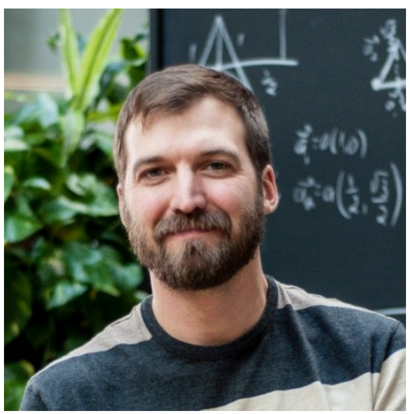
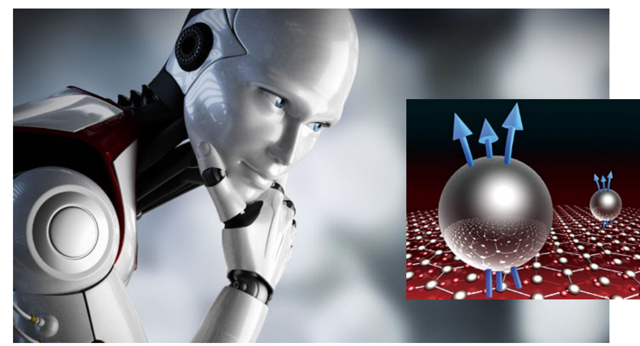
Quantum Loop Topography for Machine Learning

- on topological phases, phase transitions, and beyond

Yi Zhang (Frank)
Cornell University





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Aug. 31, 2017 @ KITP

Plain language version

- 1. Machine learning quantum systems made possible

Quantum system ⟷ Carefully chosen operators ⟷ Machine learning

Plain language version

- 1. Machine learning quantum systems made possible

Quantum system $\leftrightarrow$ Carefully chosen operators $\leftrightarrow$ Machine learning

- 2. Phase identification made reliable via machine learning – study from the system, by the system, for the system

Known systems $\rightarrow$ Machine learning $\rightarrow$ Unknown system

Plain language version

- 1. Machine learning quantum systems made <u>possible</u>

Quantum system ↔ Carefully chosen operators ↔ Machine learning

- 2. Phase identification made <u>reliable</u> via machine learning – study from the system, by the system, for the system

Known systems → Machine learning → Unknown system 1, Unknown system 2, Unknown system 3

- 3. How it changes the game and is indeed <u>not an overkill</u> – a topological phase diagram in minutes

