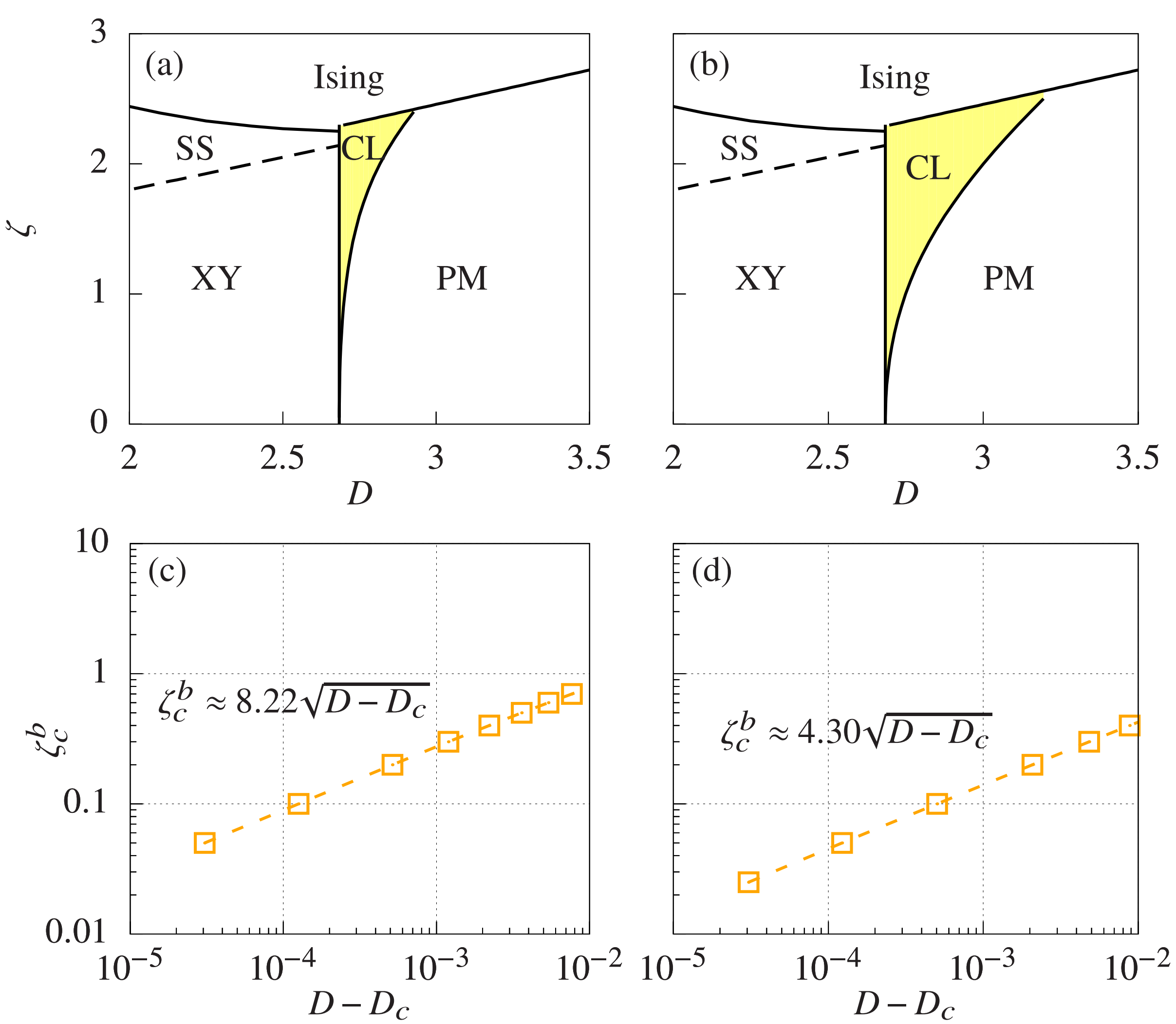
Two magnon condensation takes place **before** the single magnon one:

$$D_c^b = D_c + 0.042\alpha\zeta^2 J$$

More checks: Bethe-Salpeter equation (low-density approximation)

