Dynamical Generation of Topological Masses in Dirac Fermions

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arXiv:1705.09192

Questions

Consider a system where nontrivial topology is prohibited in the weak coupling limit (e.g. by symmetry constraints)

- How to use strong interactions to obtain a topological state?
- Can this topological phase transitions be second order?
- If yes, what are the scaling exponents?

Solution:

- Multiple pathways has been proposed
- One of them:

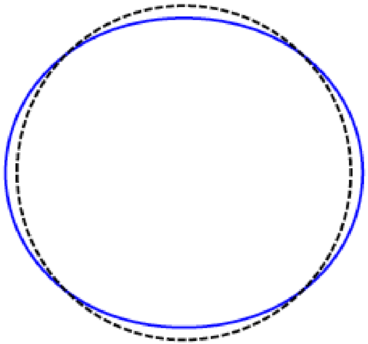
  From Fermi liquid instabilities to correlated topological states

Outline

- Instabilities in Fermi liquid
- Connections to topology
- Challenges and solutions: sign-problem free QMC

Instabilities in Fermi liquid

- BCS instability and superconductivity
  - Infinitesimal instability
  - Weak coupling theory
  - Well controlled and well understood.
- Other instabilities: e.g. Pomeranchuck instability



- Strong attractions in the angular moment channel $l = 2$
- Distortion of the Fermi surface
- Break rotational symmetry

- Key challenge:
  - Require the coupling strength to reach a threshold (i.e. strong coupling)
  - Strong-coupling nature makes it a very challenging problem
    - Competing orders: for a given system, whether or not a nematic phase will arise?
    - How to characterize the phase transitions, and what are the correct critical theory?

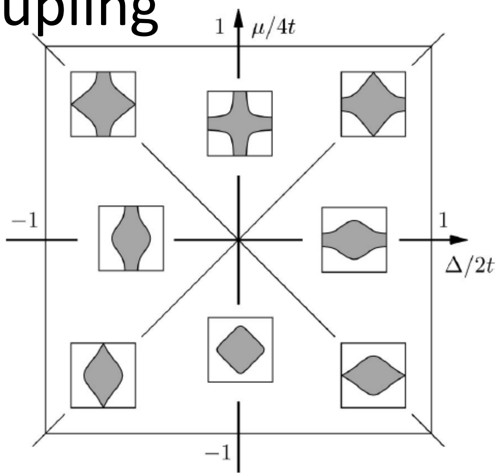
Possible approaches

- Making the phase transition arises at weak coupling

  Less competing orders to worry about

  - e.g. using van Hove singularity
  - Pomeranchuck (nematic) instability: infinitesimal

    (Khavkine, Chung, Oganesyan and Kee, PRB 2004)



- Low-energy effective theory:

$$S = S_{\text{Free Fermions}} + S_{\text{Boson }\phi^4} + S_{\text{Boson-Fermion coupling}}$$

- Relapses fermion interactions with a boson mediated interactions
  - Bosonic field: can be viewed as a Hubbard–Stratonovich auxiliary field
  - Bosonic field: severs as the order parameter
  - Bosonic field couples to fermion bilinears according to symmetry
- Naturally favors this specific order (over other competing orders)
- Still a strongly-correlated problem
- Unbiased and controlled results are challenging

An extra advantage: quantum Monte Carlo

One key challenge in QMC simulations: the sign problem

- Very effective for some systems if it is sign-problem free (e.g. Wu and Zhang 2005)
- Very inefficient for systems with the sign problem
- Interacting Fermions: interesting phases is often accompanied by the sign-problem

One solution:

- Replace the fermion interactions with a boson-mediated interactions

$$S = S_{\text{Free Fermions}} + S_{\text{Boson } \phi^4} + S_{\text{Boson-Fermion coupling}}$$

- In many cases, these type of models are sign-problem free

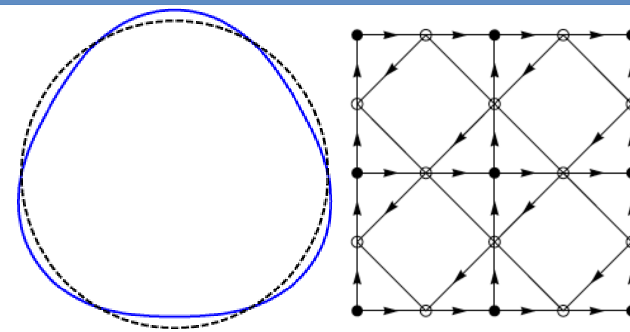
This action is the effective theory we discussed early on

Bottom line: QMC may not offer a way to solve e.g. the Hubbard model, but most effective theory that we considered in theoretical studies are sign-problem free.

