Intertwined Orders in High Temperature Superconductors

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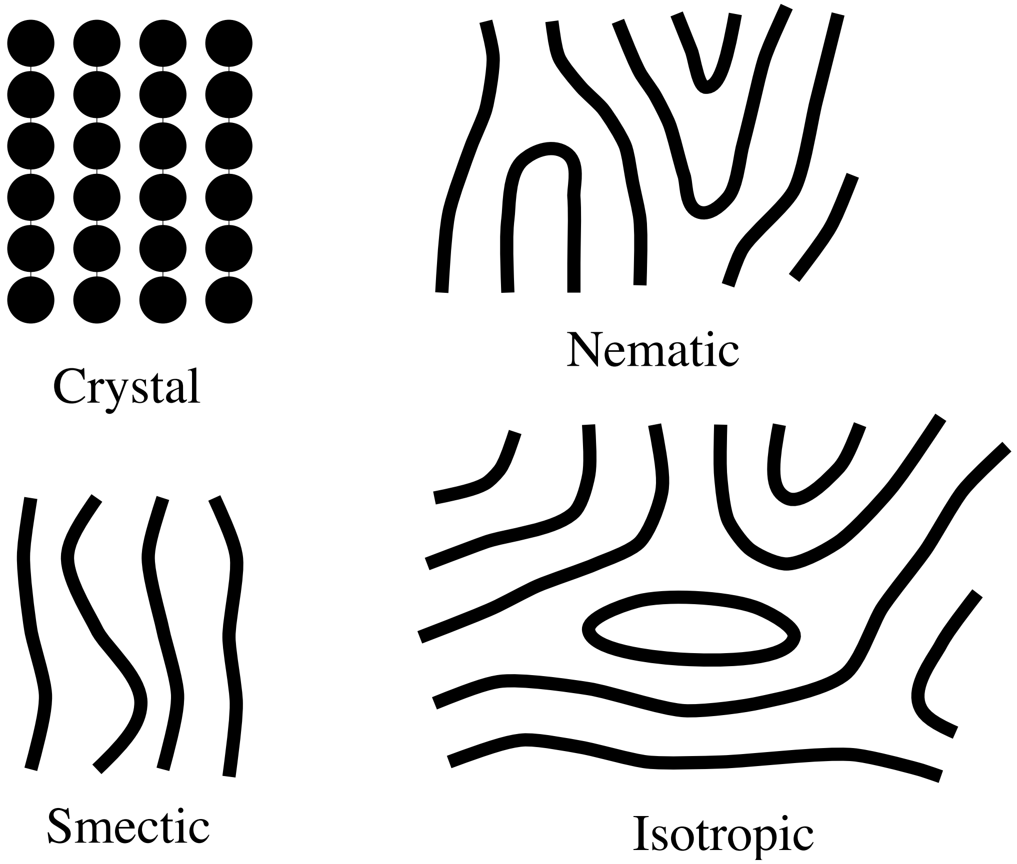
E. Fradkin, S. A. Kivelson and J. M. Tranquada, arXiv:1407.4480

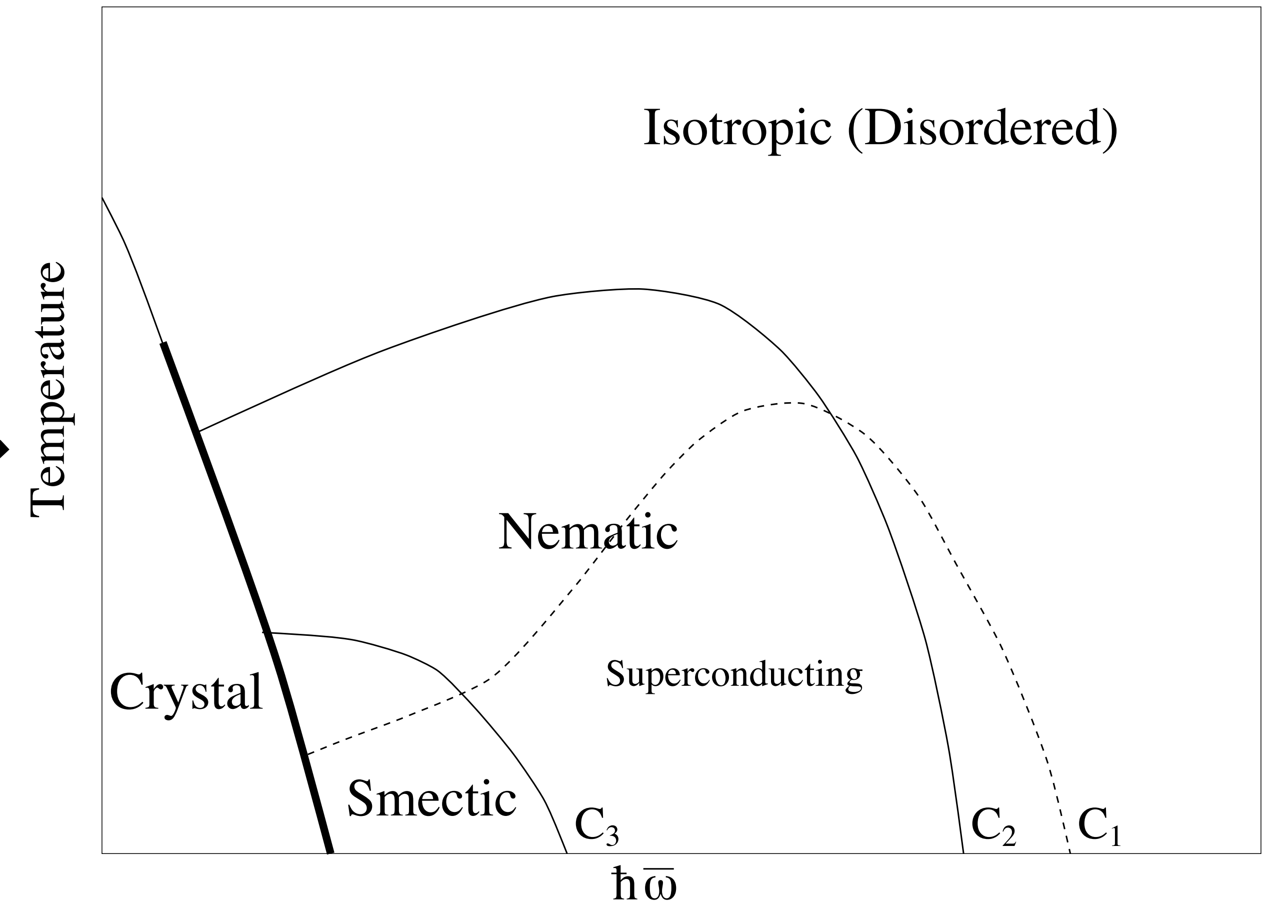
Why is High Temperature Superconductivity different

