

Hunds interaction, spin-orbit coupling and the mechanism of superconductivity in heavily hole-doped iron pnictides

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and

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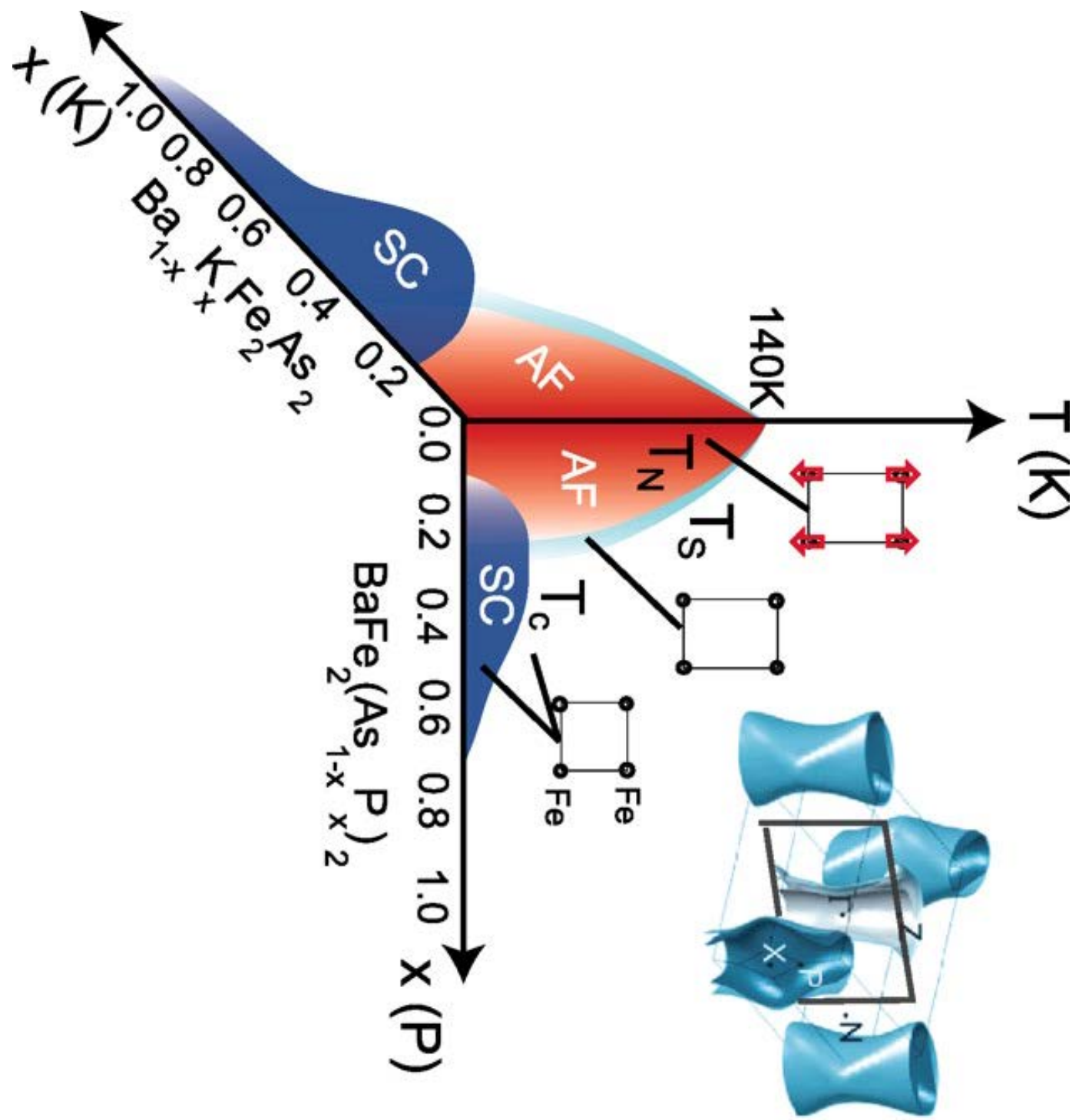
Tallahassee, FL

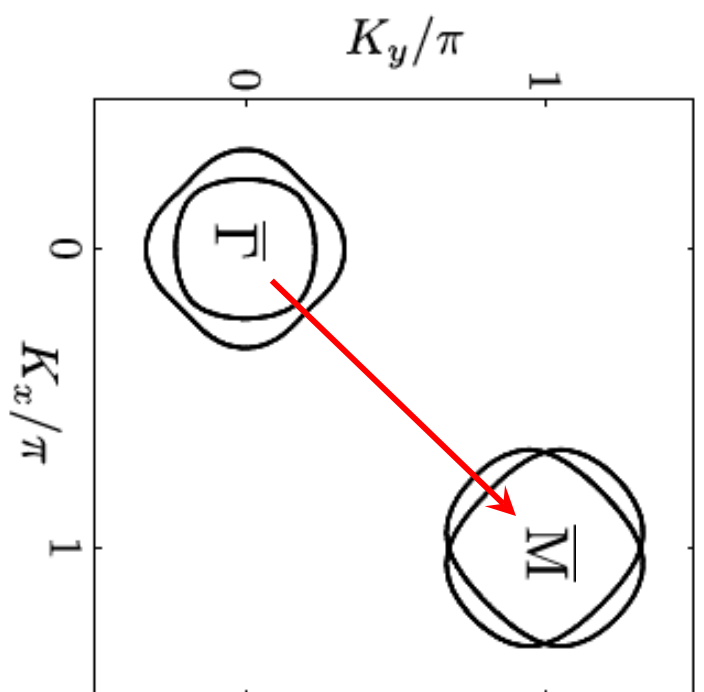
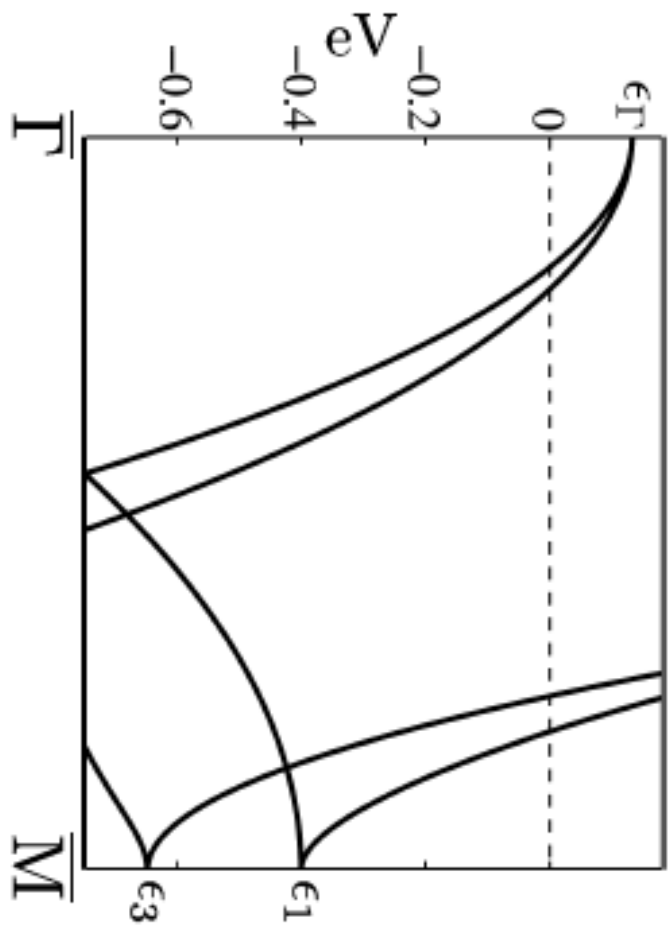