Earlier talks in this program: Yao, Schattner, Meng

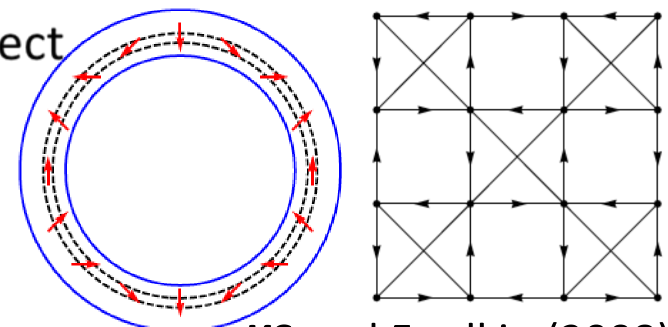
Pomeranchuck instability beyond nematic

- Break time-reversal symmetry:
  - Odd angular momentum channels (e.g. $l = 3$)
  - Corresponds to certain loop order states



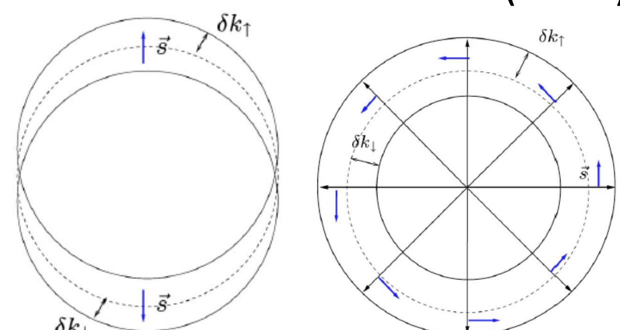
KS and Fradkin (2008)

- Break time-reversal and chiral symmetries
  - Spontaneously generated anomalous Hall effect
  - Inter-band Pomeranchuck instability
  - Require multi-orbitals (multi-bands)



KS and Fradkin (2008)

- Break SU(2) spin rotational symmetry
  - Spontaneously generated SO coupling
  - Pomeranchuck in the spin triplet channel



Wu and Zhang (2004)
Wu, **KS**, Fradkin ,Zhang (2008)

Necessary ingredients 2D Topological insulators (QHI and QSHI)

Topology from spontaneous symmetry breaking

Consider a 2D system with

- time-reversal and chiral symmetries
- weak spin-orbit couplings

Non-interacting regime:

- The system cannot be a QHI or a QSHI

How about Strong coupling:

- Is that possible to turn the system into a QHI or QSHI at strong coupling?

Answer: Yes.

- Using strong couplings to trigger a quantum phase transition, which breaks the symmetry that prevent a TI state

Pomeranchuck instability beyond nematic

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  - Odd angular momentum channels (e.g. $l = 3$)
  - Corresponds to certain loop order states
- Break time-reversal and chiral symmetries
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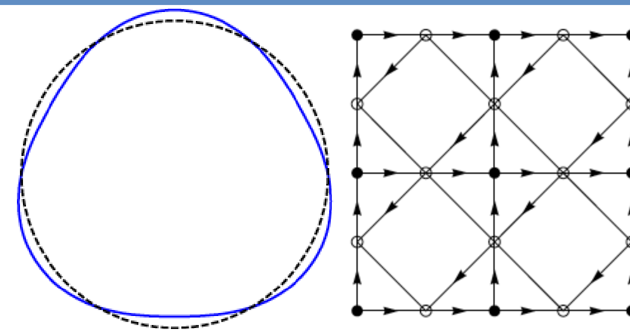

