Amplitude (Higgs) Mode and the Superfluid-Mott Glass Quantum Phase Transition

Thomas Vojta[1], Jack Crewse[1], Martin Puschmann[1,2], Daniel Arovas[3]

[1]Department of Physics, Missouri University of Science and Technology, USA

[2]Institute of Physics, Technische Universität Chemnitz, Germany

[3]Department of Physics, University of California San Diego, USA

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Outline

- Amplitude (Higgs) mode in condensed matter
- Phases and phase transitions of disordered interacting bosons
- Superfluid-Mott glass quantum phase transition
- Fate of the amplitude mode at the superfluid-Mott glass transition
- Conclusions

Support:

NSF

DAAD
Deutscher Akademischer Austausch Dienst
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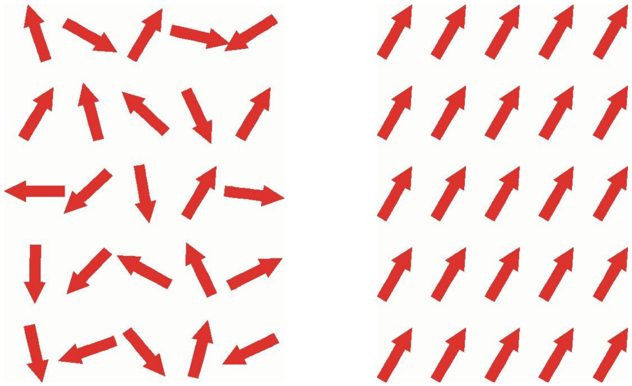
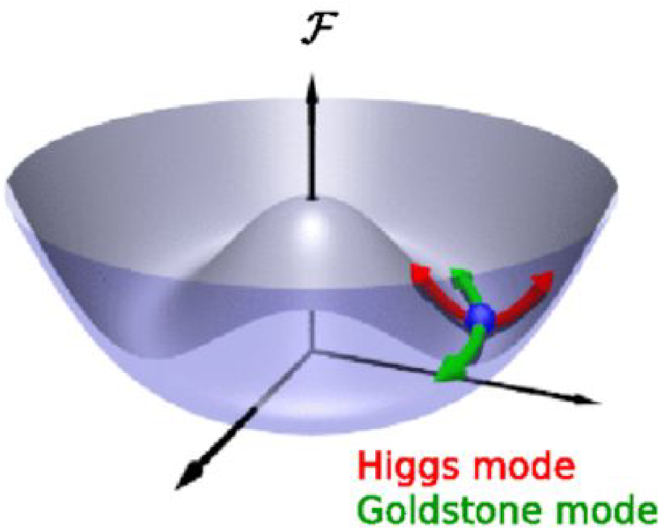
Pegasus IV HPC cluster
at Missouri S&T

What is the amplitude (Higgs) mode?

- collective excitation in systems with **broken continuous symmetry**, e.g.,
  – planar magnet breaks O(2) rotation symmetry
  – superfluid wave function breaks U(1) symmetry
- **Higgs mode**: corresponds to fluctuations of order parameter **amplitude**
- **Goldstone mode**: corresponds to fluctuations of order parameter **phase**
- **Higgs mode** is condensed matter analogue of famous **Higgs boson**



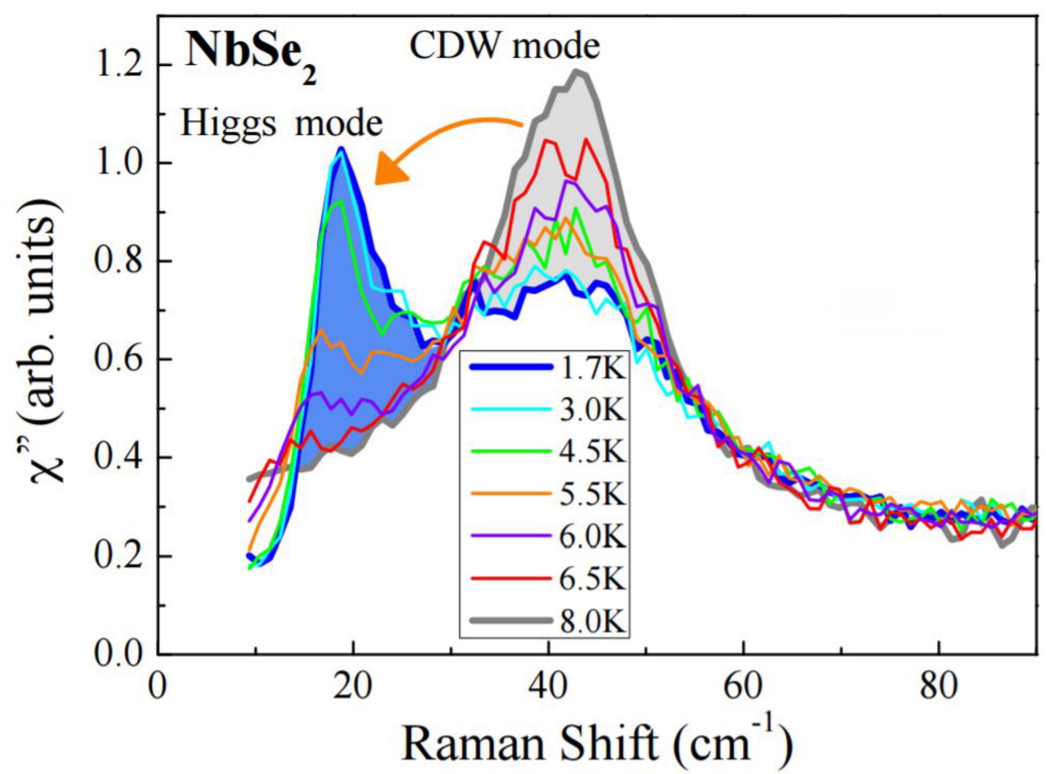


effective potential for order parameter in symmetry-broken phase

Amplitude (Higgs) mode in condensed matter?

- Is the Higgs mode a **sharp, particle-like** excitation or is it overdamped because it decays into other modes
- Does the Higgs mode remain **sharp** even in the presence of **disorder, i.e., impurities and defects**?

Raman scattering data for $NbSe_2$
[from Measson et.al., Phys. Rev. B **89**, 060503 (2014)]



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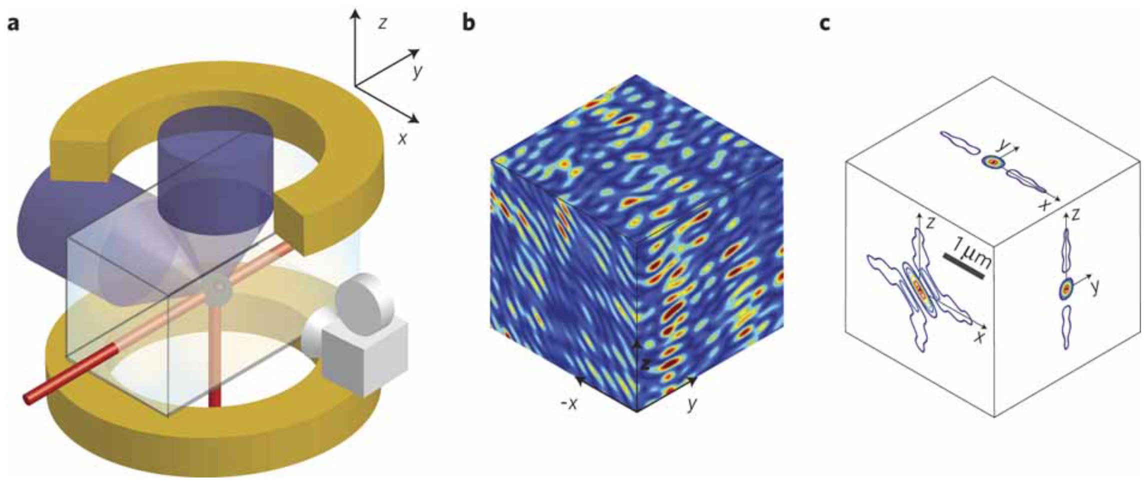
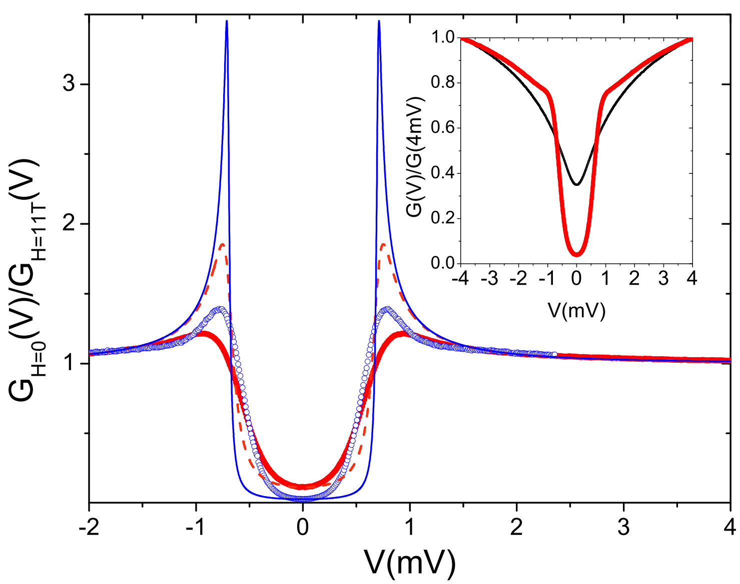
Disordered interacting bosons