I. Machine learning

Learning by examples

#1: …

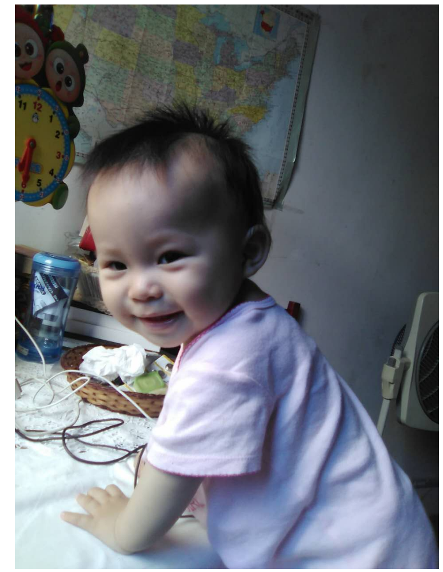
#2: …

#3: …

…

__Training set__

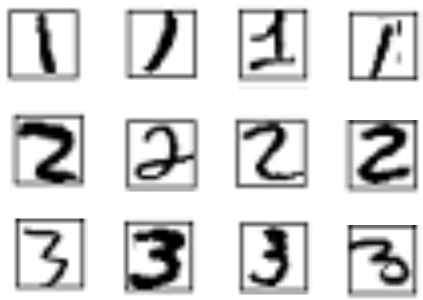
1. learning



2. Application

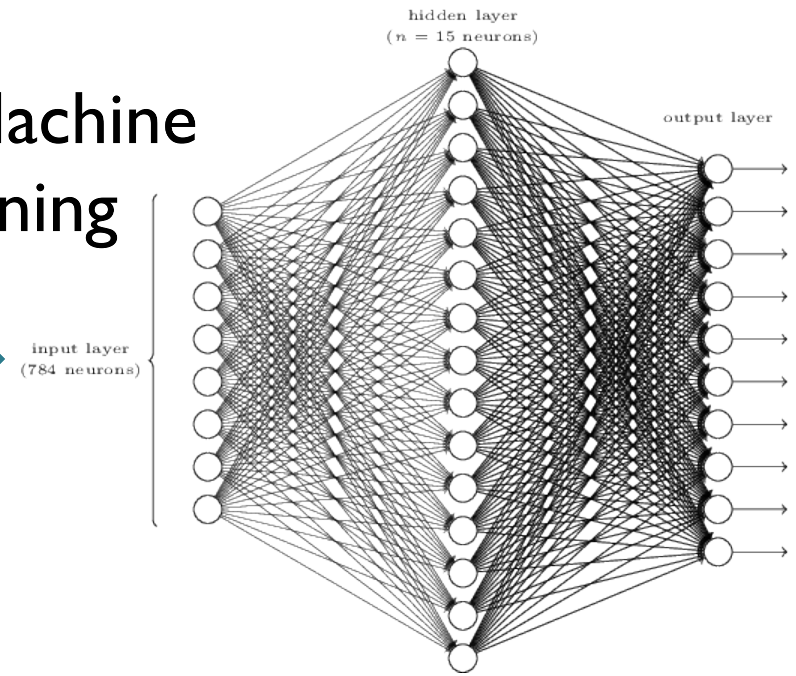
Neural network for image recognition

#1: … #2: … #3: … …



Training set

1. Machine learning

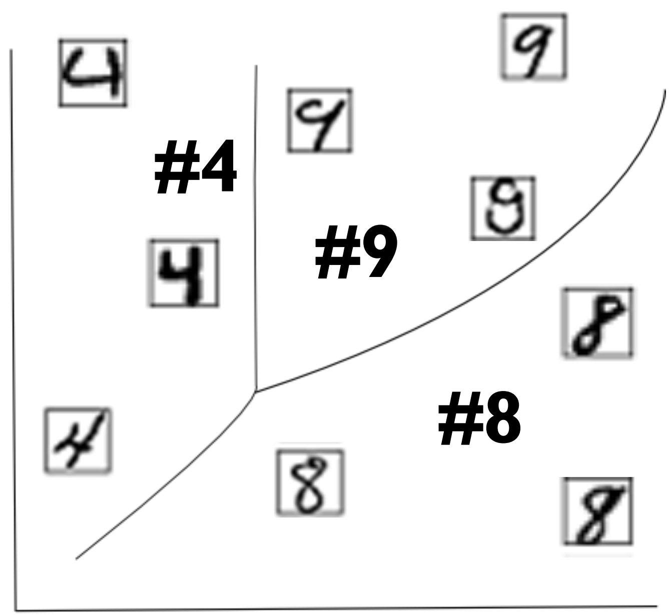


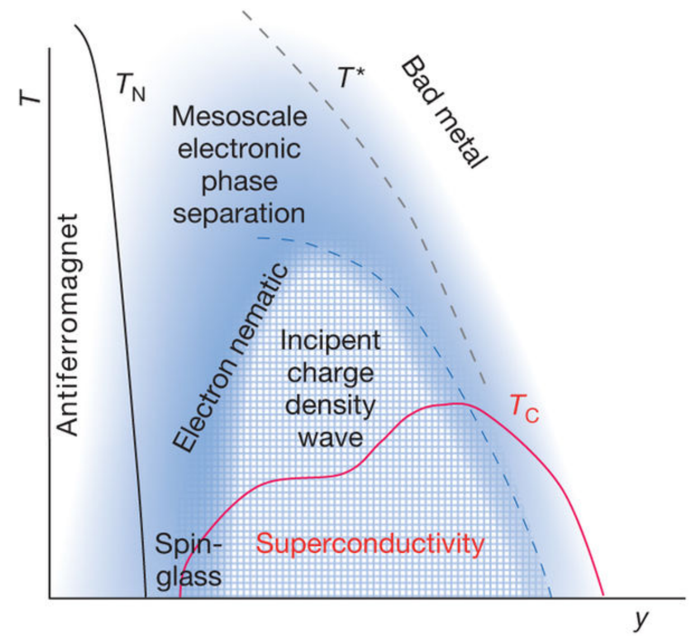
Optimal ***weights*** and ***biases***

2. Application

Online book: http://neuralnetworksanddeeplearning.com/

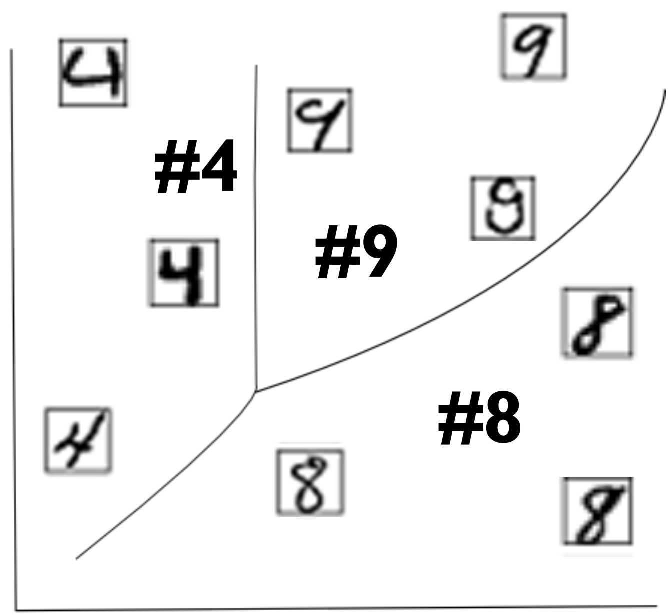
Machine learning as phase classification