- High T_c superconductors are quasi-two dimensional materials
- The "normal" phase is not a good metal: the electronic quasiparticles are not well defined
- The dominant interactions are repulsive, not attractive
- The SC state arises from doping a strongly correlated Mott insulator
- A host of other ordered (or almost ordered) states are also seen

Electronic Liquid Crystal Phases of Doped Mott Insulators

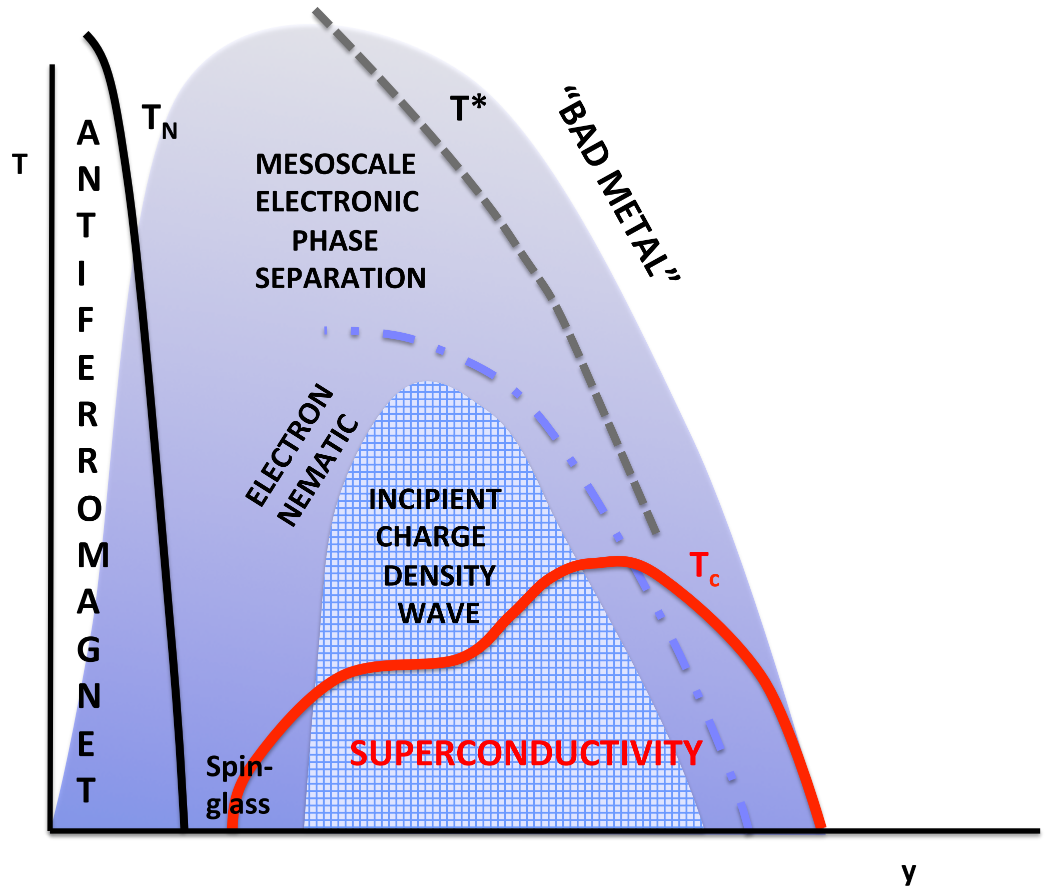
S.A. Kivelson, E. Fradkin and V. J. Emery (1998)





- Break translation and rotational invariance to varying degrees
- Smectic (stripe): breaks translational symmetry in one direction
- Nematic: uniform and anisotropic metallic or SC phase; breaks the point group symmetry
- How are they related to SC?

The YBCO Phase Diagram



E. Fradkin and S.A. Kivelson
Nature Physics, 2012

It applies generally to all the cuprates, LBCO, YBCO, LSCO… Similarity with the Fe SCs

- T_c vs x has a plateau with an anomaly at "1/8" (Bonn & Hardy)
- INS finds nematic order for y~6.45 with T_{cN}~150 K
- NMR measurements at high fields finds a charge stripe signature in agreement with quantum oscillations (dHvA) with a T_{cdw}~60 K
- Transport and Peltier experiments find nematic order in the pseudogap regime, with T_V~150 K
- RIXS, X ray diffraction, and ultrasound anomalies find static stripe charge order near x=1/8 with a T_{cdw}~150 K

The Competing Orders Scenario

- Landau Theory of Phase Transitions with several order parameters: one order is strongest and the others are suppressed
- T_c's for the different phases are quite different
- Regimes in which orders have similar T_c's are exceptional and require fine tuning (multicritical point)

Why are the orders of comparable strength?