Collaborator

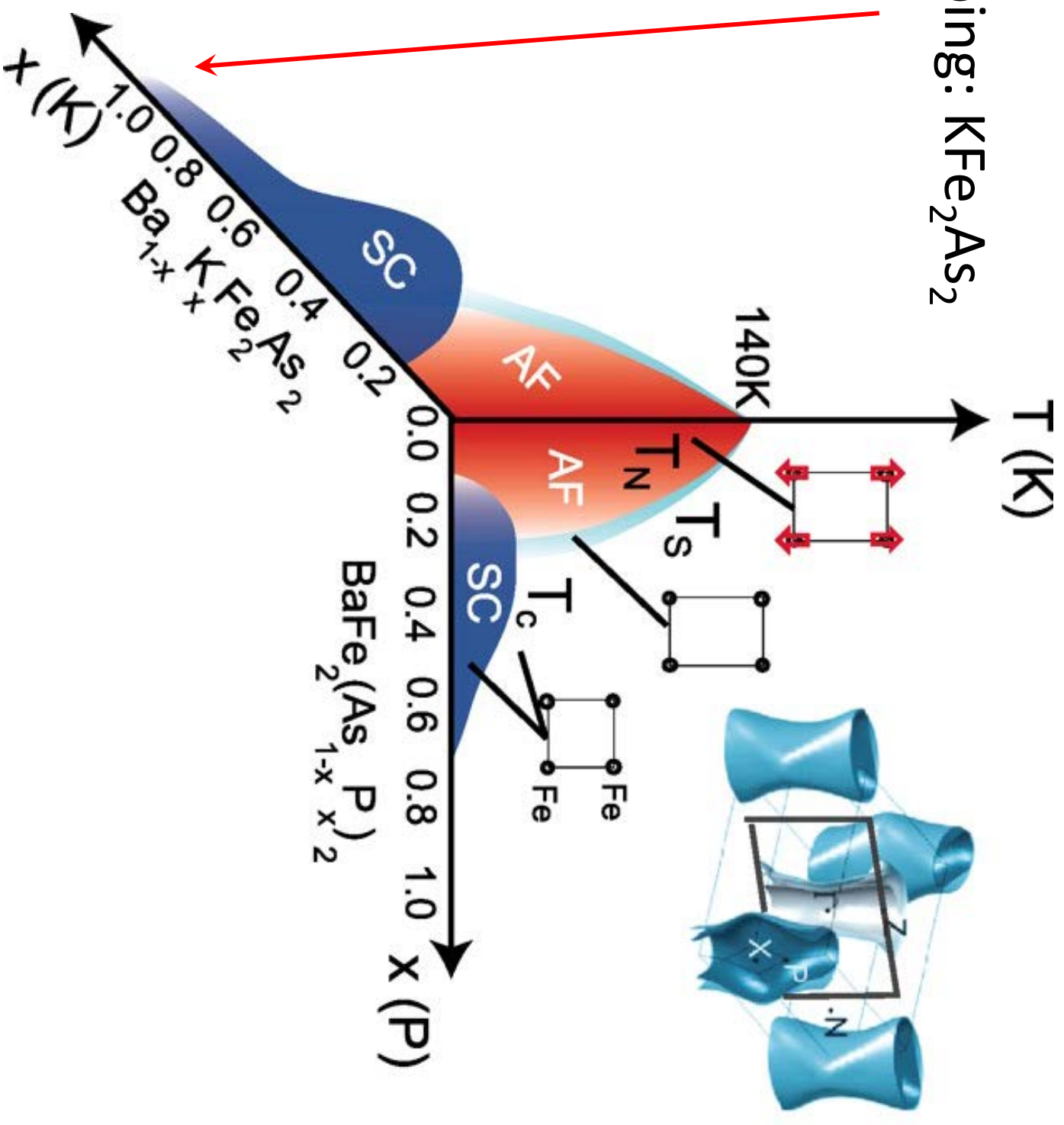


Andrey Chubukov

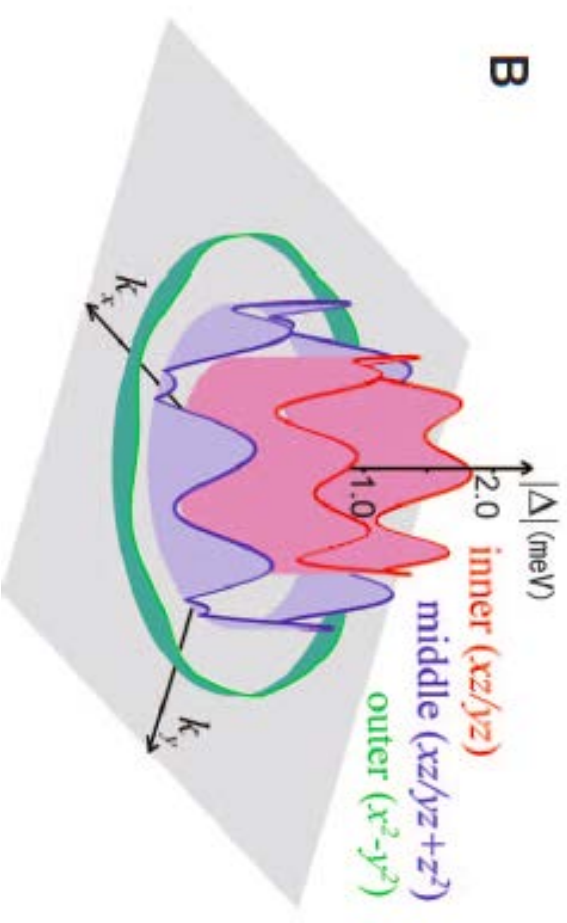
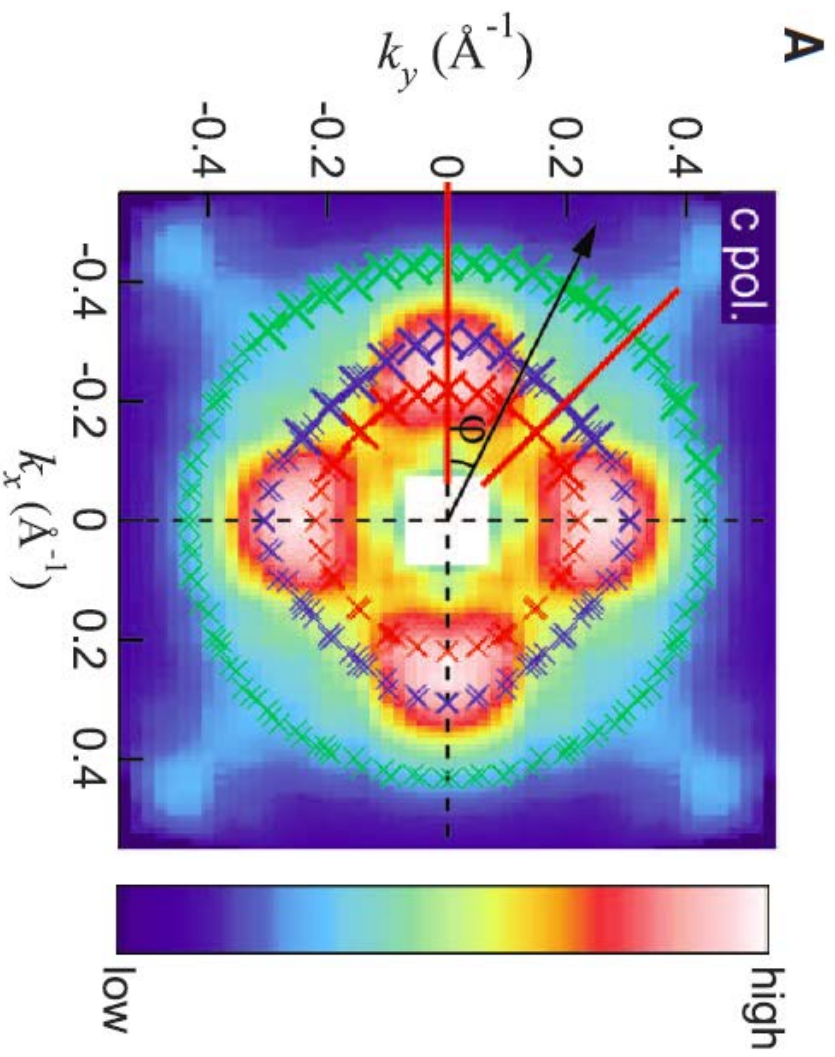




Strong hole doping: KFe_2As_2
($T_c \approx 3\text{K}$)



KFe_2As_2



Okazaki et.al.
Science (2012)

Even if the pairing symmetry is s-wave, or more precisely A_{1g} , such experiments indicate that the Cooper pairs have a richer internal structure that those of conventional sc.