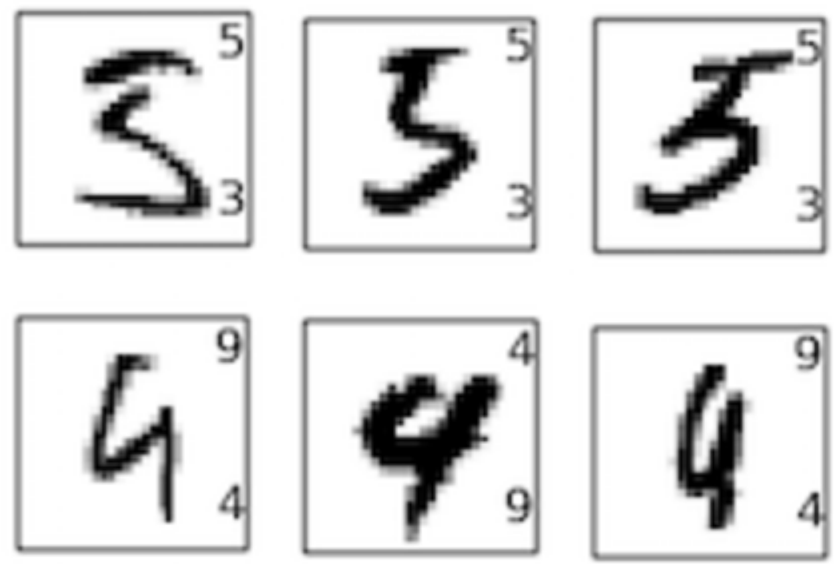




Eduardo Fradkin & Steven A. Kivelson, Nature Physics **8**, 864–866 (2012).

Machine learning as phase classification





Coexisting 'phases'

Machine learning in Condensed Matter Physics

- **Machine learning phases of matter and phase transitions (for phase diagrams)**

- Boltzmann machine as neural network states

- Algorithmic development, e.g. cluster update in Monte Carlo calculations, spectrum analysis, etc.

- Material, dynamics and molecule simulations

- Many more …

Machine learning phases of matter

- **Machine learning phases of matter and phase transitions (for phase diagrams)**

- What do we use as data?

  - How do we process it? ✓

Known systems → Machine learning → Unknown system

  - What information is relevant?

Relevant info

Total info

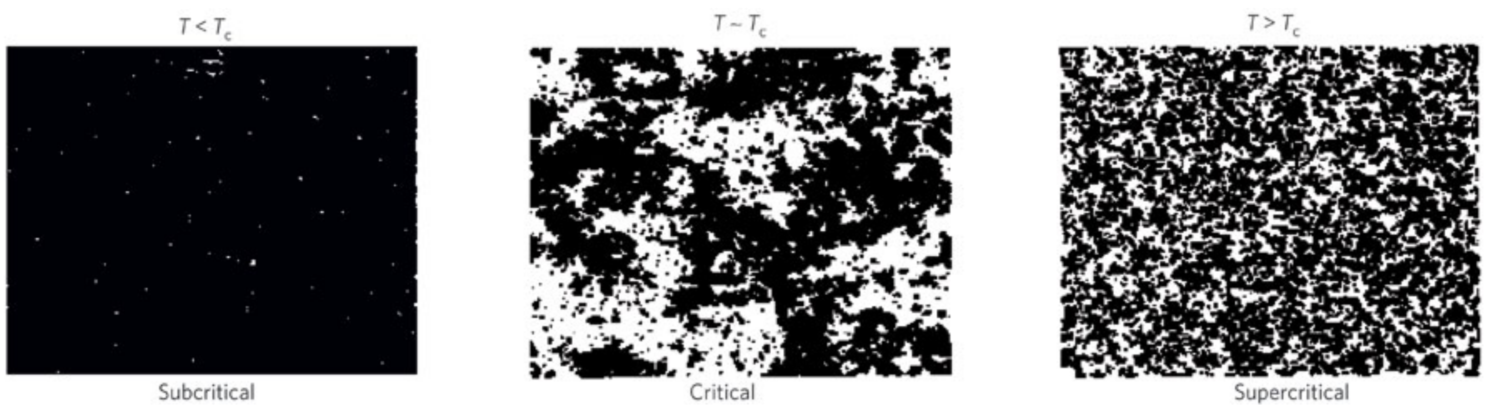
Quantum system ↔ ? ↔ Machine learning

Machine learning phases of matter

- **Machine learning phases of matter and phase transitions (for phase diagrams)**