- This is difficult to understand in terms of the competing order scenario
- This fact suggests that all the observed orders may have a common physical origin and are **intertwined**
- Strong hint: electronic inhomogeneity. STM sees stripe and nematic local order in exquisite detail in BSCCO on a broad range of temperatures, voltage and field
- This phenomenology is natural in doped Mott insulators: frustrated phase separation
- Electronic liquid crystal phases have also been seen heavy fermions and iron superconductors

The case of $La_{1-x}Ba_xCuO_4$

- LBCO, the original HTSC, is known to exhibit low energy stripe fluctuations in its superconducting state

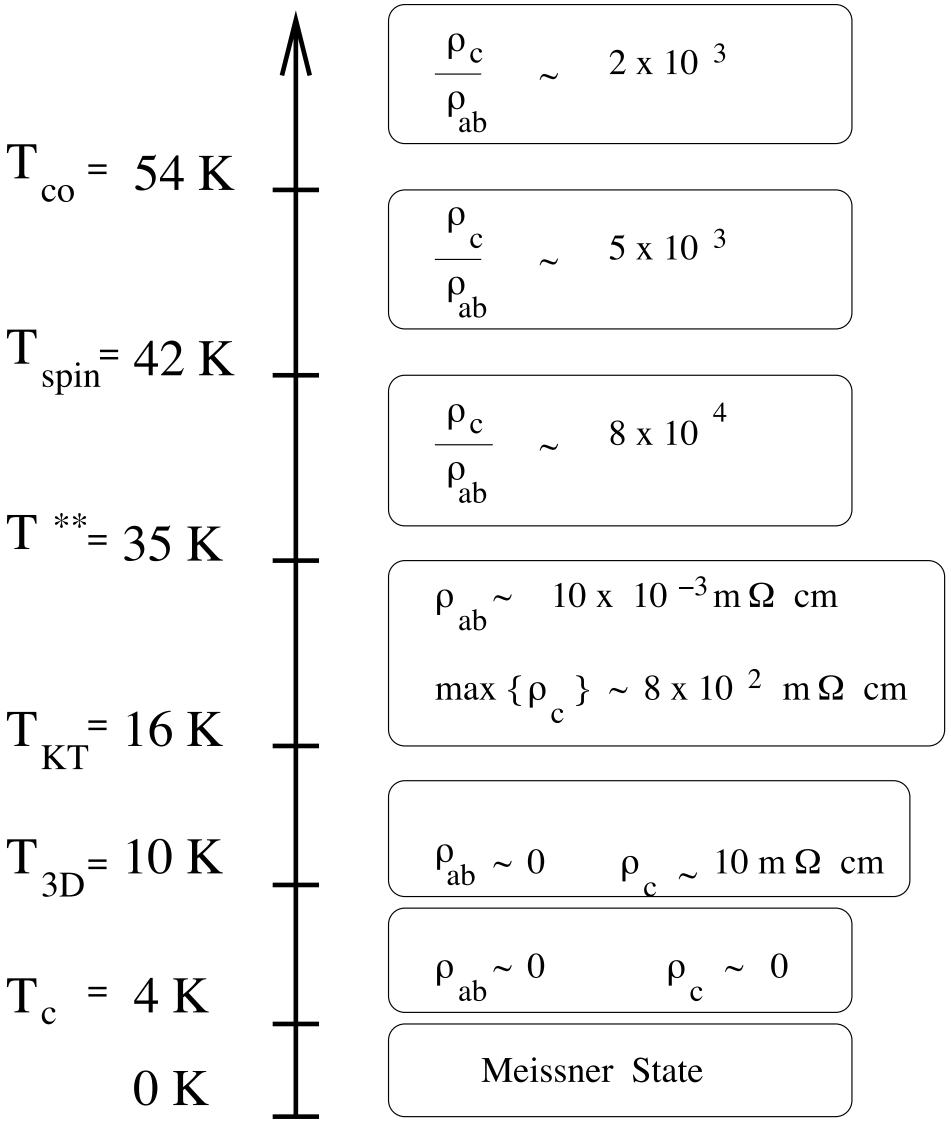
- It has a very low T_c near x=1/8 where it shows static stripe order in the LTT crystal structure

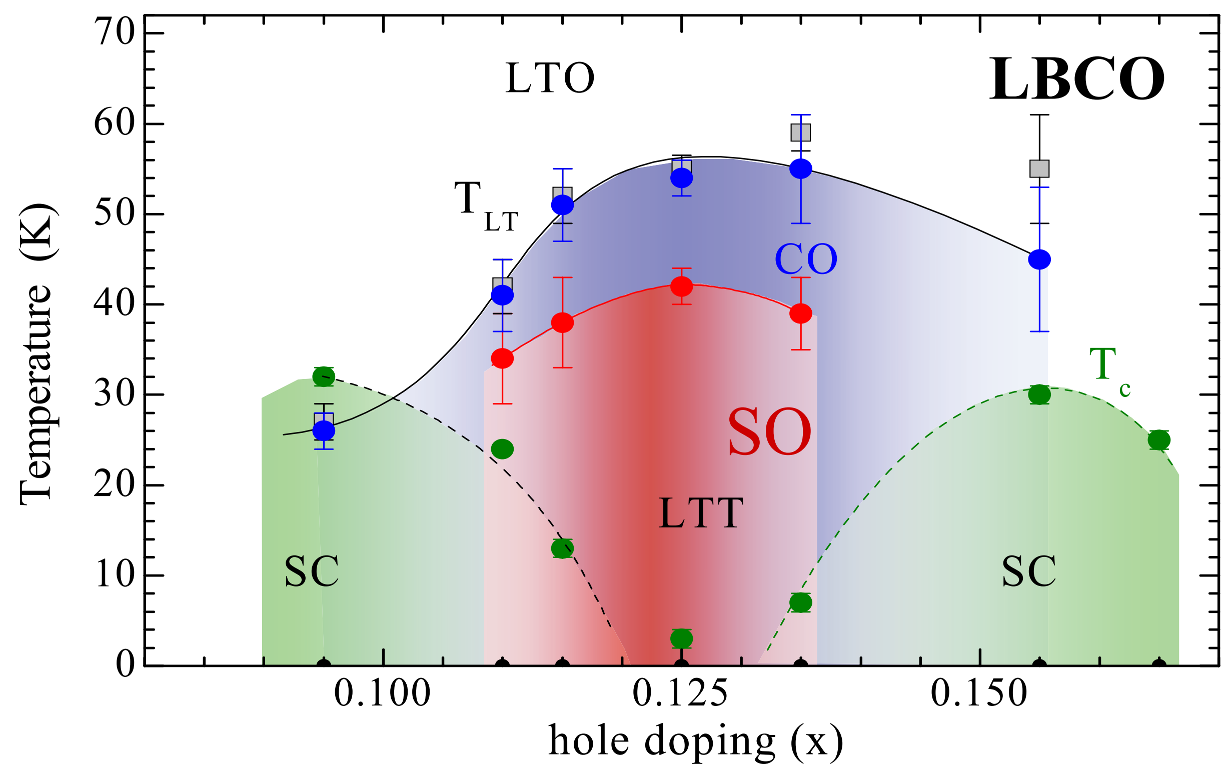
- Superconducting layer decoupling in LBCO at x=1/8 (Li *et al*)

- Layer decoupling, long range charge and spin stripe order and superconductivity: a novel striped superconducting state, a ***Pair Density Wave***, in which charge, spin, and superconducting orders are intertwined!