Ultracold atoms in optical potentials:

- disorder: speckle laser field
- interactions: tuned by Feshbach resonance and/or density



Nature Physics 8, 398403 (2012)



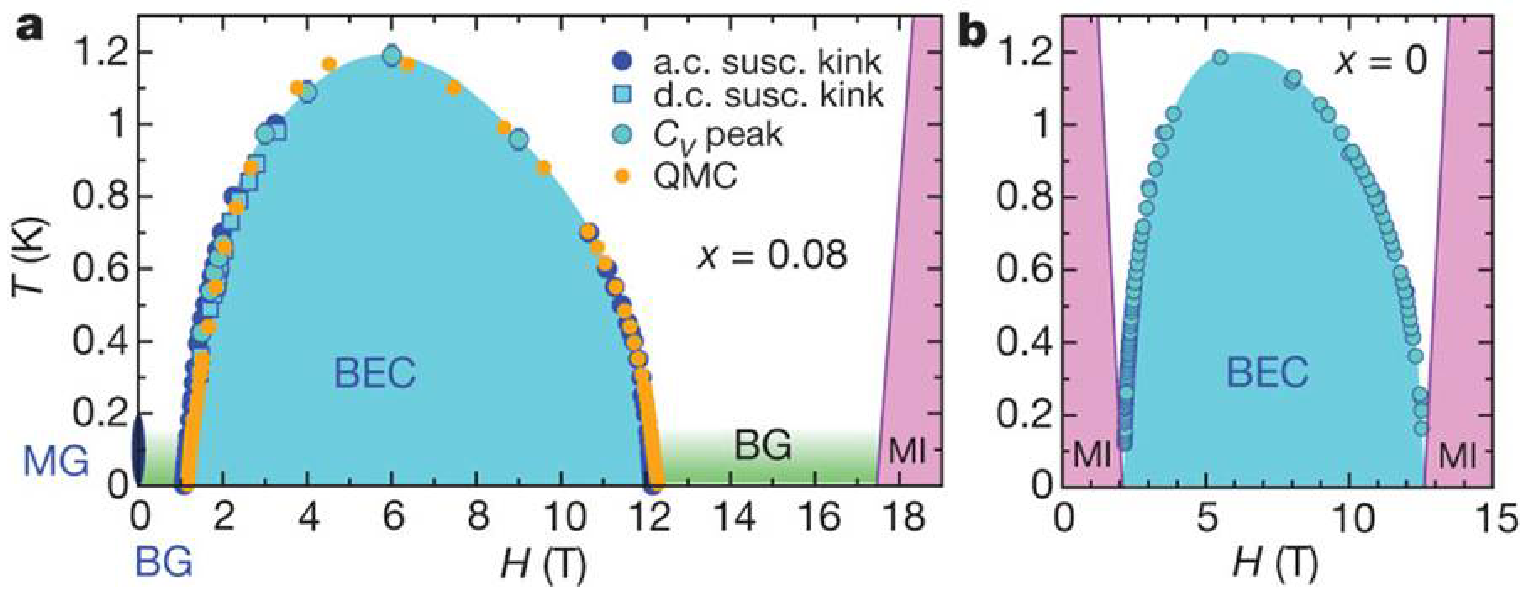
Phys. Rev. Lett. 108, 177006 (2012)

Disordered superconducting films:

- energy gap in **insulating** as well as **superconducting** phase
- preformed Cooper pairs ⇒ superconducting transition is bosonic

Disordered interacting bosons

Bosonic quasiparticles in doped quantum magnets:



Nature 489, 379 (2012)

- bromine-doped dichloro-tetrakis-thiourea-nickel (DTN)
- coupled antiferromagnetic chains of $S = 1$ Ni^{2+} ions
- $S = 1$ spin states can be mapped onto bosonic states with $n = m_s + 1$

Quantum rotor model

Josephson junction array:

$$H = \frac{U}{2}\sum_i (\hat{n}_i - \bar{n}_i)^2 - \sum_{\langle i,j \rangle} J_{ij} \cos(\hat{\theta}_i - \hat{\theta}_j)$$

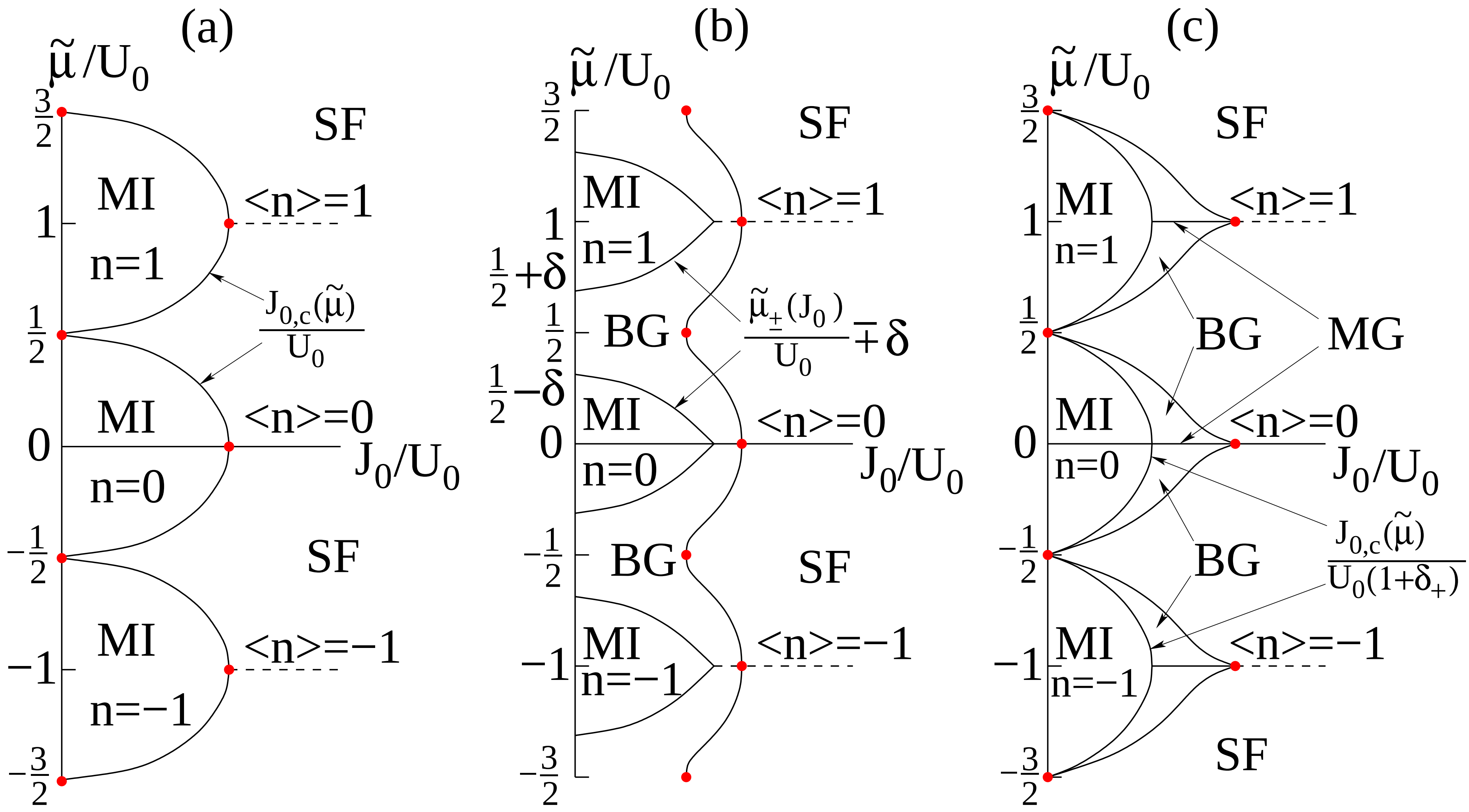
- $\hat{n}_i$: **number operator**, $\hat{\theta}_i$: **phase operator**
- superfluid ground state if **Josephson couplings** J_{ij} dominate
- insulating ground state if **charging energy** U dominates
- chemical potential $\mu_i = U\bar{n}_i$