(Caricature of the...) Basic idea (Γ point)

$$\psi_{\sigma}^{\dagger} = (d_{xz,\sigma}^{\dagger}, d_{yz,\sigma}^{\dagger})$$

Cooper pair: $\psi_{\alpha}^{\dagger} M \psi_{\beta}^*$

Kanamori Hamiltonian

s++ wave (A_{1g}) $\Delta_0 \psi_{\alpha}^{\dagger} 1 i \sigma_2^{\alpha\beta} \psi_{\beta}^*$ $(U + J')/2$

d_{xy} wave (B_{2g}) $\Delta_1 \psi_{\alpha}^{\dagger} \tau_1 i \sigma_2^{\alpha\beta} \psi_{\beta}^*$ $(U' + J)/2$

$d_{x^2-y^2}$ wave (B_{1g}) $\Delta_3 \psi_{\alpha}^{\dagger} \tau_3 i \sigma_2^{\alpha\beta} \psi_{\beta}^*$ $(U - J')/2$

triplet (A_{2g}) $\vec{\Delta}_2 \cdot \psi_{\alpha}^{\dagger} \tau_2 (i \sigma_2^{\vec{\sigma}})^{\alpha\beta} \psi_{\beta}^*$ $(U' - J)/2$

(Caricature of the...) Basic idea (Γ point)

$$\psi_{\sigma}^{\dagger} = (d_{xz,\sigma}^{\dagger}, d_{yz,\sigma}^{\dagger})$$

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s++ wave (A_{1g}) $\Delta_0 \psi_{\alpha}^{\dagger} 1 i \sigma_2^{\alpha\beta} \psi_{\beta}^*$

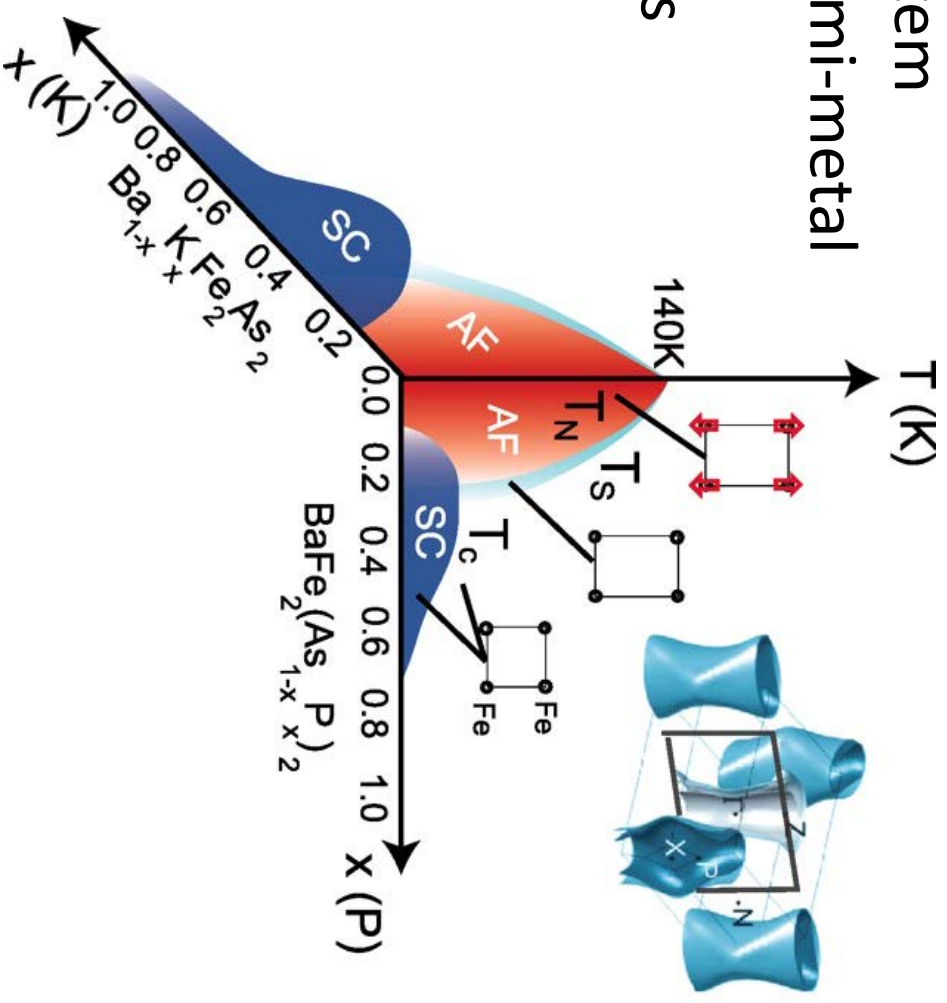
d_{xy} wave (B_{1g}) $\Delta_1 \psi_{\alpha}^{\dagger} \tau_1 i \sigma_2^{\alpha\beta} \psi_{\beta}^*$

$d_{x^2-y^2}$ wave (B_{2g}) $\Delta_3 \psi_{\alpha}^{\dagger} \tau_3 i \sigma_2^{\alpha\beta} \psi_{\beta}^*$

triplet (A_{2g}) $\vec{\Delta}_2 \cdot \psi_{\alpha}^{\dagger} \tau_2 (i \sigma_2^{\vec{\sigma}})^{\alpha\beta} \psi_{\beta}^*$ → s+- wave (A_{1g})
Spin orbit coupling + Eg

Low carrier density, quasi 2D system
parent state is a compensated semi-metal

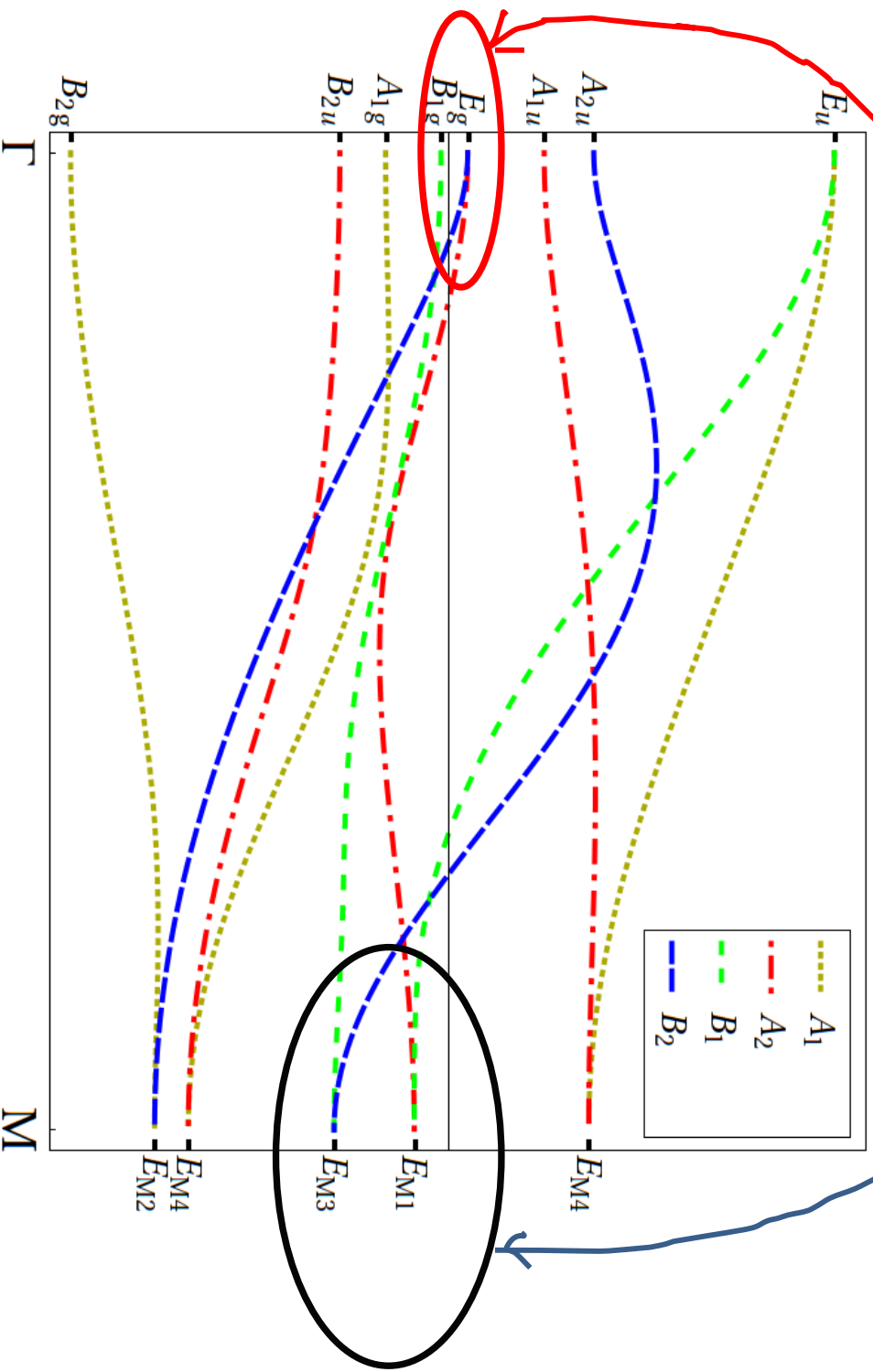
=> Luttinger's theory of invariants



Full tight banding band structure and the low energy 'spinor'

$$\psi_{\sigma}(\mathbf{k}) =$$

$$\begin{pmatrix} \psi_{X,\sigma}(\mathbf{k}) \\ \psi_{Y,\sigma}(\mathbf{k}) \\ \psi_{\Gamma,\sigma}(\mathbf{k}) \end{pmatrix}$$