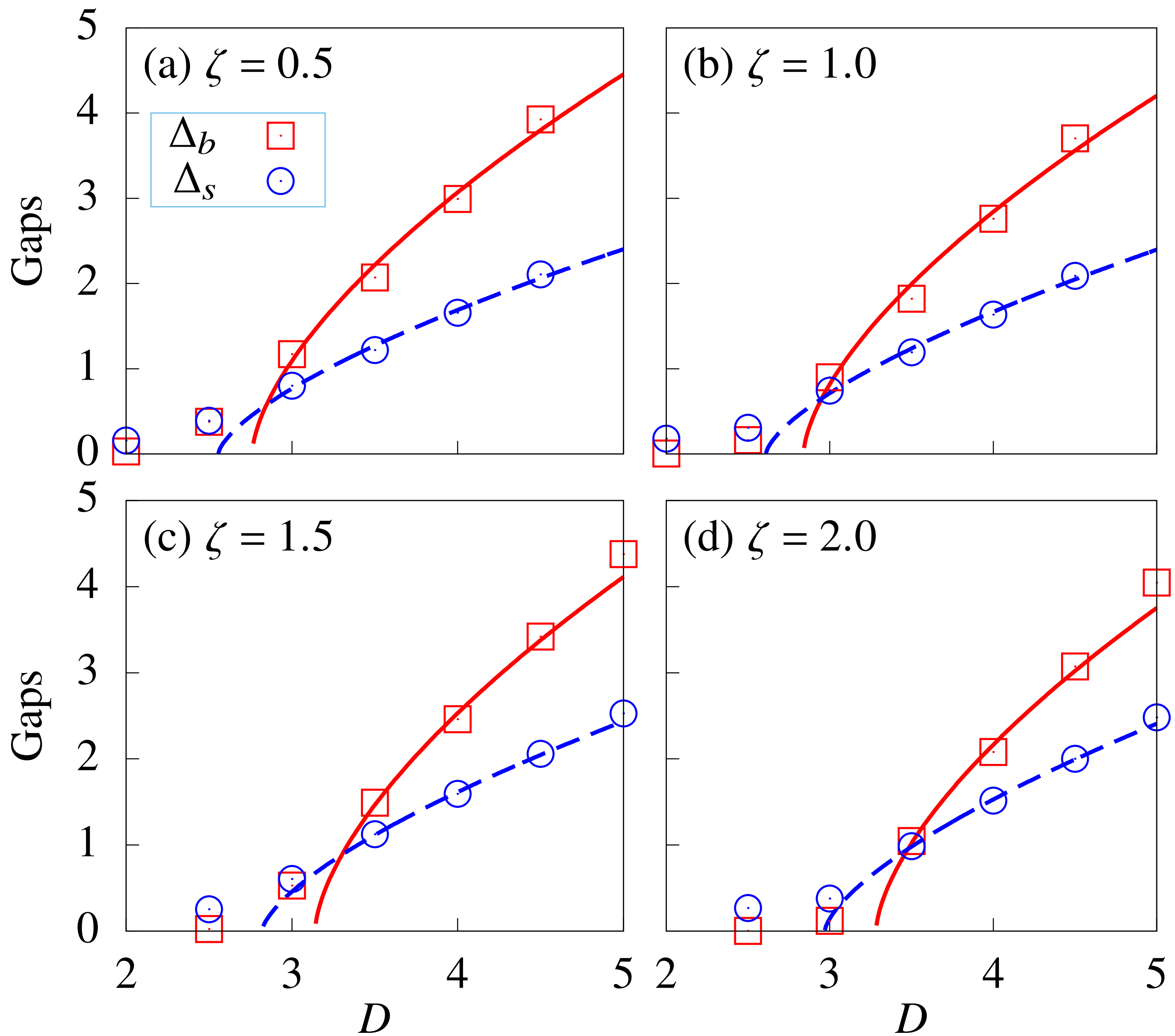


Only normal (2 -> 2) vertices



Full problem

DMRG on 6x6 triangular lattice



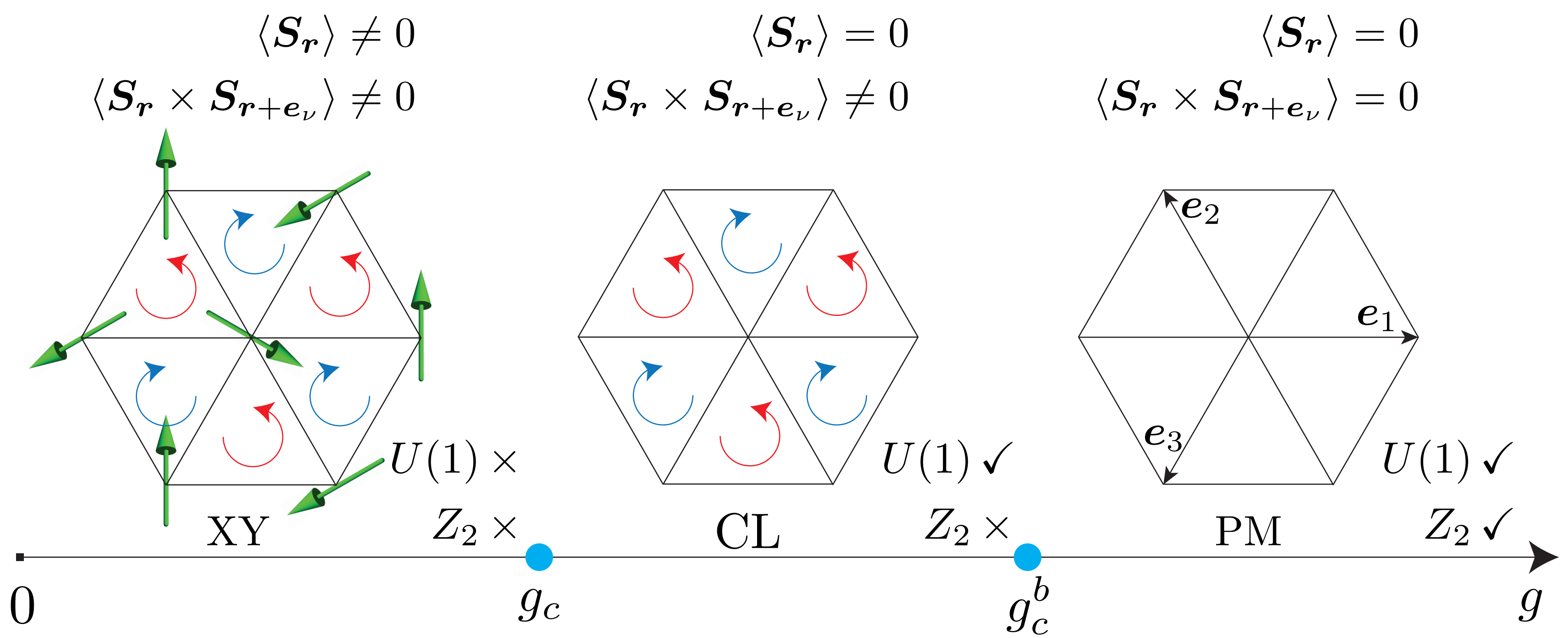
Gap crossing:
VC order via Ising transition
before
U(1) order via XY transition

Two-magnon gap $\Delta_b = E^{(1)}_{S_z=0} - E^{(0)}_{S_z=0} = c_b(D - D_b)^{\nu_{\text{Ising}}},$

Single magnon gap $\Delta_s = E^{(0)}_{S_z=1} - E^{(0)}_{S_z=0} = c_s(D - D_s)^{\nu_{\text{XY}}},$