- What do we use as data?
    - Snapshots of the order parameter field

J. Carrasquilla and R. G. Melko (2016)



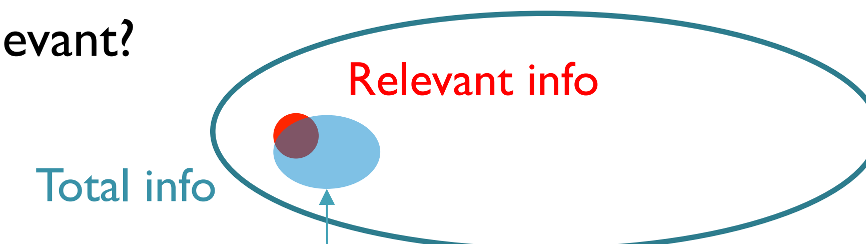
Machine learning phases of matter

- **Machine learning phases of matter and phase transitions (for phase diagrams)**

- What do we use as data?
  - How do we process it? ✓

Known systems → Machine learning → Unknown system

  - What information is relevant?



Quantum system ↔ Carefully chosen operators ↔ Machine learning

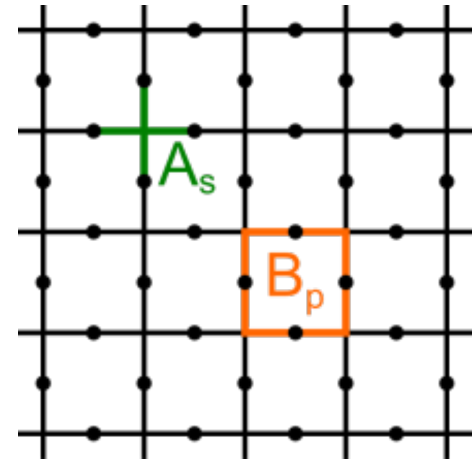
II. A physical problem

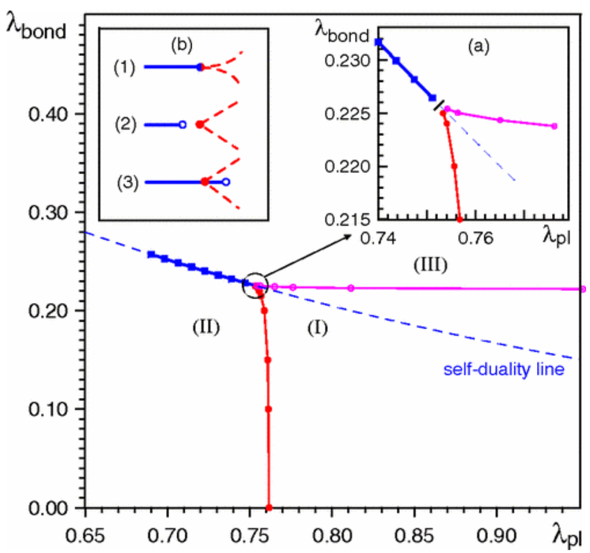
A model for Z_2 quantum spin liquid

Kitaev's toric code in magnetic field

$$H_{2D} = -J_x \sum_s A_s - J_z \sum_p B_p - h_x \sum_j \sigma_j^x - h_z \sum_j \sigma_j^z$$

$$A_s = \prod_{j\in s} \sigma_j^x \text{ and } B_p = \prod_{j\in p} \sigma_j^z$$



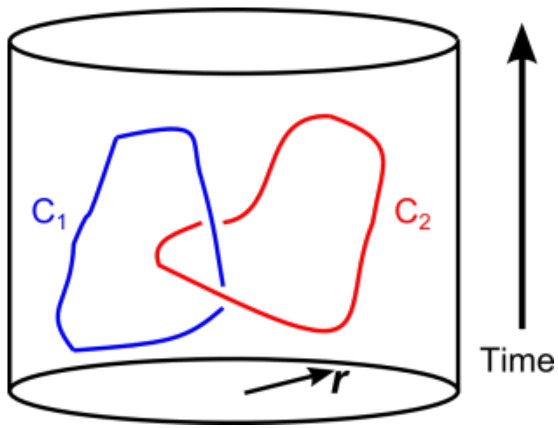


Equivalent Z_2 gauge Higgs model on a 3D cubic lattice

$$\beta H_{3D} = -\lambda_b \sum_j S_j - \lambda_p \sum_p \prod_{j\in p} S_j$$

I. S. Tupitsyn, A. Kitaev, N.V. Prokof'ev, and P. C. E. Stamp (2010), etc.

Topological quantum field theory for the Z_2 quantum spin liquid



Ideal TQFT:

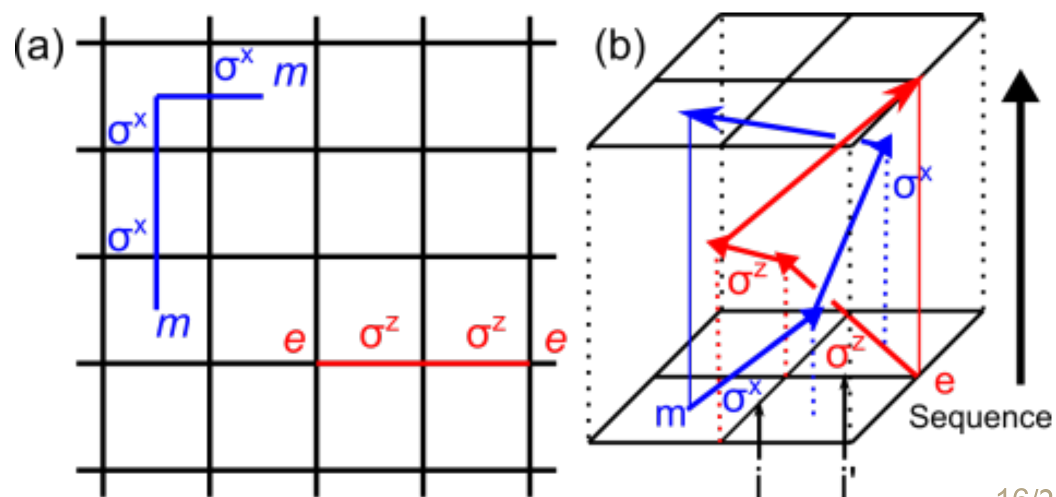
$$\mathcal{L}_{CS} = \frac{K_{IJ}}{4\pi}\epsilon^{\mu\nu\lambda}a^I_\mu\partial_\nu a^J_\lambda - a^I_\mu j^\mu_I \qquad K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$$\mathcal{P}W^I_{C_1}W^J_{C_2} = e^{2\pi i K^{-1}_{IJ}L(C_1,C_2)}$$

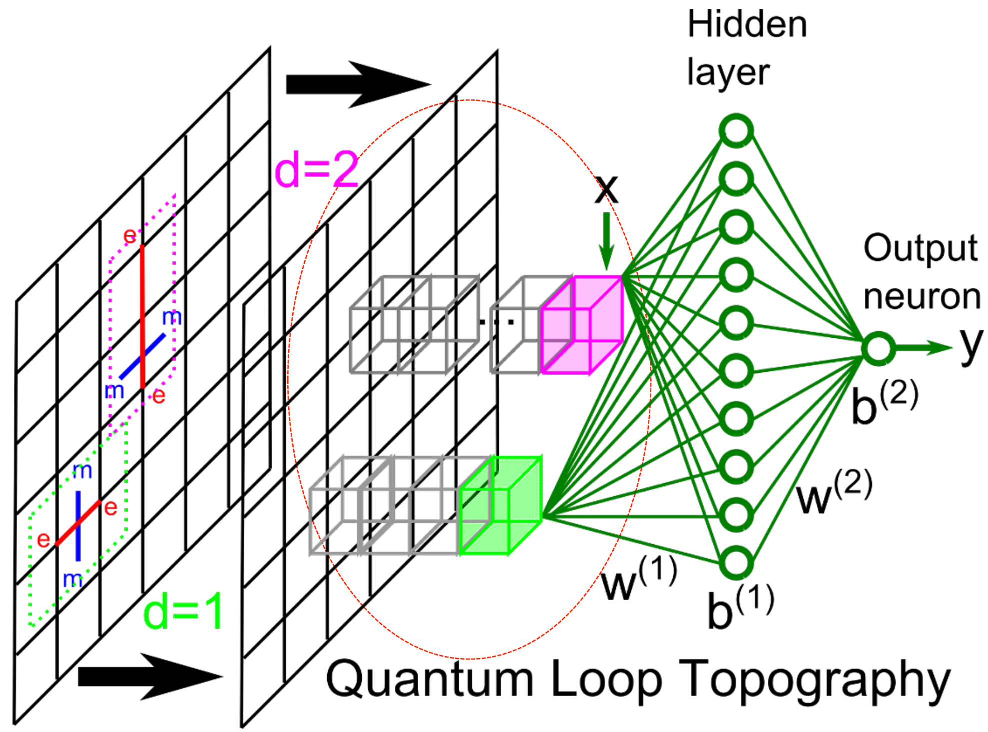
$$W^I_C \equiv \mathcal{P}\exp\left(i\oint_C a^I_l dl\right)$$

Reality in a lattice Hamiltonian:

- Discrete lattice
- Finite correlation length
- Cut off, fluctuation and uncertainty in measurement



III. A love story



Selected for a Viewpoint in *Physics*

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Quantum Loop Topography for Machine Learning