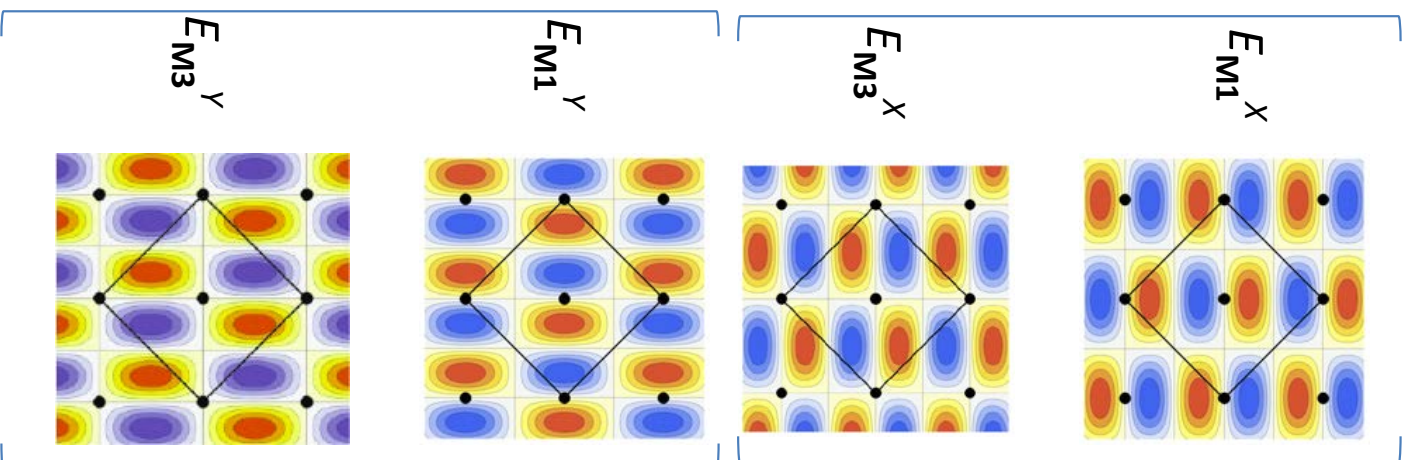
Low-energy effective theory

Low-energy spinor

(Γ : E_g states; M : E_{M1} and E_{M3} states):

$$\psi_{\sigma}(\mathbf{k}) = \begin{pmatrix} \psi_{X,\sigma}(\mathbf{k}) \\ \psi_{Y,\sigma}(\mathbf{k}) \\ \psi_{\Gamma,\sigma}(\mathbf{k}) \end{pmatrix}$$

$$\begin{bmatrix} YZ_A + YZ_B \\ -XZ_A - XZ_B \end{bmatrix}$$

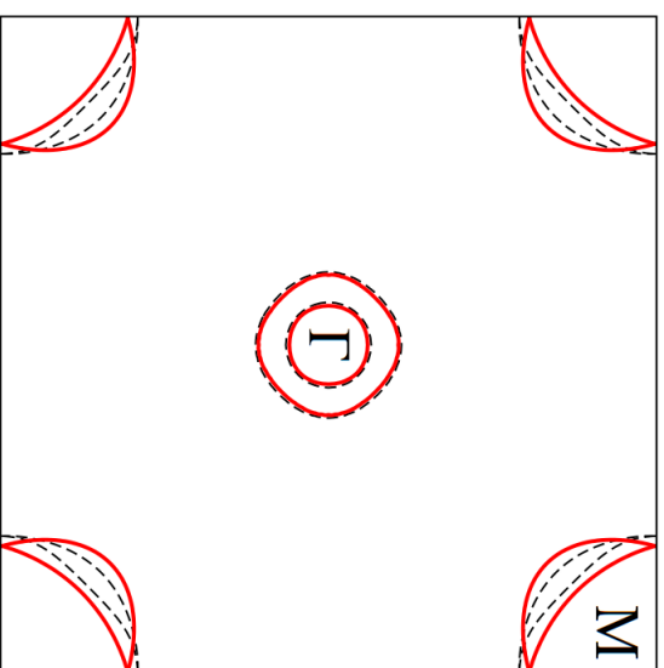
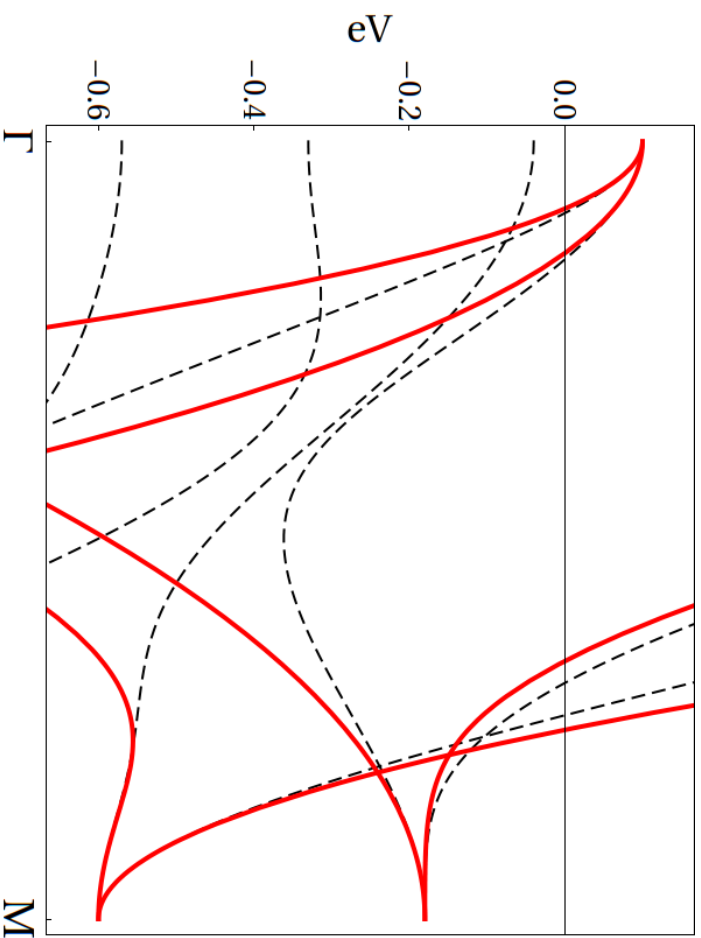


ODD under n-glide

EVEN under n-glide

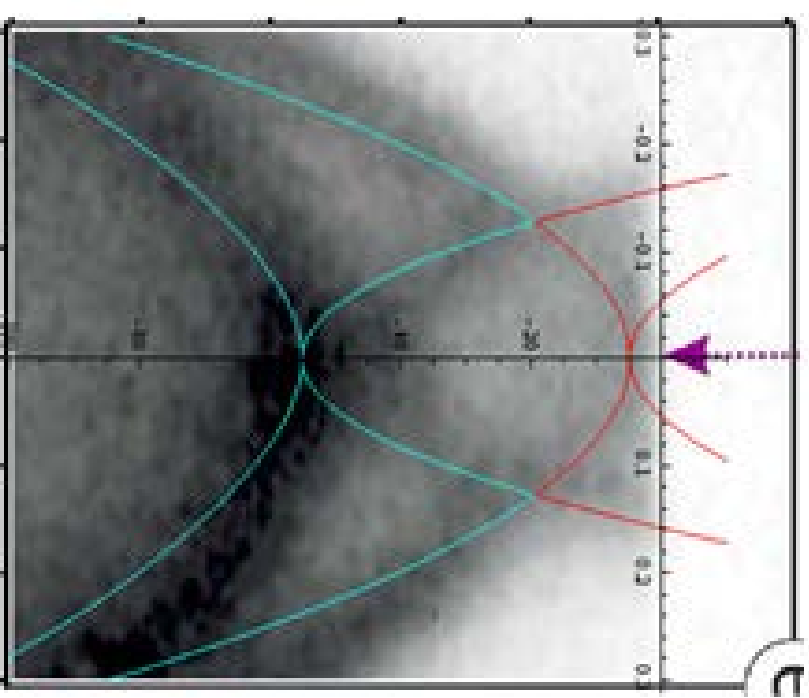
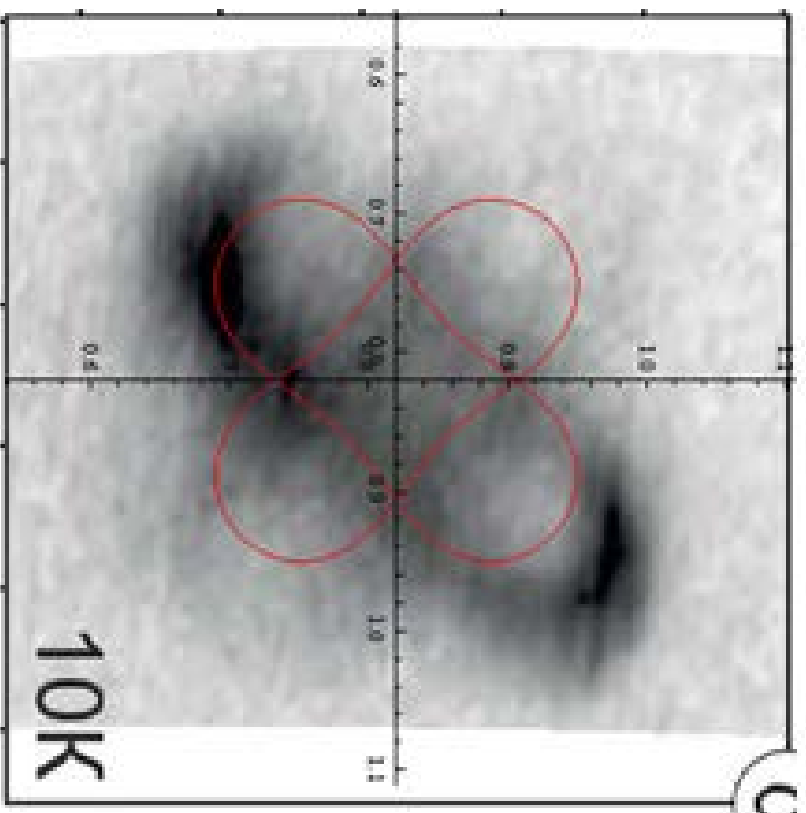
=> Do not mix (at $k_z=0$)