Particle-hole symmetry:

- if $\mu_i \equiv kU/2$, ($\bar{n}_i = k/2$) with integer k

  $\Rightarrow$ Hamiltonian **invariant** under $\hat{n}_i \to k - \hat{n}_i$ and $\hat{\theta}_i \to -\hat{\theta}_i$

Phase diagrams



clean

random potentials

random couplings

Phys. Rev. B 7, 214516 (2008)

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From quantum rotors to classical XY model

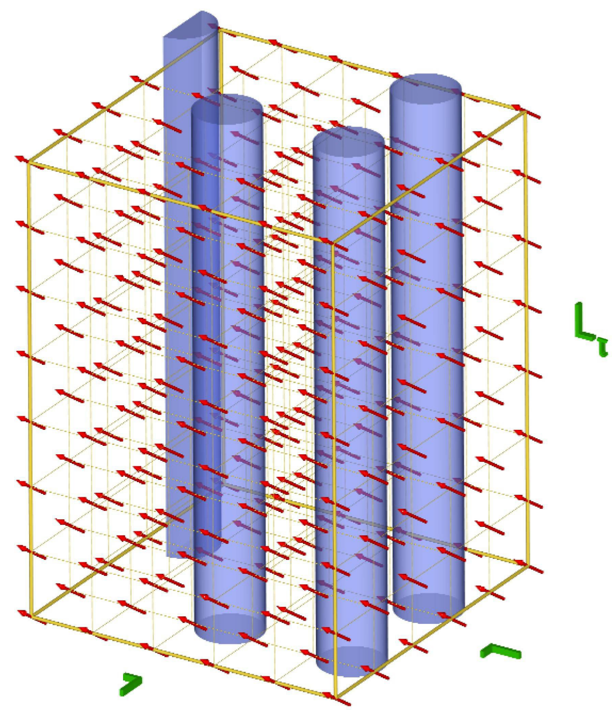
- site-diluted quantum rotor model on square lattice ($\epsilon_i = 0$ or 1 with probabilities p and $1-p$)

$$H = \frac{U}{2}\sum_i \epsilon_i(\hat{n}_i - \bar{n}_i)^2 - J\sum_{\langle ij\rangle}\epsilon_i\epsilon_j\cos(\hat{\theta}_i - \hat{\theta}_j)$$

- can be mapped onto classical (2+1)-dimensional XY model for $\bar{n}_i = k$ (**particle-hole symmetric** case)

$$H_{\mathrm{cl}} = -J_\tau \sum_{i,t}\epsilon_i \mathbf{S}_{i,t}\cdot\mathbf{S}_{i,t+1} - J_s\sum_{\langle i,j\rangle,t}\epsilon_i\epsilon_j\mathbf{S}_{i,t}\cdot\mathbf{S}_{j,t}$$

$J_\tau = J_s = 1$ (values not important for critical behavior because of **universality**)



columnar disorder in classical XY model, correlated in imaginary time

Stability of clean quantum critical behavior

Clean superfluid-Mott insulator quantum phase transition:

- for dilution $p = 0$, transition is in 3D XY universality class
- correlation length critical exponent $\nu \approx 0.6717$

Harris criterion:

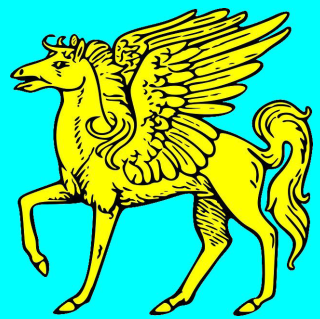
- clean critical point stable against disorder if $d_\perp \nu > 2$
- for 3D XY model with columnar disorder, $d_\perp = 2$
- clean correlation length critical exponent **violates Harris criterion**

$\Rightarrow$ 3D XY critical point unstable against columnar disorder

Critical behavior of superfluid-Mott glass transition must be in new universality class

Monte Carlo simulations

- combine **Wolff** cluster algorithm and conventional **Metropolis** updates
- **Wolff** algorithm greatly reduces critical slowing down
- **Metropolis** updates equilibrate small disconnected clusters (important for high dilutions)
- system sizes up to $L = 150$ and $L_\tau = 1792$
- dilutions $p = 0, \ 1/8, \ 1/5, \ 2/7, \ 1/3, \ 9/25$ and percolation threshold $p_c = 0.407253$
- averages over 10 000 to 50 000 disorder configurations



Pegasus IV Cluster:

- diskless HPC cluster
- designed and built in Missouri S&T Physics Department
- highly cost effective, total cost below US $140 per CPU core

web.mst.edu/∼vojtat/pegasus/home.htm

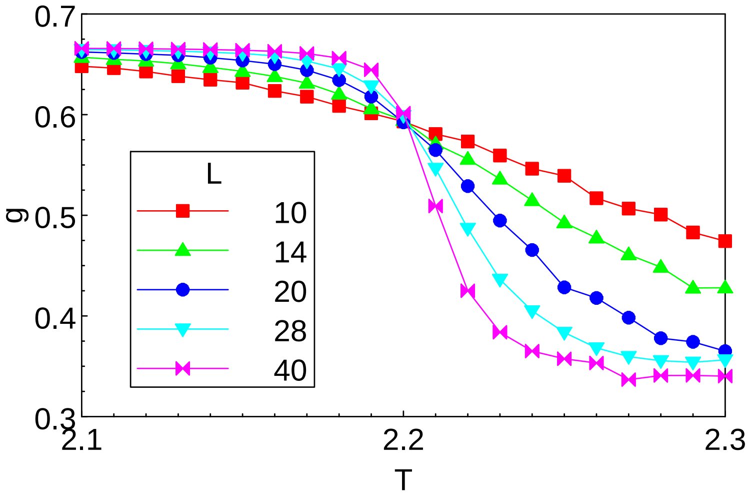
Anisotropic finite-size scaling

Binder cumulant:

$$g_{\rm av} = \left[1 - \frac{\langle|\vec{\phi}|^4\rangle}{3\langle|\vec{\phi}|^2\rangle^2}\right]_{\rm dis}$$

Isotropic systems:

- scaling form: $g_{\rm av}(r, L) = X(rL^{1/\nu})$
  $[r = (T - T_c)/T_c]$
- $g_{\rm av}$ vs. T curves for different L cross at T_c with value $g_{\rm av}(0, L) = X(0)$



Anisotropic systems:

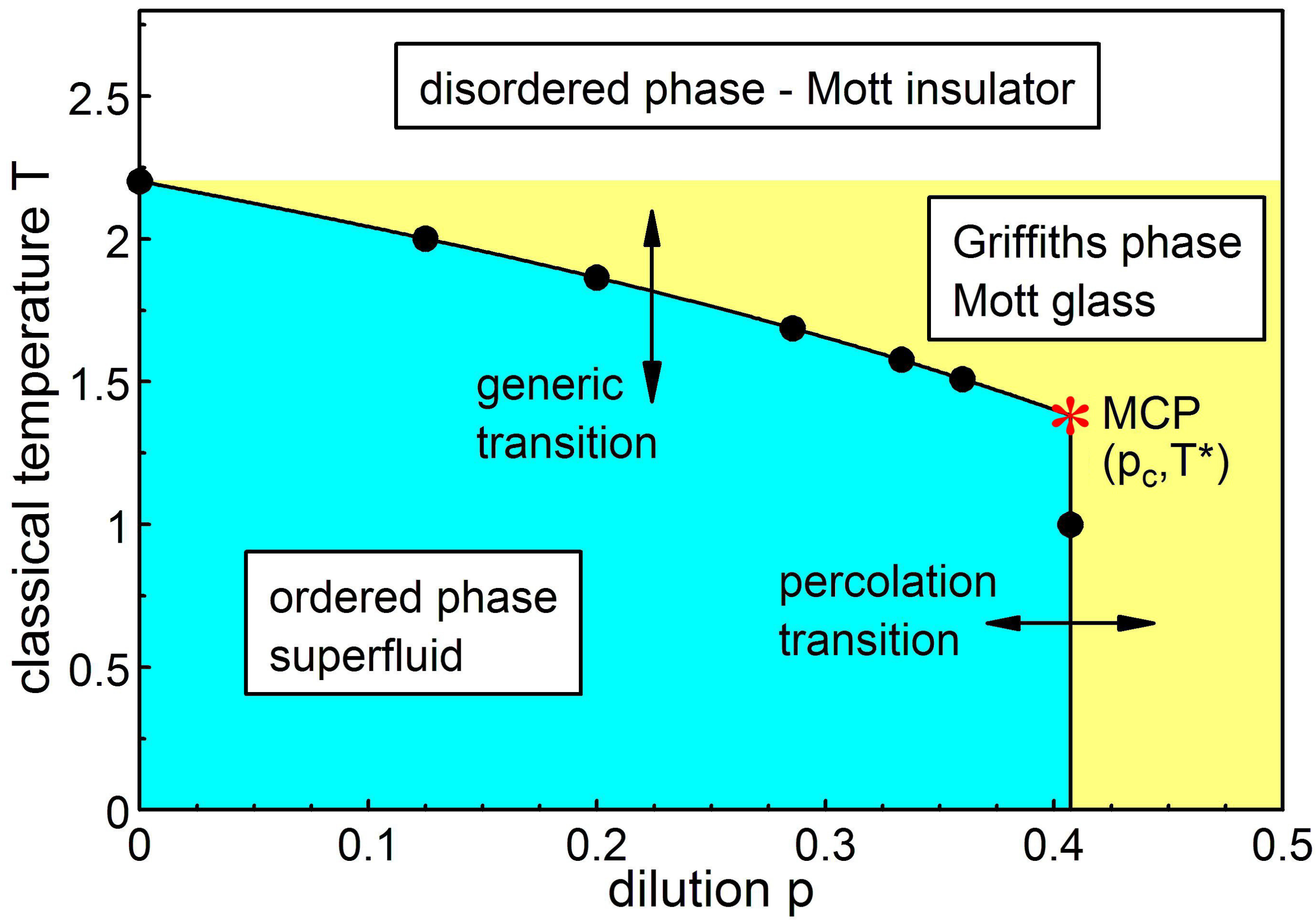
- L and L_τ are not equivalent, scaling form: $g_{\rm av}(r, L, L_\tau) = X(rL^{1/\nu}, L_\tau/L^z)$

How to choose correct sample shapes if dynamical exponent z is not known?

$\Rightarrow$ $g_{\rm av}$ vs. L_τ has maximum at **optimal shape** (L_τ/L equals correlation length ratio ξ_τ/ξ)

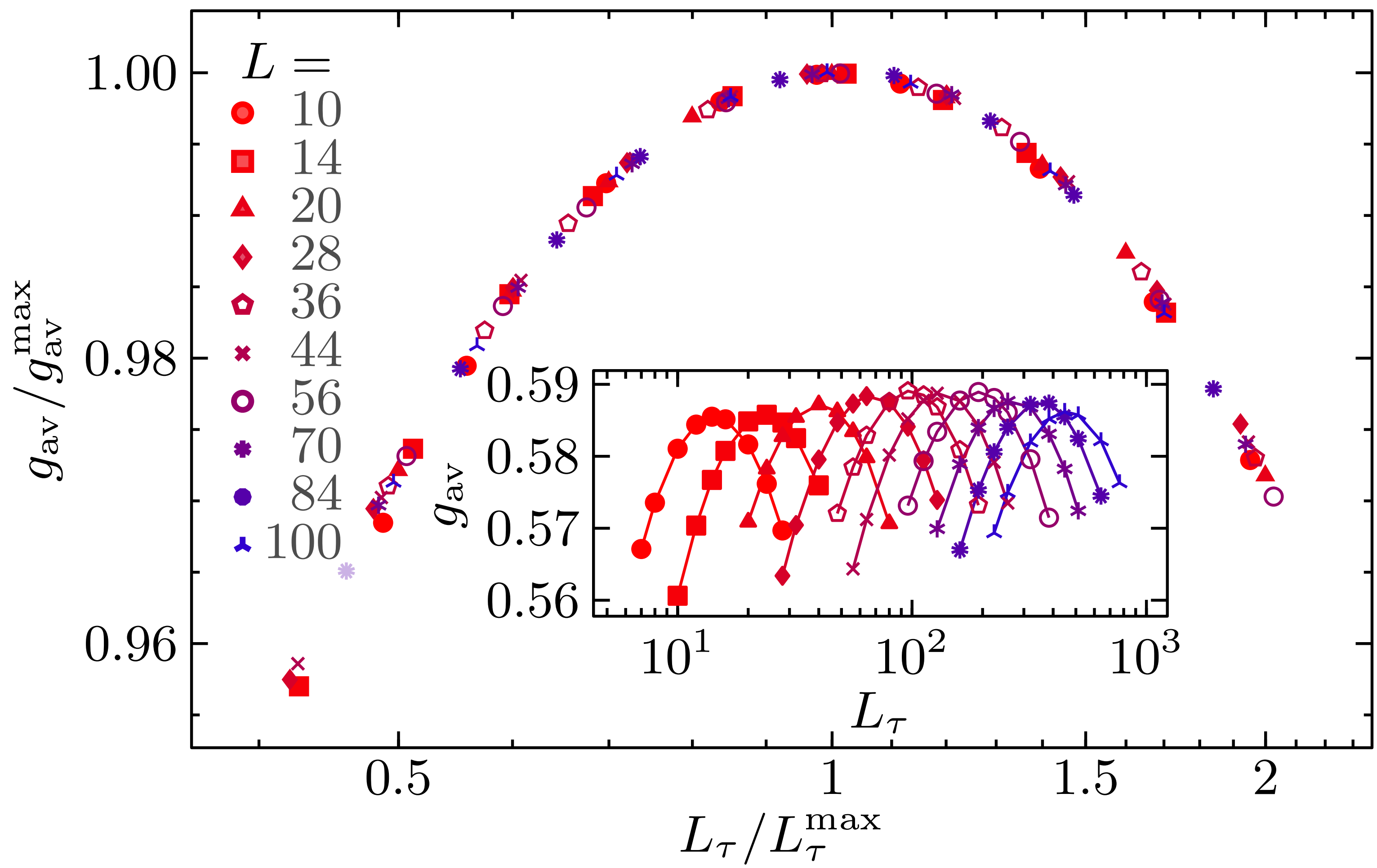
$\Rightarrow$ $g_{\rm av}$ vs. T for **optimal shapes** cross at T_c: $\quad g_{\rm av}(0, L, L_\tau^{\max}) = X(0, c)$