- This can only happen if there is a special symmetry of the superconductor in the striped state that **frustrates** the c-axis Josephson coupling.

Scales of LBCO at x=1/8: Dynamical Layer Decoupling

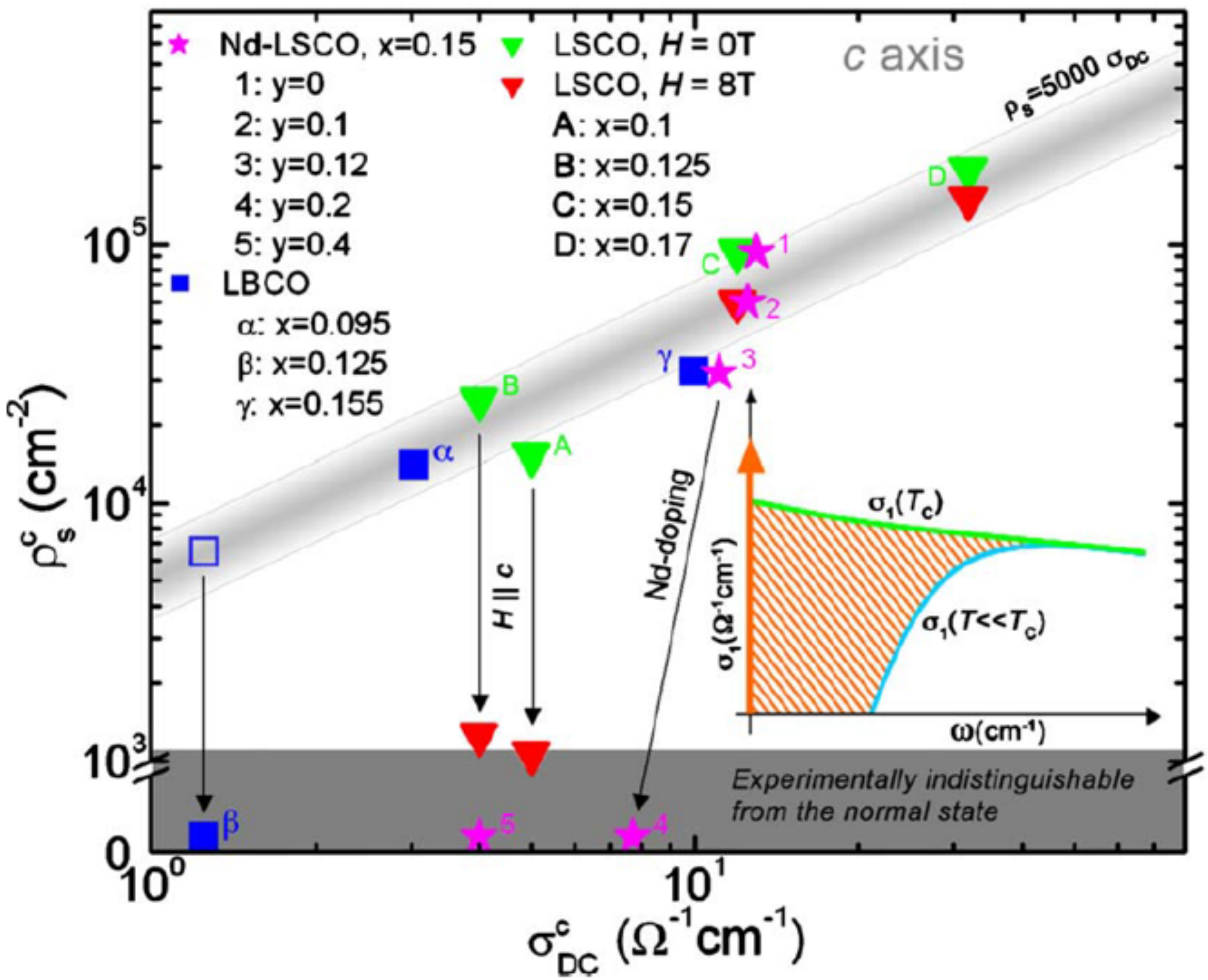




Li *et al* (2007)