Comparison of the low-energy effective theory and the tight binding models



K. Kuroki, *et al.*, PRL **101**, 087004 (2008)

Fit to the bulk FeSe ARPES (assumed tetragonal)



Effective Hamiltonian

$$\mathcal{H} = H_0 + H_{int}$$

$$H_0 = \sum_{\mathbf{k}} \sum_{\alpha, \beta = \uparrow, \downarrow} \psi_{\mathbf{k}, \alpha}^\dagger (h_{\mathbf{k}} \delta_{\alpha\beta} + h^{SO} s_{\alpha\beta}^z) \psi_{\mathbf{k}, \beta}$$

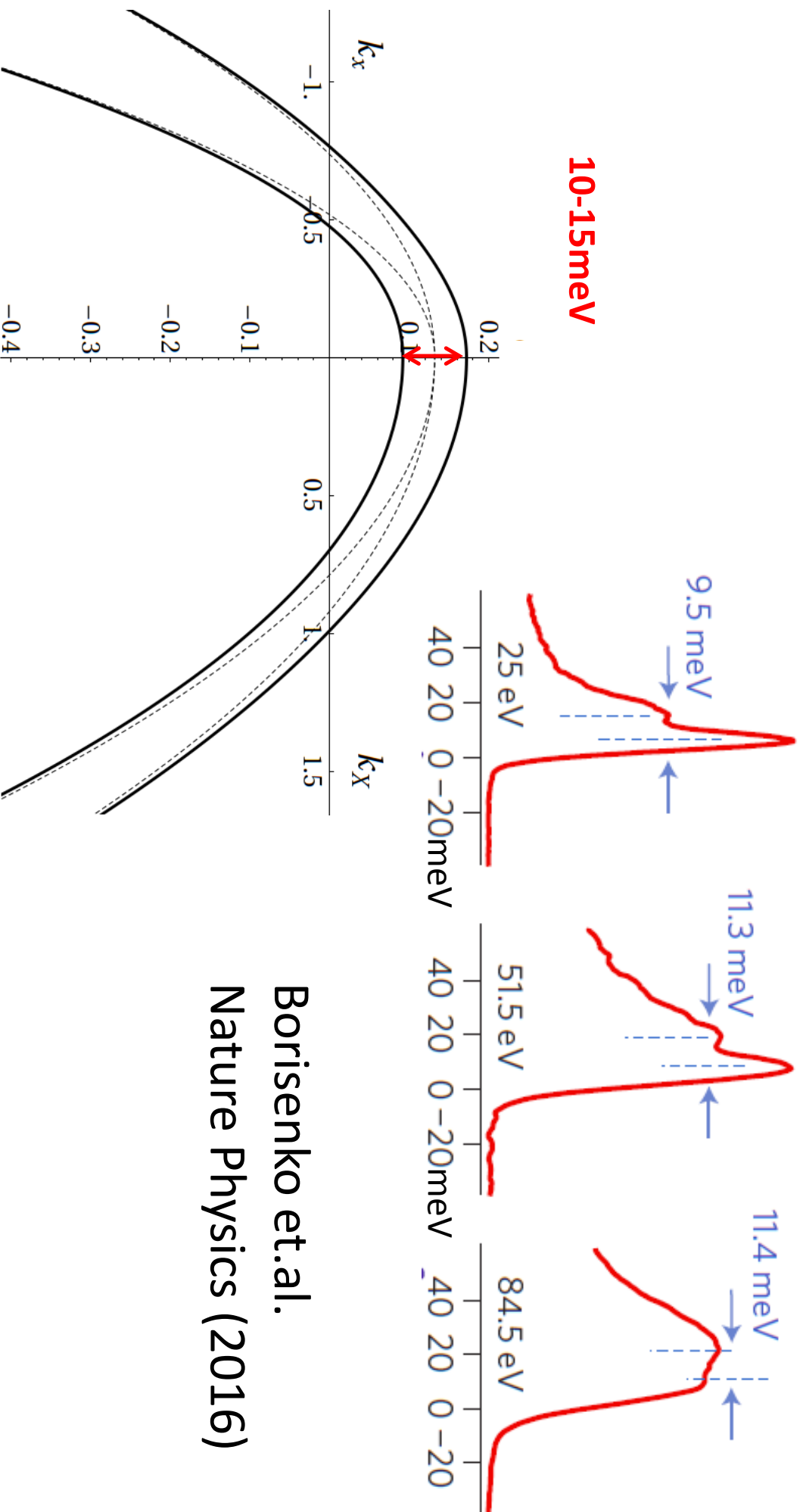
$$h_{\mathbf{k}} = \begin{pmatrix} \mu - \frac{\mathbf{k}^2}{2m} + bk_x k_y & c(k_x^2 - k_y^2) \\ c(k_x^2 - k_y^2) & \mu - \frac{\mathbf{k}^2}{2m} - bk_x k_y \end{pmatrix}$$

$$h^{SO} = \lambda \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$H_{int} = \sum_{j=0}^3 \frac{g_j}{2} \int d^2 \mathbf{r} : \psi_{\sigma}^\dagger(\mathbf{r}) \tau_j \psi_{\sigma}(\mathbf{r}) \psi_{\sigma'}^\dagger(\mathbf{r}) \tau_j \psi_{\sigma'}(\mathbf{r}) :$$

Spin-orbit interaction in the low-energy effective theory

The effect on the spectrum



Borisenko et.al.

Nature Physics (2016)

Interaction Hamiltonian

$$\begin{aligned} H_{int} &= \sum_{j=0}^3 \frac{g_j}{2} \int d^2\mathbf{r} : \psi_{\sigma}^{\dagger}(\mathbf{r}) \tau_j \psi_{\sigma}(\mathbf{r}) \psi_{\sigma'}^{\dagger}(\mathbf{r}) \tau_j \psi_{\sigma'}(\mathbf{r}) : \\ g_0 &= \frac{1}{2} (U + U') + \dots > 0 \\ g_1 &= J + \dots > 0 \\ g_2 &= 0 + \dots > 0 \\ g_3 &= \frac{1}{2} (U - U') + \dots > 0 \end{aligned}$$

intra-orbital Hubbard repulsion: U

inter-orbital Hubbard repulsion: U'

Hunds coupling: J

Fierz transformation of the interaction Hamiltonian

$$H_{int} = \sum_{j=0}^3 \frac{\tilde{g}_j}{2} \int d^2\mathbf{r} (\psi_{\sigma}^{\dagger}(\mathbf{r}) \tau_j \psi_{\sigma'}^*(\mathbf{r})) (\psi_{\sigma'}^T(\mathbf{r}) \tau_j \psi_{\sigma}(\mathbf{r}))$$

where

$$\begin{pmatrix} \tilde{g}_0 \\ \tilde{g}_1 \\ \tilde{g}_2 \\ \tilde{g}_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{pmatrix}$$

avoid large repulsive U:

$$\tilde{g}_1 = \frac{1}{2}(U' + J) + \dots > 0$$

$$\tilde{g}_2 = \frac{1}{2}(U' - J) + \dots$$

→ could be attractive!