Phase diagram



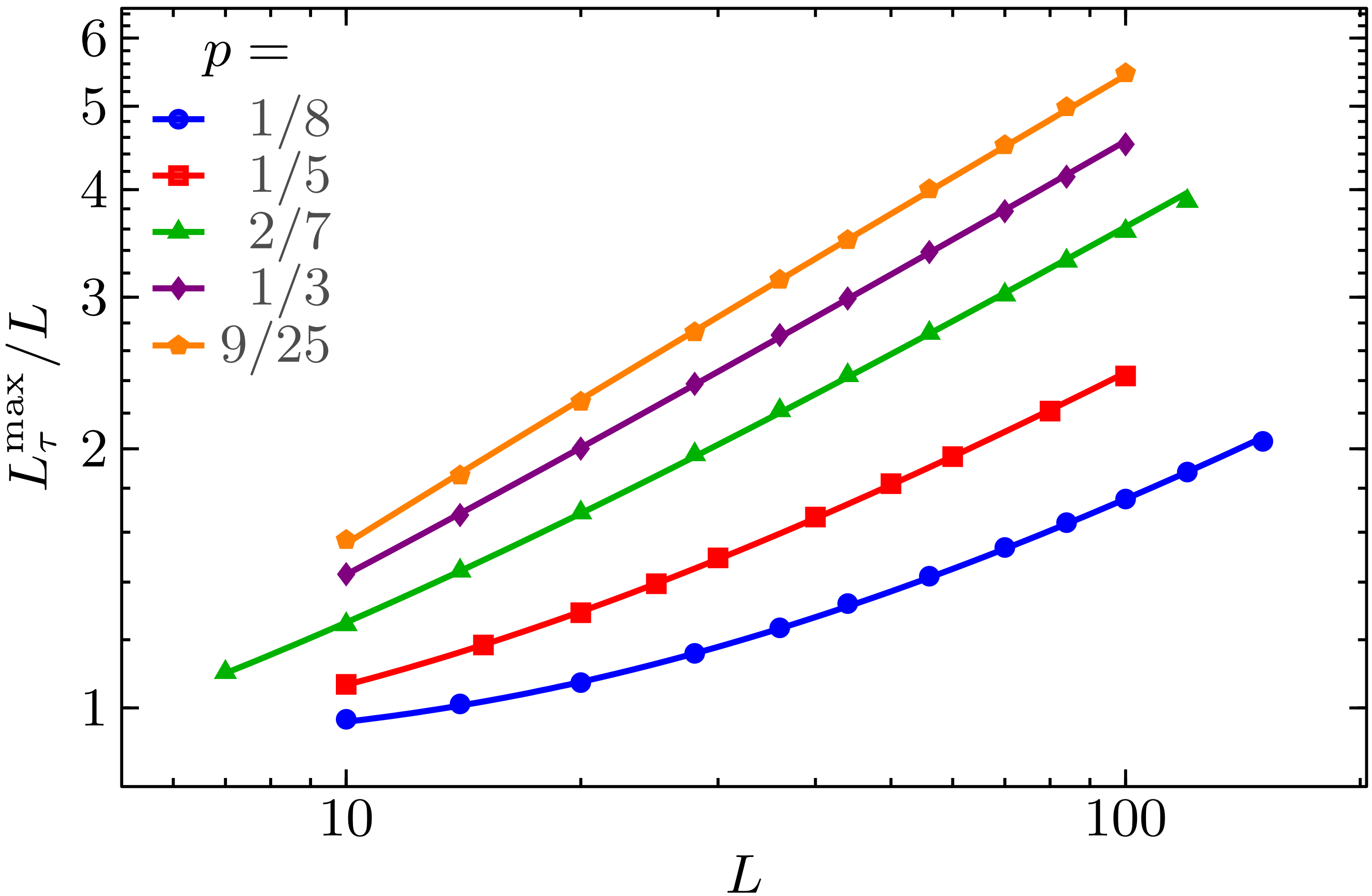
- classical temperature T represents ratio U/J of quantum rotor Hamiltonian (physical temperature of quantum system is zero)

Finding the optimal shapes



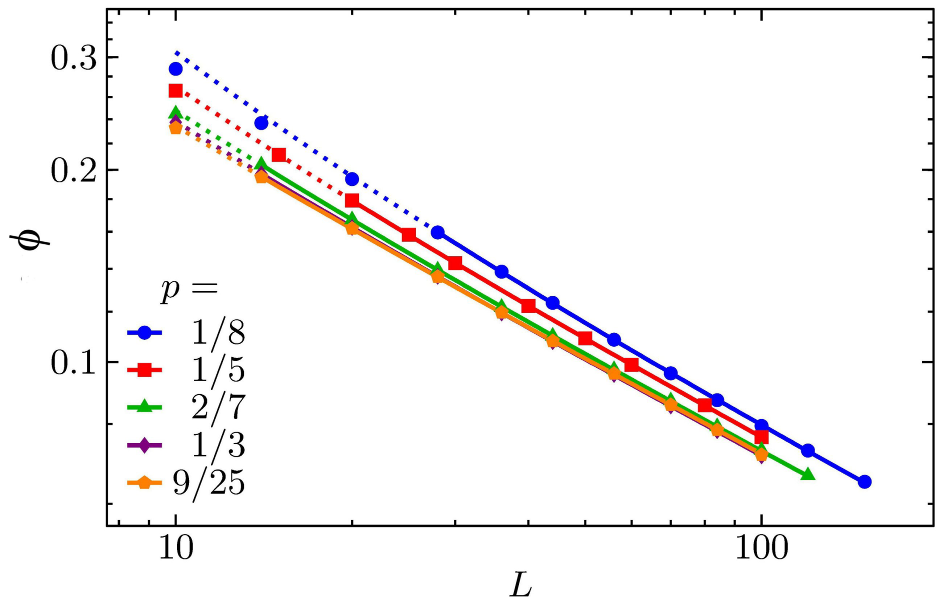
- dilution $p = 1/3$, classical temperature $T = 1.577$

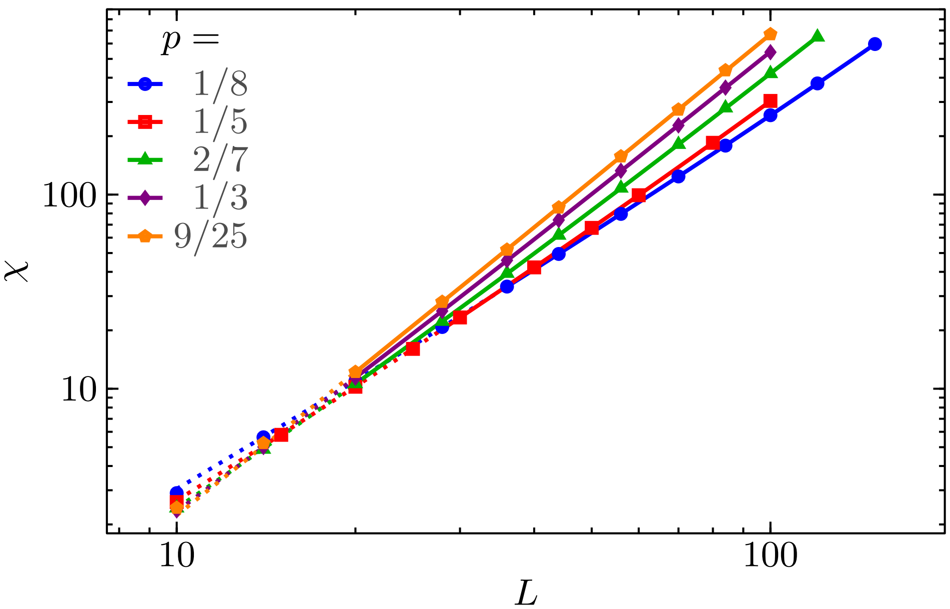
Dynamical critical exponent z



- significant deviations from pure power laws ⇒ **corrections to scaling**
- fit to ansatz $L_\tau^{\max} = aL^z(1 + bL^{-\omega})$ with universal z and ω
- **dynamical critical exponent** $z = 1.52(3)$

Order parameter ϕ and susceptibility χ





Scaling forms:

$$\phi = L^{-\beta/\nu} X_\phi(rL^{1/\nu}, L_\tau/L^z) \, ,$$
$$\chi = L^{\gamma/\nu} X_\chi(rL^{1/\nu}, L_\tau/L^z)$$

- fit data at criticality to

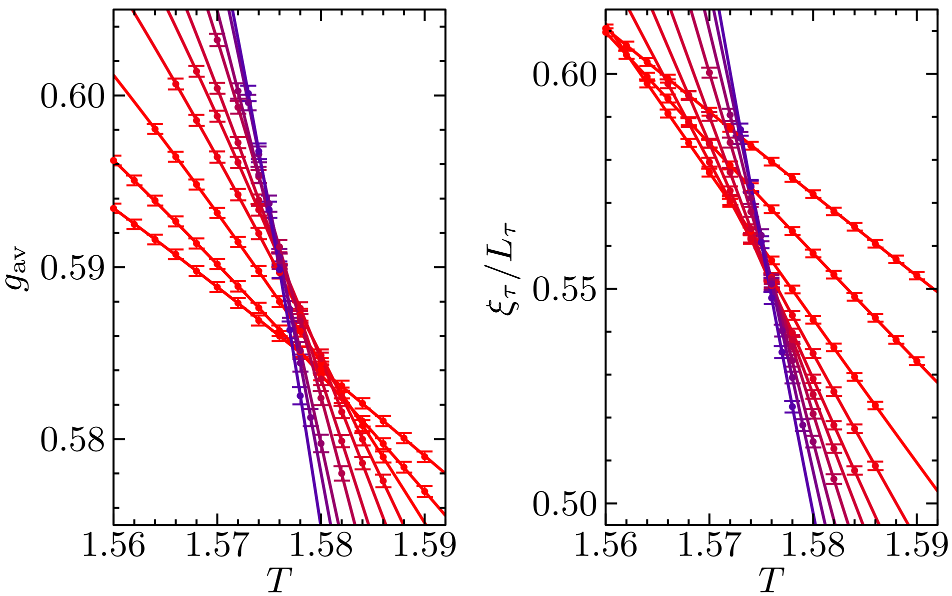
  $\phi = aL^{-\beta/\nu}(1 + bL^{-\omega})$