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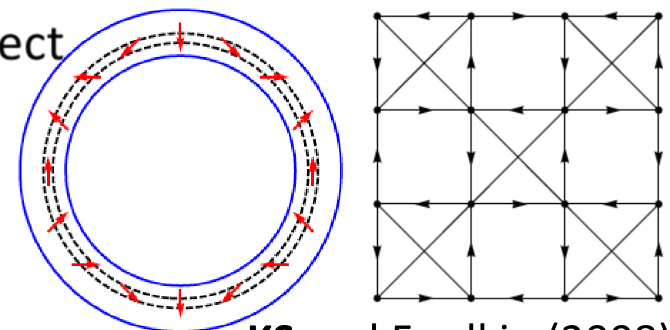

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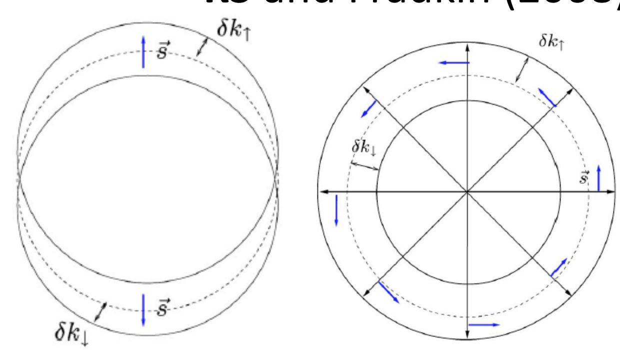
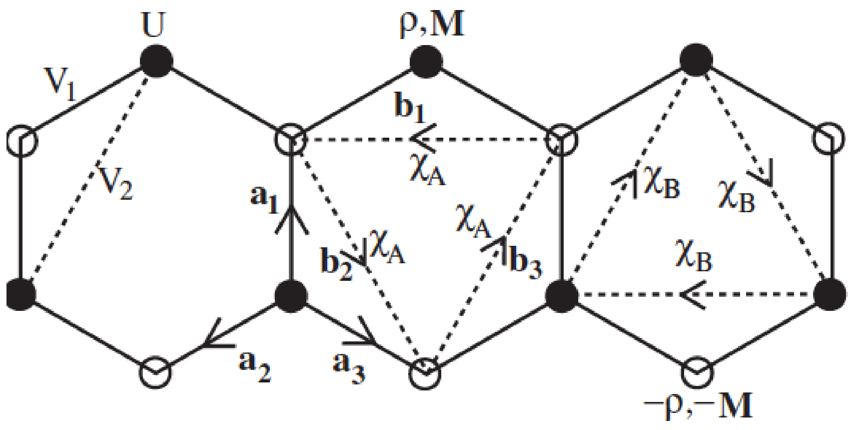


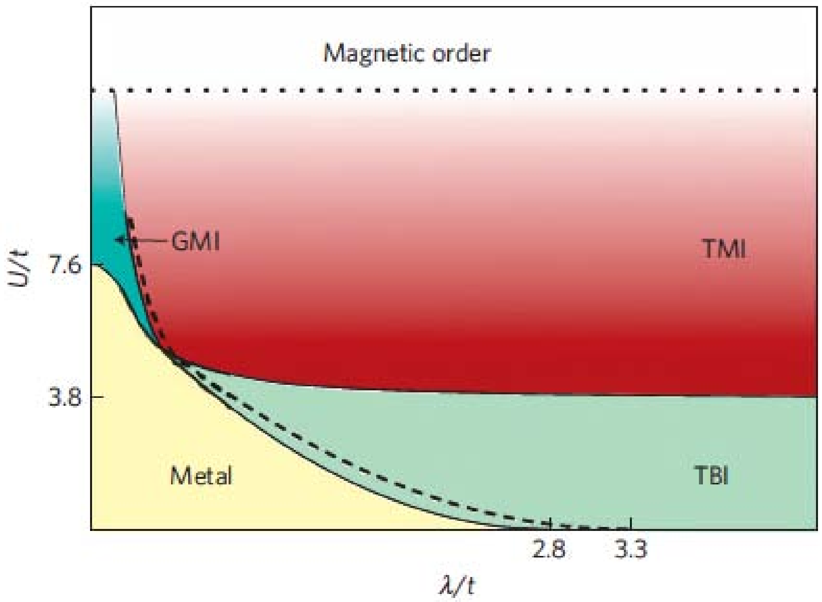
Wu and Zhang , 2004

Wu, **KS**, Fradkin ,Zhang (2008)