Symmetry of the pairing

$$\begin{aligned} & (\psi_{\sigma}^{\dagger}(\mathbf{r})\tau_2\psi_{\sigma'}^{*}(\mathbf{r})) (\psi_{\sigma'}^T(\mathbf{r})\tau_2\psi_{\sigma}(\mathbf{r})) = \\ & \frac{1}{2}\psi_{\alpha}^{\dagger}(\mathbf{r})\tau_2(\mathbf{s}^y\vec{s})_{\alpha\beta}\psi_{\beta}^{*}(\mathbf{r}) \cdot \psi_{\alpha'}^T(\mathbf{r})\tau_2(\vec{s}s^y)_{\alpha'\beta'}\psi_{\beta'}(\mathbf{r}). \end{aligned}$$

pairing channel seemingly corresponds to spin triplet A_{2g}

However, the spin-orbit coupling term in H_0 changes the symmetry to A_{1g} (i.e. s-wave)

Order parameters

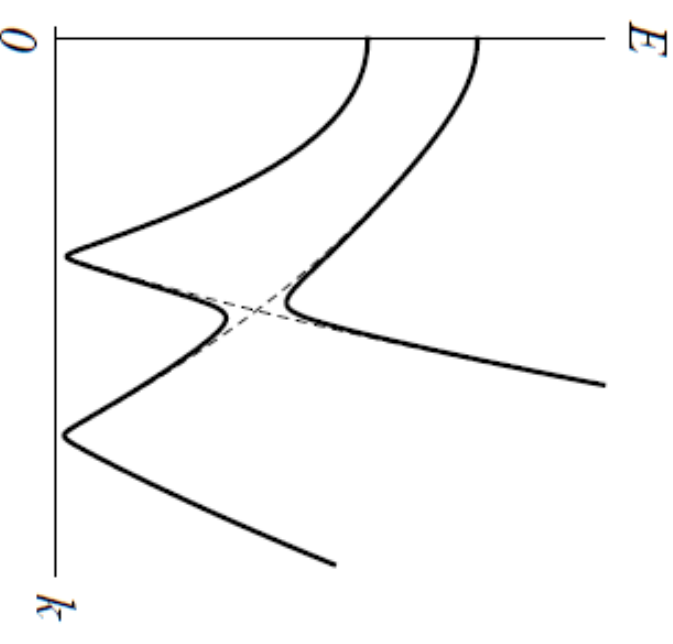
$$\Delta_0 = \frac{1}{2} \tilde{g}_0 \langle \psi_\alpha^T(\mathbf{r}) \mathbf{1} (-is^y)_{\alpha\beta} \psi_\beta(\mathbf{r}) \rangle$$

$$\Delta_2 = \frac{1}{2} \tilde{g}_2 \langle \psi_\alpha^T(\mathbf{r}) \tau_2 (is^z s^y)_{\alpha\beta} \psi_\beta(\mathbf{r}) \rangle$$

Order parameters and spectrum

$$\mathcal{U}^\dagger H_{\text{BdG}}(\mathbf{k})\mathcal{U} = \begin{pmatrix} \xi_+ & 0 & \Delta_0 + \frac{\lambda}{|\vec{B}_k|} \Delta_2 & -i\Delta_2 \sqrt{1 - \frac{\lambda^2}{\vec{B}_k^2}} \\ 0 & \xi_- & i\Delta_2 \sqrt{1 - \frac{\lambda^2}{\vec{B}_k^2}} & \Delta_0 - \frac{\lambda}{|\vec{B}_k|} \Delta_2 \\ \Delta_0 + \frac{\lambda}{|\vec{B}_k|} \Delta_2 & -i\Delta_2 \sqrt{1 - \frac{\lambda^2}{\vec{B}_k^2}} & -\xi_+ & 0 \\ i\Delta_2 \sqrt{1 - \frac{\lambda^2}{\vec{B}_k^2}} & \Delta_0 - \frac{\lambda}{|\vec{B}_k|} \Delta_2 & 0 & -\xi_- \end{pmatrix}$$

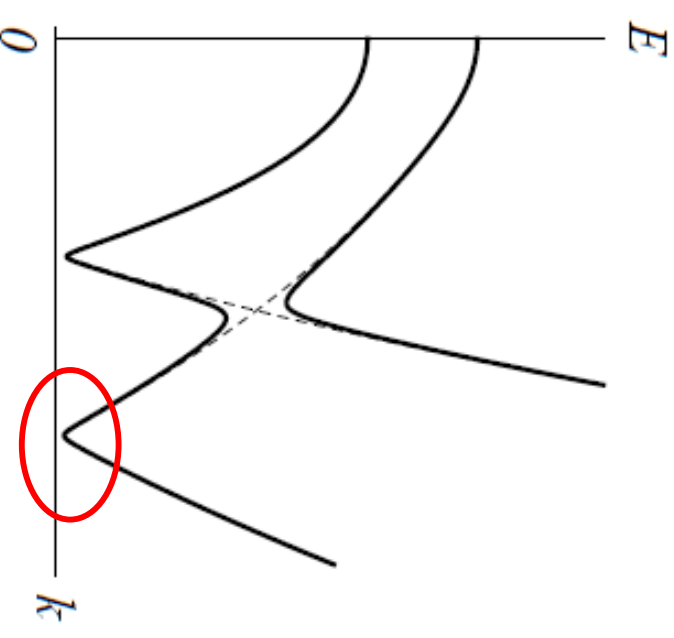
$$h_k + \lambda \tau_2 = A_k \mathbf{1} + \vec{B}_k \cdot \vec{\tau}$$



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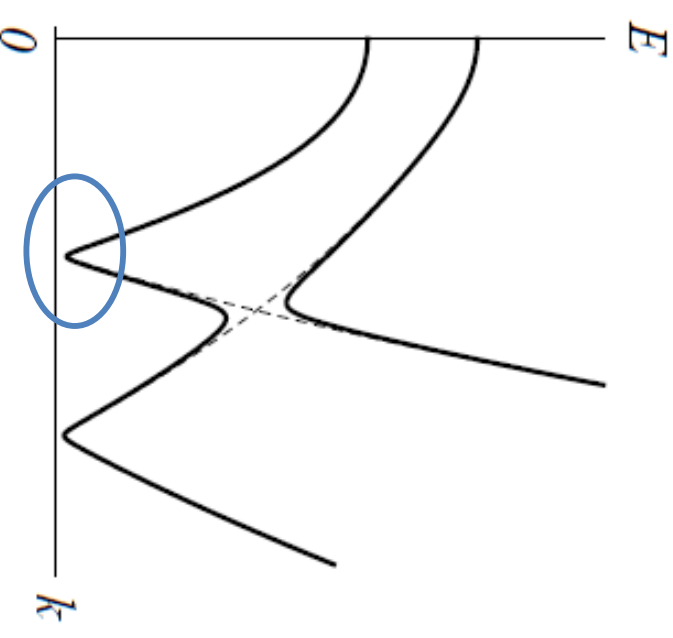
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Order parameters and spectrum

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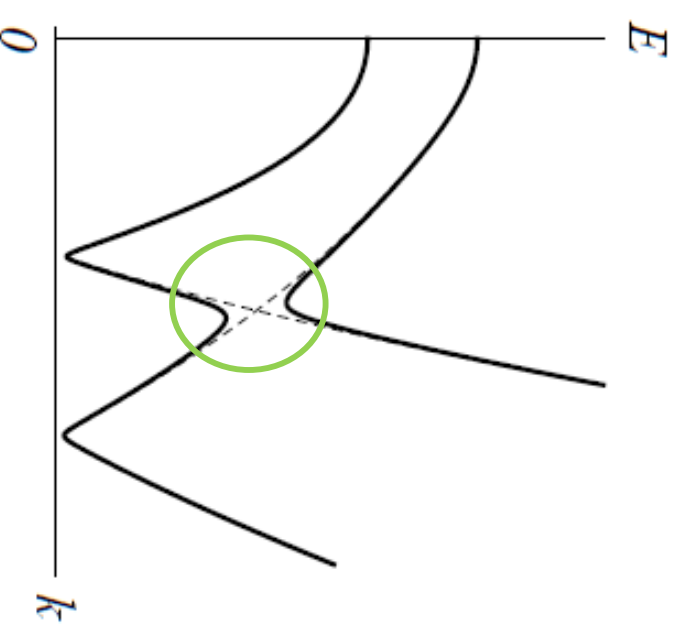
$$h_k + \lambda \vec{\tau}_2 = A_k \mathbf{1} + \vec{B}_k \cdot \vec{\tau}$$



Order parameters and spectrum

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$$h_k + \lambda \tau_2 = A_k \mathbf{1} + \vec{B}_k \cdot \vec{\tau}$$