  $\chi = aL^{\gamma/\nu}(1 + bL^{-\omega})$

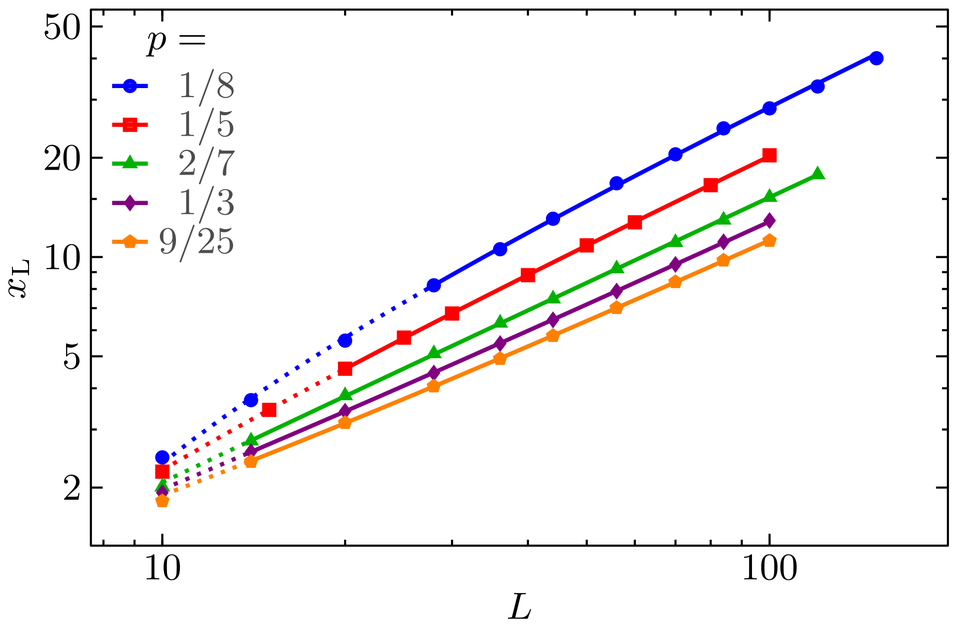
- **critical exponents:**

  $\beta/\nu = 0.48(2)$ and $\gamma/\nu = 2.52(4)$

Correlation length critical exponent ν



(dilution $p = 1/3$)



- **scaling at criticality:**

  $(d/dT)g_{\mathrm{av}} \sim (d/dT)\xi_\tau/L_\tau \sim L^{1/\nu}$
- fit data to $L^{1/\nu}(1 + bL^{-\omega})$
- **critical exponent:** $\nu = 1.16(5)$

exponent	clean	disordered
z	1	1.52
ν	0.6717	1.16
β/ν	0.518	0.48
γ/ν	1.96	2.52

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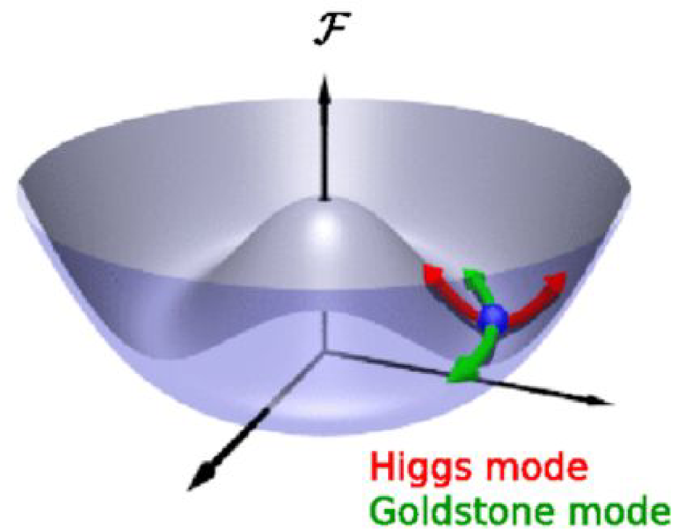
Scalar susceptibility

- parameterize order parameter fluctuations into **amplitude** and **direction**

$$\vec{\phi} = \phi_0(1 + \rho)\hat{\mathbf{n}}$$



- Higgs mode is associated with **scalar** susceptibility

$$\chi_{\rho\rho}(\vec{x}, t) = i\Theta(t) \langle [\rho(\vec{x}, t), \rho(0, 0)] \rangle$$

- Monte-Carlo simulations compute **imaginary time** correlation function

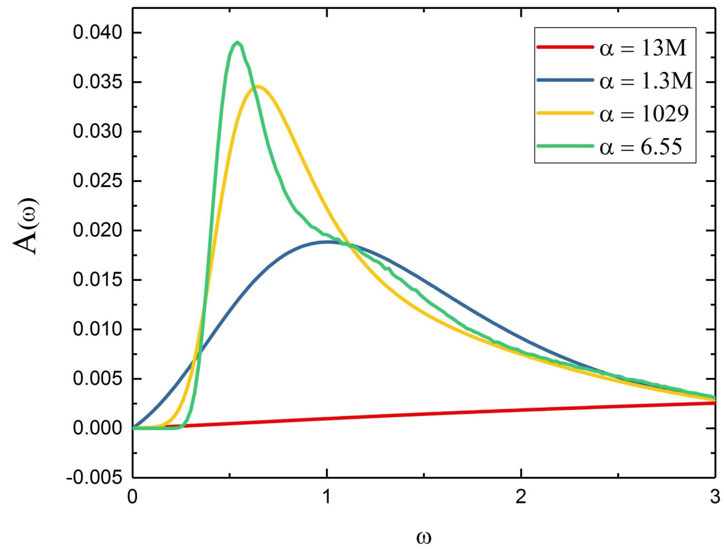
$$\chi_{\rho\rho}(\vec{x}, \tau) = \langle \rho(\vec{x}, \tau)\rho(0, 0) \rangle$$

- **Wick rotation** required: analytical continuation from imaginary to real times/frequencies

Analytic continuation - maximum entropy method

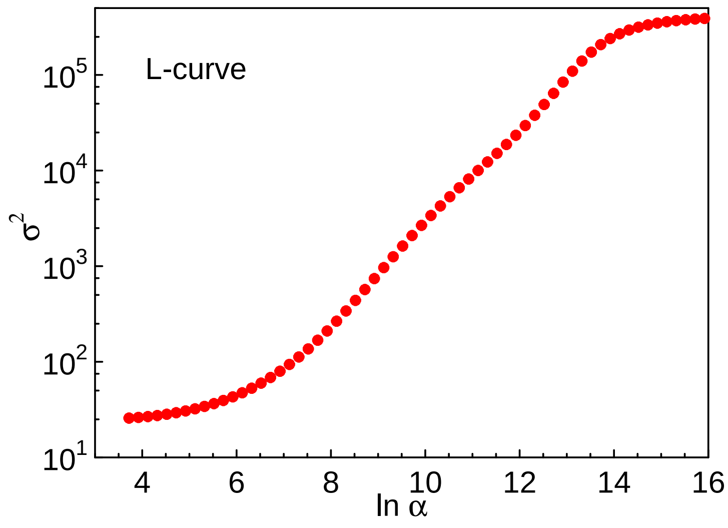
- **Matsubara susceptibility** $\chi_{\rho\rho}(i\omega_m)$ vs. **spectral function** $A(\omega) = \chi''_{\rho\rho}(\omega)/\pi$

$$\chi_{\rho\rho}(i\omega_m) = \int_0^\infty d\omega A(\omega) \frac{2\omega}{\omega_m^2 + \omega^2}$$