M. Hücker et al (2009)

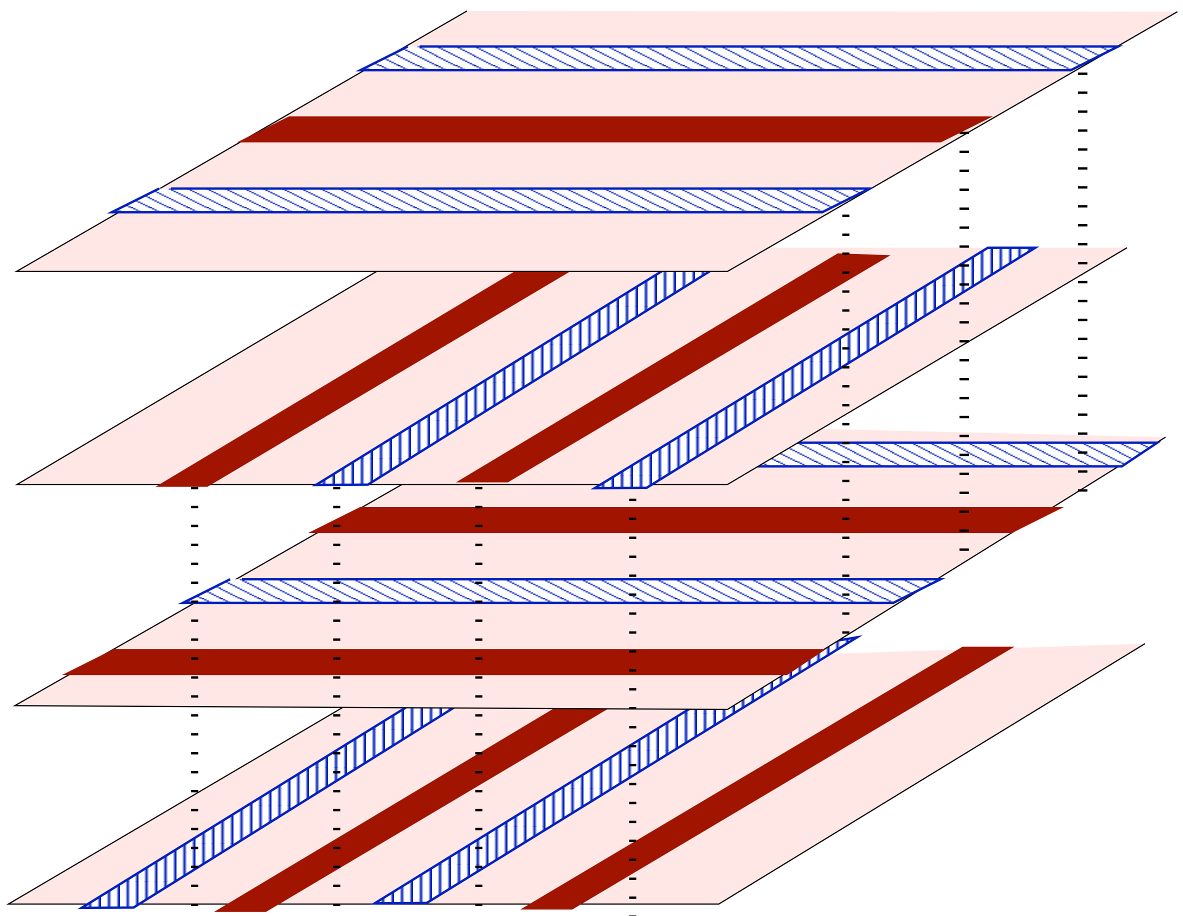
Dynamical Layer Decoupling: Josephson Resonance in LSCO, LBCO and LNSCO

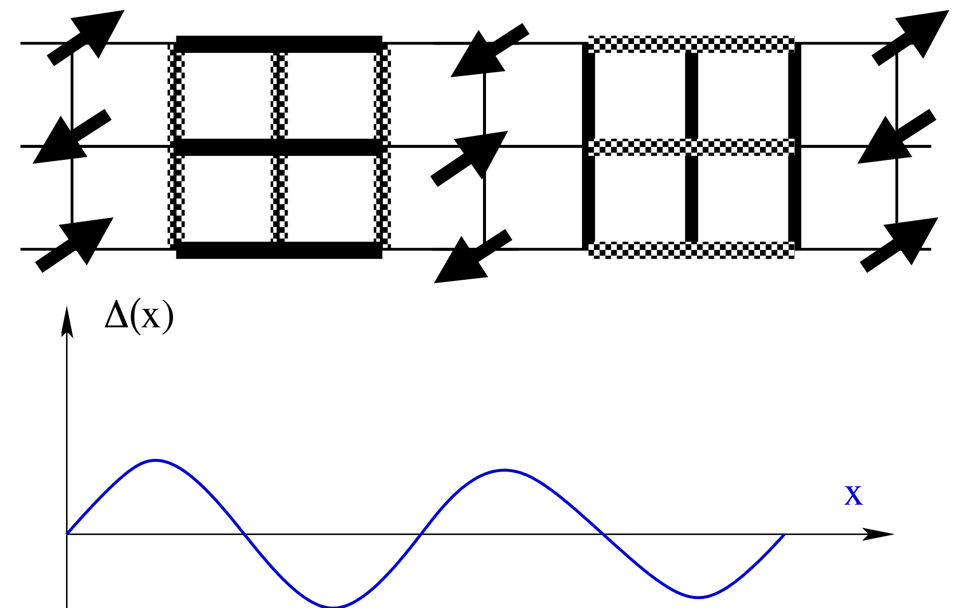


Schafgans et al. 2010

Period 4 Pair density wave Superconducting State

E. Berg et al, 2007





This state is locally d-wave intertwined with charge (and spin) order.

It is a Larkin-Ovchinnikov state without a magnetic field

C-axis Josephson effect is not allowed by symmetry!

Landau-Ginzburg Theory of the striped SC: Order Parameters

- Striped SC: $\Delta(\mathbf{r})=\Delta_Q(\mathbf{r})\, e^{i\,Q.r}+\Delta_{-Q}(\mathbf{r})\, e^{-iQ.r}$, complex charge 2e singlet pair condensate with wave vector, (*i.e.* an FFLO type state at zero magnetic field)
- Two complex SC order parameters: $\Delta_Q(\mathbf{r})$ and $\Delta_{-Q}(\mathbf{r})$
- Two amplitude and two phase fields
- Nematic: real neutral pseudo-scalar order parameter N
- Charge stripe: ρ_K, unidirectional charge stripe with wave vector $\mathbf{K}$
- Spin stripe order parameter: $\mathbf{S}_\mathbf{Q}$, charge neutral complex spin vector order parameter, $\mathbf{K}=2\mathbf{Q}$

Ginzburg-Landau Free Energy Functional

- $F = F_2 + F_3 + F_4 + \ldots$

- The quadratic and quartic terms are standard

- $F_3 = \gamma_s\, \rho_K^*\, \mathbf{S}_Q \cdot \mathbf{S}_Q + \pi/2 \text{ rotation} + \text{c.c.}$
  $+\gamma_\Delta\, \rho_K^*\, \Delta_{-Q}^*\, \Delta_Q + \pi/2 \text{ rotation} + \text{c.c.}$
  $+g_\Delta\, N\, (\Delta_Q^*\, \Delta_Q + \Delta_{-Q}^*\, \Delta_{-Q} - \pi/2 \text{ rotation}) + \text{c.c.}$
  $+g_s\, N\, (\mathbf{S}_Q^* \cdot \mathbf{S}_Q - \pi/2 \text{ rotation})$
  $+g_c\, N\, (\rho_K^*\, \rho_K - \pi/2 \text{ rotation})$

At the level of the Landau theory these orders will not develop with comparable strength unless all the parameters are fine-tuned to a multicritical point with **very large** symmetry!

