Enhancement of superconductivity near a nematic quantum critical point

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Abstract

Motivated by the evidence for Ising nematic quantum criticality in the iron-based and cuprate high temperature superconductors, we consider a metallic superconductor weakly coupled to the soft fluctuations associated with a nematic quantum critical point (NQCP). We show that:

1) A BCS-Eliashberg treatment remains valid outside of a parametrically narrow interval about the NQCP
2) The symmetry of the superconducting state (d-wave, s-wave, p-wave) is determined by the non-critical interactions, but T_C is enhanced by the nematic fluctuations in all channels.
3) In 2D, this enhancement grows upon approach to criticality up to the point at which the weak coupling approach breaks down, but in 3D the enhancement is much weaker.

Motivation

- Evidence for quantum critical points under the super-conducting dome in both hole-doped cuprate and Fe-based high T_C superconductors
- QCPs seem to occur at a critical doping x_c near optimal doping
- Nematic character of QCP confirmed in pnictides by elasto-resistance measurements
- In cuprates, strong nematic tendencies established by transport anisotropy measurements
- Other forms of criticality unlikely:
  - Large spin gap → no SDW criticality
  - Incommensurate, disorder, short correlation lengths → no CDW criticality

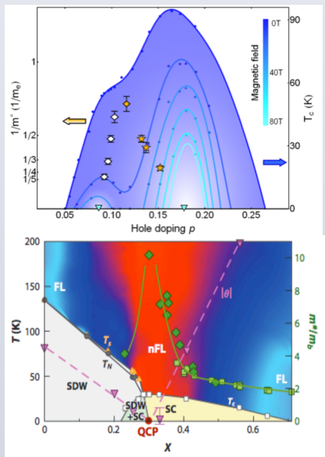


Fig. 1: Evidence for quantum criticality. In the upper panel[2], an effective mass divergence, and therefore a quantum critical point, is inferred from quantum oscillation measurements in underdoped YBCO. In the lower panel[3], similar findings for the iron pnictide superconductor $BaFe_2(As_{1-x}P_x)_2$.

Nematic quantum critical points are likely present in both families of high-T_C superconductors.

What are the consequences of nematic quantum critical fluctuations for superconductivity?

Effective action

$$S[\phi,\bar\psi,\psi] = S_{el}[\bar\psi,\psi] + S_{nem}[\phi] + S_{int}[\phi,\bar\psi,\psi]$$

- S_{el} : Fermi liquid with cutoff $W \ll E_F$, weak interactions $\Gamma^{(0)}(\vec k,\vec k')$ in the Cooper channel
- S_{nem} : d+1-dimensional Ising critical theory of nematic boson (scalar ϕ^4 theory)
  - Tuning parameter x with QCP at $x = x_c$
  - Standard critical exponents (Wilson-Fisher for d=2, mean-field for d=3), vanishing energy scale $\Omega \sim |x-x_c|^{\nu z} \ll W$
- $S_{int} = \alpha \int d\tau \frac{d\vec k}{(2\pi)^d}\frac{d\vec q}{(2\pi)^d} f(\vec q,\vec k)\phi_{\vec q}\bar\psi_{\vec k+\vec q/2}\psi_{\vec k-\vec q/2}$
- $f(\vec q,\vec k)$: $d_{x^2-y^2}$ form factor, e.g. $\cos(k_x)-\cos(k_y)$. (NB: this encodes the symmetry of the nematic order and has nothing to do with superconductivity).
- **Main assumption:** $\alpha \ll 1$ (needed for mathematical control)

Phase diagram

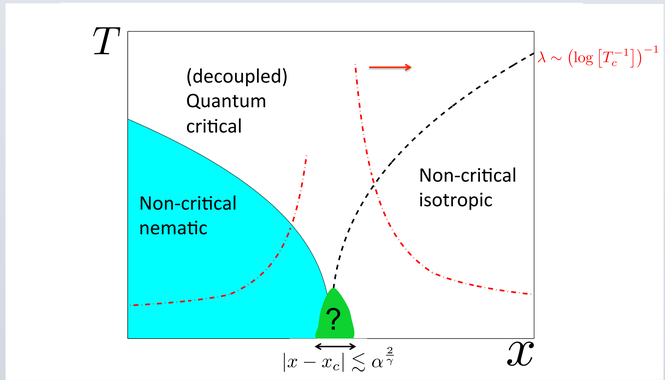


Fig. 2: Phase diagram. When the Yukawa coupling α is small, the singular effects of the fermions on the boson dynamics are confined to a parametrically small region about the quantum critical point, here shown in green. We do not work in this regime, but in the entire rest of the phase diagram. Within this regime of control, there are only small quantitative renormalizations of the $\alpha = 0$ boson phase diagram, and we can perturbatively compute the effect of the bosons on the fermionic pairing interaction. The interactions remain weak, allowing us to use a a BCS (or equivalently perturbative renormalization group) approach to analyze the superconductivity. Though T_C remains small throughout our region of control, it can increase drastically on approach to criticality.