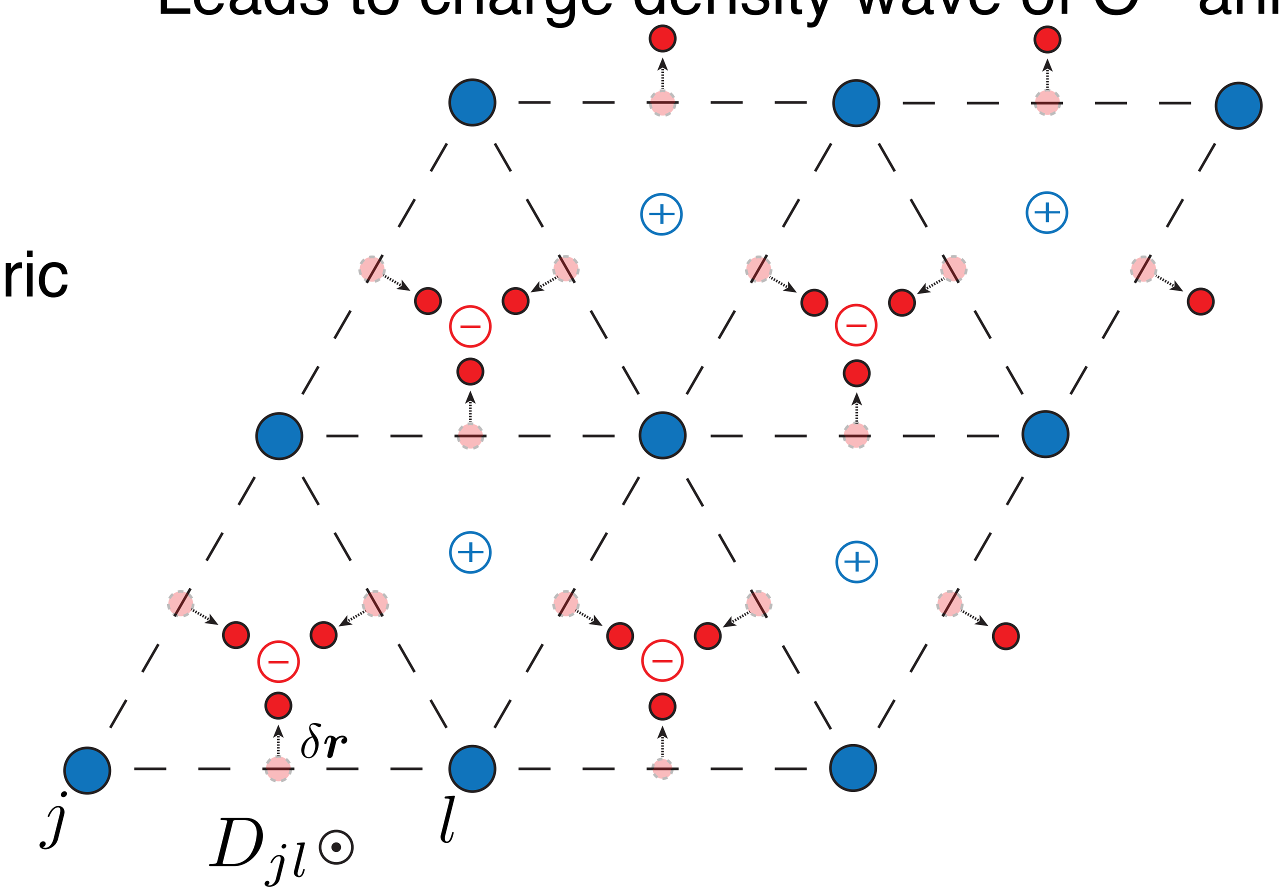
$$\nu_{\text{Ising}} = 0.63, \nu_{\text{XY}} = 0.67$$

Summary



Chiral liquid can be detected via inverse Dzyaloshinskii-Moriya effect:
Leads to charge density wave of O^{2-} anions

Magneto-electric phenomena



Conclusions

Mott -> **superfluid** transition in frustrated lattice requires U(1) x Z_2 breaking.