Some Consequences of the GL theory

- The symmetry of the term coupling charge and spin order parameters requires the condition **K= 2Q**
- PDW SC order implies charge stripe order with 1/2 the period, and nematic order
- PDW state has pockets with a finite DOS of of Bogoliubov quasiparticles
- Striped SC order (PDW) $\Rightarrow$ uniform charge 4e SC order
- $F'_3 = g_4\,[\Delta_4^* (\Delta_Q \Delta_{-Q} + \text{rotation}) + \text{c.c.}]$
- PDW $\Rightarrow$ quartet condensate!
- The PDW state couples strongly to disorder: fragile

Coexisting uniform and striped SC order

- PDW order Δ_Q and uniform SC order Δ_0
- $F_{3,u}=\Upsilon_\Delta\, \Delta_0^* \, \rho_Q \, \Delta_{-Q}+\rho_{-Q}\, \Delta_Q+g_\rho \, \rho_{-2Q}\, \rho_Q^2$ +rotation+c.c.
- If $\Delta_0\neq 0$ and $\Delta_Q\neq 0 \Rightarrow$ there is a ρ_Q component of the charge order!
- The small uniform component Δ_0 removes the sensitivity to quenched disorder of the PDW state

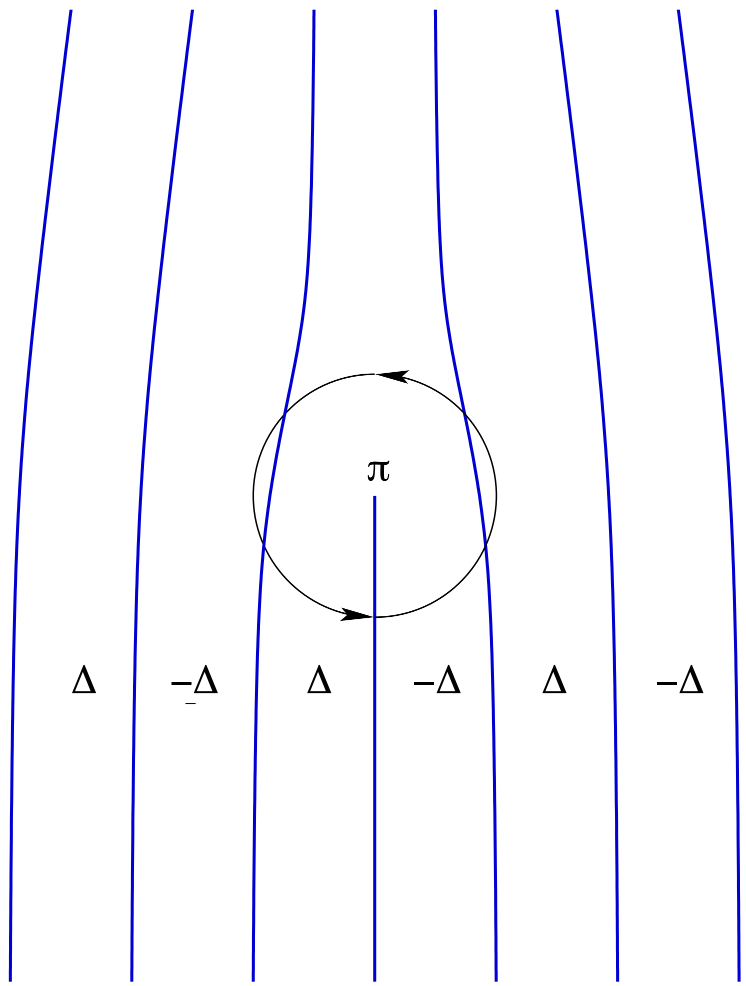
Topological Excitations of the PDW SC

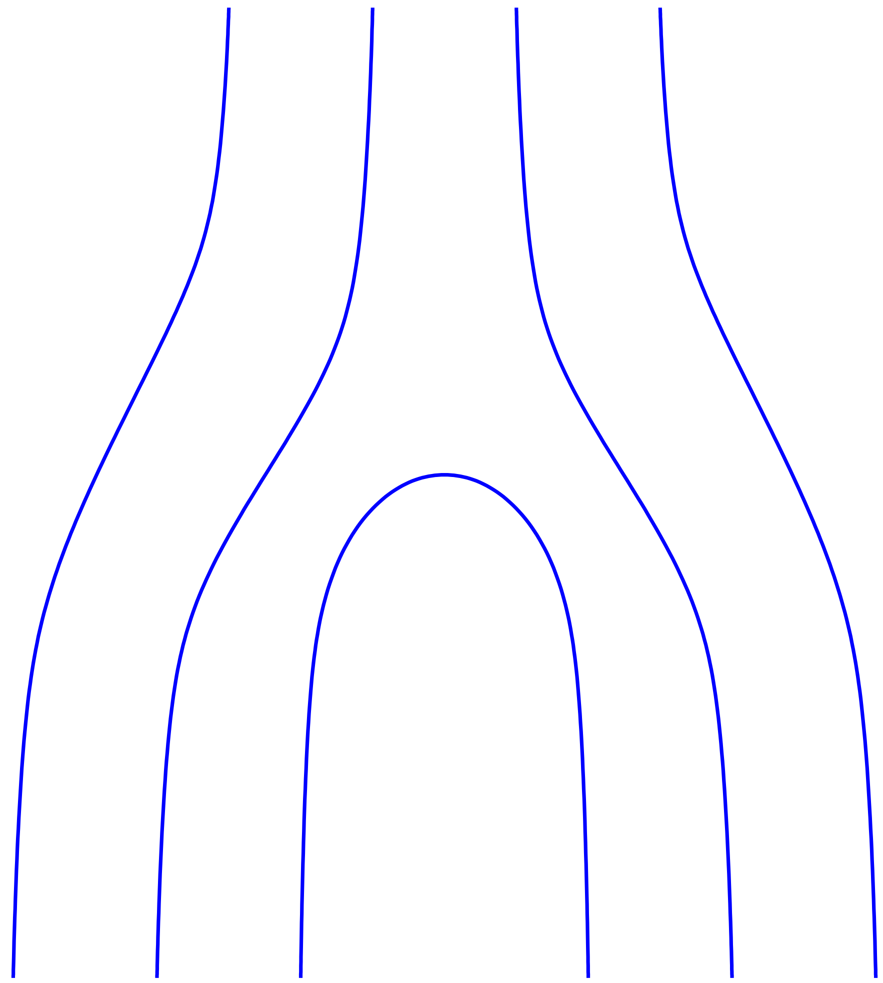
E. Berg. E. Fradkin and S. A. Kivelson