Maximum entropy method:

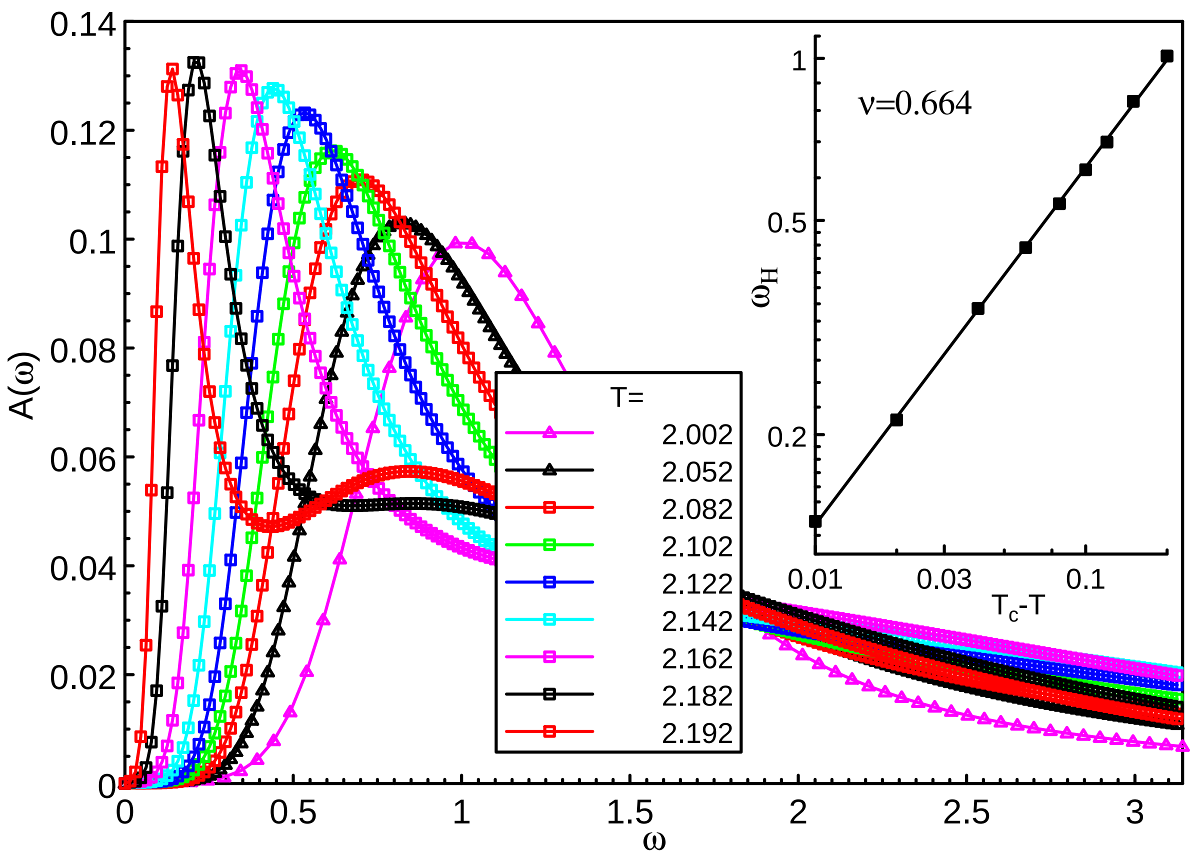
- inversion is ill-posed problem, highly sensitive to noise
- fit $A(\omega)$ to $\chi_{\rho\rho}(i\omega_m)$ MC data by minimizing $Q = \frac{1}{2}\sigma^2 - \alpha S$
- parameter α balances between fit error σ^2 and entropy S of $A(\omega)$, i.e., between fitting information and noise
- best α value chosen by L-curve method [see Bergeron et al., PRE 94, 023303 (2016)]



Higgs mode in clean undiluted system

Scaling form of the scalar susceptibility: [Podolsky + Sachdev, PRB 86, 054508 (2012)]

$$\chi_{\rho\rho}(\omega) = |r|^{3\nu-2} X(\omega |r|^{-\nu})$$



sharp Higgs peak in spectral function, Higgs mass ω_H (peak position) scales as expected with $r = T - T_c$ [confirms older results, Gazit et al. PRL 110, 140401 (2013)]

Higgs mode in disordered system

Scaling form for arbitrary d and z:

$$\chi_{\rho\rho}(\omega) = |r|^{(d+z)\nu-2} X(\omega |r|^{-z\nu})$$

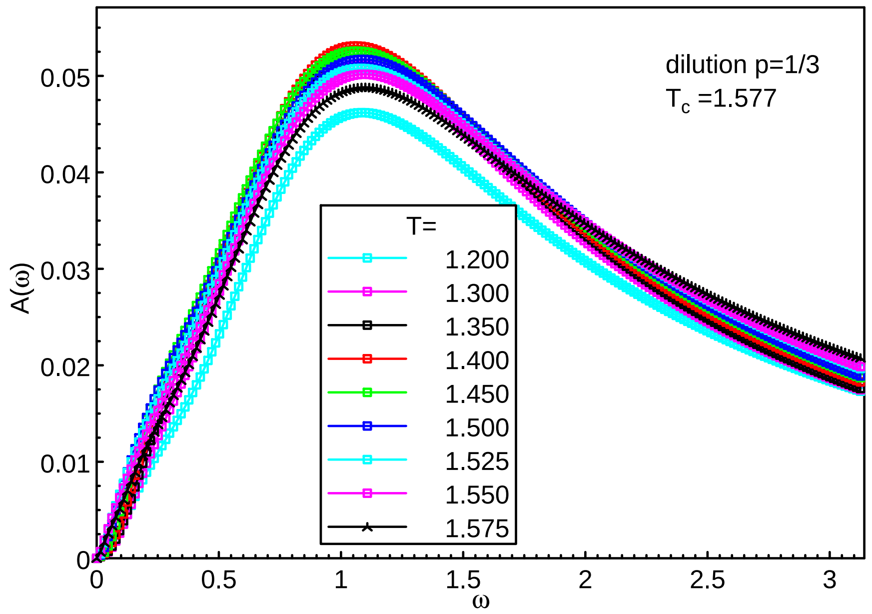
- spectral function shows broad peak near $\omega = 1$
- peak is noncritical: it does not move as T_c is approached
- Higgs mode is **not visible**

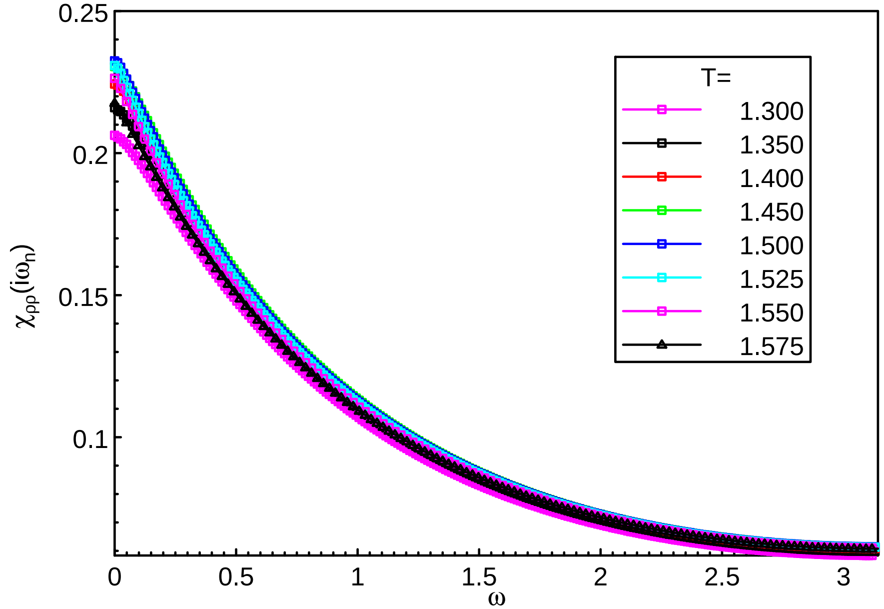
Interpretation:

- amplitude of Higgs mode proportional to $|r|^{(d+z)\nu-2} \approx r^{2.1}$

$\Rightarrow$ Higgs mode suppressed as $r \to 0$

- scalar response dominated by local excitations (clusters?)