This proceeds via intermediate **spin-current** (**chiral Mott**) phase (breaking Z_2 only).

$$|\Psi_{\rm CL}\rangle \sim e^{u \sum_{\boldsymbol{k}} \phi(\boldsymbol{k}) S^+_{\boldsymbol{k}} S^-_{\boldsymbol{k}}} |0\rangle$$

Spontaneously breaks spatial inversion.

$$\phi(\mathbf{k}) = -\phi(-\mathbf{k})$$

But preserves time-reversal $u \in \mathbb{R}$

All single particle excitations are gapped.

Thank you!

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Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersesyan,[1] Alexander O. Gogolin,[2] and Fabian H.L. Eßler[3]

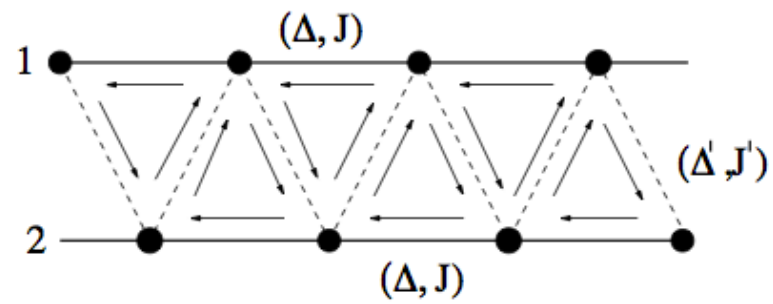

FIG. 3. Structure of the spin currents in the spin nematic phase.

PHYSICAL REVIEW B **87**, 174501 (2013)

Chiral Mott insulator with staggered loop currents in the fully frustrated Bose-Hubbard model

Arya Dhar,[1] Tapan Mishra,[2] Maheswar Maji,[3] R. V. Pai,[4] Subroto Mukerjee,[3,5] and Arun Paramekanti[2,3,6,7]

Spin-current phase = chiral Mott insulator

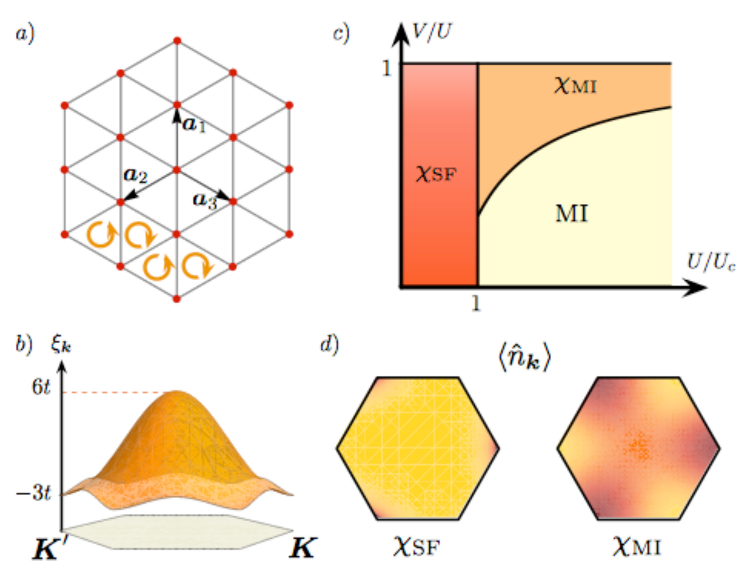

FIG. 1. **Bosons on the Frustrated Triangular Lattice.** (a) Lattice, coordinate system and sample current pattern in the χMI; (b) single-particle dispersion ξ_k, with minima at the K, K' points of the BZ; (c) Variational mean-field phase diagram showing χSF, χMI and MI phases tuned by the on site repulsion U and nearest neighbor repulsion V; (d) Momentum distribution $\langle \hat{n}_k \rangle$ for the chiral phases.

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Chiral bosonic Mott insulator on the frustrated triangular lattice

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gapped single particles;
but
spontaneously broken time-reversal
= spontaneous circulating
currents