- Strongly layered system: 2D thermal *phase fluctuations* play a key role
- Unidirectional PDW SC
- Two phase fields: the SC phase θ_+ and the CDW phase ϕ
- $H=(\rho_s (\nabla\theta_+)^2 + \kappa (\nabla\phi)^2)/2$
- SC vortex with $\Delta\theta_+ = 2\pi$ and $\Delta\phi=0$
- Bound state of a 1/2 vortex and a CDW dislocation

  $\Delta\theta_+ = \pi, \Delta\phi= 2\pi$
- Double CDW dislocation, $\Delta\theta_+ = 0, \Delta\phi= 4\pi$

Topological Excitations of the PDW





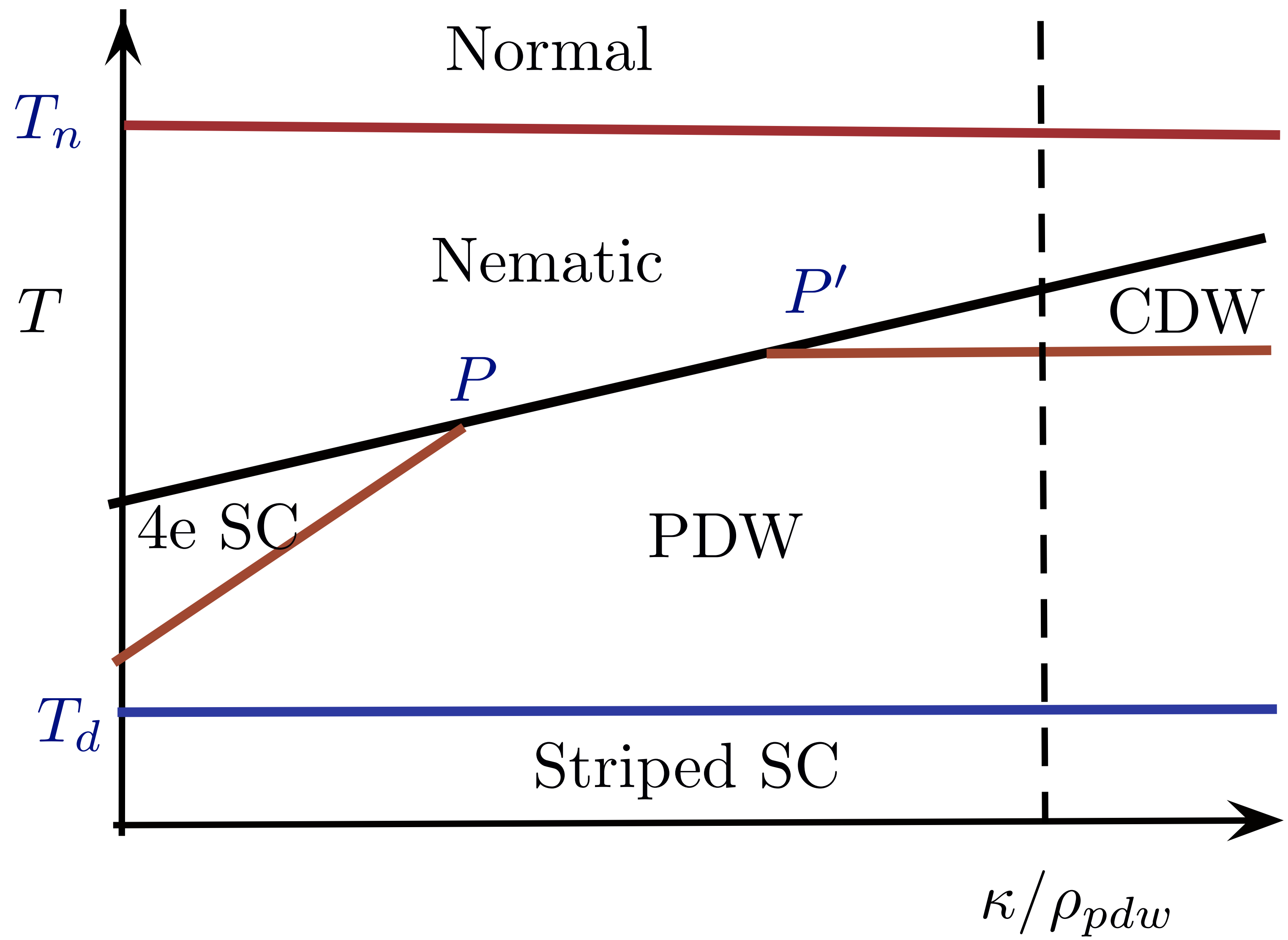
Half-vortex and a Dislocation

Double Dislocation

Thermal melting of the PDW state

- Three paths for thermal melting of the PDW state by a generalized Kosterlitz-Thouless mechanism
- Three types of topological excitations: (1,0) (SC vortex), (0,1) (double dislocation), (±1/2, ±1/2) (1/2 vortex, single dislocation bound pair)
- Scaling dimensions: $\Delta_{p,q}=\pi(\rho_{sc}\, p^2+\kappa_{cdw}\, q^2)/T$
- If $\Delta_{p,q}=2$ the defects proliferate (there is a free energy gain)
- Resulting phases: uniform SC, PDW, Charge 4e SC, CDW, and normal (Ising nematic)

Schematic Thermal Phase Diagram



Intertwined Orders!