Pairing interaction

We integrate out the nematic boson at the outset, yielding a perturbative expansion for the nematic-mediated contribution to the pairing interaction. In our regime of control, we need consider only the lowest order diagram:

$$+ \ \dots \Leftrightarrow \Gamma^{(ind)}_{\vec k,\vec k',\omega-\omega'} = -\frac{\alpha^2}{4}\left|f\left(\vec k-\vec k',\frac{\vec k+\vec k'}{2}\right)\right|^2 \frac{\chi(\vec k-\vec k',\omega-\omega')}{\sqrt{v_{\vec k}v_{\vec k'}}} + \mathcal{O}(\alpha^4)$$

where χ is the $\alpha = 0$ boson susceptibility, which has critical dependence on the tuning parameter. The interaction is fully attractive, and sharply peaked at small momentum and energy transfer—it scatters an incoming cooper pair only to an outgoing one with similar relative momentum. As such, for purposes of intuition can be usefully approximated as

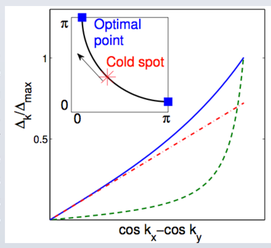
1) $$\Gamma^{(ind)}_{\hat k,\hat k'} \approx -\lambda^{(ind)}\left|f(0,\hat k)\right|^2\delta(\hat k-\hat k')$$

$$\lambda^{(ind)} \equiv \frac{\alpha^2}{4}\int\frac{d\hat k}{v_{\hat k}}\chi(\hat k,0) \sim \alpha^2\rho(E_F)\chi(0,0)(k_F\xi)^{1-d}$$

2) $$\sim \begin{cases}\alpha^2(k_F\xi)^{1-\eta} & d=2\\ \alpha^2\log[k_F\xi] & d=3\end{cases}$$

Here ξ is the nematic correlation length, which diverges at criticality. The full pairing interaction consists of this plus a suitably renormalized $\Gamma^{(0)}$.

Fig. 3: Numerical eigenstates. Gap function along the Fermi surface for d=2. The non-critical interaction $\Gamma^{(0)}$ is taken to favor a nearest-neighbor d-wave gap (dot-dashed line). The solid and dashed lines are, respectively, for $k_F\xi = 10$ and $k_F\xi = 100$. **Inset:** Cartoon of one quadrant of the Fermi surface of the cuprates showing an "optimal point" where $\Gamma^{(ind)}$ takes its maximum and the "cold spot" where it vanishes by symmetry.



Superconducting properties

The standard BCS prescription applies—we view the interaction as a matrix in momentum space, and diagonalize it over the Fermi surface. The critical temperature and gap function are determined by the leading eigenvalue and its eigenfunction:

$$T_c \sim \Omega\exp[-1/\lambda]\ , \quad \Delta_{\hat k} \propto \sqrt{v_{\hat k}}\phi_{\hat k}$$

The diagonalization is easily performed numerically, as shown in Fig. 3. Some qualitative consequences also follow from analytic calculations:

- From eq. 1), $\Gamma^{(ind)}$ is a diagonal matrix with negative eigenvalues, adding it to $\Gamma^{(0)}$ enhances all pairing eigenvalues, so **nematic fluctuations enhance T_C in all channels.**
- From eq. 2) **the enhancement grows on approach to criticality, much more rapidly in 2D than 3D**
- An eigenstate of $\Gamma^{(ind)}$ can be represented as a peak at a single Fermi surface position. Its eigenvalues have approximate degeneracy corresponding to the rotational symmetry. Thus, close to criticality where $\Gamma^{(ind)}$ dominates:
  1. The gap function is highly anisotropic.
  2. **The symmetry of the order parameter is determined by the non-critical interactions $\Gamma^{(0)}$.**

References

1. S. Lederer, Y. Schattner, E. Berg, and S. A. Kivelson, arxiv: 1406.1193
2. B.J. Ramshaw et al, arxiv: 1409.3990
3. T. Shibauchi, A. Carrington and Y. Matsuda, *Annu. Rev. Condens. Matter Phys.* 5, 113 (2014)