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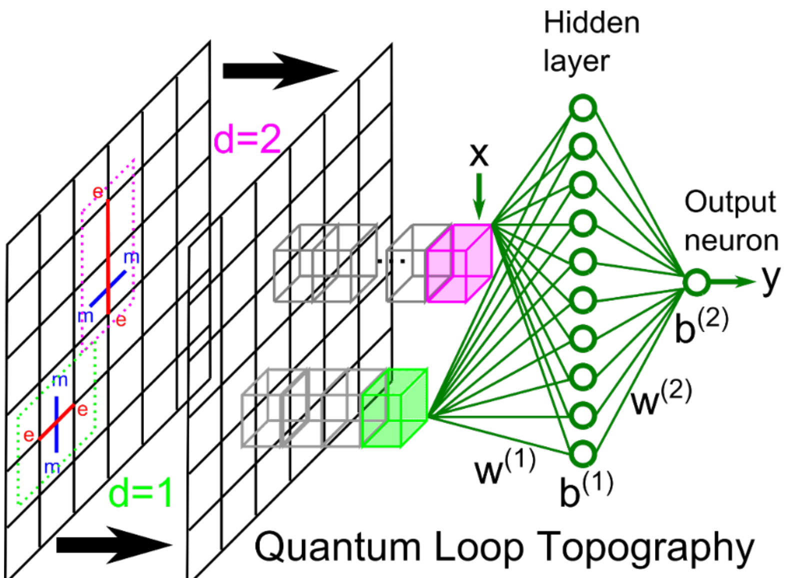
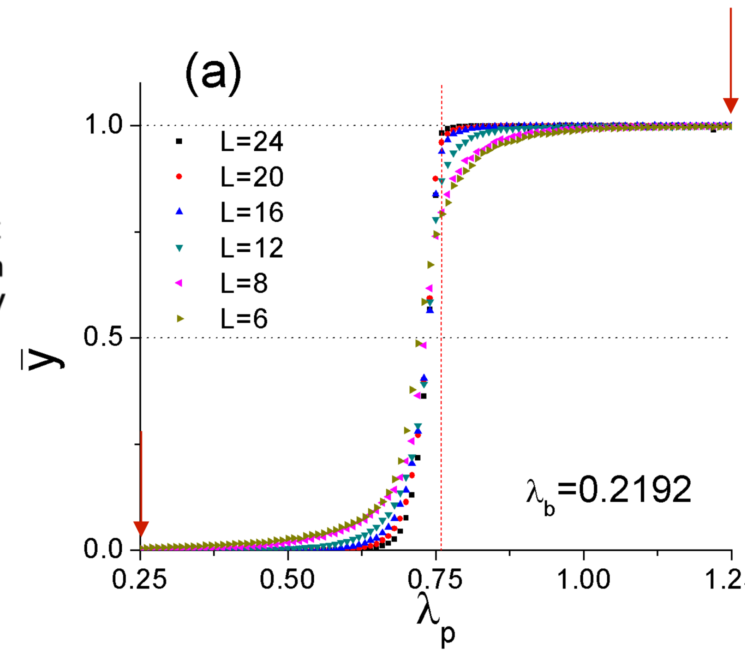
Machine learning Z_2 quantum spin liquid