Topological Mott insulators

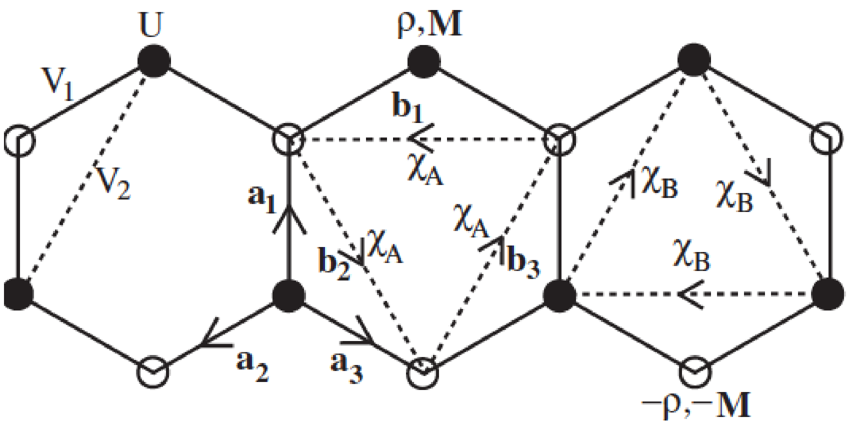


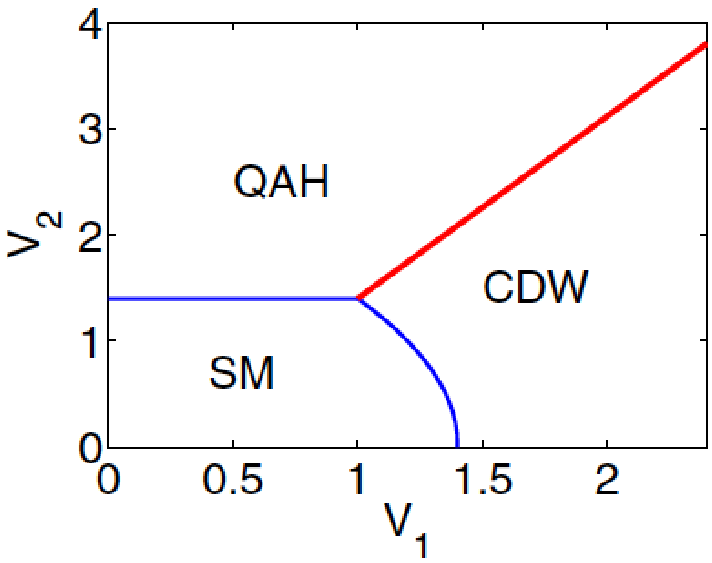
Raghu, Qi, Honerkamp and Zhang, PRL, (2008)



Pesin and Balents, Nat. Phys. (2010)

Topological Mott insulators





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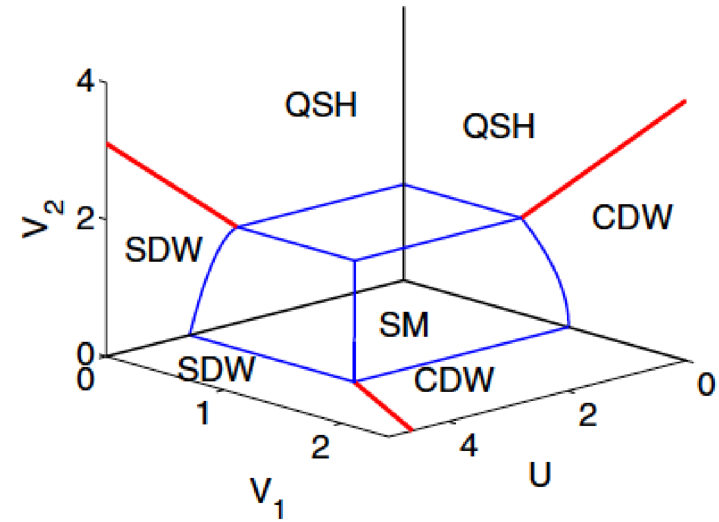