Order parameters and spectrum

$$\frac{\Delta_0(T_c)}{\Delta_2(T_c)} = -\frac{\chi_{02}(T_c)}{\frac{1}{g_0} + \chi_{00}(T_c)} = -C \frac{\lambda}{\mu} \quad C > 0$$

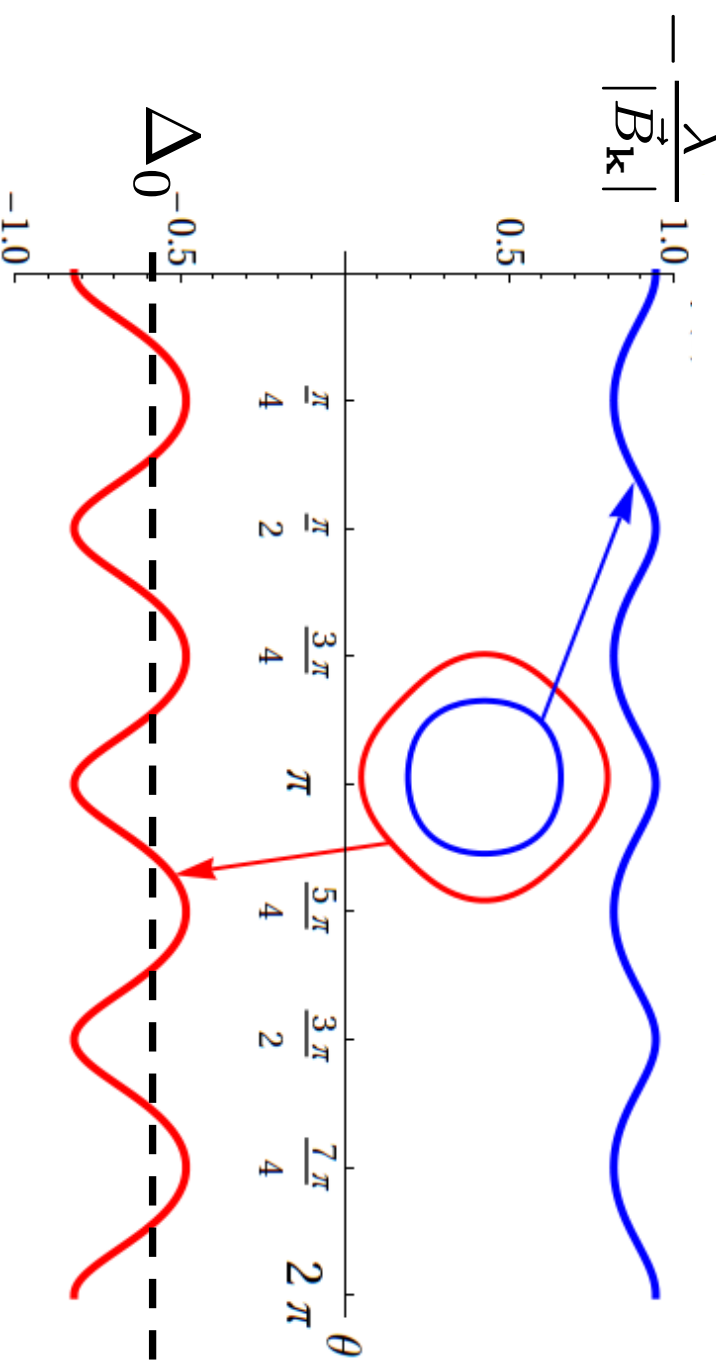
For dominant Δ_2

$$\frac{\Delta_0(T=0)}{\Delta_2(T=0)} = -C' \frac{\lambda}{\mu} \quad C' > 0$$

$$\Rightarrow \Delta_0 \pm \frac{\lambda}{|\vec{B}_k|} \Delta_2 = \Delta_2 \left(-C \frac{\lambda}{\mu} \pm \frac{\lambda}{|\vec{B}_k|} \right)$$

The anisotropic gap on the outer Fermi surface is (further) reduced by the admixture Δ_0

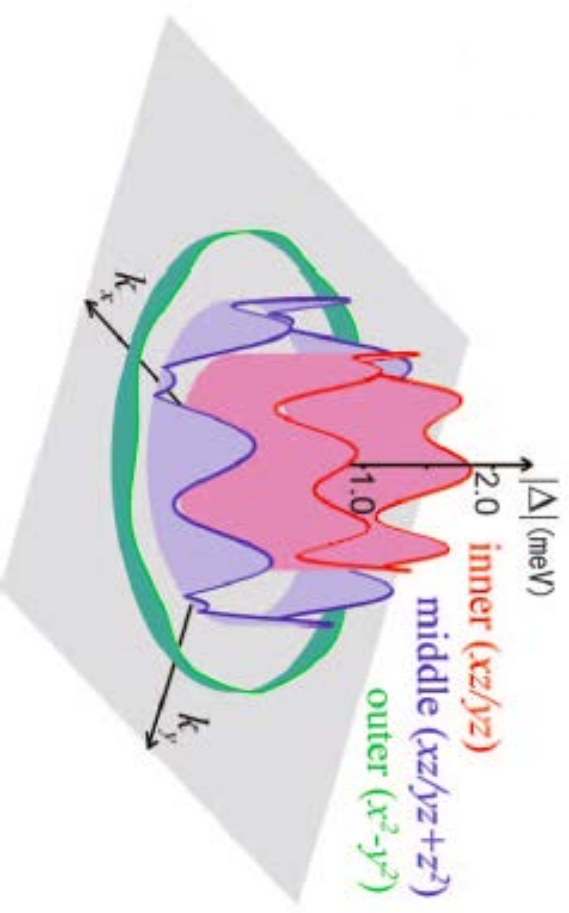
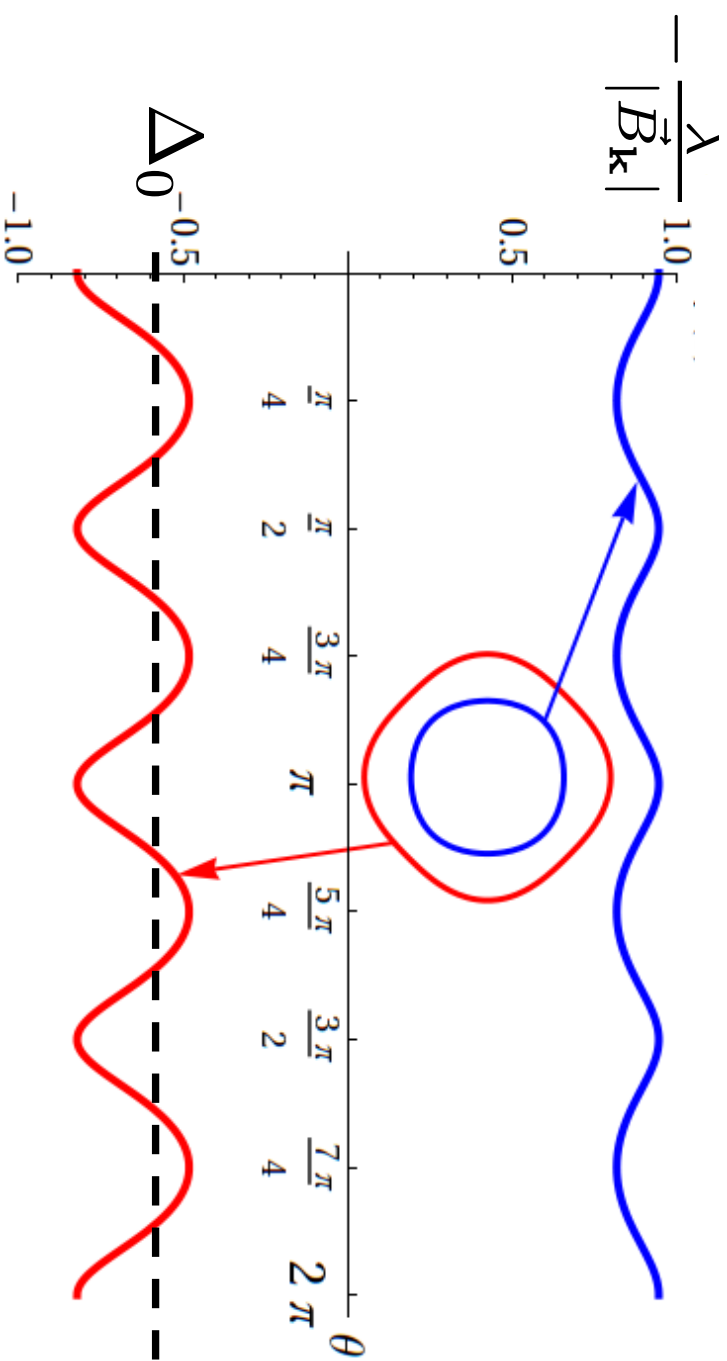
Order parameters and spectrum



$$\Rightarrow \Delta_0 \pm \frac{\lambda}{|\vec{B}_k|} \Delta_2 = \Delta_2 \left(-C \frac{\lambda}{\mu} \pm \frac{\lambda}{|\vec{B}_k|} \right) \quad C > 0$$