Information flow

1.Training

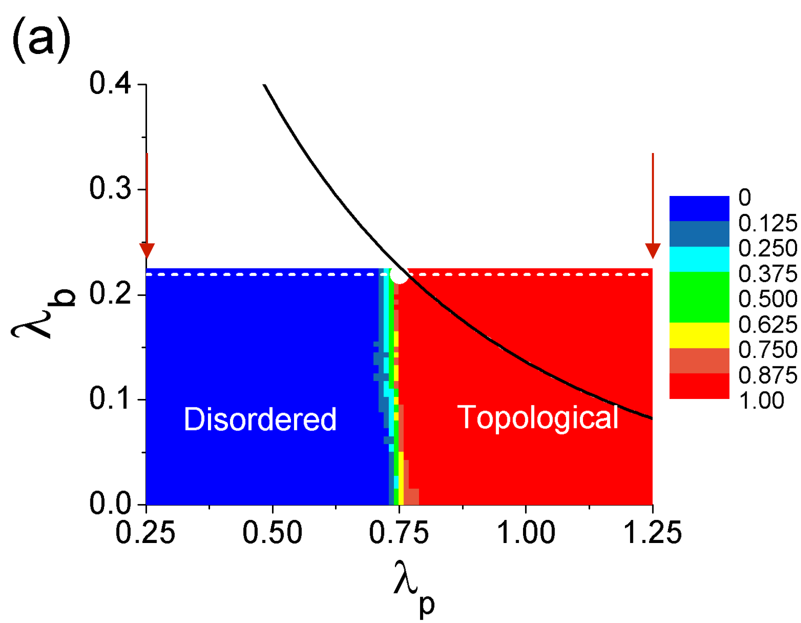
1.Training

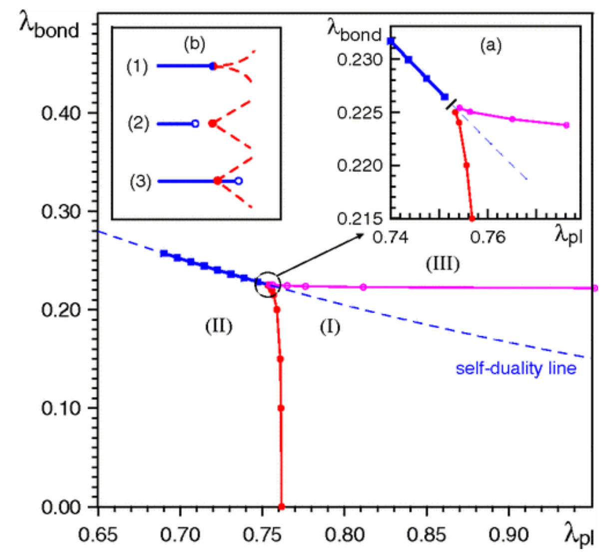
Information flow

2. Interpolating

YZ, R.G. Melko, Eun-Ah Kim (2017)

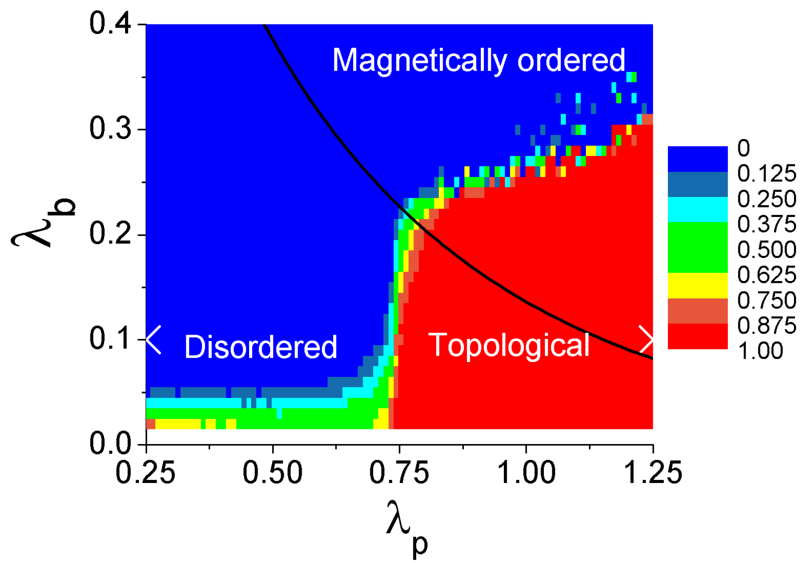
Machine learning Z_2 quantum spin liquid

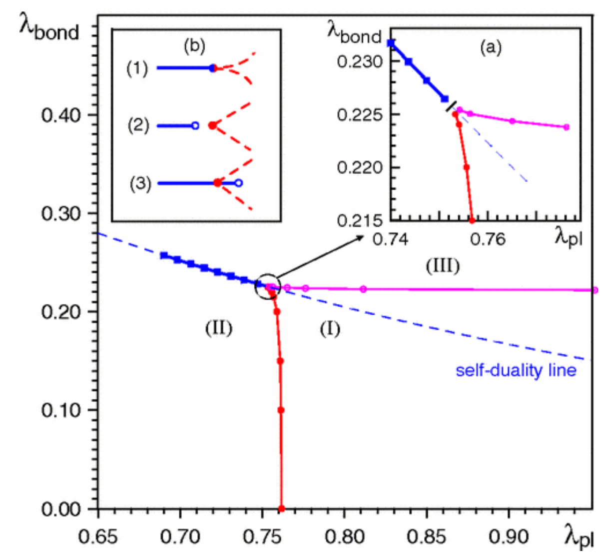




YZ, R.G. Melko, Eun-Ah Kim (2017)

Machine learning Z_2 quantum spin liquid



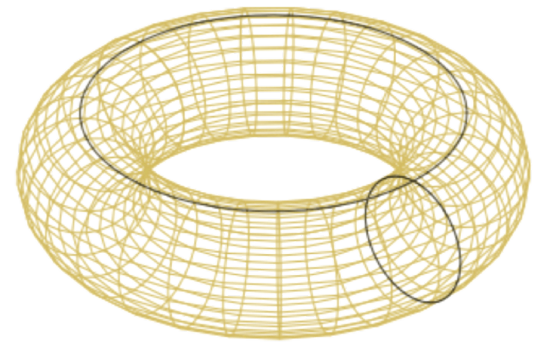


Efficiency: one training for the entire multi-parameter phase diagram

YZ, R.G. Melko, Eun-Ah Kim (2017)

Calculations take days, if not years

- We can resort to critical scaling, or long-range behavior
- E.g. minimum entropy states, but
  - Constraints
    - All degenerate ground states
    - Nontrivial manifold
  - Cost