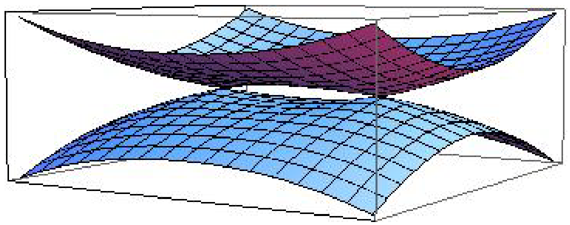
Challenges

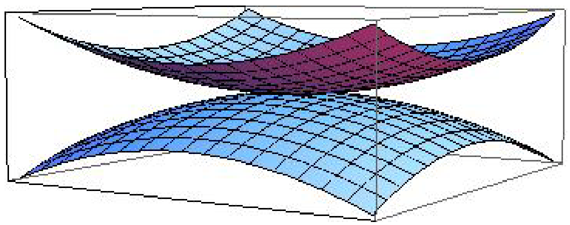
Same as the Pomeranchuck instability, requires strong coupling

- Competing orders
  - For a specific system, it is challenging to determine whether TMI is the preferred ground state or not (from unbiased methods)
  - Numerical efforts (exact diagonalization) report native results
- Detailed knowledge:
  - First order or second order
  - Scaling exponents

One alternative approach

Quadratic band crossing point:





$$S = \int dt d^d\mathbf{r}\ \bar{\Psi} i \left(\gamma_0 \partial_0 - \gamma_1 \partial_x - \gamma_2 \partial_y\right) \Psi$$

$$S = \int dt d^d\mathbf{r}\ \bar{\Psi} \left[\gamma_0 \left(i\partial_0 + t_0 \nabla^2\right) + \gamma_1 t_1 \left(\partial_x^2 - \partial_y^2\right) + \gamma_2 \left(2\partial_x \partial_y\right)\right] \Psi$$

Shor-range repulsion:

- Irrelevant for Dirac points
- marginally relevant for QBCP

Instability for QBCP is infinitesimal like BCS

- Well controlled approaches
- Numerics: Wu, et. al. PRL, 2017

KS, Yao, Fradkin and Kivelson, (2009)

Dirac points?

- Can we get TMIs from the original proposal:

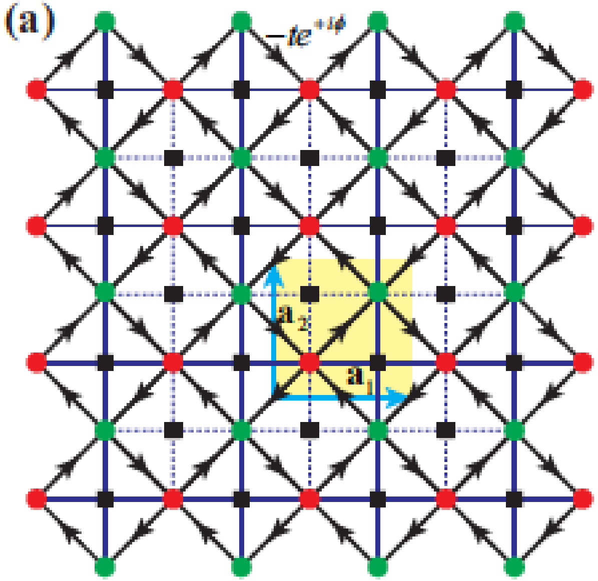
  Dirac points + interactions

- In comparison with QBCP
  - More theoretically challenge
  - May lead to a **quantum critical point** (transition for the QBCP is an essential singularity point, instead of a QCP)

Similar to Pomeranchuck instability,

- The original model suffers from competing order and the sign-problem
- Maybe we can borrow the same approach
  - Use bosons to mediate interactions
  - Making the model sign-problem free in QMC

Model



A square lattice with pi-flux (disks)

- ➢ Dirac points at the two X points

An Ising spin on each dual lattice points (squares)

- ➢ Transvers-field Ising model

Ising spins couple to the fermion NNN hopping strength

- ➢ Mediate interactions between fermions

Sign-problem free in QMC

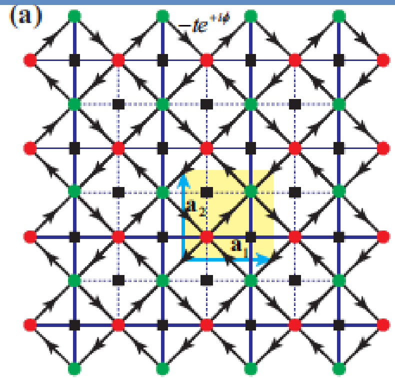
$$H = H_{\text{Fermion}} + H_{\text{Ising}} + H_{\text{Coupling}},$$