- LBCO exhibits **intertwined orders!**
- The orders melt in different sequences, they appear essentially with similar strength
- In **quasi 2D systems** it is natural to get complex phase diagrams with **comparable critical temperatures**!
- The structure of the phase diagrams owes as much to strong correlation physics as to quasi two dimensionality
- Natural candidate for competing order with the uniform d wave SC in the cuprates
- Disorder plays a crucial role in all orders that break translation and rotation invariance

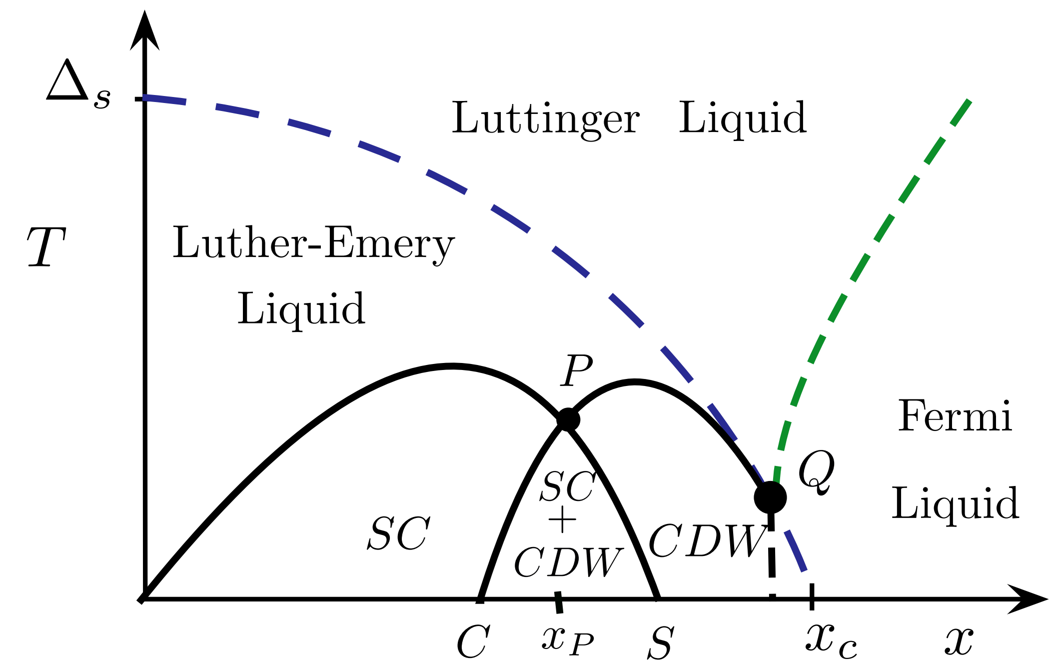
Intertwined Orders in Microscopic Models

- In BCS (i.e. in FFLO) a PDW state is found with Zeeman coupling if the nesting condition is nearly satisfied
- PDW instability only arises at a finite (typically large) coupling (see Kampf *et al*, 2009, 2010)
- BCS is a weak coupling theory that does not predict reliably finite coupling condensed phases
- Intertwined orders require intermediate coupling physics
- This type of order appears naturally in the 1D Kondo-Heisenberg chain (Berg, Fradkin and Kivelson, 2010) and in two-leg ladders (Jaefari and Fradkin, 2012)
- Intertwined orders arise in the vicinity of a nematic triplet QCP (R. Soto Garrido and E. Fradkin, 2014)

Intertwined orders in two-leg ladder arrays

Jaefari, Lal and Fradkin (2010); Fradkin Kivelson and Tranquada (2014)

- Generalized Hubbard-Heisenberg models on 2-leg ladders: broad range of parameters with a spin gap Δ_s and d-wave SC power law correlations
- Only two relevant inter-ladder interactions: coupling between the d-wave SC order parameters and coupling between the CDW order parameters
- The relative relevance of these couplings changes with the charge Luttinger parameter K_c (tuned by doping)
- The T_c's for SC and CDW are comparable and O(1)
- There is a special value of parameters (K_c =1) for multicritical point P where the couplings are equally relevant



Evidence for PDW Order in the t-J model

- Corboz, Rice and Troyer did numerical simulations of the 2D t-J model using a large-scale tensor network approach (iPEPS)
- They found that as the degree of quantum entanglement increases the uniform d wave SC, the PDW (`anti-phase') state, and a d wave SC coexisting with in-phase stripes have essentially the same ground state energy
- This effective degeneracy is seen over a broad range of doping and for intermediate values of J/t
- The ordering wave vectors and the stripe filling fraction also vary with parameters
- Earlier variational MC calculations by Ogata *et al* 2004 and MFT (Poilblanc *et al* 2007) found a similar degeneracy
- Recent work by P.A. Lee on an RVB PDW state (arXiv:1401:0519)

Conclusions and Questions

- Static stripe order seen in LBCO and other LSCO cuprates can be understood in terms of the PDW, a state in which charge, spin and superconducting orders are intertwined rather than competing
- This SC state is locally very similar to and competes with the uniform d-wave state
- Nematic and charge stripe order is seen in the pseudogap regime of YBCO and in BSCCO. How is it related to superconductivity?
- The observed nematic order is a state with "fluctuating stripe order", i.e. it is a state with melted stripe order. Is the nematic state a melted PDW?
- Is the PDW peculiar to LBCO? If it is generic, why? How is the charge orders seen in YBCO related to the phenomenology of LBCO? They must arise from the same mechanism!
- A microscopic theory of the PDW is needed. This state is not accessible by a weak coupling BCS approach. Strong evidence in the 2D t-J model and 1d Kondo-Heisenberg chain and in ladders, and tensor networks calculations in the t-J model.