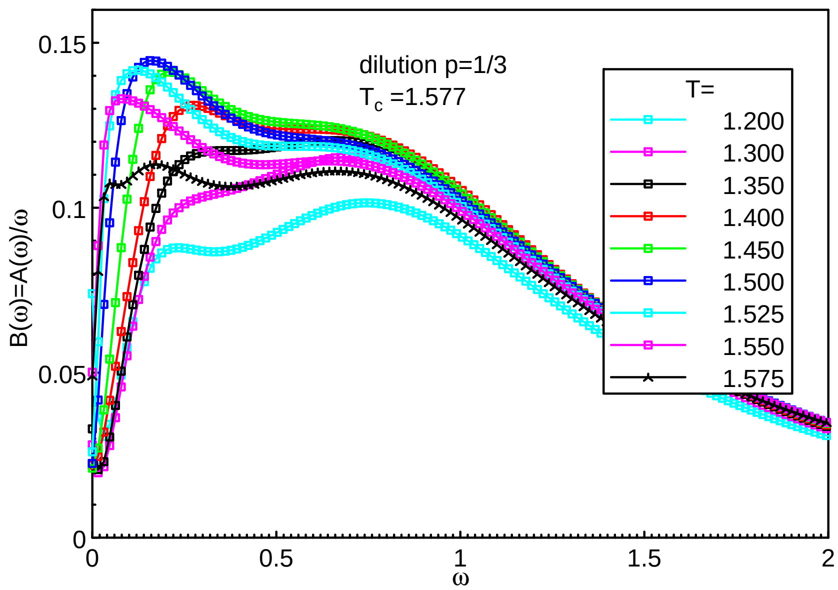


Higgs mode in disordered system II

Shoulder feature:

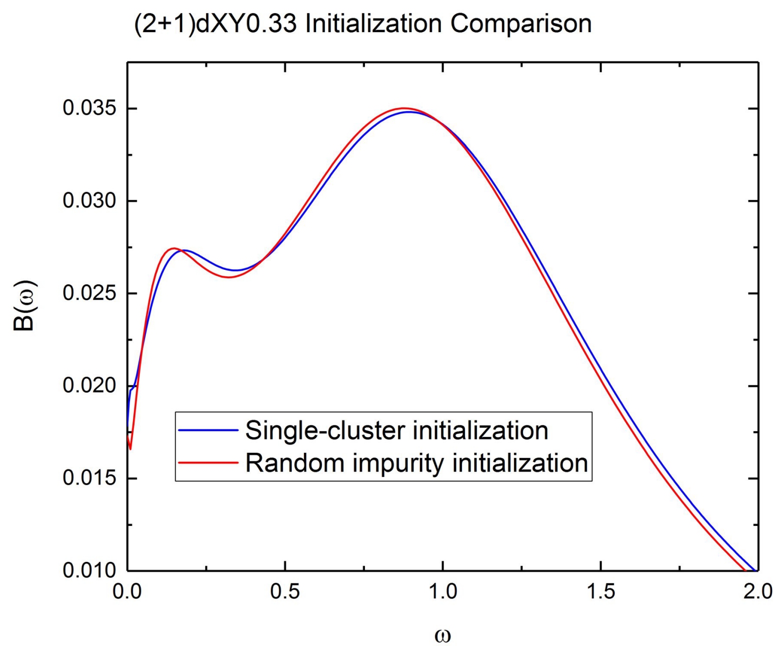
- scalar spectral function has weak **"shoulder"** at low frequencies
- better visible in $B(\omega) = A(\omega)/\omega$
- **Is this the Higgs mode?**

$\Rightarrow$ **unlikely**, peak energy does not scale when critical point is approached



Isolated percolation clusters:

- Is broad peak in scalar response caused by isolated finite-size percolation clusters?



Conclusions

- disordered interacting bosons (with particle hole symmetry) undergo quantum phase transition between **superfluid** state and **insulating Mott glass** state
- critical behavior can be studied by mapping quantum Hamiltonian onto **classical (2+1)-dimensional XY model** and applying Monte Carlo simulations
- conventional **power-law dynamical scaling** $\xi_\tau \sim \xi^z$ rather than exotic activated scaling $\ln \xi_\tau \sim \xi^\psi$ [see classification in T.V., J. Phys. A **39**, R143 (2006)]
- **universal** critical exponents $z = 1.52(3)$, $\beta/\nu = 0.48(2)$, $\gamma/\nu = 2.52(4)$, $\nu = 1.16(5)$ fulfill hyperscaling relation $2\beta/\nu + \gamma/\nu = d + z$
- scalar susceptibility of **clean**, undiluted system shows **sharp** Higgs mode that survives all the way to the quantum phase transition
- Higgs mode is **not visible in diluted system**, scalar response appears to be dominated by local excitations

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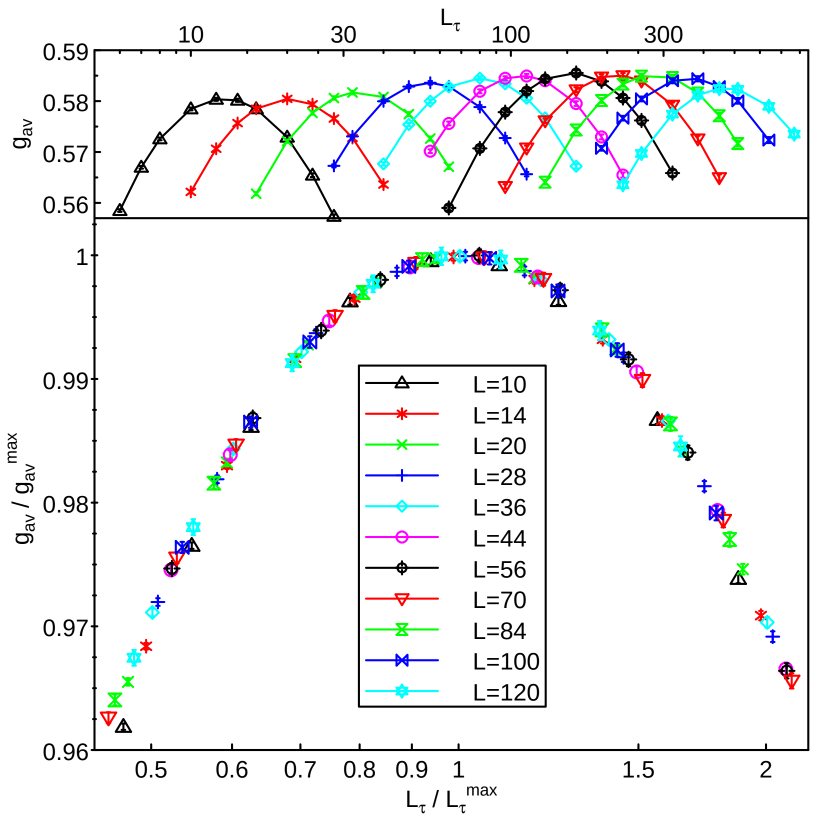
T.V., Jack Crewse, Martin Puschmann, Daniel Arovas, and Yury Kiselev, PRB 94, 134501 (2016)

Anisotropic finite-size scaling

- $g_{\rm av}$ vs L_τ has maximum at **optimal shape** (L_τ/L equals correlation length ratio ξ_τ/ξ)
- at criticality, $L_\tau^{\rm max} \sim L^z$
- for samples of optimal shape, scaling combination $L_\tau/L^z = c = \rm const$
- $g_{\rm av}$ vs. T curves for optimal shape samples cross at T_c: $g_{\rm av}(0, L, L_\tau^{\rm max}) = X(0, c)$



Iterative procedure:

- guess z and corresponding "optimal" sample shapes
- find estimate of T_c from approximate crossing of $g_{\rm av}$ vs. T curves
- find $L_\tau^{\rm max}$ which gives improved optimal shapes

Percolation transition across p_c

- driven by **critical geometry** of the lattice
- dynamical fluctuations "go along for the ride"
- theory predicts **exact** exponent values
  $\beta = 5/36,\ \gamma = 59/12,\ \nu = 4/3,\ z = 91/48$
  [T.V. +J. Schmalian, PRL 95, 237206 (2005)]
- simulation data at $T = 1.0$ and $p = p_c$ agree nearly perfectly with predictions