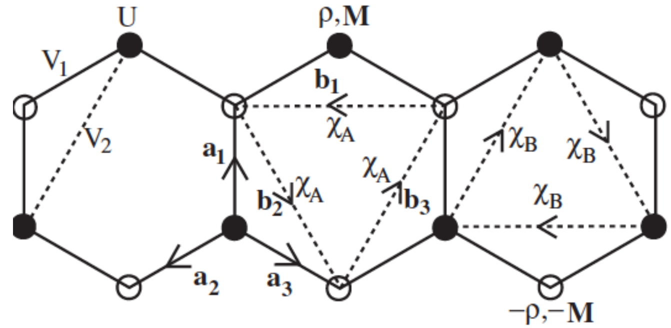
$$H_{\text{Fermion}} = -t \sum_{\langle ij \rangle \sigma} (e^{+i\sigma\phi} c^{\dagger}_{i\sigma} c_{j\sigma} + e^{-i\sigma\phi} c^{\dagger}_{j\sigma} c_{i\sigma}),$$

$$H_{\text{Ising}} = -J \sum_{\langle pq \rangle} s^z_p s^z_q - h \sum_p s^x_p,$$

$$H_{\text{Coupling}} = \sum_{\langle\langle ij \rangle\rangle \sigma} \xi_{ij} s^z_p (c^{\dagger}_{i\sigma} c_{j\sigma} + c^{\dagger}_{j\sigma} c_{i\sigma}).$$

Compare with previous models





Common features:

- ➢ Symmetry prohibits topological states (QSHI or QHI) at non-interacting regime
- ➢ Nontrivial topology can only emerge at strong coupling via many-body effects

Differences:

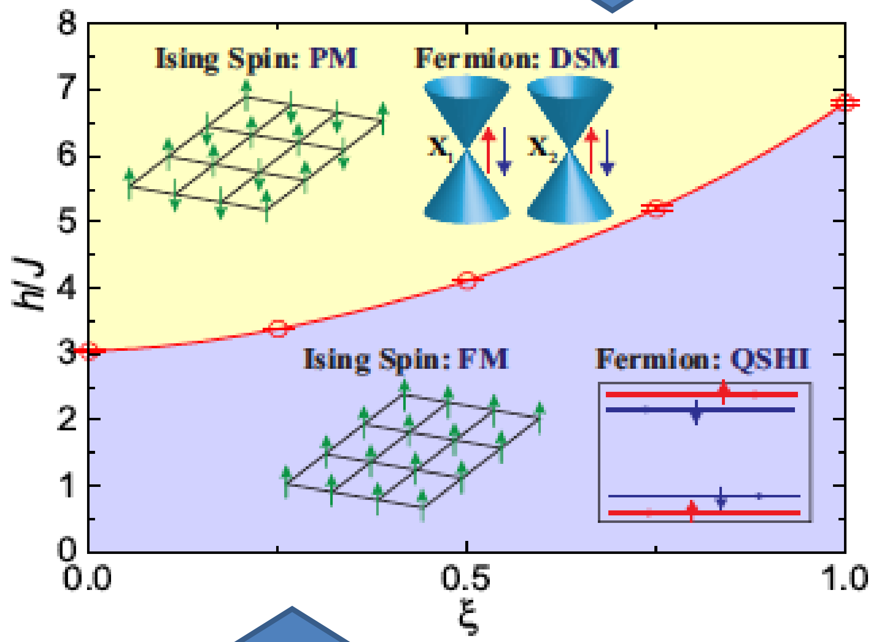
- ➢ Different lattice structures (irrelevant to topology)
- ➢ Different spin symmetry: Ising vs SU(2):
  - ❖ irrelevant for topology but changes the universality class at QCP
- ➢ Different interactions (boson mediated vs four-Fermi)
  - ❖ Competing order is suppressed by boson mediated interactions
- ➢ QMC sign problem
  - ❖ Large scale, unbiased simulation is possible

Same physics can now be studied with reliable method with less competing orders

Exactly solvable limits

At $\xi = 0$,
Bosons and fermions decouple
- Transvers Ising model
  - PM to FM transition
- Free Dirac electrons

At $h = +\infty$,
- Paramagnetic (with a large gapped)
- Weak coupling limit for fermions
  - Weakly-interacting Dirac semi-metal