Entanglement entropy $S = -\log\langle Swap \rangle$ computation*:

Differences between several S for TEE γ

Various trials for maxima of γ

Various points in the phase diagram

Parallel MC

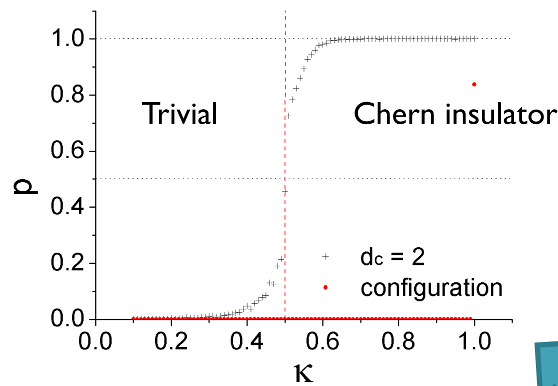
$\frac{105 CPUmins \times 100 \times 100 / 2}{56 CPUs}$

= ***7.5 years***…

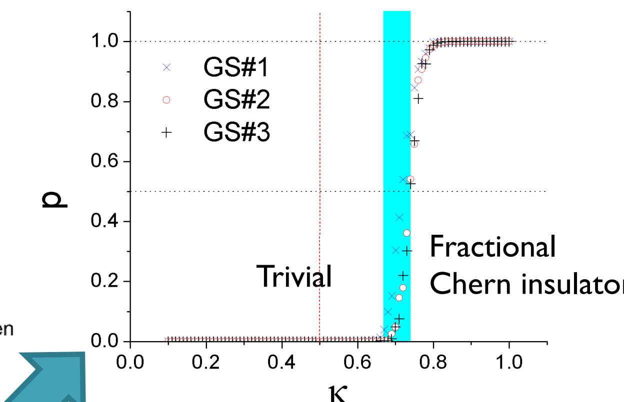
* Assuming 100 trails for maxima and 100 parameters in phase space

YZ, Tarun Grover, Ari Turner, Masaki Oshikawa, and Ashvin Vishwanath (2012).

Not years, not days, just ***minutes***



YZ, Eun-Ah Kim(2016)



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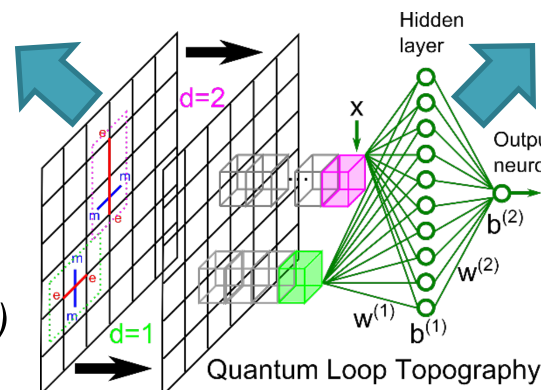

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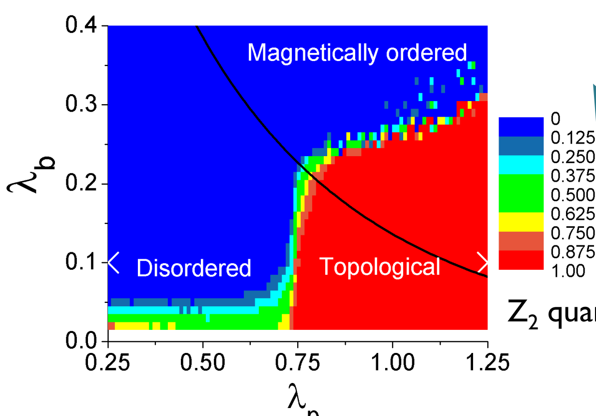
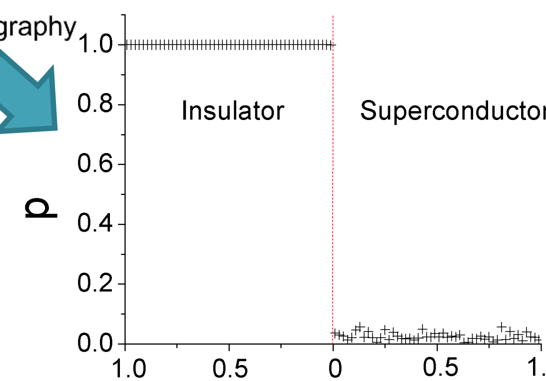


Z_2 quantum spin liquid

In preparation



Summary

- Quantum Loop Topography as a bridge between the physical systems and machine learning technology
- Machine learning as a novel approach for physical problems, such as the phase diagram with Z_2 quantum spin liquid
- Advantages:
  - Accuracy
  - Efficiency
  - Versatility
- The story is just beginning…