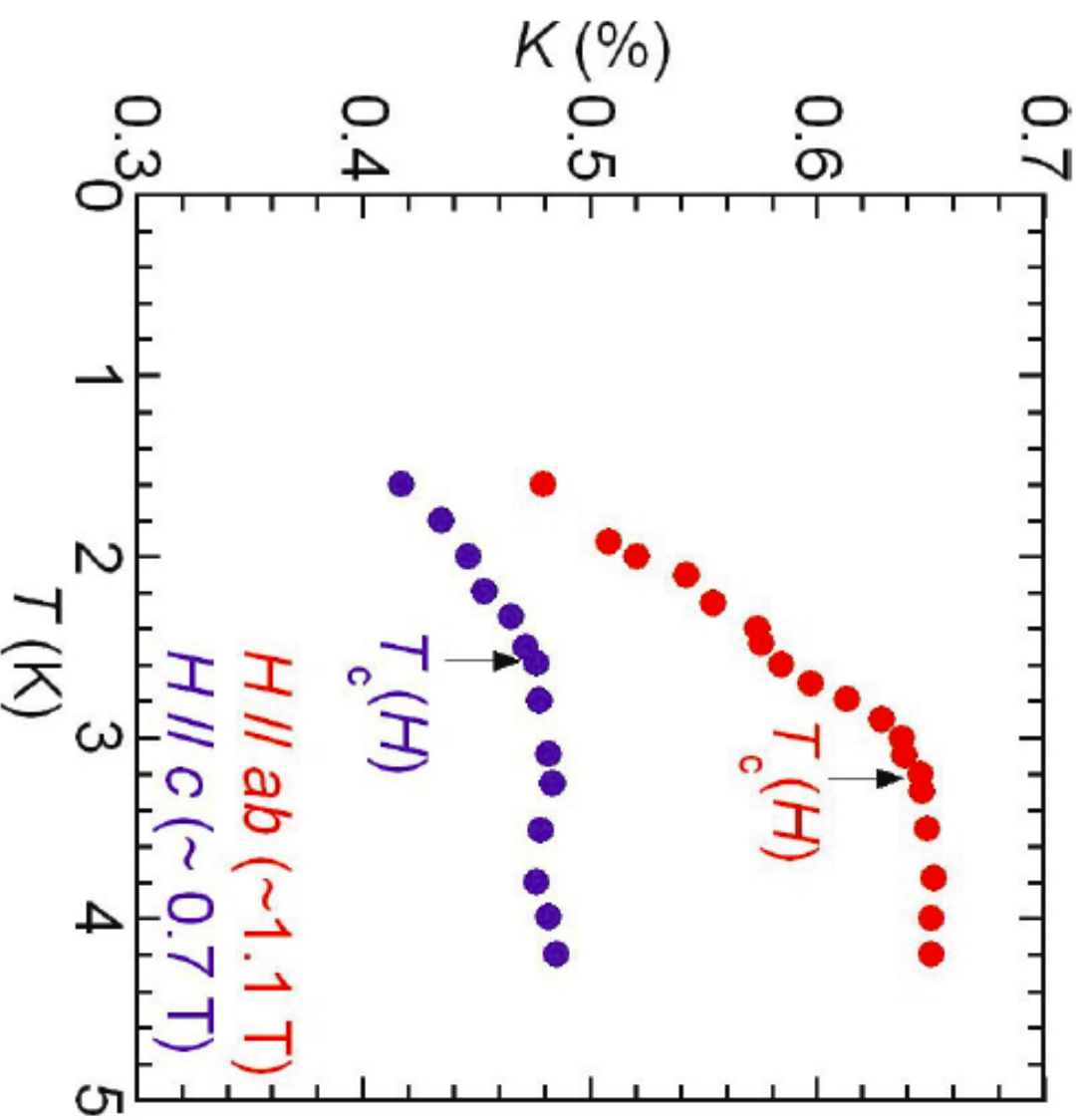
Possibility of accidental nodes but only on the outer Fermi surface
 The fourfold anisotropy leads to 8 such nodes

Order parameters and spectrum

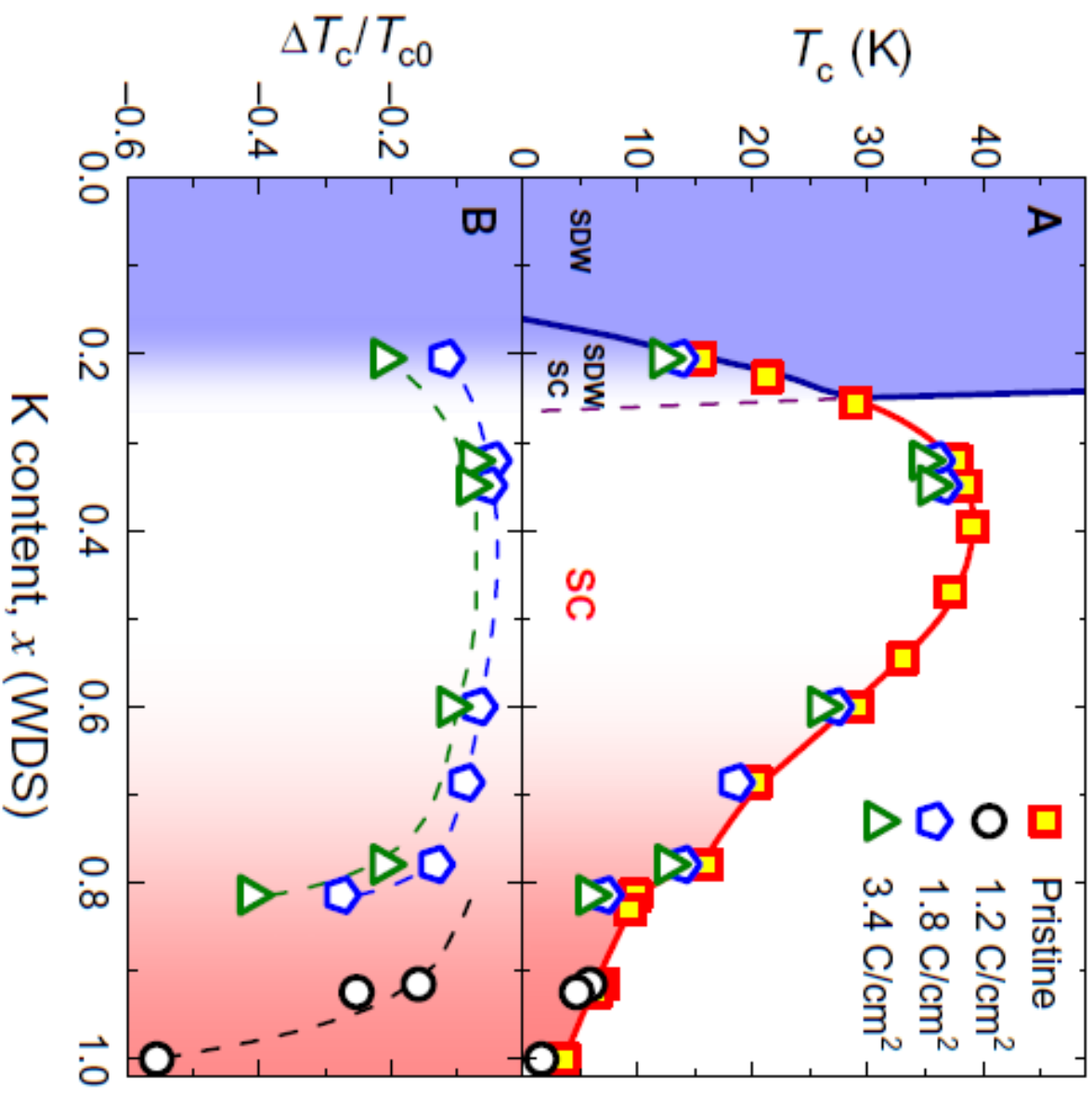


Okazaki et.al.
Science (2012)

Knight shift $K\text{Fe}_2\text{As}_2$



Ba_{1-x}K_xFe₂As₂



$$T_c \sim e^{-\frac{\mu^2}{\chi^2} \frac{1}{N_0(J-U')}}}$$

Open questions

- How to (reliably) determine the parameters in the low energy effective Hamiltonian from first principles?
- Under what conditions do we achieve $J_{\text{eff}} \gg U_{\text{eff}}$? Phonons?
- Interplay of hole and electron pockets?
- How to incorporate the additional ordering tendencies at smaller doping?