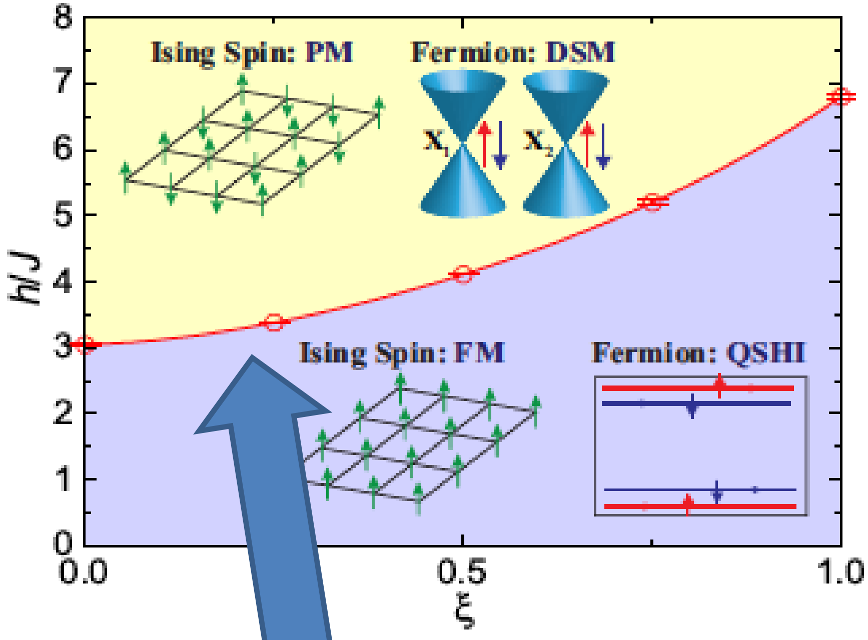


QMC:
- DSM and QSHI
- A direct second-order phase transition

At $h = 0$,
No quantum fluctuations for Ising spins
- An Ising ferromagnetic state
- FM order induce SO coupling for fermions: an QSH insulator

Universality Class

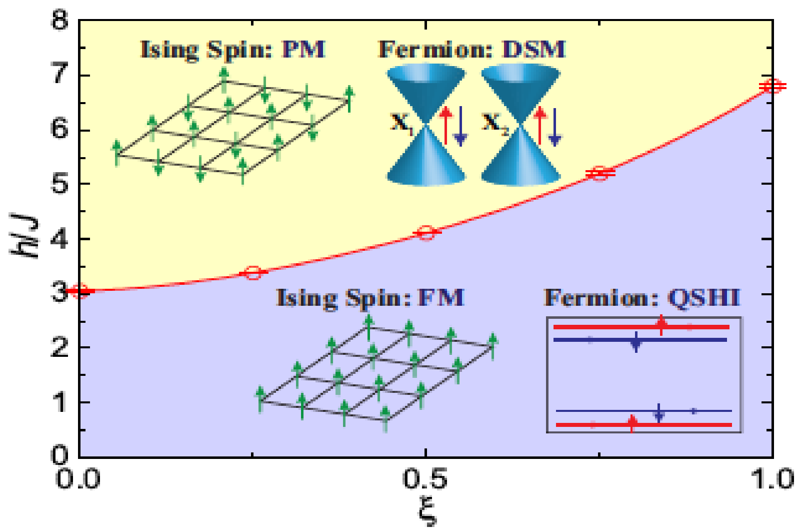
At $\xi = 0$, 2+1D Ising



Any where $\xi \neq 0$, N=8 components chiral-Ising universality class

Crossover at small ξ

	2+1D Ising	N=8 chiral Ising [1]	Our model
ν	0.630	0.83(1)	0.85(2)
η	0.036	0.62(1)	0.61(7)



2+1D Ising

N=8 Chiral Ising

Length scale

Cross-over

[1] Chandrasekharan and Li, PRD, 2013

Beyond sign-free QMC

Are these conclusions universal?

- What if we add extra terms that lead to sign-problem?

Topological phase is stable against any perturbations

- Fully gapped (both fermions and bosons)
- This is because our model has Ising symmetry (no Goldstone mode)

Phase transition

- Either remain second order or become first order
- If remains second order, should remain in the same universality class